ECE4880J: Computer Vision July 22, 2022

Homework 6: Graph neural networks

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Q1. Permutation-equivalent property of PointNet

1. $y \in \mathbb{R}^{1\times d_2}$
2. $y = y^{'}$. Since the permutation matrix only changes the order of all data points in X, when taking the max pool which downsamples the result with maximum, the output does not change.
3. W_1 and W_2 does not affect the property. As long as the base dimension N does not change, the max pool will not be affected.

Q2. Oversmoothing issue of GCNs

1. $Y \in \mathbb{R}^{N\times D}$
2. $\text{ReLU}(A\dots\text{ReLU}(A^2\text{ReLU}(A\dots(\text{ReLU}(AXW_0))\dots W_{i-1})W_iW_{i+1})\dots W_{k-1})$
3. $A = \begin{pmatrix} 0 & 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0 & 0 & 0 & 0 \\ 0.25 & 0 & 0 & 0 & 0.75 \\ 0.25 & 0 & 0 & 0 & 0 \\ 0.25 & 0 & 0.75 & 0 & 0 \end{pmatrix}$
4. $A^{10} = \begin{pmatrix} 0.0666027 & 0.0182018 & 0.103005 & 0.0182018 & 0.103005 \\ 0.0182018 & 0.00503349 & 0.0282679 & 0.00503349 & 0.0282679 \\ 0.103005 & 0.0282679 & 0.187698 & 0.0282679 & 0.131385 \\ 0.0182018 & 0.00503349 & 0.0282679 & 0.00503349 & 0.0282679 \\ 0.103005 & 0.0282679 & 0.131385 & 0.0282679 & 0.187698 \end{pmatrix}$

$A^{100} = \begin{pmatrix} 0.0000157917 & 4.33154e-6 & 0.0000244548 & 4.33154e-6 & 0.0000244548 \\ 4.33154e-6 & 1.18811e-6 & 6.70775e-6 & 1.18811e-6 & 6.70775e-6 \\ 0.0000244548 & 6.70775e-6 & 0.0000378703 & 6.70775e-6 & 0.0000378703 \\ 4.33154e-6 & 1.18811e-6 & 6.70775e-6 & 1.18811e-6 & 6.70775e-6 \\ 0.0000244548 & 6.70775e-6 & 0.0000378703 & 6.70775e-6 & 0.0000378703 \end{pmatrix}$

$A^{1000} = \begin{pmatrix} 8.96961e-42 & 2.46029e-42 & 1.38902e-41 & 2.46029e-42 & 1.38902e-41 \\ 2.46029e-42 & 6.74838e-43 & 3.80997e-42 & 6.74838e-43 & 3.80997e-42 \\ 1.38902e-41 & 3.80997e-42 & 2.15101e-41 & 3.80997e-42 & 2.15101e-41 \\ 2.46029e-42 & 6.74838e-43 & 3.80997e-42 & 6.74838e-43 & 3.80997e-42 \\ 1.38902e-41 & 3.80997e-42 & 2.15101e-41 & 3.80997e-42 & 2.15101e-41 \end{pmatrix}$

5. $A^{1000}x_1 = \begin{pmatrix} 8.96961e-39 \\ 2.46029e-39 \\ 1.38902e-38 \\ 2.46029e-39 \\ 1.38902e-38 \end{pmatrix}$ $A^{1000}x_2 = \begin{pmatrix} 2.46029e-39 \\ 6.74838e-40 \\ 3.80997e-39 \\ 6.74838e-40 \\ 3.80997e-39 \end{pmatrix}$