

# Computational Physics Project 1: Pendulum

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February 12, 2017

## The Simple Pendulum

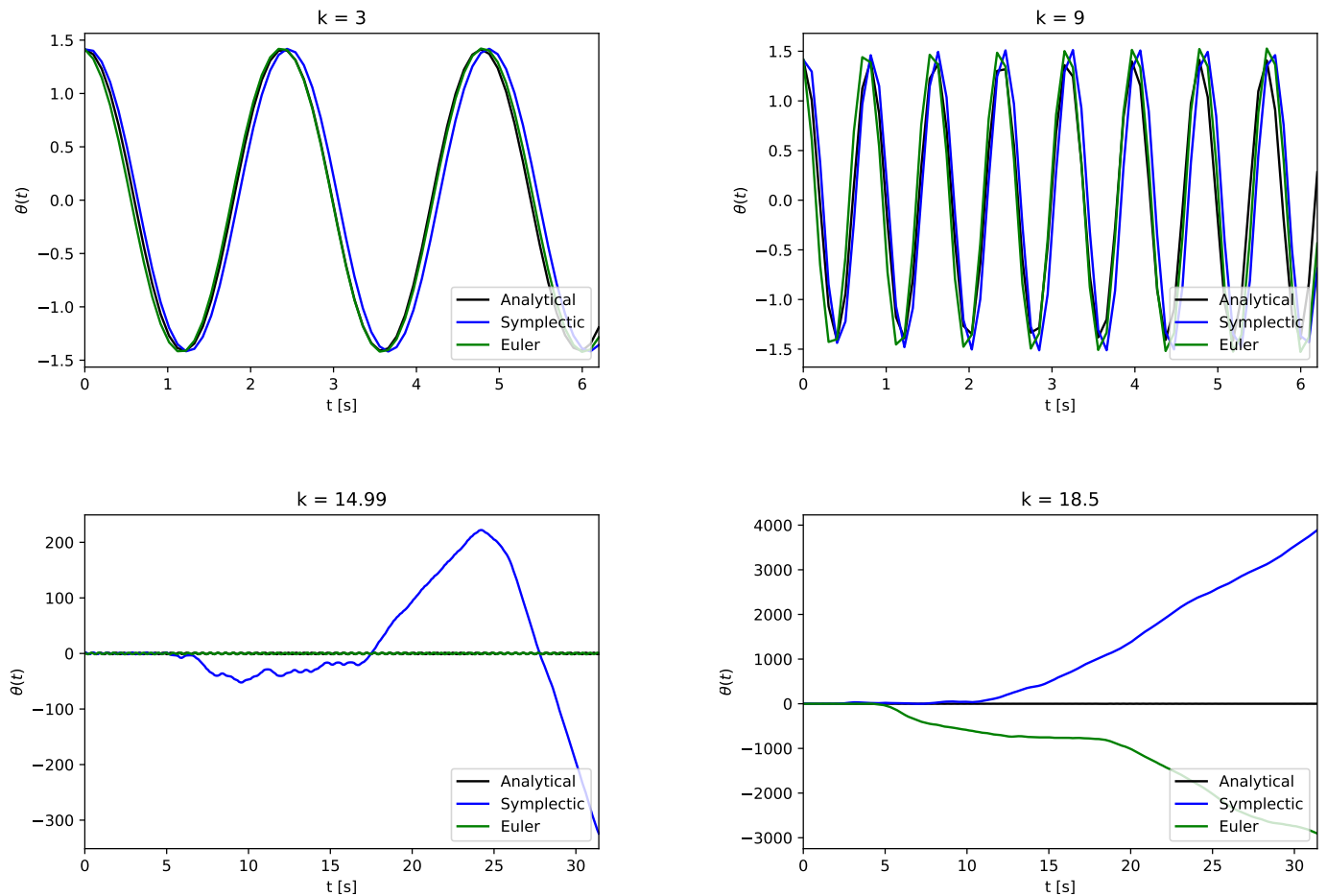


Figure 1: Trajectory for various  $\dot{\theta}$

# Route to Chaos

## 1. Phase space of nonlinear pendulum

As  $\theta_0$  approaches  $\pi$ , the trajectories go from an ellipse to a more "lemon" shape, as seen in figure 2.

For  $\theta_0 = 0$ , varying  $\dot{\theta}_0 \in [0, \pi]$ , the phase shows two different behaviors. For  $\dot{\theta}_0$  between 0 and approximately  $0.6\pi$  the phase space appears similar to the previous one, as seen in figure 3a. If  $\dot{\theta}_0$  is greater than that, the motion is no longer periodic, and  $\theta$  increases indefinitely, as seen in figure 3b.

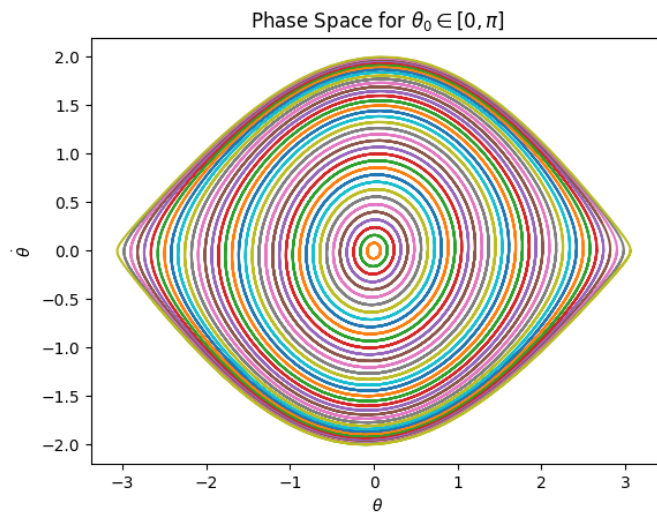


Figure 2: Plots of trajectory  $(\theta, \dot{\theta})$ , for many values of  $\theta_0 \in [0, \pi]$

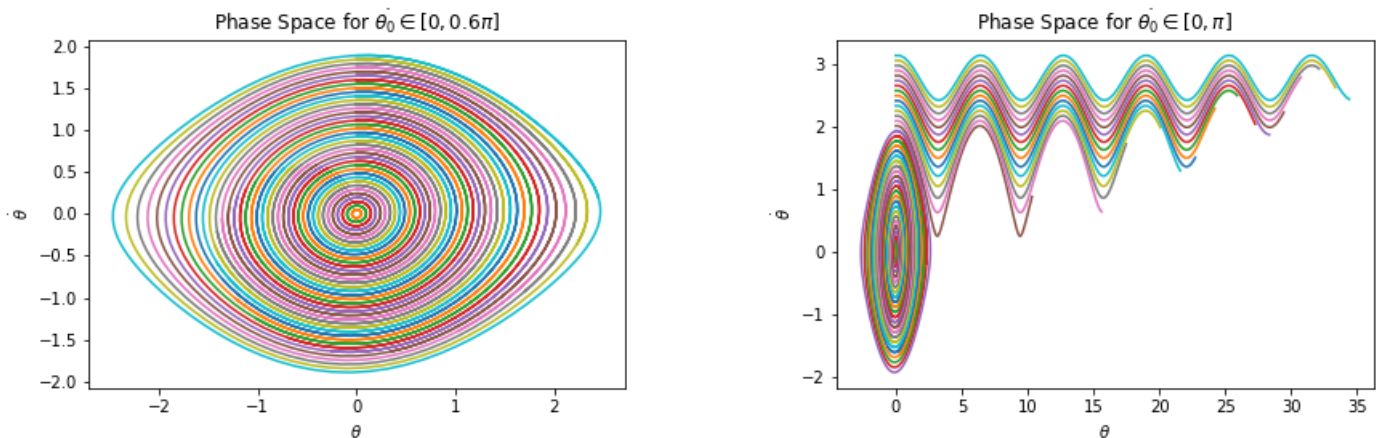


Figure 3: Trajectory for various  $\dot{\theta}$

## 2. Phase space of linear pendulum

For the linear pendulum, the phase space trajectory remains elliptical for all values of  $\theta_0$ , as seen in figure 4

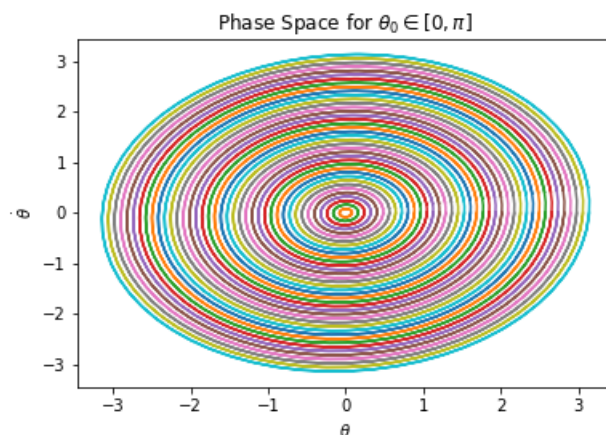


Figure 4: Plots of linearized trajectory  $(\theta, \dot{\theta})$ , for many values of  $\theta_0 \in [0, \pi]$

## 3. Pendulum with driving force, $\gamma k^2 \cos(\omega t)$

If a periodic driving force with  $\omega = k$  is added, the frequency stays the same, but the amplitude varies periodically, as seen in figure 5.

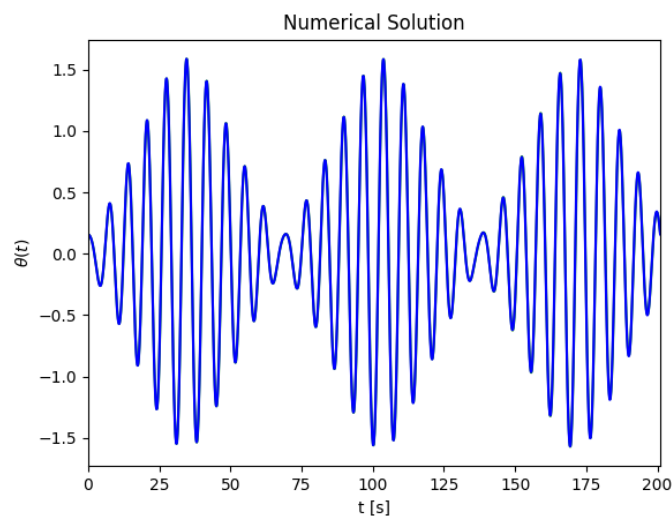


Figure 5: Solution for driven undamped pendulum

## 4. Exploration of driven system

Figure 6 shows how the trajectories vary with  $\gamma$ .

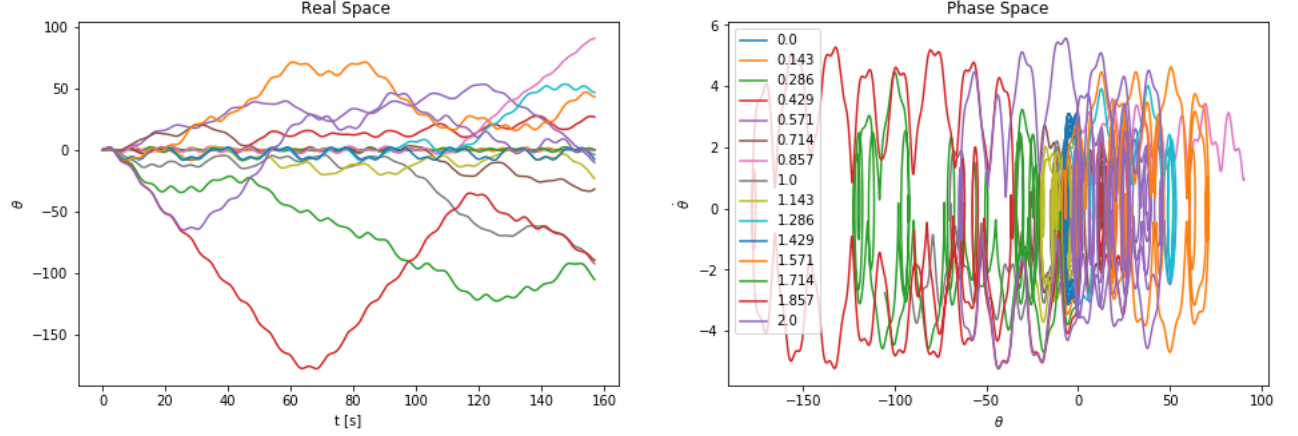


Figure 6: Trajectories for undamped driven pendulum for  $\gamma \in [0, 2]$

### 5. Identifying $(\theta_0, \gamma)$ for which the motion diverges

Figure 7 shows the phase plot for  $(\theta_0, \gamma)$  for  $\theta_0 \in [0, \pi]$ , and  $\gamma \in [0, 6]$ , after a time interval of  $8\pi$  seconds. The white regions indicate values for which the motion remained periodic. The blue regions indicate values for which the motion diverged. The darker the color, the greater the value of  $\theta$  at the end of the time interval.

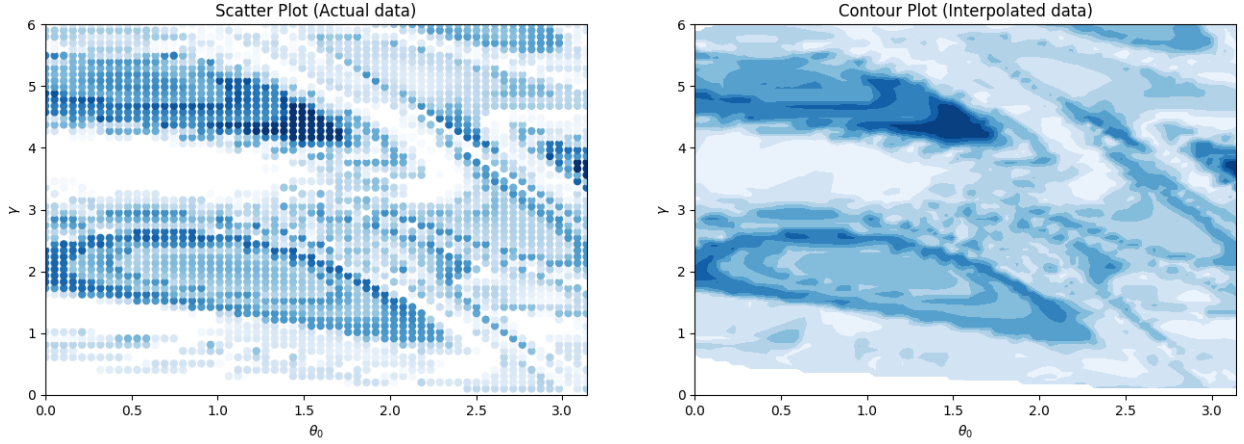


Figure 7: Phase plot for  $(\theta_0, \gamma)$ . On the left is the actual data that was calculated, and on the right is an interpolated contour plot.

### 6. Driven pendulum with damping $\ddot{\theta} + 2\beta\dot{\theta} + k^2\sin\theta = \gamma k^2\cos(\omega t)$

As  $\gamma$  increases, the pendulum transitions to chaos, as seen in figure 8. This transition is known as *period doubling*, which becomes apparent from observing the plots.

### 7. Fourier analysis

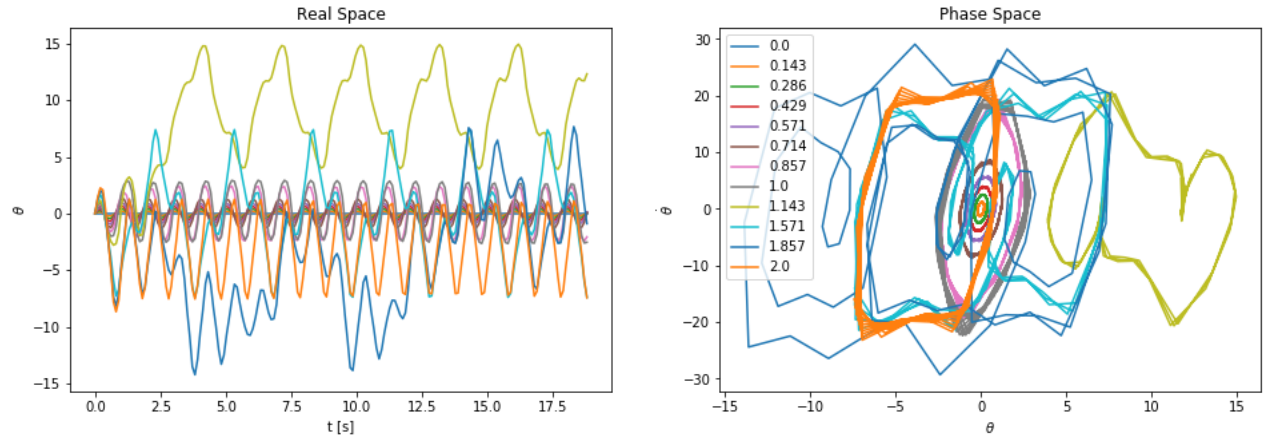


Figure 8: Plots of driven damped pendulum for various values of  $\gamma \in [0, 2]$

## The Double Pendulum