## Probabilities and Games

## November 1, 2019

## 1 Probability

- Probability is a way to measure the likelihood of a random event.
- We start with a set of events, E. For instance, for rolling a standard six sided die:  $E = \{ \boxdot, \boxdot, \boxdot, \boxdot, \boxdot, \boxdot, \boxdot, \boxdot \}$
- The probability of any event e in E is computed by:

$$P(e) = \frac{\text{number of occurrences}}{\text{number of trials}}$$

- We often express probabilities as ratios, decimals, or percents.
- The probability of every event in a set of outcomes is called the probability distribution of the set.
- If each outcome is equally likely, we call the distribution a uniform probability distribution.
- The dice example is a uniform distribution, with each number having a 1/6 probability of occurring.
- The sum of all the probabilities in a distribution is always 1. Why?

# 2 The Game of Craps

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	2	3	4	5	6	7
	3	4	5	6	7	8
<b>.</b>	4	5	6	7	8	9
	5	6	7	8	9	10
	6	7	8	9	10	11
	7	8	9	10	11	12

- Craps is played with a pair of dice.
- There is a designated shooter who rolls the dice.
- The shooter will have one of two outcomes "Pass" (win) or "Don't Pass" (lose).
- The other players bet on the outcome. (There are other bets in casino play, but we will focus on the simple win/lose outcomes.)
- Each round consists of two phases, the come-out roll and the point phase.

### 2.1 Come-Out Roll

- The shooter rolls the dice once.
- A roll of 7 or 11 is a "natural" and the pass line wins.
- A roll of 2, 3, or 12 is "crapping out" and the don't pass line wins.
- Any other roll sets the point value. The dealer marks the value of the roll, and then play proceeds to the point phase.

#### 2.2 Point Phase

- The shooter rolls the dice.
- If the shooter rolls the point value established in the come-out roll ("hit"), the "pass" line wins and the round ends.
- If the shooter rolls a 7 ("seven-out"), the "don't pass" line wins and the round ends.
- If the shooter rolls any other number, the point phase continues and they roll again.

### 2.3 Exercises

- 1. What is the probability of rolling a natural?
- 2. What is the probability of crapping out?
- 3. What is the probability of proceeding to the point phase?
- 4. What is the probability of a seven-out?
- 5. What is the probability of hitting each of the possible point values?
- 6. In groups of three, play a few rounds of craps. Log the outcome of each game. Do your observations match your predictions?
- 7. Do the following names of the craps rolls make sense?

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	Snake Eyes					
	Ace Deuce	Hard Four				
	Easy Four	Fever Five	Hard Six			
<b>B</b>	Fever Five	Easy Six	Natural	Hard Eight		
₩	Easy Six	Natural	Easy Eight	Nina	Hard Ten	
<b>B</b>	Natural	Easy Eight	Nina	Easy Ten	Yo-leven	Boxcars or Midnight