Probability (Addition Rule)

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  - Consider rolling a six-sided die, the event of rolling a six is disjoint from the event rolling an odd number
- For two disjoint events, we can find the probability of *unions* by addition
  - $P(A \text{ or } B) = P(A) + P(B)$
  - For a six-sided die, $P(\text{Six or Odd Number}) = P(\text{Six}) + P(\text{Odd Number}) = 1/6 + 3/6 = 2/3$

Disjoint Events

It's easy to visually confirm this example by looking at a simple representation of the sample space:

1	2	3
4	5	6

Disjoint Events

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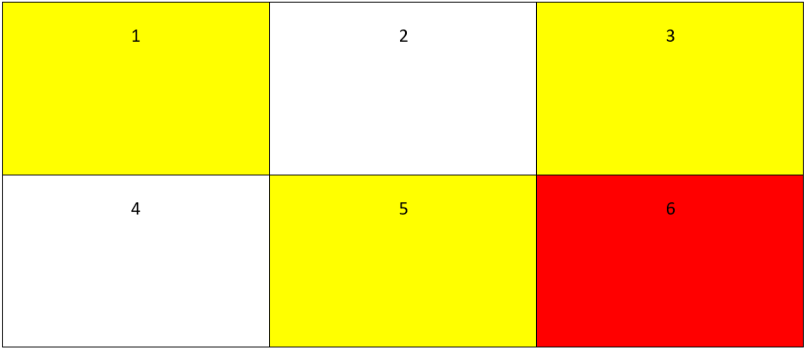
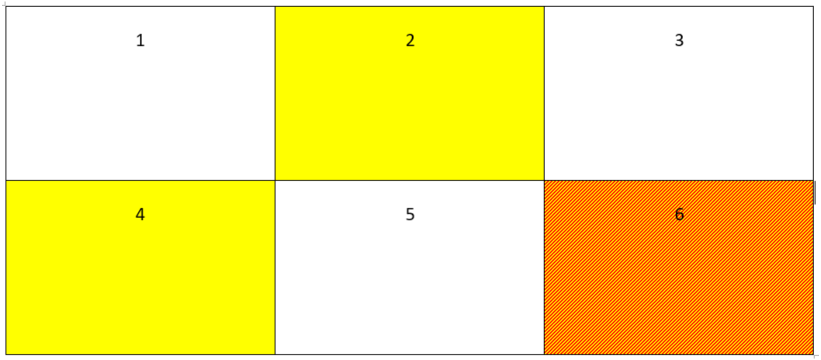


Non-disjoint events

In contrast, consider $P(\text{Six or Even Number})$, clearly these events are *not disjoint*, so adding their probabilities would be a mistake



The Addition Rule

- In general, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
  - This is known as the **addition rule**
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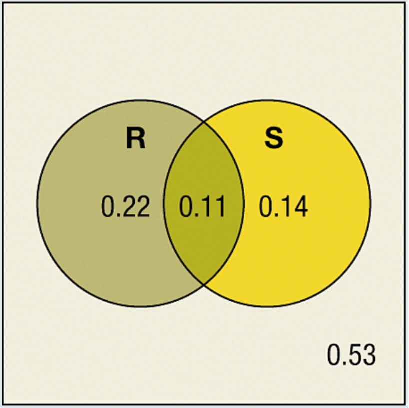
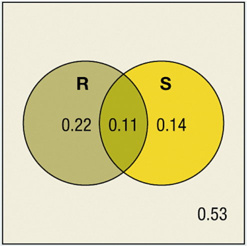
$P(\text{Six or Even Number}) = P(\text{Six}) + P(\text{Even Number}) - P(\text{Six and Even Number}) = 1/6 + 3/6 - 1/6 = 1/2$

Venn Diagrams

- Venn diagrams are frequently used as a visual aid when learning the addition and complement rules
- The diagram below depicts survey results where 33% of college students were in a relationship (R), 25% were involved in sports (S), and 11% were in both



Venn Diagrams - Example



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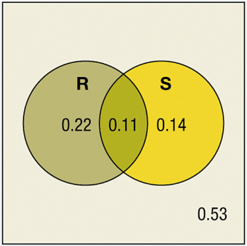

1) $P(R \text{ or } S) = 0.22 + 0.11 + 0.14 = 0.47$ (direct calculation)
2) $P(R \text{ or } S) = 0.33 + 0.25 - 0.11 = 0.47$ (addition rule)

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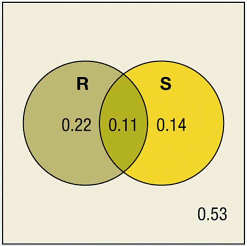

1) $P(R \text{ or } S) = 0.22 + 0.11 + 0.14 = 0.47$ (direct calculation)
2) $P(R \text{ or } S) = 0.33 + 0.25 - 0.11 = 0.47$ (addition rule)
3) $P(R \text{ or } S) = 1 - P(\text{Neither}) = 1 - 0.53 = 0.47$ (complement rule)

Conclusion

We use the *addition rule* to find the probability of the union of any two events:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

If the events are independent, we know that there intersection is zero, meaning $P(A \text{ and } B) = 0$ and the union of the events is simply the sum of their individual probabilities