

Probability (Complement Rule)

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 - ▶ So, please never report a probability outside of this range on an exam or homework assignment (if necessary leave your incorrect work and don't circle a final answer)
- ▶ An event with a probability of 1 will *always* occur
 - ▶ For example, if we flip a single coin: $P(\text{Heads or Tails}) = 1$
- ▶ An event with a probability of 0 will *never* occur
 - ▶ For example, if we sample a random adult and measure their height: $P(6'0) = 0$
 - ▶ This might seem odd, but there are infinitely many different heights, so the probability of getting someone who's *exactly* 6 feet and 0.00000... inches tall is zero

Probability of a Sample Space

- ▶ The probability of the intersection of all *outcomes* in a sample space is 1
 - ▶ If we flip a single coin: $P(\text{Heads or Tails}) = 1$
 - ▶ If we randomly sample an exam: $P(A \text{ or } B \text{ or } C \text{ or } D \text{ or } F) = 1$

The Complement Rule

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- ▶ More formally we describe these **complementary events** using the following notation
 - ▶ If A is an event, then A^C is the complement of event A
 - ▶ Thus, $P(A) + P(A^C) = 1$
 - ▶ It's sometimes useful to rearrange this expression:
$$P(A^C) = 1 - P(A)$$

Example

- ▶ Driving home from work, Professor Miller approaches a traffic light that he knows will be Green with probability 0.4, Yellow with probability 0.1, or Red with probability 0.5
 - ▶ What is the probability the light is *not Red*?
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- ▶ You might have also thought to calculate this probability by adding up the other outcomes in the sample space
 - ▶ $P(\text{Green}) + P(\text{Yellow}) = 0.5$
- ▶ There's nothing wrong with this approach, but be aware that it only works for *disjoint events*
 - ▶ By definition all *outcomes* in a sample space are *disjoint*, so it's not a problem here
 - ▶ However, many *events* are not disjoint, and in the next presentation we'll discuss this idea in greater detail (the addition rule)

Conclusion

- ▶ A *sample space* is the collection of all possible *outcomes* of a trial
- ▶ The probability of the intersection of all outcomes in a sample space is 1
- ▶ The probabilities of an event occurring, and that event not occurring, must sum to 1 (complementary events)
 - ▶ $P(A) + P(A^C) = 1$