

Sorting Algorithms and Complexity

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Outline

- 1 Intuitive Sorting
- 2 Sorting Algorithms
- 3 Time Complexity

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Activity: Sort your cards

- 1 Layout the cards in front of you.
- 2 Using some technique which looks at one or two cards at once, put the cards into sorted order.
- 3 Repeat this a few times, until you are conscious of how you do it.

Activity: How did you sort?

- 1 Think about how you sorted your cards.
- 2 Try to write down the general idea about how it was done.

Activity: Produce Pseudocode of your technique

- 1 Create a directory `labs/week10`
- 2 In this directory, using your favorite editor, create a file named `mysort.txt`
- 3 Write pseudocode to describe the sorting method that you used.
- 4 Compare sorting methods with your neighbors. Who's sort looks better?

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Sorting Algorithms

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- Many “official” sorting algorithms exist.
- These include:
 - Selection Sort
 - Bubble Sort
 - Merge Sort

Selection Sort

```
selection_sort(ar)
  for i = 0 to ar
    min=i
    for j = i+1 to ar.size()-1
      if ar[j] < ar[min]
        min = j
      end if
    end for
    swap ar[i] and ar[min]
  end for
```

- Carry out selection sort with your cards.

Selection Sort

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- Carry out selection sort with your cards.
- Intuitively, what is selection sort doing?

Bubble Sort

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bubble_sort(ar)
do
    swapped = false
    for i=0 to ar.size()-2
        if ar[i+1] < ar[i]
            swap ar[i+1] and ar[i]
            swapped = true
        end if
    end for
while swapped
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    end for
while swapped
```

- Carry out selection sort with your cards.
- Intuitively, what is bubble sort doing?

Merge Sort

```
merge_sort(ar)
  if ar.size() <= 1
    return
  end if

  mid = ar.size() / 2
  merge_sort(ar[0..mid])
  merge_sort(ar[mid+1 .. ar.size()-1])
  merge(ar[0..mid], ar[mid+1 .. ar.size()-1])
```

```
merge(left, right)
  While left and right are not empty
    take the smallest of the first element in
    left and right
  end while
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- 1 Which sorting method seemed closer to what you did?
- 2 Which sorting method seemed more efficient?

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- We typically evaluate an algorithm based on how long it will take to execute.
- The standard is to rate the algorithm by the number of steps necessary to solve a problem of n size.

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- We instead compute an asymptotic bound.
- An algorithm is $O(g(n))$ if $f(n) \leq cg(n)$ for some constant c for all n above some threshold.
- Intuitively, $O(g(n))$ gives us the “worst case” run time.

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 - Exponential - $O(2^n)$
- Using a spreadsheet, let's see how these functions relate to each other.

Selection Sort Complexity

Find the time complexity of the following:

```
selection_sort(ar)
  for i = 0 to ar
    min=i
    for j = i+1 to ar.size()-1
      if ar[j] < ar[min]
        min = j
      end if
    end for
    swap ar[i] and ar[min]
  end for
```


Bubble Sort Complexity

Find the time complexity of the following:

```
bubble_sort(ar)
do
    swapped = false
    for i=0 to ar.size()-2
        if ar[i+1] < ar[i]
            swap ar[i+1] and ar[i]
            swapped = true
        end if
    end for
while swapped
```

Merge Sort Complexity

Find the time complexity of the following:

```
merge_sort(ar)
    if ar.size() <= 1
        return
    end if

    mid = ar.size() / 2
    merge_sort(ar[0..mid])
    merge_sort(ar[mid+1 .. ar.size()-1])
    merge(ar[0..mid], ar[mid+1 .. ar.size()-1])

merge(left, right)
    While left and right are not empty
        take the smallest of the first elements
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```

Activity: What is the complexity of your search algorithm?

- 1 Compute the time complexity of your own sorting algorithm.
- 2 Include your runtime and justification of your complexity in your text file.

Activity: Code a Sorting Algorithm

- ➊ Code your favorite sorting algorithm.
- ➋ Your program should do the following:
 - ➊ Ask for 10 numbers, which it adds to a vector.
 - ➋ Run your sorting algorithm.
 - ➌ Print the sorted vector.