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Host-Parasitoid Interactions

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Abbildungsverzeichnis

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The project was implemented in Julia and uses the DrWatson package. To reproduce the results look at README.md on github.com

1 Introduction

The development of models to describe host-parasitoid interactions began as early as the beginning of the 20th century. Motivated by the possibility of regulating pests in agriculture by the controlled application of parasitoids, various approaches were developed. Today, as then, research faces a number of challenges. Although the data has grown over the years and numerous models have been developed, there is still a lack of mechanistic understanding of the interactions and complex dynamics that can occur in such a system.

Since the models can exhibit highly non-linear behavior, analytical mathematic quickly reaches its limits. Thanks to the rapid development of computer technology computers are now available to researchers. In recent years, therefore numerical algorithms have been developed to investigate complex system behavior.

2 Model and Methods

2.1 The Model

The modeling of host-parasitoid interactions has a long history. Since insects often have separate generations, discrete-time models are suitable. In general all our models base on the following assumptions:

$$\begin{aligned} N_{t+1} &= g(N_t) f(N_t, P_t) \\ P_{t+1} &= c N_t [1 - f(N_t, P_t)] \end{aligned} \tag{1}$$

Given this equations $g(N_t)$ describes the growth of the hosts, $f(N_t, P_t)$ the proportion of hosts that is not infested by parasites and c indicates the average number of parasites that emerge from a parasitized host.

Previos research often build on the assumption that the growth of host following the Ricker map given by:

$$g(N_t) = N_t e^{r(1-N_t)} \tag{2}$$

About this Ricker Model is known that in respect to the intrinsic growth rate r the behavior changes from stable, over period doubling to chaos. However we want to reduce this impact, therefore we use the Beverton-Holt map as a also well known model. This model is also a discrete-time equivalent to logistic growth, which shows stable behavior and approaches the carring capacity in every permitted parameter set.

The Beverton-Holt Model ist given by the following equation

$$g(N_t) = \frac{\lambda N_t}{1 + \frac{(\lambda-1)N_t}{K}} \tag{3}$$

with $\lambda > 0$ as the intrinsic growth rate and $K > 0$ as the capacity.

Assuming that the interaction between host and parasitoid is density-dependent with a Holling type III results:

$$f(N_t, P_t) = 1 - e^{-\frac{N_{enc}}{N_t}} = 1 - e^{\left[-\frac{aTN_tP_t}{1+cN+aT_hN_t^2}\right]} \tag{4}$$

with the total time per generation T, the handling time T_h and the parasitization rate a.

Through dedimensionalization with

$$n_t = \frac{N_t}{K} \qquad p_t = \frac{P_t}{bK} \qquad \alpha = abTK^2 \qquad \mu = aT_hK^2 \tag{5}$$

one receives:

$$\begin{aligned} n_{t+1} &= \frac{\lambda n_t}{1+(\lambda-1)n_t} e^{\frac{-\alpha n_t p_t}{1-\mu n_t^2}} \\ p_{t+1} &= n_t \left(1 - e^{\frac{-\alpha n_t p_t}{1+\mu n_t^2}}\right) \end{aligned} \tag{6}$$

with the scaled growth rate $\lambda > 0$, the scaled parasitization rate $\alpha > 0$ and the scaled handling time $\mu > 0$

2.2 Analysemethoden

The model shows the following complex behavior:

- Non-unique dynamics, which mean coexisting attractors
- Basin of attraction
- Intermittency
- Supertransient behavior

2.2.1 Implementations

Because of the complex behavior the existance and stability of an Attractor is hardly analytical shown. Therefore we investigate this behavior by using a few numerical methods. We choose the Julia programming Language because many required functionalities are already implemented, with easy readable code and great performance.

For numerically evolving the given System we use the `trajectory` function from the `DynamicalSystems` libary in the Julia programming language.

For Attractor detection we use the `AttractorsViaRecurrences` function from the `Attractors` libary by Datseries and Wagemakers which relies on a method described by Nusse and Yorke.

Given this set of Attractors one can match the inital conditions with one of the Attractors to recieve the basin of attraction. This is also implemented in the `Attractors` libary with the function `basins_of_attraction`.

2.2.2 Example

To get the basin of attraction we use a finate state machine. The state space is devided into a discrete grid initialized with a value `u` which means *unvisited*

For each inital condition we track the trajectory of the System. Once an Attractor hits a cell the value changes to `v` for *visited*. When the trajectory hits consecutive times previos visited cells an Attractor is detected and the inital condition is attached to the Attractor.

For a more detailed description see 'Effortless estimation of basin of attraction' by Datseries.

3 Ergebnisse

4 Diskursion

The given System is a fully deterministic Model. Therefore fractal properties do not directly influence the stability of an Attractor. But under respect of random fluctuations, mixed up basins with close proximaty of different Attractors may disturb the System. A metric on the basin based on the neighborhood could quantify this insecurity.