Solving model with simplex

Rewriting model

$$\begin{array}{rrrrcr} \min\ z & = & -2x_1 & -x_2 & & \\ & \text{s.c.} & x_1 & -x_2 & \geq & -2 \\ & & -x_1 & -x_2 & \geq & -6 \\ & & 2x_1 & & \geq & 4 \\ & & x_1, & x_2 & \geq & 0 \end{array}$$

Objective function from min to max

$$\begin{array}{rrrrcr} \max\ z & = & 2x_1 & +x_2 & & \\ & \text{s.c.} & x_1 & -x_2 & \geq & -2 \\ & & -x_1 & -x_2 & \geq & -6 \\ & & 2x_1 & & \geq & 4 \\ & & x_1, & x_2 & \geq & 0 \end{array}$$

Positive right hand sides

$$\begin{array}{rrrrcr} \max\ z & = & 2x_1 & +x_2 & & \\ & \text{s.c.} & -x_1 & +x_2 & \leq & 2 \\ & & x_1 & +x_2 & \leq & 6 \\ & & 2x_1 & & \geq & 4 \\ & & x_1, & x_2 & \geq & 0 \end{array}$$

Add slack variables

$$\begin{array}{rrrrrrrcr} \max\ z & = & 2x_1 & +x_2 & & & & & \\ & \text{s.c.} & -x_1 & +x_2 & +e_1 & & & = & 2 \\ & & x_1 & +x_2 & & +e_2 & & = & 6 \\ & & 2x_1 & & & & -e_3 & = & 4 \\ & & x_1, & x_2, & e_1, & e_2, & e_3 & \geq & 0 \end{array}$$

Add artificial variables

$$\begin{array}{rrrrrrrrcr} \max\ z & = & 2x_1 & +x_2 & & & & & & \\ & \text{s.c.} & -x_1 & +x_2 & +e_1 & & & & = & 2 \\ & & x_1 & +x_2 & & +e_2 & & & = & 6 \\ & & 2x_1 & & & & -e_3 & +v_3 & = & 4 \\ & & x_1, & x_2, & e_1, & e_2, & e_3, & v_3 & \geq & 0 \end{array}$$

Model to simplex table

	x_1	x_2	e_1	e_2	e_3	v_3	
c_a	0	0	0	0	0	1	0
c	2	1	0	0	0	0	0
e_1	-1	1	1	0	0	0	2
e_2	1	1	0	1	0	0	6
v_3	2	0	0	0	-1	1	4

Solving

Repair simplex table

	x_1	x_2	e_1	e_2	e_3	v_3	
c_a	-2	0	0	0	1	0	-4
c	2	1	0	0	0	0	0
e_1	-1	1	1	0	0	0	2
e_2	1	1	0	1	0	0	6
v_3	2	0	0	0	-1	1	4

Entering variable: x_1 – Leaving variable: v_3

$$L_3 \leftarrow \frac{1}{2}L_3$$

	x_1	x_2	e_1	e_2	e_3	v_3	
c_a	-2	0	0	0	1	0	-4
c	2	1	0	0	0	0	0
e_1	-1	1	1	0	0	0	2
e_2	1	1	0	1	0	0	6
v_3	1	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	2

$$\begin{aligned}
L_{c_a} &\leftarrow L_{c_a} + 2L_3 \\
L_c &\leftarrow L_c - 2L_3 \\
L_1 &\leftarrow L_1 + L_3 \\
L_2 &\leftarrow L_2 - L_3
\end{aligned}$$

	x_1	x_2	e_1	e_2	e_3	v_3	
c_a	0	0	0	0	0	1	0
c	0	1	0	0	1	-1	-4
e_1	0	1	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	4
e_2	0	1	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	4
x_1	1	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	2

Remove artificial variables

	x_1	x_2	e_1	e_2	e_3	
c	0	1	0	0	1	-4
e_1	0	1	1	0	$-\frac{1}{2}$	4
e_2	0	1	0	1	$\frac{1}{2}$	4
x_1	1	0	0	0	$-\frac{1}{2}$	2

Entering variable: x_2
Multiple possible leaving variables – Uses Bland's rule.
Entering variable: x_2 – Leaving variable: e_1

$$L_c \leftarrow L_c - L_1$$
$$L_2 \leftarrow L_2 - L_1$$
$$L_3 \leftarrow L_3$$

	x_1	x_2	e_1	e_2	e_3	
c	0	0	-1	0	$\frac{3}{2}$	-8
x_2	0	1	1	0	$-\frac{1}{2}$	4
e_2	0	0	-1	1	1	0
x_1	1	0	0	0	$-\frac{1}{2}$	2

Entering variable: e_3 – Leaving variable: e_2

$$L_c \leftarrow L_c - \frac{3}{2}L_2$$
$$L_1 \leftarrow L_1 + \frac{1}{2}L_2$$
$$L_3 \leftarrow L_3 + \frac{1}{2}L_2$$

$$
\begin{array}{c|ccccc|c}
 & x_1 & x_2 & e_1 & e_2 & e_3 & \\
c & 0 & 0 & \frac{1}{2} & -\frac{3}{2} & 0 & -8 \\
\hline
x_2 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & 0 & 4 \\
e_3 & 0 & 0 & -1 & 1 & 1 & 0 \\
x_1 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & 2
\end{array}
$$

Entering variable: e_1 – Leaving variable: x_2

$$L_1 \leftarrow 2L_1$$

$$
\begin{array}{c|ccccc|c}
 & x_1 & x_2 & e_1 & e_2 & e_3 & \\
c & 0 & 0 & \frac{1}{2} & -\frac{3}{2} & 0 & -8 \\
\hline
x_2 & 0 & 2 & 1 & 1 & 0 & 8 \\
e_3 & 0 & 0 & -1 & 1 & 1 & 0 \\
x_1 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & 2
\end{array}
$$

$$
\begin{aligned}
L_c &\leftarrow L_c - \frac{1}{2}L_1 \\
L_2 &\leftarrow L_2 + L_1 \\
L_3 &\leftarrow L_3 + \frac{1}{2}L_1
\end{aligned}
$$

$$
\begin{array}{c|ccccc|c}
 & x_1 & x_2 & e_1 & e_2 & e_3 & \\
c & 0 & -1 & 0 & -2 & 0 & -12 \\
\hline
e_1 & 0 & 2 & 1 & 1 & 0 & 8 \\
e_3 & 0 & 2 & 0 & 2 & 1 & 8 \\
x_1 & 1 & 1 & 0 & 1 & 0 & 6
\end{array}
$$

Sensitivity analysis

Computing sensitivity of c_i

Sensitivity intervals for the variables:

Variable	Reduced cost	Min	Current objective value	Max
x_1	0	–	2	–
x_2	-1	–	1	–

Compute sensitivity interval for variable x_1

$$
\begin{aligned}
-1 - \Delta_{x_1} &\leq 0 \\
-2 - \Delta_{x_1} &\leq 0
\end{aligned}
$$

Thus $\Delta_{x_1} \in [-1, \infty]$

Variable	Reduced cost	Min	Current objective value	Max
x_1	0	1	2	∞
x_2	-1	–	1	–

Compute sensitivity interval for variable x_2

$$-1 + \Delta_{x_2} \leq 0$$

Thus $\Delta_{x_2} \in [-\infty, 1]$

Variable	Reduced cost	Min	Current objective value	Max
x_1	0	1	2	∞
x_2	-1	$-\infty$	1	2

Computing sensitivity of b_j

Sensitivity intervals for the constraints:

Constraint	Dual cost	Min	Current rhs value	Max
(1)	0	–	2	–
(2)	2	–	6	–
(3)	0	–	4	–

Compute sensitivity interval for constraint (1)

$$8 - \Delta_{(1)} \geq 0$$

Thus $\Delta_{(1)} \in [-\infty, 8]$

Constraint	Dual cost	Min	Current rhs value	Max
(1)	0	$-\infty$	2	10
(2)	2	–	6	–
(3)	0	–	4	–

Compute sensitivity interval for constraint (2)

$$8 - \Delta_{(2)} \geq 0$$
$$8 - 2\Delta_{(2)} \geq 0$$
$$6 - \Delta_{(2)} \geq 0$$

Thus $\Delta_{(2)} \in [-\infty, 4]$

Constraint	Dual cost	Min	Current rhs value	Max
(1)	0	$-\infty$	2	10
(2)	2	$-\infty$	6	10
(3)	0	$-$	4	$-$

Compute sensitivity interval for constraint (3)

$$8 - \Delta_{(3)} \geq 0$$

Thus $\Delta_{(3)} \in [-\infty, 8]$

Constraint	Dual cost	Min	Current rhs value	Max
(1)	0	$-\infty$	2	10
(2)	2	$-\infty$	6	10
(3)	0	$-\infty$	4	12