Lecture 1 - Numbers and Notation

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Outline

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Why You Are Bad at Math

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- Being bad at math was socially acceptable, and you seized the opportunity because memorizing rules and procedures is boring.

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- Formal mathematics, as we know it today, really started about 3000 years ago

Ancient Numeral Systems - Roman Numerals

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- Representing Numbers as Figures

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- Representing Numbers as Figures
- Example: Roman Numeral System

Numerals		Transitions	
I	1		
V	5	IV	4
X	10	IX	9
L	50	XL	40
C	100	XC	90
D	500	CD	400
M	100	CM	900

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    3. $\mathrm{XV} - \mathrm{V} = ?$
    4. $\mathrm{V} - \mathrm{I} = ?$
    5. $\mathrm{V} \times \mathrm{IV} = ?$

The Arabic/Indian Numeral System

Image Source:

`https://www.mathematics-monster.com/glossary/Al-Khwarizmi.html`

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The Arabic/Indian Numeral System

- Introduced to the Western world by Al-Khwarizmi, but was invented in India
- Digits 0-9
- Positional value system

10^3	10^2	10^1	10^0

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The Arabic/Indian Numeral System

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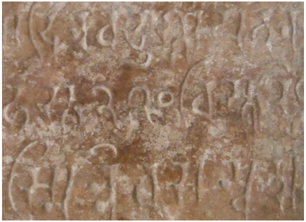

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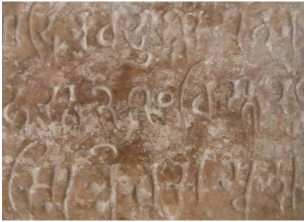

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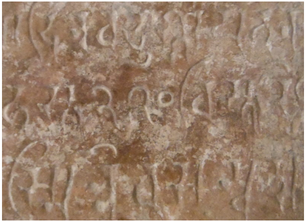

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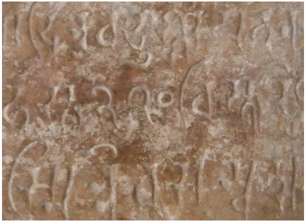

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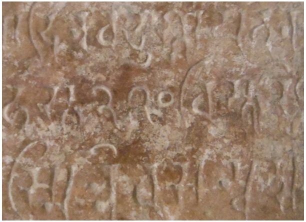

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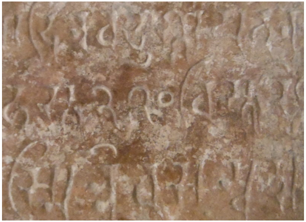

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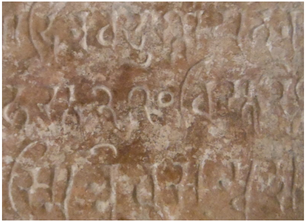

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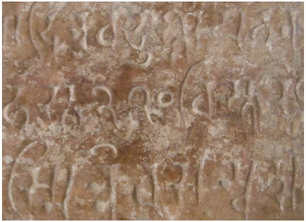
  2. $3 + 1 = ?$
  3. $15 + 5 = ?$
  4. $5 - 1 = ?$
  5. $5 \times 4 = ?$



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- Alternate notations for division: $4 \div 2$, $\frac{4}{2}$, $2\overline{)4}$, $4/2$

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  $$P \quad E \quad \begin{matrix} M \\ D \end{matrix} \quad \begin{matrix} A \\ S \end{matrix}$$

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- Example: $3^2 + 4 \times 2 - 16 \div (2 + 2)$

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- Small numbers of 0's between the decimal point and nonzero digits. This is effectively dividing by 10:

$$0.0000012 = 1.2 \times 10^{-6}$$