04 - Testing and Manipulating Grammars

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Outline

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1. Grammars and Recursion
2. LL(1) Grammars

Sample Grammar G

For this discussion, we will be using the following grammar (found on page 39 of your textbook):

$$
\begin{aligned}
S &\rightarrow E \\
E &\rightarrow T \mid E + T \\
T &\rightarrow F \mid T * F \\
F &\rightarrow U \mid (E) \\
U &\rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9
\end{aligned}
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4. Keep repeating the process until there are either no non-terminal candidates or until there are no non-terminals left in the sentence.

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$$\begin{array}{ll} T & 1+2*3 \\ F & 1+2*3 \\ U & 1+2*3 \\ 1 & 1+2*3 \\ \lambda & +2*3 \\ \text{Mismatch! Backtrack!} & \end{array}$$

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And there's the loop again...

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- We need some way to determine what production we must have based on the symbols being examined.

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- All of the above have a look-ahead buffer of k terminals.
- We are really interested in $\mathrm{LL}(1)$ grammars.

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- We can also have a set of symbols which immediately identify as an error if they are encountered.

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- Considering $\alpha_i = \beta_1\beta_2 \ldots \beta_r$ then $t \in \mathrm{START}(\beta_i) \implies t \in D(A \to \alpha_i)$

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