04 - Ratio and Proportion

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Outline

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A Complex Sounding Problem

If 8 workers in 24 days working 10 hours a day can reap 48 acres of wheat, how many acres could 12 workers reap in 20 days of 12 hours each?

Comparing and Relating Quantities

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- Ratios express the relationship between two concrete quantities.

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- A ratio is analgous to a fraction, thus $1 : 2$, $1 \div 2$, $\frac{1}{2}$, and 0.5 are all the same ratio.

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- Ratios are abstract numbers. Why?

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2. Proportion

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- We often use the word “in proportion” to describe two equal ratios.
- Proportions are the mathematical equivalent of analogies: a is to b as c is to d.

Proportion Notation and Properties

- There are two main ways to write proportions $a : b :: c : d$ or $a : b = c : d$ where a, b, c, and d are the numbers which make up the proportion. Example: $1 : 2 :: 2 : 4$ or $1 : 2 = 2 : 4$.

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- Examples: $1 : 2 :: 2 : 4$, $1 : 3 :: 6 : 18$
- Discuss: Why must these two products be the same?

The Rule of Three

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38 &\approx x
\end{aligned}
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Example 2

A besieged town, containing 22,400 inhabitants, has provisions to last 3 weeks; how many must be sent away that they may be able to hold out 7 weeks? Transcribed from: *A Treatise on Arithmetic* by J. H. Smith. 1878

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12,800 people must be sent away.

Compound Proportions

- A compound proportion is a set of three or more ratios given where one is incomplete.
- You produce solutions to compound proportions by multiplying corresponding terms together, and then solve as in a simple proportion.
- Our opening example is a compound proportion.

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