

# Lecture 1 - Numbers and Notation

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# Outline

1 Quantitative Language

2 Evaluating Expressions

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2 Evaluating Expressions

# Why You Are Bad at Math

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- Your textbooks were lacking in text. Lots of color, lots of problems, no substance!
- Being bad at math was socially acceptable, and you seized the opportunity because memorizing rules and procedures is boring.

# A Brief History of Counting



Image Source:

[https://www.maa.org/press/  
periodicals/convergence/  
mathematical-treasure-ishango-bone](https://www.maa.org/press/periodicals/convergence/mathematical-treasure-ishango-bone)



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# A Brief History of Counting

- Tally Marks 40,000 years old
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- Formal mathematics, as we know it today, really started about 3000 years ago



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# Ancient Numeral Systems - Roman Numerals

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- Example: Roman Numeral System

Numerals		Transitions	
I	1		
V	5	IV	4
X	10	IX	9
L	50	XL	40
C	100	XC	90
D	500	CD	400
M	1000	CM	900



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  - 3  $XV - V = ?$
  - 4  $V - I = ?$
  - 5  $V \times IV = ?$

# The Arabic/Indian Numeral System



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- Digits 0-9
- Positional value system

$10^3$	$10^2$	$10^1$	$10^0$

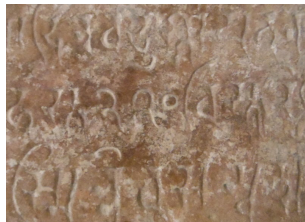


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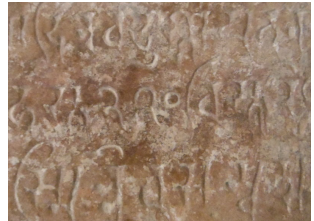
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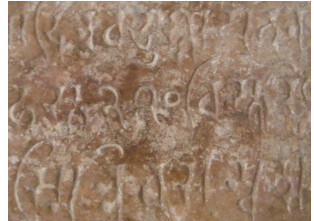


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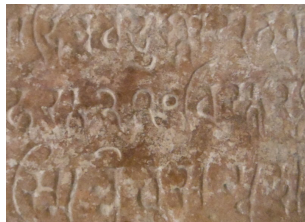
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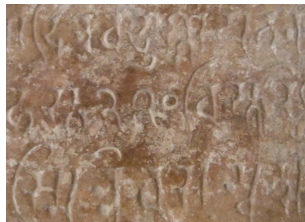


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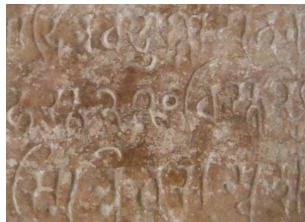




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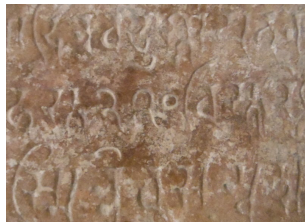
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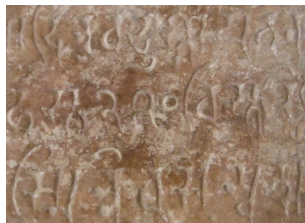
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- 2  $3 + 1 = ?$
- 3  $15 + 5 = ?$
- 4  $5 - 1 = ?$
- 5  $5 \times 4 = ?$



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- Alternate notations for multiplication:  $3 \times 5$ ,  $3 \cdot 5$ ,  $3(5)$ ,  $3 * 5$
- Alternate notations for division:  $4 \div 2$ ,  $\frac{4}{2}$ ,  $2\overline{)4}$ ,  $4/2$



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- Example:  $3^2 + 4 \times 2 - 16 \div (2 + 2)$

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- Small numbers of 0's between the decimal point and nonzero digits. This is effectively dividing by 10:

$$0.0000012 = 1.2 \times 10^{-6}$$