EFFECT OF PIECEWISE LINEAR COST FUNCTIONS ON ACCURACY AND COMPUTATIONAL PERFORMANCE

This technical note investigates how modeling fuel costs using a piecewise linear (PWL) approximation affects the accuracy and computational performance of the considered UC/CUC formulations. The objective is to assess whether the relative behavior of the formulations remains consistent when moving from simplified cost representations to a more detailed PWL cost model, and to report the corresponding numerical observations in a self-contained manner. The performance of all formulations was evaluated on the IEEE 24-bus and IEEE 300-bus test systems under renewable penetration levels of 12% and 25%. In every experiment, each unit's nonlinear fuel-cost curve was approximated by a piecewise linear (PWL) function using k = 5 segments of equal width over the unit's operating range.

To implement the PWL model, the fuel-cost term in the objective function is approximated using a piecewise linear representation. The corresponding variables and constraints (for each cluster) are defined as follows. The first term of the objective function (fuel cost) is rewritten as shown in (1). In this formulation:

- τ represents the power output values at the breakpoints.
- $\lambda_{c,k,t}$ represents the power dispatched on segment k for cluster c at time t (segment length).
- $S_{c,k}$ denotes the incremental cost (slope) of each segment.

Equations (1)–(9) define the fuel-cost term and the constraints governing the segment limits and the total power-output calculation [5]:

$$f = \sum_{t}\sum_{c}\left(F_c^{pmin}.\,u_{c,t} + \sum_{k} S_{c,k}.\,\lambda_{c,k,t}\right), \tag{1}$$

$$F_c^{pmin} = a_c.\,{P_c^{min}}^2 + b_c.\,P_c^{min}, \tag{2}$$

$$S_{c,1} = \frac{a_c.\,{\tau_{c,1}}^2 + b_c.\,\tau_{c,1} - a_c.\,{P_c^{min}}^2 - b_c.\,P_c^{min}}{\tau_{c,1} - P_c^{min}}, \tag{3}$$

$$S_{c,2} = \frac{a_c.\,{\tau_{c,2}}^2 + b_c.\,\tau_{c,2} - a_c.\,{\tau_{c,1}}^2 - b_c.\,\tau_{c,1}}{\tau_{c,2} - \tau_{c,1}}, \tag{4}$$

$$\ldots$$

$$S_{c,5} = \frac{a_c.\,{P_c^{max}}^2 + b_c.\,P_c^{max} - a_c.\,{\tau_{c,4}}^2 - b_c.\,\tau_{c,4}}{P_c^{max} - \tau_{c,4}}, \tag{5}$$

$$\lambda_{c,1,t} \le \left(\tau_{c,1} - P_c^{min}\right).\,u_{c,t} \tag{6}$$

$$\lambda_{c,2,t} \le \left(\tau_{c,2} - \tau_{c,1}\right).\,u_{c,t} \tag{7}$$

$$\ldots$$

$$\lambda_{c,5,t} \le \left(P_c^{max} - \tau_{c,4}\right).\,u_{c,t} \tag{8}$$

$$p_{c,t} = u_{c,t}.\,P_c^{min} + \sum_{k} \lambda_{c,k,t} \tag{9}$$

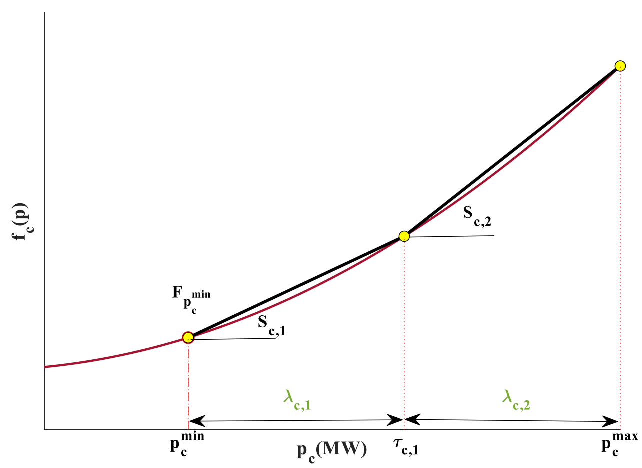

Fig. 1. Piecewise-linear approximation of the cluster cost function (example with k=2 segments)

The estimation of the production cost for each segment is illustrated in Fig. 1 (for the case of $k = 2$). The values of parameters a and b for the IEEE 24-bus and IEEE 300-bus test systems are reported in Table I and II.The results of these new numerical cases are reported in Table III. In all scenarios, all models are executed until they reach an optimality gap of 0.01%, provided they do not exceed a time limit of 30,000 seconds. Consequently, if the UC model fails to converge within this 30,000-second limit, the optimality gap achieved by the UC model is adopted as the stopping criterion for the remaining models.

The key observations are as follows: When using the PWL cost function, the relative error of almost all clustering methods decreases compared to the single-segment linear case. Consistent with previous results, ICUC remains the fastest method in all scenarios and provides strictly higher accuracy than the classical CUC in every case. Similar to the trends observed in the original manuscript (Table V), the PCUC method exhibits the highest computational time in three cases. Furthermore, its error remains high in the IEEE 24-bus system due to the large number of units with minimum up time of one hour, which PCUC struggles to model efficiently. The BCUC model demonstrates the highest accuracy in almost all cases while maintaining a short computational time. This confirms that the model's superiority is robust to the choice of cost function. Also, the accuracy of CUC+UC and TSCUC is consistently lower than that of BCUC. This is because these methods rely on the classical CUC formulation in the first stage, which introduces larger inherent errors than ICUC. Furthermore, the error of TSCUC is higher than that of CUC+UC due to the equality constraint (Eq. 22 in [2]), which forces the second stage to strictly follow the potentially flawed commitment profile of the first stage.

In conclusion, while the inclusion of PWL costs adds realism, it does not alter the relative ranking of the models, and BCUC and ICUC remain the most effective formulations in terms of the trade off between accuracy and speed.

Table I. Values of parameters aand bfor the IEEE 24-bus test system.

clusters / parameters	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
a	0.007612	0.01188	0.003934	0.0021	0.048764	0.062064	0.00308	0.006842	0.006842	0.01176
b	12.8216	11.264	13.7335	11.5555	26.367	25.8775	21.4599	19.0124	18.6824	18.6087
clusters / parameters	C11	C12	C13	C14	C15	C16	C17	C18	C19	
a	0.05964	0.0061	0.0074	0.0173	0.00896	0.02709	0.03836	0.007	0.03206	
b	25.5332	8.5085	7.1379	27.0864	19.5547	27.5264	26.7872	19.437	26.796	

Table II. Values of parameters aand bfor the IEEE 300-bus test system.

clusters / parameters	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14
a	0.0045	0.0045	0.00495	0.0046	0.00405	0.012	0.02903	0.01552	0.06617	0.03846	0.00233	0.01875	0.045	0.0045
b	23.7	23.73	23.67	23.742	23.67	10.392	14.4	9.762	15.6	13.92	4.68	10.578	24.3144	24.26
clusters / parameters	C15	C16	C17	C18	C19	C20	C21	C22	C23	C24	C25	C26	C27	C28
a	0.016	0.00646	0.05357	0.02074	0.0439	0.01209	0.0208	0.0045	0.02195	0.0045	0.01973	0.05357	0.0225	0.00375
b	14.1	7.74	18	14.8176	15.438	8.07	13.86	24.39	15.828	24.3576	13.944	16.458	15.438	5.4
clusters / parameters	C29	C30	C31	C32	C33	C34	C35	C36	C37	C38	C39	C40	C41	C42
a	0.00375	0.00947	0.0023	0.0106	0.01654	0.045	0.01	0.018	0.01485	0.013	0.015	0.0075	0.018	0.00818
b	5.4	7.86	4.698	7.8	15.84	16.8	7.74	16.14	19.137	18.576	15.462	6	15.06	7.56
clusters / parameters	C43	C44	C45	C46	C47	C48	C49	C50	C51	C52	C53	C54	C55	C56
a	0.00782	0.0264	0.05357	0.00963	0.00722	0.0037	0.0192	0.0121	0.0136	0.02432	0.0109	0.009	0.12162	0.0045
b	6.588	12.78	17.922	9.6	7.02	7.2	11.88	9.36	13.56	21.9	7.8	7.68	24.3246	4.26
clusters / parameters	C57	C58	C59	C60	C61	C62	C63	C64	C65	C66	C67	C68	C69	
a	0.1	0.02727	0.01125	0.01125	0.0388	0.00348	0.00643	0.00813	0.0004	0.0022	0.00135	0.09	0.5625	
b	22.8264	15.24	7.2	7.2	13.8	3.48	4.8	13.668	24.336	24.444	24.3576	22.08	48	

Table III. Performance comparison of the models with piecewise-linear costs on IEEE-24 and IEEE-300 test systems.

			BUC	CUC	PCUC	CUC+UC	TSCUC	ICUC	BCUC
IEEE-24 bus	12%Renewable	Cost [M$]	6.5998	6.5956	6.6168	6.60288	6.63538	6.5990	6.6000
		Cost Error [%]	-	-0.0634	0.2569	0.0450	0.5382	-0.0117	**0.0016**
		Schedule Error [%]	-	4.58	7.34	1.87	2.61	**0.76**	0.86
		Generation Error [%]	-	2.71	2.48	1.66	4.23	**0.43**	0.46
		Ramp Error [%]	-	15.5	15.1	9.6	16.8	**3.6**	4.1
		CPU Time [s]	30000	4.9	204.3	37.9	20.4	**2.84**	32.4
	25%Renewable	Cost [M$]	5.8535	5.8309	5.8603	5.8670	5.8881	5.8500	5.8545
		Cost Error [%]	-	-0.3858	0.1163	0.2313	0.5901	-0.0598	**0.0199**
		Schedule Error [%]	-	7.45	7.01	7.29	8.18	1.57	**1.26**
		Generation Error [%]	-	5.11	2.68	5.89	7.92	2.17	**0.94**
		Ramp Error [%]	-	19.8	12.9	18.3	25.5	9	**6.1**
		CPU Time [s]	30000	1168	825	1447	76.1	**5.13**	965.7
IEEE-300 bus	12%Renewable	Cost [M$]	31.2842	31.2725	31.2782	31.2872	31.2908	31.2782	31.2861
		Cost Error [%]	-	-0.0373	-0.0192	0.0094	0.0209	-0.0192	**0.0061**
		Schedule Error [%]	-	2.21	2.14	1.92	1.92	1.97	**1.83**
		Generation Error [%]	-	1.27	1.24	0.98	0.91	0.74	**0.71**
		Ramp Error [%]	-	17.1	14.3	13.9	13.5	10.3	**9.2**
		CPU Time [s]	2612	15.4	619	127.8	24.2	**14.6**	45.25
	25%Renewable	Cost [M$]	27.2584	27.2348	27.2529	27.2678	27.2880	27.2483	27.2643
		Cost Error [%]	-	-0.0864	**-0.0201**	0.0345	0.1088	-0.0369	0.0216
		Schedule Error [%]	-	3.68	2.77	2.81	3.84	2.77	**2.34**
		Generation Error [%]	-	1.85	**1.27**	1.60	1.55	1.41	1.40
		Ramp Error [%]	-	18.5	13.2	15.7	16.4	12.9	**12.6**
		CPU Time [s]	694	25.6	960	68.9	47.95	**19.5**	55.5