Lecture 1 - Numbers and Notation

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1 Quantitative Language

1.1 Why you are bad at math

- You have been taught a litany of rules and procedures, but no ideas.
- Your textbooks were lacking in text. Lots of color, lots of problems, no substance!
- Being bad at math was socially acceptable, and you seized the opportunity because memorizing rules and procedures is boring.

1.2 A Brief History of Counting

- Tally Marks 40,000 years old
- Ishango Bone 20,000 years old, may have been a rudimentary calculator
- Formal mathematics, as we know it today, really started about 3000 years ago

1.3 Ancient Numeral Systems

- Representing Numbers as Figures
- Example: Roman Numeral System

Numerals		Transitions	
I	1		
V	5	IV	4
X	10	IX	9
L	50	XL	40
C	100	XC	90
D	500	CD	400
M	1000	CM	900

- Arithmetic was usually done with some sort of manipulative aid (counting board, abacus, etc).
- Roman numeral arithmetic is difficult. (Let's Try it)
  1. I $+$ I $=$?
  2. III $+$ I $=$?
  3. XV $-$ V $=$?
  4. V $-$ I $=$?
  5. V $\times$ IV $=$?

1.4 The Arabic/Indian Numeral System

- Introduced to the Western world by Al-Kwharizmi, but was invented in India
- Digits 0-9
- Positional value system

10^3	10^2	10^1	10^0

- Works very well for arithmetic!
  1. $1 + 1 =?$
  2. $3 + 1 =?$
  3. $15 + 5 =?$
  4. $5 - 1 =?$
  5. $5 \times 4 =?$

2 Evaluating Expressions

2.1 Fundamental Operations of Arithmetic

- Fundamental operations: $+$, $-$, $\times$, $\div$
- Alternate notations for multiplication: 3×5, $3 \cdot 5$, $3(5)$, $3 * 5$
- Alternate notations for division: $4 \div 2$, $\frac{4}{2}$, $2\overline{)4}$, $4/2$

2.2 Order of Operations and Reduction

- Convention PEMDAS - Parenthesis, Exponent, Multiply, Divide, Add, Subtract
- Multiplication and Division are the same operation, so is Add and Subtract
  $$\begin{array}{cccc} \text{P} & \text{E} & \begin{array}{c}\text{M}\\ \text{D}\end{array} & \begin{array}{c}\text{A}\\ \text{S}\end{array}\end{array}$$
  Ties are broken left to right
- Example: $3^2 + 4 \times 2 - 16 \div (2 + 2)$

2.3 Scientific Notation

- Writing very large or very small numbers is very error prone.
- We usually only really care about the first few values (more on this later).
- Base 10 gives us a way to do this!
- Large numbers have 0's at the right hand side. This is effectively multiplying by 10. So we can use exponents:
  $$1,200,000 = 1.2 \times 10^6$$
- Small numbers of 0's between the decimal point and nonzero digits. This is effectively dividing by 10:
  $$0.0000012 = 1.2 \times 10^{-6}$$