ident, $x, y, y_p, y_f, -$, abbrev, r, α subscripts: p for pointers, f for functions

n, i, j index variables

 $impl_const$ implementation-defined constant member C struct/union member name

Ott-hack, ignore (annotations)

nat OCaml arbitrary-width natural number

mem_ptr abstract pointer value
mem_val abstract memory value

Ott-hack, ignore (locations)

mem_iv_c OCaml type for memory constraints on integer values

 UB_name undefined behaviour

string OCaml string

Ott-hack, ignore (OCaml type variable TY)
Ott-hack, ignore (OCaml Symbol.prefix)

mem_order, _ OCaml type for memory order

linux_mem_order OCaml type for Linux memory order

Ott-hack, ignore (OCaml type variable bt)

```
Sctypes_{-}t, \tau
                                                 C type
                                                    pointer to type \tau
tag
                                                 OCaml type for struct/union tag
                     ::=
                           ident
β, _
                                                 base types
                     ::=
                                                    unit
                           unit
                           bool
                                                    boolean
                                                    integer
                           integer
                                                    rational numbers?
                           real
                                                   location
                           loc
                           \operatorname{array} \beta
                                                    array
                           \mathtt{list}\, eta
                                                    list
                                                    tuple
                           \mathtt{struct}\,tag
                                                    struct
                           \operatorname{\mathfrak{set}} \beta
                                                    \operatorname{set}
                           opt(\beta)
                                                    option
                                                   parameter types
                           \beta \to \beta'
                           \beta_{\tau}
                                           Μ
                                                    of a C type
binop
                                                 binary operators
                                                    addition
                                                    subtraction
                                                    multiplication
                                                    division
                                                    modulus
                                                    remainder
                           rem_f
                                                    exponentiation
                                                    equality, defined both for integer and C types
```

	!= > < >= <= /\	inequality, similiarly defined greater than, similarly defined less than, similarly defined greater than or equal to, similarly defined less than or equal to, similarly defined conjunction disjunction
$binop_{arith}$::=	arithmentic binary operators
$binop_{rel}$::=	relational binary operators
$binop_{bool}$::= 	boolean binary operators
mem_int	::=	memory integer value

		1 0	M M	
$object_value$::=	$\begin{array}{l} mem_int \\ mem_ptr \\ \operatorname{array}\left(\overline{loaded_value_i}^i\right) \\ (\operatorname{struct} ident)\{\overline{.member_i:\tau_i = mem_val_i}^i\} \\ (\operatorname{union} ident)\{.member = mem_val\} \end{array}$		C object values (inhabitants of object types), which can be read/stored integer value pointer value C array value C struct value C union value
$loaded_value$::= 	$\verb specified object_value $		potentially unspecified C object values specified loaded value
value	::=	$object_value \ loaded_value \ Unit \ True \ False \ eta[\overline{value_i}^i] \ (\overline{value_i}^i)$		Core values C object value loaded C object value unit boolean true boolean false list tuple
$bool_value$::= 	True False		Core booleans boolean true boolean false
$ctor_val$::=	$\begin{array}{c} \operatorname{Nil}\beta\\ \operatorname{Cons}\\ \operatorname{Tuple} \end{array}$		data constructors empty list list cons tuple

		Array Specified	C array non-unspecified loaded value
	ı	Specifica	-
$ctor_expr$::=		data constructors
		Ivmax	max integer value
		Ivmin	min integer value
		Ivsizeof	sizeof value
		Ivalignof	alignof value
		IvCOMPL	bitwise complement
		IvAND	bitwise AND
		IvOR	bitwise OR
		IvXOR	bitwise XOR
		Fvfromint	cast integer to floating value
		Ivfromfloat	cast floating to integer value
name	::=		
name	—	ident	Core identifier
		$impl_const$	implementation-defined constant
	'	1	•
pval	::=		pure values
		ident	Core identifier
		$impl_const$	implementation-defined constant
		value	Core values
		$\mathtt{constrained}(\overline{mem_iv_c_i,pval_i}^{i})$	constrained value
		$\mathtt{error}\left(string, pval ight)$	impl-defined static error
		$ctor_val(\overline{pval_i}^i)$	data constructor application
		$(\mathtt{struct}ident)\{\overline{.member_i=pval_i}^{i}\}$	C struct expression
		$(\verb"union" ident") \{ .member = pval \}$	C union expression
tpval	::=		top-level pure values
cpout			top tevel pure variets

		$\begin{array}{ll} {\tt undef} \ \ UB_name \\ {\tt done} \ pval \end{array}$		undefined behaviour pure done
$ident_opt_eta$::= 	$_{::}eta \ ident:eta$	$binders = \{\}$ $binders = ident$	type annotated optional identifier
pattern	::= 	$ident_opt_eta \ ctor_val(\overline{pattern_i}^i)$	$\begin{aligned} & \text{binders} = \text{binders}(ident_opt_\beta) \\ & \text{binders} = \text{binders}(\overline{pattern}_i^{\ i}) \end{aligned}$	
z	::=	$i \\ mem_int \\ size_of(au) \\ offset_of_{tag}(member) \\ ptr_size \\ max_int_{ au} \\ min_int_{ au}$	M M M M M M	OCaml arbitrary-width integer literal integer size of a C type offset of a struct member size of a pointer maximum value of int of type τ minimum value of int of type τ
$\mathbb{Q},\ q,\ _{-}$::=	$rac{int_1}{int_2}$		OCaml type for rational numbers
lit	::=	$ident$ unit $bool$ z \mathbb{Q}		

```
ident\_or\_pattern
                                 ident
                                                                           binders = ident
                                                                           binders = binders(pattern)
                                 pattern
bool\_op
                                 \neg term
                                 term_1 = term_2
                                 term_1 \rightarrow term_2
                                \bigwedge(\overline{term_i}^i)
                                 \bigvee (\overline{term_i}^i)
                                 term_1 \ binop_{bool} \ term_2
                                                                           Μ
                                 if term_1 then term_2 else term_3
arith\_op
                          ::=
                                 term_1 + term_2
                                 term_1 - term_2
                                 term_1 \times term_2
                                 term_1/term_2
                                 term_1 \, {\tt rem\_t} \, term_2
                                 term_1 \, {\tt rem\_f} \, term_2
                                 term_1 \hat{} term_2
                                 term_1 \ binop_{arith} \ term_2
                                                                           Μ
cmp\_op
                                 term_1 < term_2
                                                                                                                  less than
                                 term_1 \leq term_2
                                                                                                                  less than or equal
                                 term_1 binop_{rel} term_2
                                                                           Μ
list\_op
                                 nil
```

```
term_1 :: term_2
                           \mathtt{tl}\, term
                           term^{(int)}
tuple\_op
                    ::=
                            (\overline{term_i}^i)
                           term^{(int)}
pointer\_op
                    ::=
                           mem\_ptr
                           term_1 +_{ptr} term_2
                           {\tt cast\_int\_to\_ptr}\, term
                           {\tt cast\_ptr\_to\_int}\, term
array\_op
                           [\mid \overline{term_i}^i \mid]
                           term_1[term_2]
param\_op
                    ::=
                           ident:\beta.\ term
                           term(term_1, ..., term_n)
struct\_op
                    ::=
                           term.member \\
ct\_pred
                    ::=
                           \texttt{representable}\left(\tau, term\right)
                           aligned(\tau, term)
                           \texttt{alignedI}\left(term_1, term_2
ight)
```

```
term, -
                     lit
                     arith\_op
                     bool\_op
                     cmp\_op
                     tuple\_op
                     struct\_op
                    pointer\_op
                     list\_op
                     array\_op
                     ct\_pred
                    param\_op
                     (term)
                                                                 S
                                                                        parentheses
                    \sigma(term)
                                                                 Μ
                                                                        simul-sub \sigma in term
                                                                 Μ
                     pval
                                                                      pure expressions
pexpr
                    pval
                                                                        pure values
                    ctor\_expr(\overline{pval_i}^i)
                                                                         data constructor application
                     array\_shift(pval_1, \tau, pval_2)
                                                                        pointer array shift
                                                                        pointer struct/union member shift
                    member\_shift(pval, ident, member)
                    \mathtt{not}\left(pval\right)
                                                                        boolean not
                    pval_1 \ binop \ pval_2
                                                                        binary operations
                    memberof(ident, member, pval)
                                                                         C struct/union member access
                    name(\overline{pval_i}^i)
                                                                        pure function call
                     assert\_undef(pval, UB\_name)
                     bool\_to\_integer(pval)
                     \mathtt{conv\_int}\left(	au,pval
ight)
                     \mathtt{wrapI}\left( 	au,pval 
ight)
```

tpexpr	::=	$tpval$ $\texttt{case} \ pval \ \texttt{of} \ \overline{\mid tpexpr_case_branch_i}^i \ \texttt{end}$ $\texttt{let} \ ident_or_pattern = pexpr \ \texttt{in} \ tpexpr$ $\texttt{let} \ ident_or_pattern: (y_1:\beta_1. \ term_1) = tpexpr_1 \ \texttt{in} \ tpexpr_2$ $\texttt{if} \ pval \ \texttt{then} \ tpexpr_1 \ \texttt{else} \ tpexpr_2$ $\sigma(tpexpr)$	bind binders $(ident_or_pattern)$ in $tpexpr$ bind binders $(ident_or_pattern)$ in $tpexpr_2$ bind y_1 in $term_1$	top-level pure expressions top-level pure values pattern matching pure let annoted pure let pure if simul-sub σ in $tpexpr$
$tpexpr_case_branch$::=	$pattern \Rightarrow tpexpr$	bind $binders(pattern)$ in $tpexpr$	pure top-level case expression top-level case expression br
m_kill_kind	::= 	$\begin{array}{l} \operatorname{dynamic} \\ \operatorname{static} \tau \end{array}$		
bool, _	::=	true false		OCaml booleans
$int,$ _	::=	i		OCaml fixed-width integer literal integer
res_term	::=	$\begin{array}{l} \texttt{emp} \\ points_to \\ ident \\ \langle res_term_1, res_term_2 \rangle \\ \texttt{pack} \left(pval, res_term \right) \\ \texttt{fold} \left(res_term \right) \end{array}$		resource terms empty heap single-cell heap variable seperating-conjunction pair packing for existentials fold into recursive res. pred

```
\sigma(res\_term)
                                                                                                              substitution for resource terms
                                                                                                            memory actions
mem\_action
                             create(pval, \tau)
                             create_readonly (pval_1, \tau, pval_2)
                             alloc(pval_1, pval_2)
                            kill(m_kill_kind, pval, pt)
                             store(bool, \tau, pval_1, pval_2, mem\_order, pt)
                                                                                                              true means store is locking
                            load(\tau, pval, mem\_order, pt)
                            rmw(\tau, pval_1, pval_2, pval_3, mem\_order_1, mem\_order_2)
                            fence(mem\_order)
                             cmp_exch_strong(\tau, pval_1, pval_2, pval_3, mem_order_1, mem_order_2)
                             cmp_exch_weak(\tau, pval_1, pval_2, pval_3, mem_order_1, mem_order_2)
                            linux_fence (linux_mem_order)
                            linux\_load(\tau, pval, linux\_mem\_order)
                            linux\_store(\tau, pval_1, pval_2, linux\_mem\_order)
                             linux_rmw(\tau, pval_1, pval_2, linux_mem_order)
                                                                                                            polarities for memory actions
polarity
                       ::=
                                                                                                               (pos) sequenced by let weak and let strong
                                                                                                              only sequenced by let strong
                             neg
pol\_mem\_action
                                                                                                            memory actions with polarity
                             polarity\ mem\_action
                                                                                                            operations involving the memory state
mem\_op
                                                                                                              pointer relational binary operations
                            pval_1 \ binop_{rel} \ pval_2
                            pval_1 -_{\tau} pval_2
                                                                                                              pointer subtraction
                            \mathtt{intFromPtr}\left(	au_{1},	au_{2},pval
ight)
                                                                                                              cast of pointer value to integer value
                             ptrFromInt (\tau_1, \tau_2, pval)
                                                                                                              cast of integer value to pointer value
```

```
ptrValidForDeref(\tau, pval, pt)
                                                                                                                             dereferencing validity predicate
                       ptrWellAligned(\tau, pval)
                       ptrArrayShift(pval_1, \tau, pval_2)
                       memcpy(pval_1, pval_2, pval_3)
                       memcmp(pval_1, pval_2, pval_3)
                       realloc(pval_1, pval_2, pval_3)
                       va\_start(pval_1, pval_2)
                       va\_copy(pval)
                       va\_arg(pval, \tau)
                       va_-end(pval)
                                                                                                                          spine element
spine\_elem
                 ::=
                                                                                                                             pure or logical value
                       pval
                       res\_term
                                                                                                                             resource value
                       \sigma(spine\_elem)
                                                            М
                                                                                                                             substitution for spine elements / return values
spine
                 ::=
                                                                                                                          spine
                       \overline{spine\_elem_i}
tval
                                                                                                                          (effectful) top-level values
                 ::=
                       {\tt done}\, spine
                                                                                                                             end of top-level expression
                       undef UB\_name
                                                                                                                             undefined behaviour
res\_pattern
                                                                                                                          resource terms
                                                            binders = \{\}
                                                                                                                             empty heap
                       emp
                                                            binders = ident
                       ident
                                                                                                                             variable
                       fold(res_pattern)
                                                            binders = \{\}
                                                                                                                             unfold (recursive) predicate
                       \langle res\_pattern_1, res\_pattern_2 \rangle
                                                            binders = binders(res\_pattern_1) \cup binders(res\_pattern_2)
                                                                                                                             seperating-conjunction pair
                       pack (ident, res_pattern)
                                                            binders = ident \cup binders(res\_pattern)
                                                                                                                             packing for existentials
```

$ret_pattern$::=	$egin{array}{l} { t comp} \ ident_or_pattern \ { t log} \ ident \ { t res} \ res_pattern \end{array}$	$\begin{aligned} & \text{binders} = \text{binders}(ident_or_pattern) \\ & \text{binders} = ident \\ & \text{binders} = \text{binders}(res_pattern) \end{aligned}$	return pattern computational variable logical variable resource variable
init,	::= 	✓ ×		initialisation status initialised uninitalised
$points_to, pt$::=	$term_1 \stackrel{init}{\mapsto}_{\tau} term_2$		points-to separation logic predicate
res	::=	emp $points_to$ $res_1 * res_2$ $\exists ident: \beta. res$ $term \land res$ if $term$ then res_1 else res_2 $\alpha(\overrightarrow{pval_i}^i)$ $\sigma(res)$	M	resources empty heap points-top heap pred. seperating conjunction existential logical conjuction ordered disjuction predicate simul-sub σ in res
ret , _	::=	$\Sigma ident: \beta. \ ret$ $\exists ident: \beta. \ ret$ $res \otimes ret$ $term \wedge ret$ I $\sigma(ret)$	M	return types return a computational value return a logical value return a resource value return a predicate (post-condition) end return list simul-sub σ in ret

seq_expr	::= 	$\begin{array}{l} \mathtt{ccall}\left(\tau, ident, spine\right) \\ \mathtt{pcall}\left(name, spine\right) \end{array}$		sequential (effectful) expressions C function call procedure call
seq_texpr	::=	$tval \\ \operatorname{run} ident \overline{pval_i}^i \\ \operatorname{let} ident_or_pattern = pexpr \operatorname{in} texpr \\ \operatorname{let} ident_or_pattern: (y_1:\beta_1.\ term_1) = tpexpr \operatorname{in} texpr \\ \operatorname{let} \overline{ret_pattern_i}^i = seq_expr \operatorname{in} texpr \\ \operatorname{let} \overline{ret_pattern_i}^i : ret = texpr_1 \operatorname{in} texpr_2 \\ \operatorname{case} pval \operatorname{of} \overline{\mid texpr_case_branch_i}^i \operatorname{end} \\ \operatorname{if} pval \operatorname{then} texpr_1 \operatorname{else} texpr_2 \\ \operatorname{bound} [int] (is_texpr) \\ \end{aligned}$	bind binders($ident_or_pattern$) in $texpr$ bind binders($ident_or_pattern$) in $texpr$ bind y_1 in $term_1$ bind binders($\overline{ret_pattern_i}^i$) in $texpr$ bind binders($\overline{ret_pattern_i}^i$) in $texpr_2$	sequential top-level (effectful) expres (effectful) top-level values run from label pure let annotated pure let bind return patterns annotated bind return patterns pattern matching conditional limit scope of indet seq behaviour
$texpr_case_branch$::=	$pattern \Rightarrow texpr$	bind $binders(pattern)$ in $texpr$	top-level case expression branch top-level case expression branch
is_expr	::= 	$tval \\ exttt{memop} (mem_op) \\ pol_mem_action$		indet seq (effectful) expressions (effectful) top-level values pointer op involving memory memory action
is_texpr	::= 	$\begin{array}{l} \texttt{let weak}\overline{ret_pattern_i}^{\;i} = is_expr\texttt{in}texpr\\ \texttt{let strong}\overline{ret_pattern_i}^{\;i} = is_expr\texttt{in}texpr \end{array}$	bind binders $(\overline{ret_pattern_i}^i)$ in $texpr$ bind binders $(\overline{ret_pattern_i}^i)$ in $texpr$	indet seq top-level (effectful) express weak sequencing strong sequencing
texpr	::=	seq_texpr		top-level (effectful) expressions sequential (effectful) expressions

```
indet seq (effectful) expressions
                       is\_texpr
                       \sigma(texpr)
                                                  Μ
                                                          simul-sub \sigma in texpr
                                                       argument/function types
arg
                       \Pi ident:\beta. arg
                       \forall ident: \beta. arg
                       res \multimap arg
                       term \supset arg
                       ret
                       \sigma(arg)
                                                           simul-sub \sigma in arg
                                                       pure argument/function types
pure\_arg
                       \Pi ident:\beta. pure\_arg
                       term \supset pure\_arg
                       pure\_ret
pure\_ret
                                                       pure return types
                       \Sigma ident:\beta. pure_ret
                       term \land pure\_ret
\mathcal{C}
                                                       computational var env
                       C, ident: \beta
\mathcal{L}
                                                       logical var env
```

```
constraints env
\mathcal R
                                                                                                                                   resources env
                                \frac{\mathcal{R}, ident:res}{\mathcal{R}_i}^i
                                                                                                                                   substitutions
\sigma, \psi
                          spine\_elem/ident, \sigma
                                term/ident, \sigma \ \overline{\sigma_i}^i
                                   \sigma(\psi)
                                                                                                                         Μ
                                                                                                                                        apply \sigma to all elements in \psi
typing
                                   \mathtt{smt}\left(\Phi\Rightarrow term\right)
                                ident:\beta \in \mathcal{C}
                         | ident.\beta \in \mathcal{C} 
| ident:\beta \in \mathcal{L} 
| struct tag & \overline{member_i:\tau_i}^i \in \texttt{Globals} 
| \alpha \equiv \overline{x_i:\beta_i}^i \mapsto res \in \texttt{Globals} 
| \overline{\mathcal{C}_i;\mathcal{L}_i;\Phi_i} \vdash mem\_val_i \Rightarrow mem \beta_i}^i
                                                                                                                                        recursive resource predicate
                                                                                                                                        dependent on memory object model
opsem
                                  \forall i < j. \ \mathtt{not} \ (pattern_i = pval \leadsto \sigma_i)
                                   \mathtt{fresh}\left(mem\_ptr
ight)
                                   term
                                   pval:\beta
```

```
formula
                                               judgement
                                                typing
                                                opsem
                                               term \equiv term'
                                               name:pure\_arg \equiv \overline{x_i}^i \mapsto tpexpr \in \texttt{Globals} name:arg \equiv \overline{x_i}^i \mapsto texpr \in \texttt{Globals}
heap, h
                                                                                                                                    heaps
                                      ::=
                                               h + \{points\_to\}
lemma\_jtype
                                       res\_jtype
                                               \Phi \vdash res \equiv res'
                                              \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathit{res\_term} \Leftarrow \mathit{res}
object\_value\_jtype
                                               C; \mathcal{L}; \Phi \vdash object\_value \Rightarrow \mathsf{obj}\,\beta
pval\_jtype
                                               C; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta
spine\_jtype
                                              C; \mathcal{L}; \Phi; \mathcal{R} \vdash \overline{x_i = spine\_elem_i}^i :: arg \gg \sigma; ret
```

```
pexpr\_jtype
                                                       C; \mathcal{L}; \Phi \vdash pexpr \Rightarrow ident: \beta. term
comp\_pattern\_jtype
                                             ::=
                                                       pattern: eta \leadsto \mathcal{C} 	ext{ with } term
                                                       ident\_or\_pattern:\beta \leadsto \mathcal{C} \text{ with } term
res\_pattern\_jtype
                                                       \mathcal{L}; \Phi \vdash res' = \mathtt{strip\_ifs}(res)
                                                       \mathcal{L}; \Phi \vdash res \text{ as } res\_pattern \leadsto \mathcal{L}'; \Phi'; \mathcal{R}'
                                                       \mathcal{L}; \Phi \vdash res\_pattern:res \leadsto \mathcal{L}'; \Phi'; \mathcal{R}'
ret\_pattern\_jtype
                                             ::=
                                                       \mathcal{L}; \Phi \vdash \overline{ret\_pattern_i}^i : ret \leadsto \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'
tpval\_jtype
                                             ::=
                                                       C; \mathcal{L}; \Phi \vdash tpval \Leftarrow ident: \beta. term
tpexpr\_jtype
                                             ::=
                                                       C; \mathcal{L}; \Phi \vdash tpexpr \Leftarrow ident: \beta. term
action\_jtype
                                             ::=
                                                       C; \mathcal{L}; \Phi; \mathcal{R} \vdash mem\_action \Rightarrow ret
memop\_jtype
                                             ::=
                                                       C; \mathcal{L}; \Phi; \mathcal{R} \vdash mem\_op \Rightarrow ret
tval\_jtype
```

 $C; \mathcal{L}; \Phi; \mathcal{R} \vdash tval \Leftarrow ret$

```
seq\_expr\_jtype
                                                         C; \mathcal{L}; \Phi; \mathcal{R} \vdash seq\_expr \Rightarrow ret
is\_expr\_jtype
                                              ::=
                                                        C; \mathcal{L}; \Phi; \mathcal{R} \vdash is\_expr \Rightarrow ret
texpr\_jtype
                                                        C; \mathcal{L}; \Phi; \mathcal{R} \vdash seq\_texpr \Leftarrow ret
                                                        C; \mathcal{L}; \Phi; \mathcal{R} \vdash is\_texpr \Leftarrow ret
                                                         C; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftarrow ret
subs\_jtype
                                                         pattern = pval \leadsto \sigma
                                                         ident\_or\_pattern = pval \leadsto \sigma
                                                         res\_pattern = res\_term \leadsto \sigma
                                                         \overline{ret\_pattern_i = spine\_elem_i}^i \leadsto \sigma
                                                         \overline{x_i = spine\_elem_i}^i :: arg \gg \sigma; ret
pure\_opsem\_jtype
                                                         \langle pexpr \rangle \longrightarrow \langle pexpr' \rangle
                                                         \langle pexpr \rangle \longrightarrow \langle tpexpr:(y:\beta.\ term) \rangle
                                                         \langle tpexpr \rangle \longrightarrow \langle tpexpr' \rangle
opsem\_jtype
                                                         \langle h; seq\_expr \rangle \longrightarrow \langle h'; texpr:ret \rangle
                                                         \langle h; seq\_texpr \rangle \longrightarrow \langle h'; texpr \rangle
                                                         \langle h; mem\_op \rangle \longrightarrow \langle h'; tval \rangle
                                                         \langle h; mem\_action \rangle \longrightarrow \langle h'; tval \rangle
                                                         \langle h; is\_expr \rangle \longrightarrow \langle h'; is\_expr' \rangle
                                                         \langle h; is\_texpr \rangle \longrightarrow \langle h'; texpr \rangle
```

$$\langle h; texpr \rangle \longrightarrow \langle h'; texpr' \rangle$$

$$\overline{x_i}^i :: arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \mid ret$$

$$\frac{}{::ret \leadsto :; :; : | ret} \quad \text{Arg_Env_Ret}$$

$$\frac{\overline{x_i}^i :: arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \mid ret}{x, \overline{x_i}^i :: \Pi \, x: \beta. \, arg \leadsto \mathcal{C}, x: \beta; \mathcal{L}; \Phi; \mathcal{R} \mid ret} \quad \text{Arg_Env_Comp}$$

$$\frac{\overline{x_i}^i :: arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \mid ret}{x, \overline{x_i}^i :: \forall x : \beta. arg \leadsto \mathcal{C}; \mathcal{L}, x : \beta; \Phi; \mathcal{R} \mid ret} \quad \text{Arg_Env_Log}$$

$$\frac{\overline{x_i}^{\;i} \,:: arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \mid ret}{\overline{x_i}^{\;i} \,:: term \supset arg \leadsto \mathcal{C}; \mathcal{L}; \Phi, term; \mathcal{R} \mid ret} \quad \text{Arg_Env_Phi}$$

$$\frac{\overline{x_i}^i :: arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \mid ret}{x, \overline{x_i}^i :: res \multimap arg \leadsto \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}, x: res \mid ret} \quad \text{Arg_Env_Res}$$

 $C; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'$

$$\frac{}{\cdot;\cdot;\cdot;\cdot\sqsubseteq\cdot;\cdot;\cdot;}\quad \text{Weak_Empty}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}, x{:}\beta; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}', x{:}\beta; \mathcal{L}'; \Phi'; \mathcal{R}'} \quad \text{Weak_Cons_Comp}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}, x:\beta; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}', x:\beta; \Phi'; \mathcal{R}'} \quad \text{Weak_Cons_Log}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}; \Phi, term; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi', term; \mathcal{R}'} \quad \text{Weak_Cons_Phi}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}, x : res \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}', x : res} \quad \text{Weak_Cons_Res}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}', x:\beta; \mathcal{L}'; \Phi'; \mathcal{R}'} \quad \text{Weak_Skip_Comp}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}', x : \beta; \Phi'; \mathcal{R}'} \quad \text{Weak_Skip_Log}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \sqsubseteq \mathcal{C}'; \mathcal{L}'; \Phi', term; \mathcal{R}'} \quad \text{Weak_Skip_Phi}$$

 $\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \overline{(\sigma) : (\mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}')}$

$$\mathcal{C}; \mathcal{L}; \Phi; \cdot \vdash (\cdot) : (\cdot; \cdot; \cdot; \cdot)$$
 TY_SUBS_EMPTY

$$\begin{array}{c} \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash (\sigma) : (\mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}') \\ \hline \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash (pval/x, \sigma) : (\mathcal{C}', x : \beta; \mathcal{L}'; \Phi'; \mathcal{R}') \end{array} \quad \text{Ty_Subs_Cons_Comp} \\ \end{array}$$

$$\begin{array}{l}
\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash (\sigma) : (\mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}') \\
\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta \\
\mathcal{C}; \mathcal{L}; \Phi \vdash (pval/x, \sigma) : (\mathcal{C}'; \mathcal{L}', x : \beta : \Phi'; \mathcal{R}')
\end{array}$$
TY_SUBS_CONS_LOG

$$\begin{split} & \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash (\sigma) : (\mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}') \\ & \underbrace{\mathsf{smt} \; (\Phi \Rightarrow term)}_{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash (\sigma) : (\mathcal{C}'; \mathcal{L}'; \Phi', term; \mathcal{R}')} \quad \mathsf{TY_Subs_Cons_Phi} \\ & \underbrace{ \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash (\sigma) : (\mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}') }_{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term \; \Leftarrow \; \sigma(res)} \\ & \underbrace{ \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash res_term \; \Leftarrow \; \sigma(res)}_{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}, \mathcal{R}_1 \vdash (res_term/x, \sigma) : (\mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}', x : res)} \quad \mathsf{TY_Subs_Cons_Res} \end{split}$$

 $\Phi \vdash \mathit{res} \, \equiv \, \mathit{res'}$

$$\frac{\operatorname{smt}\left(\Phi\Rightarrow\left(term_{1}=term'_{1}\right)\wedge\left(term_{2}=term'_{2}\right)\right)}{\Phi\vdash\operatorname{term}_{1}\overset{init}{\mapsto}_{\tau}\operatorname{term}_{2}\equiv\operatorname{term}'_{1}\overset{init}{\mapsto}_{\tau}\operatorname{term}'_{2}} \quad \operatorname{Ty_Res_Eq_PointsTo}$$

$$\frac{\Phi\vdash\operatorname{res}_{1}\equiv\operatorname{res}'_{1}}{\Phi\vdash\operatorname{res}_{1}*\operatorname{res}_{2}\equiv\operatorname{res}'_{1}*\operatorname{res}'_{2}} \quad \operatorname{Ty_Res_Eq_SepConj}$$

$$\frac{\Phi\vdash\operatorname{res}\equiv\operatorname{res}'}{\Phi\vdash\operatorname{\exists}\operatorname{ident}:\beta.\operatorname{res}\equiv\operatorname{\exists}\operatorname{ident}:\beta.\operatorname{res}'} \quad \operatorname{Ty_Res_Eq_Exists}$$

$$\frac{\operatorname{smt}\left(\Phi\Rightarrow\left(\operatorname{term}\to\operatorname{term}'\right)\wedge\left(\operatorname{term}'\to\operatorname{term}\right)\right)}{\Phi\vdash\operatorname{res}\equiv\operatorname{res}'} \quad \operatorname{Ty_Res_Eq_Exists}$$

$$\frac{\operatorname{smt}\left(\Phi\Rightarrow\left(\operatorname{term}\to\operatorname{term}'\right)\wedge\left(\operatorname{term}'\to\operatorname{term}\right)\right)}{\Phi\vdash\operatorname{term}\wedge\operatorname{res}\equiv\operatorname{term}'\wedge\operatorname{res}'} \quad \operatorname{Ty_Res_Eq_Term}$$

$$\frac{\operatorname{smt}\left(\Phi\Rightarrow\left(\operatorname{term}_{1}\to\operatorname{term}_{2}\right)\wedge\left(\operatorname{term}_{2}\to\operatorname{term}_{1}\right)\right)}{\Phi\vdash\operatorname{res}_{11}\equiv\operatorname{res}_{21}} \quad \Phi\vdash\operatorname{res}_{21}\equiv\operatorname{res}_{22} \quad \operatorname{Ty_Res_Eq_OrdDisj}$$

$$\frac{\Phi\vdash\operatorname{if}\operatorname{term}_{1}\operatorname{then}\operatorname{res}_{11}\operatorname{else}\operatorname{res}_{12}\equiv\operatorname{if}\operatorname{term}_{2}\operatorname{then}\operatorname{res}_{21}\operatorname{else}\operatorname{res}_{22}} \quad \operatorname{Ty_Res_Eq_OrdDisj}$$

$$\overline{\Phi \vdash \alpha(\overline{\mathit{pval}_i}^i) \equiv \alpha(\overline{\mathit{pval}_i}^i)} \quad \text{TY_RES_EQ_PRED}$$

$$C; \mathcal{L}; \Phi; \mathcal{R} \vdash res_term \Leftarrow res$$

$$\overline{\mathcal{C};\mathcal{L};\Phi; \vdash \mathsf{emp}} \Leftarrow \mathsf{emp} \qquad \mathsf{TY_RES_EMP}$$

$$\frac{\Phi \vdash points_to' \equiv points_to'}{\Phi \vdash points_to' \equiv points_to'} \qquad \mathsf{TY_RES_POINTSTO}$$

$$\overline{\mathcal{C};\mathcal{L};\Phi; \cdot, ...points_to \vdash points_to'} \Leftrightarrow \mathsf{points_to'} \qquad \mathsf{TY_RES_POINTSTO}$$

$$\mathcal{L};\Phi \vdash res'_1 = \mathsf{strip_ifs} \ (res_1)$$

$$\mathcal{L};\Phi \vdash res'_2 = \mathsf{strip_ifs} \ (res_2)$$

$$\frac{\Phi \vdash res'_1 \equiv res'_2}{\mathcal{C};\mathcal{L};\Phi; \cdot, r:res_1 \vdash r \Leftarrow res_2} \qquad \mathsf{TY_RES_VAR}$$

$$\frac{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}_1 \vdash res_term_1 \Leftarrow res_1}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}_2 \vdash res_term_2 \Leftarrow res_2} \qquad \mathsf{TY_RES_SEPCONJ}$$

$$\frac{\mathsf{smt} \ (\Phi \Rightarrow term)}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash res_term \Leftarrow res} \qquad \mathsf{TY_RES_SEPCONJ}$$

$$\frac{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash res_term \Leftarrow res}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash res_term \Leftarrow term \land res} \qquad \mathsf{TY_RES_CONJ}$$

$$\frac{\mathcal{C};\mathcal{L};\Phi \vdash pval \Rightarrow \beta}{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash res_term \Leftarrow pval/y, \cdot (res)} \qquad \mathsf{TY_RES_PACK}$$

$$\begin{array}{l} \alpha \equiv \overline{x_i : \beta_i}^i \mapsto res \in \texttt{Globals} \\ \overline{\mathcal{C}; \mathcal{L}; \Phi \vdash pval_i \Rightarrow \beta_i}^i \\ \mathcal{L}; \Phi \vdash res' = \texttt{strip_ifs}\left(\overline{pval_i/x_i, \cdot}^i(res)\right) \\ \overline{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash res_term \Leftarrow res'} \\ \overline{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \texttt{fold}\left(res_term\right) \Leftarrow \alpha\left(\overline{pval_i}^i\right)} \end{array} \quad \text{Ty_Res_Fold}$$

 $h:\mathcal{R}$

$$\frac{h:\mathcal{R}}{\frac{\cdot;\cdot;\cdot;\mathcal{R}'\vdash pt \Leftarrow pt}{h+\{pt\}:\mathcal{R},\mathcal{R}'}} \quad \text{TY_HEAP_POINTSTO}$$

 $\mathcal{C}; \mathcal{L}; \Phi \vdash object_value \Rightarrow \mathsf{obj}\,\beta$

$$\frac{}{\mathcal{C};\mathcal{L};\Phi \vdash mem_int \Rightarrow \mathtt{objinteger}} \quad \text{TY_PVAL_OBJ_INT}$$

$$\overline{\mathcal{C};\mathcal{L};\Phi \vdash mem_ptr \Rightarrow \mathtt{objloc}} \quad \mathrm{TY_PVAL_OBJ_PTR}$$

$$\frac{\overline{\mathcal{C};\mathcal{L};\Phi \vdash loaded_value_i \Rightarrow \beta}^i}{\mathcal{C};\mathcal{L};\Phi \vdash \mathtt{array}\left(\overline{loaded_value_i}^i\right) \Rightarrow \mathtt{obj}\,\mathtt{array}\,\beta} \quad \mathsf{TY_PVAL_OBJ_ARR}$$

$$\frac{\text{struct} \, tag \, \& \, \overline{member_i : \tau_i}^{\, i} \, \in \, \text{Globals}}{\mathcal{C}; \mathcal{L}; \Phi \vdash mem_val_i \, \Rightarrow \, \text{mem} \, \beta_{\tau_i}^{\, i}} \\ \frac{\mathcal{C}; \mathcal{L}; \Phi \vdash (\text{struct} \, tag) \{ \overline{.member_i : \tau_i = mem_val_i}^{\, i} \} \, \Rightarrow \, \text{obj struct} \, tag} \\ \text{Ty_PVAL_OBJ_STRUCT}$$

$$C; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta$$

$$\frac{x : \beta \in \mathcal{C}}{\mathcal{C}; \mathcal{L}; \Phi \vdash x \Rightarrow \beta} \quad \text{Ty_Pval_Var_Comp}$$

$$\frac{x : \beta \in \mathcal{L}}{\mathcal{C}; \mathcal{L}; \Phi \vdash x \Rightarrow \beta} \quad \text{Ty_Pval_Var_Log}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash object_value \Rightarrow obj \beta}{\mathcal{C}; \mathcal{L}; \Phi \vdash object_value \Rightarrow \beta} \quad \text{Ty_Pval_Obj}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash object_value \Rightarrow obj \beta}{\mathcal{C}; \mathcal{L}; \Phi \vdash object_value \Rightarrow \beta} \quad \text{Ty_Pval_Loaded}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash \text{Specified} object_value \Rightarrow \beta}{\mathcal{C}; \mathcal{L}; \Phi \vdash \text{Unit} \Rightarrow \text{unit}} \quad \text{Ty_Pval_Unit}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash \text{Unit} \Rightarrow \text{bool}}{\mathcal{C}; \mathcal{L}; \Phi \vdash \text{True} \Rightarrow \text{bool}} \quad \text{Ty_Pval_True}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash \text{False} \Rightarrow \text{bool}}{\mathcal{C}; \mathcal{L}; \Phi \vdash \beta[\overline{value_i}^i] \Rightarrow \text{list} \beta} \quad \text{Ty_Pval_List}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash \psi \text{alue}_i \Rightarrow \beta_i^i}{\mathcal{C}; \mathcal{L}; \Phi \vdash (\overline{value_i}^i) \Rightarrow \overline{\beta_i}^i} \quad \text{Ty_Pval_Tuple}$$

$$\frac{\operatorname{smt}\left(\Phi\Rightarrow\operatorname{false}\right)}{C;\mathcal{L};\Phi\vdash\operatorname{error}\left(\operatorname{string},\operatorname{pval}\right)\Rightarrow\beta} \quad \operatorname{Ty_Pval_Error}$$

$$\frac{C;\mathcal{L};\Phi\vdash\operatorname{error}\left(\operatorname{string},\operatorname{pval}\right)\Rightarrow\beta}{C;\mathcal{L};\Phi\vdash\operatorname{nil}\beta\left(\right)\Rightarrow\operatorname{list}\beta} \quad \operatorname{Ty_Pval_CTor_Nil}$$

$$\frac{C;\mathcal{L};\Phi\vdash\operatorname{pval}_1\Rightarrow\beta}{C;\mathcal{L};\Phi\vdash\operatorname{pval}_2\Rightarrow\operatorname{list}\beta} \quad \operatorname{Ty_Pval_CTor_Cons}$$

$$\frac{C;\mathcal{L};\Phi\vdash\operatorname{pval}_i\Rightarrow\beta_i{}^i}{C;\mathcal{L};\Phi\vdash\operatorname{Tuple}\left(\overline{\operatorname{pval}_i{}^i}\right)\Rightarrow\overline{\beta_i{}^i}} \quad \operatorname{Ty_Pval_CTor_Tuple}$$

$$\frac{\overline{C};\mathcal{L};\Phi\vdash\operatorname{pval}_i\Rightarrow\beta_i{}^i}{C;\mathcal{L};\Phi\vdash\operatorname{Array}\left(\overline{\operatorname{pval}_i{}^i}\right)\Rightarrow\operatorname{array}\beta} \quad \operatorname{Ty_Pval_CTor_Array}$$

$$\frac{C;\mathcal{L};\Phi\vdash\operatorname{pval}\Rightarrow\beta}{C;\mathcal{L};\Phi\vdash\operatorname{Specified}\left(\operatorname{pval}\right)\Rightarrow\beta} \quad \operatorname{Ty_Pval_CTor_Specified}$$

$$\frac{\operatorname{struct} tag \ \&\overline{\operatorname{member}_i:\tau_i{}^i}\in\operatorname{Globals}}{\overline{C};\mathcal{L};\Phi\vdash\operatorname{pval}_i\Rightarrow\beta_{\tau_i{}^i}} \quad \operatorname{Ty_Pval_CTor_Specified}$$

$$\frac{\operatorname{struct} tag \ \&\overline{\operatorname{member}_i:\tau_i{}^i}\in\operatorname{Globals}}{C;\mathcal{L};\Phi\vdash\operatorname{pval}_i\Rightarrow\beta_{\tau_i{}^i}} \quad \operatorname{Ty_Pval_Struct}$$

$$\overline{C};\mathcal{L};\Phi\vdash\left(\operatorname{struct} tag\right)\{\overline{\operatorname{member}_i=\operatorname{pval}_i{}^i}\}\Rightarrow\operatorname{struct} tag} \quad \operatorname{Ty_Pval_Struct}$$

$$\overline{C};\mathcal{L};\Phi\vdash\left(\operatorname{struct} tag\right)\{\overline{\operatorname{member}_i=\operatorname{pval}_i{}^i}\}\Rightarrow\operatorname{struct} tag} \quad \operatorname{Ty_Pval_Struct}$$

$$\overline{\mathcal{C};\mathcal{L};\Phi;\cdot\vdash ::\! \mathit{ret} \gg \cdot;\mathit{ret}} \quad \mathsf{TY_SPINE_EMPTY}$$

$$C;\mathcal{L};\Phi\vdash pval\Rightarrow\beta\\ C;\mathcal{L};\Phi;\mathcal{R}\vdash \overline{x_i}=spine_elem_i^{\ i}::arg\gg\sigma;ret\\ \hline C;\mathcal{L};\Phi;\mathcal{R}\vdash x=pval, \ \overline{x_i}=spine_elem_i^{\ i}::\Pi\,x;\beta.\,arg\gg pval/x,\sigma;ret\\ \hline \\ C;\mathcal{L};\Phi\vdash pval\Rightarrow\beta\\ \hline \\ C;\mathcal{L};\Phi\vdash \mathcal{R}\vdash x=pval, \ \overline{x_i}=spine_elem_i^{\ i}::arg\gg\sigma;ret\\ \hline \\ C;\mathcal{L};\Phi;\mathcal{R}\vdash x=pval, \ \overline{x_i}=spine_elem_i^{\ i}::vx:\beta.\,arg\gg pval/x,\sigma;ret\\ \hline \\ C;\mathcal{L};\Phi;\mathcal{R}\vdash x=pval, \ \overline{x_i}=spine_elem_i^{\ i}::vx:\beta.\,arg\gg pval/x,\sigma;ret\\ \hline \\ C;\mathcal{L};\Phi;\mathcal{R}_1\vdash res_term\Leftarrow res\\ \hline \\ C;\mathcal{L};\Phi;\mathcal{R}_2\vdash \overline{x_i}=spine_elem_i^{\ i}::arg\gg\sigma;ret\\ \hline \\ C;\mathcal{L};\Phi;\mathcal{R}_1,\mathcal{R}_2\vdash x=res_term, \ \overline{x_i}=spine_elem_i^{\ i}::res\multimap arg\gg res_term/x,\sigma;ret\\ \hline \\ \frac{smt}{C;\mathcal{L};\Phi;\mathcal{R}\vdash x_i}=spine_elem_i^{\ i}::arg\gg\sigma;ret\\ \hline \\ C;\mathcal{L};\Phi;\mathcal{R}\vdash x_i=spine_elem_i^{\ i}::arg\gg\sigma;ret\\ \hline \\ C;\mathcal{L};\Phi\vdash x_i=spine_elem_i^{\ i}::term\supset arg\gg\sigma;ret\\ \hline \\ C;\mathcal{L};\Phi\vdash pval\Rightarrow\beta;p=pval\\ \hline \\ C;\mathcal{L};\Phi\vdash pval\Rightarrow\beta;\beta,y=pval\\ \hline \\ C;\mathcal{L};\Phi\vdash pval\Rightarrow\beta;\beta,y=pval\\ \hline \\ C;\mathcal{L};\Phi\vdash pval_\Rightarrow\beta;\beta,y=pval\\ \hline \\$$

$$\begin{split} \mathcal{C}; \mathcal{L}; \Phi \vdash pval_1 \Rightarrow \texttt{loc} \\ \mathcal{C}; \mathcal{L}; \Phi \vdash pval_2 \Rightarrow \texttt{integer} \\ \hline \mathcal{C}; \mathcal{L}; \Phi \vdash \texttt{array_shift} (pval_1, \tau, pval_2) \Rightarrow y : \texttt{loc.} \ y = pval_1 +_{\texttt{ptr}} (pval_2 \times \texttt{size_of}(\tau)) \end{split} \quad \texttt{TY_PE_Array_Shift}$$

$$\begin{array}{c} \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \mathtt{loc} \\ & \mathtt{struct} \ tag \ \& \ \overline{member_i : \tau_i}^i \in \mathtt{Globals} \\ \hline \mathcal{C}; \mathcal{L}; \Phi \vdash \mathtt{member_shift} \ (pval, tag, member_j) \Rightarrow y : \mathtt{loc.} \ y = pval +_{\mathtt{ptr}} \ \mathtt{offset_of}_{tag} (member_j) \end{array} \\ \text{TY_PE_MEMBER_SHIFT}$$

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\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \texttt{bool}}{\mathcal{C}; \mathcal{L}; \Phi \vdash \texttt{not} (pval) \Rightarrow y \texttt{:bool}. \ y = \neg pval} \quad \text{TY\_PE\_NOT}
                                                                 \mathcal{C}: \mathcal{L}: \Phi \vdash pval_1 \Rightarrow \mathtt{integer}
                                                                C; \mathcal{L}; \Phi \vdash pval_2 \Rightarrow \mathtt{integer}
                                                                                                                                                                                  TY_PE_ARITH_BINOP
               \mathcal{C}: \mathcal{L}: \Phi \vdash pval_1 \ binop_{arith} \ pval_2 \Rightarrow y: integer. \ y = (pval_1 \ binop_{arith} \ pval_2)
                                                                   C; \mathcal{L}; \Phi \vdash pval_1 \Rightarrow \mathtt{integer}
                                                                   \mathcal{C}; \mathcal{L}; \Phi \vdash pval_2 \Rightarrow \mathtt{integer}
                                                                                                                                                                            TY_PE_REL_BINOP
                        \overline{\mathcal{C}; \mathcal{L}; \Phi \vdash pval_1 \ binop_{rel} \ pval_2 \Rightarrow y: bool. \ y = (pval_1 \ binop_{rel} \ pval_2)}
                                                                     \mathcal{C}: \mathcal{L}: \Phi \vdash pval_1 \Rightarrow bool
                                                                     \mathcal{C}: \mathcal{L}: \Phi \vdash pval_2 \Rightarrow bool
                                                                                                                                                                            TY_PE_BOOL_BINOP
                     \mathcal{C}: \mathcal{L}: \Phi \vdash pval_1 \ binop_{bool} \ pval_2 \Rightarrow y: bool. \ y = (pval_1 \ binop_{bool} \ pval_2)
                                            name:pure\_arg \equiv \overline{x_i}^i \mapsto tpexpr \in Globals
                                          \frac{\mathcal{C}; \mathcal{L}; \Phi; \cdot \vdash \overline{x_i = pval_i}^i :: pure\_arg \gg \sigma; \Sigma y : \beta. \ term \wedge I}{\mathcal{C}; \mathcal{L}; \Phi \vdash name(\overline{pval_i}^i) \Rightarrow y : \beta. \ \sigma(term)}  TY_PE_CALL
                                                                    \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow bool
                                                                    smt(\Phi \Rightarrow pval)
                                                                                                                                                                   Ty_PE_Assert_Under
                         \overline{\mathcal{C};\mathcal{L};\Phi \vdash \mathtt{assert\_undef}\left(pval,\ UB\_name\right)} \Rightarrow y\mathtt{:unit}.\ y = \mathtt{unit}
                                                               C; \mathcal{L}; \Phi \vdash pval \Rightarrow bool
                                                                                                                                                                          Ty_PE_Bool_To_Integer
         \overline{\mathcal{C};\mathcal{L};\Phi \vdash \mathtt{bool\_to\_integer}\,(pval)} \Rightarrow y : \mathtt{integer}.\ y = \mathtt{if}\ pval\ \mathtt{then}\ 1\ \mathtt{else}\ 0
                                                               \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \mathtt{integer}
                                                               abbrev_1 \equiv \max_{\cdot} \inf_{\tau} - \min_{\cdot} \inf_{\tau} + 1
                                                               abbrev_2 \equiv pval \, rem_f \, abbrev_1
                                                                                                                                                                                                              Ty_PE_WrapI
\overline{\mathcal{C};\mathcal{L};\Phi\vdash \mathtt{wrapI}\left(\tau,pval\right)\Rightarrow y:\beta.\;y=\mathtt{if}\;abbrev_2\leq \mathtt{max\_int}_{\tau}\,\mathtt{then}\;abbrev_2\,\mathtt{else}\;abbrev_2-abbrev_1}
```

 $pattern: \beta \leadsto \mathcal{C} \text{ with } term$

$$\overline{x:\beta:\beta \leadsto \text{with}} = \text{TY-PAT_COMP_No_SYM_ANNOT}$$

$$\overline{x:\beta:\beta \leadsto \cdot x:\beta \text{ with}} x = \text{TY-PAT_COMP_SYM_ANNOT}$$

$$\overline{nil}\,\beta():\text{list}\,\beta \leadsto \cdot \text{withnil}} = \text{TY_PAT_COMP_NIL}$$

$$\frac{pattern_1:\beta \leadsto \mathcal{C}_1 \text{ with} term_1}{pattern_2:\text{list}\,\beta \leadsto \mathcal{C}_2 \text{ with} term_2}} = \text{TY_PAT_COMP_CONS}$$

$$\frac{pattern_1:\beta \leadsto \mathcal{C}_1 \text{ with} term_i^{-i}}{\text{Tuple}(\overline{pattern}_i^{-i}):\overline{\beta_i}^{-i} \leadsto \overline{\mathcal{C}_i}^{-i} \text{ with} (\overline{term}_i^{-i})}} = \text{TY_PAT_COMP_TUPLE}$$

$$\frac{pattern_i:\beta \leadsto \mathcal{C}_i \text{ with} term_i^{-i}}{\text{Array}(\overline{pattern}_i^{-i}):\text{array}\,\beta \leadsto \overline{\mathcal{C}_i}^{-i} \text{ with} [||\overline{term}_i^{-i}||]}} = \text{TY_PAT_COMP_ARRAY}$$

$$\frac{pattern:\beta \leadsto \mathcal{C} \text{ with} term}{\text{Specified}(pattern):\beta \leadsto \mathcal{C} \text{ with} term}} = \text{TY_PAT_COMP_SPECIFIED}$$

$$\frac{pattern:\beta \leadsto \mathcal{C} \text{ with} term}{x:\beta \leadsto \cdot \cdot \cdot x:\beta \text{ with}\,x}} = \text{TY_PAT_SYM_OR_PATTERN_SYM}$$

$$\frac{pattern:\beta \leadsto \mathcal{C} \text{ with} term}{pattern:\beta \leadsto \mathcal{C} \text{ with} term}} = \text{TY_PAT_SYM_OR_PATTERN_PATTERN}$$

$$\mathcal{L}; \Phi \vdash res' = \mathtt{strip_ifs}(res)$$

$$\mathcal{L}; \Phi \vdash \mathsf{emp} = \mathsf{strip_ifs}(\mathsf{emp})$$
 TY_PAT_RES_STRIPIFS_EMPTY

$$\overline{\mathcal{L}; \Phi \vdash pt = \mathtt{strip_ifs}\left(pt\right)} \quad \text{TY_PAT_RES_STRIPIFS_POINTSTO}$$

$$\overline{\mathcal{L}; \Phi \vdash \mathit{res}_1 * \mathit{res}_2 = \mathtt{strip_ifs} \left(\mathit{res}_1 * \mathit{res}_2\right)} \quad \mathsf{TY_PAT_RES_STRIPIFS_SEPCONJ}$$

$$\mathcal{L}; \Phi \vdash \exists x : \beta. \ res = \text{strip_ifs}(\exists x : \beta. \ res)$$
 TY_PAT_RES_STRIPIFS_EXISTS

$$\overline{\mathcal{L}; \Phi \vdash term \land res = \mathtt{strip_ifs}(term \land res)} \quad \text{Ty_Pat_Res_StripIfs_TermConj}$$

$$\frac{\texttt{smt}\,(\Phi\Rightarrow term)}{\mathcal{L};\Phi\vdash res_1'=\,\texttt{strip_ifs}\,(res_1')}\\ \frac{\mathcal{L};\Phi\vdash res_1'=\,\texttt{strip_ifs}\,(\texttt{if}\,term\,\texttt{then}\,res_1\,\texttt{else}\,res_2)}{\mathcal{L};\Phi\vdash res_1'=\,\texttt{strip_ifs}\,(\texttt{if}\,term\,\texttt{then}\,res_1\,\texttt{else}\,res_2)}$$

$$\begin{array}{c} \mathtt{smt}\left(\Phi\Rightarrow\neg\,term\right) \\ \mathcal{L};\Phi\vdash\mathit{res}_2'=\mathtt{strip_ifs}\left(\mathit{res}_2\right) \\ \hline \mathcal{L};\Phi\vdash\mathit{res}_2'=\mathtt{strip_ifs}\left(\mathtt{if}\,\mathit{term}\,\mathtt{then}\,\mathit{res}_1\,\mathtt{else}\,\mathit{res}_2\right) \end{array} \\ \mathrm{TY_PAT_RES_STRIPIFS_FALSE}$$

 \mathcal{L} ; $\Phi \vdash \text{if } term \text{ then } res_1 \text{ else } res_2 = \text{strip_ifs} (\text{if } term \text{ then } res_1 \text{ else } res_2)$

$$\frac{1}{\mathcal{L}; \Phi \vdash \alpha(\overline{\textit{pval}_i}^i) = \texttt{strip_ifs}(\alpha(\overline{\textit{pval}_i}^i))} \quad \text{Ty_Pat_Res_StripIfs_Pred}$$

 $\mathcal{L}; \Phi \vdash res \text{ as } res_pattern \leadsto \mathcal{L}'; \Phi'; \mathcal{R}'$

$$\begin{split} & \frac{}{\mathcal{L};\Phi \vdash \mathsf{emp} \, \mathsf{as} \, \mathsf{emp} \, \leadsto \, \cdot; \cdot; \cdot} \quad \mathsf{TY_PAT_RES_MATCH_EMPTY} \\ \\ & \frac{}{\mathcal{L};\Phi \vdash \mathit{res} \, \mathsf{as} \, r \, \leadsto \, \cdot; \cdot; \cdot, r{:}\mathit{res}} \quad \mathsf{TY_PAT_RES_MATCH_VAR} \end{split}$$

 $\begin{array}{c} \mathcal{L}; \Phi \vdash \mathit{res_pattern}_1 : \mathit{res}_1 \leadsto \mathcal{L}_1; \Phi_1; \mathcal{R}_1 \\ \mathcal{L}; \Phi \vdash \mathit{res_pattern}_2 : \mathit{res}_2 \leadsto \mathcal{L}_2; \Phi_2; \mathcal{R}_2 \\ \hline \mathcal{L}; \Phi \vdash \mathit{res}_1 * \mathit{res}_2 \text{ as } \langle \mathit{res_pattern}_1, \mathit{res_pattern}_2 \rangle \leadsto \mathcal{L}_1, \mathcal{L}_2; \Phi_1, \Phi_2; \mathcal{R}_1, \mathcal{R}_2 \end{array} \quad \text{Ty_Pat_Res_Match_SepConj}$

 $\frac{\mathcal{L}; \Phi \vdash res_pattern: res \leadsto \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{L}; \Phi \vdash term \land res \text{ as } res_pattern \leadsto \mathcal{L}'; \Phi', term; \mathcal{R}'} \quad \text{Ty_Pat_Res_Match_Conj}$

 $\frac{\mathcal{L}; \Phi \vdash res_pattern: x/y, \cdot (res) \leadsto \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{L}; \Phi \vdash \exists y: \beta. res \text{ as pack } (x, res_pattern) \leadsto \mathcal{L}', x: \beta; \Phi'; \mathcal{R}'} \quad \text{Ty_Pat_Res_Match_Pack}$

 $\alpha \equiv \overline{x_i : \beta_i}^i \mapsto res \in \texttt{Globals}$ $\frac{\mathcal{L}; \Phi \vdash res_pattern : \overline{pval_i/x_i}, i}{\mathcal{L}; \Phi \vdash \alpha(\overline{pval_i}^i) \text{ as fold } (res_pattern) \leadsto \mathcal{L}'; \Phi'; \mathcal{R}'}$ $Ty_Pat_Res_Match_Fold$

 $\mathcal{L}; \Phi \vdash \mathit{res_pattern:res} \leadsto \mathcal{L}'; \Phi'; \mathcal{R}'$

 $\begin{array}{l} \mathcal{L}; \Phi \vdash \mathit{res'} = \mathtt{strip_ifs}\,(\mathit{res}) \\ \underline{\mathcal{L}}; \Phi \vdash \mathit{res'}\,\mathtt{as}\,\mathit{res_pattern} \leadsto \mathcal{L'}; \Phi'; \mathcal{R'} \\ \hline \\ \mathcal{L}; \Phi \vdash \mathit{res_pattern}: \mathit{res} \leadsto \mathcal{L'}; \Phi'; \mathcal{R'} \end{array} \quad \text{TY_PAT_RES_STRIP_IFS}$

 $\mathcal{L}; \Phi \vdash \overline{\mathit{ret_pattern_i}}^i : \mathit{ret} \leadsto \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'$

 $\overline{\mathcal{L}; \Phi \vdash : \texttt{I} \leadsto \cdot; \cdot; \cdot; \cdot} \quad \text{TY_PAT_RET_EMPTY}$

$$\frac{ident_or_pattern:\beta \leadsto \mathcal{C}_1 \, \text{with} \, term_1}{\mathcal{L}; \Phi \vdash \overline{ret_pattern_i}^i : term_1/y, \cdot (ret) \leadsto \mathcal{C}_2; \mathcal{L}_2; \Phi_2; \mathcal{R}_2} \\ \frac{\mathcal{L}; \Phi \vdash \mathsf{comp} \, ident_or_pattern, \, \overline{ret_pattern_i}^i : \Sigma \, y : \beta. \, ret \leadsto \mathcal{C}_1, \mathcal{C}_2; \mathcal{L}_2; \Phi_2; \mathcal{R}_2} \\ \mathcal{L}; \Phi \vdash \mathsf{comp} \, ident_or_pattern, \, \overline{ret_pattern_i}^i : \Sigma \, y : \beta. \, ret \leadsto \mathcal{C}_1, \mathcal{C}_2; \mathcal{L}_2; \Phi_2; \mathcal{R}_2}$$

$$\frac{\mathcal{L}; \Phi \vdash \overline{ret_pattern_i}^i : ret \leadsto \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{L}; \Phi \vdash \log y, \overline{ret_pattern_i}^i : \exists \ y : \beta. \ ret \leadsto \mathcal{C}'; \mathcal{L}', \ y : \beta; \Phi'; \mathcal{R}'} \quad \text{TY_PAT_RET_LOG}$$

$$\begin{array}{c} \mathcal{L}; \Phi \vdash res_pattern:res \leadsto \mathcal{L}_{1}; \Phi_{1}; \mathcal{R}_{1} \\ \mathcal{L}; \Phi \vdash \overline{ret_pattern_{i}}^{i}:ret \leadsto \mathcal{C}_{2}; \mathcal{L}_{2}; \Phi_{2}; \mathcal{R}_{2} \\ \hline \mathcal{L}; \Phi \vdash res_res_pattern, \overline{ret_pattern_{i}}^{i}:res \otimes ret \leadsto \mathcal{C}_{2}; \mathcal{L}_{1}, \mathcal{L}_{2}; \Phi_{1}, \Phi_{2}; \mathcal{R}_{1}, \mathcal{R}_{2} \end{array} \quad \text{TY_PAT_RET_RES}$$

$$\frac{\mathcal{L}; \Phi \vdash \overline{ret_pattern_i}^i : ret \leadsto \mathcal{C}'; \mathcal{L}'; \Phi'; \mathcal{R}'}{\mathcal{L}; \Phi \vdash \overline{ret_pattern_i}^i : term \land ret \leadsto \mathcal{C}'; \mathcal{L}'; \Phi', term; \mathcal{R}'} \quad \text{Ty_Pat_Ret_Phi}$$

 $C; \mathcal{L}; \Phi \vdash tpval \Leftarrow ident: \beta. term$

$$\frac{\mathtt{smt}\left(\Phi\Rightarrow\mathtt{false}\right)}{\mathcal{C};\mathcal{L};\Phi\vdash\mathtt{undef}\ \mathit{UB_name}\Leftarrow\mathit{y:}\beta.\mathit{term}}\quad \mathsf{TY_TPVAL_UNDEF}$$

$$\begin{split} & \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta \\ & \underbrace{\mathsf{smt}\left(\Phi \Rightarrow pval/y, \cdot (term)\right)}_{\mathcal{C}; \mathcal{L}; \Phi \vdash \mathsf{done}\; pval \Leftarrow y : \beta.\; term} \quad \mathsf{TY_TPVAL_DONE} \end{split}$$

 $C; \mathcal{L}; \Phi \vdash tpexpr \Leftarrow ident: \beta. term$

$$\begin{split} \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \texttt{bool} \\ \mathcal{C}; \mathcal{L}; \Phi, pval &= \texttt{true} \vdash tpexpr_1 \Leftarrow y : \beta. \ term \\ \mathcal{C}; \mathcal{L}; \Phi, pval &= \texttt{false} \vdash tpexpr_2 \Leftarrow y : \beta. \ term \\ \hline \mathcal{C}; \mathcal{L}; \Phi \vdash \texttt{if} \ pval \ \texttt{then} \ tpexpr_1 \ \texttt{else} \ tpexpr_2 \Leftarrow y : \beta. \ term \end{split} \qquad \texttt{TY_TPE_IF}$$

 $\mathcal{C}:\mathcal{L}:\Phi\vdash pexpr\Rightarrow y_1:\beta_1.\ term_1$

```
\mathcal{C}: \mathcal{L}: \Phi \vdash pval_0 \Rightarrow \mathsf{loc}
                                                                                                                                                                                                                                                                                                                                                                                                                      \mathcal{C}: \mathcal{L}: \Phi \vdash pval_1 \Rightarrow \beta_{\tau}
                                                                                                                                                                                                                                                                                                                                                                                                                         \operatorname{smt}(\Phi \Rightarrow \operatorname{representable}(\tau, pval_1))
                                                                                                                                                                                                                                                                                                                                                                                                                         \operatorname{smt} (\Phi \Rightarrow pval_2 = pval_0)
                                                                                                                                                                                                                                                                                                                                                                                                                      \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash pval_2 \mapsto_{\tau} \ = pval_2 \mapsto_{\tau} \ =
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      TY_ACTION_STORE
                                                                                                                                                                                                      \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathtt{store} \left( \_, \tau, pval_0, pval_1, \_, pval_2 \mapsto_{\tau} \_ \right) \Rightarrow \Sigma \_: \mathtt{unit}. \ pval_2 \mapsto_{\tau} \overline{pval_1 \otimes \mathtt{I}}
                                                                                                                                                                                                                                                                                                                                                                                          \mathcal{C}: \mathcal{L}: \Phi \vdash pval_0 \Rightarrow \mathsf{loc}
                                                                                                                                                                                                                                                                                                                                                                                          \operatorname{smt} (\Phi \Rightarrow pval_0 = pval_1)
                                                                                                                                                                                                                                                                                                                                                                                          \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash pval_1 \mapsto_{\tau} = pval_1 \mapsto_{\tau}
                                                                                                                                                                                                                                                                         \frac{\mathcal{C}; \mathcal{L}; \Psi; \kappa \vdash pval_1 \mapsto_{\tau -} \Leftarrow pval_1 \mapsto_{\tau -}}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \text{kill (static } \tau, pval_0, pval_1 \mapsto_{\tau -}) \Rightarrow \Sigma_{\cdot} \text{:unit. I}} \quad \text{Ty\_Action\_Kill\_Static}
C; \mathcal{L}; \Phi; \mathcal{R} \vdash mem\_op \Rightarrow ret
                                                                                                                                                                                                                                                                                                                                                                                                                                                                       \mathcal{C}: \mathcal{L}: \Phi \vdash pval_1 \Rightarrow \mathsf{loc}
                                                                                                                                                                                                                 \frac{\mathsf{C}; \mathcal{L}; \Phi \vdash pval_2 \Rightarrow \mathsf{loc}}{\mathcal{C}; \mathcal{L}; \Phi; \vdash pval_1 \ binop_{rel} \ pval_2 \Rightarrow \Sigma \ y : \mathsf{bool}. \ y = (pval_1 \ binop_{rel} \ pval_2) \land \mathsf{I}} \quad \mathsf{TY\_MEMOP\_REL\_BINOP}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                     C; \mathcal{L}; \Phi \vdash pval_2 \Rightarrow \mathsf{loc}
                                                                                                                                                                                                                                                                                                                                                                                                                                                             \mathcal{C}: \mathcal{L}: \Phi \vdash pval \Rightarrow \mathsf{loc}
                                                                                                                                                             \frac{\mathcal{C};\mathcal{L};\Phi\vdash pval\Rightarrow \mathtt{loc}}{\mathcal{C};\mathcal{L};\Phi;\cdot\vdash \mathtt{intFromPtr}\left(\tau_{1},\tau_{2},pval\right)\Rightarrow \Sigma\,y\mathtt{:integer}.\,\,y=\mathtt{cast\_ptr\_to\_int}\,pval\wedge\mathtt{I}}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       Ty_Memop_IntFromPtr
                                                                                                                                                                                                                                                                                                                                                                                                                                     \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \mathtt{integer}
                                                                                                                                                                                  \begin{array}{c} \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \mathtt{integer} \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \cdot \vdash \mathtt{ptrFromInt} \left(\tau_1, \tau_2, pval\right) \Rightarrow \Sigma \ y : \mathtt{loc.} \ y = \mathtt{cast\_int\_to\_ptr} \ pval \wedge \mathtt{I} \end{array}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     Ty_Memop_PtrFromInt
                                                                                                                                                                                                                                                                                                                                             \mathcal{C}: \mathcal{L}: \Phi \vdash pval_0 \Rightarrow \mathsf{loc}
                                                                                                                                                                                                                                                                                                                                               \operatorname{smt} (\Phi \Rightarrow pval_1 = pval_0)
                                                                                                                                                                                                                                                                                                                                             \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash pval_1 \stackrel{\checkmark}{\mapsto}_{\tau} = pval_1 \stackrel{\lor}{\mapsto}_{\tau} = pval_2 \stackrel{\lor}{\mapsto}_{\tau} = pval_2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  Ty_Memop_PtrValidForDeref
          \overline{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \mathsf{ptrValidForDeref}\left(\tau,pval_0,pval_1\overset{\checkmark}{\mapsto}_{\tau^{-}}\right)\Rightarrow \Sigma\,y{:}\mathsf{bool}.\,\,y=\mathsf{aligned}\left(\tau,pval_1\right)\wedge pval_1\overset{\checkmark}{\mapsto}_{\tau^{-}}\otimes \mathsf{I}}
```

$$\frac{\mathcal{C}; \mathcal{L}; \Phi \vdash pval_1 \Rightarrow \mathtt{loc}}{\mathcal{C}; \mathcal{L}; \Phi; \cdot \vdash \mathtt{ptrWellAligned}\left(\tau, pval\right) \Rightarrow \Sigma \ y : \mathtt{bool}. \ y = \mathtt{aligned}\left(\tau, pval\right) \wedge \mathtt{I}} \quad \mathsf{TY_MEMOP_PTRWellAligneD}$$

$$\mathcal{C}; \mathcal{L}; \Phi \vdash pval_1 \Rightarrow \mathtt{loc} \ \mathcal{C}; \mathcal{L}; \Phi \vdash pval_2 \Rightarrow \mathtt{integer}$$

 $\frac{\mathcal{C},\mathcal{L};\Phi; \cdot \vdash \mathsf{ptrArrayShift} \ (pval_1,\tau,pval_2) \Rightarrow \Sigma \ y : \mathsf{loc}. \ y = pval_1 +_{\mathsf{ptr}} (pval_2 \times \mathsf{size_of}(\tau)) \land \mathsf{I}}{\mathcal{C};\mathcal{L};\Phi; \cdot \vdash \mathsf{ptrArrayShift} \ (pval_1,\tau,pval_2) \Rightarrow \Sigma \ y : \mathsf{loc}. \ y = pval_1 +_{\mathsf{ptr}} (pval_2 \times \mathsf{size_of}(\tau)) \land \mathsf{I}}$

 $C; \mathcal{L}; \Phi; \mathcal{R} \vdash tval \Leftarrow ret$

$$\overline{\mathcal{C};\mathcal{L};\Phi;\cdot\vdash \mathtt{done} \; \Leftarrow \, \mathtt{I}} \quad \mathtt{TY_TVAL_I}$$

$$\begin{split} & \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta \\ & \frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{done} \ \overline{spine_elem_i}^{\ i} \Leftarrow pval/y, \cdot (ret)}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{done} \ pval, \ \overline{spine_elem_i}^{\ i} \Leftarrow \Sigma \ y : \beta. \ ret} \end{split} \qquad \text{TY_TVAL_COMP}$$

$$\begin{split} & \mathcal{C}; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta \\ & \frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{done} \ \overline{spine_elem_i}^{\ i} \Leftarrow pval/y, \cdot (ret)}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{done} \ pval, \ \overline{spine_elem_i}^{\ i} \Leftarrow \exists \ y : \beta. \ ret} \end{split} \quad \mathsf{TY_TVAL_LOG}$$

$$\begin{array}{l} \operatorname{smt}\left(\Phi\Rightarrow term\right) \\ \mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash\operatorname{done}spine\Leftarrow ret \\ \overline{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash\operatorname{done}spine\Leftarrow term\wedge ret} \end{array} \quad \text{Ty_TVal_Phi} \\ \end{array}$$

$$\begin{split} & \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_1 \vdash \mathit{res_term} \Leftarrow \mathit{res} \\ & \frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_2 \vdash \mathsf{done} \, \overline{\mathit{spine_elem}_i}^i \Leftarrow \mathit{ret} }{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}_1, \mathcal{R}_2 \vdash \mathsf{done} \, \mathit{res_term}, \, \overline{\mathit{spine_elem}}^i \Leftarrow \mathit{res} \otimes \mathit{ret} } \end{split}$$
 TY_TVAL_RES

$$\frac{\mathtt{smt}\,(\Phi\Rightarrow\mathtt{false})}{\mathcal{C};\mathcal{L};\Phi;\cdot\vdash\mathtt{undef}\ \mathit{UB_name} \Leftarrow\mathit{ret}}\quad \mathtt{TY_TVAL_UNDEF}$$

$$C; \mathcal{L}; \Phi; \mathcal{R} \vdash seq_expr \Rightarrow ret$$

$$\begin{split} ident: & arg \equiv \overline{x_i}^i \mapsto texpr \in \texttt{Globals} \\ & \frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \texttt{ccall}\left(\tau, ident, \overline{spine_elem_i}^i\right) \Rightarrow \sigma(ret)} \end{split} \quad \texttt{TY_SEQ_E_CCALL} \end{split}$$

$$\begin{array}{l} \mathit{name} : \mathit{arg} \; \equiv \; \overline{x_i}^i \; \mapsto \mathit{texpr} \; \in \; \mathsf{Globals} \\ \\ \frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \; \overline{x_i = \mathit{spine_elem}_i}^i \; :: \; \mathit{arg} \gg \sigma; \; \mathit{ret}}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{pcall} \left(\mathit{name}, \overline{\mathit{spine_elem}_i}^i \right) \Rightarrow \sigma(\mathit{ret})} \end{array} \quad \mathsf{TY_Seq_E_Proc}$$

$C; \mathcal{L}; \Phi; \mathcal{R} \vdash is_expr \Rightarrow ret$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash mem_op \Rightarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash memop \, (mem_op) \Rightarrow ret} \quad \text{TY_IS_E_MEMOP}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash mem_action \Rightarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash mem_action \Rightarrow ret} \quad \text{Ty_Is_E_ACTION}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash mem_action \Rightarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash neg\ mem_action \Rightarrow ret} \quad \text{Ty_Is_E_Neg_Action}$$

$$C; \mathcal{L}; \Phi; \mathcal{R} \vdash seq_texpr \Leftarrow ret$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash tval \Leftarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash tval \Leftarrow ret} \quad \text{TY_SEQ_TE_TVAL}$$

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\mathcal{C}:\mathcal{L}:\Phi\vdash pexpr\Rightarrow y:\beta.\ term
                                    ident\_or\_pattern: \beta \leadsto C_1 \text{ with } term_1
                                   \mathcal{C}, \mathcal{C}_1; \mathcal{L}; \Phi, term_1/y, \cdot (term); \mathcal{R} \vdash texpr \Leftarrow ret
                   \overline{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \mathtt{let}\,\mathit{ident\_or\_pattern} = \mathit{pexpr}\,\mathtt{in}\,\mathit{texpr} \Leftarrow \mathit{ret}} \quad \mathtt{TY\_SeQ\_TE\_LetP}
                                C; L; \Phi \vdash tpexpr \Leftarrow y:\beta. term
                                ident\_or\_pattern:\beta \leadsto \mathcal{C}_1 \text{ with } term_1
                                \mathcal{C}, \mathcal{C}_1; \mathcal{L}; \Phi, term_1/y, \cdot (term); \mathcal{R} \vdash texpr \Leftarrow ret
\overline{\mathcal{C};\mathcal{L};\Phi;\mathcal{R}\vdash \mathtt{let}\,ident\_or\_pattern:(y:\beta.\,\,term) = tpexpr\,\mathtt{in}\,texpr \Leftarrow ret} \quad \mathsf{TY\_SeQ\_TE\_LetPT}
                                          \mathcal{C}: \mathcal{L}: \Phi: \mathcal{R}' \vdash seg\_expr \Rightarrow ret_1
                                          \mathcal{L}: \Phi \vdash \overline{ret\_pattern_i}^i : ret_1 \leadsto \mathcal{C}_1: \mathcal{L}_1: \Phi_1: \mathcal{R}_1
                                         \mathcal{C}, \mathcal{C}_1; \mathcal{L}, \mathcal{L}_1; \Phi, \Phi_1; \mathcal{R}, \mathcal{R}_1 \vdash texpr \Leftarrow ret_2
\text{TY_SEQ_TE_LET}
                 \mathcal{C}: \mathcal{L}: \Phi: \mathcal{R}', \mathcal{R} \vdash \text{let } \overline{ret\_pattern_i}^i = sea\_expr \text{ in } texpr \Leftarrow ret_2
                                       \mathcal{C}: \mathcal{L}: \Phi: \mathcal{R}' \vdash texpr_1 \Leftarrow ret_1
                                        \mathcal{L}: \Phi \vdash \overline{ret\_pattern_i}^i : ret_1 \leadsto \mathcal{C}_1: \mathcal{L}_1: \Phi_1: \mathcal{R}_1
          \frac{\mathcal{C}, \mathcal{C}_1; \mathcal{L}, \mathcal{L}_1; \Phi, \Phi_1; \mathcal{R}, \mathcal{R}_1 \vdash texpr_2 \Leftarrow ret_2}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}', \mathcal{R} \vdash \mathsf{let} \overline{ret\_pattern_i}^i : ret_1 = texpr_1 \mathsf{in} \, texpr_2 \Leftarrow ret_2} \quad \mathsf{TY\_SEQ\_TE\_LETT}
                                     C; \mathcal{L}; \Phi \vdash pval \Rightarrow \beta_1
                    \frac{\frac{\mathcal{C}, \mathcal{L}, \Psi \vdash ptat \Rightarrow \beta_1}{pattern_i : \beta_1 \rightsquigarrow \mathcal{C}_i \text{ with } term_i}^i}{\frac{\mathcal{C}, \mathcal{C}_i; \mathcal{L}; \Phi, term_i = pval; \mathcal{R} \vdash texpr_i \Leftarrow ret}^i}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{case} \, pval \, \mathsf{of} \, \overline{\mid pattern_i \Rightarrow texpr_i}^i \, \mathsf{end} \Leftarrow ret} \quad \mathsf{TY\_SeQ\_TE\_CASE}
                                                C; \mathcal{L}; \Phi \vdash pval \Rightarrow bool
                                                C; \mathcal{L}; \Phi, pval = \mathsf{true}; \mathcal{R} \vdash texpr_1 \Leftarrow ret
                                                C; \mathcal{L}; \Phi, pval = \mathtt{false}; \mathcal{R} \vdash texpr_2 \Leftarrow ret
                                                                                                                                                                            Ty_Seo_TE_IF
                                  \overline{\mathcal{C}:\mathcal{L}:\Phi:\mathcal{R}\vdash \mathtt{if}\ pval}\ \mathtt{then}\ texpr_1\ \mathtt{else}\ texpr_2} \Leftarrow ret
```

$$\label{eq:continuous_continuous$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash is_texpr \Leftarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash \mathsf{bound} [int] (is_texpr) \Leftarrow ret} \quad \mathsf{TY_SEQ_TE_BOUND}$$

 $C; \mathcal{L}; \Phi; \mathcal{R} \vdash is_texpr \Leftarrow ret$

$$\begin{split} \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}' \vdash is_expr &\Rightarrow ret_1 \\ \mathcal{L}; \Phi \vdash \overline{ret_pattern_i}^i : ret_1 \leadsto \mathcal{C}_1; \mathcal{L}_1; \Phi_1; \mathcal{R}_1 \\ \mathcal{C}, \mathcal{C}_1; \mathcal{L}, \mathcal{L}_1; \Phi, \Phi_1; \mathcal{R}, \mathcal{R}_1 \vdash texpr \Leftarrow ret_2 \\ \hline \mathcal{C}; \mathcal{L}; \Phi; \mathcal{R}', \mathcal{R} \vdash \mathtt{let} \, \mathtt{strong} \, \overline{ret_pattern_i}^i = is_expr \, \mathtt{in} \, texpr \Leftarrow ret_2 \end{split}$$

 $C; \mathcal{L}; \Phi; \mathcal{R} \vdash texpr \Leftarrow ret$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash is_texpr \Leftarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash is_texpr \Leftarrow ret} \quad \text{TY_TE_IS}$$

$$\frac{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash seq_texpr \Leftarrow ret}{\mathcal{C}; \mathcal{L}; \Phi; \mathcal{R} \vdash seq_texpr \Leftarrow ret} \quad \text{TY_TE_SeQ}$$

 $pattern = pval \leadsto \sigma$

$$\underline{} := pval \leadsto \cdot$$
 Subs_Decons_Value_No_Sym_Annot

$$\overline{x:_=pval \leadsto pval/x, \cdot} \quad \text{Subs_Decons_Value_Sym_Annot}$$

$$\frac{pattern_1 = pval_1 \leadsto \sigma_1}{pattern_2 = pval_2 \leadsto \sigma_2} \qquad \text{Subs_Decons_Value_Cons} \\ \frac{\overline{pattern_i} = pval_i \leadsto \overline{\sigma_i}^i}{\text{Cons}(pvatern_i^i) = \text{Tuple}(\overline{pval_i}^i) \leadsto \overline{\sigma_i}^i} \qquad \text{Subs_Decons_Value_Tuple} \\ \frac{\overline{pattern_i} = pval_i \leadsto \overline{\sigma_i}^i}{\text{Tuple}(\overline{pattern_i}^i) = \text{Tuple}(\overline{pval_i}^i) \leadsto \overline{\sigma_i}^i} \qquad \text{Subs_Decons_Value_Array} \\ \frac{\overline{pattern_i} = pval_i \leadsto \overline{\sigma_i}^i}{\text{Array}(\overline{pattern_i}^i) = \text{Array}(\overline{pval_i}^i) \leadsto \overline{\sigma_i}^i} \qquad \text{Subs_Decons_Value_Array} \\ \frac{pattern = pval \leadsto \sigma}{\text{Specified}(pattern) = pval \leadsto \sigma} \qquad \text{Subs_Decons_Value_Specified} \\ \frac{ident_or_pattern = pval \leadsto \sigma}{\overline{pattern} = pval \leadsto \sigma} \qquad \text{Subs_Decons_Value'_Sym} \\ \frac{pattern = pval \leadsto \sigma}{\overline{pattern} = pval \leadsto \sigma} \qquad \text{Subs_Decons_Value'_Pattern} \\ \frac{pattern = pval \leadsto \sigma}{\overline{pattern} = pval \leadsto \sigma} \qquad \text{Subs_Decons_Value'_Pattern} \\ \frac{pattern = pval \leadsto \sigma}{\overline{pattern} = pval \leadsto \sigma} \qquad \text{Subs_Decons_Value'_Pattern} \\ \frac{pattern = pval \leadsto \sigma}{\overline{pattern} = pval \leadsto \sigma} \qquad \text{Subs_Decons_Value'_Pattern} \\ \frac{pattern = pval \leadsto \sigma}{\overline{pattern} = pval \leadsto \sigma} \qquad \text{Subs_Decons_Value'_Pattern} \\ \frac{pattern = pval \leadsto \sigma}{\overline{pattern} = pval \leadsto \sigma}} \qquad \text{Subs_Decons_Value'_Pattern} \\ \frac{pattern = pval \leadsto \sigma}{\overline{pattern} = pval \leadsto \sigma} \qquad \text{Subs_Decons_Value'_Pattern} \\ \frac{pattern = pval \leadsto \sigma}{\overline{pattern} = pval \leadsto \sigma}} \qquad \text{Subs_Decons_Value'_Pattern} \\ \frac{pattern = pval \leadsto \sigma}{\overline{pattern} = pval \leadsto \sigma}} \qquad \text{Subs_Decons_Value'_Pattern} \\ \frac{pattern = pval \leadsto \sigma}{\overline{pattern} = pval \leadsto \sigma}} \qquad \text{Subs_Decons_Value'_Pattern} \\ \frac{pattern}{\overline{pattern} = pval \leadsto \sigma}}$$

 $\frac{}{\text{emp} = \text{emp} \leadsto}$ Subs_Decons_Res_Emp

 $\overline{ident = res_term \leadsto res_term/ident, \cdot} \quad \text{Subs_Decons_Res_Var}$

$$res_pattern_1 = res_term_1 \leadsto \sigma_1 \\ res_pattern_2 = res_term_2 \leadsto \sigma_2 \\ \overline{\langle res_pattern_1, res_pattern_2 \rangle} = \overline{\langle res_term_1, res_term_2 \rangle} \leadsto \sigma_1, \sigma_2$$
 Subs_Decons_Res_Pair

$$\frac{res_pattern = res_term \leadsto \sigma}{\texttt{pack} \, (ident, res_pattern) = \texttt{pack} \, (pval, res_term) \leadsto pval/ident, \sigma} \quad \texttt{Subs_Decons_Res_Pack}$$

$$\frac{\mathit{res_pattern} = \mathit{res_term} \leadsto \sigma}{\mathtt{fold}\left(\mathit{res_pattern}\right) = \mathit{res_term} \leadsto \sigma} \quad \mathtt{Subs_Decons_Res_Fold}$$

 $\overline{ret_pattern_i = spine_elem_i}^i \leadsto \sigma$

 \longrightarrow Subs_Decons_Ret_Empty

$$\frac{ident_or_pattern = pval \leadsto \sigma}{ret_pattern_i = spine_elem_i{}^i \leadsto \psi} \\ \frac{comp\: ident_or_pattern = pval, \: \overline{ret_pattern_i = spine_elem_i{}^i} \leadsto \sigma, \psi}{comp\: ident_or_pattern = pval, \: \overline{ret_pattern_i = spine_elem_i{}^i} \leadsto \sigma, \psi} \\ \\ Subs_Decons_Ret_Comp$$

$$\frac{\overline{ret_pattern_i = spine_elem_i}^i \leadsto \psi}{\log ident = pval, \ \overline{ret_pattern_i = spine_elem_i}^i \leadsto pval/ident, \psi} \quad \text{Subs_Decons_Ret_Log}$$

$$\frac{res_pattern = res_term \leadsto \sigma}{ret_pattern_i = spine_elem_i^{\ i} \leadsto \psi}$$

$$\frac{res_pattern = res_term, \ \overline{ret_pattern_i} = spine_elem_i^{\ i} \leadsto \psi}{res_res_pattern = res_term, \ \overline{ret_pattern_i} = spine_elem_i^{\ i} \leadsto \sigma, \psi}$$
 Subs_Decons_Ret_Res

 $\overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret$

$$\frac{}{::ret \gg \cdot; ret} \quad \text{Subs_Decons_Arg_Empty}$$

$$\frac{\overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret}{x = pval, \ \overline{x_i = spine_elem_i}^i :: \Pi \, x:\beta. \ arg \gg pval/x, \sigma; ret} \quad \text{Subs_Decons_Arg_Comp}$$

$$\frac{\overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret}{x = pval, \overline{x_i = spine_elem_i}^i :: \forall x : \beta. arg \gg pval/x, \sigma; ret}$$
 Subs_Decons_Arg_Log

$$\frac{\overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret}{x = res_term, \overline{x_i = spine_elem_i}^i :: res \multimap arg \gg res_term/x, \sigma; ret}$$
 Subs_Decons_Arg_Res

$$\frac{\overline{x_i = spine_elem_i}^i :: arg \gg \sigma; ret}{\overline{x_i = spine_elem_i}^i :: term \supset arg \gg \sigma; ret} \quad \text{Subs_Decons_Arg_Phi}$$

 $\langle pexpr\rangle \longrightarrow \langle pexpr'\rangle$

$$\frac{mem_ptr' \equiv mem_ptr +_{\text{ptr}} mem_int \times \text{size_of}(\tau)}{\langle \texttt{array_shift} \, (mem_ptr, \tau, mem_int) \rangle \longrightarrow \langle mem_ptr' \rangle} \quad \text{Op_PE_PE_ArrayShift}$$

$$\frac{mem_ptr' \equiv mem_ptr +_{ptr} \text{ offset_of}_{tag}(member)}{\langle \text{member_shift} (mem_ptr, tag, member) \rangle \longrightarrow \langle mem_ptr' \rangle} \quad \text{OP_PE_PE_MEMBERSHIFT}$$

$$\frac{}{\langle \mathtt{not}\,(\mathtt{True})\rangle \longrightarrow \langle \mathtt{False}\rangle} \quad \mathrm{OP_PE_PE_NOT_TRUE}$$

$$\overline{\langle \mathtt{not}\,(\mathtt{False})\rangle \longrightarrow \langle \mathtt{True}\rangle} \quad \operatorname{Op_PE_PE_Not_False}$$

```
mem\_int \equiv mem\_int_1 \, binop_{arith} \, mem\_int_2
                                                                                                                                                                 OP_PE_PE_ARITH_BINOP
                                                                 bool\_value \equiv mem\_int_1 \, binop_{rel} \, mem\_int_2
                                                                                                                                                                   OP_PE_PE_REL_BINOP
                                                                   \overline{\langle mem\_int_1 \ binop_{rel} \ mem\_int_2 \rangle \longrightarrow \langle bool\_value \rangle}
                                                              \frac{bool\_value \equiv bool\_value_1 \, binop_{bool} \, bool\_value_2}{\langle bool\_value_1 \, binop_{bool} \, bool\_value_2 \rangle \longrightarrow \langle bool\_value \rangle}
                                                                                                                                                                     OP_PE_PE_BOOL_BINOP
                                                                                                                                                            OP_PE_PE_ASSERT_UNDEF
                                                                  \overline{\langle \mathtt{assert\_undef}\,(\mathtt{True},\, \mathit{UB\_name})\rangle \longrightarrow \langle \mathtt{Unit}\rangle}
                                                                  \frac{}{\langle \texttt{bool\_to\_integer}\,(\texttt{True})\rangle \longrightarrow \langle 1\rangle} \quad \text{Op\_PE\_PE\_Bool\_To\_INTEGER\_TRUE}
                                                                                                                                       Op_PE_PE_Bool_To_Integer_False
                                                                 \overline{\langle \texttt{bool\_to\_integer}\left(\texttt{False}\right)\rangle \longrightarrow \langle 0\rangle}
                                                  abbrev_1 \equiv \max_{\cdot} \inf_{\tau} - \min_{\cdot} \inf_{\tau} + 1
                                                  abbrev_2 \equiv pval \, \texttt{rem\_f} \, abbrev_1
                                                 mem\_int' \equiv \mathtt{if} \; abbrev_2 \leq \mathtt{max\_int}_{\tau} \, \mathtt{then} \; abbrev_2 \, \mathtt{else} \; abbrev_2 - \, abbrev_1
                                                                                                                                                                                              Op_PE_PE_WrapI
                                                                                 \langle \mathtt{wrapI} \ (\tau, mem\_int) \rangle \longrightarrow \langle mem\_int' \rangle
\langle pexpr \rangle \longrightarrow \langle tpexpr:(y:\beta.\ term) \rangle
                                                                           name:pure\_arg \equiv \overline{x_i}^i \mapsto tpexpr \in \texttt{Globals}
                                                                        \frac{\overline{x_i = pval_i}^i :: pure\_arg \gg \sigma; \Sigma y: \beta. \ term \land \mathtt{I}}{\langle name(\overline{pval_i}^i) \rangle \longrightarrow \langle \sigma(tpexpr): (y: \beta. \ \sigma(term)) \rangle}
                                                                                                                                                                      OP_PE_TPE_CALL
\langle tpexpr \rangle \longrightarrow \langle tpexpr' \rangle
```

```
pattern_i = pval \leadsto \sigma_i
                                                                             \frac{\forall \, i < j. \, \, \text{not} \, (pattern_i = pval \leadsto \sigma_i)}{\langle \text{case} \, pval \, \text{of} \, \overline{\mid pattern_i \Rightarrow tpexpr_i}^i \, \text{end} \rangle \longrightarrow \langle \sigma_j(tpexpr_j) \rangle} \quad \text{Op\_TPE\_TPE\_CASE}
                                                                           \frac{ident\_or\_pattern = pval \leadsto \sigma}{\langle \texttt{let}\, ident\_or\_pattern = pval \, \texttt{in} \, tpexpr \rangle \longrightarrow \langle \sigma(tpexpr) \rangle} \quad \mathsf{OP\_TPE\_TPE\_LET\_SUB}
                                     \frac{\langle pexpr\rangle \longrightarrow \langle pexpr'\rangle}{\langle \texttt{let}\, ident\_or\_pattern = pexpr\, \texttt{in}\, tpexpr\rangle \longrightarrow \langle \texttt{let}\, ident\_or\_pattern = pexpr'\, \texttt{in}\, tpexpr\rangle} \quad \text{Op\_TPE\_TPE\_LET\_LET}
                  \frac{\langle pexpr\rangle \longrightarrow \langle tpexpr_1 : (y : \beta. \ term)\rangle}{\langle \text{let} \ ident\_or\_pattern = pexpr \ in} \underbrace{tpexpr_2}\rangle \longrightarrow \langle \text{let} \ ident\_or\_pattern : (y : \beta. \ term) = tpexpr_1 \ in} \underbrace{tpexpr_2 \ in} \underbrace{tpexpr_2} \\ \text{OP\_TPE\_TPE\_LET\_LETT}
                                                                                                        ident\_or\_pattern = pval \leadsto \sigma
                                                    \frac{\textit{lae}\textit{ident\_or\_pattern} = \textit{pval} \leadsto \sigma}{\langle \texttt{let}\textit{ident\_or\_pattern} : (y:\beta.\; term) = \texttt{done}\;\textit{pval}\; \texttt{in}\; tpexpr \rangle \longrightarrow \langle \sigma(tpexpr) \rangle} \quad \text{Op\_TPE\_TPE\_Lett\_Sub}
\frac{\langle tpexpr_1 \rangle \longrightarrow \langle tpexpr_1' \rangle}{\langle \text{let } ident\_or\_pattern: (y:\beta. \ term) = tpexpr_1 \ \text{in } tpexpr_2 \rangle \longrightarrow \langle \text{let } ident\_or\_pattern: (y:\beta. \ term) = tpexpr_1' \ \text{in } tpexpr_2 \rangle}
                                                                                                                                                                                                                                                                                       OP_TPE_TPE_LETT_LETT
                                                                                                                                                                                                             Op_TPE_TPE_IF_True
                                                                                   \overline{\langle \text{if True then } tpexpr_1 \text{ else } tpexpr_2 \rangle \longrightarrow \langle tpexpr_1 \rangle}
                                                                                                                                                                                                             Op_TPE_TPE_IF_False
                                                                                 \overline{\langle \text{if False then } tpexpr_1 \text{ else } tpexpr_2 \rangle \longrightarrow \langle tpexpr_2 \rangle}
  \langle h; seq\_expr \rangle \longrightarrow \langle h'; texpr:ret \rangle
```

$$\frac{ident:arg \equiv \overline{x_i}^i \rightarrow texpr \in \texttt{Globals}}{z_i = spine.clem_i}^i : arg \gg \sigma_i ret} \qquad \texttt{OP_SE_TE_CCALL}$$

$$\frac{name:arg \equiv \overline{x_i}^i \rightarrow texpr \in \texttt{Globals}}{z_i = spine.elem_i}^i : arg \gg \sigma_i ret} \qquad \texttt{OP_SE_TE_CCALL}$$

$$\frac{name:arg \equiv \overline{x_i}^i \rightarrow texpr \in \texttt{Globals}}{z_i = spine.elem_i}^i : arg \gg \sigma_i ret} \qquad \texttt{OP_SE_TE_PCALL}$$

$$\frac{ident:arg \equiv \overline{x_i}^i \rightarrow texpr \in \texttt{Globals}}{z_i = pval_i}^i : arg \gg \sigma_i false \land 1} \qquad \texttt{OP_STE_TE_RUN}$$

$$\frac{ident:arg \equiv \overline{x_i}^i \rightarrow texpr \in \texttt{Globals}}{z_i = pval_i}^i : arg \gg \sigma_i false \land 1} \qquad \texttt{OP_STE_TE_RUN}$$

$$\frac{pattern_j = pval \leadsto \sigma_j}{\langle h; run \, ident \, \overline{pval_i}^i \rangle \rightarrow \langle h; \sigma(texpr) \rangle} \qquad \texttt{OP_STE_TE_CASE}$$

$$\frac{pattern_j = pval \leadsto \sigma_j}{\langle h; case \, pval \, of \, [\, pattern_i \Rightarrow texpr_i^i \, end \rangle \rightarrow \langle h; \sigma_j(texpr_j) \rangle} \qquad \texttt{OP_STE_TE_CASE}$$

$$\frac{ident.or_pattern = pval \leadsto \sigma}{\langle h; let \, ident.or_pattern = pval \, in \, texpr_i^i \, end \rangle \rightarrow \langle h; \sigma(texpr_j) \rangle} \qquad \texttt{OP_STE_TE_LETP_SUB}$$

$$\frac{(pexpr) \longrightarrow \langle pexpr' \rangle}{\langle h; let \, ident.or_pattern = pexpr \, in \, texpr' \rangle \rightarrow \langle h; let \, ident.or_pattern = pexpr' \, in \, texpr}} \qquad \texttt{OP_STE_TE_LETP_LETP}$$

$$\frac{\langle pexpr \rangle \longrightarrow \langle pexpr' \rangle}{\langle h; let \, ident.or_pattern = pexpr \, in \, texpr \rangle \rightarrow \langle h; let \, ident.or_pattern = texpr \rangle} \qquad \texttt{OP_STE_TE_LETP_LETP}$$

```
\frac{ident\_or\_pattern = pval \leadsto \sigma}{\langle h; \texttt{let} ident\_or\_pattern: (y:\beta. \ term) = \texttt{done} \ pval \ \texttt{in} \ texpr \rangle \longrightarrow \langle h; \sigma(texpr) \rangle} \quad \text{Op\_STE\_TE\_LetTP\_Sub}
\frac{\langle tpexpr\rangle \longrightarrow \langle tpexpr'\rangle}{\langle h; \mathtt{let}\, ident\_or\_pattern: (y:\beta.\,\, term) = tpexpr\, \mathtt{in}\, texpr\rangle \longrightarrow \langle h; \mathtt{let}\, ident\_or\_pattern: (y:\beta.\,\, term) = tpexpr'\, \mathtt{in}\, texpr\rangle} \quad \text{Op\_STE\_TE\_LetTP\_LetTP}
                                                          \frac{\overline{ret\_pattern_i = spine\_elem_i}^i \leadsto \sigma}{\langle h; \mathtt{let}\, \overline{ret\_pattern_i}^i : ret = \mathtt{done}\, \overline{spine\_elem_i}^i \, \mathtt{in}\, texpr \rangle \longrightarrow \langle h; \sigma(texpr) \rangle} \quad \mathsf{OP\_STE\_TE\_LETT\_SUB}
                                   \frac{\langle h; seq\_expr \rangle \longrightarrow \langle h; texpr_1 : ret \rangle}{\langle h; \mathsf{let} \ \overline{ret\_pattern_i}^i = seq\_expr \ \mathsf{in} \ texpr_2 \rangle \longrightarrow \langle h; \mathsf{let} \ \overline{ret\_pattern_i}^i : ret = texpr_1 \ \mathsf{in} \ texpr_2 \rangle} \quad \mathsf{OP\_STE\_TE\_LET\_LETT}
                               \frac{\langle h; texpr_1 \rangle \longrightarrow \langle h'; texpr_1' \rangle}{\langle h; \mathsf{let} \, \overline{ret\_pattern_i}^{\,\, i} : ret = texpr_1 \, \mathsf{in} \, texpr_2 \rangle \longrightarrow \langle h'; \mathsf{let} \, \overline{ret\_pattern_i}^{\,\, i} : ret = texpr_1' \, \mathsf{in} \, texpr_2 \rangle} \quad \mathsf{OP\_STE\_TE\_LETT\_LETT}
                                                                                       \overline{\langle h; \text{if True then } texpr_1 \text{ else } texpr_2 \rangle \longrightarrow \langle h; texpr_1 \rangle} OP_STE_TE_IF_TRUE
                                                                                    \overline{\langle h; \text{if False then } texpr_1 \text{ else } texpr_2 \rangle \longrightarrow \langle h; texpr_2 \rangle}
                                                                                                                                                                                                                         OP_STE_TE_IF_FALSE
                                                                                                   \frac{}{\langle h; \mathtt{bound} \, [int] (is\_texpr) \rangle \longrightarrow \langle h; is\_texpr \rangle} \quad \mathsf{OP\_STE\_TE\_BOUND}
 \langle h; mem\_op \rangle \longrightarrow \langle h'; tval \rangle
                                                                    \frac{bool\_value \equiv mem\_int_1 \ binop_{rel} \ mem\_int_2}{\langle h; mem\_int_1 \ binop_{rel} \ mem\_int_2 \rangle \longrightarrow \langle h; \texttt{done} \ bool\_value \rangle}
                                                                                                                                                                                                                      Op_Memop_TVal_Rel_Binop
```

```
mem\_int \equiv \texttt{cast\_ptr\_to\_int} \ mem\_ptr
                                                                                                                                                                                           Op_Memop_TVal_IntFromPtr
                                                             \overline{\langle h; \mathtt{intFromPtr} \left(\tau_1, \tau_2, mem\_ptr\right)\rangle \longrightarrow \langle h; \mathtt{done} \ mem\_int\rangle}
                                                            \frac{mem\_ptr \equiv \texttt{cast\_ptr\_to\_int} \ mem\_int}{\langle h; \texttt{ptrFromInt} \ (\tau_1, \tau_2, mem\_int) \rangle \longrightarrow \langle h; \texttt{done} \ mem\_ptr \rangle}
                                                                                                                                                                                           OP_MEMOP_TVAL_PTRFROMINT
                                                                                                     bool\_value \equiv aligned(\tau, mem\_ptr)
\frac{\textit{bool\_value} = \texttt{aligned}\left(\tau, \textit{mem\_ptr}\right)}{\langle h + \{\textit{mem\_ptr} \overset{\checkmark}{\mapsto}_{\tau -} \}; \texttt{ptrValidForDeref}\left(\tau, \textit{mem\_ptr}, \textit{mem\_ptr} \overset{\checkmark}{\mapsto}_{\tau -} \right) \rangle \longrightarrow \langle h + \{\textit{mem\_ptr} \overset{\checkmark}{\mapsto}_{\tau -} \}; \texttt{done}\, \textit{bool\_value}, \textit{mem\_ptr} \overset{\checkmark}{\mapsto}_{\tau -} \rangle}
                                                                                                                                                                                                                                                                                             OP_MEMOP_TVAL_PTRVALID
                                                                            bool\_value \equiv \mathtt{aligned}\left(\tau, mem\_ptr\right)
                                                    \overline{\langle h; \mathtt{ptrWellAligned} \left(\tau, mem\_ptr\right) \rangle \longrightarrow \langle h; \mathtt{done} \, bool\_value \rangle}
                                                                                                                                                                                     Op_Memop_TVal_PtrWellAligned
                                             \frac{mem\_ptr' \equiv mem\_ptr +_{ptr} (mem\_int \times size\_of(\tau))}{\langle h; ptrArrayShift (mem\_ptr, \tau, mem\_int) \rangle \longrightarrow \langle h; done mem\_ptr' \rangle}
                                                                                                                                                                                                 Op_Memop_TVal_PtrArrayShift
   \langle h; mem\_action \rangle \longrightarrow \langle h'; tval \rangle
                                                                                               fresh(mem_ptr)
                                                                                               representable (\tau *, mem\_ptr)
                                                                                               alignedI (mem_int, mem_ptr)
                                                                                                                                                                                                                                         OP_ACTION_TVAL_CREATE
                           \overline{\langle h; \mathtt{create}\,(mem\_int,\tau)\rangle \longrightarrow \langle h + \{mem\_ptr \overset{\times}{\mapsto}_{\tau}\,pval\}; \mathtt{done}\,mem\_ptr,pval,mem\_ptr \overset{\times}{\mapsto}_{\tau}\,pval\rangle}
                                                                                                                                                                                                                                                                                   OP_ACTION_TVAL_LOAD
\frac{}{\langle h + \{mem\_ptr \overset{\checkmark}{\mapsto}_{\tau} \ pval\}; \texttt{load} \ (\tau, mem\_ptr, \_, mem\_ptr \overset{\checkmark}{\mapsto}_{\tau} \ pval) \rangle} \longrightarrow \langle h + \{mem\_ptr \overset{\checkmark}{\mapsto}_{\tau} \ pval\}; \texttt{done} \ pval, mem\_ptr \overset{\checkmark}{\mapsto}_{\tau} \ pval \rangle}
                                                                                                                                                                                                                                                                                       OP_ACTION_TVAL_STORE
\frac{}{\langle h + \{mem\_ptr \overset{\checkmark}{\mapsto}_{\tau} \_\}; \mathtt{store} \left( \_, \tau, mem\_ptr, pval, \_, mem\_ptr \overset{\checkmark}{\mapsto}_{\tau} \_ \right) \rangle} \longrightarrow \langle h + \{mem\_ptr \overset{\checkmark}{\mapsto}_{\tau} \ pval\}; \mathtt{done} \ \mathtt{Unit}, mem\_ptr \overset{\checkmark}{\mapsto}_{\tau} \ pval \rangle}
```

 $\frac{}{\langle h + \{mem_ptr \mapsto_{\tau_-}\}; \texttt{kill} (\texttt{static} \, \tau, mem_ptr, mem_ptr \mapsto_{\tau_-}) \rangle \longrightarrow \langle h; \texttt{done} \, \texttt{Unit} \rangle} \quad \text{OP_ACTION_TVAL_KILL_STATIC}$

 $\langle h; is_expr \rangle \longrightarrow \langle h'; is_expr' \rangle$

$$\frac{\langle h; mem_op \rangle \longrightarrow \langle h; tval \rangle}{\langle h; memop (mem_op) \rangle \longrightarrow \langle h; tval \rangle} \quad \text{Op_ISE_ISE_MEMOP}$$

$$\frac{\langle h; mem_action \rangle \longrightarrow \langle h'; tval \rangle}{\langle h; mem_action \rangle \longrightarrow \langle h'; tval \rangle} \quad \text{Op_IsE_IsE_Action}$$

$$\frac{\langle h; mem_action \rangle \longrightarrow \langle h'; tval \rangle}{\langle h; \mathsf{neg}\, mem_action \rangle \longrightarrow \langle h'; tval \rangle} \quad \mathsf{OP_ISE_ISE_NEG_ACTION}$$

 $\langle h; is_texpr \rangle \longrightarrow \langle h'; texpr \rangle$

$$\frac{\overline{ret_pattern_i = spine_elem_i}^i \leadsto \sigma}{\langle h; \mathtt{let strong} \, \overline{ret_pattern_i}^i = \mathtt{done} \, \overline{spine_elem_i}^i \, \mathtt{in} \, texpr \rangle \longrightarrow \langle h; \sigma(texpr) \rangle} \quad \mathsf{OP_ISTE_ISTE_LETS_SUB}$$

$$\frac{\langle h; is_expr\rangle \longrightarrow \langle h'; is_expr'\rangle}{\langle h; \mathsf{let} \, \mathsf{strong} \, \overline{ret_pattern_i}^{\,\, i} \, = is_expr \, \mathsf{in} \, texpr\rangle \longrightarrow \langle h'; \mathsf{let} \, \mathsf{strong} \, \overline{ret_pattern_i}^{\,\, i} \, = is_expr' \, \mathsf{in} \, texpr\rangle} \quad \mathsf{OP_ISTE_ISTE_LETS_LETS}$$

 $\overline{\langle h; texpr \rangle} \longrightarrow \langle h'; texpr' \rangle$

$$\frac{\langle h; seq_texpr\rangle \longrightarrow \langle h; texpr\rangle}{\langle h; seq_texpr\rangle \longrightarrow \langle h; texpr\rangle} \quad \text{Op_TE_TE_SEQ}$$

$$\frac{\langle h; is_texpr\rangle \longrightarrow \langle h'; texpr\rangle}{\langle h; is_texpr\rangle \longrightarrow \langle h'; texpr\rangle} \quad \text{Op_TE_TE_IS}$$

Definition rules: 213 good 0 bad Definition rule clauses: 476 good 0 bad