Sample Spaces and Events

Def sample set S is the set of all possible outcomes of an experiment (random thing)

① tossing a coin

$$S = \{H, T\}$$

tossing a coin twice

$$S = \{\underbrace{HH}, \underbrace{HT}, \underbrace{TH}, \underbrace{TT}\}$$

outcome

② results of a horse race, horses #'ed $1, \ldots, 7$

$$S = \{\text{all orderings of } (1, \ldots, 7)\}$$

$(1, 2, 3, 4, 5, 6, 7)$

$(3, 7, 1, 2, 4, 5, 6)$

(3) The amount of a drug that must be given to a patient so they react positively.

$S = (0, \infty)$

<u>Def</u> Event E is any subset of S

If the outcome of an experiment is contained in E, then E has occured.

<u>Ex</u> $E = \{HH\}$

$E = \{$all of the outcomes when the first coin comes up H$\}$

$= \{HH, HT\}$

<u>Ex</u> Horse racing

$E = \{$all of the outcomes in S starts with 3$\}$

$= \{\}$ using the same

<u>Def</u> (union) E & F (events),

$E \cup F$ is the set of all outcomes contained in either E or F or both.

$\Rightarrow$ $E \cup F$ occurs, if E or F or both occur.

$E = \{$all outcomes starting with 6$\}$

$F = \{$all outcomes with 6 in the second place$\}$

$E \cup F = \{$horse six comes in 1st or 2nd$\}$

<u>Def</u> (Intersection) E & F

$E \cap F$ is the set of all outcomes where both E and F occur.

for above: $E \cap F = \{\phi\}$

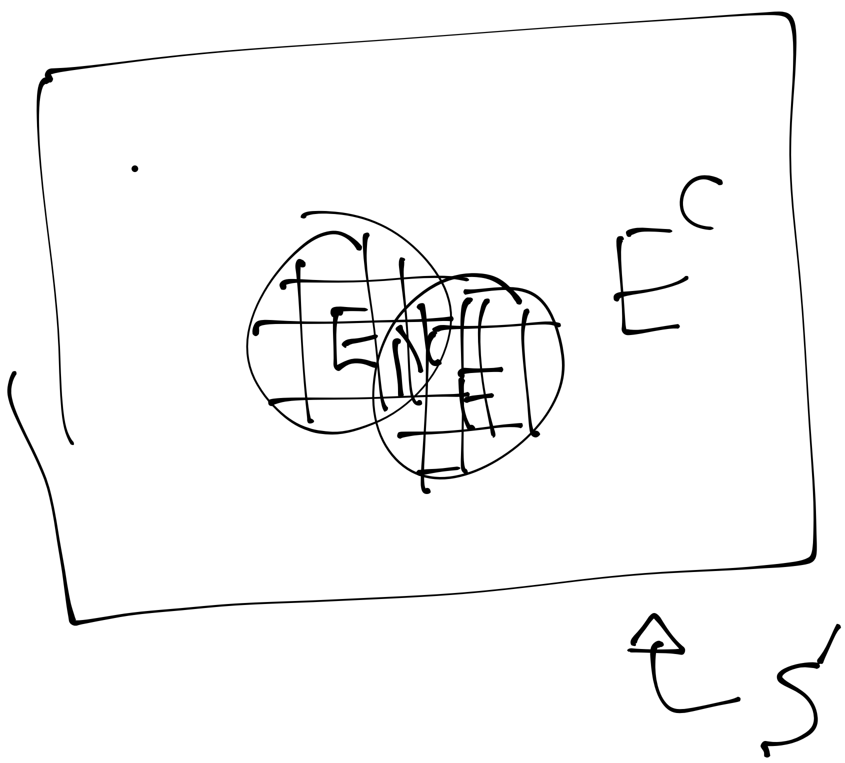
$E = (0, 5) \Rightarrow$ minimum dose is less than 5

$F = (2, 10) \Rightarrow$ between 2 and 10

$E \cap F \Rightarrow (2, 5)$

<u>Def</u> the complement of a set E

$E^c = \{$all of the events not in $E\}$



$E_1, \dots, E_n$ $\quad E_1 \cup E_2 \cup \dots \cup E_n$

= at least one occurs

$E_1 \cap E_2 \cap \dots \cap E_n$

= all occured

<u>Axioms of Probability</u>

Probability measure $\Rightarrow$ function P that maps event in the sample to the real

#s

E

Axiom 1 $0 \le P(E) \le 1$

Axiom 2 : $P(S) = 1$

Axiom 3: $E_1, E_2, \ldots$ mutually exclusive

$$E_i \cap E_j = \phi$$

$$\underbrace{P(\bigcup_{i=1}^{n} E_i)}_{E_1 \cup E_2 \cup \ldots \cup E_n} = \sum_{i=1}^{n} P(E_i)$$

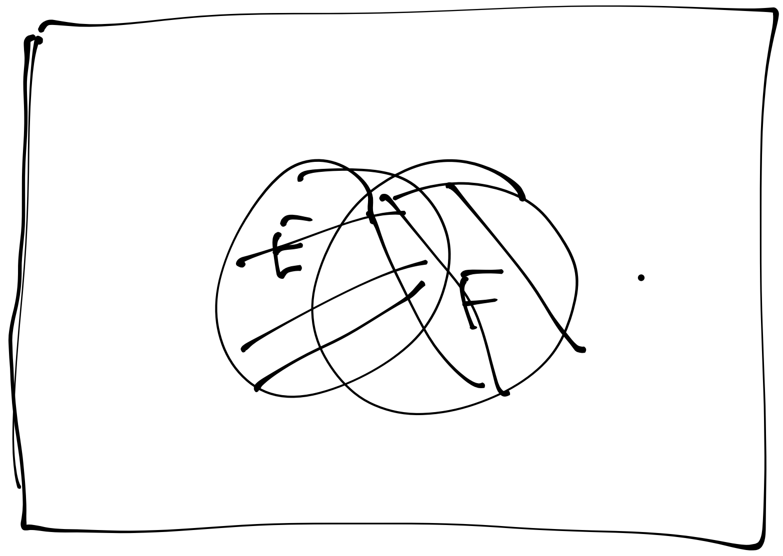
Claim $P(E^c) = 1 - P(E)$

$$E \cup E^c = S$$

$$E \cap E^c = \{\phi\}$$

$$1 = P(S) = P(E) + P(E^c)$$

Prop $P(E \cup F) = P(E) + P(F) - P(E \cap F)$



Ex 28% of males in Nevada smoke cigarettes, 6% smoke cigars and 3% smoke both.

What percent smoke neither?

$$P(\text{neither}) = 1 - P(\text{cigars} \cup \text{cigarettes})$$
$$= 1 - (P(\text{cigars}) + P(\text{cigarrets}) - P(\text{both}))$$

Conditional Probability

Suppose we have S, P, events E and F. We learn that F has occured? What can we then say chat E's likelihood of occuring?

$$P(E|F) = \frac{P(E\cap F)}{P(F)}$$

Ex Rolling a pair of dice

$S = \{ (i,j), \; i=1,\dots,6, \; j=1,\dots,6 \}$

$\Rightarrow$ if dice 1 comes up 3, what is the probability the sum of 2 dice is 8?

$E \cap F = \{ (3,5) \} \Rightarrow P(E\cap F) = \frac{1}{36}$

$P(E) = 1/6$

$F = \left.\begin{matrix} (2,6) \\ (3,5) \\ (4,4) \\ (5,3) \\ (6,2) \end{matrix}\right\} \Rightarrow \frac{5}{36}$

$$P(F|E) = \frac{P(E\cap F)}{P(F)}$$

$$= \frac{1/36}{1/6} = 1/6$$

$P(E|F)$ = probability that dice 1 is a 3 given that the sum is 8

$\hookrightarrow 1/5$

Ex If someone has two children, and at least one of them is a boy, what is the probability they are both boys?

BB BG

GB GG

$1/3$

a useful formula:

$$P(E|F) = \frac{P(E\cap F)}{P(F)}$$

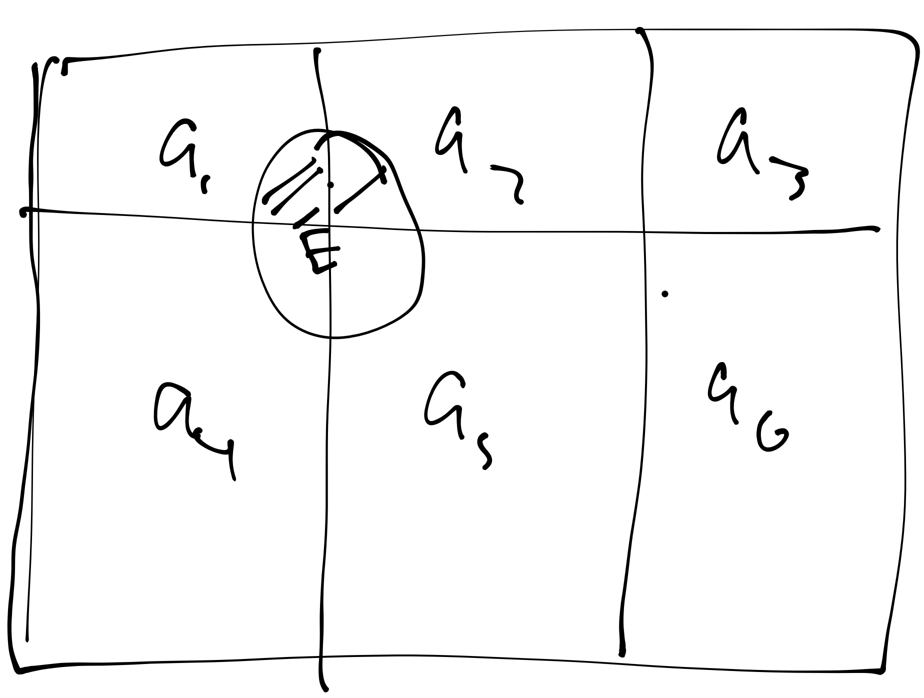
$$P(E\cap F) = P(E|F)\,P(F)$$

Law of total probabilities

$a_1, \ldots, a_n$ mutually exclusive

$$P(E) = \sum_{i=1}^{n} P(E|a_i)\,P(a_i)$$

$$\left(= P(E\cap a_i) \right)$$



Ex Auto-insurance company:

people are either accident-prone or not

(1) accident-prone have an accident in their first year with this insurance with $p = .4$

not accident-prone $p = .2$

(2) if 30% of the population is accident-prone, what is the probability a randomly selected new policy holder will have an accident?

$$P(\text{accident}) = P(\text{accident} \mid \text{prone}) \cdot P(\text{prone}) + P(\text{accident} \mid \text{not prone}) \cdot P(\text{not prone})$$

$$= (.4)(.3) + (.2)(.7) = .26$$

sps the policy holder has an accident

what is the probability they are

accident prone?

$$P(\text{accident prone} \mid \text{accident}) = \frac{\overbrace{P(\text{accident prone } \& \text{ accident})}}{P(\text{accident})}$$

$$= \frac{P(\text{prone})\, P(\text{accident} \mid \text{prone})}{P(\text{accident})}$$

$$= \frac{(.3)(.4)}{.26} \approx .462$$

<u>Bayes Rule</u>

$$P(A \mid B)P(B) = P(A \cap B)$$

$$= P(B \mid A)P(A).$$

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

Ex A laboratory blood test correctly identifies someone as having a disease 99% of the time if they have. If they don't have the disease, 1% of the time it comes back positive.

If .5 percent of the population has the disease, what is the probability of having the disease given a positive test?

$$P(D \mid pos) = \frac{P(pos \mid D)\, P(D)}{P(pos)}$$

← true positives + false positives

$$= \frac{(.99)(.005)}{P(pos \mid D)\, P(D) + P(pos \mid D^c)\, P(D^c)}$$

$$= \frac{(.99)(.005)}{(.99)(.005) + (.01)(.995)}$$

$= .332$

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