Lecture 1 - Numbers and Notation

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Outline

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Why You Are Bad at Math

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- Your textbooks were lacking in text. Lots of color, lots of problems, no substance!
- Being bad at math was socially acceptable, and you seized the opportunity because memorizing rules and procedures is boring.

A Brief History of Counting

Image Source:

https://www.maa.org/press/periodicals/convergence/mathematical-treasure-ishango-bone

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A Brief History of Counting

- Tally Marks 40,000 years old
- Ishango Bone 20,000 years old, may have been a rudimentary calculator
- Formal mathematics, as we know it today, really started about 3000 years ago

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Ancient Numeral Systems - Roman Numerals

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- Representing Numbers as Figures

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- Example: Roman Numeral System

Numerals		Transitions	
I	1		
V	5	IV	4
X	10	IX	9
L	50	XL	40
C	100	XC	90
D	500	CD	400
M	1000	CM	900

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    3. $\mathrm{XV} - \mathrm{V} = ?$
    4. $\mathrm{V} - \mathrm{I} = ?$
    5. $\mathrm{V} \times \mathrm{IV} = ?$

The Arabic/Indian Numeral System

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- Digits 0-9
- Positional value system

10^3	10^2	10^1	10^0

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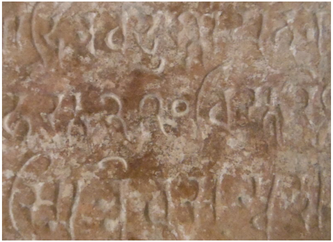

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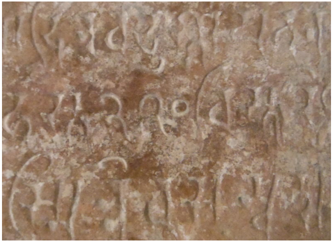

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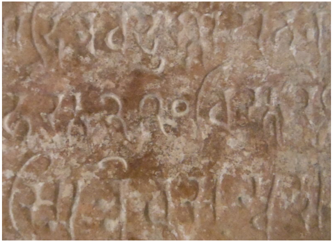

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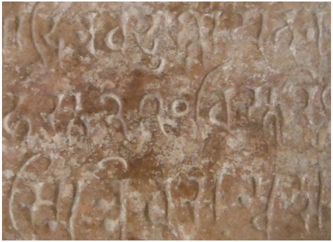

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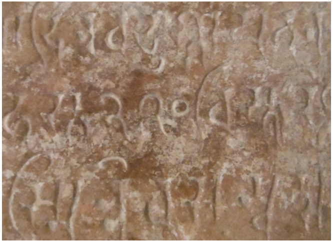

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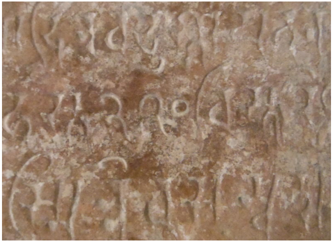

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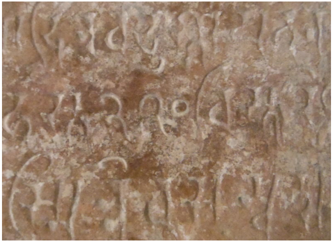

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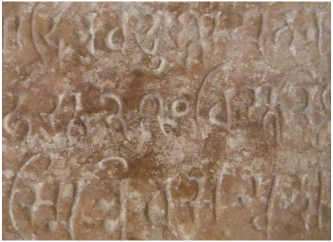
  2. $3 + 1 = ?$
  3. $15 + 5 = ?$
  4. $5 - 1 = ?$
  5. $5 \times 4 = ?$



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Fundamental Operations of Arithmetic

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- Alternate notations for multiplication: 3×5, $3 \cdot 5$, $3(5)$, $3 * 5$
- Alternate notations for division: $4 \div 2$, $\frac{4}{2}$, $2\overline{)4}$, $4/2$

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  $$\begin{array}{cccc} P & E & \begin{array}{c} M \\ D \end{array} & \begin{array}{c} A \\ S \end{array} \end{array}$$

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- Example: $3^2 + 4 \times 2 - 16 \div (2 + 2)$

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- Small numbers of 0's between the decimal point and nonzero digits. This is effectively dividing by 10:

$$0.0000012 = 1.2 \times 10^{-6}$$