Ruin Forest-Colin-Ghamnia-Grivet-simulations

December 11, 2023

1 Imports

```
[]: import multiprocessing
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import plotly.graph_objects as go
from plotly.subplots import make_subplots
from typing import List, Callable, Optional
from scipy.stats import genpareto, poisson, expon, uniform
```

2 Functions defintion

```
[]: def simulhpp(mean:float, T_max:int) -> List[float] :
    """
    Simulate a homogenous Poisson Process (hPP) with fixed T \\
    Input :
        mean : float : average number of events
        T_max : int : max time T
    Output :
        List[float] : times of occurence until T_max
        """
    T = expon.rvs(scale=1/mean, size=T_max)
    sT = np.cumsum(T)
    sT = sT[sT <= T_max]
    return np.round(sT)

# Second method to simulate an hPP
    n = np.random.poisson(mean*T_max, 1)
    hpp = np.random.uniform(0, T_max, n)
    return np.round(np.sort(hpp))</pre>
```

```
[]: def simulipp(lambda_:List[float], M:float) -> List[float] :
    """
    Simulate a inhomogenous Poisson Process (iPP) with fixed T \\
    Input :
```

```
lambda_ : List[float] : intensity function values at times 1, ..., T_max
    M : float : upper-bound of lambda_
        T_max : int : max time T

Output :
        List[float] : times of occurence until T_max

"""

T_max = len(lambda_) - 1
    ti = simulhpp(M, T_max)
    n = len(ti)
    u = uniform.rvs(loc=0, scale=M, size=n)
    lambda_ti = [lambda_[int(t)] for t in ti]
    index_ti = u <= lambda_ti
    return sorted(ti[index_ti])</pre>
```

```
[]: def S(Ah:float, lambda_lowercase:float, xi:float, sigma:float, u:float) ->__
      ⇔float :
       11 11 11
       Compute damage cost of the year \\
       Input:
         Ah : float : normalizing constant that translates the climate hazard
      \rightarrow conveyed by X_k into damage to R(t)
         lambda_lowercase : float : number of dry days
         xi : float : shape parameter of the GPD
         sigma : float : scale parameter of the GPD
         u : float : threshold of the GPD
       Output :
         float : damage cost of the year : Ah * sum(Xk) where Xk follow a pareto_\sqcup
      \hookrightarrow distribution with paremter xi, sigma, u
       Nt = poisson.rvs(lambda_lowercase)
       damage_by_days = genpareto.rvs(c=xi, loc=u, scale=sigma, size=Nt)
       return Ah * sum(damage_by_days)
```

```
[]: def R(R_t_minus_one:float, S_t : float, p_t : float, b:float, R_max:float) ->__
      ⊶float :
       Compute the reserve (non-structural carbohydrates) that allows growth of tree,
      \rightarrowat the beginning of vegetative period for year t \\
       Input:
         R_t_minus_one : float : previous reserve
         S_t: float: damages of the current year
         p_t : float : NPP (tree Net Primary Production) of the current year
         b: float: fraction of previous resources devoted to growth
         R_max: float: maximal value of reserve that a tree can store
       Output :
         float : reserve of the year : max(min((1-b)*R_t_minus_one + p_t - S_t_{, \sqcup}))
      \hookrightarrow R_{max}, 0)
       if R_t_minus_one == 0 : # Already ruined
         return 0
                                # So we can't produce anymore
       else :
         current_reserve = min((1-b)*R_t_minus_one + p_t - S_t, R_max)
         return max(current_reserve, 0)
[]: def compute_Ah_constant(B:float=0) -> float :
         Compute Ah : the normalizing constant that translates the climate hazard \sqcup
      \Rightarrow conveyed by Xk into damage to R(t) \setminus 
         Input:
             B: float: memory parameter value (not use here beacuse Ah is constant)
         Output :
             float : 0.6
         return 0.6
[]: def compute_Ah(B:float) -> float :
         11 11 11
         Compute Ah : the normalizing constant that translates the climate hazard \Box
      \Rightarrow conveyed by Xk into damage to R(t) \setminus \{
         Input:
             B: float: memory parameter value
         Output :
             float : 1.2 / (1+B)
         return 1.2 / (1+B)
[]: class SimuleTrajectories :
         def __init__(self, p0:float, b:float, R_max:float, sigma:float, xi:float, u:
      →float, compute_Ah:Callable,
```

```
lambda_lowercase:np.ndarray, lambda_uppercase:np.ndarray, B:
11 11 11
      p0 : float : optimum average yearly NPP
      b : float : fraction of previous resources devoted to growth
      R max: float: maximum amount of reserve that a tree can store
      sigma : float : scale parameter of the GPD
      xo : float : shape parameter of the GPD
      u : float : threshold parameter of the GPD
      compute_Ah : Callable : The function to compute Ah function of B
      lambda\_lowercase : List[float] : average number of dry days in a year_{\sqcup}
\hookrightarrow of heatwave
      lambda_uppercase : float : average return period of heatwave
      B : List[float] : list of memory parameter values
      r0 : float : initial reserve capital
      s0 : float : damage of the year before the simulation start
      self.n_year = None
      self.dataframes = []
      self.p0 = p0
      self.b = b
      self.R_max = R_max
      self.sigma = sigma
      self.xi = xi
      self.u = u
      self.compute_Ah = compute_Ah
      self.lambda lowercase = lambda lowercase
      self.lambda_uppercase = lambda_uppercase
      self.B = B
      self.r0 = r0
      self.s0 = s0
      self.nb B = len(self.B)
      self.df = pd.DataFrame()
      self.df variate = pd.DataFrame()
      self.n_sample = 0
      self.n_year = 0
  def __build_df(self) :
      return pd.DataFrame(columns=["B", "R", "S", "tau", "u", "sigma", "xi", [

¬"lambda", "Lambda"])
  def simulate_trajectory(self, *args, **kwargs) :
      Simulate reserve and damage over time
      11 11 11
      df = self.__build_df()
      for index_B, current_B in enumerate(self.B) :
```

```
Ah = self.compute_Ah(current_B)
           year_of_heat_waves = simulipp(self.lambda_uppercase, max(self.
→lambda_uppercase))
           S t minus one = self.s0
           R_t_minus_one = self.r0
           R list = []
           S list = []
           tau = np.inf
           for year in range(1, self.n_year+1):
               if R_t_minus_one == 0 :
                   S_year = 0
                   R_year = 0
               else :
                   p_year = p(p0=self.p0, B=current_B,__
→S_t_minus_one=S_t_minus_one)
                   if year in year_of_heat_waves :
                        S_year = S(Ah=Ah, lambda_lowercase=self.
alambda_lowercase[year-1], xi=self.xi, sigma=self.sigma, u=self.u)
                   else:
                        S_year = 0
                   R_year = R(R_t_minus_one=R_t_minus_one, S_t=S_year,__
→p_t=p_year, b=self.b, R_max=self.R_max)
               R list.append(R year)
               S_list.append(S_year)
               S_t_minus_one = S_year
               R_t_minus_one = R_year
               if R_year == 0 and tau == np.inf :
                   tau = year
           df.loc[index_B] = [current_B, R_list, S_list, tau, self.u, self.
⇒sigma, self.xi, self.lambda_lowercase[0], 1/self.lambda_uppercase[0]]
      return df
  def sample_trajectories(self, n_sample:int, t_max:int, variate:
⇔Optional[str]=None) :
       Do n_sample simulations of reserve and damage over time from 1 to t_max_{\!\scriptscriptstyle \perp}
→and store it in self.df or in self.df_variate if variate != None \\
       Input:
           n_sample : int : number of simulation
           t_{max}: int: time to simulate per simulation
           variate : Optionnal[str] : indicate the parameter variating, None_{\sqcup}
\hookrightarrow if none
       11 11 11
      self.n_sample = n_sample
      self.n_year = t_max
       if variate :
```

```
result_list = []
          for _ in range(n_sample) : # multiprocessing does not work with_
⇔variate... Don't know why...
              result list.append(self.simulate trajectory())
          df_variate = pd.concat(result_list)
          df variate["variation"] = variate
          if self.df variate.empty :
              self.df_variate = df_variate
          else:
              self.df_variate = pd.concat([self.df_variate, df_variate])
      else :
          p = multiprocessing.Pool(processes=multiprocessing.cpu_count())
          result_list = p.starmap(self.simulate_trajectory, [() for _ in_
→range(n_sample)])
          p.close()
          p.join()
          self.dataframes = result list
          self.df = pd.concat(list(self.dataframes), ignore_index=True)
          self.df["R_mean"] = self.df["R"].apply(np.mean)
      del result list[:]
      del self.dataframes[:]
  def sample_variate_params(self, n_sample:int, t_max:int, u_list:
xi_list:List[float]=[], lambda_list:
Varies each parameter independently and store result in self.df_variate_
\hookrightarrow \ \
      Input:
          n_sample : int : number of simulation per parameter combination
          t max : int : time to simulate per simulation
          u_list : List[float] : List of values for parameter u
          sigma list : List[float] : List of values for parameter sigma
          xi_list : List[float] : List of values for parameter xi
          lambda\_list: List[float]: List of values for parameter_{\sqcup}
⇒lambda_lowercase (i.e. hPP only)
          Lambda\_list : List[float] : List of values for parameter_{\sqcup}
\neg lambda\_uppercase (i.e. hPP only)
      self.df_variate = self.__build_df()
      self.df_variate = self.df_variate.assign(variation=None) # Création de_
⇔la colonne variation à None
      self.n_sample_variate = n_sample
      self.n_year_variate = t_max
```

```
u_base = self.u
      sigma_base = self.sigma
      xi_base = self.xi
      lambda_base = self.lambda_lowercase
      Lambda_base = self.lambda_uppercase
      old_n_sample = self.n_sample
      old_n_year = self.n_year
      for u in u list:
          self.u = u
           self.sample_trajectories(n_sample=n_sample, t_max=t_max,__
⇔variate="u")
      self.u = u_base
      for sigma in sigma_list :
          self.sigma = sigma
          self.sample_trajectories(n_sample=n_sample, t_max=t_max,__
⇔variate="sigma")
      self.sigma = sigma_base
      for xi in xi_list :
          self.xi = xi
          self.sample_trajectories(n_sample=n_sample, t_max=t_max,__
⇔variate="xi")
      self.xi = xi base
      for lambda_lowercase in lambda_list :
           self.lambda_lowercase = lambda_lowercase * np.ones(t_max)
          self.sample_trajectories(n_sample=n_sample, t_max=t_max,__
⇔variate="lambda")
      self.lambda_lowercase = lambda_base
      for lambda uppercase in Lambda list :
          self.lambda_uppercase = np.ones(t_max) / lambda_uppercase
          self.sample_trajectories(n_sample=n_sample, t_max=t_max,__
⇔variate="Lambda")
      self.lambda_uppercase = Lambda_base
      self.n_sample = old_n_sample
      self.n_year = old_n_year
  def plot_sample_trajectories(self, quantiles:List[float], colors:
→List[float], R_max:float) :
       n n n
```

```
Plot the damages (firt row) and the reserve (second row) for each value_
⇔of B (one column per value) \\
      The lines are the quantiles in quantiles of the average of R \setminus A
      The colors of each line correspond to colors \\
      Also print the mean reserve for each quantile \\
      Input:
          quantiles : List[float] : list of quantile to be plotted
          colors : List[float] : list of the color of each line plotted
          R_max : float : maximum reserve value (to set the plot ylim)
      if self.df.empty :
          raise ValueError("You have to simulate trajectories before plotting ⊔
⇔them :)")
      plt.figure(figsize=(8, 8))
      for i, current_B in enumerate(self.B) :
          if i > 0:
              print("----")
          df_crop = self.df[self.df["B"] == current_B].copy()
          # Sort of the rows to find the quantiles
          df_crop = df_crop.sort_values(by=["R_mean", "tau"],__
→ignore_index=True)
          ruined = df_crop[df_crop["tau"] < np.inf]</pre>
          if ruined.empty:
              index_ruined = df_crop.index[0]
          else:
              index_ruined = ruined.index[0]
          for quantile, color in zip(quantiles, colors) :
              if color == "red" and quantile == 0 :
                  index = index_ruined
                  lw = 3
                  index = df_crop.index[int(self.n_sample*quantile)]
                  lw = 1
              current_S = df_crop.loc[index, "S"]
              current_R = df_crop.loc[index, "R"]
              current_R_mean = df_crop.loc[index, "R_mean"]
              print(f"B = {current_B}, mean reserve of {int(quantile*100)}th_

¬quantile = {round(current_R_mean, 1)}")
              # Damages
              plt.subplot(2, self.nb_B, 1+i)
              plt.plot(current_S, c=color, lw=lw)
              plt.xlabel("Time (Yr)")
              plt.ylabel("Damages [S(t)]")
              # Reserve
              plt.subplot(2, self.nb_B, self.nb_B+i+1)
              plt.plot(current_R, c=color, lw=lw)
```

```
plt.plot(current_R_mean*np.ones(self.n_year),__
⇔linestyle="dotted", c=color)
               plt.ylim((0, R_max))
              plt.xlabel("Time (Yr)")
              plt.ylabel("Reserve [R(t)]")
      plt.tight layout()
      plt.show()
  def plot_sample_variate_params(self) :
      Show the boxplot of the parameter varation
      if self.df_variate.empty :
           raise ValueError("You have to simulate variations before plotting...
⇔them :)")
      for current_B in self.B :
           fig = make_subplots(rows=3, cols=2, vertical_spacing=0.1)
           df_crop = self.df_variate[self.df_variate["B"] == current_B].copy()
           df_crop["tau"] = df_crop["tau"].apply(lambda tau : min(tau, self.
→n_year_variate+3))
           sub_df_Lambda = df_crop[df_crop["variation"] == "Lambda"]
           trace0 = go.Box(
               y=sub_df_Lambda["tau"],
               x=sub df Lambda["Lambda"]
           )
           sub_df_lambda = df_crop[df_crop["variation"] == "lambda"]
           trace1 = go.Box(
               y=sub_df_lambda["tau"],
               x=sub_df_lambda["lambda"]
           )
           sub_df_sigma = df_crop[df_crop["variation"] == "sigma"]
           trace2 = go.Box(
               y=sub_df_sigma["tau"],
               x=sub_df_sigma["sigma"]
           )
           sub_df_xi = df_crop[df_crop["variation"] == "xi"]
           trace3 = go.Box(
               y=sub_df_xi["tau"],
               x=sub_df_xi["xi"]
           sub_df_u = df_crop[df_crop["variation"] == "u"]
           trace4 = go.Box(
```

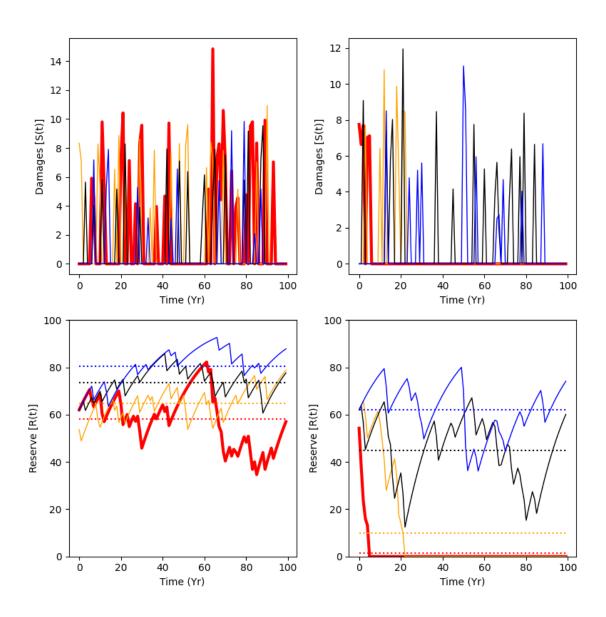
```
y=sub_df_u["tau"],
              x=sub_df_u["u"],
          )
          fig.append_trace(trace0, 1, 1)
          fig.append_trace(trace1, 1, 2)
          fig.append trace(trace2, 2, 1)
          fig.append_trace(trace3, 2, 2)
          fig.append_trace(trace4, 3, 1)
          fig.update layout(width=1000, height=800, showlegend=False,
⊸margin=dict(t=50, b=20, r=20), title_text=f"<b>Impact of parameters for⊔
\rightarrowB={current B}</b>", title x=0.5)
          fig.update yaxes(range=[0, self.n_year_variate+2], title_text="Ruin_
⇔vear")
          fig.update_xaxes(title_text="Return period of HW (Lambda)", __
fig.update_xaxes(title_text="Nb of dry days (lambda)",

→tickvals=sub_df_lambda["lambda"], row=1, col=2)
          fig.update xaxes(title text="GPD scale", ...
fig.update xaxes(title text="GPD shape", tickvals=sub df xi["xi"], u
\rightarrowrow=2, col=2)
          fig.update_xaxes(title_text="Threshold u", tickvals=sub_df_u["u"],_
\rightarrowrow=3, col=1)
          fig.show()
  def compute_proba_ruin(self) :
      Print the ruin probability for each value of B
      B_list = self.df["B"].unique()
      n sample = len(self.df) / len(B list)
      for current B in B list :
          df crop = self.df[self.df["B"] == current B]
          nb_ruined = df_crop[df_crop["tau"] < np.inf]["tau"].count() # Count_
→the number of tau < np.inf i.e. number of simulation ruined
          print(f"Ruin proba for B = {current_B:.4g} : {100*nb_ruined/
\neg n_sample:.4g}")
  def compute_mean_reserve_at_final_time(self) :
      Print the mean reserve at t max for each value of B
      B_list = self.df["B"].unique()
```

3 Part 1: Article Model

```
[]: lambda_lowercase = 10 * np.ones(100)
     lambda uppercase = np.ones(100) / 5
[]: params_article = dict(p0 = 5,
                           b = 0.05,
                           R_{max} = 100,
                           sigma = 0.1,
                           xi = -0.2,
                           u = 1,
                           compute_Ah = compute_Ah_constant,
                           lambda_lowercase = lambda_lowercase,
                           lambda_uppercase = lambda_uppercase,
                           B = [0, 1.5],
                           r0 = 60,
                           s0 = 0)
     n_sample = int(1e4)
     n_year = 100
     trajectories_article = SimuleTrajectories(**params_article)
     trajectories_article.sample_trajectories(n_sample, n_year)
     quantiles = [0, 0.05, 0.5, 0.95]
     colors = ["red", "orange", "black", "blue"]
     trajectories_article.plot_sample_trajectories(quantiles, colors,_
      ⇔params_article["R_max"])
```

B = 0, mean reserve of 0th quantile = 58.1

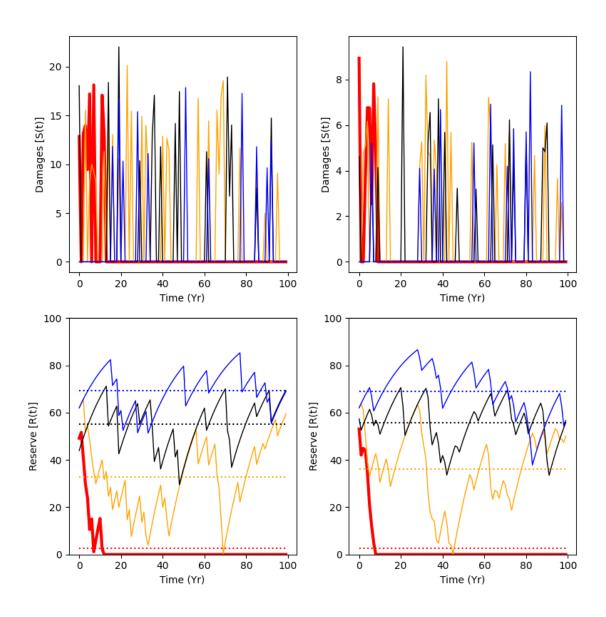


[]: trajectories_article.compute_mean_reserve_at_final_time() trajectories_article.compute_proba_ruin()

Mean reserve at t = 100 for B = 0 : 77.73 Mean reserve at t = 100 for B = 1.5 : 32.42 Ruin proba for B = 0 : 0% Ruin proba for B = 1.5 : 33.71%

4 Part 2: Change Ah to be function of B

```
[]: lambda lowercase = 10 * np.ones(100)
     lambda_uppercase = np.ones(100) / 5
[]: params_Ah_function = dict(p0 = 5,
                               b = 0.05,
                               R_{max} = 100,
                               sigma = 0.1,
                               xi = -0.2,
                               u = 1,
                               compute_Ah = compute_Ah,
                               lambda_lowercase = lambda_lowercase,
                               lambda_uppercase = lambda_uppercase,
                               B = [0, 1.5],
                               r0 = 60,
                               s0 = 0)
     n_sample = int(1e4)
     n_year = 100
     trajectories_Ah_function = SimuleTrajectories(**params_Ah_function)
     trajectories_Ah_function.sample_trajectories(n_sample, n_year)
     quantiles = [0, 0.05, 0.5, 0.95]
     colors = ["red", "orange", "black", "blue"]
     trajectories_Ah_function.plot_sample_trajectories(quantiles, colors,_
      →params_Ah_function["R_max"])
    B = 0, mean reserve of 0th quantile = 2.6
    B = 0, mean reserve of 5th quantile = 32.7
    B = 0, mean reserve of 50th quantile = 55.1
    B = 0, mean reserve of 95th quantile = 69.3
    B = 1.5, mean reserve of 0th quantile = 2.6
    B = 1.5, mean reserve of 5th quantile = 36.0
    B = 1.5, mean reserve of 50th quantile = 55.6
    B = 1.5, mean reserve of 95th quantile = 69.1
```



[]: trajectories_Ah_function.compute_mean_reserve_at_final_time() trajectories_Ah_function.compute_proba_ruin()

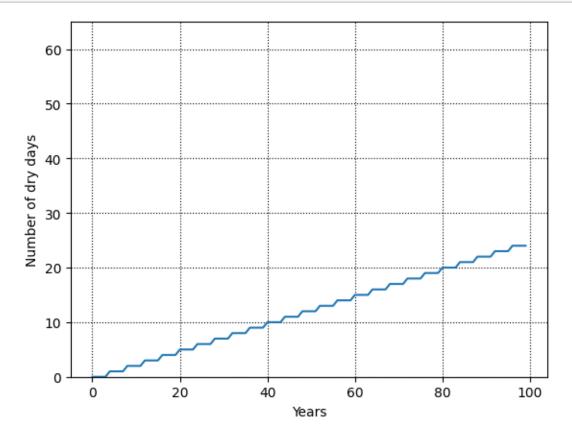
Mean reserve at t = 100 for B = 0 : 53.58 Mean reserve at t = 100 for B = 1.5 : 52.49 Ruin proba for B = 0 : 6.32% Ruin proba for B = 1.5 : 5.32%

5 Part 3: Taking climate change into account

5.1 A. Increasing number of dry days

```
[]: lambda_lowercase = np.array([0 + 2.5 * i//10 for i in range(100)])
lambda_uppercase = np.ones(100) / 5
```

```
[]: plt.plot(lambda_lowercase)
  plt.ylabel("Number of dry days")
  plt.xlabel("Years")
  plt.ylim([0, 65])
  plt.grid(linestyle="dotted", c="black")
```

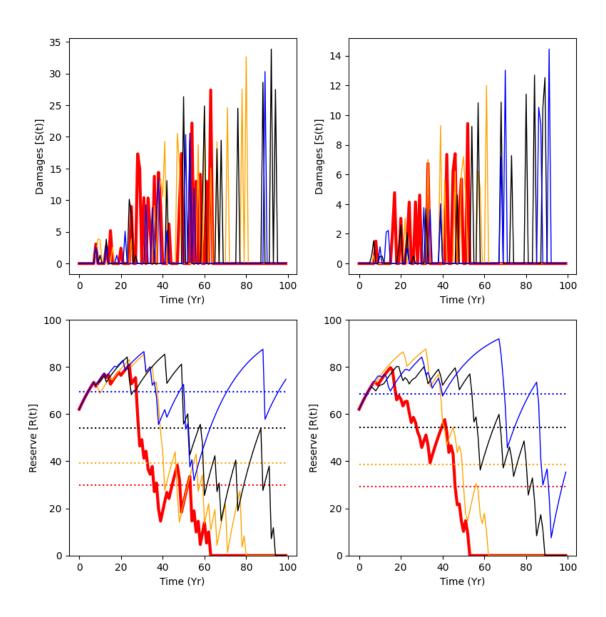


```
lambda_uppercase = lambda_uppercase,
                                  B = [0, 1.5],
                                  r0 = 60,
                                  s0 = 0)
n_sample = int(1e4)
n_year = 100
trajectories_ipp_lambda_lowercase =_

→SimuleTrajectories(**params_ipp_lambda_lowercase)

trajectories ipp_lambda lowercase.sample_trajectories(n_sample, n_year)
quantiles = [0, 0.05, 0.5, 0.95]
colors = ["red", "orange", "black", "blue"]
trajectories_ipp_lambda_lowercase.plot_sample_trajectories(quantiles, colors,_
  →params_Ah_function["R_max"])
B = 0, mean reserve of 0th quantile = 29.9
B = 0, mean reserve of 5th quantile = 39.2
B = 0, mean reserve of 50th quantile = 54.1
B = 0, mean reserve of 95th quantile = 69.4
_____
B = 1.5, mean reserve of 0th quantile = 29.2
```

B = 1.5, mean reserve of 5th quantile = 38.6 B = 1.5, mean reserve of 50th quantile = 54.5 B = 1.5, mean reserve of 95th quantile = 68.5



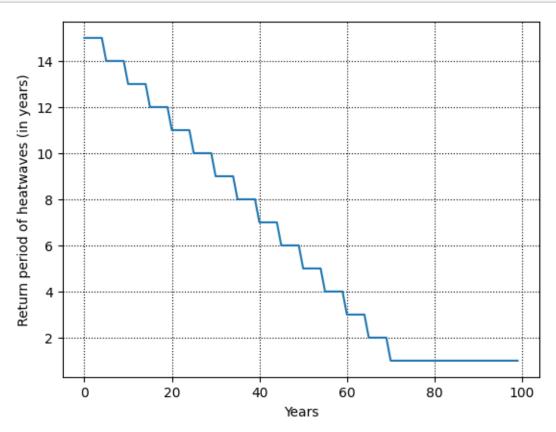
[]: trajectories_ipp_lambda_lowercase.compute_mean_reserve_at_final_time() trajectories_ipp_lambda_lowercase.compute_proba_ruin()

Mean reserve at t = 100 for B = 0 : 14.62 Mean reserve at t = 100 for B = 1.5 : 14.6 Ruin proba for B = 0 : 69.2% Ruin proba for B = 1.5 : 65.36%

5.2 B. Decreasing return period of heatwaves

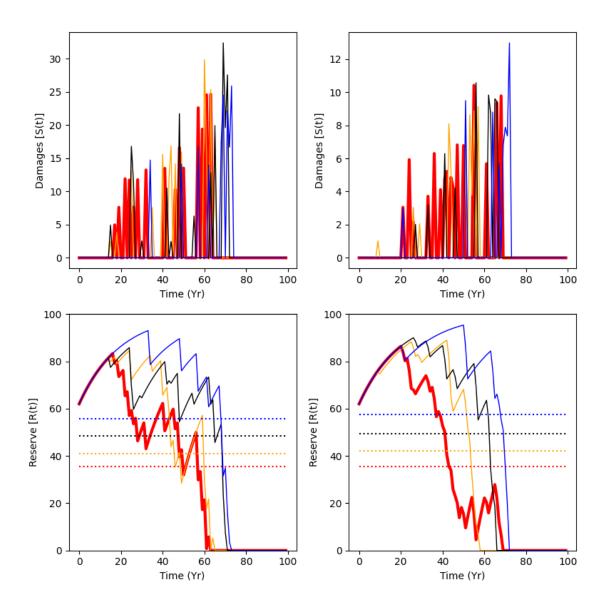
```
[]: lambda_lowercase = np.array([0 + 2.5 * i//10 for i in range(100)])
lambda_uppercase = 1 / np.array([max(15 - 2 * i//10, 1) for i in range(100)])
```

```
[]: plt.plot(1/lambda_uppercase)
   plt.ylabel("Return period of heatwaves (in years)")
   plt.xlabel("Years")
   plt.grid(linestyle="dotted", c="black")
```



```
n_sample = int(1e4)
n_year = 100
trajectories_ipp_both_lambda = SimuleTrajectories(**params_ipp_both_lambda)
trajectories_ipp_both_lambda.sample_trajectories(n_sample, n_year)

quantiles = [0, 0.05, 0.5, 0.95]
colors = ["red", "orange", "black", "blue"]
trajectories_ipp_both_lambda.plot_sample_trajectories(quantiles, colors,u_aparams_Ah_function["R_max"])
```



```
[]: trajectories_ipp_both_lambda.compute_mean_reserve_at_final_time() trajectories_ipp_both_lambda.compute_proba_ruin()
```

```
Mean reserve at t = 100 for B = 0 : 0 Mean reserve at t = 100 for B = 1.5 : 0 Ruin proba for B = 0 : 100% Ruin proba for B = 1.5 : 100%
```

6 Part 4: Influence of each parameter on the ruin time

Takes about 27min to run as it cannot be parallelized

[]: trajectories_variation.plot_sample_variate_params()