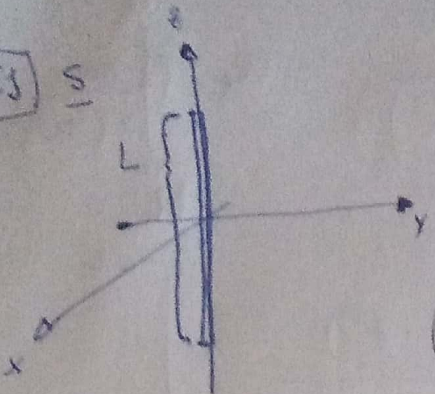
EJ 5



Cargado uniformemente con carga total q

¿Como es el campo Electrico?

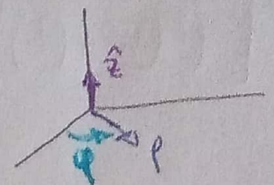
(1) El hilo tiene simetria cilindrica

⇒ ~~Cada Componente~~

$x = \rho \cos\varphi$

$y = \rho \sin\varphi$

$z = z.$



(2)

$\Delta q_i = \lambda . \Delta z$ → Porcion de carga del hilo. ⇒ $\bar{r}_i' = z\,\hat{e}_z$; $z \in \left[-\frac{L}{2}, \frac{L}{2}\right]$

$\bar{r} = (x, y, z)$.

Superposición de los campos de cada Δq_i → $\bar{E}(\bar{r}) = \sum_{i=1}^{N} k_e . \lambda \, \Delta z' \dfrac{\bar{r} - z'\hat{e}_z}{\|\bar{r} - z'\hat{e}_z\|^3}$

Pasando al continuo → tomo $\lim_{N \to \infty} \sum_{i=1}^{N}$ ⇒ $\bar{E}(\bar{r}) = \int_{-\frac{L}{2}}^{\frac{L}{2}} dz' \, k_e \lambda \dfrac{\bar{r} - z'\hat{e}_z}{\|\bar{r} - z'\hat{e}_z\|^3}$.

Como tenemos simetria cilindrica, es conveniente hacer el pase de cartesianas a cilindricas

$\hat{e}_\rho(\varphi) = \cos\varphi \, \hat{e}_x + \sin \hat{e}_y$

→ Notar que por la simetria cilindrica no hay componente φ del campo.

$$\Rightarrow \bar{E}(\bar{r}) = \int_{-\frac{L}{2}}^{\frac{L}{2}} k_e \lambda \, dz' \frac{\rho\,\hat{e}_\rho + z\,\hat{e}_z - z'\hat{e}_z}{\|\rho\hat{e}_\rho + z\hat{e}_z - z'\hat{e}_z\|^3}$$

$$\rightarrow E_\rho = k_e \lambda \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\rho}{\|\rho\hat{\rho} + z\hat{z} - z'\hat{z}\|^3} dz'$$

$$= k_e \lambda \rho \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{1}{(\rho^2 + (z - z')^2)^{\frac{3}{2}}} dz'$$

$$\int \frac{1}{(\rho^2 + (z - z')^2)^{\frac{3}{2}}} dz'$$

$z - z' = u$

$du = -dz'$

$$-\int_a^b \frac{du}{(\rho^2 + u^2)^{\frac{3}{2}}}$$

$\|$ Tabla

$-\int_a^b = \int_b^a$

$$\rightarrow \left[\frac{u}{\rho^2(\rho^2 + u^2)^{\frac{1}{2}}}\right]_b^a$$

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$$E_\rho = K_e \lambda \rho \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dz'}{\left(\rho^2 + (z-z')^2\right)^{\frac{3}{2}}} \underset{u=(z-z')}{=} K_e \lambda \rho \int_{-\tilde{u}}^{\tilde{u}} \frac{-du}{\left(\rho^2 + u^2\right)^{\frac{3}{2}}} = K_e \lambda \rho \int_{\tilde{u}}^{-\tilde{u}} \frac{du}{\left(\rho^2 + u^2\right)^{\frac{3}{2}}}$$

$$\underset{\text{tabla}}{=} K_e \lambda \rho \left[\frac{u}{\rho^2 (u^2 + \rho^2)^{\frac{1}{2}}} \right]_{\tilde{u}}^{-\tilde{u}} = K_e \lambda \rho \left[\frac{(z-z')}{\rho^2 \left((z-z')^2 + \rho^2\right)^{\frac{1}{2}}} \right]_{\frac{L}{2}}^{-\frac{L}{2}}$$

$$= K_e \lambda \rho \left[\frac{(z+\frac{L}{2})}{\rho^2 \left((z+\frac{L}{2})^2 + \rho^2\right)^{\frac{1}{2}}} - \frac{(z-\frac{L}{2})}{\rho^2 \left((z-\frac{L}{2})^2 + \rho^2\right)^{\frac{1}{2}}} \right] =$$

$$= \boxed{\frac{K_e \lambda}{\rho} \left[\frac{2z + L}{\left((2z+L)^2 + 4\rho^2\right)^{\frac{1}{2}}} - \frac{(2z - L)}{\left((2z-L)^2 + 4\rho\right)^{\frac{1}{2}}} \right] = E_\rho}$$

Componente ρ del campo.

saco factor comun ρ^2 y multiplico por 2 arriba y abajo en las fracciones.

Ahora calculemos E_z

$$E_z = K_e \lambda \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{z-z' \, dz'}{\left(\rho^2 + (z-z')^2\right)^{\frac{3}{2}}} \underset{u=z-z'}{=}$$

$$= K_e \lambda \int_{-\tilde{u}}^{\tilde{u}} \frac{-u\,du}{\left(\rho^2 + u^2\right)^{\frac{3}{2}}} = ⊛$$

$$\int_{-\tilde{u}}^{u} -du = \int_{\tilde{u}}^{-\tilde{u}} du.$$

⊛ $\underset{\text{tabla}}{=}$

Tabla.

$$\int \frac{u}{(u^2+c^2)^{\frac{3}{2}}} du = -\frac{1}{\sqrt{u^2+c^2}} + c.$$

$$K_e \lambda \left[\frac{2}{\left[(L-2z)^2 + 4\rho^2\right]^{\frac{1}{2}}} - \frac{2}{\left[(L+2z)^2 + 4\rho^2\right]^{\frac{1}{2}}} \right]$$

$$\Rightarrow \boxed{E_z = 2 K_e \lambda \left[\frac{1}{\left[(L-2z^2) + 4\rho^2\right]^{\frac{1}{2}}} - \frac{1}{\left[(L+2z)^2 + 4\rho^2\right]^{\frac{1}{2}}} \right]}$$

Notar que si el hilo es infinito $L \to \infty$.

$\Rightarrow E_z \to 0.$

y nos queda $\bar{E} = E_\rho (L=\infty)$

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