Graphing and Projection Using Linear Functions

February 10, 2020

Linear Functions

- Recall that a linear pattern can be written recursively:

$$P_n = P_{n-1} + (P_2 - P_1)$$

- Suppose we carry this out for some arbitrary number of iterations:

$$\begin{aligned}
&P_x = P_{x-1} + (P_2 - P_1)\\
&P_x = (P_{x-2} + (P_2 - P_1)) + (P_2 - P_1)\\
&P_x = ((P_{x-3} + (P_2 - P_1)) + (P_2 - P_1)) + (P_2 - P_1)\\
&\dots\\
&P_x = ((P_1 + \dots + (P_2 - P_1)) + (P_2 - P_1)) + (P_2 - P_1)
\end{aligned}$$

- Regrouping by using the associative property of addition, we get:

$$P_x = P_1 + (P_2 - P_1) + \dots + (P_2 - P_1)$$

  Where $(P_2 - P_1)$ is repeated x times.

- By the definition of multiplication, this gives us :

$$P_x = P_1 + x(P_2 - P_1)$$

- Moreover, because each iteration increases n by 1, we can make the claim that $P_2 - P_1$ is the slope because:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- This gives us:

$$P_x = P_1 + xm$$

- If we want to standardize the starting point to be where $x = 0$ on a cartesian plane, we can do this as:

$$P_x = b + xm$$

Where $b = P_0$.

- This gives us the familiar equation of a line:

$$\boxed{y = mx + b}$$

- Of course, b is rarely given to us, so we must solve for this:

$$\begin{aligned} y &= mx + b \\ y - b &= mx \\ -b &= mx - y \\ b &= -mx + y \end{aligned}$$

Therefore:

$$\boxed{b = y - mx}$$

- We find b by substituting an arbitrary (x, y) pair into the above.
- Thus we can write a linear pattern as a function:

$$\boxed{f(x) = mx + b}$$

Graphing Linear Funcions

- To graph a linear function, simply choose two points and draw them on the graph.
- Draw a straight line through both points.
- This will project the linear function backwards and forewards.
- We can also make predictions based on the numeric value of the linear function.
- Also, we can use a spreadsheet to find the equation of a line from a data series by using the "trendline" feature.