How Many 3×3 Magic Squares Are There? Sunday Puzzle

Posted November 8, 2015 By Presh Talwalkar. Read about me, or email me.

A magic square is a 3×3 grid where every row, column, and diagonal sum to the same number. How many magic squares are there using each the numbers 1 to 9 exactly once?

Prove there are no other possibilities.

I've posted a solution in a video.

How many 3×3 magic squares are there?

Or scroll below for a text/image summary.

.

.

"All will be well if you use your mind for your decisions, and mind only your decisions." Since 2007, I have devoted my life to sharing the joy of game theory and mathematics. MindYourDecisions now has over 1,000 free articles with no ads thanks to community support! Help out and get early access to posts with a pledge on Patreon.

.

.

.

.

.

.

D
.
Y
O
U
R
.
D
E
C
I
S
I
O
N
S
.
S
U
N
D
A
Y
.
P
U
Z
Z
L
E
.

.

Answer To: How Many 3×3 Magic Squares Are There?

There are 8 ways to make a 3×3 magic square.

In fact, there is really only one pattern. Every other pattern is a rotation or reflection. From the upper left, the first square on the right is a reflection through the center (transposes columns 1 and 3), for example.

Now let's prove these are the only possibilities.

The proof proceeds in 5 steps.

Step 1: The magic sum is 15

By definition, every row, column, and diagonal has the same sum M.

Thus each of first row, second row, and third row has a sum of M. So the first 3 rows sum to $3M$.

On the other hand, if we sum up all 9 elements, we must have the sum of the numbers 1 to 9

If a magic square exists, then each row, column and diagonal has to be 15.

Step 2: Combinations that sum to 15

Suppose you use the numbers 1 and 2.

1 + 2 = 3

You would need 12 in order to make 15. This is not possible since we are using the numbers 1 to 9. So we can't combine 1 and 2 to make 15.

Since 9 is the largest number, we have 1 + 9 = 10, which means our other number has to be 5.

1 + 5 + 9 = 15

Then the next largest number is 8, so we can have:

1 + 6 + 8 = 15

We can't make a sum with 7, since we would need two 7's to make that possible:

1 + 7 + 7 = 15

We can go through this exercise and find there are only 8 ways to get to 15.

Step 3: the center square is 5

The center square is involved in the middle row, the middle column, and both diagonals.

Which number is involved in 4 sums of 15? There is only one! This is the number 5.

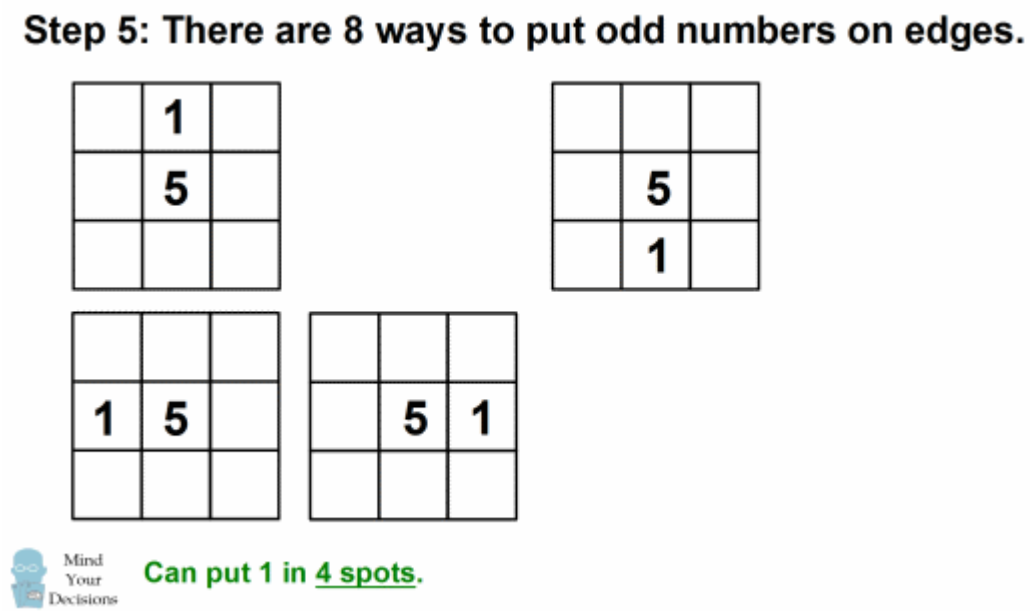
Step 4: the corners are even; the edges are odd (1, 3, 5, 7)

Similarly, each corner square is involved in a row, a column, and a diagonal sum. The only numbers involved in 3 sums are even numbers.

By the same logic, the edge squares could be the odd numbers 1, 3, 5, and 7.

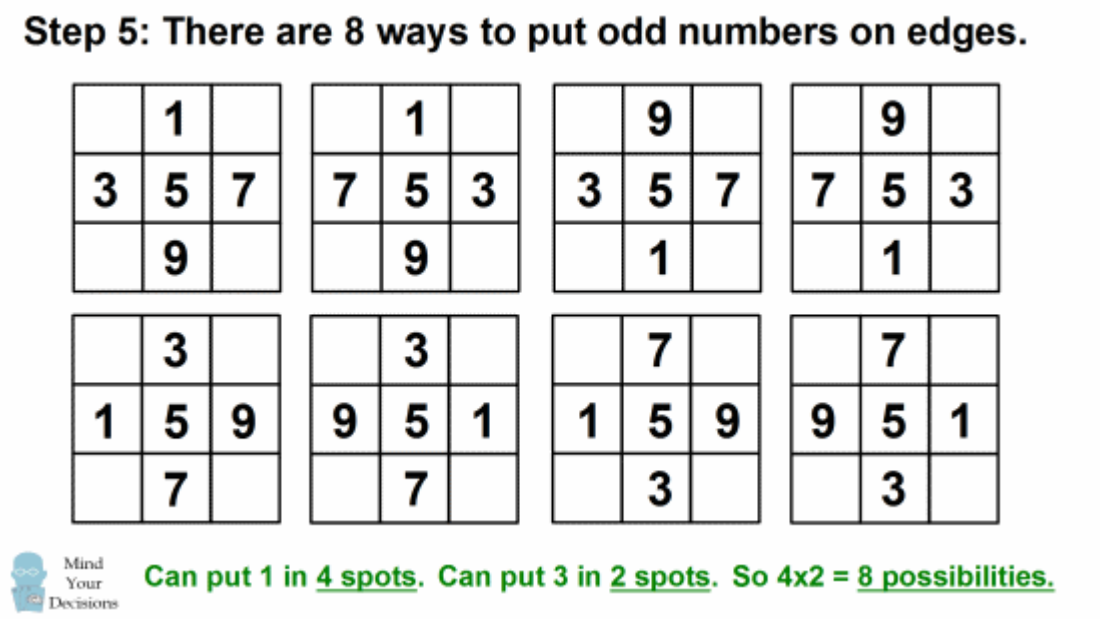
Step 5: enumerating the 8 possibilities

The number 1 can go in any of the 4 edges.

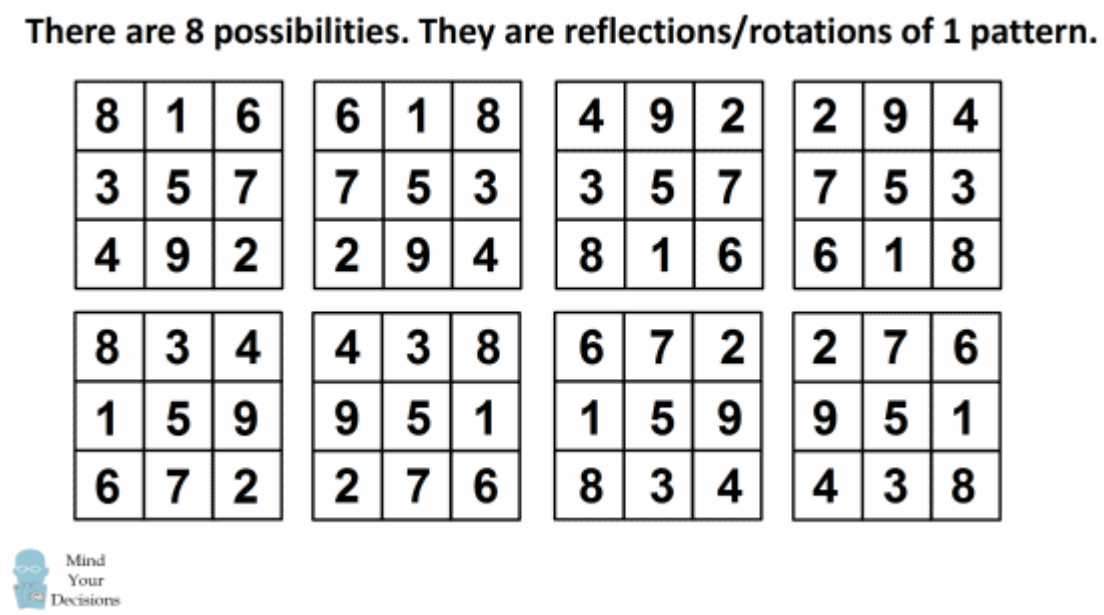


The number 9 will be forced to be opposite.

The number 3 must go in the perpendicular row to the left or right side of the 1, or in the perpendicular column to above or below the 1.



In all there are 4×2 = 8 possibilities. Since each odd number is involved in two sums, the remaining numbers are forced by these choices.



If you look at the first square, the other 7 squares are rotations or reflections. (you can see an animation of this idea in my video).

So there is 1 unique magic square. The eight patterns are rotations and reflections that correspond to symmetries of a square (the dihedral group of order 8).

It's a pretty neat proof, and we didn't have to test all 9! = 362880 possible ways to place the numbers in the square.

Relevant Math Stack Exchange

PUBLISHED BY

PRESH TALWALKAR

I run the MindYourDecisions channel on YouTube, which has over 1 million subscribers and 200 million views. I am also the author of The Joy of Game Theory: An Introduction

pay.)

By way of history, I started the Mind Your Decisions blog back in 2007 to share a bit of math, personal finance, personal thoughts, and game theory. It's been quite a journey! I thank everyone that has shared my work, and I am very grateful for coverage in the press, including the Shorty Awards, The Telegraph, Freakonomics, and many other popular outlets.

I studied Economics and Mathematics at Stanford University.

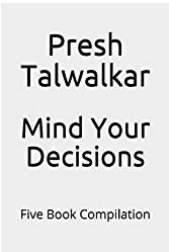
People often ask how I make the videos. Like many YouTubers I use popular software to prepare my videos. You can search for animation software tutorials on YouTube to learn how to make videos. Be prepared--animation is time consuming and software can be expensive!

Feel free to send me an email presh@mindyourdecisions.com. I get so many emails that I may not reply, but I save all suggestions for puzzles/video topics.

MY BOOKS
If you purchase through these links, I may be compensated for purchases made on Amazon. As an Amazon Associate I earn from qualifying purchases. This does not affect the price you pay.

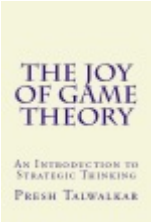
Book ratings are from June 2021.

(US and worldwide links)
https://mindyourdecisions.com/blog/my-books

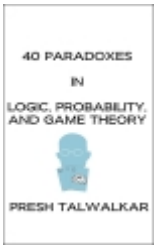


Mind Your Decisions is a compilation of 5 books:

(3) The Irrationality Illusion: How To Make Smart Decisions And Overcome Bias
(4) The Best Mental Math Tricks
(5) Multiply Numbers By Drawing Lines



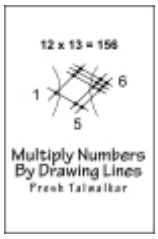
The Joy of Game Theory shows how you can use math to out-think your competition. (rated 4.2/5 stars on 200 reviews)



40 Paradoxes in Logic, Probability, and Game Theory contains thought-provoking and counter-intuitive results. (rated 4.1/5 stars on 30 reviews)

The Irrationality Illusion: How To Make Smart Decisions And Overcome Bias is a handbook that explains the many ways we are biased about decision-making and offers techniques to make smart decisions. (rated 4/5 stars on 17 reviews)

The Best Mental Math Tricks teaches how you can look like a math genius by solving



Multiply Numbers By Drawing Lines This book is a reference guide for my video that has over 1 million views on a geometric method to multiply numbers. (rated 4.1/5 stars on 23 reviews)

Mind Your Puzzles is a collection of the three "Math Puzzles" books, volumes 1, 2, and 3. The puzzles topics include the mathematical subjects including geometry, probability, logic, and game theory.

Math Puzzles Volume 1 features classic brain teasers and riddles with complete solutions for problems in counting, geometry, probability, and game theory. Volume 1 is rated 4.4/5 stars on 75 reviews.

Math Puzzles Volume 2 is a sequel book with more great problems. (rated 4.3/5 stars on 21 reviews)

Math Puzzles Volume 3 is the third in the series. (rated 4.3/5 stars on 17 reviews)

KINDLE UNLIMITED
Teachers and students around the world often email me about the books. Since education can have such a huge impact, I try to make the ebooks available as widely as possible at as low a price as possible.

Currently you can read most of my ebooks through Amazon's "Kindle Unlimited" program. Included in the subscription you will get access to millions of ebooks. You don't need a Kindle device: you can install the Kindle app on any

US, list of my books (US)
UK, list of my books (UK)
Canada, book results (CA)
Germany, list of my books (DE)
France, list of my books (FR)
India, list of my books (IN)
Australia, book results (AU)
Italy, list of my books (IT)
Spain, list of my books (ES)
Japan, list of my books (JP)
Brazil, book results (BR)
Mexico, book results (MX)

MERCHANDISE
Grab a mug, tshirt, and more at the official site for merchandise: **Mind Your Decisions at Teespring**.