An article about something[*]

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Abstract

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This pdf contains internal links: clicking on a notion leads to its definition.[1]

The notion of first-order formula is introduced in Section 2, Page 3.

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TODO: Look at my nice brackets: $[\![0, n]\!]$!

1. Categorical preliminaries

1.1. The powerset monad

The *powerset* of X is denoted by $\mathcal{P}(X)$. The categories of sets, monoids and semigroups are denoted by Set, Mon and Sgp, respectively. This notion is unknown. We define this *unknown* stuff, and then call it again: unknown.

[*]Merci à l'ANR pour tout l'argent !

[1] This result was achieved by using the `knowledge` package and its companion tool `knowledge-clustering`.

$$\begin{tikzcd}
& X \arrow[dl, "f"'] \arrow[d, dotted, "f\times g"] \arrow[dr, "g"] & \\
A & A \times B \arrow[l, "\pi_1"] \arrow[r, "\pi_2"'] & B
\end{tikzcd}$$

Figure 1.1: *Cartesian product.*

Category	Arrows
Set	Functions
Mon	Monoid morphisms
Sgp	Semigroup morphisms

TABLE 1.1: *Some table*

In a given category $\mathcal{C}$, a *cartesian product* of two objects X and $Y \in \mathcal{C}$ is another object $X \times Y \in \mathcal{C}$ together with a pair of arrows $\pi_1 : X \times Y \to X$, $\pi_2 : X \times Y \to Y$ such that for every object $U \in \mathcal{C}$, for every arrows $f : U \to X$ and $g : U \to Y$, there exists a unique arrow $f \times g : U \to X \times Y$ making the diagram in Figure 1.1 commute. A *cartesian category* is a triple $(\mathcal{C}, \times, 1)$ where:

- $\times$ is a map that associates a cartesian product to each pair of objects of $\mathcal{C}$, and
- 1 is a terminal object of $\mathcal{C}$—i.e. an object such that for all $X \in \mathcal{C}$.

Note that terminal objects and cartesian products are unique, up to isomorphism, and that $X \times 1$ and $1 \times X$ are isomorphic to X for all $X \in \mathcal{C}$.

It is routine to check that Set, Mon and Sgp are cartesian. Moreover, the forgetful functors from Mon $\to$ Sgp and Sgp $\to$ Set preserve cartesian products.

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1.2. Other stuff

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2. First-order logic

Given a signature[2] $\mathfrak{s}$ without any function symbol, the set of *first-order formulæ* over some fixed infinite set of variables is defined by the grammar

$$\varphi ::= x \mid P_{(n)}(x, \dots, x) \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \varphi \Rightarrow \varphi \mid \forall x.\, \varphi \mid \exists x.\, \varphi,$$

where x ranges over the set of variables and $P_{(n)}$ over the set of predicates symbols of arity $n \in \mathbb{N}$.

Theorem 2.1 (Schützenberger-McNaughton-Papert's theorem)**.** Some nice theorem about first-order logic.

Fact 2.2. Some fact.

Fact 2.3. Some other fact.

By Facts 2.2 and 2.3, we have (...).

We will maintly focus on the first-order theory of linear orderings [Ros82]: we assume that the signature $\mathfrak{s}$ consists of exactly one predicate of arity 2, denoted $<$, and that this predicate is always interpreted, in every model, as a linear order.

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[2] Recall that a signature is a set of function symbols and a set of predicate symbols.

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References

[Ros82] Joseph G Rosenstein. *Linear orderings*. Academic press, 1982. URL: https://libgen.lc/search.php?req=+Linear+orderings.