NEW HYBRID TECHNIQUES FOR LARGE SCALE MULTI-OBJECTIVE OPTIMIZATION

by

Emirkan Karabulut
Muhammet Çetinkaya
Emre Eren

CSE4197 / CSE4198 Engineering Project report submitted to Faculty of Engineering
in partial fulfillment of the requirements for the degree of

BACHELOR OF SCIENCE

Supervised by:
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ABSTRACT

This thesis presents a study on enhancing the performance on large-scale multi-objective optimization problems through the hybridization of multiple techniques and analyzing the performance by benchmark tests. Three different hybridizations were proposed, namely the PCX Crossover, Starting Point revision on reproduction operator, and the vector adaptation. The combination of all three extensions was also considered in our experiments.

We consider two key performance metrics, Hypervolume (HV) and Inverted Generational Distance (IGD). These metrics evaluate the effectiveness of our extension algorithms in improving convergence and diversity during the optimization process.

The experimentation phase includes testing our hybridizations nine different benchmark problems exclusively crafted for large-scale multi-objective optimization algorithms (also known as LSMOP 1 to LSMOP 9). These benchmark problems provide a diverse range of challenges bounding different landscapes and different separability levels.

Results indicate a significant improvement in performance on the LSMOP9 problem. This shows the effectiveness of our hybridizations on the complex optimization problems. Also, there was a great performance increase in the terms of early convergence in the LSMOP8 and the LSMOP9 problems. However, it was observed that the performance of hybridizations drops significantly on LSMOP3 problem which has multimodal landscape and mixed separability. This result needs further investigation by considering to behavior of the algorithm.

ACKNOWLEDGEMENTS

We want to sincerely thank our advisor Prof. Dr. Haluk Rahmi Topçuoğlu for his valuable guidance, support, and encouragement throughout this research. He played a crucial role in shaping our work. We also want to express our appreciation to our family for their constant love and understanding. Their unwavering support and encouragement have been incredibly important to us. We are truly grateful to our supervisor and family for their invaluable contributions.

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1. INTRODUCTION

1.1 Problem Description and Motivation

Optimization problems take a significant place in science, industry and real life. They are encountered in many fields of science and engineering. Novel optimization problems are even harder, more complex and costlier. These complex and costly problems are named as large-scale optimization problems (LSMOPs). To say LSMOP to an optimization problem, it should have more than 100 decision variables [1] and the number of objective functions has to be equal or more than 2 [2]. When there is a multiple objective large-scale problem, there will not be a single optimal solution. There would be Pareto optimal solutions [3]. The mathematical definition of LSMOPs is as follows:

$$F(x) \in Y, x \in \Omega$$

$$min\, f(x) = (f_1(x), f_2(x), f_3(x), \dots, f_m(x)) \qquad \textbf{(Equation 1.1)}$$

$$x = (x_1, x_2, \dots, x_n)$$

Where f(x) is the objective vector usually conflicting in objective space Y and x is the decision vector usually interacting in search space Ω [4]. The value of n indicates the number of decision numbers that exist in the search space and m is the number of objective function numbers, it must be more than or equal to 2 ($m \geq 2$) for being a multi-objective problem. The dimension of x vector n should be equal or higher than 100 [1] as written before.

LSMOPs have high computational complexity and are usually not solvable in acceptable time or memory space with limited resources. Even hardware and software technology are also getting developed a lot. As the dimension of the problem increases, search space grows exponentially which is called the curse of dimensionality [5]. Conventional algorithms and techniques are not capable of solving LSMOPs with good diversity and convergence because they are mostly designed for large scale single-objective optimization problems.

Some of the examples for these types of problems are aerodynamic design, drug design [6], TSP, scheduling and assignment problems etc. Therefore, there is a need to design and develop effective algorithms to solve these problems. The reason that we want to do this project is to develop a new and more efficient algorithm to solve Large Scale Multi-Objective Optimization Problems. Working on LSOP is one of the new research areas and there is still a lot of area to develop. There are methods like Adaptive Population Differential Evolution [7], Sparse Population Sampling [8] and Surrogate-Assisted Evolutionary Computation [6].

On the other hand, hybridization of multiple techniques is a good way to provide the strengths of various algorithms and techniques. Although hybridization techniques are considered for multi-objective hybridization optimization problems, there are only a few hybridization techniques that are targeted for large scale multi-objective optimization problems (LSMOPs). Therefore, developing a new method with hybridization is a new approach and more progressable research area with the help of hybridization, we aimed to balance exploitation and exploration. The advantage of hybridization of different techniques is to combine different techniques to take advantages of all of them. Thus, we have high diversity in solution space and fast convergence.

1.1 Main Goal and Objectives of the Project

We tried to propose a framework that uses sequential techniques. The main goal is to present hybridization techniques to provide better results with respect to several performance indicators on a selected set of benchmarks for large scale multi-objective optimization problems.

Our objectives for our project:

Objective-I: Proposing hybridization techniques to provide faster convergence rate to optimal solution. Convergence rate is the speed of closing to a solution during iterations.

Objective-II: Providing better diversity than existing study for the solutions created by the algorithm and protecting diversity of problems.

Objective-III: When providing better performance according to performance indicators, the running time should be tolerable.

2. DEFINITION OF THE PROJECT

2.1 Scope of the Project

We handled only optimization problems that have at least 100 decision variables that we generally use as LSMOPs [1] and multi-objective ones [1, 9]. There is no certain decision variable number for being a large-scale problem. In our test sets there exist features of optimization of the problem e.g., multimodal, convex, separable. We handled these features as a subset of LSMOPS.

Our project was supposed to investigate a novel search strategy on the whole decision space. We assumed that it is a better approach than other alternative approaches. Our hardware resources are sufficient for experimental work. We aimed to test only benchmark test suits. This study references the DGEA [21] paper. 3 different extensions are implemented. We observed the benchmark result and compared the performance metric result with reference study.

2.2 Success Factors

The proposed method tested on the test suites and evaluated using the metrics below:

(i) Success factor for Objective 1-2: Accuracy and diversity of algorithm will be measured by two indicators.

Inverted Generational Distance (IGD): The metric is measuring the accuracy of the algorithm [30, 32]. The concept of IGD is the gap between the Pareto Front that the algorithm created and the real Pareto Front of the problem. The formula of the IGD is given below.

$$IGD(PF, A) = \frac{1}{|P|}\sum_{i=1}^{|p|} d(PF, A) \qquad \textbf{(Equation 2.1)}$$

Sum of distances calculated between every Pareto Front value and the nearest approximation calculated by the algorithm. PF presents the Pareto Front solution; d function shows Euclidean distance between approximation value point A and Pareto Front. The smaller IGD value annotates the better performance.

Hypervolume (HV): This metric is measuring the space dominated by calculated Pareto Front [31, 33]. Hypervolume provides information of both the convergence and the distribution of the solution set at the same time. The formulation of the Hypervolume is given below.

$$H(S) = \Lambda\left(\bigcup_{\substack{p \in S \\ p \leq r}} [p,\, r]\right)$$

$$[p,\, r] = \{q \in R^d : p \leq q \text{ and } q \leq r\} \qquad \textbf{(Equation 2.2)}$$

where p, r is the box bounded by points p and r. Larger Hypervolume value annotates the better quality of the solution set.

Success factor for Objective 3: When we try to achieve better performance, we can tolerate acceptable running time trade-off for performance indicator.

2.3 Professional Considerations

In terms of planning, we utilized a Gantt Chart to visualize and manage the division of responsibilities and duties among team members. This allowed us to effectively track project timelines and milestones.

For software implementation, we chose Python and Matlab programming languages. Python was particularly advantageous due to the existing implementation of optimization algorithms in the Pymoo library [25] written in Python in the early stages of research. Later, we decided to stick on Platemo library [26] in Matlab for its reliability and being the defacto standard in research purposed implementations.

To conduct experimental studies, we employed artificial benchmark test suites, specifically focusing on the LSMOPs suite. These tests provided valuable insights into the project's performance and effectiveness.

Regarding hardware, our development and testing were conducted on systems equipped with 16 GB of RAM, an i7 processor (8th generation) with 6 cores and 12 threads, and a 256GB SSD. This robust configuration ensured efficient execution and multitasking capabilities during the project.

Realistic Constraints:

i) Economical

At the beginning of the project, we thought we may need additional hardware resources for experimental studies. It can be costly if we use external resources but then our own computer was enough.

ii) Environmental

It can have an impact on the environment if it can be integrated into real life problems.

iii) Ethical

There could be violation of user privacy unintentionally if user data in real life problems is used for testing. As we did not use real life problems there will not be a violation of user privacy. Also, copyright issues of algorithms can cause difficulties.

iv) Health and Safety

There are healthy related real-world implementations in the literature. If real life problems are implemented on this project, it has potential to make a positive impact on these topics.

v) Sustainability

For our project, there is no obstacle to the continuation of working on the project. Our project can survive to live.

vi) Social

There is no defined consideration on social life.

Legal considerations: There are no legal restriction issues on this project.

2.4 Literature Survey

Large scale multi-objective optimization problems are a subset of optimization problems. Until now, many researchers have studied to solve these LSMOPs. These methodologies generally are based on Multi Objective Evolutionary Algorithms (MOEAs). These developed techniques focused on different parts of the problem's elements. There are mainly 3 subcategories for MOEAs.

Decision variable grouping methods divide our decision variables set into the subgroups [10]. These grouping ways can be linearly, ordered, randomly or differential grouping and variable analysis [11]. This approach generally tries to group related and similar characteristic variables into the same group. After the grouping mechanism operation, we get subcomponents to optimize each of them separately and combine all of them.

Another way is decision space reduction strategies [10], they are based on problem transformation or dimensionality reduction of problem. They focus on problem space dimension and problem reformatting to reduce search space and landscape complexity.

The third way to solve LSMOPs is novel search strategies. Decision variable grouping and decision space reduction-based algorithms have good results for solving LSMOPs, but they are not effective and efficient as novel search strategies. Instead of generating offspring in reduced decision space, novel operators generate offspring in the original decision space by new reproduction operations. Therefore, there is no loss of information and exploitation and exploration can be balanced. There is no decision variable based or problem reconstruction based methodologies. We have only the original search space to search. They can be easily embedded to other MOEAs because most of the operators are independent from the algorithms. Thus, we want to focus on this. Related work for novel search strategies as follows.

In this study [12], they proposed a new selection strategy called preselection strategy to maintain balance between convergence and diversity of parent population. Also, they proposed direction vector guided reproduction strategy which can lead to generation to promising solution by the help of preselection strategy. With the help of proposed reproduction strategy, exploitation and exploration can be balanced. Then they embedded these operators to some well-known state-of-the-art algorithms. Their test results show that algorithms with proposed strategies have better solutions on basic versions.

SparseEA [13] is developed for LSMOPs with high sparsity in which the most decision variables on decision space are 0. A new population initialization and new genetic operations were developed to guarantee the sparsity of solutions and simulated binary crossover and polynomial mutation was used. The success of SparseEA on solving sparse LSMOPs is remarkable.

In this study [14], they tested a non-dominated sorting genetic algorithm 3 (NSGA-III) with 3 different crossover operators. The crossover operators are SBX, UC (uniform crossover) and SI with their different types (i.e., MUC, MSI). The test is based on Electroencephalogram (EEG) which records brain signals. This test has thousands of decision variables. Therefore, it is very suitable to benchmark. Results of this test show that the best crossover operator for NSGA-III is UC although other crossover operators are also well.

MOEA-CSOD [15] is an estimation distribution algorithm (EDA) instead of an evolutionary algorithm. To create offspring, optimize model and population, a possibility model was used. Six steps of EDA are initialization, stopping condition, sampling, selecting, updating and setting generation. To generate offspring with high convergence quickly, Distributional Adversarial Networks (DAN) was used. For the selection of the solution, Reference Vector Guided Evolutionary Algorithm (RVEA) that is based on EDA was used. The tests showed that MOEA-CSOD has good performance, but it can have overfitting issues.

In this research [16], a new method was developed to solve big data problems which can be solved as optimization problems called MOSM. MOSM is a stochastic algorithm. Hybridization was used in MOSM to ensure the balance between exploitation and exploration. Five different hybridization algorithms were proposed. First is bat and sine-cosine. Second is the center of mass crossover, assimilation operator and mutation operator. Third is particle swarm optimization and simulated binary crossover. Fourth is whale optimization algorithm and trader algorithm. The last one is heat transfer search and moth-flame optimization algorithm. EEG was used for the test dataset. For most dataset, config-3 had the best HV results.

All these works are similar to our project as being novel search strategies. They have different techniques and methods to increase diversity of the solutions, to obtain fast convergence to the optimum and to get better results. We mainly studied DGEA study. This study was implemented on LSMOPs and it has common performance metrics with DGEA paper. We focused on preselection and reproduction algorithm parts. LSMOPs benchmark test sets are chosen. There is no crossover implementation on that. We implemented crossover, starting point revision and a kind of vector adaptation.

3. SYSTEM DESIGN AND SOFTWARE ARCHITECTURE

3.1 System Design

3.1.1 System Model

Our goal was to develop novel hybrid operators and for this, we require a foundational algorithm to build on. We have chosen to use the DGEA Algorithm [21] as our foundation and we implemented our extensions on it.

3.1.2 Flowchart for Proposed Algorithms

We used DGEA as our baseline algorithm. DGEA Algorithm [21] is a multi-objective optimization algorithm which generates offspring adaptively [21] with two operators which are preselection and reproduction. Flowchart for this algorithm is given in the Figure 3.1. The details of the algorithm were explained in Section 4.2.

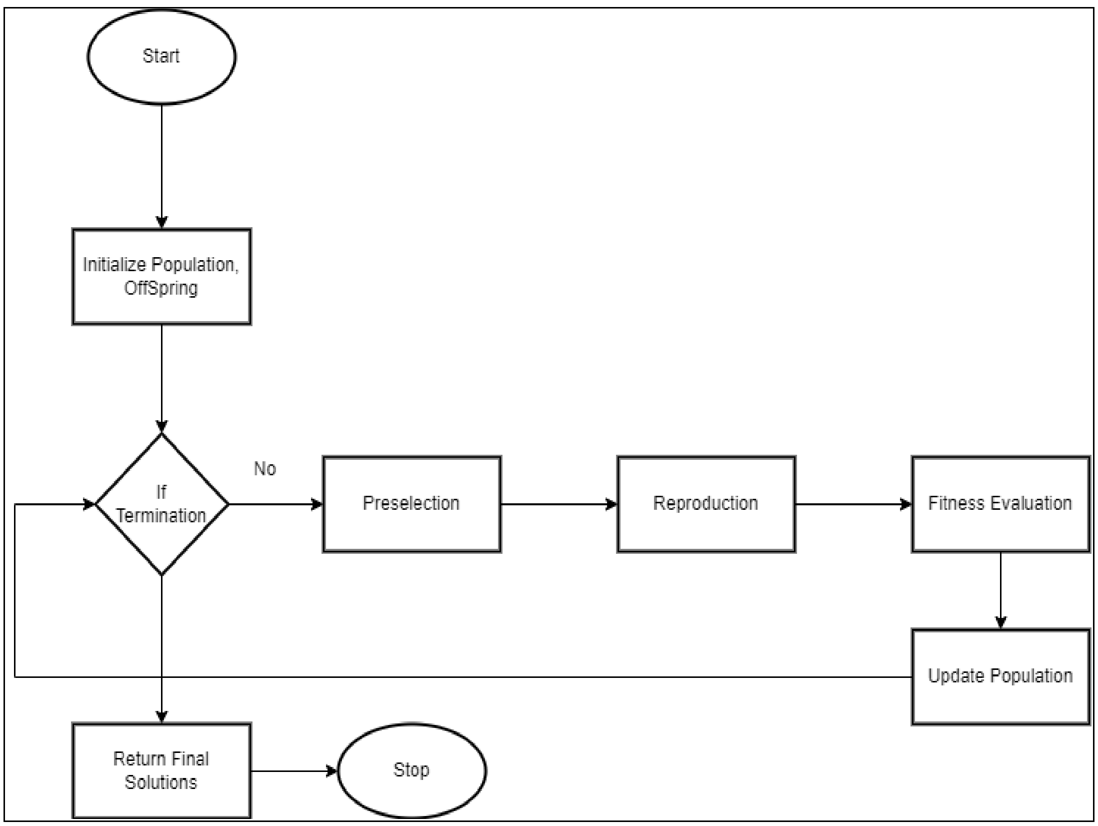

Figure 3.1 Flowchart of the DGEA Algorithm [21].

3.1.3 Comparison Metrics

Inverted Generational Distance (IGD): The metric is measuring the accuracy of the algorithm [17, 18]. The concept of IGD is the gap between the Pareto Front that the algorithm created and the real Pareto Front of the problem. The formula of the IGD is given below [17].

$$IGD(PF, A) = \frac{1}{|P|}\sum_{i=1}^{|p|} d(PF, A) \qquad \textbf{(Equation 3.1)}$$

Sum of distances calculated between every Pareto Front value and the nearest approximation calculated by the algorithm. PF presents the Pareto Front solution; d function shows Euclidean distance between approximation value point A and Pareto Front. The smaller IGD value annotates the better performance.

Hypervolume (HV): This metric is measuring the space dominated by calculated Pareto Front [19, 20]. Hypervolume provides information of both the convergence and the distribution of the solution set at the same time. The formulation of the Hypervolume is given below [19].

$$\mathrm{H}(S) = \Lambda\,(\{\mathrm{q} \in R^d \mid \exists \mathrm{p} \in S : \mathrm{p} \leq \mathrm{q} \textbf{ and } \mathrm{q} \leq \mathrm{r}\}) \qquad \textbf{(Equation 3.2)}$$

where $[p, r]$ is the box bounded by points p and r. Λ shows the Lebesgue measure. Larger Hypervolume value annotates the better quality of the solution set.

3.1.4 Benchmarks

Most of the benchmark problems are mainly developed for small scale problems. Therefore, they are not suitable for most real-world problems. LSMOPs was proposed to satisfy this need in the literature [7,24]. LSMOPs are large-scale and have many objective optimization problems. There are 9 different problems which have different challenging features. Recent studies on MOEAs highly use LSMOPs as a benchmark problem. Table 3.1 shows the modality and separability features of all LSMOPs. Basic definition of the LSMOPs are given below.

	Pareto Front Type	Landscape Type	Separability
LSMOP1	Linear	Unimodal	Separable
LSMOP2	Linear	Mixed	Partially Separable
LSMOP3	Linear	Multimodal	Mixed
LSMOP4	Linear	Mixed	Mixed
LSMOP5	Concave	Unimodal	Separable
LSMOP6	Concave	Mixed	Partially Separable
LSMOP7	Concave	Multimodal	Mixed
LSMOP8	Concave	Mixed	Mixed
LSMOP9	Disconnected	Mixed	Separable

Table 3.1. Characteristics of LSMOP benchmark functions [21].

There are 3 types of pareto front for LSMOPs which are linear, concave and disconnected.

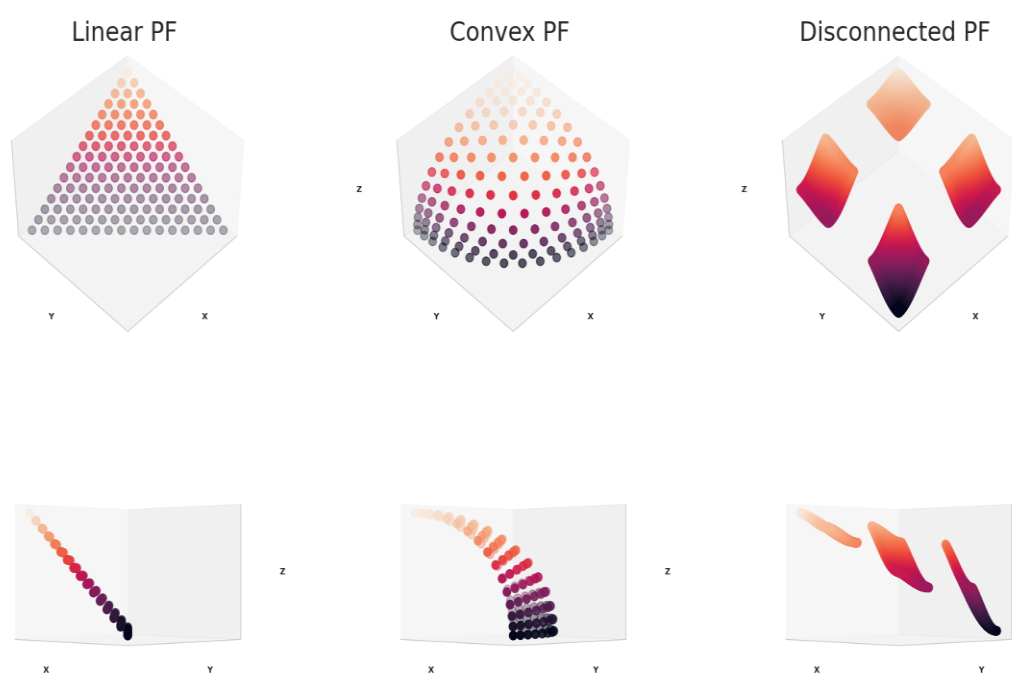

Figure 3.2 Pareto Fronts [24]

Linear Pareto Front:

$$H_1(x^f):\begin{cases} h_1(x^f) = x_1^f \dots x_{M-1}^f \\ h_2(x^f) = x_1^f \dots \left(1 - x_{M-1}^f\right) \\ \dots \\ h_{M-1}(x^f) = x_1^f\left(1 - x_2^f\right) \\ h_M(x^f) = \left(1 - x_1^f\right) \end{cases} \qquad \textbf{(Equation 3.3)}$$

Convex Pareto Front:

$$H_2(x^f):\begin{cases} h_1(x^f) = cos\left(\frac{\pi}{2}x_1^f\right) \dots cos\left(\frac{\pi}{2}x_{M-2}^f\right) cos\left(\frac{\pi}{2}x_{M-1}^f\right) \\ h_2(x^f) = x_1^f \dots h_2(x^f) \ = cos\ \left(\frac{\pi}{2}x_1^f\right) \dots cos\ \left(\frac{\pi}{2}x_{M-2}^f\right) sin\ \left(\frac{\pi}{2}x_{M-1}^f\right) \\ \dots \\ h_3(x^f) = \cos\left(\frac{\pi}{2}x_1^f\right) \dots \sin\left(\frac{\pi}{2}x_{M-2}^f\right) \\ h_M(x^f) = \left(1 - x_1^f\right) \\ \dots \\ h_{M-1}(x^f) = cos\ \left(\frac{\pi}{2}x_1^f\right) sin\ \left(\frac{\pi}{2}x_2^f\right)\ h_M(x^f) = sin\ \left(\frac{\pi}{2}x_1^f\right) \end{cases}$$

(Equation 3.4)

Disconnected Pareto Front:

$$H_3(x):\begin{cases} h_1(\mathbf{x}^f) \ = \dfrac{x_1^f}{1 + g_1(\mathbf{x}^s)} \\ h_2(\mathbf{x}^f) \ = \dfrac{x_2^f}{1 + g_2(\mathbf{x}^s)} \\ \dots \\ h_{M-1}(\mathbf{x}^f) = \dfrac{x_{M-1}^f(\mathbf{x}^s)}{1 + g_M(\mathbf{x}^s)} \\ h_M(\mathbf{x}^f) = \left(M - \displaystyle\sum_{i=1}^{M-1} \frac{x_i^f\left(1 + \sin\left(3\pi x_i^f\right)\right)}{2 + g_M(\mathbf{x}^s)}\right) \times \dfrac{2 + g_M(\mathbf{x}^s)}{1 + g_M(\mathbf{x}^s)} \end{cases}$$

(Equation 3.5)

Six Single-Objective Problems:

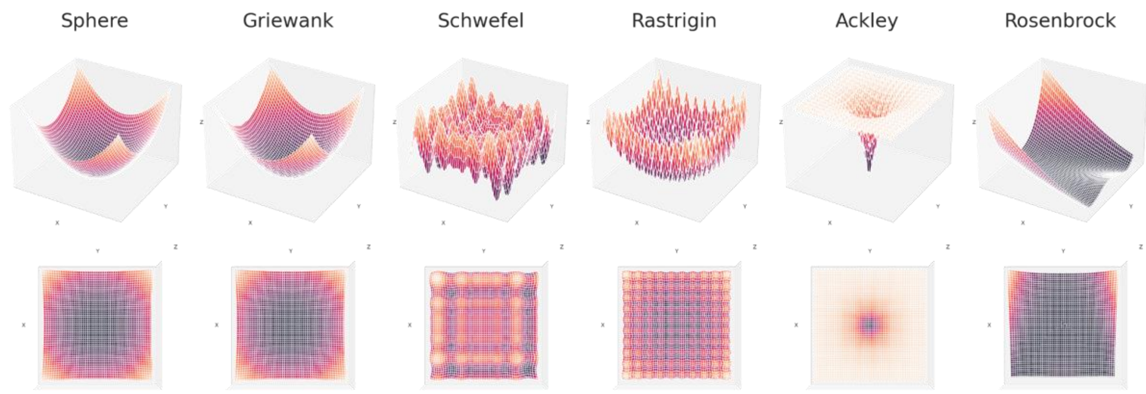

Figure 3.3 Single-Objective Problems [24]

a) Sphere Function:

$$\eta_1(x) = \sum_{i=1}^{|x|} (x_i)^2 \quad \textbf{(Equation 3.6)}$$

b) Schwefel's Problem:

$$\eta_2(x) = \max_i \{|x_i|, 1 \leq i \leq |x|\} \quad \textbf{(Equation 3.7)}$$

c) Rosenbrock's Function:

$$\eta_3(x) = \sum_{i=1}^{|x|-1} [100(x_i^2 - x_{i+1})^2 + (x_i - 1)^2] \quad \textbf{(Equation 3.8)}$$

d) Rastrigin's Function:

$$\eta_4(x) = \sum_{i=1}^{|x|} (x_i^2 - 10cos\,(2\pi x_i) + 10) \quad \textbf{(Equation 3.9)}$$

e) Griewank's Function:

$$\eta_5(x) = \sum_{i=1}^{|x|} \frac{x_i^2}{4000} - \prod_{i=1}^{|x|} cos\left(\frac{x_i}{\sqrt{i}}\right) + 1 \quad \textbf{(Equation 3.10)}$$

f) Ackley's Function:

$$\eta_6(x) = -20 * exp\,(-0.2\sqrt{\frac{1}{|x|}\sum_{i=1}^{|x|} x_i^2)} - exp\left(\frac{1}{|x|}\sum_{i=1}^{|x|} cos\,(2\pi x_i)\right) + 20 + e$$

(Equation 3.11)

Objective Functions of LSMOPs [24]:

In this section, we present three samples which are LSMOP1, LSMOP2 and LSMOP9. Others are presented at Appendix section.

LSMOP1:

$$f_1(x) = x_1^f \ldots x_{M-1}^f\left(1 + \sum_{j=1}^{M} c_{1,j} \times g^{-}{}_1(x_j^s)\right)$$

$$f_2(x) = x_1^f \ldots (1 - x_{M-1}^f)\left(1 + \sum_{j=1}^{M} c_{2,j} \times g^{-}{}_2(x_j^s)\right)$$

$$\ldots$$

$$f_{M-1}(x) = x_1^f(1 - x_2^f)\left(1 + \sum_{j=1}^{M} c_{M-1,j} \times g^{-}{}_{M-1}(x_j^s)\right)$$

$$f_M(x) = (1 - x_1^f)\left(1 + \sum_{j=1}^{M} c_{M,j} \times g^{-}{}_M(x_j^s)\right)$$

$$\begin{cases} \bar{g}_{2k-1}(\mathbf{x}_i^s) = \dfrac{1}{n_k}\displaystyle\sum_{j=1}^{n_k} \dfrac{\eta_1(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|} \\ \bar{g}_{2k}(\mathbf{x}_i^s) = \dfrac{1}{n_k}\displaystyle\sum_{j=1}^{n_k} \dfrac{\eta_1(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|} \\ k = 1, \ldots, \left\lceil \dfrac{M}{2} \right\rceil \end{cases}$$

(Equation 3.12)

LSMOP5:

$$f_1(x) = cos\left(\frac{\pi}{2}x_1^f\right)\dots cos\left(\frac{\pi}{2}x_{M-2}^f\right)cos\left(\frac{\pi}{2}x_{M-1}^f\right) \times \left(1+\sum_{j=1}^{M} c_{1,j}\times g^{-}{}_1\left(x_j^s\right)\right)$$

$$f_2(x) = cos\left(\frac{\pi}{2}x_1^f\right)\dots cos\left(\frac{\pi}{2}x_{M-2}^f\right)sin\left(\frac{\pi}{2}x_{M-1}^f\right) \times \left(1+\sum_{j=1}^{M} c_{2,j}\times g^{-}{}_2\left(x_j^s\right)\right)$$

$$\dots$$

$$f_{M-1}(x) = cos\left(\frac{\pi}{2}x_1^f\right)sin\left(\frac{\pi}{2}x_2^f\right) \times \left(1+\sum_{j=1}^{M} c_{M-1,j}\times g^{-}{}_{M-1}\left(x_j^s\right)\right)$$

$$f_M(x) = sin\left(\frac{\pi}{2}x_1^f\right)\times\left(1+\sum_{j=1}^{M} c_{M,j}g^{-}{}_M\left(x_j^s\right)\right)$$

$$\begin{cases} \bar{g}_{2k-1}(\mathbf{x}_i^s) = \dfrac{1}{n_k}\displaystyle\sum_{j=1}^{n_k}\dfrac{\eta_1\left(\mathbf{x}_{i,j}^s\right)}{\left|\mathbf{x}_{i,j}^s\right|} \\ \bar{g}_{2k}(\mathbf{x}_i^s) = \dfrac{1}{n_k}\displaystyle\sum_{j=1}^{n_k}\dfrac{\eta_1\left(\mathbf{x}_{i,j}^s\right)}{\left|\mathbf{x}_{i,j}^s\right|} \\ k = 1,\dots,\left\lceil\dfrac{M}{2}\right\rceil \end{cases}$$

(Equation 3.13)

LSMOP9:

$$f_1(x) = x_1^f$$

$$f_2(x) = x_2^f$$

$$\dots$$

$$f_{M-1}(x) = x_{M-1}^f$$

$$f_M(x) = \left(M-\sum_{i=1}^{M-1}\frac{x_i^f\left(1+sin\left(3\pi x_i^f\right)\right)}{2+\sum_{j=1}^{M} c_{M,j}\times g^{-}{}_M\left(x_j^s\right)}\right) \times\left(2+\sum_{j=1}^{M} c_{M,j}\times g^{-}{}_M\left(x_j^s\right)\right)$$

$$\begin{cases} \bar{g}_{2k-1}(\mathbf{x}_i^s) = \frac{1}{n_k}\sum_{j=1}^{n_k} \frac{\eta_1\left(\mathbf{x}_{i,j}^s\right)}{\left|\mathbf{x}_{i,j}^s\right|} \\ \bar{g}_{2k}(\mathbf{x}_i^s) = \frac{1}{n_k}\sum_{j=1}^{n_k} \frac{\eta_6\left(\mathbf{x}_{i,j}^s\right)}{\left|\mathbf{x}_{i,j}^s\right|} \\ k = 1, \dots, \left\lceil \frac{M}{2} \right\rceil \end{cases} \qquad \textbf{(Equation 3.14)}$$

3.2 System Architecture

Our test system includes nine benchmark problems, which are LSMOP1–LSMOP9 [24]. We tested both DGEA [21] and DGEA with our extensions to compare their performances. The test starts with LSMOP1 and then runs 30 times independently for both DGEA and Modified DGEA. After all independent runs finished for LSMOP1, the HV and IGD values were saved and their averages added to the results table. This process iterates for all benchmark problems. After all benchmark problems are finished, the final table, which includes averages of HVs and IGDs, returns. The flowchart of our test system is given in Figure 3.4.

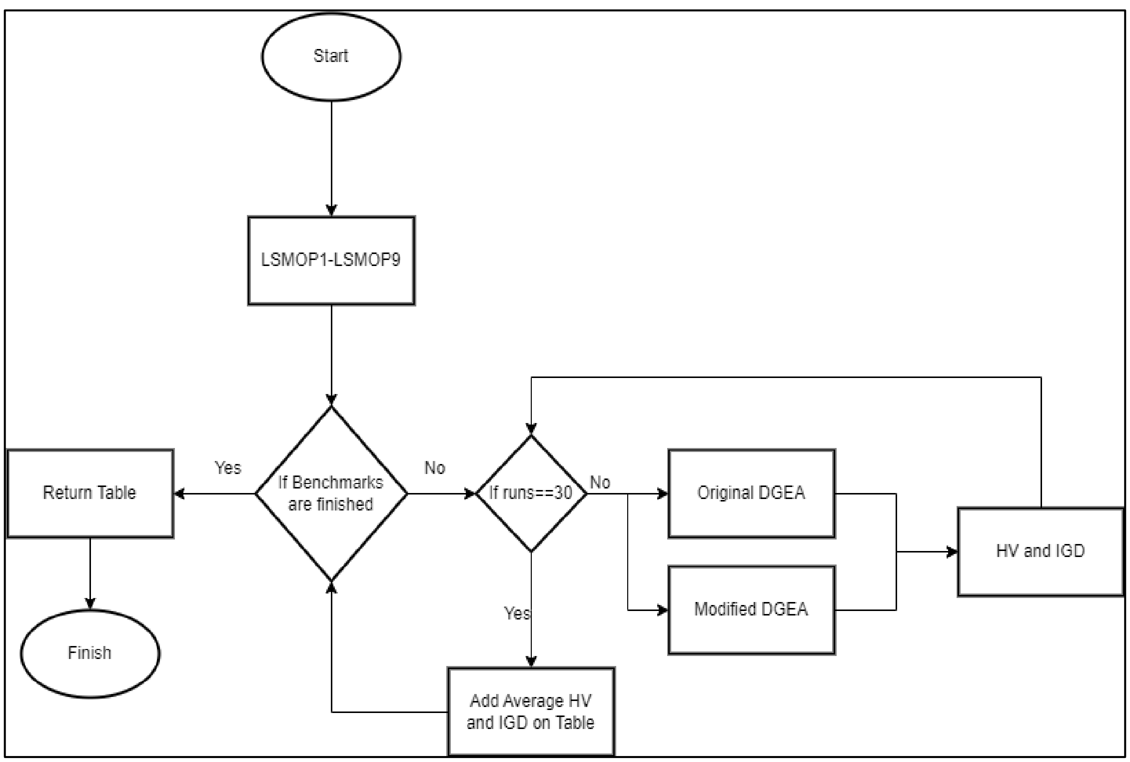

Figure 3.4 Flowchart of the Test System

4. TECHNICAL APPROACH AND IMPLEMENTATION DETAILS

4.1 Methodology

The methodology of the process is to make extensions on DGEA [21] operators which are Preselection and Reproduction. After implementing the technique, the benchmark functions run on the modified algorithm and the old algorithm and we compared using the metrics IGD and Hypervolume.

We have different categories of functions in the literature to benchmark the solution proposed. We used the LSMOP benchmark set. LSMOP is a relatively new benchmark category which mimics large-scale optimization problems.

We have a computer with 2x8GBs DDR4 3000mhz memory, 6 cores/12 threads 3.40Ghz processor and 256GBs of SSD storage.

The software technologies we used were Python and Matlab. We implemented the base algorithm and our extensions in Python. Also, we implemented LSMOP1, LSMOP2, LSMOP5, LSMOP7, and LSMOP9 on Python using Pymoo, which is a Python framework for multi-objective optimization. Then we used the PlatEMO framework in Matlab. PlatEMO is a popular framework for multi-objective evolutionary algorithms. It includes a lot of MOEAs and benchmarks. It has a GUI to make benchmarks and compare different algorithms' performances. PlatEMO includes DGEA and LSMOPs. We implemented our extensions on it and made benchmarks.

There exist 3 different approaches in the scope of the large-scale optimization problems according to survey [10]. We have decided to continue focusing on the novel search strategy-based methods.

4.1.1 Decision variable grouping based methods

This scope of the large-scale optimization problems aims to group decision variables and optimizes each group independently. There are different types of grouping algorithms. The most basic approach is that the variables can be grouped randomly. The problem with random grouping is that there can be interaction between two decision variables.

Differential grouping solves the problem of interaction between variables by putting interacting variables in the same group. This has also a drawback; the time complexity, implicitly runtime of the solution.

Random grouping and differential grouping are generally used for single objective large-scale problems. When there is a many-objective optimization problem, these methods can find a local optimum but may not be able to distinguish the Pareto front. For these types of problems there are variable analysis-based methods.

Most of the grouping methods are not able to process real-time data and it is hard to create and implement a new grouping method with real-time data process ability.

4.1.2 Decision space reduction based methods

In some decision space reduction methods, we can lose diversity of the solutions. For example, weighting frameworks that specify a weighting for a group of variables and instead of optimizing all variables [11], this weight solution vector will be optimized. As a result, in some groups we can lose diversity of solutions.

4.1.3 Novel search strategy based methods

This area mainly contains newer papers and looks promising to create new papers in the area. Recent studies on MOEAs directly solves LSMOPs by generating offspring in the decision space. Our proposed algorithm also enters this category. There are two subcategories in this approach. They are novel reproduction operator based and probability based MOEAs which have simple methods and have balance with diversity and fast convergence. The last area proposed in the survey was creating novel search strategy based algorithms. The area mainly contains newer papers and looks promising to create new papers in the area. Finally, we proposed a modified DGEA which belongs to the new novel search strategy.

4.2 Direction-Guided Evolutionary Algorithms (DGEA)

DGEA is a multi-objective evolutionary algorithm [21]. The algorithm proposes two different operators which are preselection and reproduction.

Input: N (population size), r (number of direction vectors).

Output: P (final population)

$P,Q \leftarrow$ Initialization(N)

$V \leftarrow$ Uniform_ Reference_Vector(N)

while termination criterion is not fulfilled **do**

  $Q_0, F_0 \cup \cdot\cdot\cdot \leftarrow$ Preselection $(P \cup Q, V, r)$

  $Q \leftarrow$ Reproduction$(Q_0, F_1 \cup \cdot\cdot\cdot, r, N)$

  $P \leftarrow$ Environmental_Selection$(P \cup Q, N)$

end

Figure 4.1 Direction-Guided Evolutionary Algorithm Framework (DGEA) [21]

Preselection operator makes selection based on two different strategies which are diversity-based and convergence-based. For diversity-based selection, calculates the ideal point and for a population and uses reference vectors to calculate Angle Penalized Distance (APD). Then pick the solutions with the smallest APDs. For convergence-based solutions, pick the non-dominated solutions. The main purpose of this operator is to generate better offspring in the reproduction operator [21].

```
Preselection(P, V, r)
Input: P (population), r (number of direction vectors), V (uniform reference vectors).
Output: Q0 (preselected population), Fi (solutions on the ith front).
/*****Convergence based selection*****/
F1, · · · ,Fm, · · · ← Non-dominated_Sort(P, min{|V|, |P|})
Nd ← |F1|
if Nd ≥ r then
        Q0 ← F1 ∪ · · · Fm-1
        P ← Fm
else
        Q0 ← F1 /*Non-dominated solutions*/
        P ← F2 ∪ · · · /*Dominated solutions*/
end
/*****Diversity based selection*****/
z ← Calculate the ideal point of P
for i ← 1 : |P| do
        fi' ← fi − 1
        for j ← 1 : |V| do
                cos ϑi,j ← (fj' ∗ vj) / ||fj'||
        end
        k ← argmaxj∈{1, ..., |V|} cosϑi,j
        Sk ← pk
end
for j ← 1 : |V| do
        for i ← 1 : |P| do
                dij ← APD(ϑi,j, fi')
        end
        k ← argmini∈{1, ..., |Sj|} di,j
        Q0 ← Q0∪{pk}
        F1, · · · ← Update the front numbers of Q0
end
```

Figure 4.2 Preselection Operator of Direction-Guided Evolutionary Algorithm (DGEA) [21]

Reproduction operator generates offspring with direction vectors generated from a solution to other solutions. It divides the population with non-dominated and dominated solutions. It picks a start point from non-dominated solutions randomly. Then generates directions from this start point to dominated solutions or non-dominated solutions.

For diversity maintenance, it generates solutions on generated directions between start point and non-dominated solutions. For fast convergence, it uses directions which are generated between start point and dominated solutions. To generate solutions, it uses a Gaussian distribution in these directions.

```
Reproduction(P, F1 ∪ · · · , r, N)
Input: P (population), N (population size), r (number of direction vectors), Fi (solutions on the ith front).
Output: Q (offspring population).
Pn ← F1 /*non-dominated solutions*/
Pd ← F2 ∪ · · · /*dominated solutions*/
ps ← Randomly select one solution from Pn
Nsub ← ⌊N/r⌋
if |Pd| < r then
        Q0 ← Randomly select r−|Pd| solutions from Pn\ps
        Q ← Pd ∪ Q0
else
        Q ← Randomly select r solutions from Pn
end
for i ← 1 : r do
        di ← (ps−qi) / ||ps−qi||
        for k ← 1 : |Pn| do
                λk←(pnk – ps) · dTi /*pnk is the kth vector in Pn*/
        end
        σ² ← Calculate the variance of {λ1, . . . , λ|Pn|}
        for j ← 1 : Nsub do
                γ ← Sample a value from N(0, σ²)
                sj ← γ · dj + ps
        end
        Qi ← {s1, s2, . . . , sNsub}
end
Q ← Q1 ∪ Q2 ∪ · · · ∪ Qr
```

Figure 4.3 Reproduction Operator of Direction-Guided Evolutionary Algorithm (DGEA) [21]

We applied three extensions on DGEA which are start point revision, Parent-Centric Crossover (PCX) and vector adaptation.

4.2.1 Start Point Revision

DGEA randomly chooses a start point from non-dominated solutions in the reproduction operator for a generation. Then uses this start point to generate solutions for every subpopulation. Instead of choosing only one start point for every subpopulation, choosing different start points for each subpopulation gives better results.

4.2.2 Crossover Extension

Original DGEA does not include any crossover technique. We applied Parent-Centric Crossover [22] for generating off-springs. Parent-Centric Crossover is a multi-parent crossover operator that uses a large probability rather than the center of selected parents to generate a new solution near each parent. Parent centric crossover operators are quite effective to obtain optimum solutions of real parametric problems. The definition of PCX is:

$$c = z^{(p)} + \omega_{\xi} d + \omega_{\eta} \frac{p^{(2)} - p^{(1)}}{2} \qquad \textbf{(Equation 4.1)}$$

Where:

- $^{(p)}$ is the best parent among three parents.
- $d = z^{(p)} - g$ (g is the mean of parents).
- $p^{(1)}$ and $p^{(2)}$ are the other two parents.
- $\omega_{\xi} = 0.1$ and $\omega_{\eta} = 0.1$ are the two parameters.

4.2.3 Vector Adaptation

DGEA uses Uniform Reference Vectors for calculating Angle Penalized Distance (APD). We utilized a vector adaptation method from Reference Vector Guided Evolutionary Algorithm (RVEA) [23]. The method updates the reference vectors for distributing the solutions in objective space uniformly. It updates vectors with a frequency i.e., every 100 generations for 1000 generations termination condition. We observed that updating vectors starting with higher frequency to lower frequency gives better results.

The definition of this method is:

$$v_{t+1,i} = \frac{v_{0,i} o (z_{t+1}^{max} - z_{t+1}^{min})}{\| v_{0,i} o (z_{t+1}^{max} - z_{t+1}^{min}) \|} \qquad \textbf{(Equation 4.2)}$$

• i = 1…, N where N is the population size.

• $v_{t+1,\, i}$ is the i-th adapted reference vector for next generation t+1.

• $v_{0,\, i}$ is the initialized uniform reference vector.

• z values are the max and min objective values for t+1 generation.

• o operator means element-wise production.

5. EXPERIMENTAL STUDY

5.1 Benchmarks

In this study we utilized the nine different large-scale multi-objective optimization problem (LSMOP 1 to 9) benchmarks to evaluate the performance of our extensions. LSMOP problems are commonly used in the field of large-scale multi-objective optimization as they provide a challenging environment for optimization algorithms to solve.

The LSMOP benchmarks consist of a collection of multi-objective optimization problems with various characteristics, such as varying landscapes, Pareto fronts with different shapes, and different separability levels. These benchmarks allow us to assess the effectiveness and efficiency of our proposed our extensions in solving multi-objective optimization problems across a range of scenarios.

The LSMOP benchmarks use single objective functions to shape their distinct characteristics. Each benchmark contains a pair of different single objective functions, which play a role in evaluating the fitness score of a given set of solutions. These single objective functions are categorized as either even or odd functions based on their application to the indexed variables within the solution set. Even functions are applied to the variables with even indices, while odd functions are applied to those with odd indices. This approach allows the benchmarks to cover a diverse range of landscapes of multi-objective optimization methods across various scenarios.

The Pareto fronts of the LSMOP benchmarks display different characteristics and challenges. A Pareto front represents the set of optimal solutions where improving one objective comes at the expense of another. In the case of LSMOP benchmarks, the Pareto fronts have various shapes; Linear Pareto, Convex Pareto, and Disconnected Pareto.

5.2 Comparison Metrics

In this study, we used Inverted Generational Distance (IGD) and Hypervolume (HV) comparison metrics as stated in the Section 3.1.3 to quantitatively assess our extensions' performance. In multi-objective optimization, these metrics are often used to evaluate the quality of a set of obtained solutions.

5.3 Experimental Design

In this study, we designed an experiment to evaluate the performance of our four different proposed our extensions and the original DGEA algorithm to compare performance differences. The original algorithm served as a baseline, representing a traditional optimization method commonly used in the literature.

These benchmarks were chosen to represent a wide range of challenging optimization scenarios which are good fit for our aim of improving large-scale multi-objective optimization algorithms. Performance differences compared through the performance metrics defined above.

To conduct an extensive evaluation, we utilized a set of settings used in the DGEA paper to acquire similar results on our experiment. All our extensions and the baseline algorithm run on nine different benchmark functions and 30 independent runs. Every benchmark ran with 1.000.000 function evaluations (FEs), 3 objectives, 100 decision variables, and 105 population size. This is a computationally intensive test.

While we utilized the computationally intensive test, we also ran another test with only changing the function evaluation count to 100.000 to see if our extensions had the advantage of early convergence compared to the baseline algorithm.

5.4 Results

5.4.1 Late stage

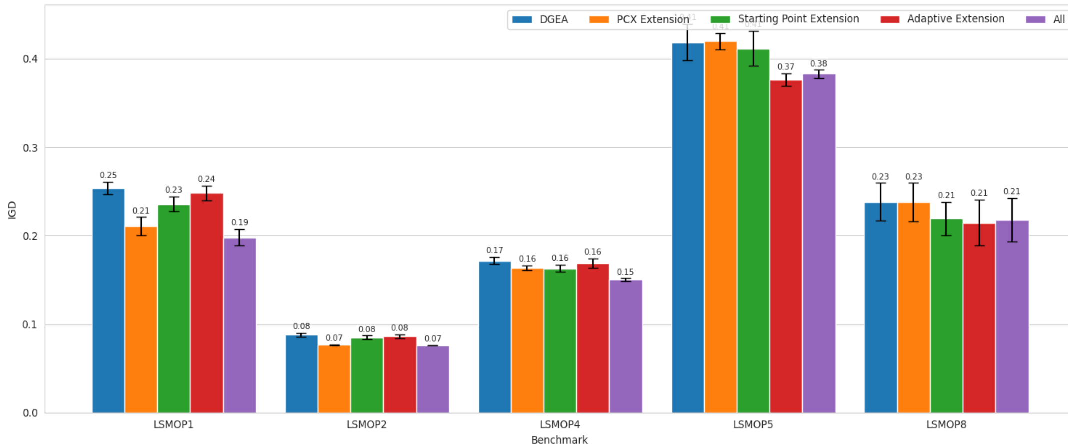

Figure 5.1 Late Stage IGD Performance of Our Extensions on the Benchmark Functions (1.000.000 Function Evaluations)

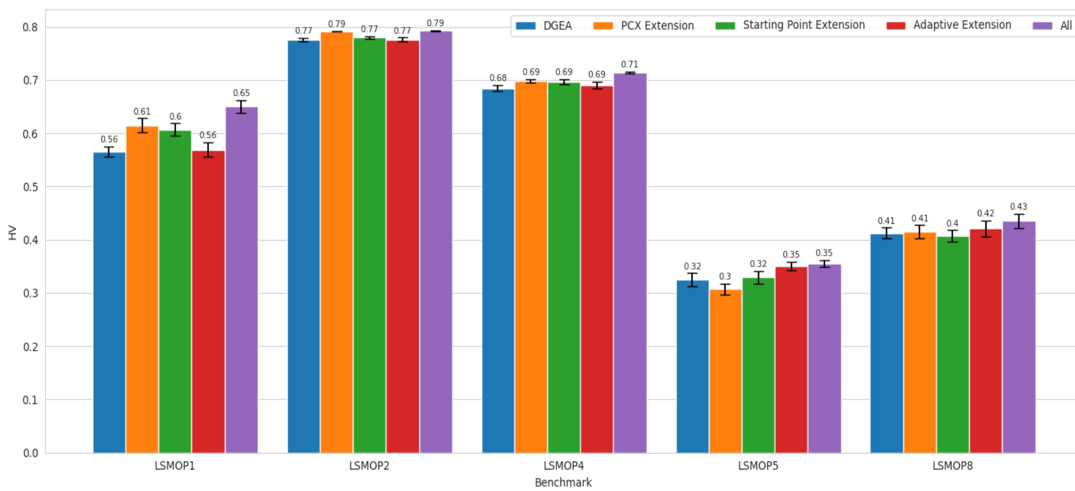

Figure 5.2 Late Stage HV Performance of Our Extensions on the Benchmark Functions (1.000.000 Function Evaluations)

In the late stage performance analysis of the proposed hybridizations, we observe a slight improvement in both the IGD and HV metrics for LSMOP1, LSMOP2, LSMOP4, LSMOP5, and LSMOP8 problems. While these performance enhancements may not be as good as those seen in LSMOP9, they still signify progress in these specific problem landscapes. The PCX extension and the mix of all extensions continue to demonstrate their effectiveness and efficiency exclusively in the LSMOP1, LSMOP2, and LSMOP4 problems, even if it is not as good as LSMOP3 and LSMOP9 we will see later. Even so, the overall evaluation shows promising late-stage improvements in the performance of our extensions, highlighting their potential on various problem landscapes.

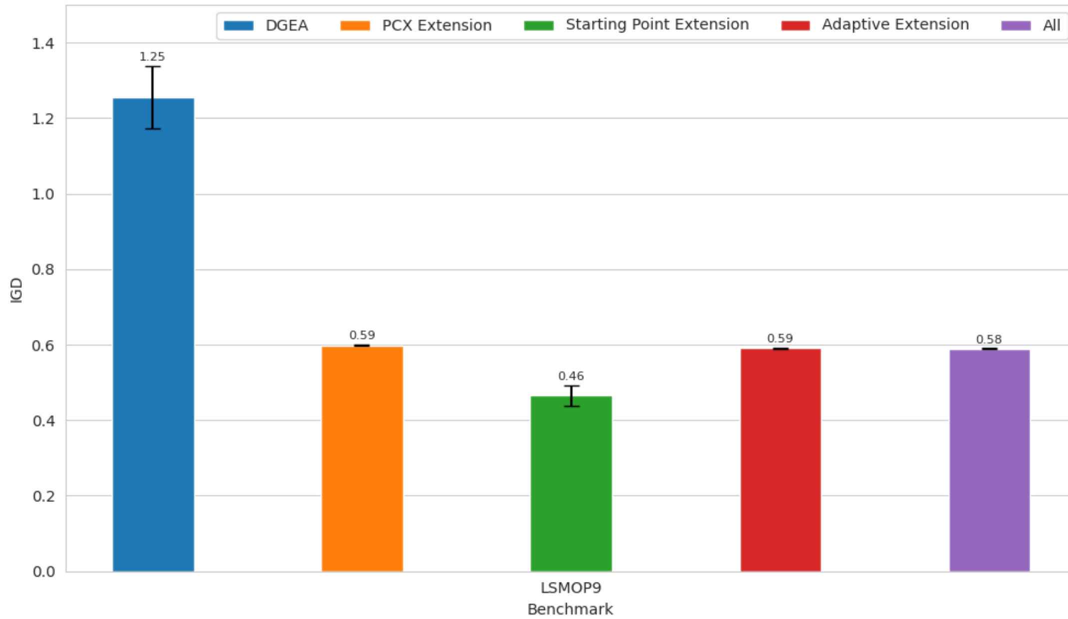

Figure 5.3 Late Stage IGD Performance of Our Extensions on the Benchmark Functions (1.000.000 Function Evaluations)

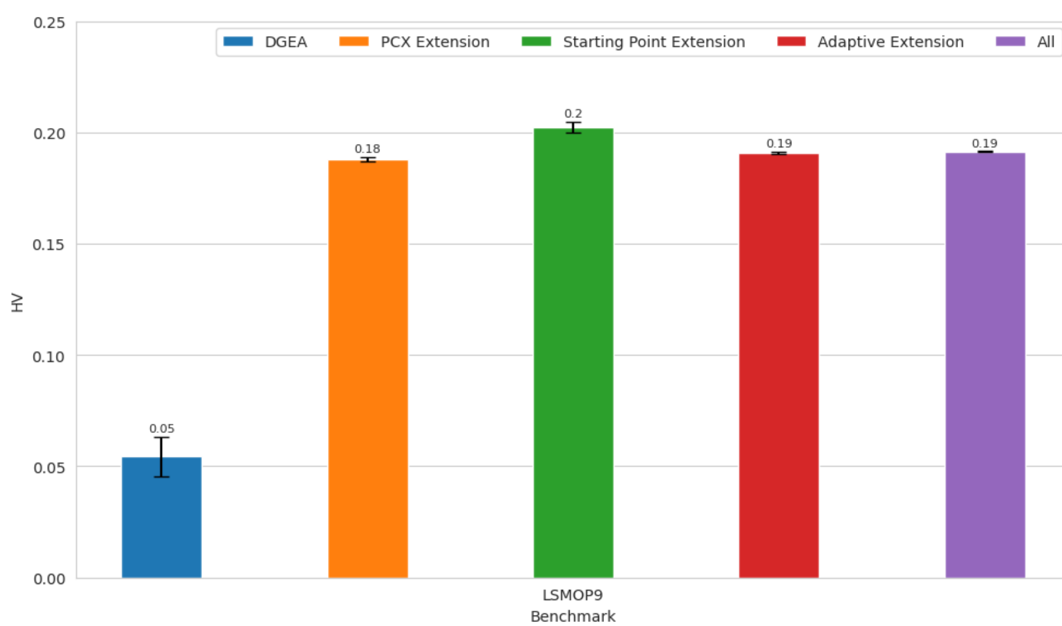

Figure 5.4 Late Stage HV Performance of Our Extensions on the Benchmark Functions (1.000.000 Function Evaluations)

In contrast to the mixed results observed for LSMOP1, LSMOP2, LSMOP4, LSMOP5, and LSMOP8, the late-stage performance of the proposed algorithms shows a substantial improvement in both the IGD and HV metrics for LSMOP9. The use of the PCX extension and the combination of all extensions proves to be highly effective and efficient in addressing the challenges posed by this particular problem landscape. The experiment results show a significant enhancement in performance on LSMOP9, indicating the effectiveness of the proposed our hybridizations.

The significant improvement in both IGD and HV metrics shows that these hybridizations are well-suited for overcoming the complexities of LSMOP9, showing their potential to optimize solutions better. The significant performance observed in LSMOP9 serves as strong evidence to the effectiveness and efficiency of our extensions proposed.

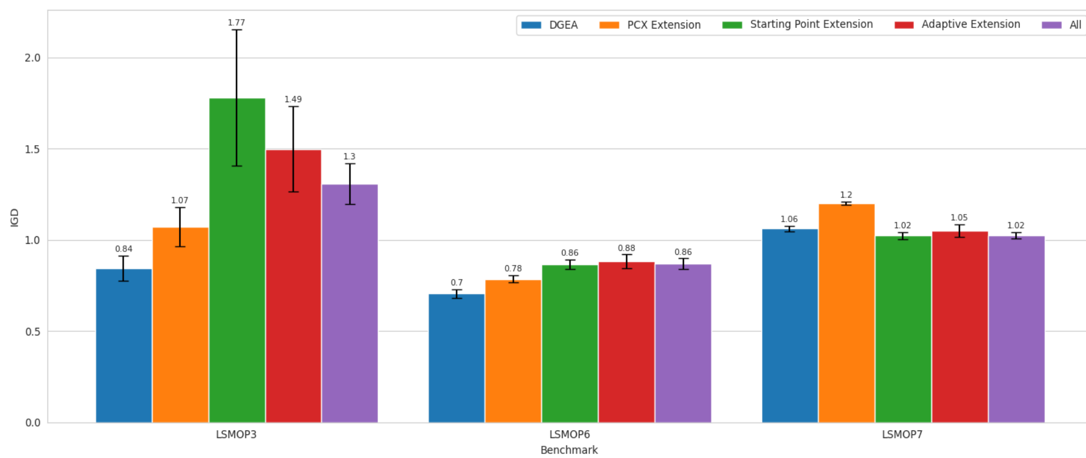

Figure 5.5 Late Stage IGD Performance of Our Extensions on the Benchmark Functions (1.000.000 Function Evaluations)

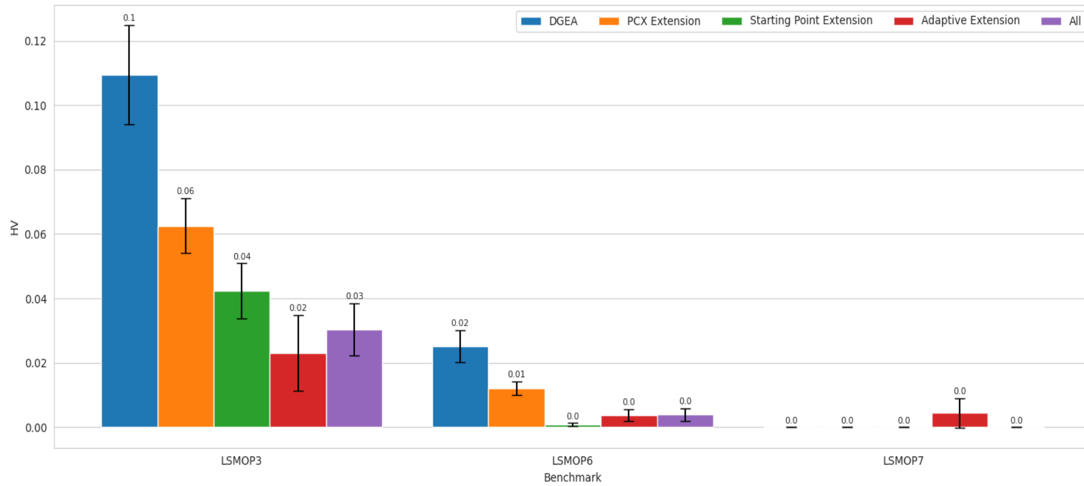

Figure 5.6 Late Stage HV Performance of Our Extensions on the Benchmark Functions (1.000.000 Function Evaluations)

In contrast to the positive outcomes observed in LSMOP9 and other problems mentioned, the late-stage performance of the proposed algorithms presents worse results for LSMOP3, LSMOP6, and LSMOP7. Among these, LSMOP3 exhibits the poorest performance, while LSMOP6 and LSMOP7 show relatively closer but still have less performant results. These problem landscapes, characterized by their multimodal nature and mixed separability, present significant challenges for our extensions proposed. As a result, the algorithms struggle to optimize and experience a drop in performance in terms of both the IGD and HV metrics.

5.4.2 Early stage

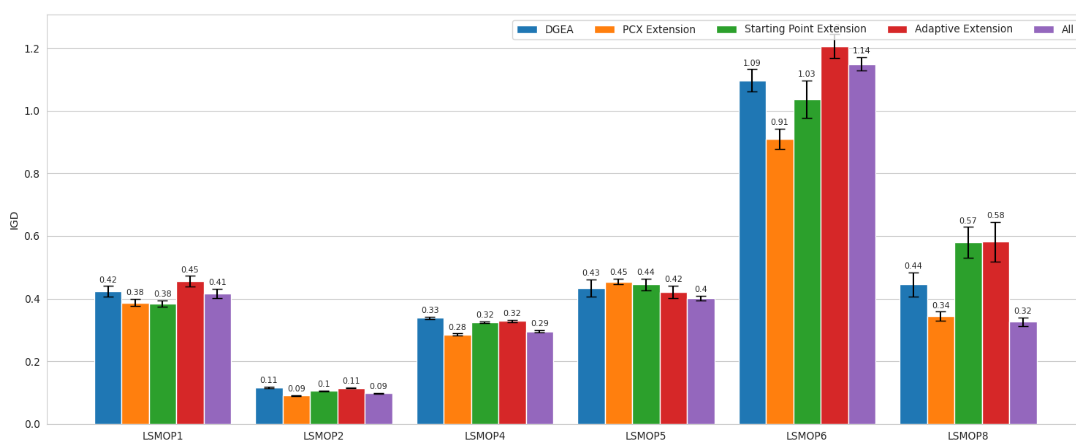

Figure 5.7 Early Stage IGD Performance of Our Extensions on the Benchmark Functions (100.000 Function Evaluations)

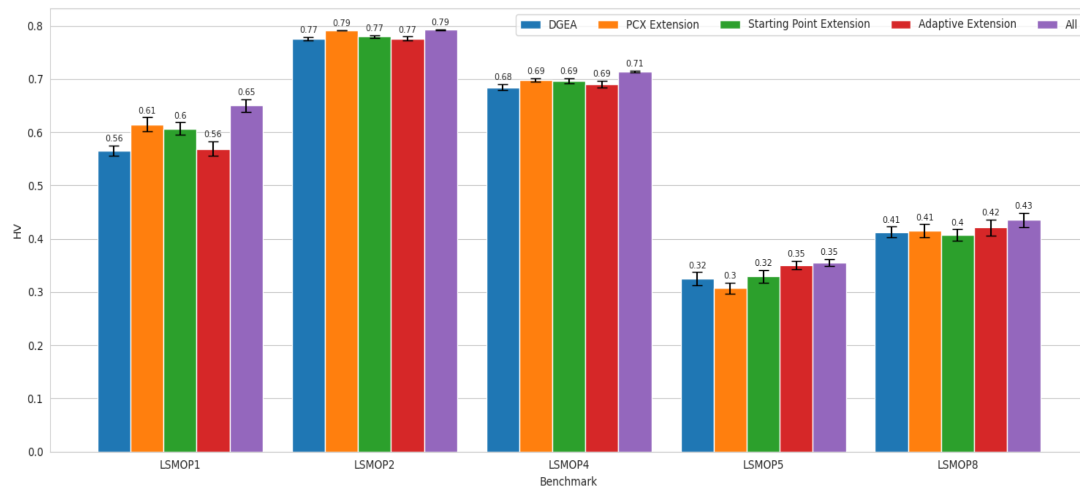

Figure 5.8 Early Stage HV Performance of Our Extensions on the Benchmark Functions (100.000 Function Evaluations)

When evaluating the early-stage performance of the proposed algorithms, both the IGD and HV metrics show notable improvements. Although the HV performance is slightly better in LSMOP1, LSMOP2, LSMOP4, LSMOP5, and LSMOP8, there is no significant performance boost compared to the baseline algorithm. Similarly, the IGD performance of all our extensions is slightly better in LSMOP1, LSMOP2, LSMOP4, and LSMOP5, but the improvements are not considered significant. However, the IGD performance of the PCX Extension in LSMOP6 and LSMOP8 demonstrates a significant improvement, indicating the effectiveness and efficiency of the extensions in these problems. Additionally, the mix of all extensions shows a significantly better IGD performance in LSMOP8, further highlighting the effectiveness and efficiency of the hybridization for this particular problem.

Overall, while there are some modest improvements in HV and IGD metrics for several problems, the PCX extension and mix of all extensions show significant improvements on the early-stage performance in LSMOP8.

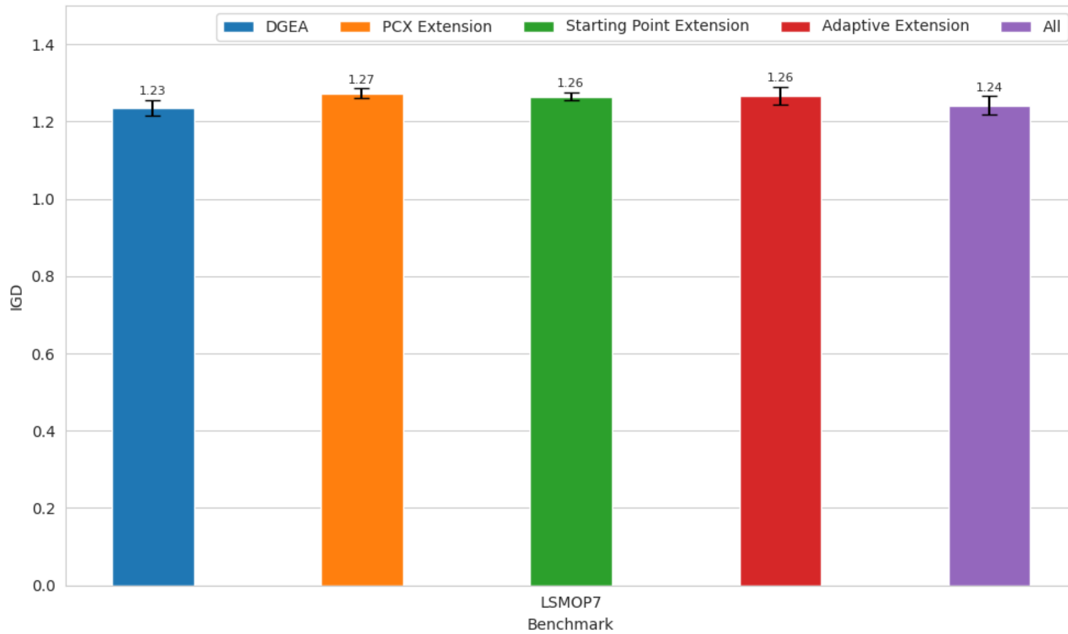

Figure 5.9 Early Stage IGD Performance of Our Extensions on the LSMOP7 (100.000 Function Evaluations)

All extensions showed similar IGD performance in LSMOP7.

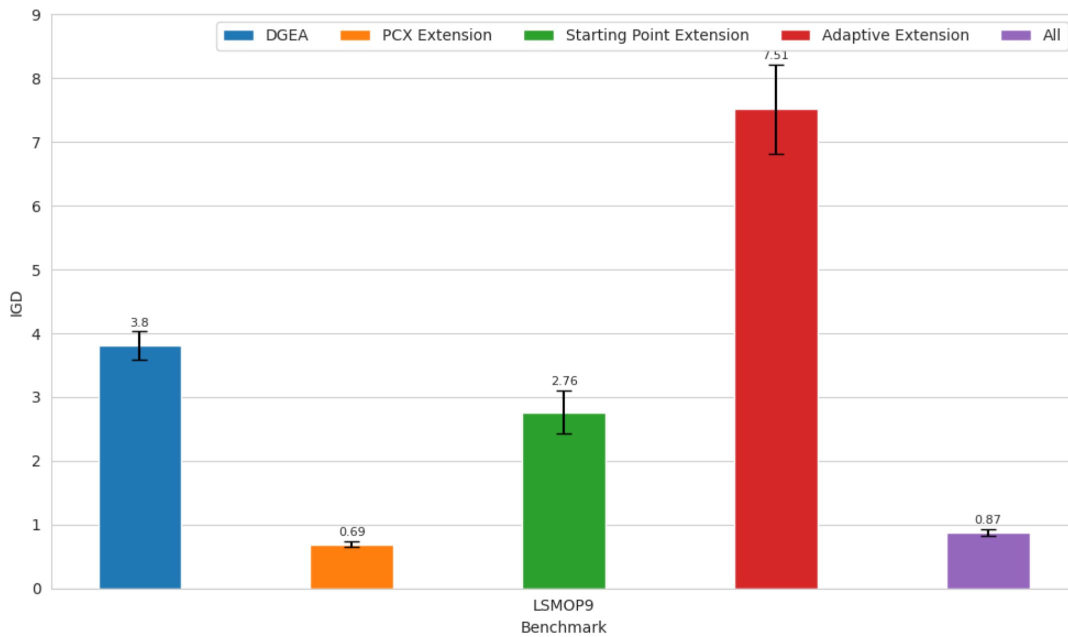

Figure 5.10 Early Stage IGD Performance of Our Extensions on the LSMOP9 (100.000 Function Evaluations)

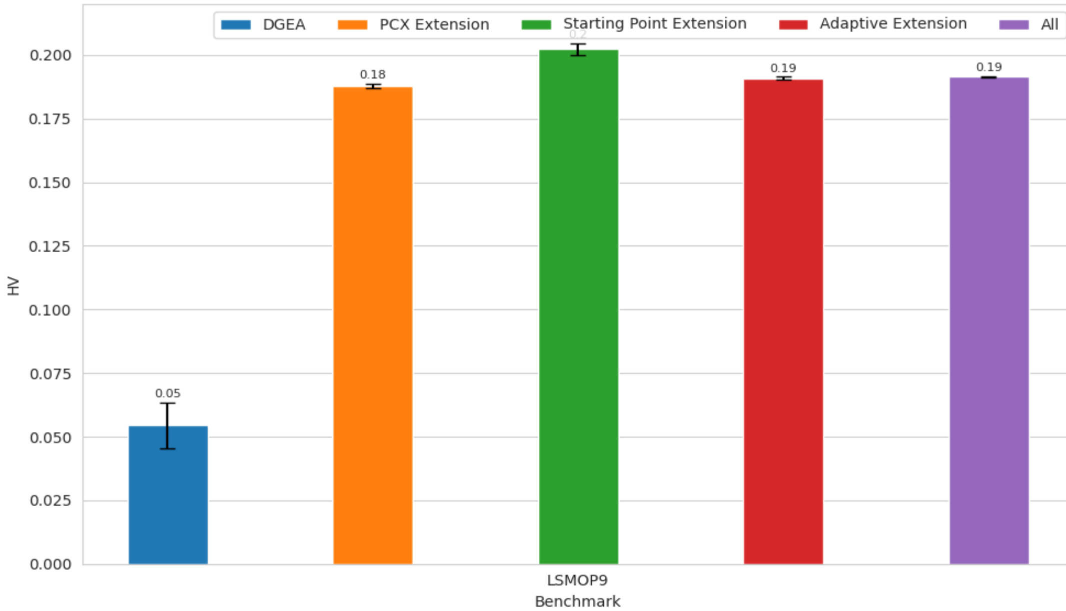

Figure 5.11 Early Stage HV Performance of Our Extensions on the LSMOP9 (100.000 Function Evaluations)

When evaluating the early-stage performance on the problem LSMOP9, there is a significant improvement observed in both the IGD and HV metrics. Our extensions demonstrate enhanced performance, indicating their effectiveness and efficiency for this particular problem LSMOP9. The improvements in both metrics suggest that the proposed algorithms have successfully handled the problem's mixed landscape. This improvement shows the potential of the proposed our extensions for addressing complex optimization problems. Only adaptive extension is less performant than the baseline algorithm in the context of IGD. This should be extensively researched.

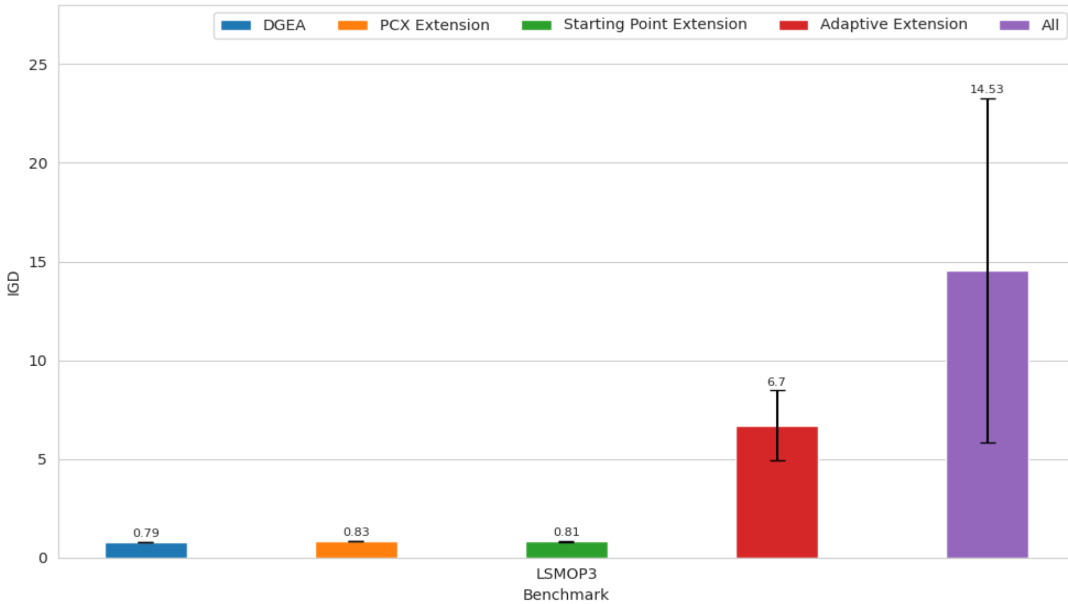

Figure 5.12 Early Stage IGD Performance of Our Extensions on the LSMOP7 (100.000 Function Evaluations)

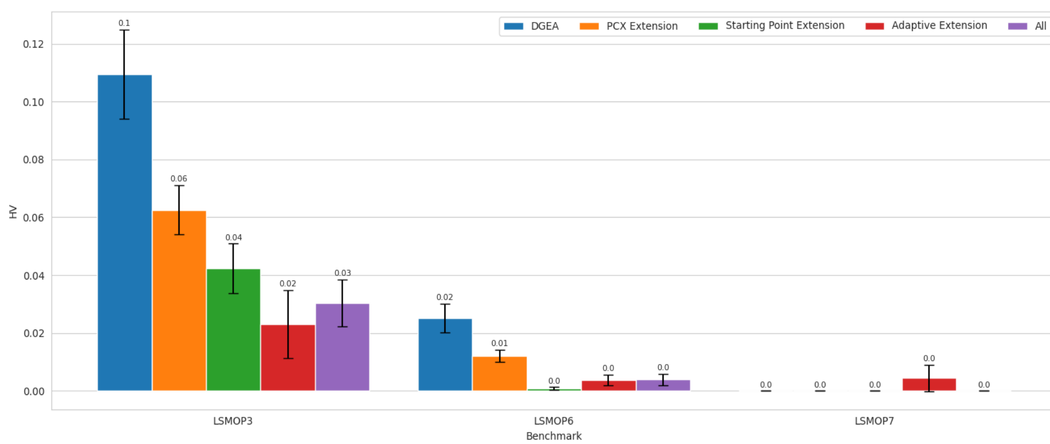

Figure 5.13 Early Stage HV Performance of Our Extensions on the LSMOP3, LSMOP6 and LSMOP7 (100.000 Function Evaluations)

In contrast to the significant performance improvement observed for LSMOP9, the early-stage performance of the our extensions on LSMOP3 shows significant drop in performance. Multimodal landscape and mixed separability of LSMOP3 benchmark is challenging for the proposed our extensions. Despite the enhancements observed in LSMOP9, our extension struggled to effectively overcome the complexities of LSMOP3, resulting in diminished performance. This indicates that our extensions may not be suitable for the specific challenges presented by LSMOP3 in the early stages.

6. BENEFITS AND IMPACT

Our study can give insight to other academic researchers. By focusing on the novel search operator, this study offers new extensions to solving the LSMOPs. This approach can give idea to researchers working on this area to improve their own studies. They can gain new perspectives on this. This study can be a stepping stone for new studies. Also, it can also be beneficial to the owners of the work that we referenced.

i. Scientific Impact: In recent years, it has become a challenging topic for the optimization world. As a result, it has become the focus of the researchers in this field. Solving LSMOPs is one of the most tackled optimization topics and our referenced study deals with that. We have 3 different types of extension on that. We do not expect publishing scientific paper, but it can give insight and ideas for other researchers.

ii. Economic/Commercial/Social Impact: For example, it can contribute to a financial institution, manufacturing companies, health, pharmaceutical companies and technology companies, are dealing with large scale multi-objective optimization problems facilitate the processes with stronger algorithms, achieving products and services with less costs, developing better products and they can serve better services and make more profit. For instance, finance companies also need to manage their investment portfolios. These study models contain more variables for maximizing profit of the portfolio [10]. Also, maybe designing and discovering a drug can take more days, months and years. This situation can be very costly and furthermore it can be a time loss. As a result of these advancing technological developments, it may lead to the production of better technological devices and to produce more economic value. But in the near future, we did not observe the absolute impact on these factors. Our project is mainly focused on academic contribution.

iii. Potential Impact on New Projects: It can give a new approach and idea for solving problems with more decision variables and multi-objective problems for research. It may be useful to people who will work on the novel search operator topic. We achieved successful solutions of some LSMOPs in our study. The results and conclusion of our study can provide valuable information to them. They can focus on the LSMOPs tests that we failed. Based on the future work we have explained, better improvements and extensions can be made.

iv. Impact on National Security: There is no related impact on national security issues.

7. CONCLUSION AND FUTURE WORK

In conclusion, this thesis focused on enhancing the performance of large-scale multi-objective optimization problems through the hybridization of multiple techniques. The proposed our hybridization, including PCX Crossover, Starting Point revision, Vector Adaptation, and these three were combined to create a fourth extension. The performance of these our hybridization was evaluated using two key metrics: Hypervolume (HV) and Inverted Generational Distance (IGD), which measure convergence and diversity during the optimization process.

The experimentation phase involved testing our hybridization on nine benchmark problems specifically designed for large-scale multi-objective optimization algorithms. The results demonstrated a significant improvement in performance, particularly on the LSMOP9 problem, indicating the effectiveness of the hybridization in complex search spaces. Additionally, the hybridization shows enhanced early convergence capabilities on the LSMOP8 and LSMOP9 problems.

However, the performance of our hybridization dropped significantly on the LSMOP3 problem, which features a multimodal landscape and mixed separability. This outcome suggests that further investigation is required to understand and improve the algorithm's behavior in such scenarios.

For future work, the reason for the performance drop on the LSMOP3 benchmark can be investigated considering its characteristics. This investigation may involve studying the behavior of individual hybridization components, such as the PCX Crossover or Vector Adaptation, to identify potential weaknesses. Secondly, exploring alternative hybridization strategies by incorporating additional techniques or modifications could lead to improved performance. For instance, investigating the integration of other crossover operators, mutation strategies, or local search mechanisms may help overcome the challenges posed by the multimodal landscape and mixed separability of LSMOP3.

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APPENDICES

Objective Functions of LSMOPs [24]

LSMOP2:

$$f_1(x) = x_1^f \ldots x_{M-1}^f \left(1 + \sum_{j=1}^{M} c_{1,j} \times g^{-}{}_1(x_j^s) \right)$$

$$f_2(x) = x_1^f \ldots \left(1 - x_{M-1}^f\right) \left(1 +. \sum_{j=1}^{M} c_{2,j} \times g^{-}{}_2(x_j^s) \right)$$

...

$$f_{M-1}(x) = x_1^f \left(1 - x_2^f\right) \left(1 + \sum_{j=1}^{M} c_{M-1,j} \times g^{-}{}_{M-1}(x_j^s) \right)$$

$$f_M(x) = \left(1 - x_1^f\right) \left(1 + \sum_{j=1}^{M} c_{M,j} \times g^{-}{}_M(x_j^s) \right)$$

$$\begin{cases} \bar{g}_{2k-1}(\mathbf{x}_i^s) = \dfrac{1}{n_k} \displaystyle\sum_{j=1}^{n_k} \dfrac{\eta_5\left(\mathbf{x}_{i,j}^s\right)}{\left|\mathbf{x}_{i,j}^s\right|} \\ \bar{g}_{2k}(\mathbf{x}_i^s) = \dfrac{1}{n_k} \displaystyle\sum_{j=1}^{n_k} \dfrac{\eta_2\left(\mathbf{x}_{i,j}^s\right)}{\left|\mathbf{x}_{i,j}^s\right|} \\ k = 1, \ldots, \left\lceil \dfrac{M}{2} \right\rceil \end{cases}$$

LSMOP3:

$$f_1(x) = x_1^f \ldots x_{M-1}^f \left(1 + \sum_{j=1}^{M} c_{1,j} \times g^{-}{}_1(x_j^s) \right)$$

$$f_2(x) = x_1^f \dots \left(1 - x_{M-1}^f\right)\left(1 + \sum_{j=1}^{M} c_{2,j} \times g^{-}{}_2\left(x_j^s\right)\right)$$

…

$$f_{M-1}(x) \ = x_1^f\left(1 - x_2^f\right)\left(1 + \sum_{j=1}^{M} c_{M-1,j} \times g^{-}{}_{M-1}\left(x_j^s\right)\right)$$

$$f_M(x) \ \ = \left(1 - x_1^f\right)\left(1 + \sum_{j=1}^{M} c_{M,j} \times g^{-}{}_M\left(x_j^s\right)\right)$$

$$\begin{cases} \bar{g}_{2k-1}(\mathbf{x}_i^s) = \dfrac{1}{n_k}\displaystyle\sum_{j=1}^{n_k} \dfrac{\eta_4\left(\mathbf{x}_{i,j}^s\right)}{\left|\mathbf{x}_{i,j}^s\right|} \\ \bar{g}_{2k}(\mathbf{x}_i^s) = \dfrac{1}{n_k}\displaystyle\sum_{j=1}^{n_k} \dfrac{\eta_3\left(\mathbf{x}_{i,j}^s\right)}{\left|\mathbf{x}_{i,j}^s\right|} \\ k = 1, \dots, \left\lceil \dfrac{M}{2} \right\rceil \end{cases}$$

LSMOP4:

$$f_1(x) \ = x_1^f \dots x_{M-1}^f\left(1 + \sum_{j=1}^{M} c_{1,j} \times g^{-}{}_1\left(x_j^s\right)\right)$$

$$f_2(x) \ = x_1^f \dots \left(1 - x_{M-1}^f\right)\left(1 + \sum_{j=1}^{M} c_{2,j} \times g^{-}{}_2\left(x_j^s\right)\right)$$

…

$$f_{M-1}(x) \ = x_1^f\left(1 - x_2^f\right)\left(1 + \sum_{j=1}^{M} c_{M-1,j} \times g^{-}{}_{M-1}\left(x_j^s\right)\right)$$

$$f_M(x) \ \ = \left(1 - x_1^f\right)\left(1 + \sum_{j=1}^{M} c_{M,j} \times g^{-}{}_M\left(x_j^s\right)\right)$$

$$\begin{cases} \bar{g}_{2k-1}(\mathbf{x}_i^s) = \frac{1}{n_k}\sum_{j=1}^{n_k} \frac{\eta_6(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|} \\ \bar{g}_{2k}(\mathbf{x}_i^s) = \frac{1}{n_k}\sum_{j=1}^{n_k} \frac{\eta_5(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|} \\ k = 1, \dots, \left\lceil\frac{M}{2}\right\rceil \end{cases}$$

LSMOP6:

$$f_1(x) \;\; = cos\ \left(\frac{\pi}{2}x_1^f\right) \dots cos\ \left(\frac{\pi}{2}x_{M-2}^f\right) cos\ \left(\frac{\pi}{2}x_{M-1}^f\right) \;\; \times \left(1 + \sum_{j=1}^{M} c_{1,j} \times g^{-}{}_{1}\left(x_j^s\right)\right)$$

$$f_2(x) \;\; = cos\ \left(\frac{\pi}{2}x_1^f\right) \dots cos\ \left(\frac{\pi}{2}x_{M-2}^f\right) sin\ \left(\frac{\pi}{2}x_{M-1}^f\right) \;\; \times \left(1 + \sum_{j=1}^{M} c_{2,j} \times g^{-}{}_{2}\left(x_j^s\right)\right)$$

$$\dots$$

$$f_{M-1}(x) \;\; = cos\ \left(\frac{\pi}{2}x_1^f\right) sin\ \left(\frac{\pi}{2}x_2^f\right) \;\; \times \left(1 + \sum_{j=1}^{M} c_{M-1,j} \times g^{-}{}_{M-1}\left(x_j^s\right)\right)$$

$$f_M(x) \;\; = sin\ \left(\frac{\pi}{2}x_1^f\right) \times \left(1 + \sum_{j=1}^{M} c_{M,j} g^{-}{}_{M}\left(x_j^s\right)\right)$$

$$\begin{cases} \bar{g}_{2k-1}(\mathbf{x}_i^s) = \frac{1}{n_k}\sum_{j=1}^{n_k} \frac{\eta_3(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|} \\ \bar{g}_{2k}(\mathbf{x}_i^s) = \frac{1}{n_k}\sum_{j=1}^{n_k} \frac{\eta_2(\mathbf{x}_{i,j}^s)}{|\mathbf{x}_{i,j}^s|} \\ k = 1, \dots, \left\lceil\frac{M}{2}\right\rceil \end{cases}$$

LSMOP7:

$$f_1(x) \;\; = cos\ \left(\tfrac{\pi}{2}x_1^f\right) \dots cos\ \left(\tfrac{\pi}{2}x_{M-2}^f\right) cos\ \left(\tfrac{\pi}{2}x_{M-1}^f\right) \;\; \times \left(1 + \textstyle\sum_{j=1}^{M} c_{1,j} \times g^{-}{}_{1}\left(x_j^s\right)\right)$$

$$f_2(x) = cos\left(\frac{\pi}{2}x_1^f\right)\dots cos\left(\frac{\pi}{2}x_{M-2}^f\right)sin\left(\frac{\pi}{2}x_{M-1}^f\right)\times\left(1+\sum_{j=1}^{M}c_{2,j}\times \bar{g}_2\left(x_j^s\right)\right)$$

$$\dots$$

$$f_{M-1}(x) = cos\left(\frac{\pi}{2}x_1^f\right)sin\left(\frac{\pi}{2}x_2^f\right) \times\left(1+\sum_{j=1}^{M}c_{M-1,j}\times \bar{g}_{M-1}\left(x_j^s\right)\right)$$

$$f_M(x) = sin\left(\frac{\pi}{2}x_1^f\right)\times\left(1+\sum_{j=1}^{M}c_{M,j}\bar{g}_M\left(x_j^s\right)\right)$$

$$\begin{cases} \bar{g}_{2k-1}(\mathbf{x}_i^s) = \dfrac{1}{n_k}\displaystyle\sum_{j=1}^{n_k}\dfrac{\eta_6\left(\mathbf{x}_{i,j}^s\right)}{\left|\mathbf{x}_{i,j}^s\right|} \\ \bar{g}_{2k}(\mathbf{x}_i^s) = \dfrac{1}{n_k}\displaystyle\sum_{j=1}^{n_k}\dfrac{\eta_3\left(\mathbf{x}_{i,j}^s\right)}{\left|\mathbf{x}_{i,j}^s\right|} \\ k = 1,\dots,\left\lceil\dfrac{M}{2}\right\rceil \end{cases}$$

LSMOP8:

$$f_1(x) = cos\left(\frac{\pi}{2}x_1^f\right)\dots cos\left(\frac{\pi}{2}x_{M-2}^f\right)cos\left(\frac{\pi}{2}x_{M-1}^f\right)\times\left(1+\sum_{j=1}^{M}c_{1,j}\times \bar{g}_1\left(x_j^s\right)\right)$$

$$f_2(x) = cos\left(\frac{\pi}{2}x_1^f\right)\dots cos\left(\frac{\pi}{2}x_{M-2}^f\right)sin\left(\frac{\pi}{2}x_{M-1}^f\right)\times\left(1+\sum_{j=1}^{M}c_{2,j}\times \bar{g}_2\left(x_j^s\right)\right)$$

$$\dots$$

$$f_{M-1}(x) = cos\left(\frac{\pi}{2}x_1^f\right)sin\left(\frac{\pi}{2}x_2^f\right) \times\left(1+\sum_{j=1}^{M}c_{M-1,j}\times \bar{g}_{M-1}\left(x_j^s\right)\right)$$

$$f_M(x) = sin\left(\frac{\pi}{2}x_1^f\right)\times\left(1+\sum_{j=1}^{M}c_{M,j}\bar{g}_M\left(x_j^s\right)\right)$$

$$\begin{cases} \bar{g}_{2k-1}(\mathbf{x}_i^s) = \dfrac{1}{n_k}\displaystyle\sum_{j=1}^{n_k}\dfrac{\eta_5\left(\mathbf{x}_{i,j}^s\right)}{\left|\mathbf{x}_{i,j}^s\right|} \\ \bar{g}_{2k}(\mathbf{x}_i^s) = \dfrac{1}{n_k}\displaystyle\sum_{j=1}^{n_k}\dfrac{\eta_1\left(\mathbf{x}_{i,j}^s\right)}{\left|\mathbf{x}_{i,j}^s\right|} \\ k = 1,\dots,\left\lceil\dfrac{M}{2}\right\rceil \end{cases}$$