Microeconomics Paper 150529

1a Let $X = \{x_1, x_2, \dots\}$ denote the set of social outcomes and $N = \{1, 2, \dots, n\}$ denote the set of individuals. Let $\succsim_i$ denote the weak preference relation of individual $i \in N$. Outcome $x \in X$ strictly Pareto-dominates outcome $x' \in X$ iff $\forall i \in N: x \succsim_i x'$ and not $x' \succsim_i x$. Outcome $x \in X$ weakly Pareto-dominates outcome $x' \in X$ iff $\forall i \in N: x \succsim_i x'$ and $\exists i \in N$ not $x' \succsim_i x$. In other words, this is iff each individual weakly prefers x to x' and some individual strictly prefers x to x'. An outcome is Pareto-optimal (i.e. satisfies the Pareto criterion) iff it is not Pareto-dominated by some other outcome.

~~A Pareto~~ Under a Pareto optimal outcome, no individual can be made better off without ~~~~ leaving another worse off. In this sense, the Pareto criterion eliminates inefficient outcomes which are such that some individual can "costlessly" (to others) be made better off.

A unique Pareto-optimal outcome does not necessarily exist, and a Pareto-optimal outcome could be extremely inequitable.

b Only points along the contract curve, at ~~the~~ points of tangency between the consumers' indifference curves, B, E, and G are Pareto efficient.

At any other point, consumers have non-equal marginal rate of substitution (MRS), and some mutually profitable trade exists which yields a Pareto improvement. For example, at A, the rate at which b is willing to exchange y for x is higher than the rate for A, then ~~~~ an exchange of some marginal amount of y ~~for some marginal amount of x~~ from b for some marginal amount of x from a ~~at~~ in some intermediate ratio leaves both a and b strictly better off. Graphically, the area bounded by the indifference curves through A represents a Pareto improvement.

B, E, and D are Pareto superior to A. Each of these points lies weakly above each ~~~~ consumer's indifference curve that passes through A, and strictly above at least one of these indifference curves. Then, at each of these points, each consumer is weakly better off and at least one consumer is strictly better off.

C is Pareto inferior to A because it lies weakly below ~~one of the two~~ each ~~~~ indifference curve through A and strictly below at least one of these.

c The ~~~~ statement is true. Consumer a is strictly worse off at G than at A and b is strictly better off at G than at A, hence neither G Pareto dominates A nor A Pareto dominates G. By the ~~above~~ argument in (b), G is Pareto efficient while A is not. G merely constitutes a Kaldor-Hicks improvement over A.

A social planner could use a Rawlsian, Atkinsonian, or Utilitarian criterion to choose between A and G. The first criterion selects the outcome ~~at~~ which ~~the worst off~~ ~~~~ maximises the minimum utility. The second maximises some weighted product of utilities, the third maximises the sum of utilities. Each criterion requires some estimate of each consumer's utility function.

If such an estimation is not plausible, the social planner could select G on the basis of its being a Kaldor-Hicks improvement over A or A on the basis that it constitutes a more equal distribution of goods.

d At the price vector given by p_0, given the initial endowment M, consumer a's demand is given ~~at the~~ by the point of tangency of the price line and the highest attainable indifference curve, which is the point H, that solves consumer a's constrained (by the budget constraint) utility maximisation problem. Likewise at p_0, consumer b has demands given by such a point of tangency which is H. Then, aggregate excess demand for each good is 0. Graphically the ~~dist~~ displacement in x from a of H gives a's demand for x and the displacement in x from M of H gives a's excess demand for x ~~and good~~. It can be seen graphically that for each good, each consumer's excess demand is the negative of the other's hence aggregate excess demand is zero.

Equivalently, B's excess demand for x is equal to A's excess supply of x and the reverse is true for y.

At p_0, aggregate excess demand for each good is zero, each market clears, hence the allocation given by M and the price vector $(p_0, 1)$ constitutes a general equilibrium.

The price vector $(p_1, 1)$ is not a price vector in ~~the~~ a general equilibrium because at this price vector, a's demand is given by c and b's demand is given by J. b's excess demand for x (the x displacement of J from M) exceeds a's excess supply (the displacement of C from M in x). Similarly for good y. Neither market clears, the resulting allocation is not part of a general equilibrium, $(p_1, 1)$ is not a price vector in a general equilibrium.

e A lump sum transfer of some amount of x from b to a could achieve K. The magnitude of this transfer can be found as follows. Compute the marginal rates of substitution ~~at K~~ (which are equal) at K. Plot the ~~pe~~ budget constraint ~~to~~ given by the price ratio equal to this marginal rate of substitution that passes through K. Find the intersection of this budget line with the ~~the sheet~~ horizontal line through M. ~~This intersection~~ The transfer is such as to move from M to this intersection.

Whether the equilibrium price ratio is greater or lower is indeterminate because this depends on the slope of the indifference curves at K.

It is possible to achieve any Pareto optimal allocation along the contract curve by some lump sum redistribution, by the second fundamental welfare theorem.

2a Horizontally differentiated products differ in nature rather than quality, such ~~differentia~~ differentiation is in some sense symmetric. ~~At~~ All other factors equal (including price), some consumers prefer one product ~~white~~ and some prefer the other. Neither product is "objectively" more valuable than the other.

Vertically differentiated products differ in quality and are in some sense asymmetric. All other factors equal, all consumers prefer the higher quality product over the lower quality product. The former is "objectively" ~~more~~ (universally) more valuable than the other.

The given demand functions represent horizontal differentiation ~~be~~ rather than vertical differentiation because where prices are equal, firms have equal demand, and it is not the case that all consumers buy ~~from one for~~ one product and not the other.

b Profit of firm 1

$\pi_1 = (p_1 - c) q_1 (p_1, p_2)$

$= (p_1 - c)(100 - \alpha p_1 + \beta p_2)$

Optimisation problem of firm 1

$\max_{p_1} \pi_1 = (p_1 - c)(100 - \alpha p_1 + \beta p_2)$

FOC: $\partial/\partial p_1 \, \pi_1 = (100 - \alpha p_1 + \beta p_2) + (p_1 - c)(-\alpha)$

$= 100 - \alpha p_1 + \beta p_2 - \alpha p_1 + \alpha c$

$= 100 - 2\alpha p_1 + \beta p_2 + \alpha c = 0$

$\Rightarrow p_1 = 100 + \beta p_2 + \alpha c / 2\alpha$

$= \cancel{100}\ c/2 + 100/2\alpha + \beta p_2 / 2\alpha$

SOC: $\partial^2/\partial p_1^2 \, \pi_1 = -2\alpha < 0$

Best response function of firm 1

$B_1(p_2) = c/2 + 100/2\alpha + \beta p_2 / 2\alpha$

~~G At~~ Given that ~~NE is at~~ the required NE is symmetric, from the fact that players play mutual best responses at NE

~~$B_1(p_1) = c/2 + 100/2\alpha +$~~

$p_1^* = B_1(p_1^*) = c/2 + 100/2\alpha + \beta p_1^* / 2\alpha$

$\Rightarrow p_1^*(1 - \beta/2\alpha) = c/2 + 100/2\alpha$

$\Rightarrow p_1^* = 100 + \alpha c / 2\alpha - \beta$

By symmetry, $p_2^* = 100 + \alpha c / 2\alpha - \beta$

$q_1^* = 100 - \alpha p_1^* + \beta p_2^*$

$= 100 - (100 + \alpha c)\alpha / 2\alpha - \beta + (100 + \alpha c)\beta / 2\alpha - \beta$

$= 200\alpha - 100\beta - 100\alpha - \alpha^2 c + 100\beta + \alpha\beta c / 2\alpha - \beta$

$= 100\alpha - \alpha(\alpha - \beta)c / 2\alpha - \beta$

$> 0 \Leftarrow 100 > (\alpha - \beta)c \Leftarrow 100 > \alpha c$

It is verified that the equilibrium quantity is positive. (By symmetry $q_2^* = q_1^* > 0$

Given $\alpha > \beta$, we have $2\alpha - \beta > 0$. Given $\alpha, c \geq 0$, we have $100 + \alpha c > 0$, hence we have $p_i^* > 0$ and by symmetry $p_j^* > 0$. It's verified that prices and quantities are positive.

c $\cancel{\partial/\partial\alpha\, p_i^* = c(-1)(2\alpha-\beta)^{-2} + (100+\alpha c)(2\alpha-\beta)^{-1}}$

$\quad = \cancel{(2\alpha-\beta)^{-1}[100 + \alpha c - (2\alpha-\beta)c]}$

$\quad = \cancel{(2\alpha-\beta)^{-1}[100 - \alpha c + \beta c]}$

$\quad \cancel{> 0 \text{ (given } 2\alpha-\beta>0,\ 100 > \alpha c)}$

$\partial/\partial\beta\, p_i^* = (100+\alpha c)(-1)(2\alpha-\beta)^{-2}(-1)$

$\quad = (100+\alpha c)(2\alpha-\beta)^{-2} > 0$

$\partial/\partial\alpha\, p_i^* = (100+\alpha c)(-1)(2\alpha-\beta)^{-2}(2)$

$\quad + c(2\alpha-\beta)^{-1}$

$\quad = (2\alpha-\beta)^{-2}[-200 - 2\alpha c + 2\alpha c - \beta c]$

$\quad = (2\alpha-\beta)^{-2}[-200 - \beta c] < 0$

$\partial/\partial c\, p_i^* = (2\alpha-\beta)^{-1}\alpha > 0$

Intuitively $\uparrow \alpha$ constitutes an increase in the price elasticity of demand, an increase in price yields the same increase in profit on infra-marginal units, but has a larger negative effect on profit at the margin because more consumers are alienated by the price increase. Firms have less incentive to increase price, so choose a lower price.

Intuitively $\uparrow \beta$ constitutes an increase in the ~~diversion ratio~~ sensitivity of demand to the rival's price. An increase in one firm's price causes a greater increase in the other's demand hence a larger increase in the other's price, which in turn increases the first firm's demand and optimal price, and so on.

Intuitively, firms pass on part of the increase in marginal costs to consumers because the duopolists have some degree of market power. An increase in cost causes a reduction in margin hence firms have less incentive to sell a high output by choosing a low price.

d $\Pi = \pi_1 + \pi_2 = (p-c)q_1 + (p-c)q_2 \cancel{= (p-c)[200}$

$\quad = (p-c)[200 - 2(\alpha-\beta)p]$

FOC $\partial/\partial p\, \Pi = (p-c)[-2(\alpha-\beta)] + [200 - 2(\alpha-\beta)p]$

$\quad = 2(\alpha-\beta)c + [200 - 2(\alpha-\beta)p - 2(\alpha-\beta)p]$

$\quad = 0$

$\cancel{\Rightarrow 200 - 4(\alpha-\beta)p + 2(\alpha-\beta)c = 0}$

$\cancel{\Rightarrow p = 200 + 2(\alpha-\beta)c / 4\alpha - \beta}$

$\cancel{= c/2 + 50/\alpha-\beta}$

$\cancel{= \alpha c/2\alpha + 100/2\alpha-2\beta}$

$\cancel{= \alpha c - \beta c/2\alpha-2\beta + 100/2\alpha-2\beta}$

$\cancel{= 100 + \alpha c - \beta c/2\alpha - 2\beta}$

SOC: $\partial^2/\partial p^2\, \Pi = -4(\alpha-\beta) < 0$

~~The monop~~

FOC $\Rightarrow p = 200 + 2(\alpha-\beta)c / 4(\alpha-\beta)$

$\quad = 100 + \alpha c - \beta c / 2\alpha - 2\beta$

The monopolist optimally chooses ~~such p~~

$p^m = 100 + \alpha c - \beta c / 2\alpha - 2\beta$

$p^m \gtreqless p^* \Leftrightarrow$

$100 + \alpha c - \beta c / 2\alpha - 2\beta \gtreqless 100 + \alpha c / 2\alpha - \beta \Leftrightarrow$

$(100+\alpha c)(2\alpha-\beta) - \beta c(2\alpha-\beta) \gtreqless (100+\alpha c)(2\alpha-\beta) - \beta(100+\alpha c)$

$-\beta c(2\alpha-\beta) \gtreqless -\beta(100+\alpha c) \Leftrightarrow$

$2\alpha\beta c - \beta^2 c \lesseqgtr 100\beta + \alpha\beta c \Leftrightarrow$

$\alpha c - \beta c \lesseqgtr 100$

$100 > \alpha c \Rightarrow 100 > \alpha c - \beta c \Rightarrow p^m > p^*$

The monopolist chooses a higher price because it internalises the mutual ~~negative~~ positive externalities between the duopolists. In choosing a ~~price~~ higher price, each duopolist has a positive effect on the other's profit which it does not account for pre-merger. This leads to each duopolist choosing a price that fails to maximise joint profit.

3a The participation constraint is

PC: $E(u(w,e)\,|\,e=0) \geq \bar{u} = 0$

$\iff \frac{3}{4}u(w_e,0) + \frac{1}{4}u(w_n,0) \geq 0$

$\iff \frac{3}{4}[4 - \frac{5}{w_e}] + \frac{1}{4}[4 - \frac{5}{w_n}] \geq 0$

$\iff 4 - \frac{15}{4w_e} - \frac{5}{4w_n} \geq 0$

where w_e is the wage in the event of an error, and w_n is the wage in the event of no error.

The incentive constraint is

IC: $E(u(w,e)\,|\,e=0) \geq E(u(w,e)\,|\,e=1)$

$\iff 4 - \frac{15}{4w_e} - \frac{5}{4w_n} \geq \frac{1}{4}[4 - \frac{5}{w_e} - 2] + \frac{3}{4}[4 - \frac{5}{w_n} - 2]$

$\iff 4 - \frac{15}{4w_e} - \frac{5}{4w_n} \geq 2 - \frac{5}{4w_e} - \frac{15}{4w_n}$

$\iff 2 + \frac{10}{4w_n} \geq \frac{10}{4w_e}$

$\cancel{\iff \frac{8}{10} + \frac{1}{w_n} \geq \frac{1}{w_e}}$

At any optimum, PC binds. Every candidate optimum such that PC does not bind fails to deviation by reducing w_n by sufficiently small amount ε, ~~which the deflect increases payoff to the principal~~ ~~P GA~~ such that PC remains satisfied. IC remains satisfied and payoff to the principal increases.

At any optimum $w_e = w_n$ hence IC is strictly satisfied. Every candidate optimum such that $w_e \neq w_n$ fails to the deviation to $w_e = w_n$ such that PC is just satisfied. By concavity of u in w, a mean-preserving contraction of expected wage increases expected utility hence P can then decrease expected wage while continuing to satisfy PC and IC.

From the above arguments,

$4 - \frac{15}{4w_e} - \frac{5}{4w_n} = 0$, $w_e = w_n \Rightarrow 4 - \frac{5}{w_e} = 0 \Rightarrow w_e = \frac{5}{4} = w_n$

~~$E(\pi \mid w_e = w_n = 4/5)$~~

$E(\pi - w\,|\,e=0) = \frac{3}{4}(0 - \frac{5}{4}) + \frac{1}{4}(12 - \frac{5}{4}) = 3 - \frac{5}{4} = \frac{7}{4}$

P pays a constant wage because P ~~need not off offer~~ does not intend to induce high effort hence need not align incentives with a variable wage scheme. Then, P chooses constant wage such that PC is just satisfied. This wage is $5/4$. P has expected gross profit of 3 hence expected net profit of $7/4$.

b PC: $E(u(w,e)\,|\,e=1) \geq \bar{u} = 0$

$\iff \frac{1}{4}(4 - \frac{5}{w_e} - 2) + \frac{3}{4}(4 - \frac{5}{w_n} - 2) \geq 0$

$\iff 2 - \frac{5}{4w_e} - \frac{15}{4w_n} \geq 0$

IC: $E(u(w,e)\,|\,e=1) \geq E(u(w,e)\,|\,e=0)$

$\iff 2 - \frac{5}{4w_e} - \frac{15}{4w_n} \geq 4 - \frac{15}{4w_e} - \frac{5}{4w_n}$

$\iff \frac{10}{4w_e} \geq 2 + \frac{10}{4w_n}$

$\iff \frac{5}{w_e} \geq 4 + \frac{5}{w_n}$

At the optimum, both constraints bind. Every candidate optimum such that PC does not bind fails to deviation by decreasing w_e by sufficiently small amount ε such that PC remains satisfied. Such deviation continues to satisfy IC and yields higher expected net profit. Every candidate optimum such that PC does not bind fails to the deviation that consists in a sufficiently small mean preserving contraction of w_e, w_n and a sufficiently small decrease in each of w_e, w_n. The former "loosens" PC, the latter increases expected net profit, and in sufficiently small magnitudes, IC remains satisfied.

From the above argument,

$2 - \frac{5}{4w_e} - \frac{15}{4w_n} = 0$

$4 - \frac{15}{4w_e} - \frac{5}{4w_n} = 0$

$\cancel{\Rightarrow 2 - \frac{5}{4w_e} - 3(4 - \frac{15}{4w_e}) = -10 + \frac{4}{w_e} = 0}$

$\cancel{\Rightarrow w_e = \frac{4}{10}}$

$\cancel{\Rightarrow 2 - \frac{50}{16} - \frac{15}{4w_n} = 0}$

$\cancel{\Rightarrow \frac{15}{4w_n} = \frac{18}{16}}$

$\cancel{\Rightarrow w_n = \frac{16}{18}\,\frac{15}{4} = \frac{20}{6}}$

$\Rightarrow 4 - \frac{15}{4w_e} - \frac{5}{4w_n} - 3(2 - \frac{5}{4w_e} - \frac{15}{4w_n}) = 0$

$\Rightarrow -2 + \frac{40}{4w_n} = -2 + \frac{10}{w_n} = 0 \Rightarrow w_n = 5$

$\Rightarrow 2 - \frac{5}{4w_e} - \frac{3}{4} = 0$

$\Rightarrow \frac{5}{4w_e} = \frac{5}{4} \Rightarrow w_e = 1$

The optimal contract to induce high effort is $w_e = 1$, $w_n = 5$

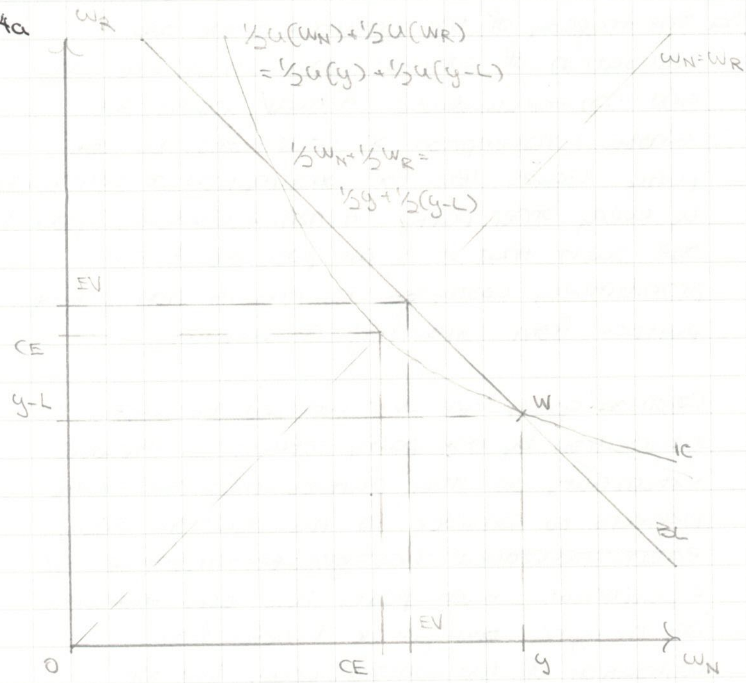
$E(\pi - w\,|\,e=1) = \frac{1}{4}(0-1) + \frac{3}{4}(12-5) = -\frac{1}{4} + \frac{21}{4} = 5$

P finds it optimal to induce high effort

c The agency cost is the increase in expected wage (equivalently, the decrease in expected net profit) in inducing high effort from the case where effort is observable to where it is not. This cost is incurred because under unobservable effort, P must offer A a variable wage scheme such that high effort is incentive compatible. Then, risk-averse A must be compensated for higher risk-bearing with higher expected wage such that participation remains individually rational.

Under observable effort, ~~or~~ the fixed wage required to induce high effort is such that PC ~~just~~ binds. $4 - \frac{5}{w} - 2 = 0 \Rightarrow w = \frac{5}{2}$. Expected wage under unobservable effort is $\frac{1}{4}1 + \frac{3}{4}5 = 4$. The agency cost is $\frac{3}{2}$.

4a



A's endowment point is given by W, certainty equivalent CE is given by the intersection of the indifference curve through W and the 45° $w_N=w_R$ certainty line, expected value EV is given by the ~~line $\frac{1}{2}w_N+\frac{1}{2}w_R$~~ intersection of the line $\frac{1}{2}w_N+\frac{1}{2}w_R = \frac{1}{2}y+\frac{1}{2}(y-L)$ with the 45° certainty line. EV > CE because A is risk averse hence has positive risk premium RP = EV - CE. Risk aversion is represented graphically by the convexity of the indifference curve IC.

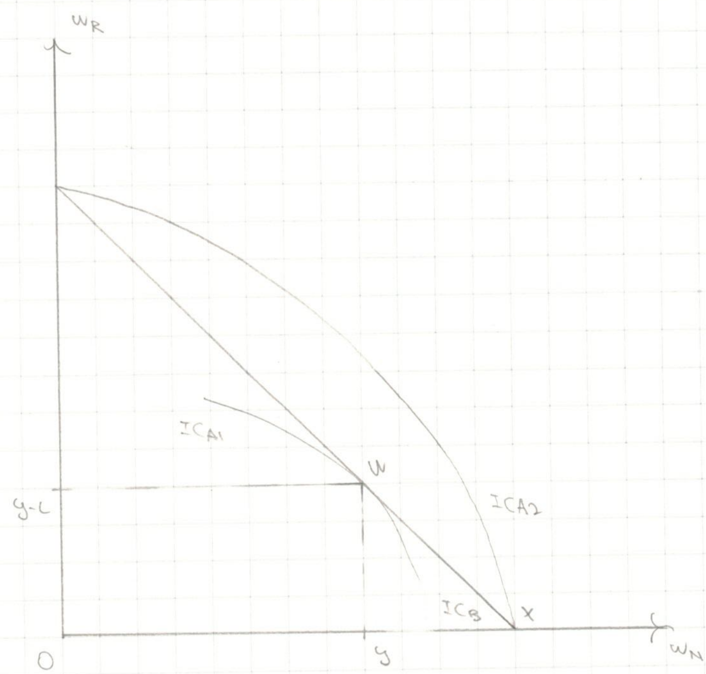
b In a competitive insurance market, supposing that insurers are risk-neutral, insurers offer ~~act~~ insurance at actuarially fair premiums, i.e. one unit of coverage in the event E is offered at the premium $p=\pi$, where π is the probability of E. ~~At any~~ ~~A~~ No higher premium is offered in equilibrium because an insurer makes positive expected profit from a higher premium and in a competitive market, insurers make zero expected profit (if risk neutral).

Given actuarially fair insurance is available, if A buys ~~a~~ coverage a in the event of rain at premium $p=\pi=\frac{1}{2}$, then A faces lottery $[\frac{1}{2},\frac{1}{2};\ y-\frac{1}{2}a,\ y-\frac{1}{2}a-L+a]$. ~~This~~ The set of such points corresponds to the points on the budget line BC.

By inspection, some points along BC lie above IC, then A is better off at these points, buying some amount of insurance, than at W, hence it is optimal for A to insure. A optimally fully insures given fair premiums. This is the point (EV, EV) which lies on the highest attainable indifference curve given constraint BC.

ii The maximum expected profit B can make is equal to EV - CE. B offering A full insurance is equivalent to B buying A's lottery for some fixed price. The minimum price A is willing to sell for is A's CE, by definition of CE, this is the lowest certain amount A ~~is~~ weakly ~~pref~~ prefers to the lottery. The expected value of the lottery is EV, hence B's maximum ~~p~~ expected profit is EV - CE.

c If A is risk seeking, A has concave indifference curves and the highest attainable indifference curve is attained at a corner where A faces an "all or nothing" lottery. The agreement that is optimal for A given that B is weakly willing to accept is such that A pays B y-L in the event of rain R and B pays A y-L in the event of no rain N.



This agreement leaves B on IC_B, B is no better off, ~~A As~~ W lies on the lower IC_{A1}, X lies on the highest attainable IC_{A2}, A is better off.

d Expected utility without pooling
$E(u) = \frac{1}{2}\ln 8 + \frac{1}{2}\ln 2 = \ln 16^{1/2} = \ln 4$

Expected utility with pooling
$E(u) = p\ln 8 + p\ln 2 + (1-2p)\ln(\frac{8+2}{2})$
$= \ln 16^p + \ln 5^{1-2p}$
$= \ln 16^p 5^{1-2p}$
$= \ln 4^{2p} 5^{1-2p} > \ln 4$ (for ~~p~~ $0 \leq p < 0.5$)

Increase in expected utility
$\ln 4^{2p} 5^{1-2p} - \ln 4$

The benefits of pooling decreases as p increases. As p increases, the correlation between the outcomes on the two farms increases, then the pooled lottery converges to the original unpooled lottery. ~~There is no reduc~~ The reduction in risk realised by pooling due to intermediate outcome being realised with non-zero probability decreases. ~~Hence the~~ The pooled lottery constitutes a smaller mean preserving contraction as p increases.

5a The tragedy of the commons is the over-utilisation of common goods which are rival and non-excludable. A rival good is one whose consumption or utilisation by one party leaves less for consumption or utilisation by every other party. A non-excludable good is one such that it is not possible if not prohibitively costly ~~to~~ to prevent non-paying parties from consuming or utilising.

Common goods are over-utilised because utilisation by one party inflicts a negative externality on other parties that the former neglects to consider in his decision. Then, ~~each individual has greater incentive~~ at equilibrium, each party has zero ~~marginal~~ net marginal ~~bet~~ private benefit from utilisation of the common good, but the ~~marginal~~ net marginal social benefit is negative, and social surplus would be increased by a reduction in consumption/utilisation.

b At equilibrium, the benefit to each driver from using the road is equal to the cost. If benefit exceeds cost, each driver not using the road has profitable deviation to using the road. If cost exceeds benefit, each driver using the road has profitable deviation to not using it (supposing a sufficiently large number of drivers and an outside option with zero net benefit).

$v = a + bN^e \Rightarrow N^e = {}^{v-a}/_b$, where e denotes equilibrium quantities.

c $W(N) = N(\cancel{\quad} v - (a+bN))$

$\max_N W(N)$

FOC: $\partial/\partial N \; W(N) = v - a - 2bN = 0 \Rightarrow N = {}^{v-a}/_{2b}$

SOC: $\partial^2/\partial N^2 \; W(N) = -2b < 0$

$N = v-a/2b$ maximises $W(N)$

The socially optimal number of drivers is $N^* = {}^{v-a}/_{2b}$

$W(N^e) = N(v - (a+bN^e)) = 0$

The ~~e~~ equilibrium number of drivers is ~~less~~ more than socially optimal because in ~~d~~ choosing whether to use the road each driver considers only the private benefit and the private cost. But road usage imposes a negative externality (congestion) on other road users that each driver fails to consider. At ~~the~~ equilibrium, by the argument in (b), each driver has zero net private benefit. Then, because of the negative externality, net social benefit of each driver's road usage is negative. Social surplus would be increased by reducing the number of drivers because the marginal driver has zero private net benefit but negative external net benefit.

At equilibrium, each driver has zero net private benefit, hence aggregate ~~be~~ net benefit, $W(N^e)$ is zero.

d The optimal toll is such that at equilibrium, the number of road users is N^*. From the argument above, at equilibrium, each road user has zero net private benefit. ~~th~~

$$v = a + bN^* + t$$
$$\Rightarrow t = v - a - bN^*$$
$$= v - a - b\left(\frac{v-a}{2b}\right)$$
$$= \frac{v-a}{2}$$

The externality imposed by each road user on each other road user is b. At the optimum, there are $\frac{v-a}{2b}$ road users, hence the total externality ~~each road~~ the marginal road user imposes on others is $\frac{v-a}{2}$.

The optimal toll is equal to the externality each road user imposes on others at the optimal utilisation, then, at the optimal utilisation, each driver's incentives coincides with the social planners and equilibrium is achieved.

e $W(N, x) = N(v - (a + bN - gx)) - x$

At equilibrium, each driver has zero net private benefit

$v = a + bN^e - gx \Rightarrow N^e = \frac{v - a + gx}{b}$

~~Welfare is the sum of net private ben~~

$$W(N^e, x) = N^e(v - (a + bN^e - gx)) - x$$
$$= N^e(v - v) - x$$
$$= -x$$

~~Social~~ Social value of road improvements net of the cost is negative. This is because the gross social value of road improvements is zero which is in turn because any road improvements are met with an increase in induced demand such that net private benefit is zero hence gross welfare is zero.

This result fails to obtain if the number of potential road users is not sufficiently large.

~~Otherwise~~