

A

## Game Theory Problem Set 7

Same time Thursday  
Due Wed 2pm

G	C	D
C	1	g
I	b	
D	b	0
g	0	

Given:  $a > 1$ ,  $b < 0$ ,  $a+b < 1$ ,  $p' + (1-p')b \geq 0$ 

In what follows, superscripts denote the period, subscripts denote the player and type, and \* denotes equilibrium values.

Best responses underlined. By inspection, D is strictly dominant for P1 and P2  $\Rightarrow s_R^2* = s_T^2* = D$ , where R denotes rational P1 and T denotes irrational tit-for-tat P1. By definition of construction of type T,  $s_T^1* = C$ ,  $s_T^2* = s_R^1*$ .

Exactly right first step

$\pi_R^2* = \pi_R^2(s_R^2* = D, s_T^2* = D) = 0$  for all  $s_R^1$   $\Rightarrow$  ~~πR's payoff~~  $\pi_R^2 = \pi_R^1 + \pi_R^2$  reduces to  $\pi_R^1(s_R^1, s_T^2)$ . In words, because rational players defect with certainty in period 2 regardless of period 1 actions, rational P1 maximises total payoff by maximising period 1 payoff. Then, D is strictly dominant  $\Rightarrow$  ~~s\_R^1\* = D~~

Exactly right second step

$$\begin{aligned} \pi_R^1(s_2^1 = C, s_2^2 = D; s_R^1*, s_T^1*) &= p'(1+a) + (1-p)b \\ \pi_R^1(s_2^1 = D, s_2^2 = D; s_R^1*, s_T^1*) &= p'(a) + (1-p)(0) \\ \# \Rightarrow p \\ \pi_R^1(s_2^1 = C, s_2^2 = D; s_R^1*, s_T^1*) &> \pi_R^1(s_2^1 = D, s_2^2 = D; s_R^1*, s_T^1*) \Leftrightarrow \\ p'(1+a) + (1-p)b &\Rightarrow p'a + (1-p)b \Leftrightarrow \\ p' + (1-p)b &\end{aligned}$$

$$\begin{aligned} \pi_R^1(s_2^1 = D, s_2^2 = D; s_R^1*, s_T^1*) &> \pi_R^1(s_2^1 = C, s_2^2 = D; s_R^1*, s_T^1*) \Leftrightarrow \\ p'a &\Leftrightarrow p'(1+a) + (1-p)b \Leftrightarrow p' + (1-p)b < 0 \\ \text{By reductio, given } p' + (1-p)b \geq 0, \pi_R^1(s_2^1 = C) &\geq \pi_R^1(s_2^1 = D) \\ \text{Then } s_R^1* = C. \text{ In words, P2's best response to } s_R^1* &\\ \text{that tit-for-tat defects with certainty in each period and} &\\ \text{s2* that plays tit-for-tat is to cooperate then} &\\ \text{defect since the probability of encountering an} &\\ \text{irrational P1 is sufficiently high such that} &\\ \text{the additional payoff of 1 from an additional period} &\\ \text{of cooperation, against with probability } p' \text{ outweighs} &\\ \text{the penalty of } b \text{ from being exploited by a rational} &\\ \text{P1 with probability } 1-p'. & \end{aligned}$$

(weak best response)

$$s_R^1* = (D, D), s_T^1* = (C, C), s_T^2* = (C, D)$$

P2's beliefs are such that at  $t=1$ , P2 believes P1 is T with probability 1 and at  $t=2$ , P2 believes P1 is R with certainty if  $s_1^1 = D$  and P2 believes P1 is T with certainty if  $s_1^1 = C$ .

b Suppose R plays C with certainty in T<sub>1</sub>, then by Bayes rule, P2's belief in period 2,  $P^2 = p'/p' + (1-p')$  =  $p'$ . In words, P1's action in equilibrium in T<sub>1</sub> is entirely uninformative.

Correct

By the result in a,  $S_R^{2*}, S_R^{3*} = D, D, S_T^{2*}, S_T^{3*} = \boxed{C, C}$   
 $S_R^{2*}, S_R^{3*} = C, D, S_T^{2*} = (C, C, C), \text{ &}$   
 $S_T^{2*} = (C, C, D)$ .  $S_R^{2*}, S_R^{3*} = D, D, S_T^{2*}, S_T^{3*} = C, C,$   
 $S_T^{2*}, S_T^{3*} = C, D \Rightarrow S_R^{*} = (C, D, D), S_T^{*} = (C, C, C), S_2^{*} = (C, C, D)$

$$\begin{aligned}\Pi_2(S_2^1 = C, S_2^{2*}, S_2^{3*}; S_T^*, S_T^*) &= p'(1+a) + (1-p) \\ &= p'(1+a) + (1-p')(1+b+a) = p'(2+a) + (1-p')(1+b) \\ \Pi_2(S_2^1 = D, S_2^{2*}, S_2^{3*}; S_T^*, S_T^*) &= \\ &\Rightarrow p'\end{aligned}$$

$$\begin{aligned}\Pi_2(S_2^1; S_R^*, S_T^*) &= p'(1+a) + (1-p')(1+b+a) = \cancel{p'(1+a)} \\ &= p'(2+a) + (1-p')(1+b)\end{aligned}$$

Check deviation for P1 from CDD to DDD

Suppose P2 deviates from this candidate eqn by choosing  $S_2^1 = S_2^1 \neq S_2^1 = C$ , then  $S_2^1 = D \Rightarrow S_2^2 = D$ ,  $S_2^3 = D$  (because if R plays C with 100% zero probability, it reveals itself and invites defection in period 3, and also receives lower payoff in period 3, and also receives lower payoff in period 2),  $S_R^3 = D$  (because D is strictly dominant),  $S_T^3 = D$  (for the same reason).  $\cancel{S_2^1 = C}$

Suppose further that  $S_2^2 = C$ , then

$$\cancel{\Pi_2} = p'(a+b+a) + (1-p')(a+b+0) = p'a + a+b$$

Suppose instead that  $S_2^2 = D$ , then

$$\Pi_2 = p'(a+0+a) + (1-p')(a+0+0) = a$$

$$\Pi_2^* \geq \Pi_2(D, C, D) \Leftrightarrow$$

$$p'(2+a) + (1-p')(1+b) \geq p'a + a+b \Leftrightarrow$$

$$2p + p - bp \geq a \Leftrightarrow p' + (1-p')b + 1 + ap \geq p'a + a + b \Leftrightarrow$$

$$1 - p - bp \geq a \Leftrightarrow p' + (1-p')b + 1 \geq a + b \cancel{a+b} \Leftrightarrow$$

$$p' + (1-p')b \geq 0 \text{ and } a + b \leq 1$$

$$\text{so } \Pi_2^* \geq \Pi_2(S_2 = (D, C, D) \rightarrow D, C, D)$$

$$\Pi_2^* \geq \Pi_2(D, D, D) \Leftrightarrow$$

$$p'(2+a) + (1-p')(1+b) \geq a \Leftrightarrow$$

$$p' + (1-p')b \leftarrow \cancel{p' + (1-p')a} - (1-p')a + 1 \geq 0 \Leftrightarrow$$

$$p' + (1-p')b \geq 0 \text{ and } (1-p')a \leq 1$$

$$\text{so } \Pi_2^* \geq \Pi_2(D, D, D)$$

$\cancel{S_2^1 = C}$  is optimal for P2. It is trivial that  $S_2^1 = C$  is optimal for R since R forgoes one period of cooperation payoff if it defects earlier.

Maybe write this out

Rational P1 preserves its reputation as a tit-for-tat player by playing C in period 1 and to extract an additional period of cooperation payoff before defecting to extract greater payoff a and irreversibly revealing itself and terminating cooperation

criticism: result contingent on the form of irrationality chosen.

Because P2 knows P1 will C in T1, P2 C in T2

2a Given  $P_1, P_2, P_3$  bid truthfully, i.e. bid equal to their respective valuations  $V_1=10, V_2=4, V_3=2$ , their respective bids are  $S_1=10, S_2=4, S_3=2$ .

Given  $\epsilon$  at the highest bidder pays the second highest bid and so on, the highest bidder,  $P_1$  and second highest bidder,  $P_2$ , have cost per click through  $c_1 = S_2 = 4$  and  $c_2 = S_3 = 2$  respectively.

Given that the highest bidder ~~receives~~ receives 200 click throughs per hour and the second highest bidder receives 100,  $P_1$  and  $P_2$  have volume of click throughs  $q_1 = 200, q_2 = 100$ . Net profits per hour  $\pi_1 = (V_1 - c_1)q_1 = 1200$ ,  $\pi_2 = (V_2 - c_2)q_2 = 200$ ,  $\pi_3 = 0$ .

Next

Next week: infinitely repeated games with punishment

b By inspection,  $\pi_1$  is independent of  $S_1$  for all  $S_1 \geq S_2 = 4$ . Suppose that  ~~$S_1 = 2, S_2 = 4$~~  (and for simplicity  $P_1$  loses in the event of a tie). Then  $c_1 = S_3 = 2, q_1 = 100$ ,  $\pi_1 = (V_1 - c_1)q_1 = (10 - 2) \times 100 = 800 < 1200$ . Suppose instead that  $S_1 \leq 2$  (and again that  $P_1$  loses in the event of a tie). Then  $q_1 = 0, \pi_1 = 0 < 1200$ . Consequently,  $P_1$  has no incentive to deviate from ~~the~~ the truthful strategy profile.

By inspection,  $\pi_2$  is independent of  $S_2$  for all  $S_1 = 10 \geq S_2 \geq S_3 = 2$ . Suppose that  $S_2 \geq 10$  (and for simplicity that  $P_2$  wins in the event of a tie). Then  $c_2 = S_1 = 10, q_2 = 200, \pi_2 = (4 - 10) \times 200 = -1200 < 200$ . Suppose instead that  $S_2 \leq 2$ , then  $q_2 = 0, \pi_2 = 0 < 200$ . Consequently,  $P_2$  has no incentive to deviate from the truthful strategy profile.

Similarly,  $\pi_3$  is independent of  $S_3$  for all  $S_3 \neq 4$  (supposing  $P_3$  wins in the event of a tie). For all  $S_3 \geq 4$ , given  $S_1 = 10, S_2 = 4$ , we have  $c_3 = 4 > V_3 = 2$ , hence  $\pi_3 = (V_3 - c_3)q_3 < 0$ . Consequently,  $P_3$  has no incentive to deviate.

No player has incentive to deviate from the truthful strategy profile, so it is an NE.

c Suppose instead that the second highest bidder receives 200 click throughs per hour. Then, at the truthful strategy profile, i.e.  $S_1=10, S_2=4, S_3=2$ , we have  $c_1=S_2=4, c_2=S_3=2, q_1=200, q_2=200, \pi_1=(V_1-c_1)q_1=1200$ ,  $\pi_2=(V_2-c_2)q_2=400$ ,  $\pi_3=0$ .

Suppose  $P_1$  deviates to  $S'_1=3$ . Then  $c_1=S_3=2$ ,  $c_2=S_1=3$ ,  $q_1=200, q_2=200, \pi_1=(V_1-c_1)q_1=1600 > 1200$ ,  $\pi_2=(V_2-c_2)q_2=200$ .  $P_1$  has incentive to deviate from the truthful strategy profile, the truthful strategy profile is not an NE under the revised example.

d A true VCG mechanism in this environment would involve two auctions (one for each sponsored

link spot, where bidders submit their bid in hourly terms (or equivalently in per click ~~#~~ through per hour, if the click through ~~rate~~ volume is equivalent between advertisers) and the winning bidder for each spot pays the second highest bid for that spot.

Google's mechanism differs because it "rolls the two auctions into one". This generates incentives for truthful reporting because players may prefer to make the second highest bid if the value of the second prize is sufficiently close to the value of the first prize. There is no such incentive in the VCG mechanism since the second highest bidder wins nothing.

Vickrey mechanism: pay externality  
P1 imposed externality on ~~B~~ of P2 of 400, P2 imposed 200 on P3

"what happens if you're not there"  
OR

"if P1 gets this unit rather than who would get it otherwise" ← this is correct to avoid double counting 

3a Given:  $0 \leq b \leq a \leq 1$

Suppose for simplicity that in the event of a tie, P2 gets, loses, wins

P2 wins one unit iff  $0 \leq b \leq a \leq x_1$  or  $0 \leq b \leq x_1 \leq a$   
 $0 \leq b \leq x_1 \leq a \Leftrightarrow b \leq x_1$ , which occurs with probability  $1-b^2$ .

P2 wins two units iff  $0 \leq x_1 \leq b \leq a$  iff  $x_1 \leq b$ ,  
which occurs with probability  $b$ .

If P2 wins one unit, P2 wins one unit of the object which P2 has valuation  $x_2$  for and P2 pays bid  $b$ , P2's payoff is  $x_2 - b$ .

If P2 wins two units, P2 has valuation  $x_2 + Y_2$  for those units, and pays  $2x_1$  which has distribution given by  $F_{X_1}(x) = x^2$

$$\begin{aligned} \text{P2's expected payoff is } & -(1-b^2)(x_2 - b) + \int_0^b (x_2 + Y_2 - 2x) \\ & (1-b^2)(x_2 - b) + \int_0^b (x_2 + Y_2 - 2x) F_{X_1}(x) dx \\ & = (1-b^2)(x_2 - b) + \int_0^b (x_2 + Y_2 - 2x) 2x dx \end{aligned}$$

✓  $\hookrightarrow x$  is price of each object

b P2 chooses By inspection, P2's expected payoff  $\Pi_2$  is independent of  $a$ . P2's optimisation problem reduces to  $\max_b \Pi_2$ .

Intuition: lower price effect dominates less winning effect

$$\begin{aligned} \text{FOC: } \frac{\partial \Pi_2}{\partial b} &= -2b(x_2 - b) + (1-b^2)x_2 - b + \frac{\partial}{\partial b} ((x_2 + Y_2) \int_0^b 2x dx) \\ &+ \int_0^b -4x^2 dx * = 0 \Leftrightarrow \\ &-2bx_2 + 2b^2 - b + b^3 + \frac{\partial}{\partial b} [(x_2 + Y_2)(b^2)] - 4b^3/3 = 0 \Leftrightarrow \\ &-2bx_2 + 2b^2 - b + b^3 + 2(x_2 + Y_2)b - 4b^2 = 0 \Leftrightarrow \\ &-2b^2 - b + b^3 + 2bY_2 = 0 \quad \cancel{b=0} \Leftrightarrow \\ &\cancel{-2b^2 - 2x_2 = 0} \quad b=0 \text{ or } \cancel{2Y_2 = 2b + 1 - b^2} \Leftrightarrow \\ &\cancel{b=0} \quad \Pi_2 = x_2 \quad b=0 \text{ or } Y_2 = -\frac{1}{2}b^2 + b + \frac{1}{2} \Leftrightarrow \\ &b=0 \text{ or } Y_2 = \frac{-1 \pm \sqrt{1-4}}{2} = 1 \text{ (reject since } P(Y_2=1)=0) \end{aligned}$$

"Fundamental theorem of calculus":  
 $\frac{d}{dy} \int_0^y F(z) dz = F(y)$

$$\begin{aligned} \text{SOC: } & 2x_2 + 4b - 1 + 3b^2 + 2x_2 + 2Y_2 - 8b \\ & = -4b - 1 + 3b^2 + 2Y_2 \end{aligned}$$

Supposing that the FOC is sufficient for a maximum, P2 maximizes  $\Pi_2$  given  $S_1 = \{0, x_1\}$  by choosing  $b=0$  and any  $a$ , so  $S_2 = \{0, x_2\}$  is a best response to  $S_1 = \{0, x_1\}$ . By symmetry,  $S_1 = \{0, x_1\}$  is a best response to  $S_2 = \{0, x_2\}$ . This strategy profile  $\Rightarrow$  The strategy profile is a BNE.

"There is some  $p$  you set the price you pay"

c The outcome is not necessarily efficient. With non-zero probability,  $x_1 > y_1 > x_2 \geq Y_2$ , i.e. P1 has higher valuation for the second unit than P2 has for the first, so it is optimal for P1 to receive both units. The outcome of the above BNE is such that each player receives one unit regardless of their valuations, so it is not necessarily efficient.

The following mechanism is efficient in this setup.

This happens with probability zero, so the assumption is not problematic (maybe not necessary)

P<sub>1</sub> and P<sub>2</sub> first bid for one unit and the highest bidder pays the second highest bid and receives one unit of the good. Then, P<sub>1</sub> and P<sub>2</sub> bid for the second unit and the highest bidder ~~pays the~~ again pays ~~for~~ the second highest bid and receives the second unit of the good. This is equivalent to the single-stage auction where each player reports two valuations, the first unit is awarded to the player with the highest valuation, who pays an amount equal to the other player's higher valuation. ~~then~~ The highest valuation is then excluded in what follows. The second unit is awarded to the player with the highest remaining valuation, who pays an amount equal to the ~~if~~ other player's highest remaining valuation.

1P. UCC

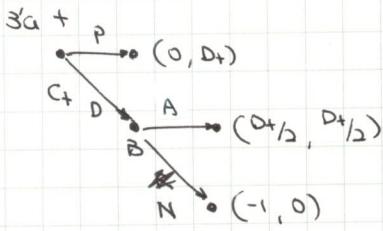
This is correct

Vickrey intuition "What is the damage (and)"

Vickrey Mechanism  $\Rightarrow$  always pay some one else's bid, your bid affects your winning p but not your payment

In a uniform price auction, each bidder submits any number of price-quantity pairs. Then the good is allocated to the highest bidder, in the ~~if~~ corresponding quantity, then to the second highest bidder, and so on until all units of the good have been allocated.

✓



For  $t \in \{1, 2\}$ , where  $C_1$  and  $C_2$  denote country 1 and country 2 respectively,  $B$  denotes the bank,  $P$  denotes pay,  $D$  denotes default,  $A$  denotes the bank's absorbing some losses to bail out the borrower, and  $\star$  denotes the bank refusing to bail out the borrower.

Given

It is natural to suppose that  $D+ > 0$ . In the ~~D-~~ subgame, solving for SPE by backward induction, in the  $D$ -subgame, by inspection, it is optimal for  $B$  to choose  $A$ . Then, ~~in the~~ it is optimal for  $C_t$  to choose  $D$ , which yields a payoff of  $D+/2$  while  $P$  yields a payoff of 0. In the one shot game,  $C_t$  defaults knowing that the bank rationally will bail ~~#~~ it out.

b The only SPE of the ~~two stage~~ two period game is such that each country defaults and the bank bails out each country.

Solving for ~~#~~ SPE by backward induction, from (a), ~~the~~ the equilibrium strategy of country 2,  $s_{C_2}^{2*} = D$  and the equilibrium action of the bank in period 2, ~~of~~  $s_B^{2*} = A$ . This is regardless of  $C_1$  and  $B$ 's actions in period 1. So  $C_2$ 's payoff and  $B$ 's payoff in period 2 are independent of earlier actions, in period 1,  $B$  maximises total payoff by maximising period 1 payoff, i.e.  $B$  treats each period as an independent one shot game. Then, by the result in (a), the SPE is  $s_B^* = AA$ , ~~and~~  $s_{C_1}^* = D$ ,  $s_{C_2}^* = D$ .

c Let  $\gamma^*$  denote the probability that  $C_t$  assigns to  $B$ 's being  $T$  (tough) at the stage where  $C_t$  is called to act. Let  $R$  denote the rational bank. In what follows, superscripts denote the period, and subscripts denote the player and type, and  $*$  denotes eqm values.

Given  $\forall t: D_t > 0 \Rightarrow D_{t/2} > 0 \Rightarrow s_R^{2*} = A$

Given:  $s_T^1 = s_T^{2*} = \star$

$$\Rightarrow \pi_C^2(s_C^2 = D | \gamma^{\star 2}) = \gamma^2(-1) + (1 - \gamma^2)D+/2$$

Given:  $\pi_C^2(s_C^2 = P) = 0$

$$s_C^2 = D \text{ if } -\gamma^2 + ((1 - \gamma^2)D+/2) > 0$$

$$s_C^2 = P \text{ if } -\gamma^2 + ((1 - \gamma^2)D+/2) \leq 0$$

If  $-\gamma^2 + ((1 - \gamma^2)D+/2) \geq 0$ ,  $C_2$  potentially maxes

Suppose  $s_R^{1*} = (1-q)A + qR$  for  $q \in [0, 1]$

$$\text{Given: } p^1 = 1/2$$

$$\text{By Baye's rule, } p^2 = \frac{1/2}{1/2 + q/2} \text{ if } \alpha_B^1 = R, \\ p^2 = \frac{1/2}{1/2 + q/2} = \frac{1}{1+q} \text{ if } \alpha_B^1 = P, p^2 = 0$$

Suppose that

$$-p_2 + (1-p^2)D_{1/2} = 0 \iff p^2(1+D_{1/2}) = D_{1/2} \iff \\ p^2 = \frac{D_{1/2}}{1+D_{1/2}}$$

$$\text{Suppose that } p^1 = 1/2 > D_{1/2}/2 + D_2. \text{ Then if } \alpha_C^1 = R, q = 1, \frac{1-q}{1+q} = 1 \\ p^2 = p^1 = 1/2 = \frac{D_{1/2}}{2+D_2} \Rightarrow \Pi_C^2(S_C^2 = P) > \Pi_C^2(S_C^2 = D | p^2) \Rightarrow \\ C \text{ chooses } P. \text{ If } q = 1, p^1 = 1/2 > D_{1/2}/2 + D_2 \Rightarrow \Pi_C^1(S_C^1 = P) > \\ \Pi_C^1(S_C^1 = D | p^1) \Rightarrow C \text{ chooses } P.$$

$$\Pi_C^1(S_C^1 = D | p^1) = (p^1 + (-p^1)q)(-1) + (1-p^1)(1-q)D_{1/2} \\ \Pi_C^1(S_C^1 = P | p^1) = 0 \\ (p^1 + (-p^1)q)(-1) + (1-p^1)(1-q)D_{1/2} = 0 \iff \\ -p^1 + p^1q = q + D_{1/2} \\ -p^1q + p^1q + (1-p^1-q+p^1q)D_{1/2} = 0 \iff \\ -q + (1-q)D_{1/2} = -p^1q + (-p^1+q^1)D_{1/2} \iff \\ -q + (1-q)D_{1/2} = p^1(1-q) + (-p^1)(1-q)D_{1/2} \iff \\ -q + (1-q)D_{1/2} = p^1((1-q) + (1-q)D_{1/2}) \iff \\ p^1 = \frac{-q + (1-q)D_{1/2}}{(1-q) + (1-q)D_{1/2}} = \frac{-2q + (1-q)D_1}{(1-q)(2+D_1)}$$

Suppose that  $\alpha_C^1 = D$ . ~~then if  $q = 1$ ,~~

$$\Pi_B^1(q=1) =$$

$$\text{Suppose} \\ -p_2 + (1-p^2)D_{1/2} = 0 \iff p^2(1+D_{1/2}) = D_{1/2} \iff p_2 = \frac{D_{1/2}}{1+D_{1/2}}$$

Suppose  $p^1 > D_{1/2}/2 + D_2$ , then if  $\alpha_C^1 = D$

$$\Pi_B^1(q=1) = D_{1/2}, \quad \Pi_B^1(q=0) = D_{1/2} + D_2/2$$

$$\pi_C^2(S_C^2 = D | \mu^2) = \pi_C^2(S_C^2 = P) \Leftrightarrow$$

$$\mu^2(-1) + (1-\mu^2)D_2/2 = 0 \Leftrightarrow$$

$$D_2/2 = \mu^2 * (1 + D_2/2) \Leftrightarrow$$

$$\mu^2 = D_2/2 + D_2 \Rightarrow \mu^2 > 1/2 \text{ (given } D_2 > 2)$$

so if R plays  $\mathbb{N}$  with certainty at  $t=1$ , then by Bayes rule,  $\mu^2 = \mu^1 = 1/2$ , then C<sub>2</sub> finds it optimal to play D, then R would be better off playing A in  $t=1$ . So at PBE, R does not play P with certainty in  $t=1$ .

If R plays A with certainty at  $t=1$ , then by Bayes rule,  $\mu^2 = 1$  if R plays  $\mathbb{N}$ , then C<sub>2</sub> finds it optimal to play P, then R would be better off playing R in  $t=1$ . So at PBE, R does not play A with certainty in  $t=1$ .

Intuitively, R has some incentive to imitate T and thereby deter efficient D of C<sub>2</sub>, but if R perfectly imitates T, then R's playing  $\mathbb{N}$  is uninformative and fails to deter C<sub>2</sub> from D-ing, perfect imitation fails to build a reputation for being T.

in  $t=1$   
 If R plays A, then by Bayes rule,  $\mu^2 = 0$ , then C<sub>2</sub> finds it optimal to D and R always A<sub>2</sub>. If R plays N in  $t=1$ , then C<sub>2</sub> mixes. If  $a_R^1 = N$  induces D with certainty, R's incentive would be better off had it chosen  $a_R^1 = A$ . If  $a_R^1 = N$  induces P with certainty, then, supposing  $D_2 > D_1$ , R has strict incentive to choose  $a_R^1 = N$  and does not mix.

Intuitively, if imitating T is entirely effective at deterring C<sub>2</sub>'s D-ing, then R has incentive to perfectly imitate T, then such imitation fails to deter C<sub>2</sub>'s D-ing. So it cannot be that imitating T is entirely effective at deterring C<sub>2</sub>'s D-ing.

q is such that C<sub>2</sub> is indifferent between P and D, then q is such that  $\mu^2 = D_2/2 + D_2$ . By Bayes rule,

$$\mu^2 = \mu^1/\mu^1 + (1-\mu^1)q. \text{ Then}$$

$$\frac{D_2}{2+D_2} = \mu^1/\mu^1 + (1-\mu^1)q \Leftrightarrow$$

$$D_2(\mu^1 + (1-\mu^1)q) = (2+D_2)\mu^1 \Leftrightarrow$$

$$D_2(\frac{1}{2} + \frac{q}{2}) = (2+D_2)\frac{\mu^1}{2} \Leftrightarrow$$

$$D_2(q+1) = 2 + D_2 \Leftrightarrow D_2q = 2 \Leftrightarrow q = 2/D_2$$

$$\begin{aligned} \pi_C^1(a_C^1 = D) &= (\mu^1 + (1-\mu^1)q)(-1) + (1-\mu^1)(1-q)D_1/2 \\ &= -(\frac{1}{2} + \frac{q}{2}) + (\frac{1}{2})(1-q)D_1/2 \\ &= -(\frac{1}{2} + \frac{1}{D_2}) + (\frac{1}{2}(D_2 - 2/D_2))D_1/2 \\ &= -(\frac{1}{2} + \frac{1}{D_2}) + (\frac{1}{2} - \frac{1}{D_2})(D_1/D_2) \end{aligned}$$

$$\pi_C^1(a_C^1 = P) = 0$$

$$\pi_C^1(a_C^1 = D) > \pi_C^1(a_C^1 = P) \Leftrightarrow$$

$$\frac{1}{2} + \frac{1}{D_2} < (\frac{1}{2} - \frac{1}{D_2})(\frac{D_1}{D_2})$$

$$D_2/2D_1 + 1/D_1 < (\frac{1}{2} - \frac{1}{D_2})$$

Not ~~definite~~ C<sub>1</sub> defaults if this condition is satisfied.

Derivation is incomplete. Strategies of C<sub>1</sub>, C<sub>2</sub>, R to be fully spelled out, and beliefs

