Lecture 4 Ratio and Proportion

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A Complex Sounding Problem

If 8 workers in 24 days working 10 hours a day can reap 48 acres of wheat, how many acres could 12 workers reap in 20 days of 12 hours each?

Ratio

Comparing and Relating Quantities

- A ratio compares quantities of like types. (days to days, dollars to dollars, workers to workers, etc.)
- Ratios express the relationship between two concrete quantities.

Notation

- The notation for a ratio is $a : b$ Where a is the first quantity and b is the second.
- This is frequently pronounced as "a to b"
- A ratio is analgous to a fraction, thus $1 : 2$, $1 \div 2$, $\frac{1}{2}$, and 0.5 are all the same ratio.

Ratios, Categories, and Properties

- Often we use ratios to categorize populations of similar items.
- Example: What is the ratio of women to men in this room? What is the ratio of men to women?
- Ratios can be reduced in the same way we reduce fractions.
- Ratios can always be compared, even when they represent ratios of disparate objects.
- Ratios are abstract numbers. Why?

Proportion

- A proportion is two ratios which represent the same fraction. (Example: $1 : 2$ and $2 : 4$.)
- We often use the word "in proportion" to describe two equal ratios.
- Proportions are the mathematical equivalent of analogies: a is to b as c is to d.

Proportion Notation and Properties

- There are two main ways to write proportions $a : b :: c : d$ or $a : b = c : d$ where a, b, c, and d are the numbers which make up the proportion. Example: $1 : 2 :: 2 : 4$ or $1 : 2 = 2 : 4$.
- The two outer numbers are called the **extremes** of the proportion. $a : b :: c : d$ has extremes a and d.
- The two inner numbers are called the **means** of the proportion. $a : b :: c : d$ has means b and c.
- In order for four numbers to be in proportion, the product of the extremes must equal the product of the means. So in $a : b :: c : d$, $a \times d = b \times c$.
- Examples: $1 : 2 :: 2 : 4$, $1 : 3 :: 6 : 18$
- Discuss: Why must these two products be the same?

The Rule of Three

- If three numbers of a proportion are known, the fourth may be found.
- Missing Mean (if we know a, c, and d)

$$\begin{aligned} a : x &:: c : d \\ x \cdot c &= a \cdot d \\ x &= \frac{a \cdot d}{c} \end{aligned}$$

- Missing Extreme (if we know b, c, d)

$$\begin{aligned} x : b &:: c : d \\ x \cdot d &= b \cdot c \\ x &= \frac{b \cdot c}{d} \end{aligned}$$

Example Problems

1. Assuming that all classes maintain the same proportion of men and women as this one, if a class had 20 men, how many women would it have?
2. A besieged town, containing 22,400 inhabitants, has provisions to last 3 weeks; how many must be sent away that they may be able to hold out 7 weeks? Transcribed from: *A Treatise on Arithmetic* by J. H. Smith. 1878

Compound Proportions

- A compound proportion is a set of three or more ratios given where one is incomplete.
- You produce solutions to compound proportions by multiplying corresponding terms together, and then solve as in a simple proportion.
- Our opening example is a compound proportion.