Q1)

a) $H = (w, b)$ $\quad x \in R^d \quad \langle w, x \rangle = b$

Let $\tilde{H} = H = \{x \in R^d \quad \langle \tilde{w}, x \rangle = \tilde{b}\}$

$\| \tilde{w} \| = \beta \qquad \| w \| = \alpha$

$\langle w, x \rangle = b$

$\frac{\alpha}{\beta} \cdot \langle \frac{w}{\alpha}, x \rangle = b$

$\langle \frac{w\beta}{\alpha}, x \rangle = \frac{\beta b}{\alpha}$

$\| \frac{\beta}{\alpha} w \|_2 = \frac{\beta}{\alpha} \alpha = \beta$

$\therefore \frac{\beta}{\alpha} w = \tilde{w} \qquad \frac{\beta}{\alpha} b = \tilde{b}$

$\therefore \tilde{H} = (\tilde{w}, \tilde{b})$ exists

where $\| \tilde{w} \|_2 = \beta$ $(\beta > 0)$ and $H = \tilde{H}$

Hence proved

b) ~~we know from~~

$H = (w^*, b) = (\tilde{w}^*, \tilde{b})$

$\| w^* \| = \alpha$

$\| \tilde{w}^* \| = 1$

$\tilde{b} = b/\alpha \; , \; \tilde{w} = w^*/\alpha$

$\therefore \quad M \leq \frac{R^2 \| w^* \|_2^2}{\gamma^2}$

$\leq \frac{R^2 \| \tilde{w}^* \|_2^2}{\gamma^2} \cdot \frac{1}{\alpha^2}$

$\leq R^2/\eta^2 \qquad (\eta = \gamma/\alpha)$

Q2)

a) $D = \{(x^i, y^i)\}$ is linearly seperable when there exists $w^* \in R^d$ & b > 0 such that:

$\langle w^*, x^t \rangle > b$ when $y^t = 1$

$\langle w^*, x^t \rangle < -b$ when $y^t = 0$