Host-parasitoid models: The interplay of density-dependent host growth and functional response of parasitoids
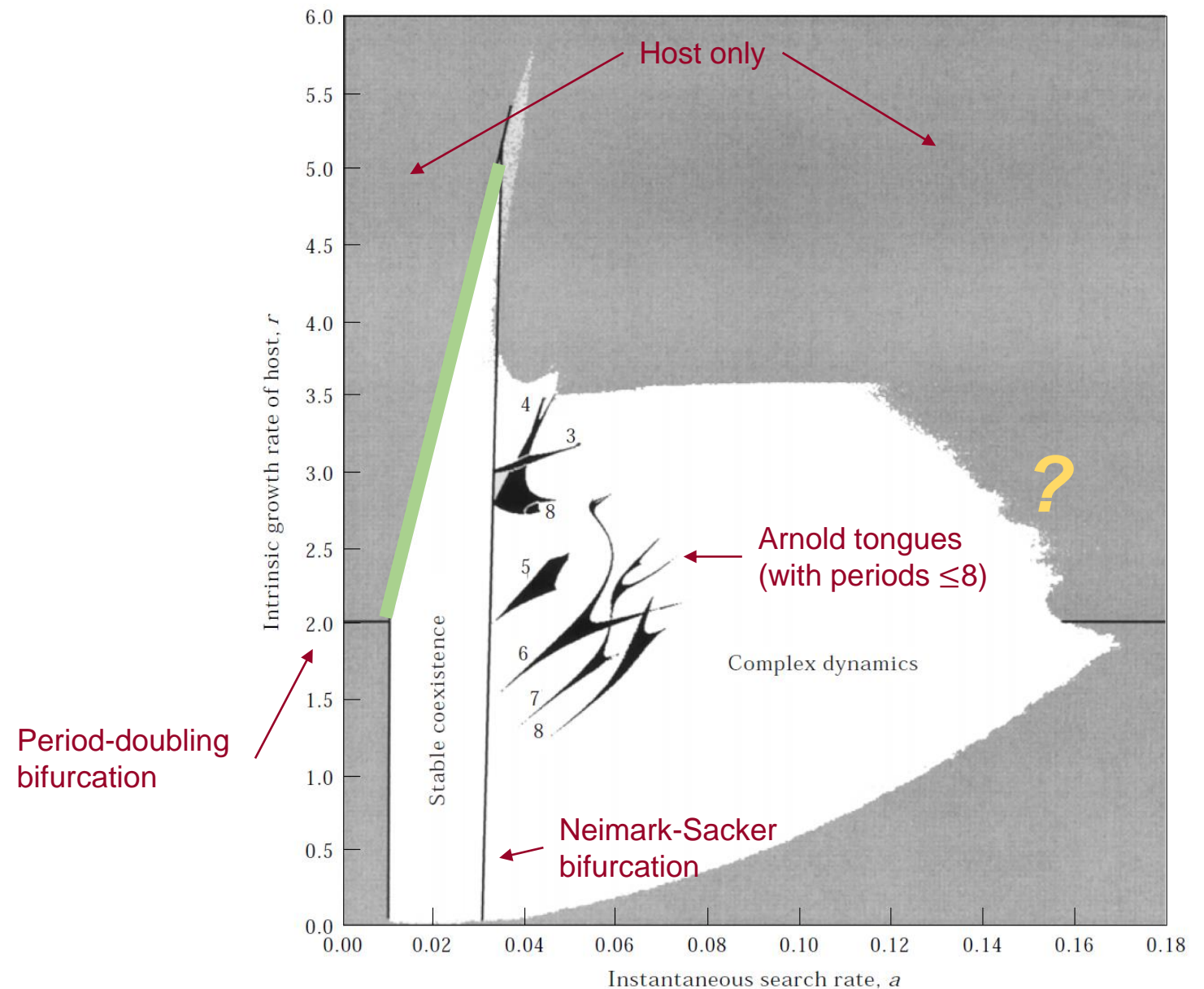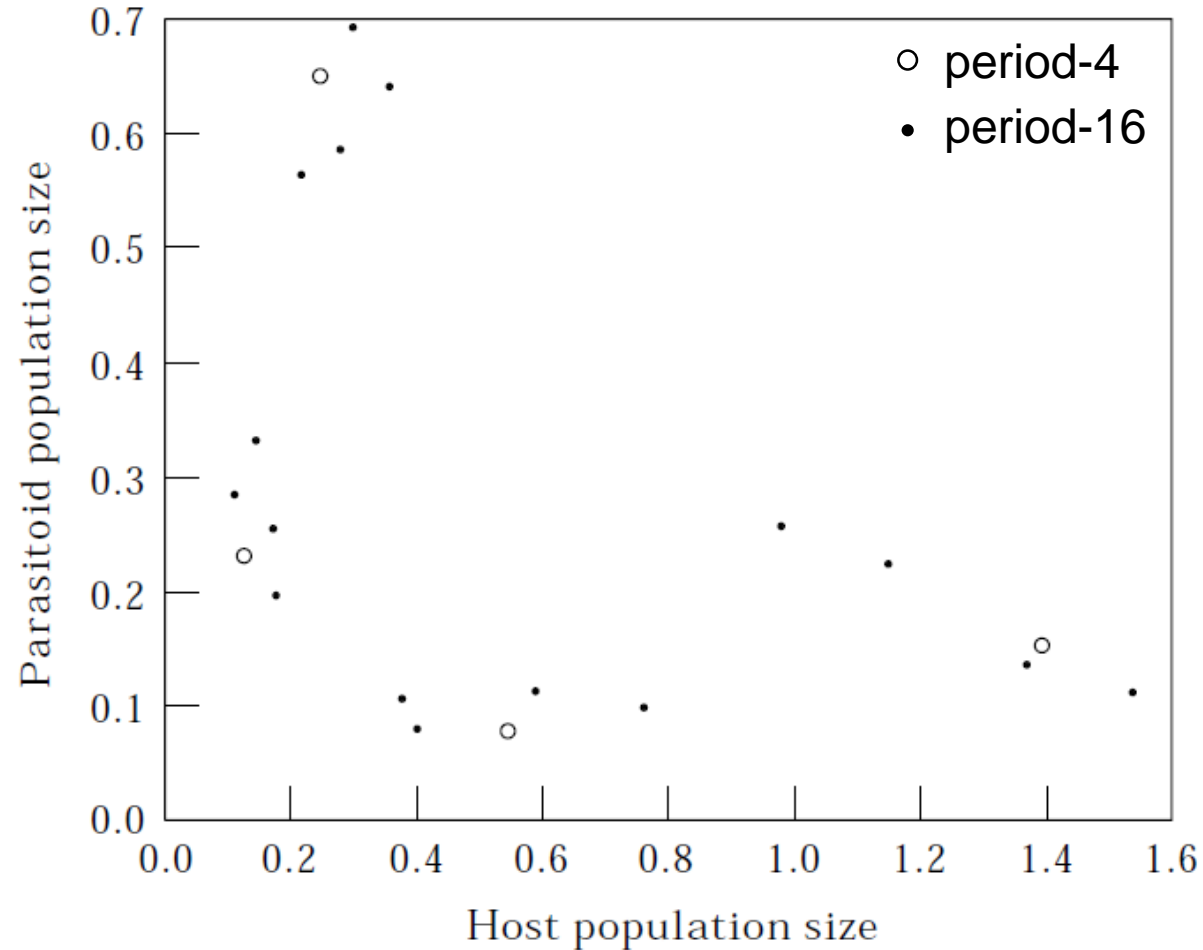
Frank M. Hilker

$$H_{t+1} = H_t e^{r(1-H_t)-\frac{aTP_t}{1+aT_hH_t}}$$

$$P_{t+1} = H_t\left(1 - e^{\frac{-aTP_t}{1+aT_hH_t}}\right)$$

$T = 100, T_h = 1$

Host only

Intrinsic growth rate of host, r

Arnold tongues
(with periods ≤8)

Period-doubling
bifurcation

Stable coexistence

Complex dynamics

Neimark-Sacker
bifurcation

Instantaneous search rate, a

?

Kaitala et al. (1999, *J Theor Biol*)

2

Coexistence of two periodic attractors
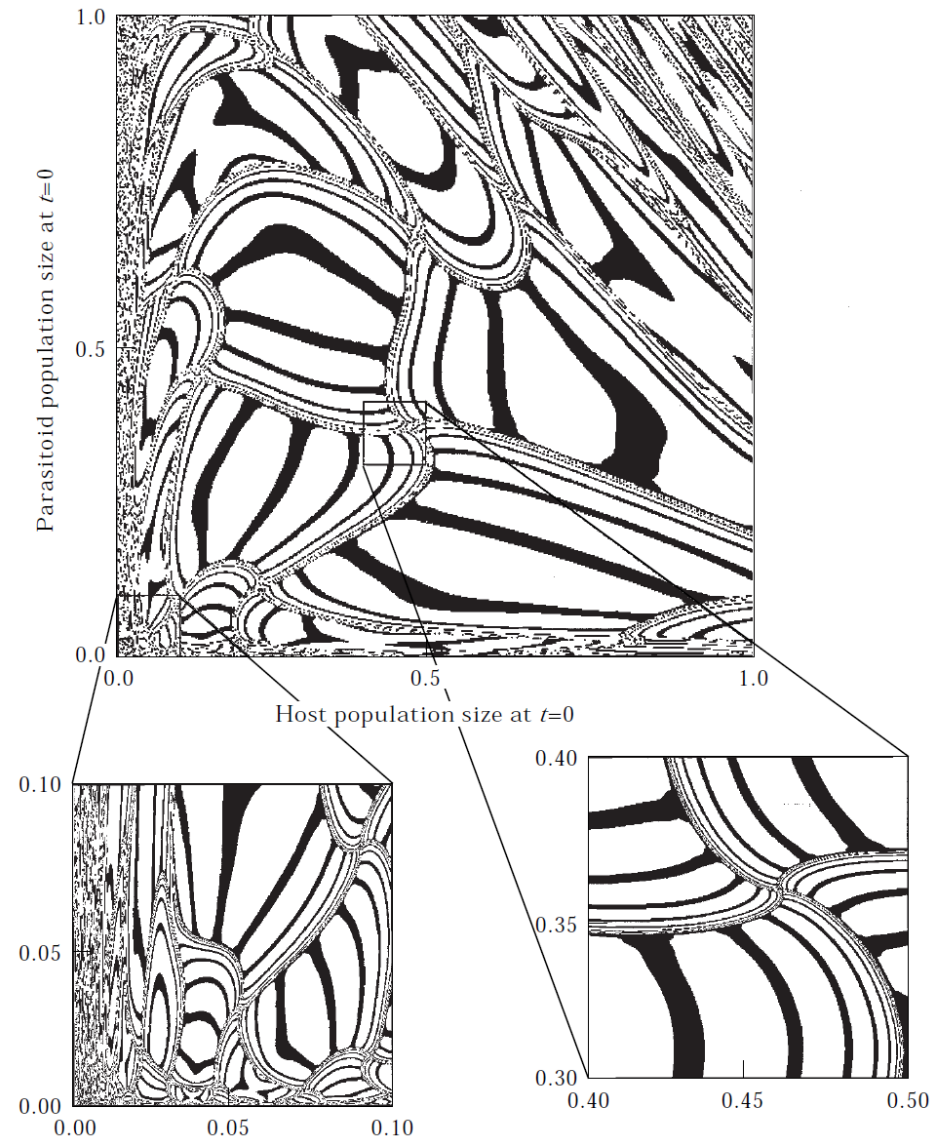

$$r = 2.8, a = 0.043, T = 100, T_h = 1$$

Other examples reported
- period-8 and period-15
- period-16 and period-45
- chaotic and six-piece chaotic

However, **no report of more than two** coexisting attractors.
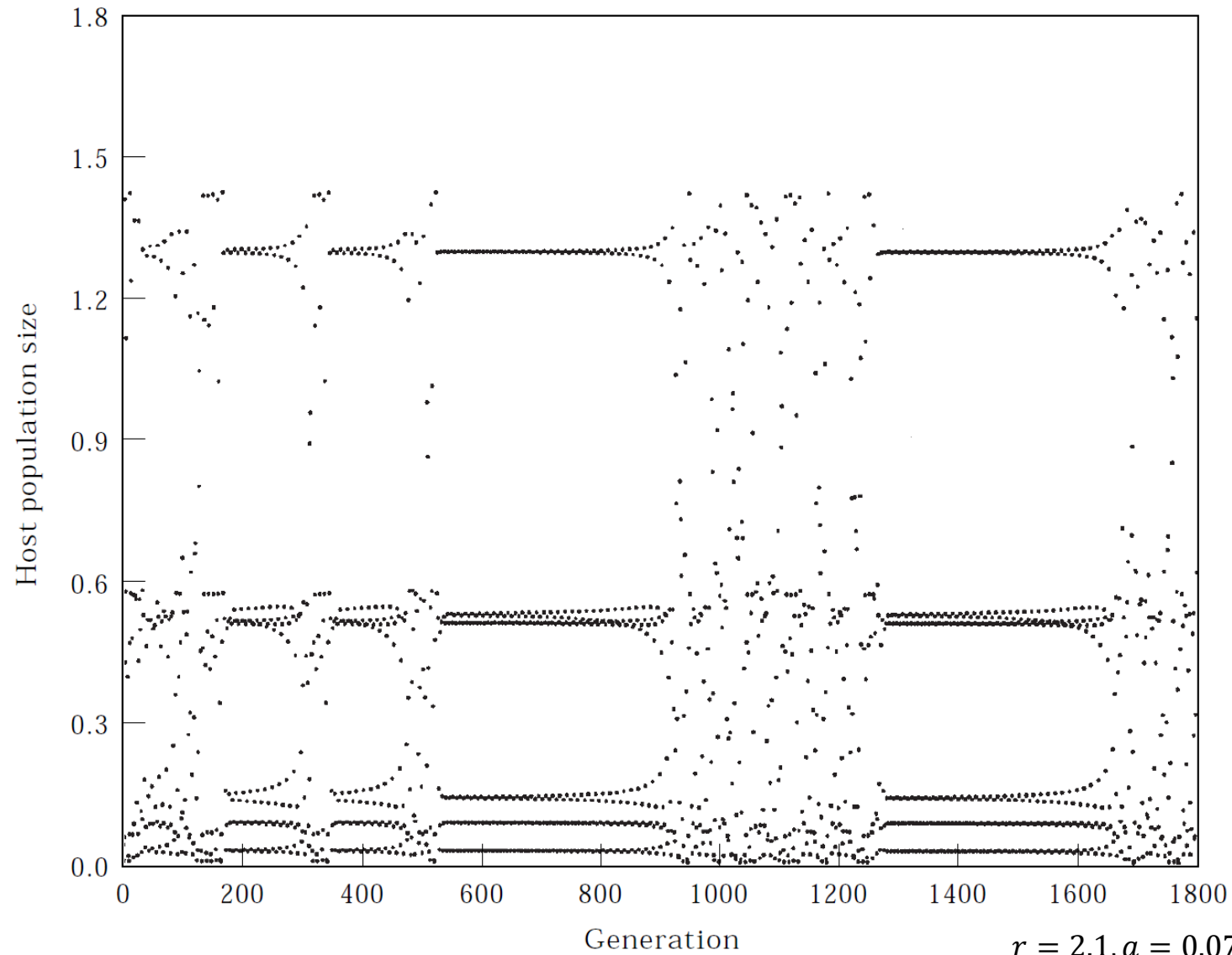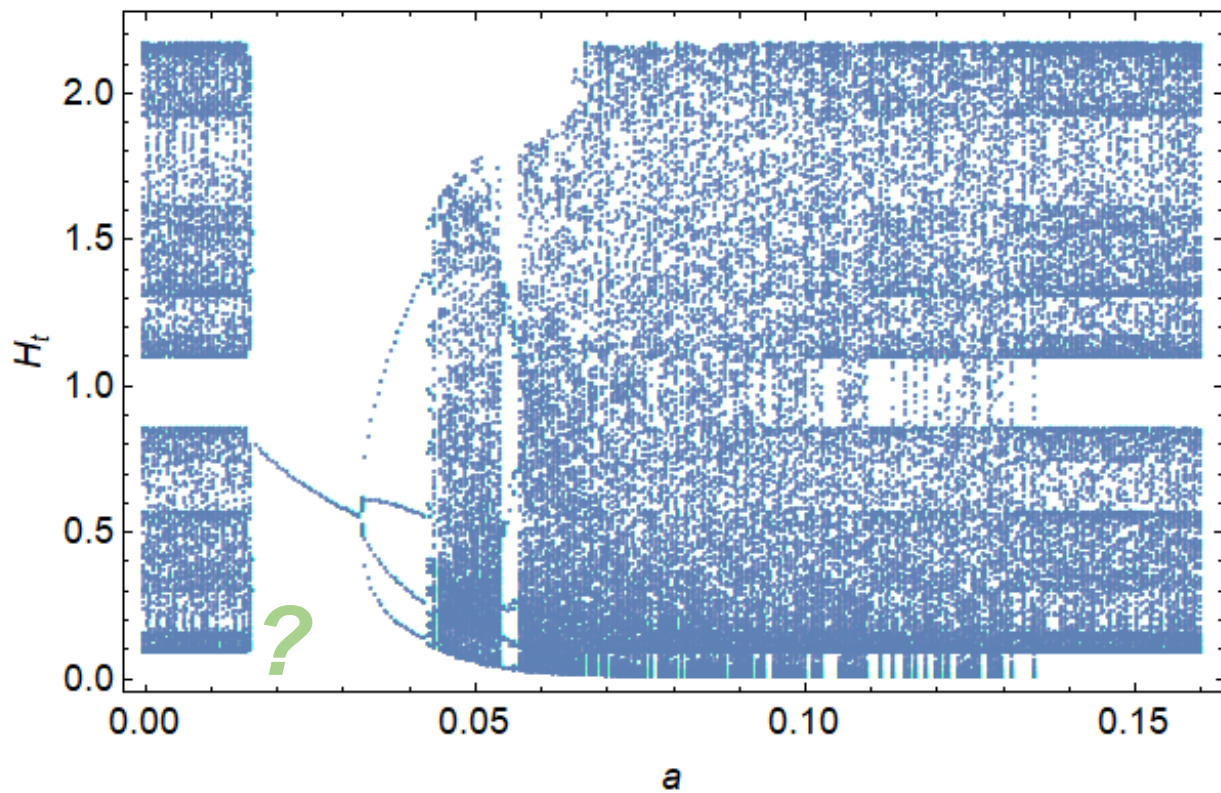
There are fractal basins of attraction

Parasitoid population size at $t=0$

Host population size at $t=0$

$r = 2.8, a = 0.043, T = 100, T_h = 1$

$r = 2.1, a = 0.0717, T = 100, T_h = 1, H_0 = P_0 = 0.5$

Kaitala et al. (1999, *J Theor Biol*)

Parasitoids stabilize the unstable host dynamics – how?
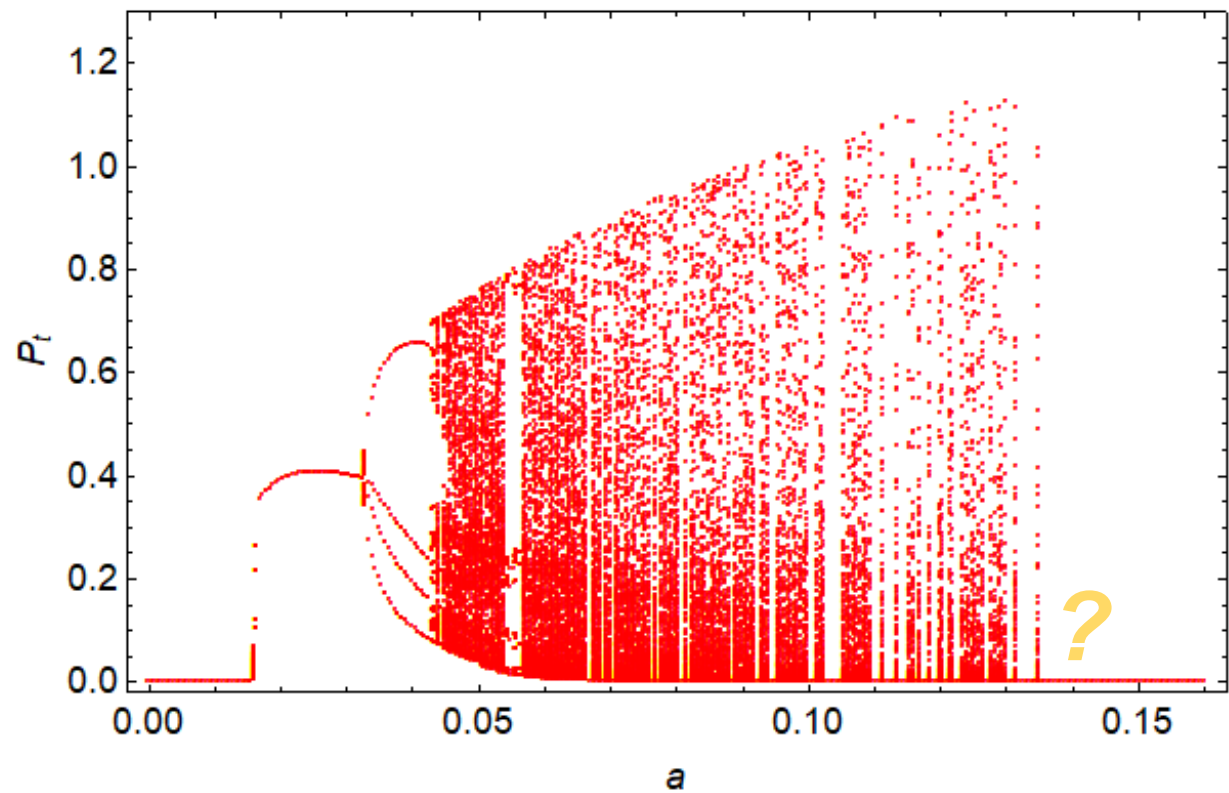
Why do the (quasi-)periodic cycles disappear?

$r = 2.8, T = 100, T_h = 1$, random ICs

$$H_{t+1} = \frac{\lambda H_t}{1 + (\lambda - 1)H_t} \, e^{\frac{-aP_t}{1+hH_t}}$$

$$P_{t+1} = H_t \left(1 - e^{\frac{-aP_t}{1+hH_t}}\right)$$

No Arnold tongues,
"just" periodic windows

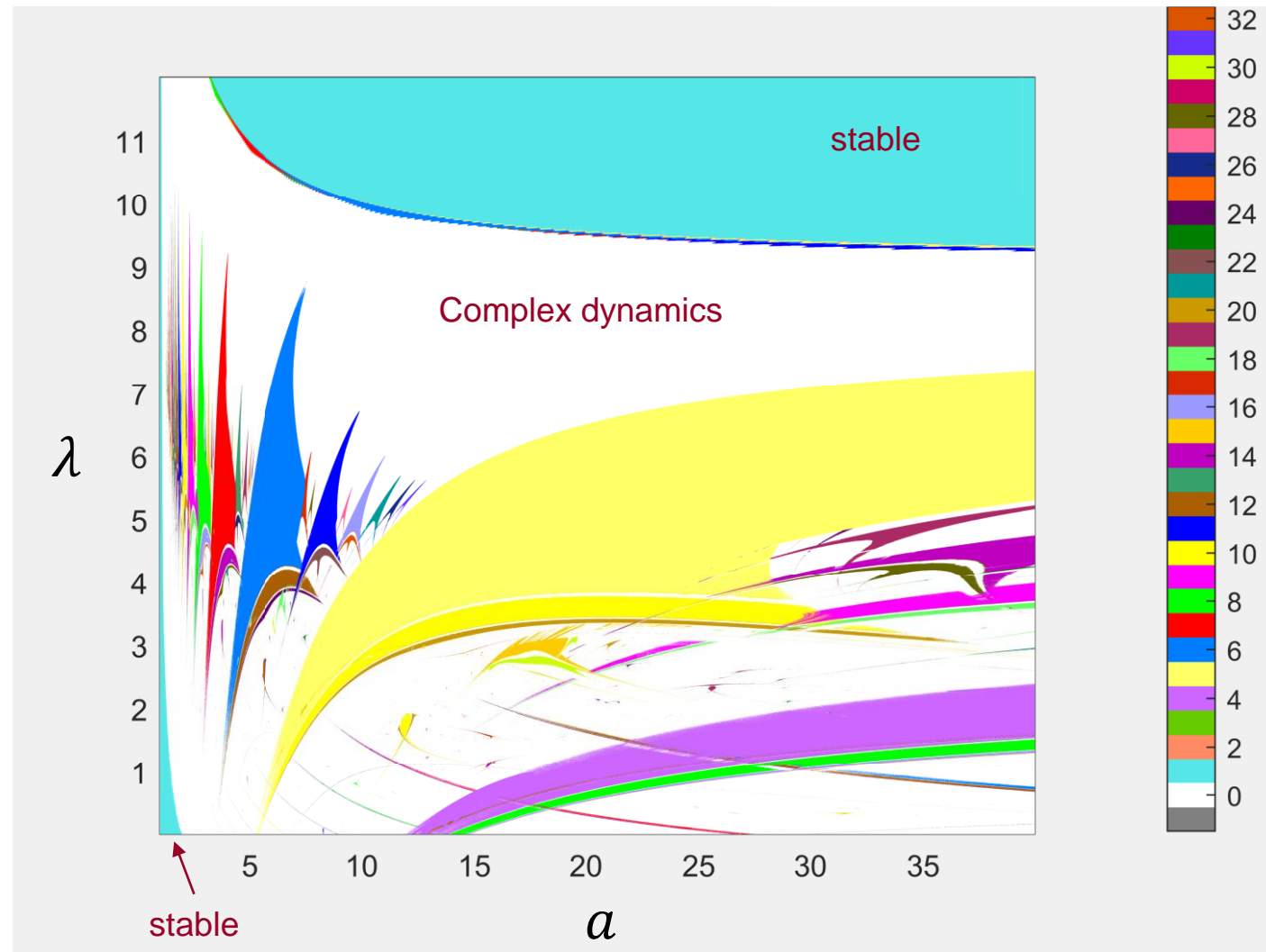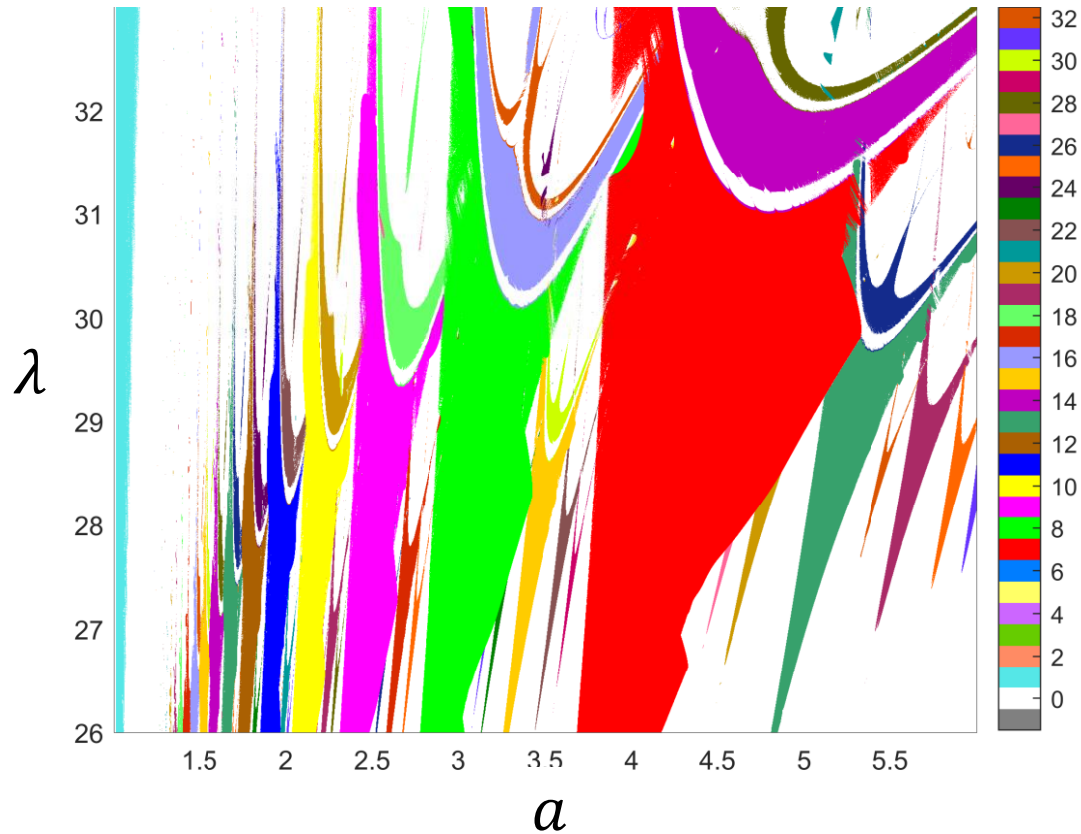$h = 7.4$
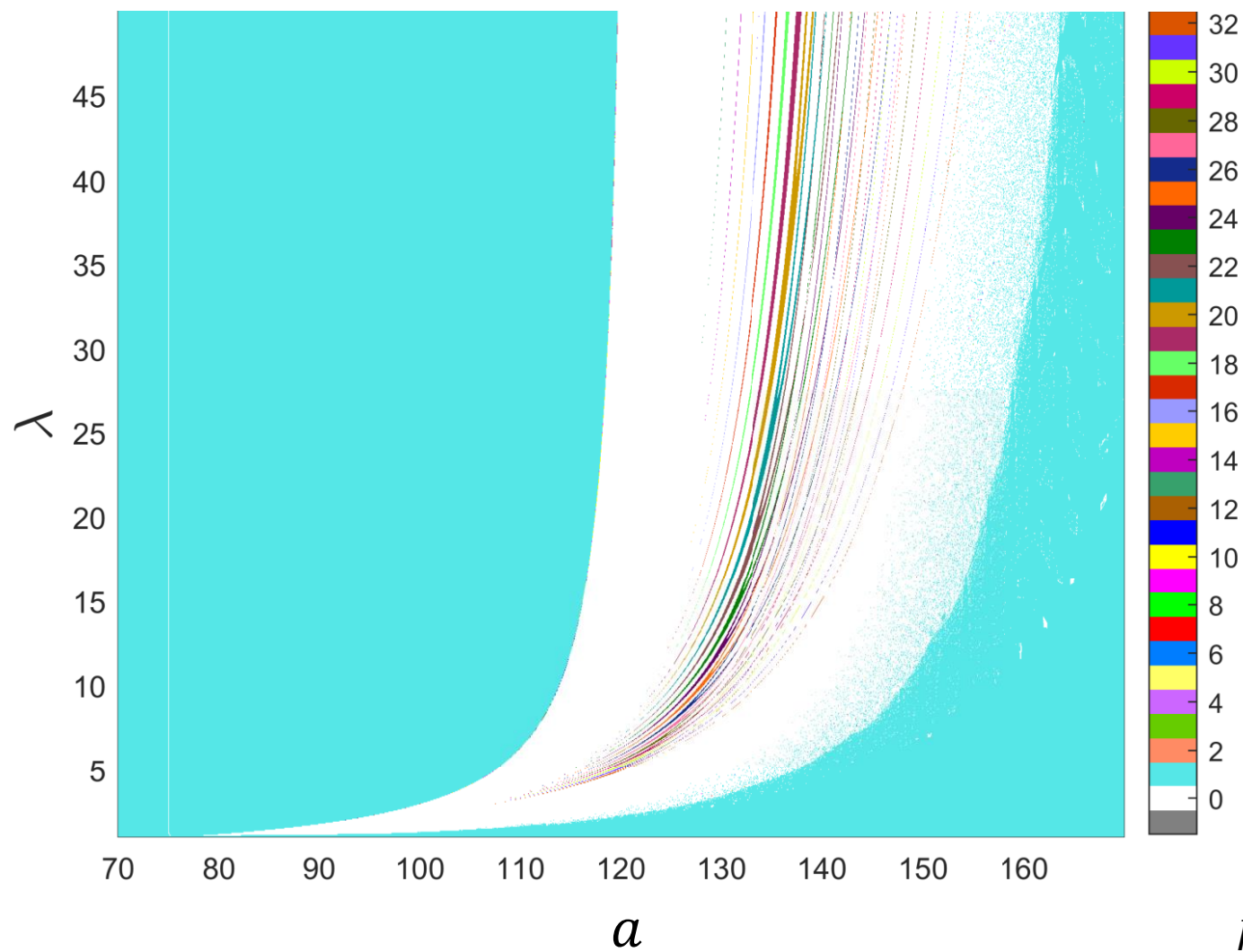
$$H_{t+1} = \frac{\lambda H_t}{1 + (\lambda - 1)H_t} \, e^{\frac{-aH_tP_t}{1+hH_t^2}}$$

$$P_{t+1} = H_t \left(1 - e^{\frac{-aH_tP_t}{1+hH_t^2}} \right)$$

Zoom

λ

a

λ

stable

Complex dynamics

stable

a

$h = 7.4$

$$H_{t+1} = H_t e^{r(1-H_t/K) - \frac{aTH_tP_t}{1+cH_t+aT_hH_t^2}}$$

$$P_{t+1} = H_t\left(1 - e^{\frac{-aTH_tP_t}{1+cH_t+aT_hH_t^2}}\right)$$
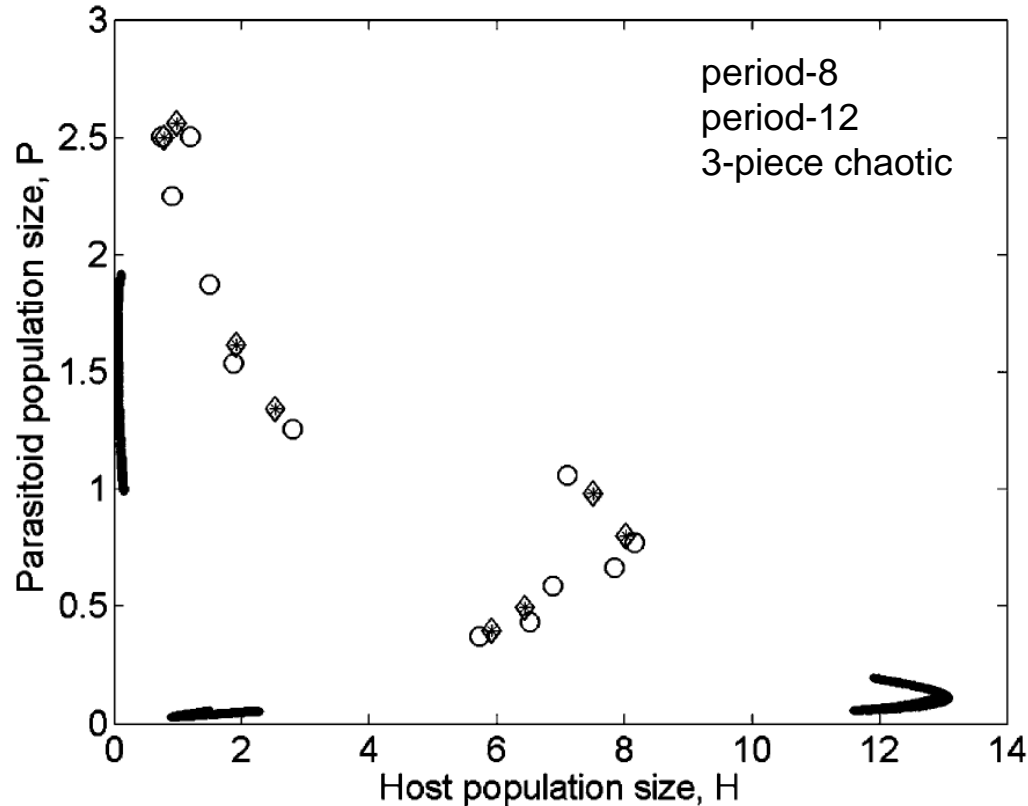
…at first glance, no particular novel dynamic behavior reported
(but more careful reading required)

Coexistence of three attractors

Ricker + parasitoid interference

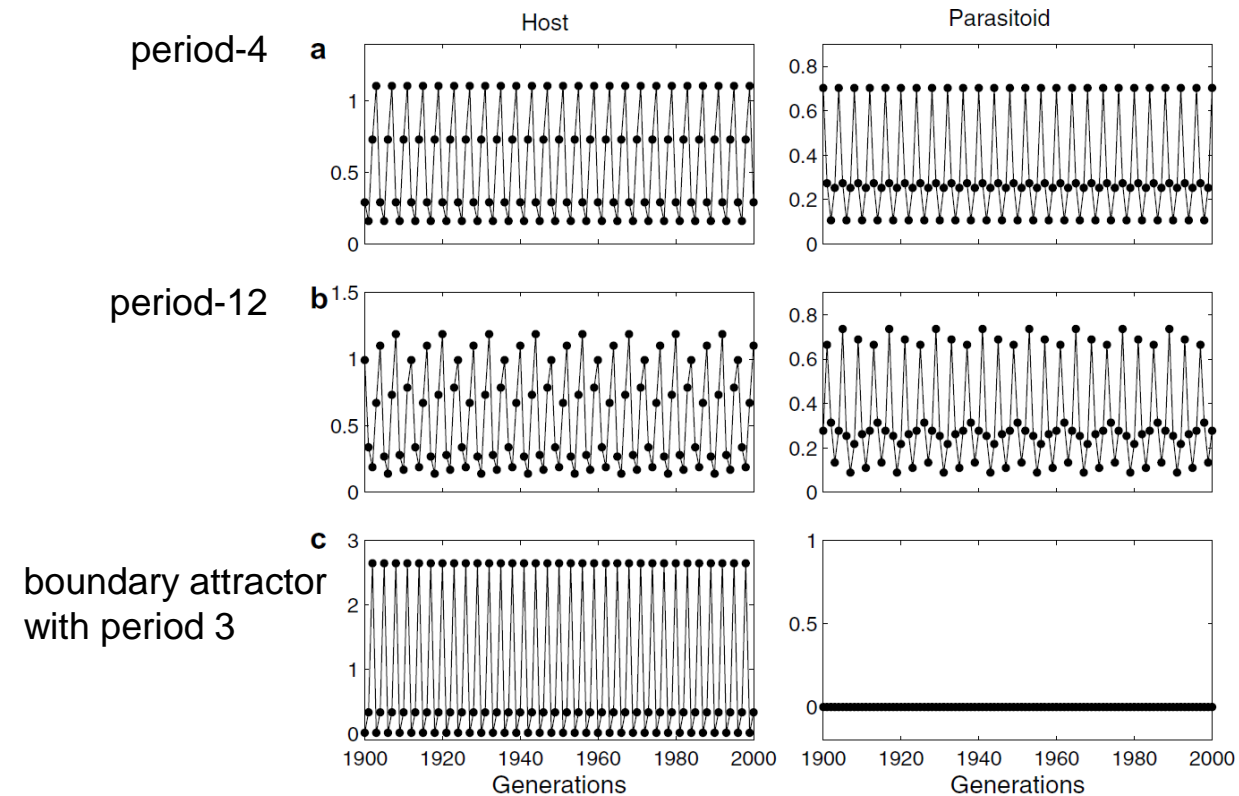$$H_{t+1} = H_t \, e^{r(1-H_t/K)-aP_t^{-m+1}}$$
$$P_{t+1} = N_t\left(1 - e^{-aP_t^{-m+1}}\right)$$

Ricker + intergenerational parasitoid survival

$$H_{t+1} = H_t \, e^{r(1-H_t/K)-aP_t}$$
$$P_{t+1} = N_t(1 - e^{-aP_t}) + \delta P_t$$

period-8
period-12
3-piece chaotic

period-4 **a**

period-12 **b**

boundary attractor with period 3 **c**

Generalized host growth

Idea: Choose a general host growth that can be undercompensatory (like Beverton-Holt)
or overcompensatory (like Ricker) depending on a parameter

$$H_{t+1} = g(H_t)\, f(H_t, P_t)$$
$$P_{t+1} = N_t(1 - f(H_t, P_t))$$

$$g(H) = \frac{\lambda H}{(1 + cH)^b}$$

Hassell (1976) map
$\lambda > 0$ intrinsic growth rate
$c > 0$ measure of intraspecific competition
$b \geq 0$ measure of compensation

$b = 0$ geometric growth
$0 < b < 1$ undercompensation
$b = 1$ Beverton-Holt
$b > 1$ overcompensation

$$H_{t+1} = \frac{\lambda H_t}{\left(1 + aH_t e^{\frac{-aTP_t}{1+aT_h H_t}}\right)^b} e^{\frac{-aTP_t}{1+aT_h H_t}}$$

$$P_{t+1} = H_t \left(1 - e^{\frac{-aTP_t}{1+aT_h H_t}}\right)$$

"something else"

$$H_{t+1} = \frac{\lambda H_t}{(1 + aH_t e^{-AP_t})^b} e^{-AP_t}$$

$$P_{t+1} = H_t(1 - e^{-AP_t})$$

$b = 1$ Beverton-Holt   Liu et al. (22008)

$b > 0$ Hassell   Liu (2009)

A =constant or

$A = aP^{-m}$ parasitoid interference (Hassel-Varley)

Liu et al. (2008) Computer simulation of the dynamic complexity of a host-parasitoid model. International Conference on Computer Science and Information Technology
Liu, Hua (22009) Intra-specific density dependent effect of a host-parasitoid interaction model.

Zhu and Zhao (2009, *Chaos, Solitons and Fractals*)

- Coexistence of non-equilibrium attractors
  - not more than two in Kaitala et al. (1999) reported
  - Is $r > 2$ needed, i.e. instability in the host dynamics alone?
  - With Beverton-Holt host growth, is there also chaos possible
    with type II or III functional response?

- Bifurcation questions
  - How do parasitoids go extinct for large attack rates a?
    (e.g., attractor crisis, homoclinic loop, numerical artefact?)
  - How do host-only oscillations become stabilized by parasitoid invasion,
    at smaller values of a? (transition from periodic or chaotic attractor on the boundary
    to a stable fixed point in the interior?)

- Type III seems to differ from type II when combined with Beverton-Holt

- Arnold tongues: structure in the periodicities?