

# AN ARTICLE ABOUT SOMETHING\*

Machin<sup>1</sup>, Truc<sup>2</sup>, and Bidule<sup>1</sup>

<sup>1</sup>INSTITUT DU FROMAGE

<sup>2</sup>BAGUETTE UNIVERSITÉ SORBONNE-EST

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## ABSTRACT

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▢ This pdf contains internal links: clicking on a notion leads to its definition.<sup>1</sup>

The notion of first-order formula is introduced in Section 2, Page 3.

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**TODO:** Look at my nice brackets:  $\llbracket 0, n \rrbracket$ !

## 1. CATEGORICAL PRELIMINARIES

### 1.1. THE POWERSET MONAD

▢ The *powerset* of  $X$  is denoted by  $\mathcal{P}(X)$ . The categories of sets, monoids and semigroups are denoted by *Set*, *Mon* and *Sgp*, respectively. This notion is *unknown*. We define this *unknown* stuff, and then call it again: *unknown*.

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\*Merci à l'ANR pour tout l'argent !

<sup>1</sup> This result was achieved by using the *knowledge* package and its companion tool *knowledge-clustering*.

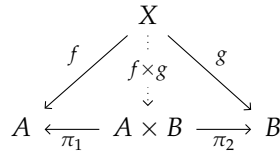


FIGURE 1.1: Cartesian product.

Category	Arrows
Set	Functions
Mon	Monoid morphisms
Sgp	Semigroup morphisms

TABLE 1.1: *Some table*

⌞ In a given category  $\mathcal{C}$ , a *cartesian product* of two objects  $X$  and  $Y \in \mathcal{C}$  is another object  $X \times Y \in \mathcal{C}$  together with a pair of arrows  $\pi_1 : X \times Y \rightarrow X$ ,  $\pi_2 : X \times Y \rightarrow Y$  such that for every object  $U \in \mathcal{C}$ , for every arrows  $f : U \rightarrow X$  and  $g : U \rightarrow Y$ , there exists a unique arrow  $f \times g : U \rightarrow X \times Y$  making the diagram in Figure 1.1 commute. A *cartesian category* is a triple  $(\mathcal{C}, \times, 1)$  where:

- $\times$  is a map that associates a cartesian product to each pair of objects of  $\mathcal{C}$ , and
- $1$  is a terminal object of  $\mathcal{C}$ —i.e. an object such that for all  $X \in \mathcal{C}$ .

Note that terminal objects and cartesian products are unique, up to isomorphism, and that  $X \times 1$  and  $1 \times X$  are isomorphic to  $X$  for all  $X \in \mathcal{C}$ .

⌞ It is routine to check that *Set*, *Mon* and *Sgp* are cartesian. Moreover, the forgetful functors from *Mon*  $\rightarrow$  *Sgp* and *Sgp*  $\rightarrow$  *Set* preserve cartesian products.

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## 1.2. OTHER STUFF

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## 2. FIRST-ORDER LOGIC

⌈ Given a signature<sup>2</sup>  $\mathfrak{s}$  without any function symbol, the set of *first-order formulæ* over some fixed infinite set of variables is defined by the grammar

$$\varphi ::= x \mid P_{(n)}(x, \dots, x) \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \varphi \Rightarrow \varphi \mid \forall x. \varphi \mid \exists x. \varphi,$$

where  $x$  ranges over the set of variables and  $P_{(n)}$  over the set of predicates symbols of arity  $n \in \mathbb{N}$ .

**Theorem 2.1** (Schützenberger-McNaughton-Papert’s theorem). Some nice theorem about first-order logic.

**Fact 2.2.** Some fact.

**Fact 2.3.** Some other fact.

By Facts 2.2 and 2.3, we have (...).

⌈ We will mainly focus on the first-order theory of linear orderings [Ros82]: we assume that the signature  $\mathfrak{s}$  consists of exactly one predicate of arity 2, denoted  $<$ , and that this predicate is always interpreted, in every model, as a linear order.

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<sup>2</sup> Recall that a signature is a set of function symbols and a set of predicate symbols.

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## REFERENCES

- [Ros82] Joseph G Rosenstein. *Linear orderings*. Academic press, 1982. URL: <https://libgen.lc/search.php?req=+Linear+orderings>.