Church - Turing Problems

April 3, 2020

Attempt each of the following problems. For this problem, you must solve at least one easy Church problem, one easy Turing problem and one hard problem. Submit your electronic response on blackboard.

1 Easy Problems that Impress Normal People

1. **Church** Using λ-calculus, show the encoding and perform the reductions of the sum $3 + 5$. (Hint: This should result in the Church-encoding of 8.)
2. **Church** Write a λ-calculus function which produces the Fibonacci sequence. (This is not so bad if you use abbreviations, as in the tutorial.)
3. **Turing** Create the table for a Turing machine which produces the sequence *ababbabbbabbba*... That is, it prints 0 b's followed by an a, 2 b's followed by an a, then 3, 4, 5, etc.
4. **Turing** Create the table for a Turing machine which determines if its tape contains an equal number of '(' and ')' symbols. Have it write "Y" if they are balanced and "N" if they are not.

2 Problems that Impress Mathematicians

1. We know that the circular problem is unsolvable for Turing Machines, but is it solvable for physical computers? That is, for a real computer (like the one you are using now), could you create a program which determines whether another program will continue printing 1's and 0's indefinitely? Prove your answer
2. Suppose we had a Turing machine which has two tapes. Can it solve any problems that are not solvable by a single tape Turing machine? Prove your answer
3. Suppose we have a Turing machine that is able to get outside help. That is, it can stop and ask the operator a question. Would this allow this machine to solve more problems than the base model Turing machine? If

so, what class of problems can it solve that a normal TM cannot? What are its limitations? Prove your answers.