(17) P(two heads)

$$= \frac{|HHT, THH, HTH|}{8} = 3/8$$

$P(\text{first head} \mid 2) = 2/3$

(18) $f(x) = \begin{cases} x+1, & -1 \le x \le 0 \\ 1-x, & 0 < x \le 1 \end{cases}$

$E[\cancel{f(x)}\, X] = 0$ b/c symmetric around 0

Var(X)=

$$E[X^2] = \int_{-1}^{0} x^2(x+1)\,dx + \int_{0}^{1} x^2(1-x)\,dx$$

$$= \left(\frac{x^4}{4} + \frac{x^3}{3}\right)\Bigg|_{-1}^{0} + \frac{x^3}{3} - \frac{x^4}{4}\Bigg|_{0}^{1}$$

$$= -\frac{1}{4} + \frac{1}{3} + \frac{1}{3} - \frac{1}{4} = \frac{2}{3} - \frac{1}{2} = 1/6$$

(19) $E[X] = 0.8$

$E[Y] = 0.3$

$E[XY] = 0.3$

$Cov(X,Y) = 0.3 - 0.3(.8)$

$= .06$

(20)

$X_1, \ldots, X_n$

$$L(\alpha \mid X_1, \ldots, X_n) = \alpha^n \prod \frac{1}{x_i^{\alpha+1}}$$

$$\ell(\alpha) = n\log(\alpha) - (\alpha+1)\sum \log(x_i)$$

$$\frac{\partial \ell}{\partial \alpha} = \frac{n}{\alpha} - \sum \log(x_i)$$

$$\hat{\alpha} = \frac{n}{\sum \log(x_i)}$$

(21)

$$\bar{x} \pm t_{.025,15} \cdot \frac{2}{4}$$

$$1 \pm 2.13 \cdot \frac{1}{2} \qquad (-.066, 2.066)$$

(22) critical region

$$|\bar{x}| > 1.96 \cdot \frac{1}{4} = 0.49$$

p-value: 3.1×10^{-5}

or equivalently CI for μ is

$$\bar{x} \pm (1.96)(1/4) = (.51, 1.49)$$

which does not contain zero

(23) Mention:

correlation / overfitting

$\sum |\beta_i|$ vs $\sum \beta_i^2$

sparse LASSO coefficients / variable selection

(24) Need to mention the idea that the parameter is within CI for 95% of samples