Linear Growth

February 17, 2020

Explanatory and Response Variables

- Oftentimes, we study a system based on its inputs and its outputs.
- An **explanatory** variable is a variable which explains some effect.
- A **response** variable is a variable which quantifies a response.
- We typically want to model how a a response variable changes when the explanatory variable change.
- We use the predict future behavior of something under study.

Linear Patterns

- To explore and model data, we look for patterns.
- A linear pattern is one where the ratio of the change in the response variable to the change in the explanatory variable remains constant.
- Suppose we have response variable y and explanatory variable x. If we pick two entries (x_1, y_1) and (x_2, y_2), then for all pairs of values, the ratio must remain the same:
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
- For example, are the following patterns linear?

x	y	x	y
1	20	1	20
2	25	2	25
4	35	4	39
6	45	6	61

Recursive Relationship

- A linear pattern can be written as a recursive relationship. The relationship is the sum of the previous value and then the constant change in the response variable.
- This can be written as follows:

$$P_n = P_{n-1} + (y_2 - y_1)$$

- Example: Write the previous linear pattern as a recursive relationship.
- Recursive relationships work well for conceptualizing how the pattern changes from one value to the next, but it is not very good for projecting future values. (Discuss: Why?)

Explicit Linear Functions

- A linear growth pattern can be characterized as a linear function.
- One of the most common ways to write a linear function is in **slope-intercept** form. This form looks like this:

$$f(x) = \mathrm{m}x + \mathrm{b}$$

- Here, m is the constant ratio of change (the slope)
- b is the value of the function when $x = 0$. This is called the "y-intercept" because this is the value of the function when it crosses the y access.
- Example: Find a linear function for the linear pattern in the previous example.
- Example: Graph this function for x values 1 through 10.