## A. Bayesian deconvolution method

**Algorithm 1** Detailed algorithm

Compute Cauchy-Schwartz bounds $M(\omega)$

choose amplitude $v_f$ and width $w$ of $V_f(\omega)$ envelope

Compute $\mathbf{\Delta W}^{(e)}_{S,n} = \left(\mathbf{H}^\top \mathbf{V}_e^{-1}\mathbf{H} + \mathbf{V}_f^{-1}\right)^{-1} \mathbf{H}^\top \mathbf{V}_e^{-1} \widetilde{\mathbf{\Delta W}}^{(e)}_{S,n}$

project under Cauchy-Schwartz bounds $\Delta W^{(e)}_{S,n}(\omega) := \min(\Delta W^{(e)}_{S,n}(\omega), M(\omega))$ and $\Delta W^{(e)}_{S,n}(\omega) := \max(\Delta W^{(e)}_{S,n}(\omega), -M(\omega))$

**repeat**

Compute gradient $\nabla\left(\mathbf{\Delta W}^{(e)}_{S,n}\right) = -\mathbf{H}^\top \mathbf{V}_e^{-1}\left(\widetilde{\mathbf{\Delta W}}^{(e)}_{S,n} - \mathbf{H}\mathbf{\Delta W}^{(e)}_{S,n}\right) + \mathbf{V}_f^{-1}\mathbf{\Delta W}^{(e)}_{S,n}$ Project gradient $\mathbf{P}\nabla\left(\mathbf{\Delta W}^{(e)}_{S,n}\right)$ **if** $|\Delta W^{(e)}_{S,n}(\omega)| \geq M(\omega)$ and $\Delta W^{(e)}_{S,n}(\omega) * \nabla\left(\Delta W^{(e)}_{S,n}\right)(\omega) \leq 0$ **then**

$\mathrm{P}\nabla\left(\Delta W^{(e)}_{S,n}\right)(\omega) = 0$

**else**

$\mathrm{P}\nabla\left(\Delta W^{(e)}_{S,n}\right)(\omega) = \nabla\left(\Delta W^{(e)}_{S,n}\right)(\omega)$

**end if**

Compute furthest accessible displacement $d_\infty$ along $\mathbf{P}\nabla\left(\mathbf{\Delta W}^{(e)}_{S,n}\right)$ direction

**for all** $\mathrm{P}\nabla\left(\Delta W^{(e)}_{S,n}\right)(\omega) \neq 0$ **do**

$d_\infty \leftarrow \min\left(\frac{M(\omega) - \Delta W^{(e)}_{S,n}(\omega)}{\mathrm{P}\nabla\left(\Delta W^{(e)}_{S,n}\right)(\omega)}, d_\infty\right)$

**end for**

Compute the optimum displacement $d_0$ along $\mathbf{P}\nabla\left(\mathbf{\Delta W}^{(e)}_{S,n}\right)$ direction

$d_0 = \left|\left|\mathbf{P}\nabla\left(\mathbf{\Delta W}^{(e)}_{S,n}\right)\right|\right|^{-2}\left(\left|\left|\mathbf{HP}\nabla\left(\mathbf{\Delta W}^{(e)}_{S,n}\right)\right|\right|^2_{\mathbf{V}_e} + \left|\left|\mathbf{P}\nabla\left(\mathbf{\Delta W}^{(e)}_{S,n}\right)\right|\right|^2_{\mathbf{V}_f}\right)$

Compute the descent gradient $\mathbf{\Delta W}^{(e)}_{S,n} \leftarrow \mathbf{\Delta W}^{(e)}_{S,n} + \min(d_0, d_\infty)\mathbf{P}\nabla\left(\mathbf{\Delta W}^{(e)}_{S,n}\right)$

**until** $-\ln\left(p\left(\mathbf{\Delta W}^{(e)}_{S,n} \middle| \widetilde{\mathbf{\Delta W}}^{(e)}_{S,n}, \mathbf{V}_e\right)\right)$ is minimized