3D Simulation of PDE and Jitter in SPAD Devices.

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Abstract

In this paper we present a full 3D simulation methodology to extract Photon Detection Probability (PDP) and Jitter of Single-Photon Avalanche Diode (SPAD) Devices. The simulation results are compared with measurements on devices and show good agreement with the experiments.

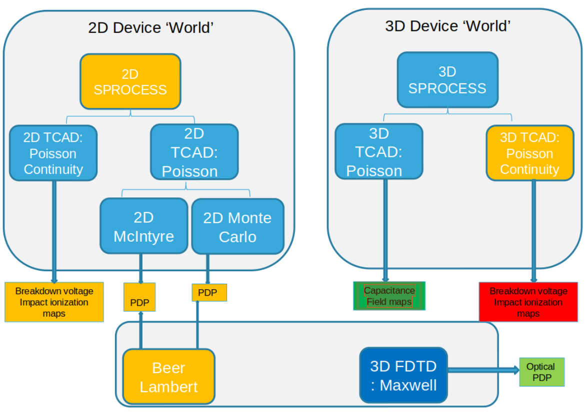
Keywords— single-photon avalanche diode (SPAD), photon detection probability (PDP), jitter, avalanche breakdown probability, breakdown voltage

1 Introduction

Single-photon avalanche diodes (SPADs) are key opto-electronics devices.

2 Device structure and TCAD simulation

Figure 1: SPAD simulation workflow



3 Avalanche breakdown probability

The avalanche breakdown probability is computed by the means of the well known McIntyre model [Oldham et al., 1972]. We briefly recall the model derivation : Let $P_e(x)$ be the probability that an electron starting at x in the depletion layer triggers an avalanche and $P_h(x)$ the same probability for an hole starting at x. Straightforwardly, the probability that neither an hole nor an electron starting at x trigger an avalanche is given by $(1-P_e(x))(1-P_h(x))$. Thus, the probability that either the hole or the electron trigger an avalanche, noted P_{pair} is :

$$\begin{aligned} P_{pair}(x) &= 1-(1-P_e(x))\,(1-P_h(x)) \\ &= P_e + P_h - P_e P_h \end{aligned}$$

Now, the probability that an electron starting at $x+dx$ triggers an avalanche is : The probability that the electron reaches the position x and triggers an avalanche in x plus the probability that it triggers an avalanche between x and $x+dx$ less the probability of the intersection of the two previous events. It writes :

$$\begin{aligned} P_e(x+dx) &= P_e(x) + \alpha_e(x)dxP_{pair}(x) - P_e(x)\alpha_e dxP_{pair}(x) \\ &= P_e(x) + \alpha_e(x)dx(P_e(x)+P_h(x)-P_e(x)P_h(x)) \\ &\quad - P_e(x)\alpha_e(x)dx(P_e(x)+P_h(x)-P_e(x)P_h(x)) \\ &= P_e(x) + dx\,\alpha_e(x)(P_e(x)+P_h(x)-P_e(x)P_h(x))(1-P_e(x)) \end{aligned}$$

Where α_e is the electron linear ionization rate : the probability by length that an electron create an impact ionization event.
One can rearrange the terms to obtain :

$$\frac{P_e(x+dx)-P_e(x)}{dx} = \alpha_e(x)(P_e(x)+P_h(x)-P_e(x)P_h(x))(1-P_e(x))$$

Which leads to the first ordinary differential equation :

$$\frac{dP_e}{dx} = (1-P_e)\alpha_e(P_e+P_h-P_eP_h)$$

The same reasoning applies to the probability that an hole starting at $x-dx$ triggers an avalanche. Which leads to the second ordinary differential equation :

$$\frac{dP_h}{dx} = -(1-P_h)\alpha_h(P_e+P_h-P_eP_h)$$

Therefore we can draw up the McIntyre system :

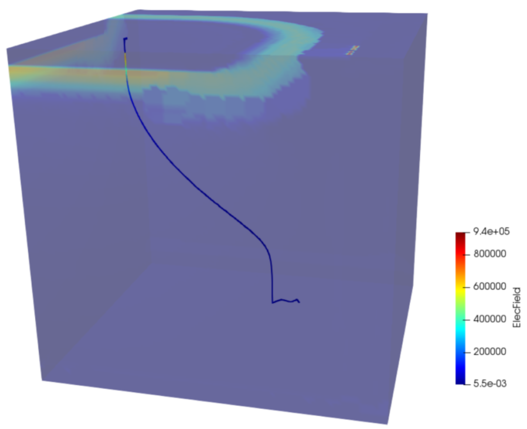
$$\begin{cases} \dfrac{dP_e}{dx} = (1-P_e)\alpha_e(P_e+P_h-P_eP_h) & (1) \\ \dfrac{dP_h}{dx} = -(1-P_h)\alpha_h(P_e+P_h-P_eP_h) & (2) \end{cases}$$

for $0 \le x \le W$.
Adding the couple of boundary value conditions :

$$\begin{cases} P_e(x=0) = 0 & (3) \\ P_h(x=W) = 0 & (4) \end{cases}$$

we have a full 1D coupled and non-linear boundary value problem. Since we have to extract this value at a large number of points, we use a self-made solver, embedded in a C++ program. This solver uses finite difference method coupled with a Newton's method to care of the non-linearity of the problem. The algorithm is different from those implemented in MatLab routine (bvp4c) or SciPy function (solve_bvp) but the comparison with these tools show no difference.

Figure 2: Field line of electric field inside the device



4 Jitter modeling

5 Results and comparisons with experiments

6 Discussion

Algorithm 1: Newton's Method Solver for BVP

input : The Boundary value Problem
input : The Mesh $\mathscr{M}$
input : An initial guess of $Y_{\mathscr{M}}$
input : A maximal tolerance TOL
input : A maximal number of iterations
output: The Solution $Y_{\mathscr{M}}$
output: The final residual error

RES $\leftarrow$ 1000 ;
$NbIterations \leftarrow 0$;
Initialize $w_{\mathscr{M}}$ as a vector of size 2N;

while $RES > TOL$ *and* $NbIterations <$ *MaxNbIterations* **do**
  for *i=1 to 2N* **do**
    Construct S_i;
    Construct R_i;
    Construct $q_i = -\mathbf{N}_{\mathscr{M}} y_i$;
  Construct A;
  Construct β;
  Solve $Aw_{\mathscr{M}} = \hat{\beta}$;
  for *i=1 to 2N* **do**
    $y_i \leftarrow y_i + (w_{\mathscr{M}})_i$
  $RES \leftarrow ||w_{\mathscr{M}}||$;
  $NbIterations \leftarrow NbIterations + 1$;
if $RES \leq TOL$ **then**
  //The method has converged ;
  return Y;
else
  //The method has converged ;
  return Error : No Convergence

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