The tables for 4*4 puzzles:

Statistics for #state:

	min. #states	max. #states	mean #states	median #states
Approach 1	1813759666	2632010551	2201380423	2266947961
Approach 2	180	996	431.2	332
Approach 3	76	372	168	149
Approach 4	18	104	50.8	39
Approach 5	10	20	15.4	16

Statistics for total time(microseconds):

	min. time	max. time	mean time	median time
Approach 1	791980877.4	1261845902	1001141061	1008996188
Approach 2	160.916	3649.849	953.407	329.91
Approach 3	183.745	8866.7	2267.238	561.917
Approach 4	150.526	2464.854	884.5964	509.382
Approach 5	420.542	1501.952	807.1028	790.536

Statistics for #states/microseconds (ms):

	min. states/ms	max. states/ms	mean states/ms	median states/ms
Approach 1	2.085841502	2.290155885	2.207206987	2.246735903
Approach 2	0.2731619856	1.199383529	0.8923031607	1.009366191
Approach 3	0.04206751102	0.419059022	0.2182994223	0.1405901583
Approach 4	0.04259887198	0.126224041	0.08143200346	0.07924982119
Approach 5	0.01131860406	0.04042402424	0.02392426158	0.02169150344

The tables for 5*5 puzzles:

Statistics for #state:

	min. #states	max. #states	mean #states	median #states
Approach 2	2055	37545	12281	3905
Approach 3	735	10682	3717.2	1234
Approach 4	170	2749	1030.2	607
Approach 5	50	602	286.4	237

Statistics for total time(microseconds):

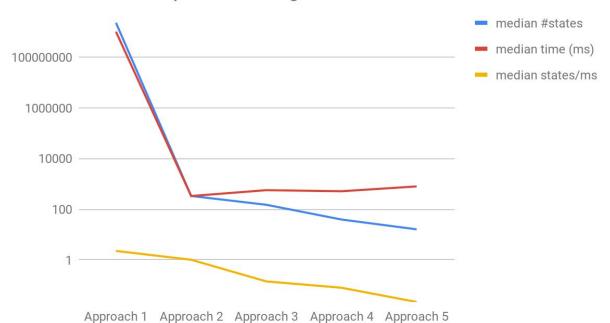
	min. time	max. time	mean time	median time
Approach 2	2364.302	60666.268	18400.4544	7397.985
Approach 3	3605.919	63708.885	19200.7558	7199.651
Approach 4	1445.371	15391.287	8263.7344	7440.061
Approach 5	956.831	15665.332	7410.3994	3079.899

Statistics for #states/microseconds (ms):

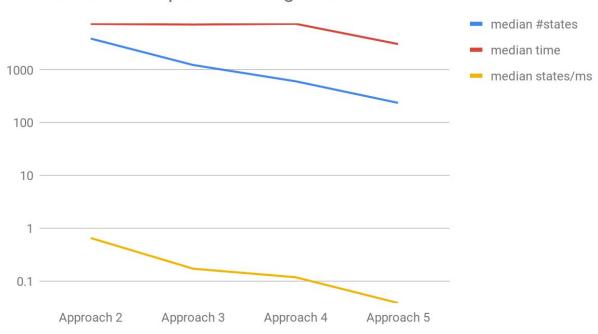
	min. states/ms	max. states/ms	mean states/ms	median states/ms
Approach 2	0.5279816058	0.8696117221	0.7079702865	0.6537626668
Approach 3	0.1235823116	0.3391991168	0.2012221987	0.1715360925
Approach 4	0.04784907005	0.2268497182	0.1198358523	0.1183087249
Approach 5	0.03099886349	0.0772752613	0.04669229809	0.03849264095

The charts in log-scale:

The chart for 4*4 puzzles in log-scale



The chart for 5*5 puzzles in log-scale



In this homework, we tried different search algorithms to find solutions for Kenken puzzles. We implemented 5 different approaches where Approach #1 searches the possible combinations of the numbers by using very little knowledge about Kenken. On the contrary, other approaches use more and more knowledge in each step about Kenken from Approach #2 to Approach #5.

As we can see from the tables and the charts, using more knowledge about Kenken decreases the number of combinations (or states) which we need to evaluate. For example, while we need to evaluate millions of states in Approach #1, even incorporating a simple rule to our search (there can't be duplicate numbers in the same row and in the same column) made just searching hundreds of states enough. The side effect of using more knowledge in an approach is that it increased our cost per state since we need to compute more to evaluate each state with our increased knowledge. Therefore, we can also see from the results that the number of states per time decreased when we use more knowledge. In other words, we are able to investigate less number of states with Approach #5 compare to Approach #1.

In summary, we need to spend more resources per state if we use more knowledge about the environment, however, the new set of knowledge can help us to search less if we are not going to find a solution in a direction. In the case of this homework, this trade-off seems not very important for 5*5 puzzles and Approach 5 solves these puzzles more quickly. However, there can be some other environments like 4*4 puzzles or completely different games and approaches where we may prefer to use less knowledge since it increases our overall time. Of course, this decision only applies if we want to optimize total execution time. However, there could be some other decision criteria like memory usage, decision time for each new state or even ease of implementation.