

Quantitative algebraic characterisations on truly infinite words

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joint works with

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Highlights '22 - Paris

I ♥ algebra!

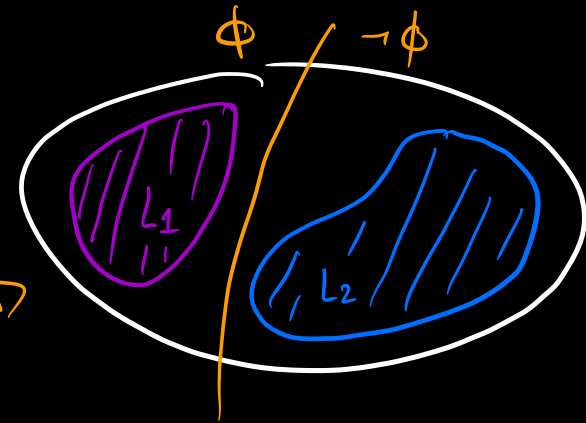
Thm [Schützenberger '65 & McNaughton-Papert '71]

regular lang. $\rightarrow L$ is definable in first-order logic
IFF
the syntactic monoid of L is aperiodic

Ex $(aa)^*$ not definable in FO

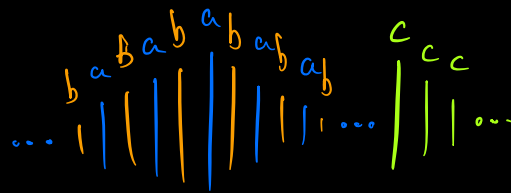
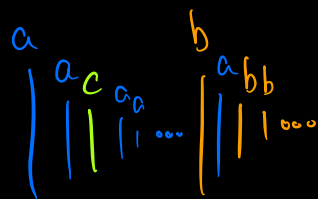
Thm [Hendickell '88]

L_1 and L_2 can be separated by first-order logic
IFF (but somewhat cute!)
some condition on a weird algebraic construction ...
(aperiodic 2-pointlike sets)



To infinity and beyond!

What about bigger words?



word over the
rationals that
shuffle a's and b's.

$$\omega\text{-words} \subseteq \text{countable ordinal words} \subseteq \text{countable scattered words} \subseteq \text{all countable words}$$

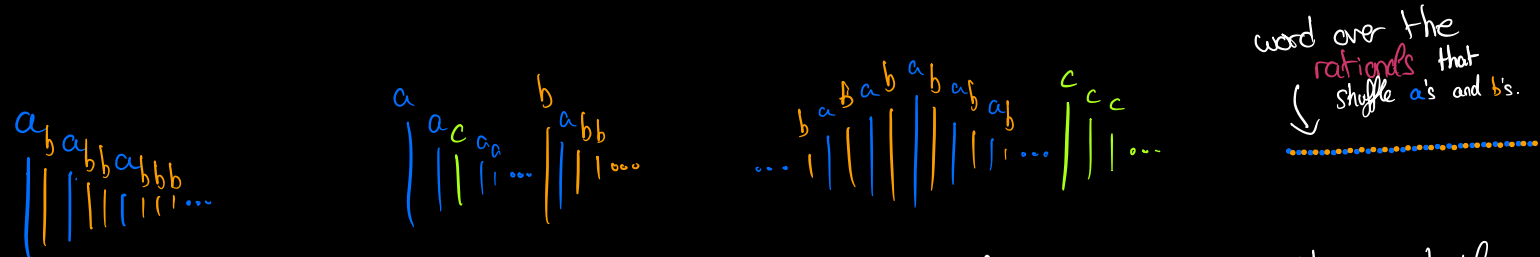
- Nice algebraic notions to study regular languages
- Nice algebraic characterisations of first-order definability!

Not a lot of results on separation.

R. Morvan

until now!

Qualitative & quantitative characterisations



$\omega\text{-words} \subseteq \text{countable ordinal words} \subseteq \text{countable scattered words} \subseteq \text{all countable words}$

Characterisation
of FO-definability

Characterisation
of FO-separability

[Perin '84]	[Beton '01]	[Bis-Curton '11]	[Colcombet - Sreejith '18]
	cannot detect <u>groups</u>	cannot detect <u>groups</u> and <u>gaps</u>	cannot detect <u>groups</u> , <u>gaps</u> and <u>shuffles</u>
[Place-Zitoun '16]	[Colcombet - van Jaarsveld - Fossacs '22]	[Colcombet - M. unpublished]	working on it...

+ We're also studying
WTSO, FO[cut], FO[ord], ...

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(have you ever heard
about knowledge-clustering?)

