Interval Estimation

Ryan Miller

Trivia Activity - Introduction

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- While μ is the most likely value of $\bar{x}$, very often the sample average isn't *exactly* μ
- So our most likely value is almost always wrong...
- A better approach might be to estimate μ using an *interval* based upon $\bar{x}$

Trivia Activity - Directions

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- In this activity, I'll ask 16 trivia questions with numeric answers
- Your goal is to come up with an interval that correctly captures the truth 80% of the time
- To prevent cheating, only 15 randomly chosen questions will count
- If 12/15 (80%) of your intervals capture the truth you'll win a prize
- Think carefully about this target accuracy rate when forming your intervals

Trivia Activity

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Question 4:

How many new (first-year) students enrolled at Grinnell College in 2016?

Trivia Activity

Question 5:

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Question 8:

In years, how long did it take Michelangelo to paint the ceiling of the Sistine Chapel?

Trivia Activity

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Question 11:

How many "days" are in a year for the planet Mars (ie: its orbital period in Earth days)?

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Question 12:

How many US presidents have died on the 4th of July?

Question 13:

> *According to Google Maps, how long (to the nearest hour) does it take to drive from Chicago IL to New York NY (assuming the fastest route and no stops)?*

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How many letters are in the longest word that is typed using only the left hand? (assuming a standard QWERTY keyboard)

Question 16:

How many feet are in a mile?

Trivia Answers

Q1: 35,000 ants (source)

Q2: 348,580 people (source)

Q3: 26.5 million (source)

Q4: 414 students (source)

Q5: 118 F / 48 C (source)

Q6: 2,240 people (source)

Q7: 30.4 years (source)

Q8: 4 years (source)

Trivia Answers

Q9: $14 (source)

Q10: 33 states (source)

Q11: 687 (source)

Q12: 3 presidents (source)

Q13: 12 hours (source)

Q14: 26 bones (source)

Q15: 12 letters “stewardesses” (source)

Q16: 5280 feet (source)

Interval Estimation

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- Point estimates are almost always wrong
- Including a **margin of error** allows for a more reasonable description of the truth
- In the trivia activity we saw how *difficult* it was to come up with a margin of error that produced 80% **coverage** of the truth
- In our defense, we almost certainly could have come up with a better estimation procedure if we had been using data

Interval Estimates

Generally speaking, an interval estimate of a population parameter has the form:

$$\text{Sample Statistic} \pm \text{Margin of Error}$$

Ideally, we'd like the margin of error to carry some *quantifiable* claim of precision (ie: 80% of the time these intervals will contain the true population parameter)

Confidence Intervals

A **confidence interval** is an interval estimate computed from sample data *using a procedure* that will capture the population parameter with a specified success rate known as the **confidence level**

- Any particular interval either will contain the parameter or won't contain the parameter
- It's the way the interval was created that is special
- The confidence level *doesn't* describe how we feel about any particular interval, it describes the procedure used to create the interval

Confidence Intervals

1. For a symmetric, bell-shaped distribution, roughly 95% of values fall within 2 standard deviations of the center. You may have noticed that *sampling distributions* tend to have this shape.
2. For a single sample, we're most likely to see an estimate at the very center of the sampling distribution

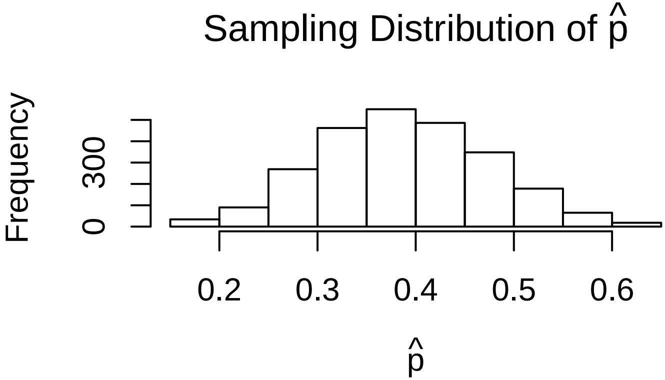
Together, these facts suggest the following procedure for constructing 95% confidence intervals:

$$\text{Sample Statistic} \pm 2 * \text{SE}$$

But does the procedure actually work?

Confidence Interval Coverage

Let's explore this procedure using the class survey data. Suppose are interested in estimating the proportion of students taking the class for fun (p). We will begin by constructing the sampling distribution of $\hat{p}$ by taking many samples of 20 students:



Confidence Interval Coverage

The standard error of $\hat{p}$ is 0.088. The first random sample had a sample proportion of $\hat{p} = 0.35$, thus the 95% confidence interval for p using this sample is:

$$0.35 \pm 2 * 0.088 = (0.174, 0.526)$$

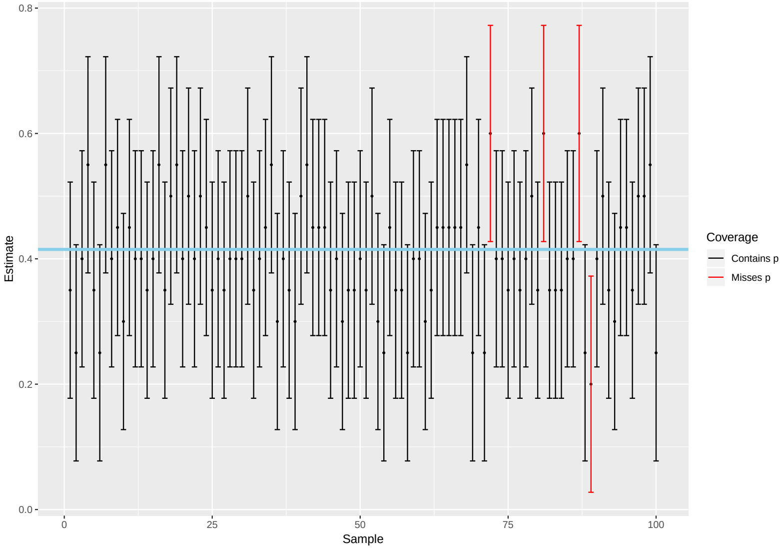
The true proportion of students taking STA-209 for fun (In Fall 2018) is 0.415, so this particular interval does indeed capture the true p

Confidence Interval Coverage

We can repeat this procedure for each random sample:

Sample ID	Sample proportion	Calculation	95% CI
1	0.35	0.35 +/- 2*0.88	(0.174,0.526)
2	0.25	0.25 +/- 2*0.88	(0.074,0.426)
3	0.4	0.4 +/- 2*0.88	(0.224,0.576)
4	0.55	0.55 +/- 2*0.88	(0.374,0.726)
5	0.35	0.35 +/- 2*0.88	(0.174,0.526)
6	0.25	0.25 +/- 2*0.88	(0.074,0.426)

Confidence Interval Coverage



96 confidence intervals from the first 100 samples contain the truth

Interpreting Confidence Intervals

- Because we only have one sample, we only end up with one confidence interval for the population parameter
- This interval either contains the parameter or it doesn't (ie: there is a 100% or 0% chance the population parameter is in this particular interval)
- For this reason we **avoid** saying things like: "There is a 95% chance that μ is between A and B"
- Instead, we speak in terms of confidence: "We are 95% confident the interval (A, B) contains the true value of μ"
- We are confident in the procedure used to make the interval, *not* necessarily the single interval

Confidence Intervals from a Single Sample?

- So far we've seen how to construct a confidence interval using the standard error (SE)
- But how do we find the standard deviation of the sampling distribution when we only have one sample??
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- We will start with a simple, seemingly too good to be true, method:
  - Repeatedly taking samples from our single sample
  - This approach is called **bootstrapping**, and we will explore it in our next lab

Conclusion

Right now you should...

1. Understand advantages of interval estimates
2. Be able to calculate a confidence interval when given a sample statistic and its standard error
3. Correctly interpret a confidence interval and recognize common misconceptions
4. Understand the difference between "margin of error", "standard error", and "standard deviation"

These notes cover Section 3.2 the textbook, I encourage you to read through the section and its examples