Assignment 4: Hypothesis Testing

Due March 26 11:59pm

Problem 1

Let $X_1, ..., X_n \sim N(\mu, \sigma^2)$. We can test:

$$H_0 : \sigma \leq 1$$
$$H_A : \sigma > 1$$

By noting that $(n-1)S^2 \sim \chi^2_{n-1}$ under H_0, the level α test is:

$$S^2 > \frac{1}{n-1}\chi^2_{\alpha,n-1}$$

Calculate the power of this test if $\alpha = .05$, $n = 10$ and $\sigma = 2$ by simulation. You can do so by generating m samples of size n from a $N(\mu, 2)$, where m is a reasonably large number, say 1000. For each such sample, you would calculate S^2, and check if it is in critical region discussed above. Your estimate of the power would be the proportion of the samples in which reject H_0.

Problem 2

Load the data from "normal_samples.csv." Suppose $X_1, ..., X_n \sim N(\mu, \sigma^2)$. Using the formulas from class, test the null hypothesis that $\mu = 0$, against the alternative that $\mu \neq 0$ under the assumption that $\sigma = 0.3$, and under the assumption that σ is unknown. Use $\alpha = .05$.

Problem 3

Use bootstrapping to test whether the sample mean of the data in "accidents.csv" is less than or equal to 2.5, against the alternative that is greater than or equal to 2.5 (you do not need to assume a parametric model). Calculate the p-value using your boostrap samples. Use $\alpha = .05$.

More specifically, you should write a function that takes in data, μ_0 (the expectation under the null hypothesis), the number of bootstrap samples, and the level of the test α. It will return whether or not it reject H_0 (true/false), and a p-value. Then pass in the data etc.