Lecture 1 - Numbers and Notation

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1 Quantitative Language

1.1 Why you are bad at math

- You have been taught a litany of rules and procedures, but no ideas.
- Your textbooks were lacking in text. Lots of color, lots of problems, no substance!
- Being bad at math was socially acceptable, and you seized the opportunity because memorizing rules and procedures is boring.

1.2 A Brief History of Counting

- Tally Marks 40,000 years old
- Ishango Bone 20,000 years old, may have been a rudimentary calculator
- Formal mathematics, as we know it today, really started about 3000 years ago

1.3 Ancient Numeral Systems

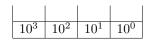
- Representing Numbers as Figures
- Example: Roman Numeral System

Numerals		Transitions	
I	1		
V	5	IV	4
X	10	IX	9
L	50	XL	40
С	100	XC	90
D	500	CD	400
Μ	1000	CM	900

- Arithmetic was usually done with some sort of manipulative aid (counting board, abacus, etc).
- Roman numeral arithmetic is difficult. (Let's Try it)
 - 1. I + I = ?
 - 2. III + I = ?
 - 3. XV V = ?
 - 4. V I = ?
 - 5. $V \times IV = ?$

1.4 The Arabic/Indian Numeral System

- Introduced to the Western world by Al-Kwharizmi, but was invented in India
- Digits 0-9
- Positional value system



- Works very well for arithmetic!
 - 1. 1+1=?
 - $2. \ 3 + 1 = ?$
 - 3. 15 + 5 = ?
 - 4. 5-1=?
 - 5. $5 \times 4 = ?$

2 Evaluating Expressions

2.1 Fundamental Operations of Arithmetic

- Fundamental operations: $+, -, \times, \div$
- Alternate notations for multiplication: 3×5 , $3 \cdot 5$, 3(5), $3 \cdot 5$
- Alternate notations for division: $4 \div 2, \frac{4}{2}, 2)4, 4/2$

2.2 Order of Operations and Reduction

- Convention PEMDAS Parenthesis, Exponent, Multiply, Divide, Add, Subtract
- Multiplication and Division are the same operation, so is Add and Subtract

$$P \quad E \quad \begin{array}{cc} M & A \\ D & S \end{array}$$

Ties are broken left to right

• Example: $3^2 + 4 \times 2 - 16 \div (2+2)$

2.3 Scientific Notation

- Writing very large or very small numbers is very error prone.
- We usually only really care about the first few values (more on this later).
- Base 10 gives us a way to do this!
- Large numbers have 0's at the right hand side. This is effectively multiplying by 10. So we can use exponents:

$$1,200,000 = 1.2 \times 10^6$$

• Small numbers of 0's between the decimal point and nonzero digits. This is effectively dividing by 10:

$$0.0000012 = 1.2 \times 10^{-6}$$