03 - Grammars

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Outline

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Metalanguages and Subject Languages

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- A grammar is sometimes also called a **syntax**.

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- P A set of productions. Set of rules which transform strings of terminal and non-terminal symbols into a valid sentence.

Example Language: Simplified Email Addresses

Example Sentence:

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& < \mathrm{domain} >, < \mathrm{tld} >\}
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- **Discuss:** Does this process have anything to do with closure?

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- Let's do this for the email grammar!

Outline

Chomsky Hierarchy

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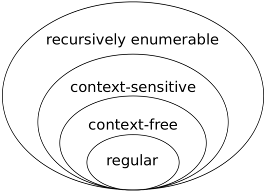

Image Source: `https://en.wikipedia.org/wiki/Chomsky_hierarchy`

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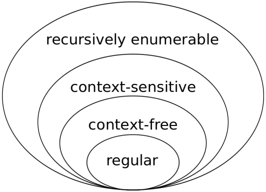

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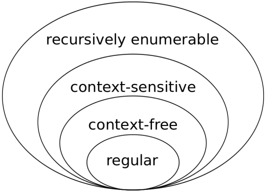

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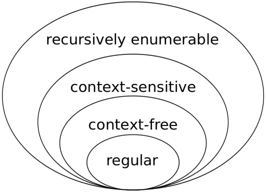

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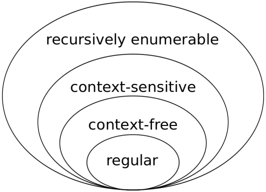

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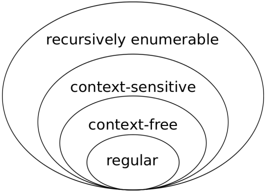
Type-2 **Context-Free**
$A \to \alpha$



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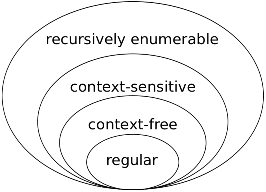
Type-3 **Regular**
$A \rightarrow a$ and $A \rightarrow aB$



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- Context-Free (Type-2) grammars will be our focus. Even in context-sensitive languages, the syntax is often expressed first as a context free grammar with additional constraints applied by the compiler.
- Regular Grammars are too limited to express general programming languages. They are typically useful for searching and general pattern matching.

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- Replacement arrows are $::=$
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- Terminal strings are enclosed in single quotes.

Example Grammar

$\langle S \rangle$ ::= <Expression>

$\langle \mathit{Expression} \rangle$ ::= <Term> | <Expression> ‘+’ <Term>

$\langle \mathit{Term} \rangle$ ::= <Factor> | <Term> ‘*’ <Factor>

$\langle \mathit{Factor} \rangle$ ::= <Unit> | (<Expression>)

$\langle \mathit{Unit} \rangle$ ::= ‘0’|‘1’|‘2’|‘3’|‘4’|‘5’|‘6’|‘7’|‘8’|‘9’

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- Named in honor of Henry Ledgard, author of *Programming Language Landscapes*.
- Essentially contains “just enough” of the elements of programming languages to explore compiler creation.

Ledgard Syntax

$\langle$*program*$\rangle$ ::= ‘program’ <decl-list> ‘begin’ <stmt-list> ‘end’ ‘;’

$\langle$*decl-list*$\rangle$ ::= <declaration> | <decl-list> <declaration>

$\langle$*declaration*$\rangle$ ::= <identifier-list> ‘:’ <type> ‘;’

$\langle$*identifier-list*$\rangle$ ::= <identifier> | <identifier-list> ‘,’ <identifier>

$\langle$*type*$\rangle$ ::= <simple-type> | <array-type>

$\langle$*simple-type*$\rangle$ ::= ‘integer’ | ‘boolean’

$\langle$*array-type*$\rangle$ ::= ‘array’ ‘[’ <bounds> ‘]’ ‘of’ <type>

$\langle$*bounds*$\rangle$ ::= <integer-literal> ‘..’ <integer-literal>

Ledgard Syntax (continued)

⟨*stmt-list*⟩ ::= <statement> | <stmt-list> <statement>

⟨*statement*⟩ ::= <assignment-stmt> | <exchange-stmt> |
<if-stmt> | <loop-stmt> | <input-stmt> | <output-stmt>

⟨*assignment-stmt*⟩ ::= <variable> ':=' <expression> ';'

⟨*exchange-stmt*⟩ ::= <variable> ':=:' <variable> ';'

⟨*if-stmt*⟩ ::= 'if' <expression> 'then' <stmt-list> 'end' 'if' ';'
| 'if' <expression> 'then' <stmt-list> 'else' <stmt-list> 'end' 'if' ';'

⟨*loop-stmt*⟩ ::= 'while' <expression> 'loop' <stmt-list> 'end' 'loop' ';'

Ledgard Syntax (continued)

⟨*input-statement*⟩ ::= ‘input’ <variable-list> ‘;’

⟨*output-statement*⟩ ::= ‘output’ <variable-list> ‘;’

⟨*variable-list*⟩ ::= <variable> | <variable-list> ‘,’ <variable>

⟨*expression*⟩ ::= <operand> | <operand> <operator>
<operand>

⟨*operand*⟩ ::= <variable> | <integer-literal> | <boolean-literal> |
‘(’ <expression> ‘)’ | ‘not’ <operand>

⟨*variable*⟩ ::= <variable> | <variable> ‘[’ <expression> ‘]’

Ledgard Syntax (continued)

⟨*boolean-literal*⟩ ::= ‘true’ | ‘false’

⟨*operator*⟩ ::= ‘<’ | ‘<=’ | ‘==’ | ‘<>’ | ‘>=’ | ‘>’ | ‘+’ | ‘-’ | ‘*’ | ‘/’ | ‘and’ | ‘or’

- An integer-literal is just a string of digits 0-9.
- Comments begin with – and continue to the end of the line.