

Computational Design of Reconfigurable Underactuated Linkages for Adaptive Grippers*

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Abstract—We present an optimization-based structural-parametric synthesis method for reconfigurable closed-chain underactuated linkages for robotic systems that physically interact with the environment with an emphasis on adaptive grasping. The key idea is to implement morphological computation concepts to keep both necessary trajectory-specific holonomic constraints and mechanism adaptivity using *variable length links* (VLL), while we evolve from a fully actuated to an underactuated system satisfying imposed design requirements. It allows to minimize the number of actuators, weight, and cost but keep high payload and endurance that are not reachable by tendon-driven designs. Despite the method is general enough, for clarity, we demonstrate its use on a number of finger mechanisms for adaptive grippers.

I. INTRODUCTION

Generally, robotic systems are intended to be versatile devices. Articulated collaborative robots such as KUKA LWR [1] can provide a wide range of trajectories within their configuration space and physically interact with the environment what makes them a useful tool for flexible automation; legged robots as MIT Cheetah [2], ANYmal robot [3], HyQ [4] are able to reproduce multiple trajectories for different gaits, such as, walking, running, galloping, or even jumping; robotics grippers such as Shadow Hand [5] able to perform various grasps to manipulate a wide range of objects with different shapes, sizes, and mass.

All mentioned robotic devices utilize simple open-chain mechanisms and complex control algorithms, focusing on motion planning, balance, and physical interaction. Besides apparent advantages such as versatility to reconfigure motion, the *software path* is related to a list of disadvantages in terms of weight, cost, and control issues. Particularly the need to configure non-linear controllers, which are also sensitive to accuracy and noise in measurements, delays, and signal sampling as well as performance of computing systems. These issues can be eliminated if we co-design hardware and software parts and put more attention to *smart* mechanisms.

We address the task of shaping system behavior algorithmically and mechanically which is very much in line with the physical intelligence concept [6], [7]. Our study follows the idea of *morphological computation*, which means that part

of 'computation' of control law is performed by mechanics. We 'program' desired kinematics and dynamics properties by means of the mechanical structure of a robot, while control effort should be as minimum as possible to excite, stabilize, or take advantage of the natural dynamics [8].

Although fully actuated open-chain mechanisms allow us to reconfigure the motion, having an individual actuator for each kinematic pair entails problems in cost, weight, and control. For the specific robotic applications, when the high level of versatility is unnecessary, it is possible to synthesize a mechanical system such as we can simplify its control system in terms of hard- and software. When there is a task to follow a predetermined trajectory with a little variation because of possible physical interaction, a light, an adaptive, and an efficient mechanism is required. Low inertia and adaptivity are crucial because of physical intersection; efficiency means using a lesser number of actuators.

With this paper, we focus on *structural-parametric synthesis* of planar closed-chain linkages for robotic applications that: (a) relocates actuators to a mechanism's base to decrease linkage inertia, (b) reduces the number of actuators by holonomic constraints, (c) introduces mechanical compliance to act stably under physical interactions. The synthesized mechanisms (i) can be driven by a few actuators with simple position control algorithms, (ii) physically interact with the environment, and (iii) can be reconfigured between 'precise tracking' and 'compliant interaction' modes. If there is no physical interaction, the synthesized mechanism follows a predefined trajectory and automatically adjusts in response to external forces in case of interaction.

The mechanism reconfiguration and adaptivity are achieved by means of *variable length links* (VLL) that can be employed in the active and the passive settings. The links' lengths change because of holonomic constraints and applied external forces. By active VLL we mean that its length can be discreetly locked; otherwise, we call a VLL passive.

Although we believe the proposed method is general and applicable for a wide range of mechanisms,¹ further we focus on adaptive finger mechanisms for the anthropomorphic grippers as a good illustrative applied example.

A. Illustrative examples and method outline

To show the benefits of the proposed method, let us consider an example of three different types of finger mechanisms that can be used for anthropomorphic grippers to stably grasp objects of different shapes (see Fig. 1).

¹For example, a leg mechanism for a galloping robot presented in [9] has been synthesized using the same method

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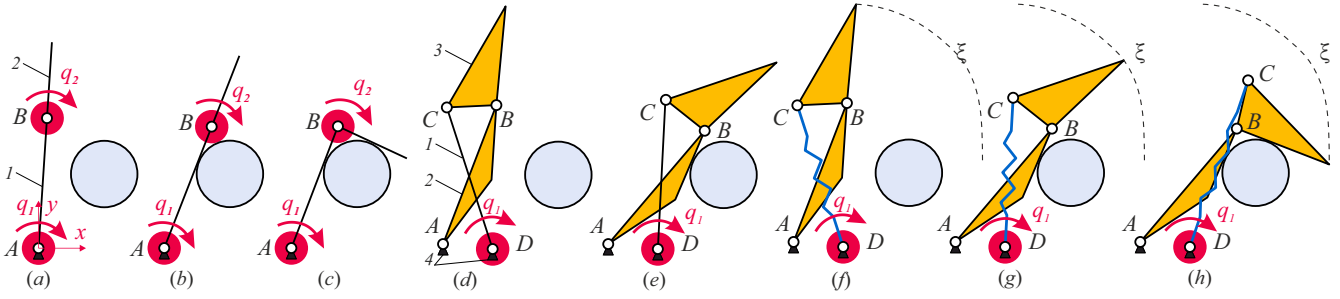


Fig. 1. Bending sequences of different planar fingers mechanisms for anthropomorphic grippers: (a) - (c) fully actuated mechanism with open chain, (d) - (e) fully actuated mechanism with closed chain, (f) - (h) underactuated mechanism with closed chain. Red circles indicate actuators. Blue spring-like body CD indicates physical variable length link.

Fig. 1, (a-c) show a grasping sequence of a fully actuated open-chain finger that grasps a cylinder-shaped object fixed in space [10]. The finger consists of two actuated links: proximal (1) and distal (2) phalanges. The main advantage of such mechanism is the ability to shape a wide range of trajectories without changing the mechanical structure. The main disadvantages are: (a) two motors with position sensors are needed (indicated by the red circles), (b) sophisticated motion planning and control system to provide both desired contact points and grasping forces, and (c) with no compliance, you need 100% accuracy, which is never available.

An alternative design with a fully actuated closed-chain mechanism is depicted in Fig. 1, (d-e). The finger is a simple four-bar mechanism and uses a single motor to steer all links. The finger consists of an input link (1) that is attached to a motor's shaft, proximal (2) and distal (3) phalanges that are called output links since they interact with an object, and a fixed link (4). Such mechanisms were suggested for prosthetic hands to follow human-like finger motion [11]. However, after the first contact with an object (Fig. 1, e), the mechanism is stuck and does not continue the motion. Thus, an adaptive grasp is not achieved.

In order to get the adaptive behavior of that finger together with the simple control system, a mechanical structure should comply with the environment. Fig. 1, (f-h) shows a grasping sequence of an *underactuated* closed-chain finger mechanism, where we use a VLL that helps to perform an encompassing grasp of the same cylinder-shaped object. Initially (Fig. 1, f) the finger behaves as in the previously considered example (Fig. 1, d). It moves in the same way until the proximal phalanx touches the object (Fig. 1, g). Then the VLL extends in reaction to the contact forces, and this allows to continue motion and finish an encompassing grasp (Fig. 1, h) with a simple position controller.

Indeed, these examples illustrate main stages of the suggested design procedure:

- 1) define a fully-actuated open-chain linkage mechanism that can accomplish the task following the predetermined trajectory;
- 2) extend the open-chain linkage by adding *groups* of rigid links to relocate/eliminate actuators keeping the desired trajectory;

- 3) replace selected rigid links from the added groups by VLL such that we get the final under-actuated mechanism that satisfies all design requirements.

At each step, we resolve existing design constraints (e.g. mechanism existence conditions, etc.) and search for the simplest solution as an outcome of the corresponding numerical optimization procedure. We elaborate on each step in the next section.

B. Contribution and relevant studies

In the aforementioned case (Fig. 1), the choice of VLL was straightforward: the mechanism contains only 3 movable parts, and 2 of them are output links that have to keep their geometry intact; thus, link CD is the only candidate to be replaced by VLL. In general, we have multiple choices which can be hard to distinguish, and a design method that navigates through these options towards an optimal solution is of high interest.

For the sake of saving space, we do not add a comprehensive review (which can be a separate paper alone), but we should say that there are contributions on anthropomorphic grippers design such as [5] or [12], and more relevant studies on analysis and synthesis of underactuated mechanisms, e.g. [13], as well as mechanisms including elements of variable geometry like elastic links and joints [14].

Structural-parametric synthesis is a highly unintuitive design process. Finding a good or even optimal structure and geometry for a closed-chain linkage structure is challenging. Here we mention a few papers related to the linkage optimization, such as [15], [16] on parametric numerical optimization of linkages, [17] on synthesis of compliant mechanical systems that exhibit large-amplitude oscillations. We also addressed related problems in our previous works on parametric optimization of a multi-link leg mechanism of a bio-inspired galloping robot, e.g. [18].

An example that inspired this work and the methodology for closed-kinematics linkage topology optimization is presented in [19]. This methodology helps a user to successfully replace the joint motors with new rigid links that mechanically couple the motions of different mechanical assembly parts. Thus, a more sophisticated mechanical system is obtained, which is actuated with a single motor.

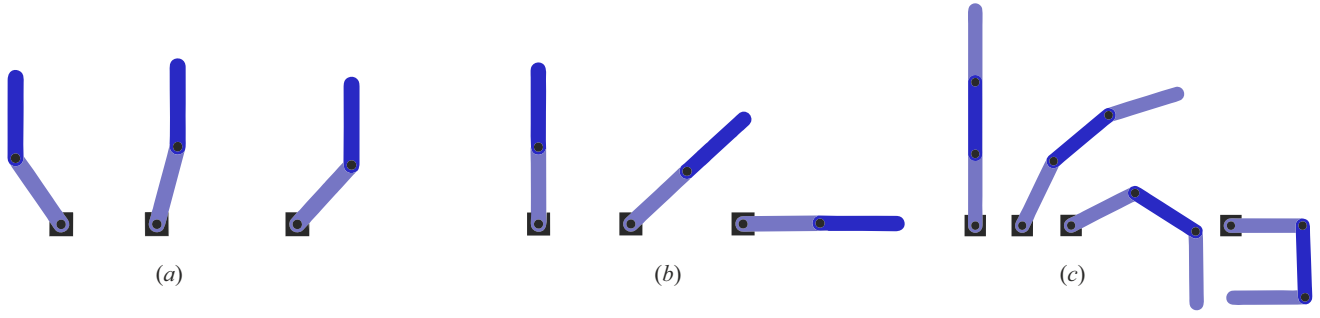


Fig. 2. Bending sequence of initial open-chain mechanisms: two phalanx finger conducts a parallel motion (a) and rotates on 90° (b); and three phalanx finger conducts a human-like motion with pre-bending (c)

Our attempt here is to augment the design method presented in [19] giving more systematic approach that extends beyond particular cases considered in the mentioned article, and further bring arguments on how VLL as a tool for creating directed mechanical compliance should be embedded. We show that the proposed method is general enough to cope with different initial kinematics and design constraints and allows to automate the synthesis's routine tasks and guide a developer through the design process.

II. DESIGN LOOP

Although mechanisms include different components such as gear-trains and cams, linkages are arguably the most challenging part to design, especially if we are talking about closed-kinematics. We further describe stages of the design procedure following the outline from Section I-A, but show how it can be applied to evolve from more complex closed-chain mechanisms.

A. Defining an open-chain linkage

The design process begins with defining a reference ('virtual') joint trajectory that must be followed if there is no physical interaction, a number of phalanges (links), and their constant geometric parameters, e.g., lengths, for a fully-actuated open-chain linkage with W_o degrees of freedom (DoF). The desired degrees of actuation, i.e., number of motors A , and underactuation, i.e., number of passive DoF.

Fig. 2 shows a few initial open-chain mechanisms with two and three phalanges. Those are the most common topologies for fingers of anthropomorphic grippers. Each joint is equipped with its own actuator. Fig. 2, (a) shows a bending sequence of a finger that conducts a parallel grasp. For example, that motion is needed for a thumb of a prosthetic hand [20] or industrial gripper [21]. Fig. 2, (b) shows a motion for an index finger that rotates on 90° . Fig. 2, (c) shows a motion of a finger with three phalanges that is similar to natural human motion. The finger with two phalanges has 2 DoF, while the one with three phalanges has 3 DoF. Those topologies and desired trajectories are used for the following examples.

B. Synthesizing of closed-chain linkage

The purpose of transforming the open-chain mechanism into the closed-chain is to relocate the actuators close to the mechanism base and reduce their number. To do that, we can replace motors with a group of rigid links by adding new holonomic constraints.

A group of additional links brings holonomic constraints, i.e., takes degrees of freedom. Let us define A as a desired number of actuators to steer the whole finger, then the additional group of links should have W_a degrees of freedom

$$W_a = -(W_o - A). \quad (1)$$

Using the Chebychev-Grübler criterion, it is feasible to calculate all possible combinations of the number of movable links and joints. The criterion for planar mechanisms is

$$W_a = 3n_a - 2P_{5a}, \quad (2)$$

where $n_a \in \mathbb{Z}$ is a number of movable links, $P_{5a} \in \mathbb{Z}$ is a number of joints with 5 constraints.

The combination of movable links n_a and 1 DoF joints P_{5a} , i.e., topology of an adding group of links, depends on how many actuators we want to eliminate. Characteristics for various topologies of adding groups $h_{n_a, P_{5a}}$ are given in Table I and Fig. 3 shows them.

TABLE I
LINKS GROUPS' POSSIBLE COMBINATIONS

Parameter	Value							
W_a	0		-1		-2		-3	
h	$h_{2,3}$	$h_{4,6}$	$h_{1,2}$	$h_{3,5}$	$h_{2,4}$	$h_{4,7}$	$h_{1,3}$	$h_{3,6}$
n_a	2	4	1	3	2	4	1	3
P_{5a}	3	6	2	5	4	7	3	6
m_a	2	3	2	3	3	3	3	4

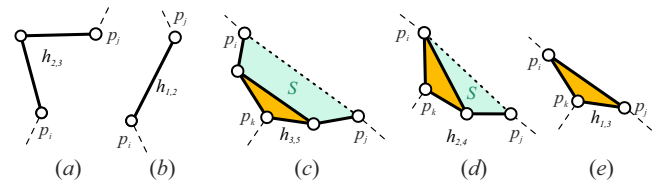


Fig. 3. Links' groups: (a) $h_{2,3}$, (b) $h_{1,2}$, (c) $h_{3,5}$, (d) $h_{2,4}$, (e) $h_{1,3}$

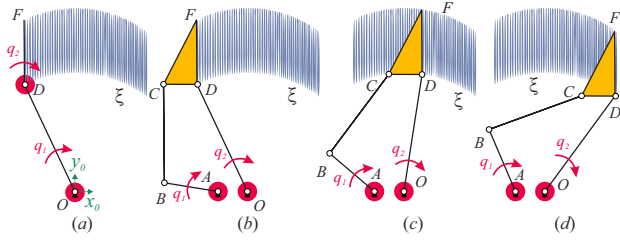


Fig. 4. Finger with two phalanges: (a) open chain linkage, (b-d) bending sequences of a closed chain five-bar linkage. Blue vertical lines represent discrete position ξ^* of a working surface FD

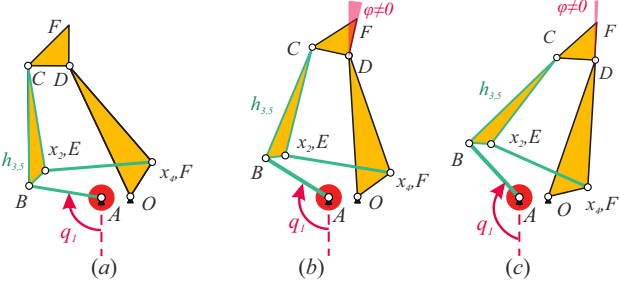


Fig. 5. A finger that consists of an initial open-chain mechanism and attached group $h_{3,5}$

Fig. 3a shows a group of links $h_{2,3}$ that consists of $n_a = 2$ movable links and $P_{5a} = 3$ joints with 1 DoF; m_a indicates a number of joints that have to be connected with the initial mechanism. That combination has been calculated using eq. (2). If the task to keep the number of actuators intact, but only to relocate them close to the finger's base, we can add group with $W_a = 0$, e.g. $h_{2,3}$ or $h_{4,6}$, since they do not change the mechanism's degree of freedom. The group $h_{2,3}$ has two joints to be added to the initial open-chain mechanism. Since we want to form a closed-chain mechanism and relocate actuators near a mechanism's bottom, one of the free joints p_j has to be added to a fixed frame, while the second one p_i to the extreme link of the initial open-chain linkage.

For example, if the task to perform a motion depicted in Fig. 2a keeping $A = 2$, we can add the group $h_{2,3}$. Fig. 4a shows an open-chain finger with 2 actuators that has been transformed to a five-bar mechanism with 2 actuators, but located close to the mechanism's base. One of motors can be replaced with a torsional spring to form a famous underactuated mechanism [21].

If the task is to reduce a number of actuators, groups' DoF has to be less than zero $W_a < 0$. The simplest structure $h_{1,2}$ that can be added to eliminate a degree of freedom consists of one movable link $n_a = 1$ and two joints $P_a = 2$ (Fig. 3b). The attaching only this group was considered in [19]. If we attach $h_{1,2}$ to a finger with two phalanges we would get a four-bar mechanism depicted in Fig. 1d.

Suppose the resulting mechanism is not satisfactory, for example, in terms of force distribution, the desired trajectory following, or there is no room for structural elements such as sensors, spring, magnets, etc. In that case, the next in line group with three links and five joints $h_{3,5}$ can be

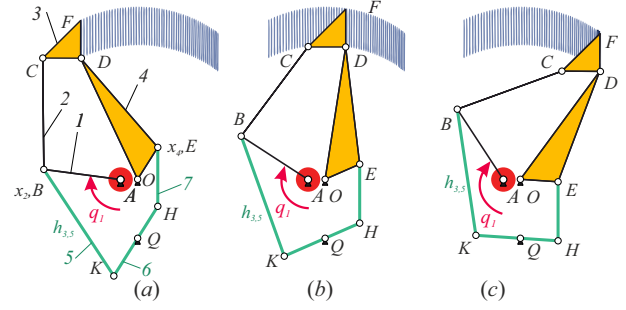


Fig. 6. A finger that consists of an initial open-chain mechanism and two attached groups $h_{3,5}$

added (Fig. 5). Here, the extreme joints p_i and p_k have to be attached to a frame and an extreme link of initial open-chain linkage, as in the previous example, while place for the last joint p_j must be found.

The idea is that for two different links a and b we look for a pair of material points x_a and x_b such as they follow the same trajectory over an entire motion cycle, that we can connect via a 1 DoF joint. In order to find these points we need to solve an optimization task described in the next subsection.

The same trick can be done with a closed-chain mechanism treated as an initial one. A five-bar mechanism with two actuators (Fig. 5) can be added with another $h_{3,5}$ group of links to form a new linkage depicted in Fig. 6. Such kinematically more complex mechanism can be synthesized due to design purposes. Section III gives a short overview for the designed prototype for a finger mechanism depicted in Fig. 6.

C. Examples of extended mechanisms

The aforementioned open-chain mechanisms with predefined behavior (Fig. 2) have been added with additional links' groups. The results are presented in Fig. 7. The presented linkages were synthesized as examples to show that the presented method is general enough. The mechanisms' aesthetics required for prosthetic devices were not considered as a parameter for the optimization task. There are countless solution for each mechanism topologies and required trajectories.

Fig. 7a show a bending sequence of a finger with two phalanges that conducts a parallel grasp and Fig. 7b shows a similar finger that rotates on 90° . Both fingers were attached with $h_{3,5}$ group as it was showed in Fig. 5.

Fig. 7c and d show a bending sequence of a finger with three phalanges that conducts a human-like motion. The first one (Fig. 7c) was attached with one $h_{2,4}$ group as we propose within this article, while (Fig. 7d) was attached with two rigid links $h_{1,2}$ according to the method presented in [19]. Those mechanisms can not be treated as optimal; however, the mechanism with $h_{2,4}$ conducts a motion that is closer to the desired one. The mechanism depicted in Fig. 7d can be used as a finger if the attached links have an arc form such as the links do not stand in front of phalanges.

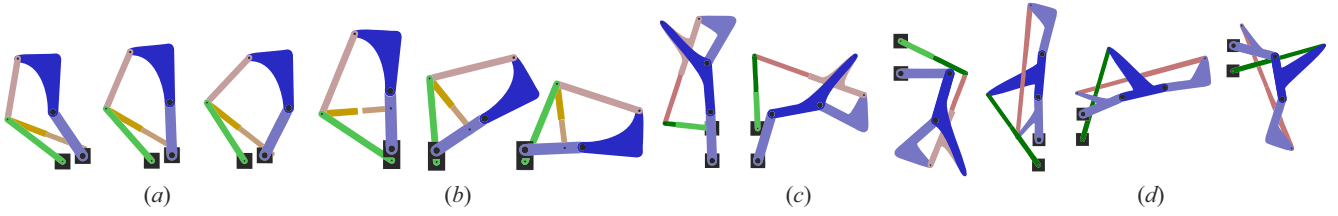


Fig. 7. Bending sequence of synthesized closed-chain mechanisms: two phalanx finger conducts a parallel motion (a) and rotates on 90° (b); and three phalanx finger conducts a human-like motion with pre-bending (c) and (d)

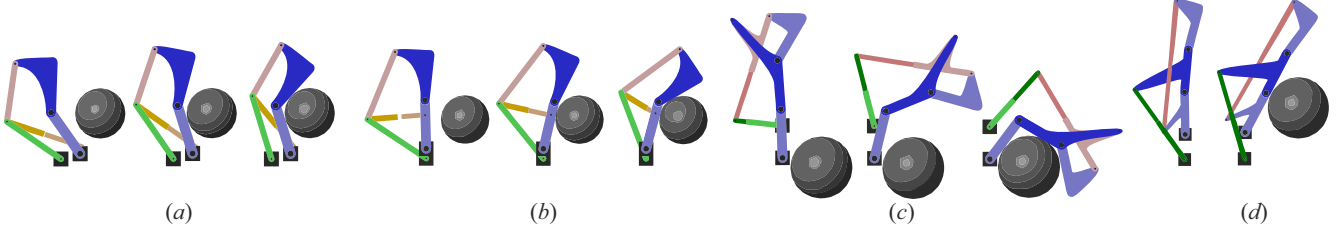


Fig. 8. Encompassing grasps: underactuated fingers with two phalanges and VLL (a) and (b), underactuated finger with three phalanges and VLL (c), and a finger with three phalanges without VLL (d)

D. Parametric optimization

In order to be general, we treat all the linkages as transformers of input links' motion to the motion of output links. By an input link, we mean the one which is actuated by an own motor, and we can control its state directly; an output link is the one that follows the desired trajectory and physically interacts with the environment.

In general, we deal with linkages, which behavior and desired properties are challenging to describe by functions. However, out of all specifications we can define an essential property that can be analytically represented by a function that maps the vector of m geometric parameters $\theta \in \mathbb{R}^m$ and n input links' coordinates $q \in \mathbb{R}^n$ describing mechanism configuration to l values characterizing output link state $\xi \in \mathbb{R}^l$:

$$\xi = f(\theta, q). \quad (3)$$

An example of such a dependency is direct kinematics, where ξ denotes Cartesian coordinates of an output link. To parametrise output link state not in a discrete point, but as a continuous evolution, we can introduce a new independent monotonically increasing path variable s and rewrite (3) as

$$\xi(s) = f(\theta, q(s)). \quad (4)$$

To give an intuition, let us consider a five-bar finger mechanism (Fig. 4). Such a mechanism is used for a number of grippers, such as [22] and [23]. Assume that we want to design a reconfigurable gripper that is able to perform a parallel grasp on a section of a working stroke of the desired length. Then, the output link CDF needs to move such that its orientation remains constant $\phi = 0$, where ϕ is the angle between DF and vertical, and the desired output link trajectory on a given segment can be parametrized as

$$\xi^*(s) = \begin{pmatrix} x^*(s) \\ y^*(s) \\ \phi^*(s) \end{pmatrix} = \begin{pmatrix} OA + OD \sin(s) \\ OD \cos(s) \\ 0 \end{pmatrix}, \quad (5)$$

where s is the angle between OD and a vertical drawn via point O measured in rad clockwise. In [20], [24], [25] we discuss the desired trajectories of an output link for adaptive grippers and anthropomorphic hands in more details.

Even thought for others examples necessary conditions and dimensionality of $\xi(s)$ can be different, we still should be able to define $\xi^*(s) \in \Xi$, where Ξ is a set of all possible output link positions.

After that we are ready to formulate and solve the problem of finding geometric parameters and reference joint trajectories for the extended mechanism as an optimization one:

$$\{\theta^*, q^*(s)\} = \underset{\theta \in C_\theta, q \in C_q}{\operatorname{argmin}} \|\xi^*(s) - f(\theta, q(s))\|, \quad (6)$$

i.e. we minimize the Euclidean distance between the desired and current path traveled by an output link, where manifolds C_θ and C_q should be defined a priori.

To deal with a well-posed problem formulation, we can augment (6) with the following arguments. Based on the design constraints, like mechanism existence, no links overlap, or maximum allowed motion envelop, it may be possible to:

- 1) Split the entire set of geometric parameters and generalized coordinates into subsets of free and constrained variables $\theta = (\theta_f, \theta_c^*)^T$, $q(s) = (q_f(s), q_c^*(s))^T$, where θ_c^* and $q_c^*(s)$ are now considered as pre-defined.
- 2) Introduce additional constraints $G(\theta_f, q_f(s)) \leq 0$, $G: \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^k$, $k \geq 0$.

Finally, we should solve the optimization task

$$\begin{aligned} \{\theta_f^*, q_f^*(s)\} = & \underset{\theta_f \in C_{\theta_f}, q_f \in C_{q_f}}{\operatorname{argmin}} \|\xi^*(s) - f(\theta_f, \theta_c^*, q_f(s), q_c^*(s))\|, \\ \text{s.t. } & G(\theta_f, q_f(s)) = 0. \end{aligned} \quad (7)$$

Here we find the complete set of desired geometric parameters θ_f^* , θ_c^* and nominal joint trajectories $q_f^*(s)$, $q_c^*(s)$ defining the extended close-chain mechanism. The optimization has been conducted in MATLAB and *patternsearch* has been used to find minimum of the cost function.

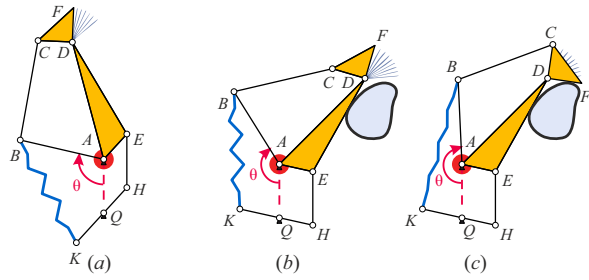


Fig. 9. The bending sequence of the gripper's underactuated finger

E. Variable length links

The result of the previous stage is a 'half-stuff' mechanism that complies with the essential requirement (4), but may not necessarily meet all desired properties, e.g., not able to perform an adaptive grasp an object. To overcome this limitation we suggest to replace one or several rigid links added at the previous stage by a VLL. By doing so we embed mechanical compliance and make system responsive to external forces.

Fig. 7 (a)-(c) show the mechanisms with embedded variable length links: an orange one for Fig. 7 (a)-(b) and both attached links (pink and green) for the finger with three phalanges Fig. 7 (c). The finger synthesized according to [19] is showed in Fig. 8 (d). It is not equipped with VLL. The figures show that the fingers follow the predefined trajectory if there is no physical interaction.

If an external force is applied to the fingertips, the fingers can grasp an object utilizing the fingertip areas only, i.e., perform a precision grasp. If the force is applied near to the joint areas or proximal/medial phalanges, the finger will automatically adapt to the object's shape. Fig. 8 (a)-(c) shows the fingers which perform encompassing grasps because of VLL. Contrary, the finger without VLL stuck if there is contact with an object.

On the other hand, the underactuation in manipulation/physical interaction is sometimes viewed with trepidation: lack of controllability and precision may be an issue. However, we have found a way to reconfigure the behaviour of an underactuated system by adding an *active* VLL that enables "tuning" the intrinsic behaviour of a mechanism.

For underactuated grippers the main disadvantage that a precision grasp is difficult to achieve unless specific design modifications are applied [21]. A fully actuated mechanism can ensure precision grasp. To combine both behaviors, underactuated adaptive and fully actuated precision grasps, in one device we need to install an *active* VLL which can be treated as a clutch to change a finger's degree of freedom.

Fig. 6 and Fig. 9 show the same finger in precision and adaptive modes respectively. In fully actuated mode, VLL BK keeps its length constant, and because of that finger has only one DOF. Thus, the finger motion is expected, and it follows the desired trajectory to perform precision grasp. In underactuated mode, the length of the link BK is getting variable; because of that, the finger has 2 DOF. Thus,

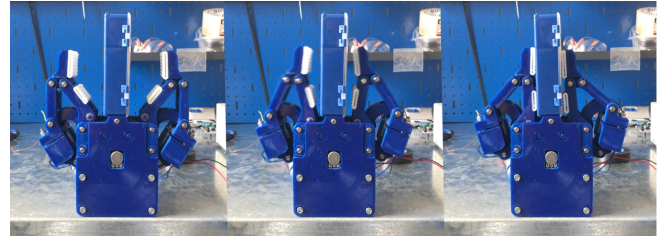


Fig. 10. Industrial gripper prototype executing pinch grasp

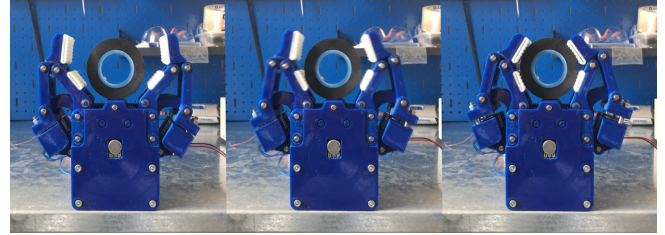


Fig. 11. Industrial gripper prototype executing adaptive grasp

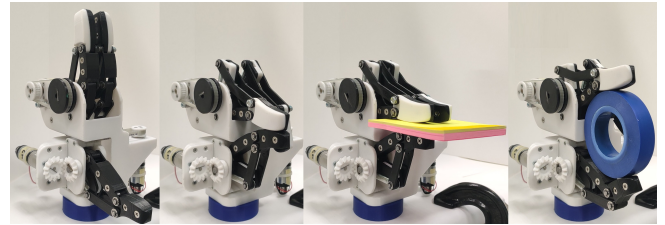


Fig. 12. Artificial three-fingered hand prototype executing different grasps

adaptive grasp is possible. It is important to highlight that only position/velocity control is needed here to actuate the whole finger. Our previous research regarding such grippers can be found in [24], [25].

The minimum and maximum VLL parameters are selected in a way that all possible trajectories of output links belong to a given working area. For recommendations on defining which link is the best choice to be replaced by VLL and on adjusting stiffness and damping parameters of that VLL, we direct the reader to our previous works on kinetostatic analysis [25] and stiffness allocation [26].

III. PROTOTYPES DESIGN

Several prototypes of grasping devices have been created based on the described method. One of that is the two fingered industrial gripper with active VLL. Fig. 10 shows how it grasps a box-shaped object in the fully actuated mode, while VLL is fixed at a minimum length using an electromagnet. To get the power grasp (Fig. 11) we switch to the underactuated mode, when the length of the VLL is determined only by the external force and internal holonomic constraints. It was possible to achieve stable interaction without applying force or impedance control. Indeed, we simply control the motion of the input link by a PID-controller, where coefficients are calculated on the basis of the stability criterion of a closed-loop system. More detailed description of control system is presented in [24].

Fig. 12 shows a prototype of an artificial hand with three fingers. The linkage topology is the same, but the VLL

is equipped with an elastic element, i.e., two rigid bodies connected via a spring-loaded prismatic joint that acts as a mechanical clutch to change a grasping mode. Thus, extra control for VLL is not needed, and VLL can change its length due to holonomic constraints and external forces applied to output links. More details are given in [20].

IV. CONCLUSIONS

Our work is inspired by morphological computations approach to design robots that are able to interact with unmodeled environment controlled by simple algorithms.

We described a method for optimisation-based structural-parametric design of underactuated closed-chain reconfigurable mechanisms that, thanks to variable length links (VLL), can combine two different operation modes: strictly follow reference trajectory or adapt configuration when external forces are applied, if needed. This is a systematic approach that extends beyond cases considered in [19] and brings arguments on how VLL should be embedded to create directed mechanical compliance.

Provided advantage is that mechanisms are driven by a single actuator. It allows to minimise weight and cost, but keep high payload and endurance that are not reachable, e.g. by tendon-driven designs.

We show that the proposed method is general enough to cope with different initial kinematics and design constraints and allows to automate the synthesis's routine tasks and guide a developer through the design process.

Design of finger mechanism for adaptive grippers was considered as the main applied example throughout the paper. We illustrated main steps with a number of different kinematics and demonstrated developed industrial two-finger gripper and three-fingered artificial hand prototypes.

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REFERENCES

- [1] G. Raiola, C. A. Cardenas, T. S. Tadele, T. de Vries, and S. Stramigioli, "Development of a safety- and energy-aware impedance controller for collaborative robots," *IEEE Robotics and Automation Letters*, vol. 3, no. 2, pp. 1237–1244, 2018.
- [2] B. Katz, J. D. Carlo, and S. Kim, "Mini cheetah: A platform for pushing the limits of dynamic quadruped control," in *2019 International Conference on Robotics and Automation (ICRA)*, 2019, pp. 6295–6301.
- [3] J. Lee, J. Hwangbo, L. Wellhausen, V. Koltun, and M. Hutter, "Learning quadrupedal locomotion over challenging terrain," *Science robotics*, vol. 5, no. 47, 2020.
- [4] S. Fahmi, G. Fink, and C. Semini, "On state estimation for legged locomotion over soft terrain," *IEEE Sensors Letters (L-SENS)*, vol. 5, no. 1, pp. 1–4, Jan. 2021.
- [5] F. Rothling, R. Haschke, J. J. Steil, and H. Ritter, "Platform portable anthropomorphic grasping with the bielefeld 20-dof shadow and 9-dof tum hand," in *2007 IEEE/RSJ International Conference on Intelligent Robots and Systems*. IEEE, 2007, pp. 2951–2956.
- [6] G. A. Folkertsma and S. Stramigioli, "Energy in robotics," *Found. Trends Robot.*, vol. 6, no. 3, p. 140–210, Oct. 2017. [Online]. Available: <https://doi.org/10.1561/23000000038>

- [7] J. Ramos, B. Katz, M. Y. M. Chuah, and S. Kim, "Facilitating model-based control through software-hardware co-design," in *2018 IEEE International Conference on Robotics and Automation (ICRA)*, 2018, pp. 566–572.
- [8] G. A. Folkertsma, A. J. van der Schaft, and S. Stramigioli, "Morphological computation in a fast-running quadruped with elastic spine," *IFAC-PapersOnLine*, vol. 48, no. 13, pp. 170–175, 2015.
- [9] I. I. Borisov, I. A. Kulagin, A. E. Larkina, A. A. Egorov, S. A. Kolyubin, and S. Stramigioli, "Study on elastic elements allocation for energy-efficient robotic cheetah leg," in *2019 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 2019, pp. 1696–1701.
- [10] W. Townsend, "The barretthand grasper—programmably flexible part handling and assembly," *Industrial Robot: an international journal*, 2000.
- [11] E. N. Haulin, A. Lakis, and R. Vinet, "Optimal synthesis of a planar four-link mechanism used in a hand prosthesis," *Mechanism and Machine Theory*, vol. 36, no. 11–12, pp. 1203–1214, 2001.
- [12] L. B. Bridgwater, C. Ihrke, M. A. Diftler, M. E. Abdallah, N. A. Radford, J. Rogers, S. Yayathi, R. S. Askew, and D. M. Linn, "The robonaut 2 hand—designed to do work with tools," in *2012 IEEE International Conference on Robotics and Automation*. IEEE, 2012, pp. 3425–3430.
- [13] G. A. Kragten, M. Baril, C. Gosselin, and J. L. Herder, "Stable precision grasps by underactuated grippers," *IEEE Transactions on Robotics*, vol. 27, no. 6, pp. 1056–1066, 2011.
- [14] A. Bicchi, "Hands for dexterous manipulation and robust grasping: A difficult road toward simplicity," *IEEE Transactions on robotics and automation*, vol. 16, no. 6, pp. 652–662, 2000.
- [15] S. Slesongsom and S. Bureerat, "Optimal synthesis of four-bar linkage path generation through evolutionary computation with a novel constraint handling technique," *Computational Intelligence and Neuroscience*, vol. 2018, 2018.
- [16] M. Bächer, S. Coros, and B. Thomaszewski, "Linkedit: interactive linkage editing using symbolic kinematics," *ACM Transactions on Graphics (TOG)*, vol. 34, no. 4, pp. 1–8, 2015.
- [17] P. Tang, J. Zehnder, S. Coros, and B. Thomaszewski, "A harmonic balance approach for designing compliant mechanical systems with nonlinear periodic motions," *ACM Transactions on Graphics (TOG)*, vol. 39, no. 6, pp. 1–14, 2020.
- [18] A. A. Egorov, I. I. Borisov, S. A. Kolyubin, and S. Stramigioli, "Cascaded constrained optimization for cheetah-inspired galloping robot leg mechanism," *IFAC-PapersOnLine*, 2020.
- [19] B. Thomaszewski, S. Coros, D. Gauge, V. Megaro, E. Grinspun, and M. Gross, "Computational design of linkage-based characters," *ACM Transactions on Graphics (TOG)*, vol. 33, no. 4, pp. 1–9, 2014.
- [20] I. I. Borisov, O. I. Borisov, D. S. Monich, T. A. Dodashvili, and S. A. Kolyubin, "Novel optimization approach to development of digit mechanism for bio-inspired prosthetic hand," in *2018 7th IEEE International Conference on Biomedical Robotics and Biomechanics (Biorob)*. IEEE, 2018, pp. 726–731.
- [21] T. Laliberté, L. Birglen, and C. Gosselin, "Underactuation in robotic grasping hands," *Machine Intelligence & Robotic Control*, vol. 4, no. 3, pp. 1–11, 2002.
- [22] S. Montambault and C. m. M. Gosselin, "Analysis of underactuated mechanical grippers," *J. Mech. Des.*, vol. 123, no. 3, pp. 367–374, 2001.
- [23] M. Suarez-Escobar, J. A. Gallego-Sanchez, and E. Rendon-Velez, "Mechanisms for linkage-driven underactuated hand exoskeletons: conceptual design including anatomical and mechanical specifications," *International Journal on Interactive Design and Manufacturing (IJIDeM)*, vol. 11, no. 1, pp. 55–75, 2017.
- [24] I. I. Borisov, O. I. Borisov, V. S. Gromov, S. M. Vlasov, D. Dobriborsci, and S. A. Kolyubin, "Design of versatile gripper with robust control," *IFAC-PapersOnLine*, vol. 51, no. 22, pp. 56–61, 2018.
- [25] I. I. Borisov, S. A. Kolyubin, and A. A. Bobtsov, "Static force analysis of a finger mechanism for a versatile gripper," in *International Conference Cyber-Physical Systems and Control*. Springer, 2019, pp. 275–289.
- [26] I. I. Borisov, I. A. Kulagin, A. E. Larkina, A. A. Egorov, S. A. Kolyubin, and S. Stramigioli, "Study on elastic elements allocation for energy-efficient robotic cheetah leg," in *2019 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2019, pp. 1696–1701.