Suppose I have two data sets:

$$\begin{cases} X_1, \dots, X_n \sim N(\mu_1, \sigma_1) \\ Y_1, \dots, Y_m \sim N(\mu_2, \sigma_2) \end{cases}$$

given som $\alpha \in (0,1)$

$\Longrightarrow$ How do I calculate $(1-\alpha)100\%$ CI for $\mu_1 - \mu_2$

Ex Testing the impact of a drug on cholesterol levels

X_i = the chge in cholestrol level for patient i in the treatment group

Y_i = the chg in cholestrol for patint i in the placebo arm

$\Longrightarrow$ calculating the estimate effect of the statin on cholesterol

$$\hat{\mu_1} - \hat{\mu_2} = \bar{X} - \bar{Y}$$

confidence interval for the difference of this

$$\bar{X} \sim N\left(\mu_1, \frac{\sigma_1}{\sqrt{n}}\right) \qquad \bar{Y} \sim N\left(\mu_2, \frac{\sigma_2}{\sqrt{m}}\right)$$

$$\bar{X} - \bar{Y} \sim N\left(\mu_1 - \mu_2, \sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}\right)$$

σ_1, σ_2 are known

$$P\left(-Z_{\alpha/2} < \frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}} < Z_{\alpha/2}\right) = 1 - \alpha$$

$$P\left(\mu_1 - \mu_2 \in \bar{X} - \bar{Y} \pm Z_{\alpha/2} \cdot \sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}}\right) = 1 - \alpha$$

$$X_1, \ldots, X_n \sim N(\mu, \sigma^2)$$

$$\bar{X} \pm Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Case $\sigma_1 = \sigma_2 = \sigma$ unknown

$$S_p^2 = \frac{(n-1)S_x^2 + (m-1)S_Y^2}{n+m-2}$$

$t_{\alpha,n}$: $P(T_n \geq t_{\alpha,n}) = \alpha$

$$\bar{X} - \bar{Y} \pm t_{\alpha/2,\, n+m-2} \cdot S_p \cdot \sqrt{1/n + 1/m}$$

Case 3: $\sigma_1 \neq \sigma_2$ σ_1, σ_2 unknown

$\rightarrow$ I wouldn't even bother doing

# of bootstrap samples I want is J

generate J bootstrap samples of size n from $X_1, \ldots, X_n$

generate J bootstrap samples of size m from $Y_1, \ldots, Y_m$

$\bar{X}_i^{BS}$ = the sample mean from BS i of the X's

$\bar{Y}_i^{BS}$ = the sample mean from BS i of the Y's

$$U_i = \bar{X}_i^{BS} - \bar{Y}_i^{BS}$$

$(1-\alpha)100\%$ CI

$\alpha/2$ - percentile

$1-\alpha/2$ percentile

Hypothesis Testing

Given $X_1, \ldots, X_n \sim f(x \mid \theta)$

↳ what is our best guess for the
value of θ?
point estimate

↳ what is a reasonable set of
possible vals for θ?
confidence intervals

↳ Can we rule out certain values of θ?
hypothesis testing

Suppose I am the manufacturer of
the statin, running clinical trial

↳ prove to FDA that drug has

→ prove
a positive effect on cholesterol level

↳ proving that it doesn't have no effect or a negative effect

Postulate "null hypothesis"

↳ the thing we hope to disprove

point $H_0: \mu = k \ (= 0)$ $H_A: \mu \neq 0$

compound $H_0: \mu \geq 0$ $H_A: \mu < 0$

$\mu \leq 0$ $H_A: \mu > 0$

⇒ how do we test a hypothesis

↳ given a test statistic, we reject H_0 if our test statistic is sufficiently unlikely to have happened if H_0 were true

↳ in our previous (stations)

$H_0: \mu \geq 0$

$\bar{X} = -20$

↳ this will depend on the sample size, the variability σ, the level of

↳ the level of the test $\alpha \in (0,1)$ is the probability of rejecting H_0 by accident if it is in fact true

Let's suppose that

$X_1, \dots, X_n \sim N(\mu, \sigma)$ σ known

$H_0: \mu = k$ test this at level

$H_A: \mu \neq k$

under H_0: $\bar{X} \sim N(k, \frac{\sigma}{\sqrt{n}})$

$$\frac{\bar{X} - k}{\sigma/\sqrt{n}} \sim N(0,1)$$

prob outside $= \alpha$

$$P\left(-z_{\alpha/2} < \frac{\bar{X}-k}{\sigma/\sqrt{n}} < z_{\alpha/2}\right) = 1-\alpha$$

$$P\left(\bar{X} \in \left(k \pm \frac{\sigma}{\sqrt{n}} z_{\alpha/2}\right)\right) = 1-\alpha$$

under H_0: $P(\bar{X}$ is outside of this interval$) \leq \alpha$

reject H_0

if $\bar{x} > k + \frac{\sigma}{\sqrt{n}} z_{\alpha/2}$

$\bar{x} < k - \frac{\sigma}{\sqrt{n}} z_{\alpha/2}$

$\alpha = .05$ $\quad z_{\alpha/2} = 1.96$

$\sigma = 10$ $\quad k = 0$

$n = 100$

$\bar{x} = -10$ $\quad 0 \pm \frac{10}{10} \cdot 1.96$

reject H_0 if

$\underline{\bar{x} < -1.96}$

or $\bar{x} > 1.96$

reject H_0.

given our sample size (100)

variability in data (10)

-10 is a very extreme value for the sample mean

if $\mu = 0$

reject H_0 if $\bar{x} < -1.96$

$\bar{x} > 1.96$

we only end with false positive 5% of the time