Computational Physics Project 1: Pendulum

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The Simple Pendulum

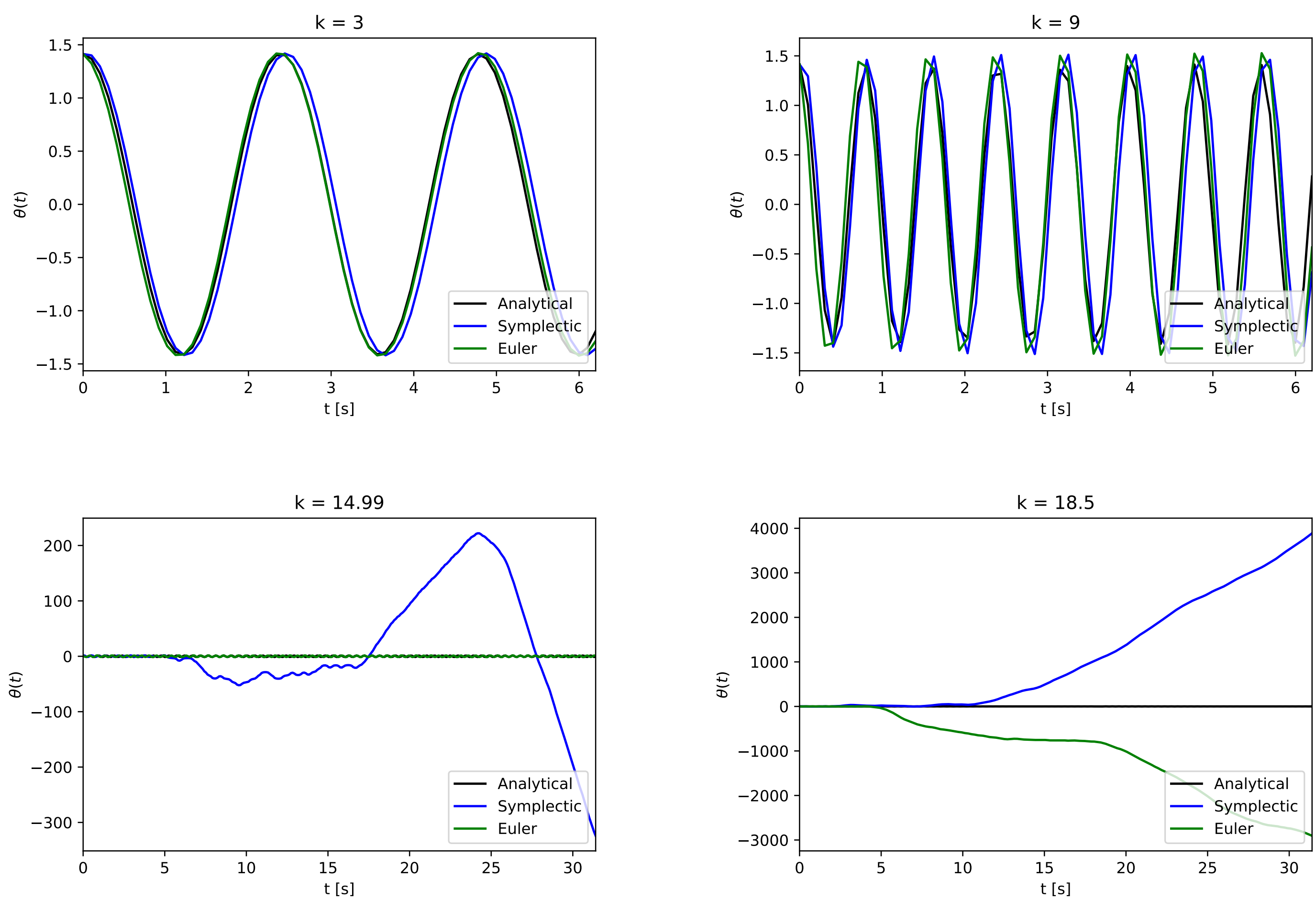

Figure 1: Trajectory for various $\dot{\theta}$

Route to Chaos

1. Phase space of nonlinear pendulum

As θ_0 approaches π, the trajectories go from an ellipse to a more "lemon" shape, as seen in figure 2.

For $\theta_0 = 0$, varying $\dot{\theta}_0 \in [0, \pi]$, the phase shows two different behaviors. For $\dot{\theta}_0$ between 0 and approximately 0.6π the phase space appears similar to the previous one, as seen in figure 3a. If $\dot{\theta}_0$ is greater than that, the motion is no longer periodic, and θ increases indefinitely, as seen in figure 3b.

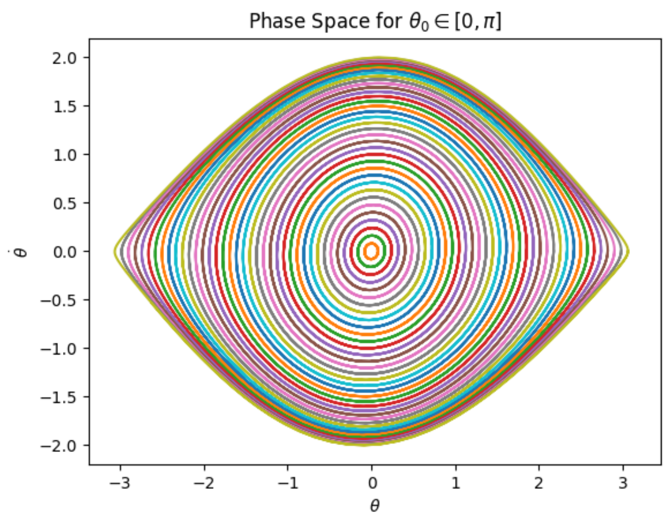

Figure 2: Plots of trajectory $(\theta, \dot{\theta})$, for many values of $\theta_0 \in [0, \pi]$

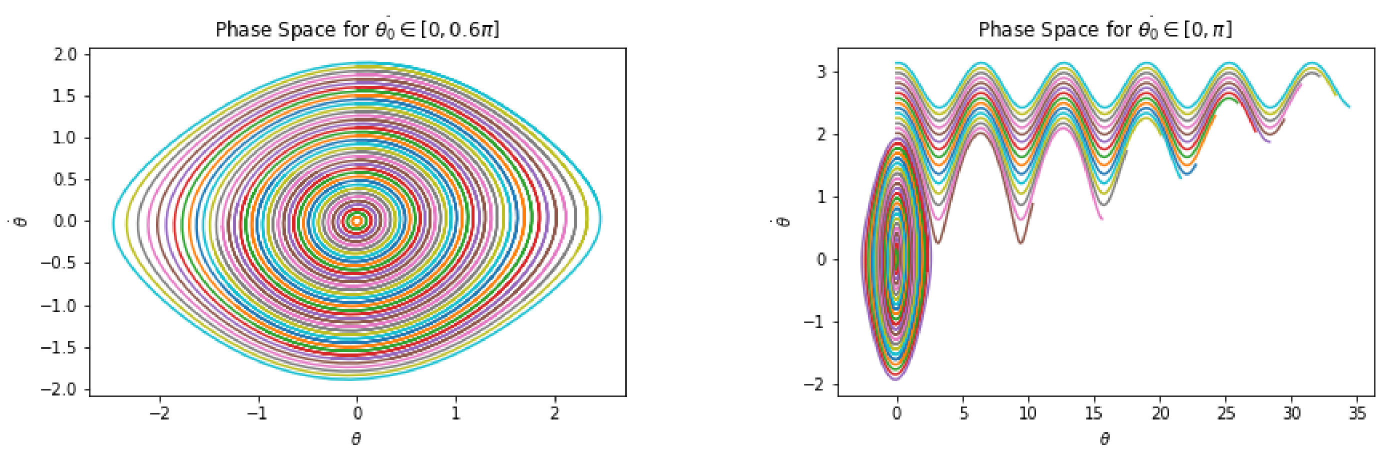

Figure 3: Trajectory for various $\dot{\theta}$

2. Phase space of linear pendulum

For the linear pendulum, the phase space trajectory remains elliptical for all values of θ_0, as seen in figure 4

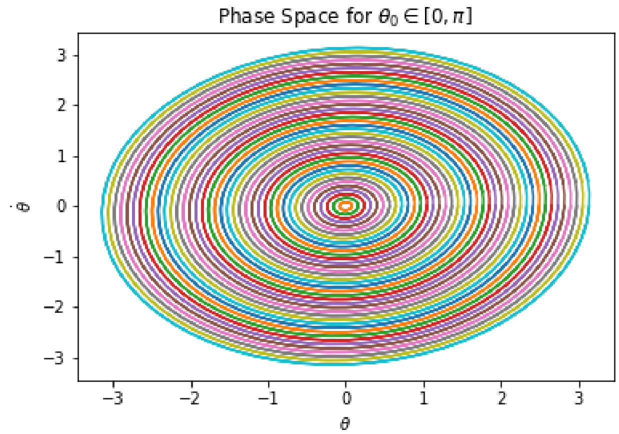

Figure 4: Plots of linearized trajectory $(\theta, \dot{\theta})$, for many values of $\theta_0 \in [0, \pi]$

3. Pendulum with driving force, $\gamma k^2 cos(\omega t)$

If a periodic driving force with $\omega = k$ is added, the frequency stays the same, but the ampltude varies periodically, as seen in figure 5.

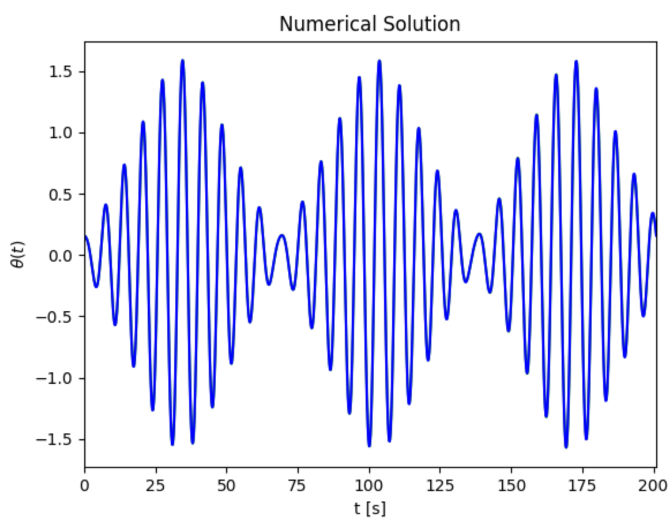

Figure 5: Solution for driven undamped pendulum

4. Exploration of driven system

Figure 6 shows how the trajectories vary with γ.

5. Identifying (θ_0, γ) for which the motion diverges

Figure 7 shows the phase plot for (θ_0, γ) for $\theta_0 \in [0, \pi]$, and $\gamma \in [0, 6]$, after a time interval of 8π seconds. The white regions indicate values for which the motion remained periodic. The blue regions indicate values for which the motion diverged. The darker the color, the greater the value of θ at the end of the time interval.

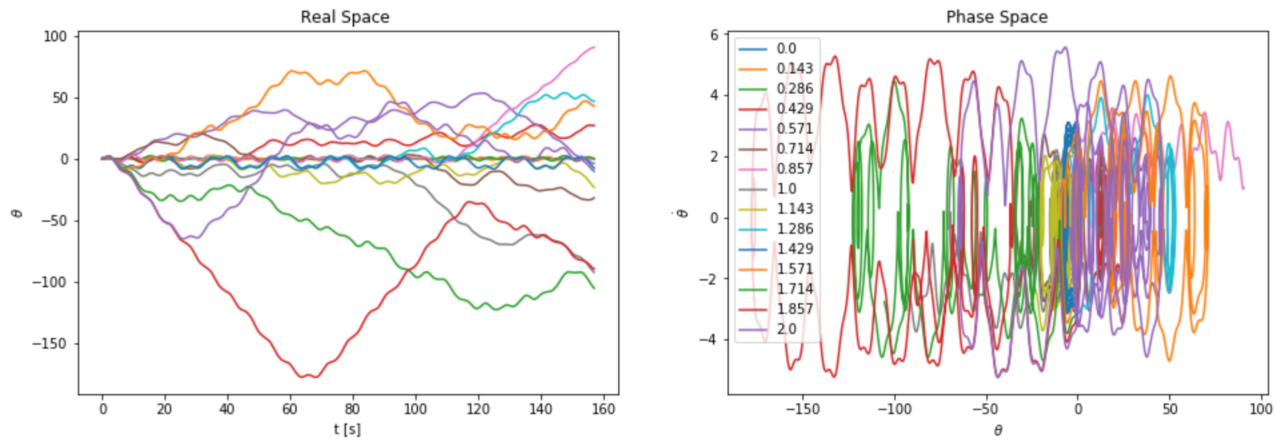

Figure 6: Trajectories for undamped driven pendulum for $\gamma \in [0, 2]$

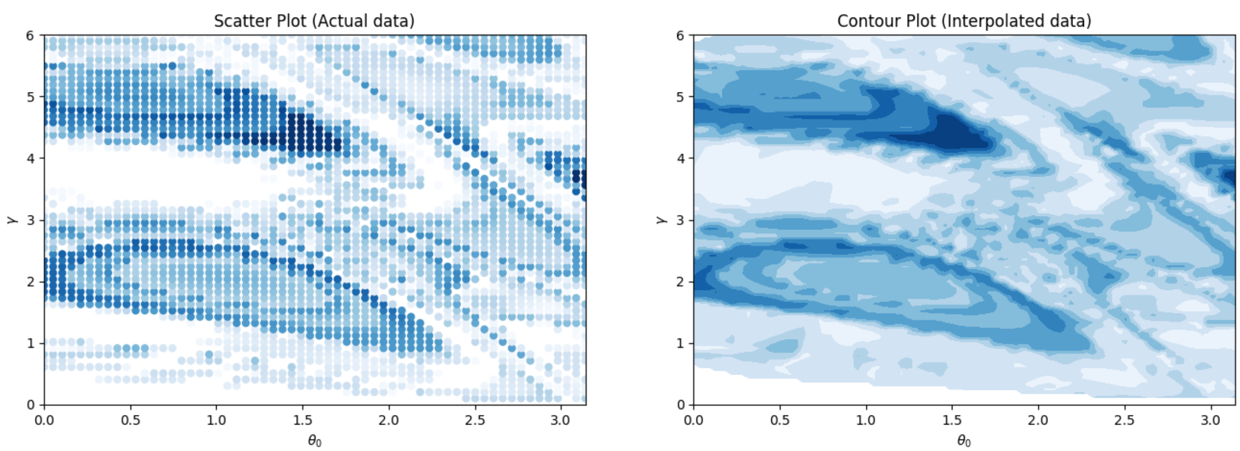

Figure 7: Phase plot for (θ_0, γ). On the left is the actual data that was calculated, and on the right is an interpolated contour plot.

6. Driven pendulum with damping $\ddot{\theta} + 2\beta\dot{\theta} + k^2 sin\theta = \gamma k^2 cos(\omega t)$

As γ increases, the pendulum transitions to chaos, as seen in figure 8. This transition is known as *period doubling*, which is becomes apparent from observing the plots. As γ increases, some trajectories diverge and become nonperiodic, while some trajectories remain stable and periodic, with a period twice as long as before. Figure 8 only shows trajectories that remained periodic.

7. Fourier analysis

The Double Pendulum

1. Visualization

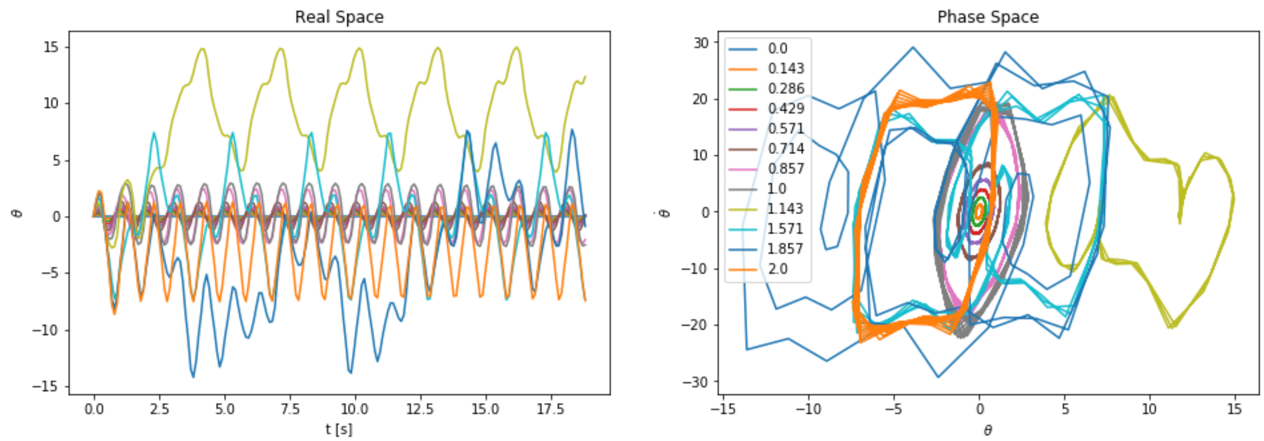

Figure 8: Plots of driven damped pendulum for trajectory various values of $\gamma \in [0, 2]$

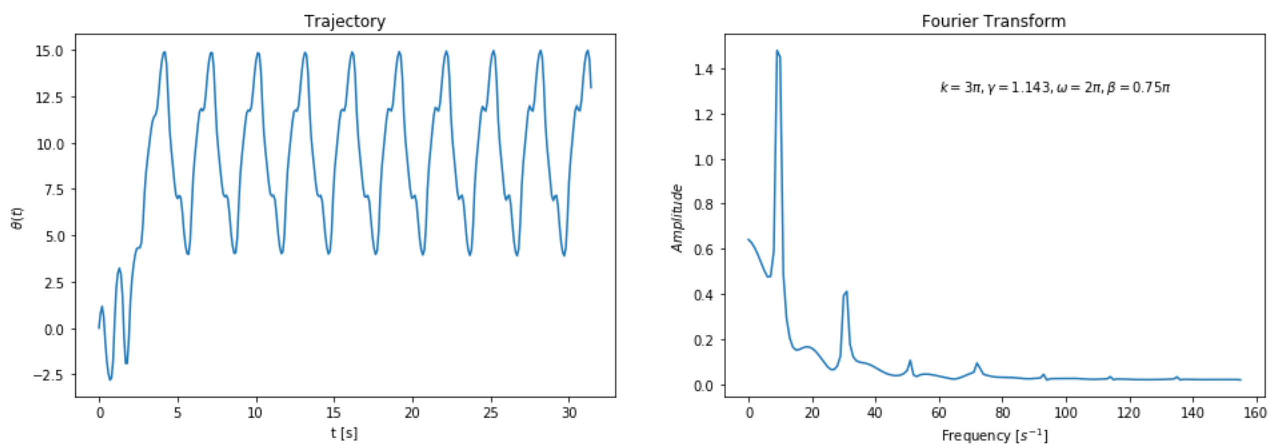

Figure 9: Fourier transform of damped driven pendulum with the parameters shown

Figure 10:

Figure 11:

2. Lyapunov Exponent

3. Transition to Chaos

Figure 12:

4. Time for First Flip

Figure 13: