Interval Estimation

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Trivia Activity - Introduction

Lately we've been exploring the uncertainty of the estimates we get from a sample:

- Based upon the sampling distribution of $\bar{x}$, we saw that μ is the most likely value of $\bar{x}$ for a single sample
- This means $\bar{x}$ from our sample is our best guess at the population parameter μ
- While μ is the most likely value of $\bar{x}$, very often the sample average isn't *exactly* μ
- So our most likely value is almost always wrong...
- A better approach might be to estimate μ using an *interval* based upon $\bar{x}$

Trivia Activity - Directions

Today we will focus on **interval estimation**. For an interval estimate to be meaningful, it should provide a specified level of accuracy. We will explore this idea using triva questions:

- In this activity, I'll ask 16 trivia questions with numeric answers
- Your goal is provide your best estimate, along with an interval that correctly captures the truth 80% of the time
- To prevent cheating, only 15 randomly chosen questions will count
- If 12/15 (80%) of your intervals capture the truth you'll win a prize
- Think about the target accuracy rate when forming your intervals

Trivia Activity

Question 1:

On average, how many ants does an anteater eat in a day?

Question 2:

What is the total population of Canada?

Question 3:

What is the diameter (in inches) of a regulation basketball rim?

Question 4:

How many new (first-year) students enrolled at Grinnell College in 2016?

Trivia Activity

Question 5:

What is the hottest temperature (either degrees F or C) ever recorded in the state of Iowa?

Question 6:

From 1918-2017, the US social Security Administration has recorded the names of 173,916,919 male and 169,671,039 female births. The most popular name, James, belongs to how many Americans over the past 100 years?

Question 7:

What is the median age of the world population (in years)?

Question 8:

In years, how long did it take Michelangelo to paint the ceiling of the Sistine Chapel?

Trivia Activity

Question 9:

The first item ever sold on eBay was a broken laser printer, how much did it sell for (in dollars)?

Question 10:

How many states were part of the United States in the year 1860?

Question 11:

According to a study conducted by the research firm, Counterpoint, Apple earned, on average, how much profit per iPhone in 2017?

Question 12:

How many US presidents have died on the 4th of July?

Trivia Activity

Question 13:

According to Google Maps, how long (to the nearest hour) does it take to drive from Chicago IL to New York NY (assuming the fastest route and no stops)?

Question 14:

How many bones are there in a human foot?

Question 15:

How many letters are in the longest word that is typed using only the left hand? (assuming a standard QWERTY keyboard)

Question 16:

The grocery store chain Hy-Vee has approximately 250 supermarkets located throughout the Midwestern United States, what is the annual revenue of Hy-Vee?

Trivia Answers

Q1: 35,000 ants (source)

Q2: 32,623,490 people (source)

Q3: 18 inches (source)

Q4: 414 students (source)

Q5: 118 F / 48 C (source)

Q6: 4,792,954 people (source)

Q7: 30.4 years (source)

Q8: 4 years (source)

Trivia Answers

Q9: $14 (source)

Q10: 33 states (source)

Q11: $151 (source)

Q12: 4 presidents (source)

Q13: 12 hours (source)

Q14: 26 bones (source)

Q15: 12 letters “stewardesses” (source)

Q16: 10 Billion dollars (source)

Interval Estimation

- Point estimates are almost always wrong
- Including a **margin of error** allows for a more reasonable description of the truth
- In the trivia activity we saw how difficult it was to construct a margin of error that produced 80% **coverage** of the truth
- In our defense, we almost certainly could have come up with a better estimation procedure if we had been using data

Interval Estimates

Generally speaking, an interval estimate of a population parameter has the form:

$$\text{Sample Statistic} \pm \text{Margin of Error}$$

Ideally we'd like the margin of error to carry some quantifiable claim of precision (ie: 80% of the time these intervals will contain the true population parameter)

Confidence Intervals

A **confidence interval** is an interval estimate computed from sample data *using a procedure* that will capture the population parameter with a specified success rate known as the **confidence level**

- Any particular interval either will contain the parameter or won't contain the parameter
- It's the way the interval was created that is special
- The confidence level *doesn't* describe how we feel about any particular interval, it describes the procedure used to create the interval

Confidence Intervals

For a symmetric, bell-shaped distribution, roughly 95% of the values fall within 2 standard deviations from the center. What does this tell us about the Sampling Distribution?

1. For a single sample, it is most likely that our estimate is the center of the sampling distribution
2. Based upon the sampling distribution, 95% of possible estimates are within 2 standard errors of the center

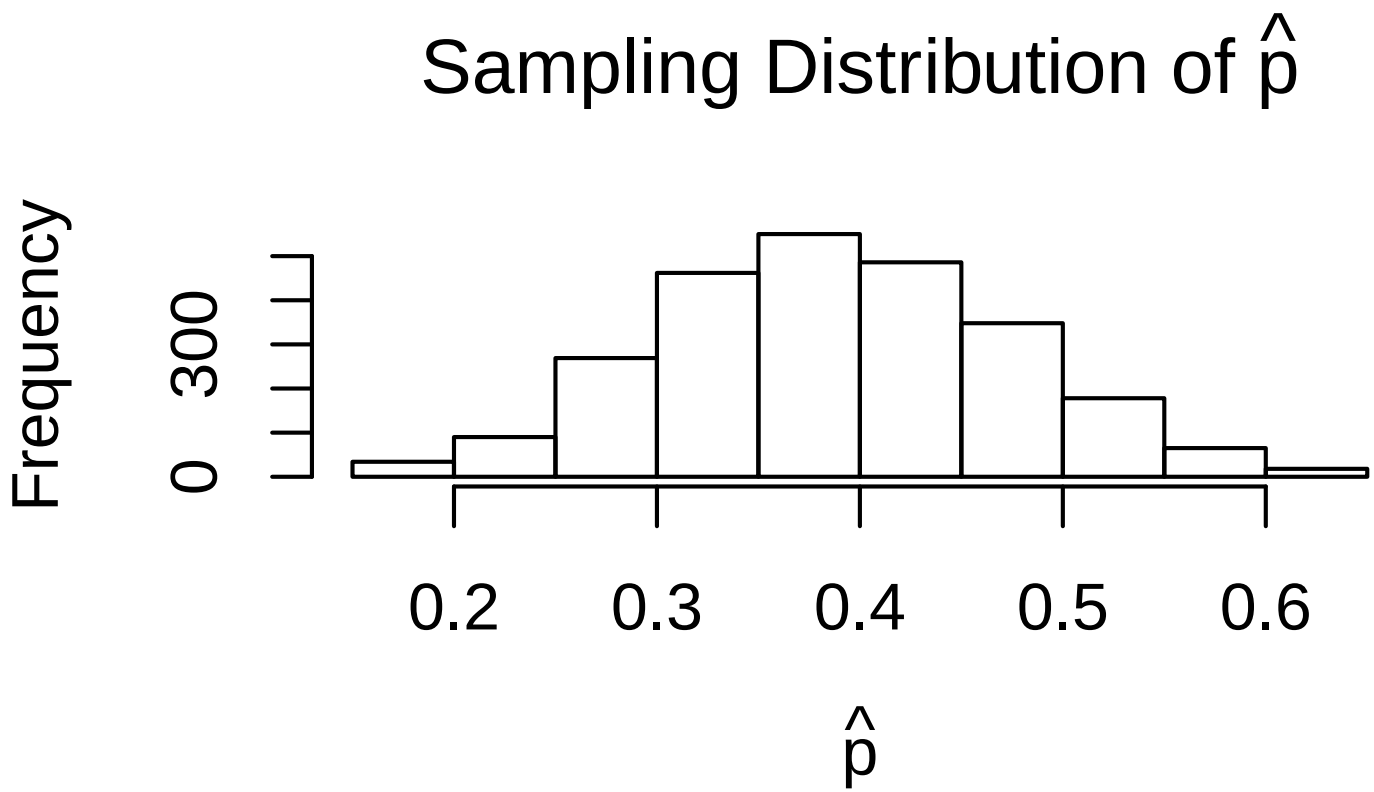
This suggests the following procedure for constructing 95% confidence intervals:

$$\text{Sample Statistic} \pm 2 * \text{SE}$$

But does the procedure actually work?

Confidence Interval Coverage

Let's explore this procedure using the class survey data. Suppose are interested in estimating the proportion of students taking the class for fun (p). We will begin by constructing the sampling distribution of $\hat{p}$ by taking many samples of 20 students:



Confidence Interval Coverage

The standard error of $\hat{p}$ is 0.088. The first random sample had a sample proportion of $\hat{p} = 0.35$, thus the 95% confidence interval for p using this sample is:

$$0.35 \pm 2 * 0.088 = (0.174, 0.526)$$

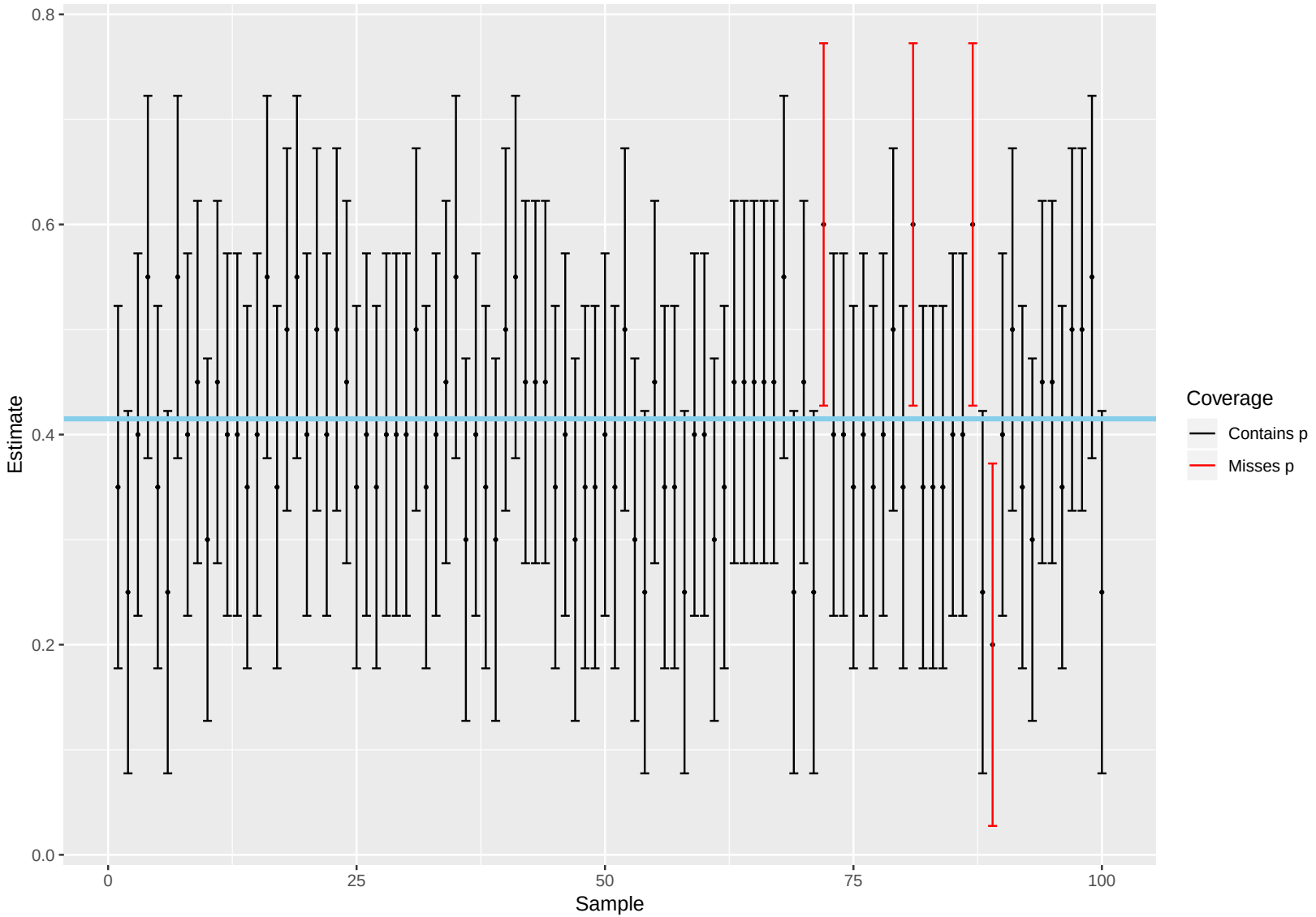
The true proportion of students taking STA-209-02/04 fun is 0.415, so this particular interval does indeeed capture the true p

Confidence Interval Coverage

We can repeat this procedure for each random sample:

Sample ID	Sample proportion	Calculation	95% CI
1	0.35	0.35 +/- 2*0.88	(0.174,0.526)
2	0.25	0.25 +/- 2*0.88	(0.074,0.426)
3	0.4	0.4 +/- 2*0.88	(0.224,0.576)
4	0.55	0.55 +/- 2*0.88	(0.374,0.726)
5	0.35	0.35 +/- 2*0.88	(0.174,0.526)
6	0.25	0.25 +/- 2*0.88	(0.074,0.426)

Confidence Interval Coverage



96 confidence intervals from the first 100 samples contain the truth

Interpreting Confidence Intervals

- Because we only have one sample, we only end up with one confidence interval for the population parameter
- This interval either contains the parameter or it doesn't (ie: there is a 100% or 0% chance the population parameter is in this particular interval)
- For this reason we **avoid** saying things like: "There is a 95% chance that μ is between A and B"
- Instead, we speak in terms of confidence: "We are 95% confident the interval (A, B) contains the true value of μ"
- We are confident in the procedure used to make the interval, not necessarily the interval itself

How do we Construct a Confidence Interval from a Single Sample?

- Right now we've seen how to construct a confidence interval using the standard error (SE)
- But how do we find the standard deviation of the sampling distribution when we only have one sample??
- For much of the remainder of the course we will explore different ways of reconstructing the sampling distribution for various situations
- We will start with a simple, seemingly too good to be true, method:
  - Repeatedly taking samples from our single sample
  - This approach is called **bootstrapping**, and we will explore it in our next lab

Conclusion

Right now you should...

1. Understand advantages of interval estimates
2. Be able to calculate a confidence interval when given a sample statistic and its standard error
3. Correctly interpret a confidence interval and recognize common misconceptions
4. Understand the difference between "margin of error", "standard error", and "standard deviation"

These notes cover Section 3.2 the textbook, I encourage you to read through the section and its examples