$$Q(\theta;\theta^i) = \sum_n \sum_k \gamma_{nk}\Big(-0.5\log(C_k)$$

$$-0.5\,(y_n-\mu_k)' C_k^{-1}(y_n-\mu_k)$$

$$+\log \omega_k\Big)$$

$$\frac{\partial Q(\theta;\theta^i)}{\partial \mu_k} = 0$$

$$\sum_n \sum_k \gamma_{nk}\left[\cancel{\frac{\partial(-0.5\log(C_k))}{\partial \mu_k}}^{\,0} + C_k^{-1}(y_n-\mu_k) + 0\right]$$

only for that particular k, so $\sum_k = 0 + 0 + \cdots + \sum_n C_k^{-1}(y_n-\mu_k) + 0 \cdots$

$$\frac{\partial Q}{\partial \mu_k} = \sum_n \gamma_{nk}\, C_k^{-1}(y_n-\mu_k) = 0$$

$$\sum_n (y_n-\mu_k)\cdot\gamma_{nk} = 0$$

$$\sum_n y_n\cdot\gamma_{nk} - \sum_n \mu_k\,\gamma_{nk} = 0$$

$$\boxed{\mu_k = \frac{\sum_n y_n\cdot\gamma_{nk}}{\sum_n \gamma_{nk}}}$$

$$\frac{\partial Q}{\partial C_k} = 0$$

$$\Rightarrow \sum_n \left[\frac{-0.5}{\cancel{|C_k|}} \cdot \cancel{|C_k|} \cdot (C_k^{-T}) \right] \cdot \gamma_{nk} \quad \frac{\partial |A|}{\partial A}^T$$

$$+ 0.5 \sum_n \gamma_{nk} \cdot \left[+ C_k^{-T} \cdot (y_n - \mu_k)(y_n - \mu_k)^T C_k^{-T} \right]$$

$$= 0$$

$$\Rightarrow \sum_n \gamma_{nk} C_k^{-T} = \sum_n \gamma_{nk} \left[C_k^{-T} (y_n - \mu_k)(y_n - \mu_k)^T C_k^{-T} \right]$$

multiply C_k^T on both sides. (pre)

$$\sum_n \gamma_{nk} \cdot \cancel{C_k^T C_k^{-T}} = \sum_n \gamma_{nk} \left[\cancel{C_k^T C_k^{-T}} (y_n - \mu_k) (y_n - \mu_k)^T \cdot C_k^{-T} \right]$$

multiply C_k^T on both sides (post)

$$\sum_n \gamma_{nk} C_k = \sum_n \gamma_{nk} \left[(y_n - \mu_k)(y_n - \mu_k)^T \right] \cancel{C_k^{-T} C_k^T}$$

const

$$\boxed{C_k = \frac{\sum_n \gamma_{nk} (y_n - \mu_k)(y_n - \mu_k)^T}{\sum_n \gamma_{nk}}}$$

$$\underset{(\omega_k)}{\arg\max}\; Q \equiv \underset{\omega_k}{\arg\max} \sum_n \sum_k \gamma_{nk} \log \omega_k$$

additional contd. $\sum_k \omega_k = 1$

using Lagrangian multiplier.

$$L = \sum_n \sum_k \gamma_{nk} \log(\omega_k) + \lambda\left(\sum_k \omega_k - 1\right)$$

$$= \sum_k \left(\log(\omega_k) \cdot \underbrace{\sum_n \gamma_{nk}}_{\gamma_k}\right) + \lambda\left(\sum_k \omega_k - 1\right)$$

$$= \sum_k \gamma_k \cdot \log \omega_k + \lambda\left(\sum_k \omega_k - 1\right)$$

$$\frac{\partial L}{\partial \omega_k} = 0 \Rightarrow \frac{1}{\omega_k} \cdot \gamma_k + \lambda = 0$$

$$\boxed{\omega_k = \frac{-\gamma_k}{\lambda}}$$

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow \sum_k \omega_k = 1$$

$$\Rightarrow \sum_k \left(-\frac{\gamma_k}{\lambda}\right) = 1$$

$$\boxed{\lambda = \sum_k \gamma_k}$$

$$\boxed{\omega_k = \frac{+\gamma_k}{+\sum_k \gamma_k}}$$

$$= \boxed{\frac{\gamma_k}{N}}$$