TIGHT AND COMPACT UNIT COMMITMENT

A. Unit Commitment

The objective function, adapted from the CUC, follows from the CUC objective function through a direct substitution of the index, replacing the cluster index with the unit-specific index. Similar to the constraints in the previous section, the TC UC formulation incorporates technical constraints for each unit and system-wide constraints. However, these constraints are explicitly reformulated for the unit-based model and indexed by (g) to represent the unit.

1) Objective function

The objective function includes fuel costs, fixed operation costs, start-up costs, shutdown costs, upward and downward reserve costs, wind curtailment costs, and load shedding costs. In this study, the fuel cost function is considered as a constant function for ease of analysis.

$$C^{TOT} = \sum_{t=1}^{T}\left[\sum_{g\in\Omega_G}\left(C_g^{VC}.p_{g,t} + C_g^{NL}.u_{g,t} + C_g^{SU}.v_{g,t} + C_g^{SD}.w_{g,t} + C_g^{R-up}.r_{g,t}^{+} + C_g^{R-down}.r_{g,t}^{-}\right) + \sum_{i\in\Omega_i^L} Cls.\ LDS_i + \sum_{i\in\Omega_w^i} C^W.P_{i,t}^C\right], \tag{1}$$

2) technical constraints of units

a) Logical constraints:

Using (2) and (3), the state transitions for each unit's commitment, start-up, and shutdown are enforced.

$$u_{g,t} - u_{g,t-1} = v_{g,t} - w_{g,t}, \forall t, g \in \Omega_G \tag{2}$$

$$v_{g,t} + w_{g,t} \le 1, \forall t, g \in \Omega_G \tag{3}$$

b) Minimum up/down time constraints:

Minimum uptime and downtime constraints, as defined in equations (4) and (5), are derived from [1-3]. As demonstrated in [3], these equations offer the most efficient computational performance.

$$\sum_{\tau=t-MUT_g+1}^{t} v_{g,\tau} \le u_{g,t}, \forall t, g \in \Omega_G \tag{4}$$

$$\sum_{\tau=t-MDT_g+1}^{t} w_{g,t} \le 1 - u_{g,t,} \forall t, g \in \Omega_G \tag{5}$$

c) Generation constraints:

The maximum generation for each unit is imposed using (6a), (6b), (7), and (8). Equations (6a) and (6b) define the maximum generation for fast-start units (minimum up time=1), while (7) represents the maximum generation for the remaining units with longer minimum up time. Equation (8) applies to all units; however, equations (6) and (7) increase the tightness of the formulation [2]. Equation (9) ensures that each unit generates at least the sum of the downward reserve and minimum generation at each hour. Equation (10) states that each unit's generation equals its minimum generation level plus the generation above the minimum level.

$$p_{g,t}{}^{*} + r_{g,t}^{+} \le \left(P_g^{max} - P_g^{min}\right).u_{g,t} - \left(P_g^{max} - SD_g\right).w_{g,t+1} - \max\left(SD_g - SU_g, 0\right) v_{g,t}, \forall t, g \in \Omega_G \tag{6a}$$

$$p_{g,t}{}^{*} + r_{g,t}^{+} \le \left(P_g^{max} - P_g^{min}\right).u_{g,t} - \max\left(SU_g - SD_g, 0\right) w_{g,t+1} - \left(P_g^{max} - SU_g\right).v_{g,t}, \forall t, g \in \Omega_G \tag{6b}$$

$$p_{g,t}{}^{*} + r_{g,t}^{+} \le \left(P_g^{max} - P_g^{min}\right).u_{g,t} - \left(P_g^{max} - SU_g\right).v_{g,t} - \left(P_g^{max} - SD_g\right).w_{g,t+1}, \forall t, g \in \Omega_G \tag{7}$$

$$p_{g,t}{}^{*} + r_{g,t}^{+} \le \left(P_g^{max} - P_g^{min}\right).u_{g,t}, \forall t, g \in \Omega_G \tag{8}$$

$$p_{g,t}{}^{*} - r_{g,t}^{-} \ge 0, \forall t, g \in \Omega_G \tag{9}$$

$$p_{g,t} = p_{g,t}{}^{*} + u_{g,t}.P_g^{min}, \forall t, g \in \Omega_G \tag{10}$$

d) Ramp rate constraints

Equations (11) and (12) enforce each unit's hourly ramp-up/down capacities. While various ramp rate formulations exist, we employ these specific formulations in this study due to their demonstrated effectiveness in improving the formulation's tightness and reducing CPU time [1].

$$p_{g,t}{}^{*} + r_{g,t}^{+} - p_{g,t-1}{}^{*} \le RU_g.\left(u_{g,t} - v_{g,t}\right) + v_{g,t}.\left(SU_g - P_g^{min}\right), t \ne 1, g \in \Omega_G \tag{11}$$

$$p_{g,t-1}{}^{*} + r_{g,t}^{-} - p_{g,t}{}^{*} \le RD_g.\left(u_{g,t-1} - w_{g,t}\right) + w_{g,t}.\left(SD_g - P_g^{min}\right), t \ne 1, g \in \Omega_G \tag{12}$$

e) System-wide constraints

System-wide constraints include DC power flow equations (13), (14a), and (14b), power balance (15), and the total up/down reserve requirements, as enforced in (16). In equation (13), B_l is the susceptance of the line l (p.u.), and S_b is the base power of the system in MVA.

$$f_{l,t} = B_l.\left(\theta_{i,t} - \theta_{j,t}\right).S_b, \forall t, l \in \Omega_i^l(+), \Omega_j^l(-) \tag{13}$$

$$-F_l^{max} \le f_{l,t} \le F_l^{max}, \forall t, \forall l \tag{14a}$$

$$-\pi \le \theta_{i,t} \le \pi, \forall t, \forall l \tag{14b}$$

$$\sum_{g\in\Omega_i^G} p_{g,t} + PW_{i,t} - P_{i,t}^C - \sum_{l\in\Omega_i^l(+)} f_{l,t} + \sum_{l\in\Omega_i^l(-)} f_{l,t} + LDS_{i,t} = D_t.\alpha_i, \forall t, \forall i \tag{15}$$

$$\sum_{g\in\Omega_G} r_{g,t}^{+} \ge RT_t^{+}, \sum_{g\in\Omega_G} r_{g,t}^{-} \ge RT_t^{-}, \forall t \tag{16}$$

[1] D. A. Tejada-Arango, S. Lumbreras, P. Sánchez-Martín, and A. Ramos, "Which unit-commitment formulation is best? A comparison framework," *IEEE Trans. Power Syst.,* vol. 35, no. 4, pp. 2926-2936, Jul. 2020.

[2] G. Morales-España, J. M. Latorre, and A. Ramos, "Tight and compact MILP formulation for the thermal unit commitment problem," *IEEE Trans. Power Syst.,* vol. 28, no. 4, pp. 4897-4908, Nov. 2013.

[3] J. Ostrowski, M. F. Anjos, and A. Vannelli, "Tight mixed integer linear programming formulations for the unit commitment problem," *IEEE Trans. Power Syst.,* vol. 27, no. 1, pp. 39-46, Feb. 2012.