$$f(t) \longrightarrow \boxed{h(t)} \longrightarrow \oplus \xleftarrow{\varepsilon(t)} \longrightarrow g(t) = h(t) * f(t) + \varepsilon(t)$$

Discrétisation.

$$g(m) = \sum_k h(k)\, f(m-k) + \varepsilon(m)$$

$$\underline{g} = H \underline{f} + \underline{\varepsilon}$$

Res. Quad. $\hat{f} = \arg\min_f J(f)$

d'ordre zero $J(\underline{f}) = \|g - Hf\|^2 + \lambda \|f\|^2$

$$\nabla J = -2H^t(g - Hf) + 2\lambda f$$

$$= 0 \Rightarrow H^tH\hat{f} + \lambda f = H^t g$$

$$\hat{f} = (H^tH + \lambda I)^{-1} H^t g$$

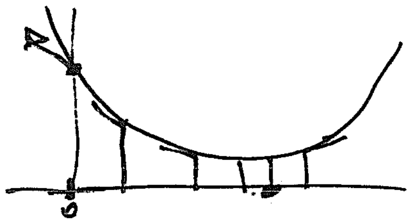
d'ordre un $J(f) = \|g - Hf\|^2 + \lambda \|Df\|^2$ $\quad D = \begin{pmatrix} 1 & & 0 \\ -1 & 1 & \\ 0 & \ddots & \ddots \\ & -1 & 1 \end{pmatrix}$

$$\nabla J = -2H^t(g - Hf) + 2\lambda D^tDf$$

$$= 0$$

$$\hat{f} = (H^tH + \lambda D^tD)^{-1} H^t g$$

Méthodes itératives



$$\hat{f} = \arg\min_f J(f)$$

$$f^{(0)} = 0$$

$$f^{(k+1)} = f^{(k)} + \alpha \left[H^t(g - Hf^{(k)}) - \lambda f^{(k)} \right]$$

Imposer les Contraintes

α fixe

α petit plus de temps mais convergence

α grand oscillation

α adaptative

Gradient
- à pas fixe
- à pas adaptative
- à plus grande pente

Contrainte $f^{(k+1)}$ ——— ———

Impose les contraintes à chaque itérations

- support
- zero sur les bords
- positivité

———

Critère avec les poids.

$$J(f) = \|g - Hf\|^2_{W_1} + \lambda \| \quad \|_{W_2}$$

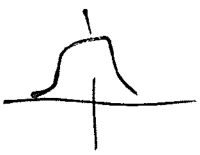
$$= \sum_i \frac{(g_i - (Hf)_i)^2}{v_{\epsilon_i}} + \lambda \sum_j \frac{f_j^2}{v_{f_j}}$$

MAP $\quad g = Hf + \epsilon$

$$\epsilon \sim N(\epsilon | 0, v_\epsilon I) \Rightarrow p(g|f) = N(g | Hf, v_\epsilon I)$$ vraisemblance

$$f \sim N(f | 0, v_f I) \qquad p(f) = N(\qquad | \qquad)$$

$$p(g|f) \propto \exp\left(-\frac{1}{2v_\epsilon}\|g - Hf\|^2\right)$$



$$\rightarrow p(f) \propto \exp\left(-\frac{1}{2v_f}\|f\|^2\right) \Longrightarrow p(f) \propto \exp\left(-\gamma \|f\|_1\right)$$

$$p(f|g) = \frac{p(g|f)\,p(f)}{p(g)} \propto p(g|f)\,p(f)$$

$$p(g) = \int p(g|f)\,p(f)\,df$$

$$p(f|g) \propto \exp\left(-\frac{1}{2v_\epsilon}\|g - Hf\|^2 - \frac{1}{2v_f}\|f\|^2\right)$$

$$-\frac{1}{2v_\epsilon} J(f)$$

$$J(f) = \|g - Hf\|^2 + \overbrace{\frac{v_\epsilon}{v_f}}^{\lambda} \|f\|^2$$

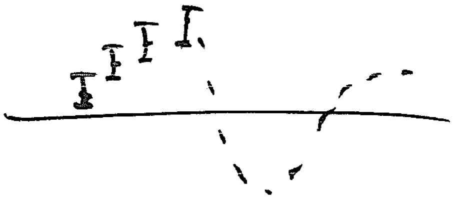
$$\hat{f}_{MAP} = \arg\max_f p(f|g) \equiv \arg\min_f J(f)$$

Bayesian advantages

$$P(f|g) \propto \exp\left(-\frac{1}{2v_\epsilon} J(f)\right) \qquad J(f) = (f-\hat{\mu})\hat{\Sigma}^{-1}(f-\hat{\mu})$$

$$= \mathcal{N}(f| \hat{\mu}, \hat{\Sigma})$$

$$\begin{cases} \hat{\mu} = \hat{f}_{MAP} = (H^tH+\lambda I)^{-1}H^tg \\ \hat{\Sigma} = v_\epsilon (H^tH+\lambda I)^{-1} \end{cases}$$ matrice covariance a posteriori



$$\begin{cases} \lambda = \frac{v_\epsilon}{v_f} \\ \hat{\Sigma} \end{cases}$$ les éléments diagonaux peuvent être utilisés pour mettre des barre d'erreur

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$$\lambda = \frac{v_\epsilon}{v_f}$$

θ: (v_ϵ, v_f) hyperparamètres v_ϵ v_f

$$P(f,\theta|g) = \frac{P(g|f,\theta_1)\, P(f|\theta_2)\, \overbrace{P(\theta)}^{P(v_\epsilon)P(v_f)}}{P(g)}$$

$$\text{JMAP}: \quad (\hat{f},\hat{\theta}) = \arg\max_{(f,\theta)} P(f,\theta|g)$$

opt. alternée

$\theta^{(0)}, f^{(0)}$

$$f^{(k+1)} = \arg\max_f P(f, \theta^{(k)}|g)$$

$$\theta^{(k+2)} = \arg\max_\theta P(f^{(k+1)}, \theta|g)$$

$\underline{\epsilon} \sim \mathcal{N}(\epsilon|\underline{0}, V_\epsilon)$ $\qquad V_\epsilon = \mathrm{diag}(v_{\epsilon 1}, \dots, v_{\epsilon M})$

$f \sim \mathcal{N}(f|0, V_f)$ $\qquad V_f = \mathrm{diag}(v_{f1}, \dots v_{fN})$

$p(\theta)$?

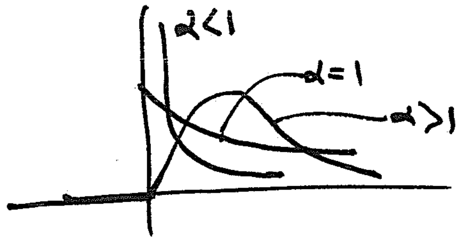
lois a priori Conjugués

$p(v_\epsilon) = IG$

$p(v_f)$

$x \sim \mathcal{G}(x|\alpha,\beta)$

$p(x|\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$



IG: $z = \frac{1}{x}$

$p(z|\alpha,\beta) = \quad z^{\alpha+1} e^{-\beta z}$

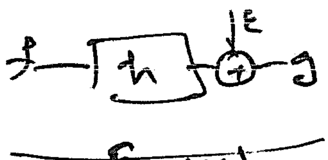
$$g = h * f + \varepsilon$$

$$\Gamma_{gg}(\tau) = \int g(t)\, g(t+\tau)\, dt \xrightarrow{\text{F.T.}} S_{gg}(\omega)$$

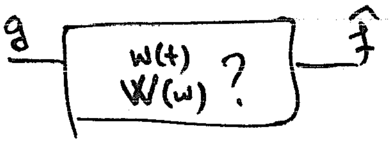
$$\Gamma_{gf}(\tau) = \int g(t)\, f(t+\tau)\, dt \longrightarrow S_{gf}(\omega)$$

$$S_{gg}(\omega) = |H(\omega)|^2 S_{ff}(\omega) + S_{\varepsilon\varepsilon}(\omega)$$

$$S_{gf}(\omega) = H^*(\omega)\, S_{ff}(\omega)$$



Forward model



$$\text{EQM} = E\{\|\hat{f} - f\|_2^2\} \quad \text{minimal}$$

$$\downarrow$$

$$w * g$$

$$W \cdot G$$

$$\frac{\partial}{\partial w} = 0 \longrightarrow W(\omega) = \frac{H^*(\omega)}{|H(\omega)|^2 + \frac{S_{\varepsilon\varepsilon}(\omega)}{S_{ff}(\omega)}}$$

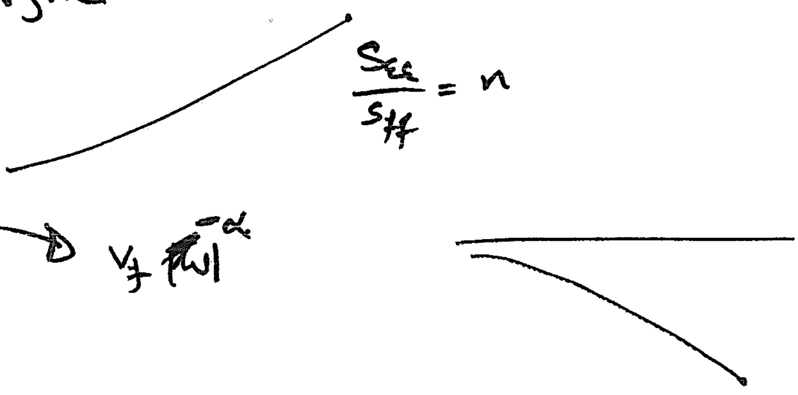
$S_{\varepsilon\varepsilon}(\omega)$ DSP du bruit

$S_{ff}(\omega)$ DSP du signal

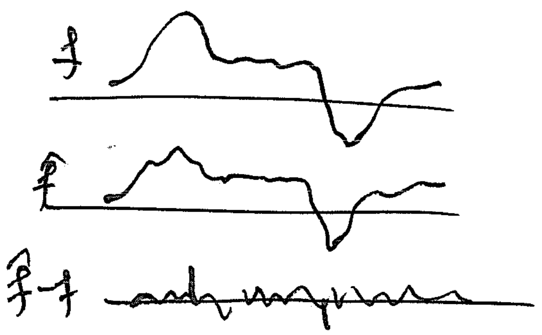
$$\frac{S_{\varepsilon\varepsilon}}{S_{ff}} = n$$

Hypothèses : $S_{\varepsilon\varepsilon}(\omega) = v_\varepsilon$

$S_{ff}(\omega) = v_f \longrightarrow v_f |\omega|^{-\alpha}$



optimale



$$\|\hat{f} - f\|_1$$