Q1)

a) $P = (3-1)/2 = 1$ as this maintains image dimentions after convolution

$\therefore M = 27 \times 27 \times 1 = 729$ (assuming $P=1$)

b) X → Input image

f^1 → Matrix containing convolution filter

f^2 →

$Z^1 = \omega^1 A^0 + b^1$ ← conv using matrix multip.

$Z^2 = \omega^2 * Z^1$ ← pooling

$Z^3 =$

Forward prop:

$Z^1 = x * f^1$

$Z^2 = \text{max pool}(Z^1)$

$Z^3 = A^1 Z^2 + b$

$Z^4 = \text{relu}(Z^3)$

$Z^5 =$ sigmoid

A = linear layer weights

b = bias of linear layer

c) Back Propogation:

input → conv → pooling → linear → relu → error

$$\frac{\partial Z^5}{\partial A^5} * \frac{\partial Z^4}{\partial Z^5} * \frac{\partial \mathcal{E}}{\partial Z^4}$$

Loss = cross entropy

pred → Z^4 actual → $Z^{4'}$

$$\therefore \frac{\partial \mathcal{E}}{\partial Z^4} = -\left(\frac{Z^{4'}}{Z^4}\right) + \left(\frac{1-Z^{4'}}{1-Z^4}\right)$$

$$\frac{\partial z^4}{\partial z^3} = \begin{cases} 1 & (z^{4,3} > 0) \\ 0 & (z^{4,3} < 0) \end{cases}$$

$$\frac{\partial z^3}{\partial A} = z^2$$

$$\frac{\partial \varepsilon}{\partial A} = z^2 \left(\frac{\partial z^4}{\partial z^3} \right) \left(\frac{-z^{4,1}}{z^4} + \frac{1 - z^{4,1}}{1 - z^4} \right)$$

Conv Layer:

d) Potential causes for vanishing gradients:

① $z^3 < 0$

② small z^2

causes for exploding gradients:

① large z^2