Game Theory Paper 220524

1a

A	L	R
U	$\underline{8}$ $\underline{5}$ 5	1 0 $\underline{3}$
M	4 $\underline{2}$ 1	$\underline{3}$ 1 $\underline{5}$
D	1 $\underline{4}$ $\underline{4}$	2 0 0

B	L	R
U	0 $\underline{3}$ $\underline{6}$	2 2 2
M	1 $\underline{1}$ 0	$\underline{4}$ 0 4
D	$\underline{3}$ $\underline{2}$ 3	3 1 $\underline{1}$

Best responses underlined.

By inspection,
$L \succ_2 R$, then $\frac{1}{2}U + \frac{1}{2}D \succ_1 M$, then no strictly dominated strategies remain.

The above game reduces to

L	A	B
U	$\underline{8}$ $\underline{5}$ 5	0 $\underline{3}$ $\underline{6}$
D	1 $\underline{4}$ $\underline{4}$	$\underline{3}$ $\underline{2}$ 3

By inspection, there are no pure NE where players play pure mutual best responses. By inspection, ~~both~~ P1 has a ~~strict best respo~~ unique pure best response against L and any of P3's pure strategies, so if P2 plays L ~~a~~ (which he always does at NE because it is strictly dominant) ~~then~~ and P3 plays a pure ~~action~~ strategy, then P1 ~~&~~ plays a pure strategy. Likewise if P2 plays L and P1 plays a pure ~~act~~ strategy, then P3 plays a pure strategy. So there are no candidate NE such that only one of P1 and P3 mix.

Suppose P1 mixes at NE σ^*. Then P1 has no profitable deviation and is indifferent between U and D (P1 plays M with zero probability because it is strictly dominated).

$\pi_1(U, \sigma_3^*) = \pi_1(D, \sigma_3^*) \Leftrightarrow$

$8r = 1r + 3(1-r) \Leftrightarrow$

$10r = 3 \Leftrightarrow$

$r = 3/10$

where r is the probability σ_3^* assigns to A.

Given that P3 mixes at NE, P3 is indifferent between A and B.

$\pi_3(A, \sigma_1^*) = \pi_3(B, \sigma_1^*) \Leftrightarrow$

$5p + 4(1-p) = 6p + 3(1-p) \Leftrightarrow$

$1 - p = p \Leftrightarrow$

$p = 1/2$

where p is the probability σ_1^* assigns to U.

Then, the unique NE is the mixed NE
$(\frac{1}{2}U + \frac{1}{2}D, L, \frac{3}{10}A + \frac{7}{10}B)$

b

A	L	R
U	$\underline{8}$ $\underline{5}$ 5	1 0 $\underline{3}$
M	4 $\underline{2}$ 1	$\underline{3}$ 1 $\underline{5}$
D	1 $\underline{4}$ 2	2 0 0

B	L	R
U	0 $\underline{3}$ $\underline{6}$	2 2 2
M	1 $\underline{1}$ 0	$\underline{4}$ 0 4
D	$\underline{3}$ $\underline{2}$ $\underline{3}$	3 1 $\underline{1}$

By inspection,
$L \succ_2 R$, then $\frac{1}{2}U + \frac{1}{2}D \succ_1 M$, then $B \succ_3 A$, then $U \succ_1 D$; then no strictly dominated strategies remain.

~~Only one strategy~~ For each player, only one strategy survives iterated strict dominance. Then, at NE, each player plays his strategy that survives iterated strict dominance. The NE is (U, L, B). At every other strategy profile, ~~as~~ given that it does not survive iterated strict dominance, at least one player has a strictly profitable deviation, so it is not a NE.

c

U	A	B
L	$\underline{8}$ $\underline{5}$ 5	0 $\underline{3}$ $\underline{6}$
R	1 0 $\underline{3}$	2 2 2

M	A	B
L	4 $\underline{2}$ $\underline{1}$	1 $\underline{1}$ 0
R	$\underline{3}$ 1 $\underline{5}$	$\underline{4}$ 0 4

D	A	B
L	1 $\underline{4}$ 2	$\underline{3}$ $\underline{2}$ $\underline{3}$
R	2 0 0	3 1 $\underline{1}$

Best responses underlined.

By inspection, in the U subgame, the unique NE is the pure NE (L, B). ~~In the M subgame~~ This yields payoff of 0 to P1. In the M subgame, the unique NE is (L, A), which yields 4. In the D subgame the unique NE is (L, B) which yields 3. In the first stage, by backward induction, P1 plays M. P1 is ~~as~~ better off than in the simultaneous game. Formally, this SPE is the strategy profile σ, where $\sigma_1 = M$, $\sigma_2 = LLL$, $\sigma_3 = BAB$, where the three letter codes denote action in the event that P1 plays U, M, D respectively.

The subgame NE are unique because L is strictly dominant for P2 in each subgame, P3 has a unique best response to L, and ~~the~~ P1 has a unique best response in the reduced game.

d

L	A	B
U	$\underline{8}$ $\underline{5}$ 5	0 $\underline{3}$ $\underline{6}$
M	4 $\underline{2}$ $\underline{1}$	1 $\underline{1}$ 0
D	1 $\underline{4}$ 2	$\underline{3}$ $\underline{2}$ $\underline{3}$

R	A	B
U	1 0 $\underline{3}$	2 2 2
M	$\underline{3}$ 1 $\underline{5}$	$\underline{4}$ 0 4
D	2 0 0	3 1 $\underline{1}$

Best responses underlined.

By inspection, in the L subgame, the NE is ~~(D, L, B) and~~ (D, B) and it is unique. In the R subgame, the NE is (M, A) and it is unique. The former yields 2 to P2 the latter yields 1. By backward induction, P2 plays L at SPE.

Formally, this SPE is the strategy profile σ where $\sigma_1 = DM$, $\sigma_2 = L$, $\sigma_3 = BA$ where the first letter in each two letter code denotes action in the event of L and the second denotes action in the event of R.

P2 is no better off than in the simultaneous game.

By inspection of the payoff table in ~~8~~ (b), the A subgame NE is (U, L) and the B subgame NE is (D, L). These are unique because L is strictly dominant for P2 and P1 has a unique best response. The former yields 5 for P3, the latter yields 3. By backward induction, P3 plays A. Formally, this is the strategy profile σ where $\sigma_1 = UD$, $\sigma_2 = LL$, $\sigma_3 = A$.

P3 (and every player) benefits from P3 moving first. This achieves an outcome that Pareto dominates that of the simultaneous game. This outcome is not achievable there because ~~A~~ P3 cannot credibly commit to playing A, if P1 and P2 play their part of this ~~state~~ outcome, it is rational for P3 to deviate.

2a

A	L_1		R_1	
T	0	1	2	0
B	3	0	0	1

B	L_2		R_2	
T	2	0	1	1
B	0	1	2	0

P1's pure strategies are $A_1 = \{T, B\}$
P2's pure strategies are A_2 ~~$= \{L_1L_2, L_1R_2, R_1L_2,$~~
$= \{L_1L_2, L_1R_2, R_1L_2, R_1R_2\}$

Each player's strategies are the complete type-contingent plans of action. P1 has one type, so each strategy is some action. P2 has two types, so each strategy specifies two actions, one in the event that P2 is type A, the other in the event that P2 is type B.

b

	L_1L_2	L_1R_2	R_1L_2	R_1R_2
T	$1 \quad \underline{1} \quad 0$	$\frac{1}{2} \quad \underline{1} \quad \underline{1}$	$\underline{2} \quad 0 \quad 0$	$\underline{\frac{3}{2}} \quad 0 \quad \underline{1}$
B	$\underline{\frac{3}{2}} \quad 0 \quad \underline{1}$	$\underline{\frac{5}{2}} \quad 0 \quad 0$	$0 \quad \underline{1} \quad \underline{1}$	$1 \quad \underline{1} \quad 0$

Payoffs in interim expectation, with payoffs of P1 given first, then payoffs of type A P2 then payoffs of type B P2.

Best responses underlined.

If P1 plays T, P2's best response in interim expectation is L_1R_2 (this is also the best response in ex ante expectation). If P1 plays B, P2's best response is R_1L_2.

Suppose for reductio that there exists a BNE such that P1 plays pure T. Then by definition, P2 plays best response L_1R_2. Then P1 plays best response B. By reductio there is no such BNE.

Suppose for reductio that there exists a BNE such that P1 plays pure B. Then by definition, P2 plays best response R_1L_2, then P1 plays best response T. By reductio, there is no such BNE.

Then, there is no BNE such that P1 plays a pure strategy.

c Let σ_2 denote P2's strategy.
$\sigma_2 = (xL_1 + (1-x)R_1,\ yL_2 + (1-y)R_2)$
$\pi_1(T, \sigma_2) = \pi_1(B, \sigma_2) \iff$
$\frac{1}{2}[0x + 2(1-x)] + \frac{1}{2}[2y + 1(1-y)]$ ~~$= \frac{1}{2}[3x + 0(1-x)]$~~
$= \frac{1}{2}[3x + 0(1-x)] + \frac{1}{2}[0y + 2(1-y)] \iff$
$[2-2x] + [1+y] = [3x] + [2-2y] \iff$
$3y = -1 + 5x \iff$
$y = \frac{5}{3}x - \frac{1}{3}$

d From (b), at any BNE, $p \neq 0, 1$.

Let σ_1 denote P1's strategy.
$\sigma_1 = (pT + (1-p)B)$

$\pi_2(L_1L_2, \sigma_1) = \frac{1}{2}p + \frac{1}{2}(1-p) = \frac{1}{2}$
$\pi_2(L_1R_2, \sigma_1) = \frac{1}{2}p + \frac{1}{2}p = p$
$\pi_2(R_1L_2, \sigma_1) = \frac{1}{2}(1-p) + \frac{1}{2}(1-p) = 1-p$
$\pi_2(R_1R_2, \sigma_1) = \frac{1}{2}(1-p) + \frac{1}{2}p = \frac{1}{2}$
in ex ante expectation
~~suppose $p > \frac{1}{2}$, then L_1R_2~~

Suppose that at BNE $p > \frac{1}{2}$, then L_1R_2 is a strict best response for P2, ~~then~~ so P2 plays L_1R_2, then P1 plays best response B, so $p = 0$. By reductio, at BNE, $p \not> \frac{1}{2}$.

Suppose that at BNE $p < \frac{1}{2}$, then R_1L_2 is a strict best response for P2, so P2 plays R_1L_2. Then P1 plays best response T, so $p = 1$. By reductio, at BNE, $p \not< \frac{1}{2}$.

There are no BNE such that $p > \frac{1}{2}$ nor such that $p < \frac{1}{2}$.

e From (b), (d), at BNE $p \neq 0, 1$, $p \not> \frac{1}{2}$, $p \not< \frac{1}{2}$, so ~~at~~ $p = \frac{1}{2}$.

Then, expected payoff from every pure strategy for P2 is equal, so every (potentially mixed) strategy is a best response for P2.

$p = \frac{1}{2}$ is a best response for P1 iff, from (c), $y = \frac{5}{3}x - \frac{1}{3}$. Given that $x, y \in [0, 1]$, this is iff $x \in [\frac{1}{5}, \frac{4}{5}]$, $y = \frac{5}{3}x - \frac{1}{3}$

P2 has equal payoff at every ~~at~~ BNE, which is equal to the payoff from any pure action, $p = \frac{1}{2}$. P2 has no preference over the ~~at~~ BNE.

Given that P1 mixes at BNE, P1's payoff is equal to the common payoff from either pure action.

$\pi_1(\sigma_1, \sigma_2)$ ~~$= \pi_1($~~
$= \pi_1(T, \sigma_2)$
$= \frac{1}{2}(2-2x) + \frac{1}{2}(1+y)$
$= \frac{1}{2}(2-2x) + \frac{1}{2}(\frac{5}{3}x + \frac{2}{3})$
$= \frac{1}{2}(\frac{8}{3} - \frac{1}{3}x)$

This is maximised at $x = \frac{1}{5}$. P1 prefers the BNE corresponding to lower x.

Date

3a Each player i maximises payoff by minimising some weighted average of squared differences. The two differences are a differene from a ~~true~~ "type" and a difference from the other player's choice.

Informally, each player has incentive to ~~choose~~ choose action close to his type and to choose action close to the other player's action.

This is ~~an~~ ~~version~~ a more elaborate Battle of the Sexes game.

~~two firms selling each serve an exclusive exclusively s~~

Each^i of two firms exclusively serves a market located at t_i chooses the location of ~~their~~ its plant x_i. The closer the plant to the market the less transport cost is incurred. The closer the plants to each other, the greater the external economies of scale. So each firm has incentive to locate its plant near its market and near the other's plant.

b $u_i = -a_i(x_i - t_i)^2 - (1-a_i)(x_i - x_j)^2$

$\max_{x_i} u_i(x_i, x_j)$

FOC: $-a_i(2)(x_i - t_i) - (1-a_i)(2)(x_i - x_j) = 0 \Leftrightarrow$

$-2a_i x_i + 2a_i t_i - 2(1-a_i)x_i + 2(1-a_i)x_j = 0 \Leftrightarrow$

$2a_i t_i + 2(1-a_i)x_j = 2x_i \Leftrightarrow$

$x_i = a_i t_i + (1-a_i)x_j$

SOC: $-2a_i - 2(1-a_i) = -2 < 0$

$a_i t_i + (1-a_i)x_j = \arg\max_{x_i} u_i(x_i, x_j)$

Each player i chooses x_i intermediate between x_j and t_i. ~~where~~ x_i is some weighted avenge of x_j and t_i with weights $(1-a_i)$ and a_i respectively.

The greater a_i, the closer the optimal x_i to t_i than to ~~x~~ x_j.

An increase in a_i can be interpreted as an increase in player i's incentive to ~~to~~ choose x_i close to ~~x~~ t_i relative to his incentive to choose x_i close to ~~t_j~~ x_j.

$BR_i(x_j) = a_i t_i + (1-a_i)x_j$.

c At NE, players play mutual best responses.

~~NE~~

~~$a_1^* = BR_1(a_2^*)$, $a_2^* = BR_2(a_1^*)$~~

$x_1^* = BR_1(x_2^*)$, $x_2^* = BR_2(x_1^*)$

By substitution,

$x_1^* = a_1 t_1 + (1-a_1)x_2^*$

$= a_1 t_1 + (1-a_1)[a_2 t_2 + (1-a_2)x_1^*] \Leftrightarrow$

$[1-(1-a_1)(1-a_2)]x_1^* = a_1 t_1 + (1-a_1)a_2 t_2 \Leftrightarrow$

$x_1^* = a_1 t_1 + (1-a_1)a_2 t_2 / 1-(1-a_1)(1-a_2)$

By symmetry

$x_2^* = a_2 t_2 + (1-a_2)a_1 t_1 / 1-(1-a_1)(1-a_2)$

$1-(1-a_1)(1-a_2) = 1 - [1 + a_1 a_2 - a_1 - a_2] = a_1 + a_2 - a_1 a_2$

Then, x_1^* is a weighted average of t_1 and t_2 with weights $a_1 / a_1 + a_2 - a_1 a_2$ and $a_2 - a_1 a_2 / a_1 + a_2 - a_1 a_2$ respectively. The greater a_1 (and the smaller a_2), the ~~closer~~ closer x_1^* to t_1 than to t_2 ~~and~~. ~~The reverse~~

A symmetrical result obtains for x_2^*.

The greater the incentive one player^i has to choose ~~a~~ x_i close to his type t_i relative to the other player's incentive, the closer both players actions to t_i at NE. At NE, both players play action intermediate between their types.

d Given that $a_1 = a_2 = a$, by substitution,

$x_1^* = a/2a - a^2 \, t_1 + a - a^2 / 2a - a^2 \, t_2$

$x_2^* = a/2a - a^2 \, t_2 + a - a^2 / 2a - a^2 \, t_2$

By inspection, as a increases, each player i chooses x_i closer to his type t_i than to the other player's t_j, although both players always choose actions intermediate between the types.

For $a = 0$, the above ~~equation~~ x_1^* and x_2^* are undefined. Any $x_1 = x_2$ constitutes a NE. Payoff maximisation reduces to minimisation of differene between actions. The game is then a coordination game.

For $a = 1$, ~~the above are~~ $x_1^* = t_1$, $x_2^* = t_2$, payoff maximisation reduces to minimisation of the difference between action and type. ~~$a_i = t$~~ $x_i = t_i$ is strictly dominant for each player i.

e Given x_1, P2 chooses $x_2 = BR_2(x_1) = a_2 t_2 + (1-a_2)x_1$

P1's reduced form payoff function is

$u_1(x_1) = -a_1(x_1 - t_1)^2 - (1-a_1)(x_1 - a_2 t_2 - (1-a_2)x_1)^2$

$\max_{x_1} u_1$

FOC: $-a_1(2)(x_1 - t_1) - (1-a_1)(2)(a_2 x_1 - a_2 t_2)(a_2) = 0 \Leftrightarrow$

$-2a_1 x_1 + 2a_1 t_1 - 2(1-a_1)a_2^2 x_1 + 2(1-a_1)a_2^2 t_2 = 0 \Leftrightarrow$

$\cancel{2(a_1 x}\ 2a_1x_1 + 2(1-a_1)a_2^2 x_1 = 2a_1t_1 + 2(1-a_1)a_2^2 t_2 \Leftrightarrow$

$x_1 = a_1t_1 + (1-a_1)a_2^2 t_2 / a_1 + (1-a_1)a_2^2$

$= a_1 / a_1 + (1-a_1)a_2^2 \, t_1 + (1-a_1)a_2^2 / a_1 + (1-a_1)a_2^2 \, t_2$

SOC: $-2a_1x_1 - 2(1-a_1)a_2^2 x_1 < 0$

The above x_1 maximises payoff to P1

The unique SPE is such that P1 plays the above x_1 and P2 plays $x_2 = BR_2(x_1)$ which, on the eqm path, is equal to

$a_2t_2 + (1-a_2)\left[a_1 / a_1 + (1-a_1)a_2^2 \, t_1 + (1-a_1)a_2^2 / a_1 + (1-a_1)a_2^2 \, t_2 \right]$

P1 chooses x_1 which is some weighted average of t_1 and t_2, with weights a_1 and $(1-a_1)a_2^2$ rather than weights a_1 and $(1-a_1)a_2$ as in the simultaneous move game. P1 chooses x_1 weakly closes to t_1 and P2 best responds by choosing x_2 closer to x_1 (hence $\to t_1$) and further from t_2. Only in the case that $a_2 = 0, 1$ there no change.

In the general case $a_2 \in (0,1)$, P1 has a first mover advantage because choosing x_1 closer to t_1 has the strategic effect of inducing P2 to choose x_2 closer to t_1 and x_1 in the sequential game. ~~There is no strategic ef.~~ This in turn makes such x_1 more profitable.

In the extreme cases, P1 chooses $x_1 = t_1$ and P2 chooses either $x_2 = x_1 = t_1$ $(a_2 = 0)$ or $x_2 = t_2$ regardless, so there is no strategic effect, hence no differene between the simultaneous and sequential games.

†a ~~At PBE~~, by definition, players play sequentially rational strategies. Then, B buys (A, for accept) if $m_2 > v$ because this yields payoff $v - m_2 > 0$ whereas rejection (R) yields payoff 0. Similarly, B plays R if $m_2 < v$.
~~Then, $m_2 > H$ is not sequentially~~

Then $m_2 > H$ is not sequentially rational. $m_2 > H$ yields payoff 0 to S because no B plays A. This fails to deviation $m_2' = \varepsilon$ for sufficiently small ε ($< L$). Then all B play A and S has payoff $\varepsilon > 0$.

$L < m_2 < H$ is not sequentially rational because only H types play A in response so this yields payoff $p_2 m_2$. This fails to deviation to $m_2 = H - \varepsilon$ for sufficiently small ε ($< H - m_2$) which yields $p_2 m_2' > p_2 m_2$.

$m_2 < L$ is not sequentially rational because all B play A in response, this yields m_2 to S and fails to deviation to $m_2' = L - \varepsilon$ for sufficiently small ε ($< L - m_2$) which yields $m_2' > m_2$.

So only $m_2 = L$ and $m_2 = H$ are sequentially rational, and only these are played at PBE.

B does not reject $m_2 = v$ at PBE. If B rejects $m_2 = v$, then it is not sequentially rational to play $m_2 = v$ because this fails to deviation to $m_2' = v - \varepsilon$ for sufficiently small ε. ~~But by the above argument, this is not sequentially rational, so no m_2 is sequentia~~ no m_2 is sequentially rational, so ~~no PBE exists~~ where B rejects ~~$m_2 = v$~~.

B plays A iff $m_2 \geq v$.

b Given that B plays A iff $m_2 \geq v$, expected payoff to S given belief $p_2(m_1)$ from $m_2 = L$ is L because all B A, and expected payoff to S from $m_2 = H$ is $p_2(m_1)H$.

If $p_2(m_1) < a$, then $p_2(m_1)H < L$, so only $m_2 = L$ is sequentially rational. If $p_2(m_1) > a$, then $p_2(m_1)H > L$, so only $m_2 = H$ is sequentially rational. If $p_2(m_1) = a$, then $p_2(m_1)H = L$, so any mix of $m_2 = L$ and $m_2 = H$ is sequentially rational.

Then at PBE only these sequentially rational strategies are played. The given claim is verified.

c $m_1 = H + L$, $m_2 = L$

H types have payoff $H - L$ from buying in either period. By an argument analogous to that in (a), at PBE, H types buy in period 1. L types have payoff $L - H < 0$ from buying m_1 in period 1 and payoff $L - L = 0$ from buying in period 2. So L types buy in period 2. By Bayes's rule, $p_2(m_1) = 0$. Then by the argument in (a), $m_2 = L$ is sequentially rational. No PBE where H and L mix because then S has π dev by $-\varepsilon$ on prices.
S has expected payoff ~~$p_1 H + (1-p_1)$~~
$p_1(H+L) + (1-p_1)L = L + p_1 H$

d $m_1 = 2L$. Need to specify m_2

Given that $m_2 \geq L$, H types have payoff $2H - 2L$ from buying in period 1 and payoff no greater than $H - L$ from buying in period 2, so all H types buy in period 1.

Then, if any L types do not buy in period 1, by Bayes's rule, ~~$p_2(m_1)$~~ $p_2(m_1) = 0$, then $m_2 = L$, the remaining L types buy in period 2, then S has profitable deviation to $m_1' = m_1 - \varepsilon$ for sufficiently small ε because additional L is captured from otherwise consumption deferring L types. But m_1' is not sequentially rational because it fails to $m_1'' = m_1 - \varepsilon/2$, so no m_1 is sequentially rational and no PBE exists if L types defer consumption. So all L types buy in period 1 at PBE.

Then, S beliefs can be freely set

S has expected payoff $2L$

e $m_2 = H$ B sequentially rational iff $p_2(m_1) \geq a$
If all H types buy in t=1, S tempted to set $m_2 = L$
By Bayes rule, $p_2(m_1) = (1-r)p_1 / (1-r)p_1 + (1-p_1)$

$p_2(m_1) \geq a \iff$
$(1-r)p_1 / (1-r)p_1 + (1-p_1) \geq a \iff$
$(1-r)p_1 \geq a(1-r)p_1 + a(1-p_1) \iff$
$p_1 - rp_1 \geq ap_1 - arp_1 + a - ap_1 \iff$
$arp_1 - rp_1 \geq -p_1 + ap_1 + a - ap_1 \iff$
$r(ap_1 - p_1) \geq (a - p_1) \iff$
$r \leq a - p_1 / ap_1 - p_1 \iff$
$r \leq p_1 - a / p_1 - ap_1$

The given claim is verified.
Expected payoff is ~~$2H(1-r) + Hr$~~ ~~$= H + rH$~~
~~$= H + (1-r)H$~~ $2Hr + H(1-r) = H + rH$

f $m_1 = H+L,\ m_2 = L \Rightarrow \pi = L + p_1 H$ $\rangle\ p_1 \geq a$

$m_1 = 2L,\ m_2 = L \Rightarrow \pi = 2L$

$m_1 = 2H,\ m_2 = H \Rightarrow \pi = \cancel{H+rH}\ p_1(H + rH)$

$\cancel{= H + \frac{p_1 - a}{p_1 - ap_1} H}$

$\cancel{= p_1 H + \frac{p_1 - a}{1-a} aH}$

$\cancel{= H + p_1 H}$

~~if p_1~~

$\cancel{p_1 \gtreqless a \Leftrightarrow L + p_1 H \gtreqless 2L}$

$\cancel{p_1 = a \Leftrightarrow H + rH = H}$

$\cancel{p_1 > a \Leftrightarrow H + rH > 2H}$

$\cancel{p_1 < a \Leftrightarrow H + rH < H}$

$= p_1 H + \frac{p_1 - a}{1-a} H$

$\cancel{= p_1 - ap_1 + p_1 - a / 1-a H}$

For low p_1, pooling with $m_1 = 2L$ is profitable. For medium p_1, separating with $m_1 = H+L, m_2 = L$ is profitable. For high p_1, separating with $m_1 = 2H, m_2 = H$ is profitable.

$\cancel{p_1 - \frac{L/H}{1 - L/H} H}$

$\cancel{\frac{p_1 H - L}{1 - a}\ L/H}$

$\cancel{\frac{p_1 H - L}{H - L/H}}$

$\cancel{H\left(\frac{p_1 H - L}{H - L}\right)}$

$p_1 r H > L$?

$p_1 r > a z$

$\frac{p_1 - a}{1 - a} > a$

$p_1 - a > a - a^2$

$p_1 > 2a - a^2$ — then $2H, H$ & $H+L, L$

$p_1 > a$ — then $H+L, L$ & $2L$

2s	C	N
C	$0.8-z$, $0.8-z$	$0.6-z$, 0.4
N	0.4, $0.6-z$	0, 0

bi $z < 0.4 \Rightarrow C,C$ is strict NE $\Rightarrow$ ESS

strict NE suff but not nec (in general)

ii $z > 0.6 \Rightarrow N,N$ is strict NE $\Rightarrow$ ESS

iii Find q (prob of C) for NE

Check against (C,C), (N,N)

In principle have to check gainst mixed mutants

c ~~NO. Nec cond~~

Nec : $p - z > 1 - m \Rightarrow m - z > \cancel{1-p}\ 1-p$

$0 > m - z$

$\Rightarrow\ \ 0 > 1-p \Rightarrow p > \cancel{1-p}\ 1-p$

6a

	L	C	R
T	3	0	4
~~B~~	3	0	1
B	1	0	0
	5	0	0

Best responses underlined.

By inspection, there are two pure NE where players play ~~both~~ pure mutual best responses, (B, L) and (T, R).

By inspection, L strictly dominates C, so P2 plays C with zero probability ~~for any NE~~. at any NE.

Suppose there exists NE σ^* such that P1 mixes, then P1 has no profitable deviation and P1 is indifferent between T and B.

$\pi_1(T, \sigma_2^*) = \pi_1(B, \sigma_2^*) \Leftrightarrow$
$3q + 1(1-q) = 5q + 0(1-q) \Leftrightarrow$
$1-q = 2q \Leftrightarrow$
$q = 1/3$

~~so P2 ...~~ where q is the probability σ_2^* assigns to L. Then P2 mixes, so P2 must be indifferent between L and R

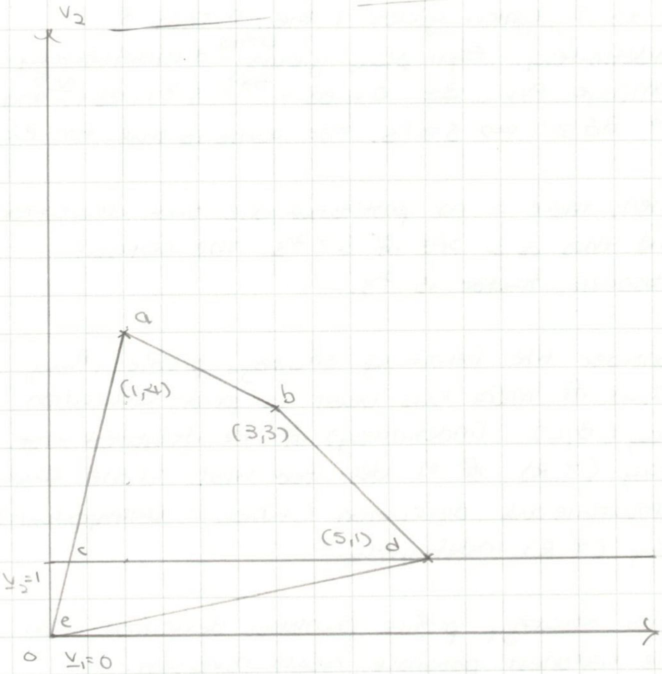
$\pi_2(L, \sigma_1^*) = \pi_2(R, \sigma_1^*) \Leftrightarrow$
$3p + 1(1-p) = 4p + 0(1-p) \Leftrightarrow$
$1-p = p \Leftrightarrow$
$p = 1/2.$

where p is the probability σ_1^* assigns to T. If P1 mixes so does P2. If P2 mixes so does P1, so no hybrid NE exists. The unique mixed NE is $(\frac{1}{2}T + \frac{1}{2}B, \frac{1}{3}L + \frac{2}{3}R)$

The NE are $(T, R), (B, L), (\frac{1}{2}T + \frac{1}{2}B, \frac{1}{3}L + \frac{2}{3}R)$.

b P2 minmaxes P1 by ~~p~~ playing C. ~~The~~ Then, P1 best responds by playing any (potentially degenerate) mix of T and B. This holds P1 to ~~pay~~ minmax payoff $\underline{v}_1 = 0$ and P1 guarantees $\underline{v}_1 = 0$ by so playing.

P1 minmaxes P2 by playing B. Then P2 best responds by playing L. P1 holds P2 to the minmax payoff $\underline{v}_2 = 1$ and P2 guarantees $\underline{v}_2 = 1$ by so playing



The set of feasible payoffs is represented by the area abde. The set of individually rational payoffs is ~~th~~ represented by the area above $v_2 = 1$ and to the right of $v_1 = 0$. The set of feasible and individually rational payoffs is represented by the intersection abdc.

c The most severe ~~punish~~ one period punishment is (B, R) (or (T, C) or (B, C)) for one period.

Consider the strategy profile represented by the following automaton.

G $(T,L) \underset{(B,R)}{\overset{\neg(T,L)}{\rightleftarrows}} (B,R) \circlearrowleft \neg(B,R)$
(T,L)

In words this strategy profile is: in every period, if it is the first period or if ~~there~~ there was no deviation in the previous period, play (T, L). Otherwise play (B, R).

At any history where there was no immediately prior deviation, optimal one shot deviation by P1 is to B which yields 5 then 0 then 3 indefinitely. Eqm play yields 3 indefinitely. Compare PV. $3 + 3\delta + 3\delta^2/1-\delta \geq 5 + 0\delta + 3\delta^2/1-\delta \Leftrightarrow$ $3\delta \geq 2 \Leftrightarrow \delta \geq 2/3$. Optimal one shot deviation by P2 is to R which yields 4 then 0 then 3 indefinitely. Eqm play yields 3 indefinitely. Compare PV. $3 + 3\delta + 3\delta^2/1-\delta \geq 4 + 0\delta + 3\delta^2/1-\delta \Leftrightarrow$ $3\delta \geq 1 \Leftrightarrow \delta \geq 1/3$.

At any history immediately following a deviation, optimal one shot deviation by P1 is to T, which yields 1 then 0 then 3 indefinitely. Eqm play yields 0 then 3 indefinitely. Compare PVs ~~3+~~ $0 + 3\delta + 3\delta^2/1-\delta \geq 1 + 0\delta + 3\delta^2/1-\delta$ $\Leftrightarrow 3\delta \geq 1 \Leftrightarrow \delta \geq 1/3$. The same is true for P2.

Then, there is no profitable one shot deviation and this is a SPE iff $\delta \geq 2/3$. The critical discount factor is $2/3$.

d Consider the following strategy profile. Play (T, L) iff there has been no prior deviation. Play (B, L) (indefinitely) iff P2 deviated first. Play (T, R) iff P1 deviated first. In the case of simultaneous deviation (which is unimportant), play (T, R) indefinitely.

This strategy profile punishes deviation with the harshest possible Nash reversion.

In the cooperation (no prior deviation) phase, optimal one shot deviation by P1 is to B, this yields 5 then 1 indefinitely. Eqm play yields 3 indefinitely. Compare PV. ~~$3+\delta/1-\delta$~~
$3 + 3\delta/1-\delta \geq 5 + \delta/1-\delta \Leftrightarrow 2\delta/1-\delta \geq 2 \Leftrightarrow 2\delta \geq 2-2\delta$ $\Leftrightarrow \delta \geq 1/2$. Optimal one shot deviation by P2 is to R which yields 4 then 1 indefinitely. Eqm play yields 3 indefinitely. Compare PV.
$3 + 3\delta/1-\delta \geq 4 + \delta/1-\delta \Leftrightarrow 2\delta/1-\delta \geq 1 \Leftrightarrow 2\delta \geq 1-\delta \Leftrightarrow$ $\delta \geq 1/3$.

In either punishment phase (prior deviation by either player) there is no profitable one shot deviation since players play a NE in each stage.

Then there is no profitable one shot deviation and this strategy profile is a NE iff $\delta \geq 1/2$.

e The construction in (c) is not renegotiation proof because at any history immediately following a deviation, where prescribed play is (B, R), each player strictly prefers to continue play instead ~~to~~ "as if" there has been no history. Play as prescribed yields 0 then 3 indefinitely for each player. Continuation ~~from~~ as if there has been no history yields 3 indefinitely for each player.

The construction in (d) is renegotiation proof because no continuation is strictly preferred by both players over any other. P1 prefers continuation from "punish P2" ~~to "cooperate"~~ ~~and~~ continuation from "cooperate" to continuation from "punish P1". The order is reversed for P2.

f At SPE, P2 has payoff weakly greater than $v_2 = 1$. Suppose for reductio that at SPE, P2 has payoff $v_2 < 1$. Then deviation to "always L" ~~≥~~ yields payoff (in ADV) of 1 for P2, and is strictly profitable. This candidate SPE is not a NE hence not a SPE. By reductio, P2 has payoff $v_2 \geq 1$ at any ~~#~~ SPE.

By inspection of the diagram in (b), the minimum payoff to v_1 (in ADV) consistent with $v_2 \geq 1$ is $1/4$. ~~This is achieved by P2 playing 3/4 L + 1/4 R and P1 playing responding optimally with T.~~

So P1 has payoff weakly greater than $1/4$ at any SPE.

By the Fudenberg-Maskin theorem, some SPE that yields ADV payoffs $(1/4, 1)$ exists since this is feasible and individually rational.

Consider the strategy profile under which players play T, L in every period iff there has been no prior deviation, (T, R) if P2 deviated first, and the above SPE if P1 deviated first.

By the argument in (d), P2 has no profitable one shot deviation in the cooperation phase iff $\delta \geq 1/3$. By the argument in (d), there is no profitable one shot deviation in the punish P2 phase for all δ

~~P1 has~~ In the cooperation phase, optimal one shot deviation by P1 is to B which yields 5 then an ADV of $1/4$ indefinitely. Eqm play yields 3 indefinitely. Compare PV. $3 + 3\delta/1-\delta \geq$ $5 + \frac{1}{4}\delta/1-\delta \Leftrightarrow 11/4(\delta/1-\delta) \geq 2 \Leftrightarrow 11/4\,\delta \geq 2-2\delta$ $\Leftrightarrow 19/4\,\delta \geq 2 \Leftrightarrow \delta \geq 8/19 \approx 0.42$. ~~P1 has~~

By definition of SPE, there is no profitable deviation in the punish P1 phase.

Then there is no profitable one shot deviation and this is a SPE iff $\delta \geq 0.42$. The claim is verified.