# Probability (Complement Rule)

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- An event with a probability of 1 will *always* occur
  - For example, if we flip a single coin: $P(\text{Heads or Tails}) = 1$
- An event with a probability of 0 will *never* occur
  - For example, if we sample a random adult and measure their height: $P(6'0) = 0$
  - This might seem odd, but there are infinitely many different heights, so the probability of getting someone who's *exactly* 6 feet and 0.00000... inches tall is zero

# Probability of a Sample Space

- The probability of the *union of all outcomes* in a sample space is 1
  - If we flip a single coin: $P(\text{Heads or Tails}) = 1$
  - If we randomly sample an exam: $P(\text{A or B or C or D or F}) = 1$

# The Complement Rule

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  - For a random exam: $P(\text{A}) + P(\text{Not A}) = 1$
- More formally we describe these **complementary events** using the following notation
  - If $A$ is an event, then $A^C$ is the complement of event $A$
  - Thus, $P(A) + P(A^C) = 1$
  - It's sometimes useful to rearrange this expression: $P(A^C) = 1 - P(A)$

# Example

- Driving home from work, Professor Miller approaches a traffic light that he knows will be Green with probability 0.4, Yellow with probability 0.1, or Red with probability 0.5
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- You might have also thought to calculate this probability by adding up the other outcomes in the sample sample
  - $P(\text{Green}) + P(\text{Yellow}) = 0.5$
- There's nothing wrong with this approach, but be aware that it only works for *disjoint events*
  - By definition all *outcomes* in a sample space are *disjoint*, so it's not a problem here
  - However, many *events* are not disjoint, and in the next presentation we'll discuss this idea in greater detail (the addition rule)

# Conclusion

- A *sample space* is the collection of all possible *outcomes* of a trial
- The probability of the intersection of all outcomes in a sample space is 1
- The probabilities of an event occurring, and that event not occurring, must sum to 1 (complementary events)
  - $P(A) + P(A^C) = 1$