Homework Assignment 3

Stream function, Potential flow, Boundary layer theory

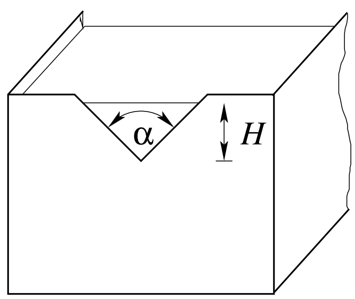
Max. marks: 50 *Due date: Mar 23, 2021*

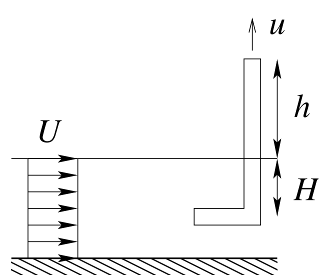
1. (5 marks) The velocity field for uniaxial extensional flow in cylindrical coordinates is given by
$$v_r = -Ar/2 \qquad v_z = Az,$$
where A is a constant. Obtain the stream function (ψ) for the flow and sketch streamlines ($\psi = C$) for the flow.

2. (10 marks) Application of Bernoulli's equation:

   (a) (5 marks) A triangular notch is cut in a plate at the end of a rectangular channel (refer to the figure below). Find the volumetric flow rate of fluid from the notch assuming the flow to be inviscid and the liquid level to be at a height H from the lowest point of the notch.

   (b) (5 marks) A right angled pipe is inserted in a stream flowing at a constant velocity U (refer to the figure below). Find the velocity with which the fluid comes out of the tube (u) assuming inviscid flow.





3. (10 marks) The velocity field for an inviscid flow is given by
$$v_x = x^2y + y^3 \qquad v_y = -x^3 - xy^2.$$
Obtain the pressure field for the flow from the inviscid flow equations neglecting gravity. Show that the velocity and vorticity vectors for the flow are tangent to the Bernoulli surfaces defined as $\chi = P + \rho v^2/2 = C$, i.e., show that $\underset{\sim}{v} \cdot \nabla\chi = \underset{\sim}{\omega} \cdot \nabla\chi = 0$, where $\underset{\sim}{\omega} = \nabla \times \underset{\sim}{v}$ is the vorticity. (Note: this flow is not irrotational, hence it is not a potential flow; χ is not constant everywhere, as in the case of potential flows.)

4. (10 marks) The velocity field for a flow is given by
$$v_x = 2x^2 - y^2 - z^2 \qquad v_y = -2xy \qquad v_z = -2xz.$$

   (a) Show that the flow conserves volume and is irrotational.

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(b) Obtain $\nabla^2 \underset{\sim}{v}$ and show that the flow is inviscid.

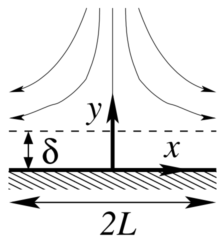
(c) Obtain the velocity potential (ϕ) for the flow.

(d) Obtain the pressure field for the flow assuming that gravity acts in the $-x$ direction.

5. (10 marks) The velocity field for an inviscid stagnation flow for fluid impinging on a plate is given by

$$u_x = ax \qquad u_y = -ay.$$

A boundary layer is formed on the plate as shown in the figure below.



(a) Show that the flow in the boundary layer is of the form

$$v_x = axf(\eta)$$

where $\eta = y/\delta$ and the boundary layer thickness (δ) is constant. (Note that the pressure varies along the boundary layer in this case.)

(b) Find δ, in terms of the system parameters, using dimensional analysis.

(c) Obtain an expression for the normal force acting on the plate, if the plate width is W and the plate length is $2L$, in terms of the scaled velocity (do not solve for the velocity field).

6. (5 marks) Fluid flows over a flat plate at a velocity $U = 3$ m/s at atmospheric pressure and 27°C. The Reynolds number at x is $Re_x = 2 \times 10^3$, where $Re_x = \rho U x/\mu$. Compute the thickness of the boundary layer at a distance x for the following fluids: (a) air (b) water (c) glycerine ($\rho = 1.4$ g/cm^3, $\mu = 980$ cP).