Probability (Examples)

Ryan Miller

Introduction

- This presentation will walk through a couple of different examples aimed at providing you additional practice with probability
- I encourage you to pause the video and try these examples on your own before watching the solutions

Example #1 - part 1

A local hospital has 22 patients staying overnight, 15 are adults and 7 are children. Among the adults, this is the first ever hospital stay for 4 of them. Among the children, this is the first ever hospital stay for 5 of them. Use this information to calculate the following probabilities:

1) A randomly selected patient is an adult
2) A randomly selected patient is an adult, given it's their first ever hospital stay
3) A randomly selected patient is in their first ever hospital stay, given they are a child
4) A randomly selected patient is in their first ever hospital stay, *or* they are a child

Example #1 - part 1 (solution)

1) $P(\text{Adult}) = 15/22$, there are 22 patients and 15 are adults

Example #1 - part 1 (solution)

1) $P(\text{Adult}) = 15/22$, there are 22 patients and 15 are adults
2) $P(\text{Adult}|\text{First}) = 4/9$, there are 9 first-time patients and 4 are adults

Example #1 - part 1 (solution)

1) $P(\text{Adult}) = 15/22$, there are 22 patients and 15 are adults
2) $P(\text{Adult}|\text{First}) = 4/9$, there are 9 first-time patients and 4 are adults
3) $P(\text{First}|\text{Child}) = 5/7$, there are 7 children and 5 of them are first-time patients

Example #1 - part 1 (solution)

1) $P(\text{Adult}) = 15/22$, there are 22 patients and 15 are adults
2) $P(\text{Adult}|\text{First}) = 4/9$, there are 9 first-time patients and 4 are adults
3) $P(\text{First}|\text{Child}) = 5/7$, there are 7 children and 5 of them are first-time patients
4) $P(\text{First or Child}) = P(\text{First}) + P(\text{Child}) - P(\text{First and Child}) = 9/22 + 7/22 - 5/22 = 0.5$, notice we could have calculated this *directly* by realizing there are 7 children and 4 first-time adults (totaling 11 of 22 patients)

Example #1 - part 2

A local hospital has 22 patients, 15 are adults and 7 are children.
Among the adults, this is the first ever hospital stay for 4 of them.
Among the children, this is the first ever hospital stay for 5 of them.
Now let's consider randomly selecting two patients sequentially:

1) What is the probability that *both* selections are adults?
2) What is the probability that *at least one* of the selections is an adult?

Example #1 - part 2 (solution)

1) Let A_1 and A_2 denote the selection of adults, then $P(A_1 \text{ and } A_2) = P(A_2|A_1) * P(A_1) = \frac{14}{21} * \frac{15}{22} = 0.45$; notice these events are not independent

Example #1 - part 2 (solution)

1) Let A_1 and A_2 denote the selection of adults, then $P(A_1 \text{ and } A_2) = P(A_2|A_1) * P(A_1) = \frac{14}{21} * \frac{15}{22} = 0.45$; notice these events are not independent
2) Using the additional rule could get pretty complicated here because the events are not independent. Instead, let C_1 and C_2 denote the selection of adults and consider $P(A_1 \text{ or } A_2) = 1 - P(\text{Neither}) = P(C_2|C_1) * P(C_1) = \frac{6}{21} * \frac{7}{22} = 1 - 0.09 = 0.91$

Example #1 - part 2 (solution)

1) Let A_1 and A_2 denote the selection of adults, then $P(A_1 \text{ and } A_2) = P(A_2|A_1) * P(A_1) = \frac{14}{21} * \frac{15}{22} = 0.45$; notice these events are not independent
2) Using the additional rule could get pretty complicated here because the events are not independent. Instead, let C_1 and C_2 denote the selection of adults and consider $P(A_1 \text{ or } A_2) = 1 - P(\text{Neither}) = P(C_2|C_1) * P(C_1) = \frac{6}{21} * \frac{7}{22} = 1 - 0.09 = 0.91$

Example #2

Consider a well-shuffled deck of 52 playing cards and the random selection of two cards, a "top" card and a "bottom" card

1) The following line of reasoning is incorrect: "Because of the addition rule, the probability that the top card is the jack of clubs *and* the bottom card is the jack of hearts is 2/52." Point out the flaw in this argument.
2) The following line of reasoning is also incorrect: "Because of the addition rule, the probability that the top card is the jack of clubs *or* the bottom card is the jack of hearts is 2/52." Point out the flaw in this argument.
3) The statements in 1 and 2 both contain flaws, but these mistakes are not equally bad. Which approach will result in an answer closer to the truth (for the situation it describes)?

Example #2 (solution)

1) The addition rule pertains to intersections or "or" statements, so it shouldn't be applied here

Example #2 (solution)

1) The addition rule pertains to intersections or "or" statements, so it shouldn't be applied here
2) The events involved are not disjoint, it is possible for the top card to be the jack of clubs and the bottom card to be the jack of hearts.

Example #2 (solution)

1) The addition rule pertains to intersections or "or" statements, so it shouldn't be applied here
2) The events involved are not disjoint, it is possible for the top card to be the jack of clubs and the bottom card to be the jack of hearts.
3) The second statement is much closer to the truth, because the possibility for both is very small ($\frac{1}{52} * \frac{1}{51}$ by the multiplication rule)