Beverton-Holt + type III

Consider the classical framework

$$\begin{aligned} N_{t+1} &= g(N_t)f(N_t,P_t) \\ P_{t+1} &= bN_t(1-f(N_t,P_t)) \end{aligned}$$

with a Beverthon-Holt map for host growth:

$$g(N_t) = \frac{\lambda N_t}{1+\frac{(\lambda-1)N_t}{K}}$$

where $\lambda > 0$ is the host reproduction rate and $K > 0$ the carrying capacity.

For the functional response, we assume type III:

$$f(N_t,P_t) = \exp\left[\frac{-aTN_tP_t}{1+cN_t+aT_h(N_t)^2}\right]$$

where a is the parasitism rate, T the total time available to parasitoids, T_h the handling time, and c a positive constant.

With the scalings

$$n_t = \frac{N_t}{K}, \qquad p_t = \frac{P_t}{bK}, \qquad \alpha = abTK^2, \qquad \mu = aT_hK^2$$

and the assumption $c = 0$ we obtain

$$\begin{aligned} n_{t+1} &= \frac{\lambda n_t}{1+(\lambda-1)n_t}\,\mathrm{e}^{\frac{-\alpha n_t p_t}{1+\mu n_t^2}} \\ p_{t+1} &= n_t\left(1-\mathrm{e}^{\frac{-\alpha n_t p_t}{1+\mu n_t^2}}\right) \end{aligned}$$

$\lambda > 0$ scaled host reproduction rate
$\alpha > 0$ scaled parasitism rate
$\mu > 0$ scaled handling time

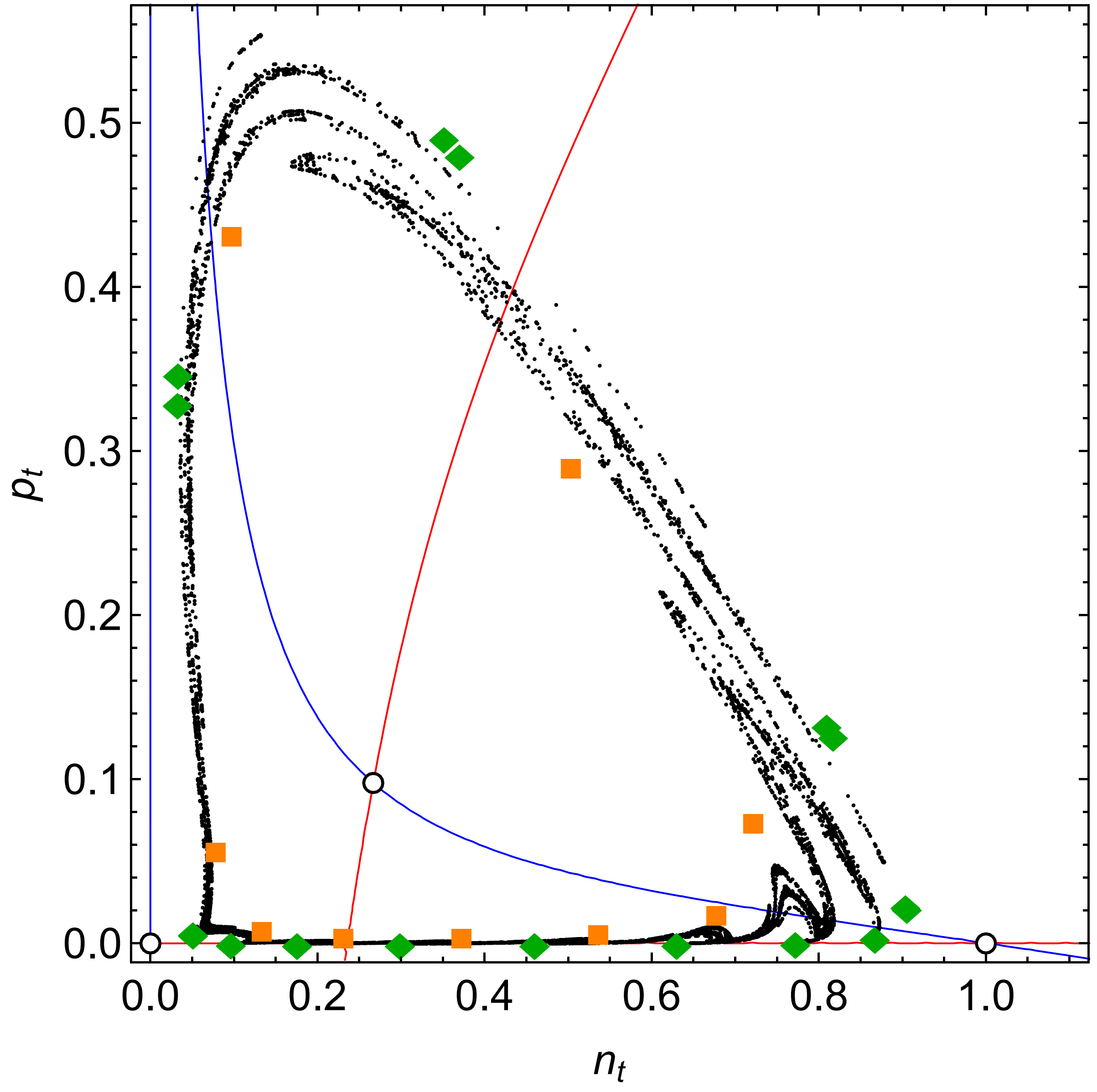
This is the model we will investigate.

The model has up to three equilibrium points

- $(0,0)$ extinction of both species
  locally asymptotically stable for $\lambda < 1$

- $(1,0)$ hosts only (parasitoid extinction)
  locally asymptotically stable for
  $\lambda > 1$ and $\alpha < \mu + 1$

- (n^*, p^*) coexistence
  no explicit solution can be found



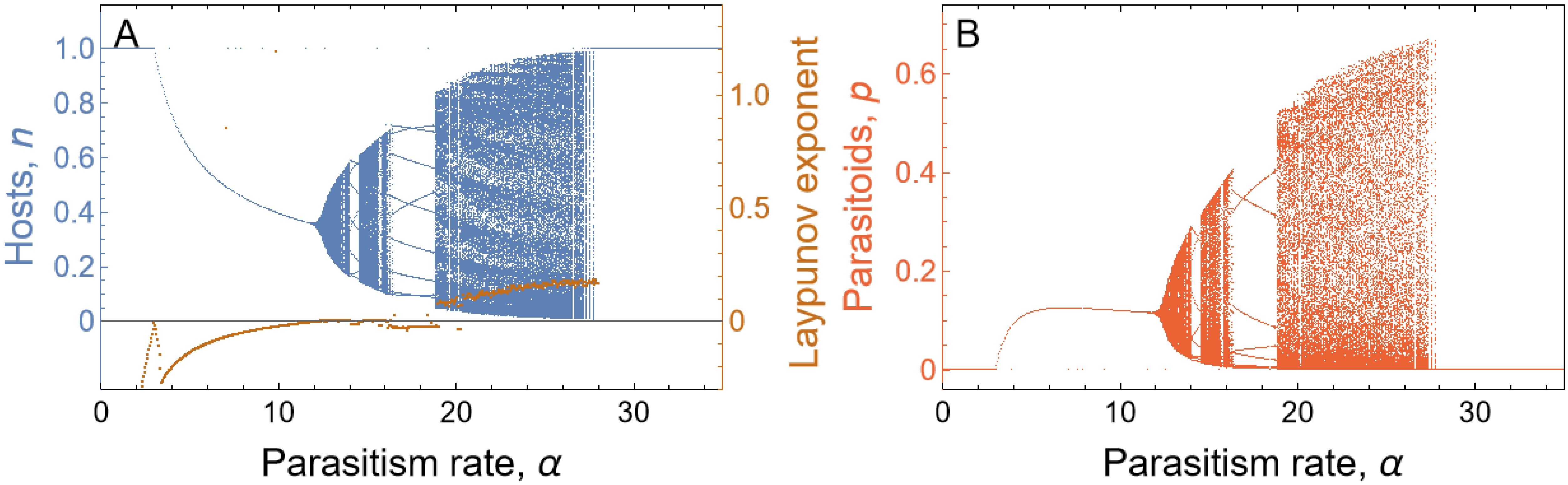
Coexistence of three attractors

In this example:

- 15-cycle (green)
- 9-cycle (orange)
- Invariant loop (black)

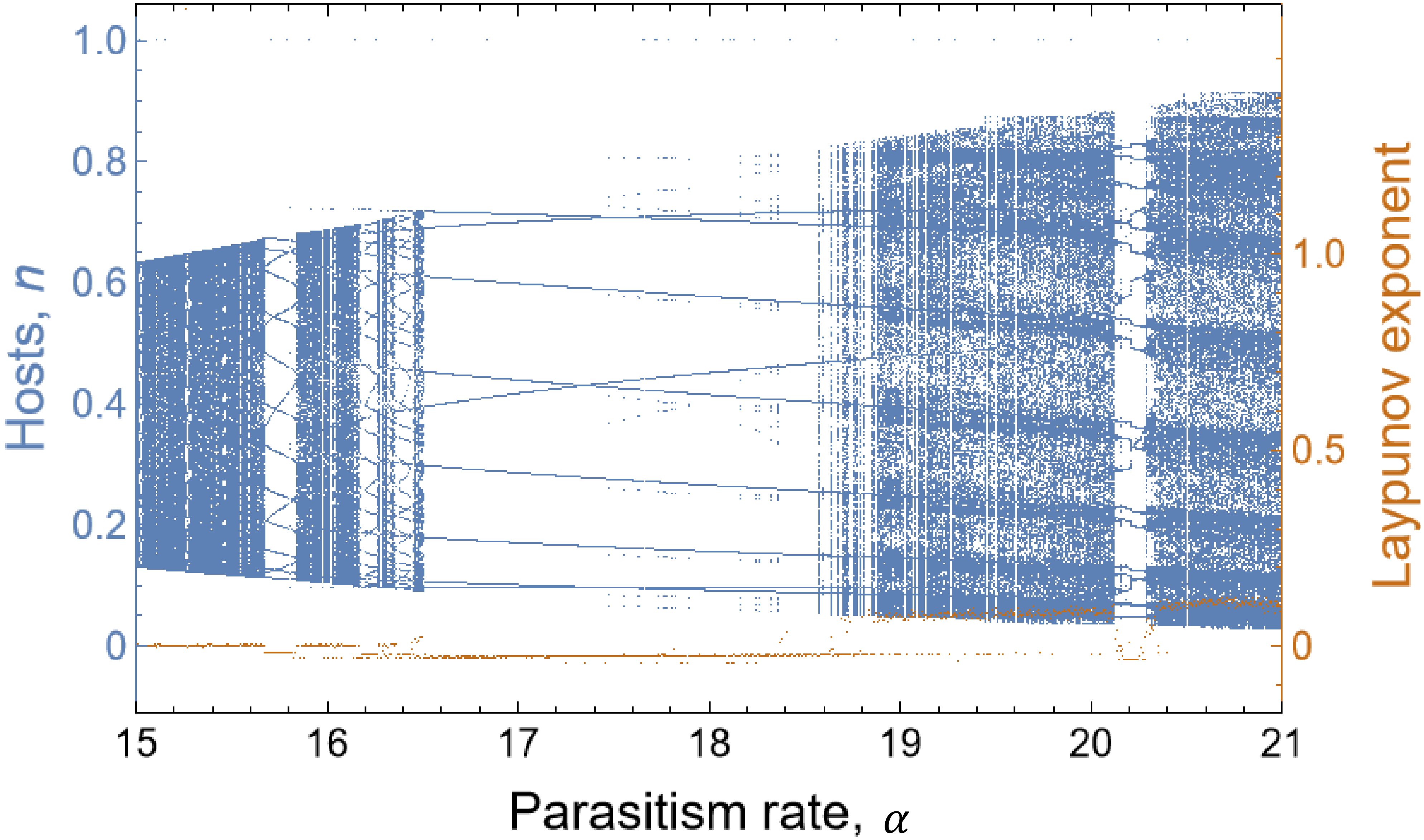
Blue and red lines are the nullclines.
Empty circles are unstable equilibria.

$\lambda = 2, \alpha = 20, \mu = 2$

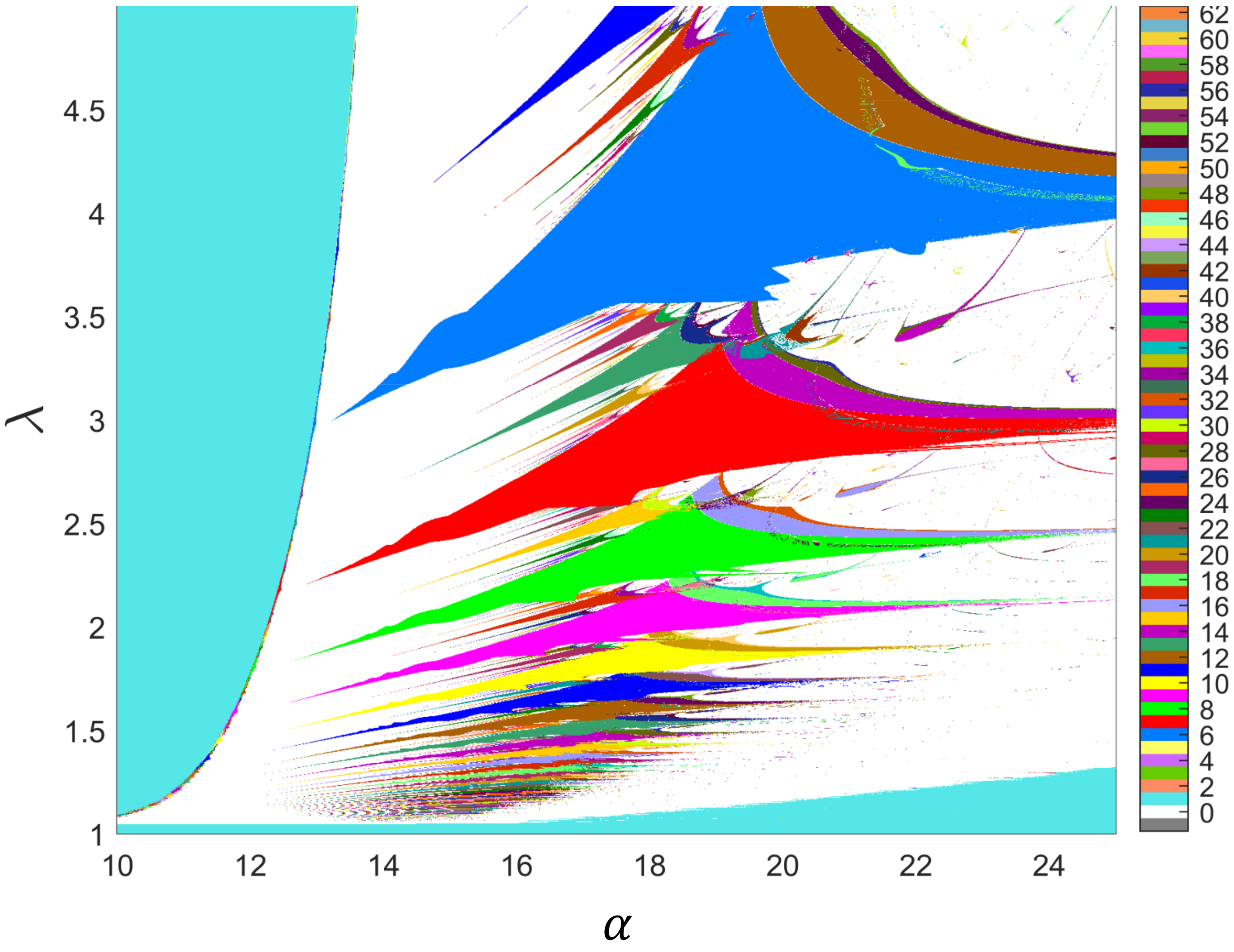

A
Hosts, n
1.0
0.8
0.6
0.4
0.2
0
Laypunov exponent
1.0
0.5
0
Parasitism rate, α
10
20
30
B
Parasitoids, p
0.6
0.4
0.2
0
Parasitism rate, α
10
20
30

$\lambda = 2, \mu = 2$

Zoom $\alpha \in [15,21]$

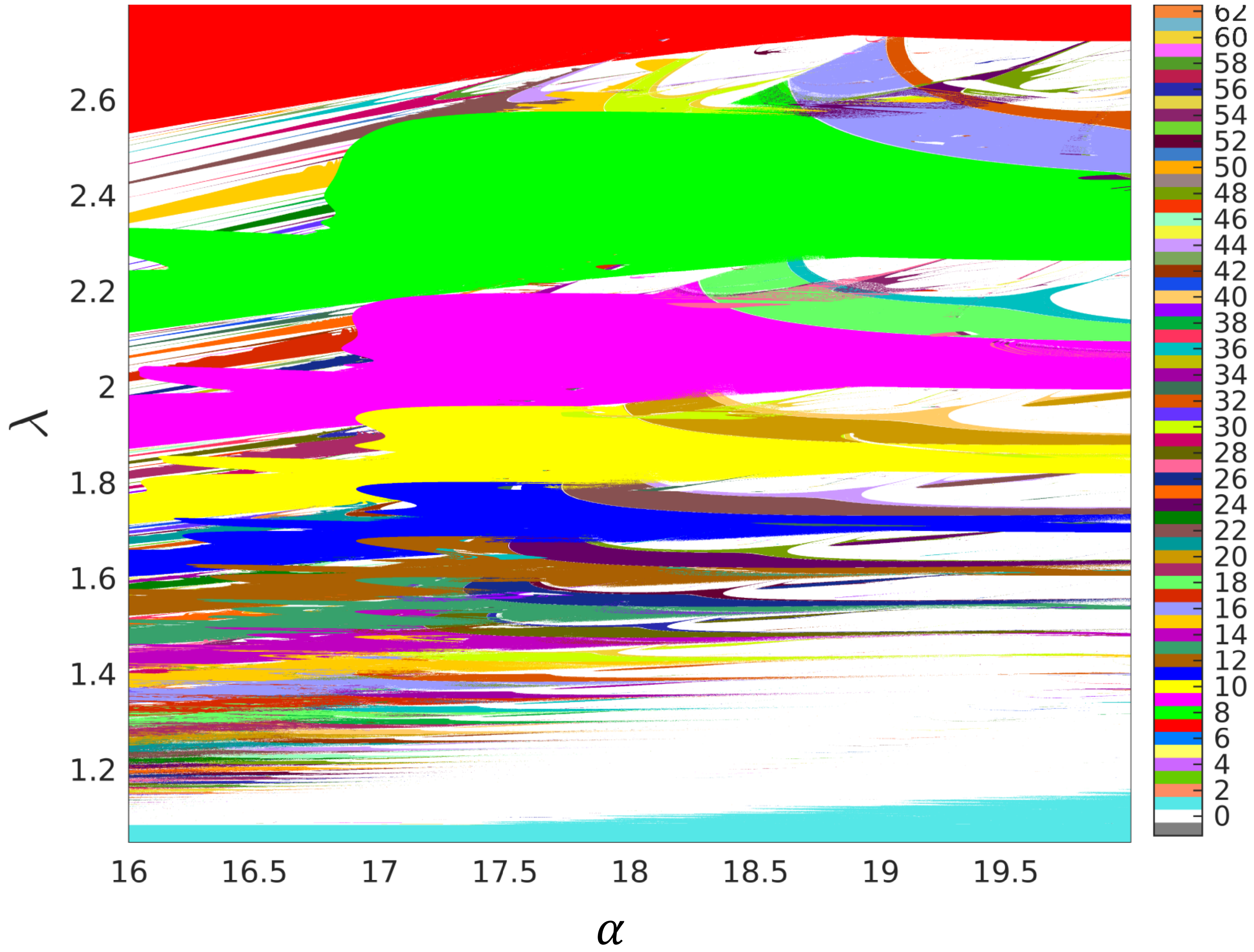


$\lambda = 2, \mu = 2$



Turquoise: stable fixed point
White: Invariant curve or cycle >62
Other colors indicate the cycle

$\mu = 2$



Zoom from previous slide

$\mu = 2$