Modal Propositional Logic Reading

Plantinga, 1978

Plantinga, A. (1978) The Nature of Necessity. Oxford, England: Clarendon Press. pp. 1-9.

- Plantinga considers and rejects candidate explanations of the distinction between necessary and contingent facts.
- The explanation that necessary facts are facts that it is impossible to deny is entirely unhelpful. When we demand an explanation of the concepts of necessity and contingency, such concepts as impossibility are also unclear to us. Impossibility is conceptually proximate to necessity and contingency that to explain the latter two in terms of the former is unenlightening.
- Plantinga offers some examples of what are commonly considered necessary facts.
  - Truths of propositional and predicate (first-order) logic are commonly considered necessary truths.
  - As are truths of arithmetic, set theory, and mathematics more generally.
  - As are more "homely" truths like "no one is taller than himself" and "no number is a human being".
- So the sense of necessity in question is broader than that captured in first-order logic.
- The sense of necessity in question is narrower than causal necessity. It is causally necessary that no person can jump over a tall building, or travel faster than a speeding bullet unaided, but these are not necessary in the relevant sense.
- The sense of necessity in question is distinct from the concept of unrevisability. A proposition is unrevisable iff it is in principle immune to revision.
  - The reason necessity is distinct from unrevisability is that no proposition is immune to revision. There are circumstances under which it would be reasonable to revise or abandon any proposition. For example, abandoning the law of excluded middle has been proposed as a means of simplifying quantum mechanics.
  - It is inappropriate to identify (or construct) a distinction between propositions that are contingent on experience and propositions that hold independently of experience. Any proposition can be maintained in the face of recalcitrant experience by sufficiently drastic adjustments elsewhere.
  - To consider some fact as necessary is not to think that it will never be abandoned. Necessity in the relevant sense is not conferred by "long term popular favour".
  - For example, even if it is the case that every person shares the belief that Plantinga is a fine fellow, and would not abandon this belief in the face of recalcitrant experience, it is not the case that Plantinga is necessarily a fine fellow.
- The sense of necessity in question is distinct from the concept of what cannot be rationally rejected.
  - For example, suppose that Plantinga computes the sum $97 + 342 + 781$ four times and each time makes the same mistake, yielding the incorrect result 1120. Under such circumstances, it is rational to reject that $97 + 342 + 781 = 1220$, but this is a necessary fact.
- The sense of necessity in question is distinct from the concept of self-evidence.
  - Intuitively, to consider some proposition self-evident is to think that the proposition is obvious to anyone or nearly anyone who understands it.
  - But that $97 + 342 + 781 = 1220$ is necessary but not self-evident in this sense.
  - Perhaps such propositions are self-evident in the extended sense that they are the consequences of self-evident truths by argument forms whose corresponding conditionals are themselves self-evident, i.e. they are deducible from self-evident propositions by self-evident steps.
  - The sense of necessity in question is distinct from self-evidence in this extended sense. Each of Goldbach's Conjecture and Fermat's Last Theorem is either necessarily true or necessarily false. But it could turn out to be that for each of these, neither it nor its denial is self-evident in the extended sense. So whether a proposition is necessary and whether it is self-evident in the extended sense are distinct questions.
  - Whether all self-evident propositions are also necessary is also not immediate and obvious, so again, these are two distinct questions.
  - Propositions may appear to be self-evident when in fact false. Each of the following statements (and some corresponding inferences) are self-evident. For every property, there is some set of all and only the things with that property. There is a property of being a set that is not a member of itself. There is no set that both is and is not a member of itself. But not all of these can be true.
- The sense of necessity in question is distinct from the concept of a priori (as opposed to a posteriori) knowledge.
  - The distinction between a priori and a posteriori knowledge is an epistemological distinction.

    - Necessary facts cannot simply be propositions that are known a priori, because some necessary facts (such as Fermat's Last Theorem or its negation) are not known at all, and a fortiori, not known a priori.
    - Necessary facts can be known a posteriori rather than a priori. For example, if knowledge of some necessary fact is grounded in credible testimony.
    - Another suggestion is that necessary facts are knowable a priori. But to consider some fact "knowable" is to think that fact is possibly known. But when we demand an explanation of the distinction between necessary and contingent facts, such concepts as possibility are obscure to us.
    - It seems possible to know contingent truths a priori. For example, it is contingent that Plantinga exists, but it seems possible (for Plantinga) to know a priori that Plantinga exists.
- Plantinga concludes that necessity fails to coincide with the concepts of narrow logical necessity, causal necessity, unrevisability, rational rejectability, self-evidence, and a priori knowledge.

Salmon, 1989

Salmon, N. (1989) "The Logic of What Might Have Been," The Philosophical Review, 98(1), pp. 3-34.

- Salmon argued that S4 and S5 are inappropriate for metaphysical modality because the characteristic axiom schema of S4, $\Box\phi \rightarrow \Box\Box\phi$, which is also an axiom schema of S5, is not a logical truth schema, and some of its instances are in fact false.
- Salmon here repeats this argument, responds to objections, and argues further that B is also inappropriate for metaphysical modality because the characteristic axiom schema $\phi \rightarrow \Box \diamond \phi$ is not a logical truth schema, even if it has no false instances, and even if its instances are necessarily true.
- The argument against the appropriateness of S4 for metaphysical modality is as follows.
    - Consider a particular wooden table, Woody, that in fact originates from the matter m^*. It seems true that Woody could have originated from slightly different matter m' (while still being Woody). In other words, it seems metaphysically possible that Woody originated from m' rather than m^*. It seems also true that Woody could not have originated from drastically different matter m''. In other words, there is some matter m'' that is sufficiently "distant" from m^* that Woody could not have originated from m''. any thing that originates from m'' would not be Woody. Suppose that m' is a limiting case, i.e. m' is as distant from m^* such that it is still true that Woody could have originated from m'. Suppose also that m'' is sufficiently close to m' that, if Woody in fact originates from m', then Woody could have originated from m'' (while still being Woody).
    - Then, if Woody in fact originates from m', then it is possible that Woody originates from m''. Because it is possible that Woody originates from m', it is possible that it is possible that Woody originates from m''. So the claim that Woody necessarily does not originate from m'' is true, but not necessarily true. In symbols, $\Box\phi$ is true, but $\Box\Box\phi$ is false (for appropriate ϕ), so $\Box\phi \rightarrow \Box\Box\phi$ is false, and $\Box\Box\phi$ is not a semantic consequence of $\Box\phi$.
    - The intricacies of Salmon's handling of vagueness and indeterminacy is neglected for simplicity.
- This argument is supplemented by Salmon's account of possible worlds.
    - Salmon maintains that metaphysical necessity is truth in every possible world and that metaphysical possibility is truth in some possible world.
    - Salmon, quite conventionally, conceives of a possible world as certain sorts of (in some sense) maximal abstract entities according to which certain things (like facts and states of affairs) obtain or do not obtain. Informally, possible worlds are "total ways things might have been".
        - Qualification of "maximal" is necessary because of indeterminacy (the sorts of cases that motivate trivalent propositional logic, such as vagueness and presupposition failure) and because of "meta-facts" about other possible worlds and sets of facts.
        - "A possible world, then, may be thought of as a set of statements of a certain restricted but still very comprehensive sort."
    - Where Salmon deviates from convention is in defending the existence of "impossible worlds". These have in common with possible worlds that they are maximal ways for things to be, but differ in that these are not ways that things could have been. So there is an impossible world in which Woody originates from m''. Other impossible worlds include the world in which Nathan Salmon is Henry Kissinger.
    - On this account, w' (the world in which Woody originates from m') is possible, w'' is impossible but possibly possible.
    - Essentially, Salmon's argument is the metaphysical accessibility relation is intransitive.
- Salmon explains the objection to his argument that S4 and S5 are inappropriate for metaphysical modality.
    - Intransitive or otherwise restricted accessibility relations are introduced in modal semantics to interpret restricted modalities, such as temporal, epistemic, or deontic modality. For example, some proposition is physically necessary

in some world w iff it is true in every possible world in which the physical laws of w apply. More generally, a proposition is (qualifiedly) necessary in some world w iff it is true in every possible world of a certain sort, where this "certain sort" depends on the sort of necessity in question.
    - By contrast, the objection goes, metaphysical necessity, or unqualified necessity is entirely unrestricted. A proposition is metaphysically necessary iff it is true in every possible world.
    - The objection alleges that Salmon's concept of a metaphysically impossible world is incoherent. Every world is metaphysically possible. So Salmon is a "closet radical anti essentialist". Salmon writes that the world where Woody originates from w'' is metaphysically impossible, but this is incoherent, and what Salmon thinks is that there is a metaphysically possible world where Woody originates from w'', and also that there is a metaphysically possible world where Nathan Salmon is Henry Kissinger.
    - Salmon's restriction on the metaphysical accessibility relation then seems ad hoc, because it does not clarify the way in which this relation is to be restricted.
- Salmon argues that the above objection rests on a conflation of the generic notion of a way for things to be, and the peculiarly modal notion of the way things could have been.
    - This confusion arises from the ambiguity in the term "possible world", which is introduced to distinguish abstract logical entities according to which certain things like facts and states of affairs obtain from the ordinary understanding of a "world" as a physical world.
    - So understood, "possible" in "possible world" does not imply that the possible world is some way that things (metaphysically) could have been, but simply a linguistic tool for distinguishing between the ordinary and the technical use of "world".
    - Salmon argues that a possible world, properly understood (as Salmon understands it, rather than as denoting a world in the technical sense), "is a total way for things to be that conforms to metaphysical constraints concerning what might have been." This is the "modal notion" as opposed to the "generic notion" above.
- Salmon argues that the above objection goes wrong in thinking that metaphysical modality is an unrestricted sort of modality. Metaphysical modality is about truth in (modally, rather than generically) possible worlds.
    - Metaphysical modality is not an unrestricted modality because it is certainly more restricted than, for example, mathematical modality.
    - "A proposition is mathematically necessary if its truth is required by the laws of mathematics alone, and mathematically possible if its truth is not precluded by the laws of mathematics alone."
    - It is metaphysically impossible that Nathan Salmon is Henry Kissinger, but not mathematically impossible that this is so. There are mathematically accessible but metaphysically inaccessible worlds in which Nathan Salmon is Henry Kissinger.
    - The metaphysically accessible worlds are the metaphysically possible worlds. Once it is clear that metaphysical possibility does not refer to generic possibility (any way that things could be), "metaphysically possible" makes sense as a restriction.
    - "If worlds include ways things metaphysically cannot be in addition to ways things metaphysically might have been, then the idea that metaphysical necessity corresponds to truth in every world whatsoever is flatly mistaken."
    - Salmon sees no reason for thinking that worlds, in the sense of ways for things to be, exclude metaphysically impossible worlds.
    - If a world is understood in the "minimal" generic sense that Salmon understands it, as some abstract entity according to which certain sorts of things obtain, then there are worlds such that Woody originates from m'', even though this is metaphysically impossible. So the opponent of Salmon's view has reversed the direction of analysis. Metaphysical modality is not to be analysed in terms of (metaphysically) possible worlds, rather, possible worlds are to be analysed in terms of metaphysical modality. Similarly, ethical modality is not to be analysed in terms of deontically possible worlds, rather, deontically possible worlds, are to be analysed in terms of ethical modality. Our understanding of metaphysical necessity and possibility are prior to our understanding of metaphysically possible worlds.
        - Metaphysical necessity and possibility are not defined in terms of truth in possible worlds, rather theorems about the relation between metaphysical necessity and possibility, and truth in possible worlds follow from the definition of relative possibility.
    - So the "required" restriction is that some world w' is metaphysically accessible from world w iff all the facts of w' are possibilities in w.
- Salmon detours to discuss logical modality (logical necessity and logical possibility). It seems to Salmon that logical modality accommodates the axioms and rules of S5, but also that S5 is too restrictive for logical modality, i.e. there are axioms and rules of logical modality that are not accommodated in S5.
- Salmon discusses the "ostrich approach" to metaphysical modality.

    - The ostrich approach that S5 represents disregards all metaphysically distant worlds, and considers only directly metaphysically accessible worlds, in evaluating necessity and possibility.
    - "The ostrich approach flies in the face of the very meanings of the words "necessary" and "possible." On any standard or conventional sense of "possible" in English, a sentence of the form "It is possible that such-and-such" is true if there is a possible (in the same sense) scenario, a way things might have been, according to which it is the case that such-and-such."
    - "Metaphysical modality appears unrestricted because the restriction to meta- physically possible worlds is already built into one's practice concerning which worlds to pay attention to and to quantify over."
    - "Specifically, the ostrich approach misconstrues the simple modal term "necessary" to mean the modally complex concept of actual necessity, or necessity according to $W_@$, where $W_@$ is the actual world. Likewise, the ostrich approach misconstrues "possible" to mean actual possibility, or possibility according to $W_@$. The simple modal concepts of necessity and possibility simpliciter-the real meanings of the simple modal terms "necessary" and "possible"-are not the same as the concepts of actual necessity and actual possibility, necessity and possibility according to the actual world."
- A similar argument can be constructed to the effect that some worlds or states of affairs are contingently possible. (The Woody argument is to the effect that some worlds are contingently impossible).
- "The friends of B modal logic commit themselves to the loaded claim that it is logically true that the property of possibly being realized (or of being a way things might have been) is an essential property of the actual world [i.e. that it necessarily obtains]. The friends of S4 modal logic commit themselves to the similarly loaded claim that it is logically true that the property of not possibly being realized is always an essential property of those worlds that have it"
    - Even if these are true, and (metaphysically) necessarily true, they do not seem to be logically true.

Williamson, 2013

Williamson, T. (2013) Identity and Discrimination. Nashville, TN: John Wiley & Sons. pp. 126-143.

Williamson, 2016

Williamson, T. (2016) "Modal Science," Canadian Journal of Philosophy, 46(4-5), pp. 453-492.