04 - Testing and Manipulating Grammars

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Outline

- Grammars and Recursion
- 2 LL(1) Grammars
- Manipulating Grammars





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Sample Grammar G

For this discussion, we will be using the following grammar (found on page 39 of your textbook):

$$S \to E$$

 $E \to T \mid E + T$
 $T \to F \mid T * F$
 $F \to U \mid (E)$
 $U \to 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$





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- Expand the next non-terminal in the sentence.
- If the target symbol does not match, backtrack and select a different non-terminal.
- Weep repeating the process until there are either no non-terminal candidates or until there are no non-terminals left in the sentence.





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$$T$$
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And there's the loop again...

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- Generally, we want parse times to be proportional to the size of the input, not exponential.
- Undoing parsing is difficult!
- We need some way to determine what production we must have based on the symbols being examined.





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- LR(k) scans from left to right, expanding left-most derivations.
- All of the above have a look-ahead buffer of *k* terminals.
- We are really interested in LL(1) grammars.



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- We can also have a set of symbols which immediately identify as an error if they are encountered.





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- The start set is START(α) = β : β <<* α
- Considering $\alpha_i = \beta_1 \beta_2 \dots \beta_r$ then $t \in START(\beta_1) \implies t \in D(A \rightarrow \alpha_i)$





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- $D(A \rightarrow A\gamma) \subseteq START(A)$





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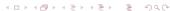
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- This function is defined recursively (where γ ∈ (N ∪ T) and δ ∈ (N ∪ T)*):

$$\begin{aligned} & \operatorname{FIRST}(\lambda) = \emptyset \\ & \operatorname{FIRST}(\gamma \delta) = \operatorname{terminals of START}(\gamma) \cup \operatorname{FIRST}(\delta) & \text{if } \gamma \stackrel{*}{\Rightarrow} \lambda \\ & \operatorname{FIRST}(\gamma \delta) = \operatorname{terminals of START}(\gamma) & \text{o.w.} \end{aligned}$$





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- the EMPTY property can be defined as follows:
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 - **1** If $\gamma \in N$ then
 - If $\exists \gamma \to \lambda$ then $\mathrm{EMPTY}(\gamma) = \mathrm{true}$
 - ② If $\exists \gamma \to \delta_1 \dots \delta_k$ where $\forall 1 \leq i \leq k$ EMPTY (δ_i) = true then EMPTY (γ) = true





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 - **6** For all other γ , EMPTY(γ) = false





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 - ② If $\exists \gamma \to \delta_1 \dots \delta_k$ where $\forall 1 \leq i \leq k$ EMPTY(δ_i) = true then EMPTY(γ) = true
 - **3** For all other γ , EMPTY(γ) = false
- Let's calculate the FIRST for the productions in G.





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- What are the complete director sets for G?





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