Problem 1

100 students

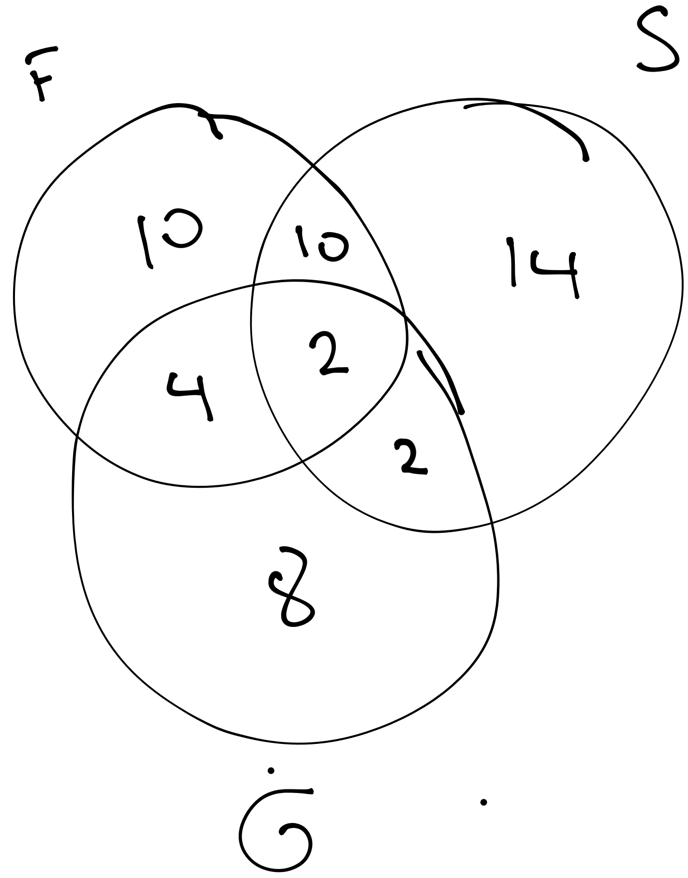
28 in spanish, 26 in french, 16 german

12 in both s & f

4 in both s & g

6 in both f & g

2 in all 3



Pt 1 $\longrightarrow$ 50 students taking at least one language

$P(\text{no lang}) = 1 - \frac{50}{100} = .5$

Pt 2 → probability of exactly one:

$$\frac{10+14+8}{100} = \frac{32}{100} = .32$$

Pt 3 → If 2 students are chosen randomly, what is the probability that at least one is taking a language class?

This is $1 - P(\text{neither})$

$$P(\text{neither}) = \left(\frac{50}{100}\right)\left(\frac{49}{99}\right) \approx .247$$

$$\Rightarrow 1 - .247 = .753$$

Problem 2

5% of men and .25% of women are color blind

if $P(M) = P(W) = 1/2$

$$P(M \mid CB) = \frac{P(CB \mid M)P(M)}{P(CB)}$$

$$= \frac{(.05)(\,^1/_2)}{(.05)(\,^1/_2) + (.0025)(\,^1/_2)}$$

$$\approx .9524$$

if $P(m) = ^2/_3$, $P(w) = ^1/_3$,

$$P(m \mid CB) = \frac{(.05)(\,^2/_3)}{(.05)(\,^2/_3) + (.0025)(\,^1/_3)}$$

$$\approx .976$$

Exercise 3

$P(S_1) = .3$

$P(S_2) = .6$

$P(s \mid S_1) = P(s \mid S_2)$

$= P(D \mid S_1) = P(S \mid S_2)$

$= 0.5$

↳ let Y = amount from custome 1

Z = amount from custome 2

let $g(y,z)$ be the pmf of Y, Z

$g(0,0) = P(Y=0, Z=0) = (.7)(.4) = .28$

$g(500,0) = P(Y=500, Z=0) = (.3)(.5)(.4) = .06$

$= (.7)(.6)(.5) = .21$

$g(0,500) = P(Y=0, Z=500) = (.7)\ldots$

$g(500,500) = P(Y=500, Z=500)$

$= (.3)(.5)(.6)(.5)$

$= .045$

$g(1000,0) = (.3)(.4)(.5) = .06$

$g(0,1000) = (.7)(.6)(.5) = 0.21$

$g(1000,500) = (.3)(.5)(.6)(.5) = .045$

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$g(1000,1000) = (.3)(.5)(.6)(.5) = .045$

$p(0) = .28$

$p(500) = .27$

$p(1000) = .315$

$p(1500) = .09$

$p(2000) = .045$

Ex 4

$$F(x) = \begin{cases} 1 - \exp(-x), & x > 0 \\ 0, & x \leq 0 \end{cases}$$

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$$E[X] = \int_0^\infty x e^{-x} \, dx$$

use integration by parts:

$$\int v \, du = uv - \int u \, dv$$

with $v = x$ $\quad dv = 1$

$du = e^{-x}$ $\quad u = -e^{-x}$

$$\int_0^\infty x e^{-x} dx = -e^{-x} x \Big|_0^\infty + \int_0^\infty e^{-x} dx$$

$$= 0 + \left(-e^{-x} \Big|_0^\infty\right)$$

$$= 1$$

$$E[\underline{X}^2] = \int_0^\infty x^2 e^{-x}\,dx \qquad v = x^2 \quad dv = 2x \qquad du = e^{-x} \quad u = -e^{-x}$$

$$= -x^2 e^{-x}\Big|_0^\infty + 2\int_0^\infty x e^{-x}\,dx$$

$$= 0 + 2\int_0^\infty x e^{-x}\,dx = 2 \quad \text{(from pt 1)}$$

$$Vr(X) = 2 - 1^2 = 1$$