$$X_1, \ldots, X_n \sim f(x \mid \theta)$$

$$(L, U)$$

$(1-\alpha) 100\%$ CI

$(1-\alpha) 100\%$ (

all samples from th
distribution of size n
th parameter $\theta \in (L, U)$

$(3.7, 4.1)$ 4.2

Couple of ways of calculating confidence
intervals if our data is normal
distributed

$$z_\alpha \qquad t_{\alpha, n}$$

z_α is the value where $P(Z \geq z_\alpha) = \alpha$

$t_{\alpha, n}$ is the equivalent for a T dist'n

with n degrees of freedom

$$Z_\alpha = \text{scipy.stats.norm.ppf}(1-\alpha)$$

$$t_{\alpha,n} = \text{scipy.stats.t.ppf}(1-\alpha, n)$$

Suppose $X_1, \ldots, X_n \sim N(\mu, \sigma)$ $\underline{\underline{\sigma \text{ known}}}$

$\underline{\underline{(1-\alpha)\,100\%}}$ CI for μ

$$\left(\bar{x} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \,,\, \bar{x} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \right)$$

if $\underline{\underline{\sigma}}$ unknown

$S = $ sample SD

$$= \sqrt{\frac{1}{n-1} \sum_{i=1}^{\hat{n}} (X_i - \bar{X})^2}$$

$$\left(\bar{x} - t_{\alpha/2, n-1} \frac{S}{\sqrt{n}} \,,\, \bar{x} + t_{\alpha/2, n-1} \frac{S}{\sqrt{n}} \right)$$

Bootstrap

4 zero A+

3 one $\bar{x} = 1.7$

1 two

1 three

1 seven
=====

$n = 10$

m bootstrp samples of size n

resample dete with replacement
man times

generate random for

0, 1, 2, 3, 7

$p(0) = 4/10$

$p(1) = 3/10$

$p(2) = 1/10$

$p(3) = 1/10$

$p(7) = 1/10$

each group of size n ⟹ calculate the
sample mean

$(1-\alpha)100\%$ CI

$\alpha/2$ percentile of the bootstrp sample means

$1-\alpha/2$ percentile of the bootstrp simple
means

$0\ 0\ 0\ 0\ 0\ 0\ 7\ 7\ 3 \longrightarrow 1.7$

$0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 2\ 2 \longrightarrow 0.7$

scipy.stats.norm.ppf$(1-\alpha/2)$

$\alpha = .05$

$\alpha/2 = .025$

$\bar{x} = 1$

$S = 2$

for ucb of σ

$\boxed{Z_{\frac{\alpha}{2}} = 1.96}$

$\boxed{t_{\alpha/2,\ 10}} = 2.29$

known $\longrightarrow \left(1 - 1.96 \cdot \dfrac{2}{\sqrt{11}}\ ,\ 1 + 1.96 \cdot \dfrac{2}{\sqrt{11}} \right)$

$\left(1 - 2.29 \cdot \dfrac{2}{\sqrt{11}}\ ,\ 1 + 2.29 \cdot \dfrac{2}{\sqrt{11}} \right)$