Hash

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Dictionary

- Collection of pairs.
  - (key, element)
  - Pairs have different keys.
- Operations.
  - Search(theKey)
  - Delete(theKey)
  - Insert(theKey, theElement)

- A <u>linear list</u> is an ordered sequence of elements.
- A <u>dictionary</u> is just a collection of pairs.

Hash Tables

- A dictionary data structure
  - Collection of pairs (key, element)
  - Search, Delete, Insert operations
- Worst-case time for Search, Insert, and Delete is O(size).
- Expected time is O(1).

Main Idea of Hash

- Associate key with index

k ? $\longrightarrow$ A[f(k)]

Ideal Hashing

- Uses an 1D array (or table) table[0:b-1].
  - Each position of this array is a bucket.
  - A bucket can normally hold only one dictionary pair.
- Uses a hash function f that converts each key k into an index in the range [0, b-1].
  - f(k) is the home bucket for key k.
- Every dictionary pair (key, element) is stored in its home bucket table[f[key]].

Ideal Hashing Example

- Pairs are: (22,a), (33,c), (3,d), (73,e), (85,f).
- Hash table is table[0:7], b = 8.
- Hash function is key/11.
- Pairs are stored in table as below:

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
(3,d)		(22,a)	(33,c)			(73,e)	(85,f)

- Search, Insert, and Delete take O(1) time.

What Can We Go Wrong?

(3,d)		(22,a)	(33,c)			(73,e)	(85,f)
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]

- Where does (26,g) go?
- Keys that have the same home bucket are synonyms.
  - 22 and 26 are synonyms with respect to the hash function that is in use.
- The home bucket for (26,g) is already occupied.
- Where does (100,h) go?

What Can We Go Wrong?

(3,d)		(22,a)	(33,c)			(73,e)	(85,f)

- A collision occurs when the home bucket for a new pair is occupied by a pair with a different key.
- An overflow occurs when there is no space in the home bucket for the new pair.
- When a bucket can hold only one pair, collisions and overflows occur together.
- We need a method to handle overflows.

Hash Table Issues

- Choice of hash function.
- Overflow handling method.

Hash Functions

- Two parts:
  - Convert key into a nonnegative integer in case the key is not an integer.
    - Done by the function hash().
  - Map an integer into a home bucket.
    - f(k) is an integer in the range [0, b-1] where b is the number of buckets in the table.

String to Integer

- Each character is 1 byte long.
- An int is 4 bytes.
- A 2 character string key[0:1] may be converted into a unique 4 byte non-negative int using the code:

```
number = (int)key[0];
number <<= 8;
number += (int)key[1];
```

- Strings that are longer than 4 characters do not have a unique non-negative int representation.

String to Integer

```
unsigned int stringToInt(char *key)
{
    int number = 0;
    while(*key)
    {
        number += (int)*key++;
        number <<= 8;
    }
    return number;
}
```

Number is shifted left 8 bits after adding to number.

Map into a Home Bucket

(3,d)		(22,a)	(33,c)			(73,e)	(85,f)
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]

- Most common method is to use modular division.

homeBucket = hash(theKey) % divisor;

- divisor equals number of buckets b.
- 0 <= homeBucket < divisor = b

Uniform Hash Function

(3,d)		(22,a)	(33,c)			(73,e)	(85,f)
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]

- Let keySpace be the set of all possible keys.
- A uniform hash function maps the keys in keySpace into buckets such that approximately the same number of keys get mapped into each bucket.
- Our goal is to use a uniform hash function.

Uniform Hash Function

(3,d)		(22,a)	(33,c)			(73,e)	(85,f)
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]

- Equivalently, the probability that a randomly selected key has bucket i as its home bucket is 1/b, 0 <= i < b.
- A uniform hash function minimizes the likelihood of an overflow when keys are selected at random.

Hashing by Division

- keySpace = all ints.
- For every b, the number of ints that get mapped (hashed) into bucket i is approximately $2^{32}/b$.
- Therefore, the division method results in a uniform hash function when keySpace = all ints.
- In practice, however, keys tend to be correlated.
- So, the choice of the divisor b affects the distribution of home buckets.

Selecting the Divisor

- Because of this correlation, applications tend to have a bias towards keys that map into odd integers (or into even ones).
- When the divisor is an even number, odd integers hash into odd home buckets and even integers into even home buckets.
  - 20%14 = 6, 30%14 = 2, 8%14 = 8
  - 15%14 = 1, 3%14 = 3, 23%14 = 9
- The <u>bias in the keys</u> results in a bias toward either the odd or even home buckets.

Selecting the Divisor

- When the divisor is an odd number, odd (even) integers may hash into any home.
  - 20%15 = 5, 30%15 = 0, 8%15 = 8
  - 15%15 = 0, 3%15 = 3, 23%15 = 8
- The bias in the keys does not result in a bias toward either the odd or even home buckets.
- Better chance of uniformly distributed home buckets.
- So <u>do not use an even divisor</u>.

Selecting the Divisor

- Similar biased distribution of home buckets is seen, in practice, when the divisor is a multiple of prime numbers such as 3, 5, 7, …
- The effect of each prime divisor p of b decreases as p gets larger.
- Ideally, choose b so that it is a **prime number**.
- Alternatively, choose b so that it has no prime factor smaller than 20.

Overflow Handling

- An overflow occurs when the home bucket for a new pair (key, element) is full.
- We may handle overflows by:
  - Search the hash table in some systematic fashion for a bucket that is not full.
    - Linear probing (linear open addressing).
    - Quadratic probing.
    - Rehashing (or Double hashing)
  - Eliminate overflows by permitting each bucket to keep a list of all pairs for which it is the home bucket.
    - Array linear list.
    - Chain.

Linear Probing – Get and Insert

- divisor = b (number of buckets) = 17.
- Home bucket = key % 17.

0				4				8				12				16
34	0	45				6	23	7			28	12	29	11	30	33

- Insert pairs whose keys are 6, 12, 34, 29, 28, 11, 23, 7, 0, 33, 30, 45

Linear Probing – Delete

0				4				8				12				16
34	0	45				6	23	7			28	12	29	11	30	33

- Delete(0)

0				4				8				12				16
34		45				6	23	7			28	12	29	11	30	33

- Search cluster for pair (if any) to fill vacated bucket.

0				4				8				12				16
34	45					6	23	7			28	12	29	11	30	33

Linear Probing – Delete(34)

0				4				8				12				16
34	0	45				6	23	7			28	12	29	11	30	33

0				4				8				12				16
	0	45				6	23	7			28	12	29	11	30	33

- Search cluster for pair (if any) to fill vacated bucket.

0				4				8				12				16
0		45				6	23	7			28	12	29	11	30	33

0				4				8				12				16
0	45					6	23	7			28	12	29	11	30	33

Linear Probing – Delete(29)

0				4				8				12				16
34	0	45				6	23	7			28	12	29	11	30	33

0				4				8				12				16
34	0	45				6	23	7			28	12		11	30	33

- Search cluster for pair (if any) to fill vacated bucket.

0				4				8				12				16
34	0	45				6	23	7			28	12	11		30	33

0				4				8				12				16
34	0	45				6	23	7			28	12	11	30		33

0				4				8				12				16
34	0					6	23	7			28	12	11	30	45	33

Performance of Linear Probing

0				4				8				12				16
34	0	45				6	23	7			28	12	29	11	30	33

- Cluster is a contiguous block of items.
- Insert and search cost depend on length of cluster
- Worst-case find/insert/erase time is $\Theta(n)$ where n is the number of pairs in the table.
  - This happens when all pairs are in the same cluster

Expected Performance

0				4				8				12				16
34	0	45				6	23	7			28	12	29	11	30	33

- Trivial: average length of cluster = α = loading density = (number of pairs)/b.
  - $\alpha = 12/17$.
- [Knuth 1962] Let $\alpha < 1$ be an average length of list

Search: $\frac{1}{2}(1 + 1/(1 - \alpha))$

Insert: $\frac{1}{2}(1 + 1/(1 - \alpha)^2)$

$\alpha <= 0.75$ is recommended.

Quadratic Probing

- Examine the hash table buckets

  $ht[f(x) \% b]$,

  $ht[(f(x) + i^2) \% b]$,

  $ht[(f(x) - i^2) \% b]$,

  for $0 \le i \le (b-1)/2$,

- reduce the average number of probes

Quadratic Probing – Insert

- divisor = b (number of buckets) = 17.
- Home bucket = key % 17.

0				4				8				12				16
34	0					6	23	7		11	28	12	29	30	45	33

- Insert pairs whose keys are 6, 12, 34, 29, 28, 11, 23, 7, 0, 33, 30, 45

Rehashing

- use a series of hashing functions

  $f_1, f_2, \cdots, f_m$

- bucket $f_i(x)$ is examined for

  $i = 1, 2, \cdots, m$

Rehashing – Insert

- F_1 = key % 17.
- F_2 = key % 7.
- F_3 = key % 7 + i.

0				4				8				12				16
34	29	23	0	11	45	6	7				28	12	30			33

- Insert pairs whose keys are 6, 12, 34, 29, 28, 11, 23, 7, 0, 33, 30, 45

Hash Table Design

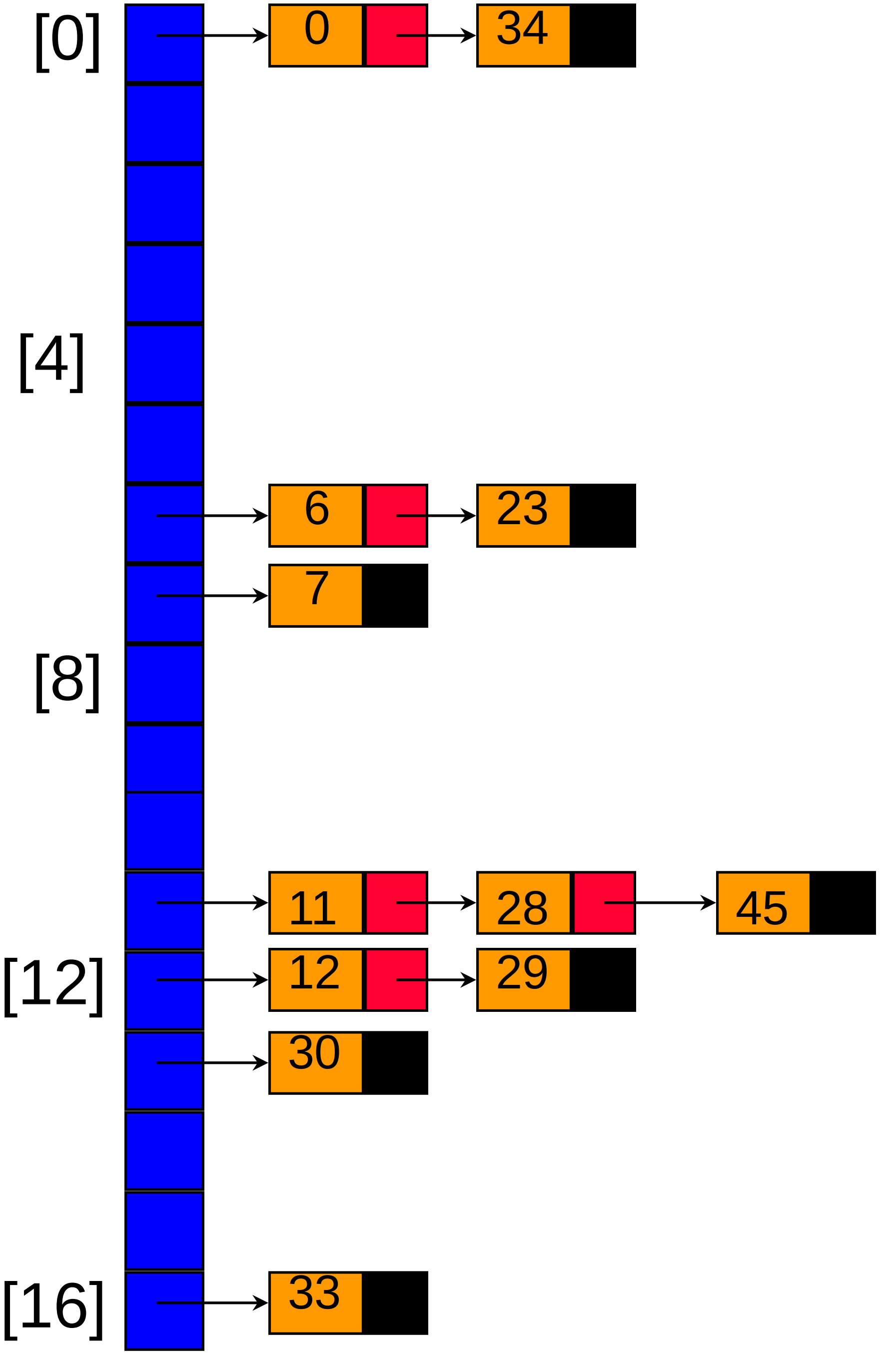
- Dynamic resizing of table.
  - Maintain the loading density <= 4/5
  - Whenever loading density exceeds a threshold (e.g., 4/5), rehash into a table of approximately twice the current size.
- Fixed table size.
  - When the maximum number of pairs is known.
  - Pick b (equal to divisor) to be a prime number or an odd number with no prime divisors smaller than 20.

Linear List of Synonyms

- Each bucket keeps a linear list of all pairs for which it is the home bucket.
- The linear list may or may not be sorted by key.
- The linear list may be an array linear list or a chain.

Sorted Chains

- Put in pairs whose keys are 6, 12, 34, 29, 28, 11, 23, 7, 0, 33, 30, 45
- Home bucket = key % 17.



Expected Performance

- Note that $\alpha >= 0$.
- Expected chain length is α.
- Search: $1 + \alpha/2$.
- Insert: $1 + \alpha/2$ for sorted, 1 for unsorted

Questions?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