

discussion about Deconvolution

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Problem

$$g(t) = h * f(t) + \epsilon(t) \quad (1)$$

Discretization:

$$g(m) = \sum_k h(m-k)f(k) + \epsilon(m) \quad (2)$$

$$\underline{g} = H\underline{f} + \underline{\epsilon} \quad (3)$$

Régularisation quadratique

$$\hat{f} = \arg \min_f J(f) \quad (4)$$

Critère d'ordre zéro

$$J(f) = \|g - Hf\|^2 + \lambda\|f\|^2 \quad (5)$$

solution

$$\nabla_f J(\hat{f}) = 0 \quad (6)$$

$$-2H^\top(g - H\hat{f}) + 2\lambda\hat{f} = 0 \quad (7)$$

$$\hat{f} = (H^\top H + \lambda Id)^{-1} H^\top g \quad (8)$$

Critère d'ordre un

$$J(f) = \|g - Hf\|^2 + \lambda\|Df\|^2 \quad (9)$$

solution

$$\nabla_f J(\hat{f}) = 0 \quad (10)$$

$$\hat{f} = (H^\top H + \lambda D^\top D)^{-1} H^\top g \quad (11)$$

Méthode itérative

$$\hat{f} = \arg \min_f J(f) \quad (12)$$

$$f^{(0)} = 0 \quad (13)$$

$$f^{(k+1)} = \alpha^{(k)}(H^\top(g - Hf^k) - \lambda f^k) \quad (14)$$

$$f^{(k+1)} = f^{(k+1)} + \text{Contraintes} \quad (15)$$

$$\hat{f} = f^{(\infty)} \quad (16)$$

Critère pondéré

$$J(f) = \|g - Hf\|_{v_\epsilon}^2 + \lambda \|Df\|_{v_f}^2 \quad (17)$$

$$J(f) = \sum_i \frac{(g_i - [Hf]_i)^2}{v_{\epsilon_i}} + \lambda \sum_j \frac{(f_j)^2}{v_{f_j}} \quad (18)$$