QE Problem Set 3

1a $educ_i = 10.36 - 0.094 sibs_i + 0.131 meduc_i + 0.210 feduc_i + u_i$

$n = 722$

We might expect that ~~the more siblings a person has, the less~~ a person with more siblings would ~~receive~~ ~~have~~ spend fewer years in education for several reasons.

Families with more children are, ~~ceteris~~ less ~~p~~ able to invest financial and other resources in the education of each child, ceteris paribus. This is because ~~the~~ a finite pool of resources must be shared among a larger number of children.

Additionally, families with many children may require the older children to enter the workforce early to support the family financially.

Further, families with many children are disproportionately low income, and hence less able to invest in their children's education.

The coefficient on sibs is expected to be negative.

Sibs has to increase by $1/0.094 = 10.6 \approx 11$ to reduce predicted years of education by one year. The magnitude of the coefficient is relatively small.

b On average, an increase in meduc by 1 year ~~increases~~ is associated with a 0.131 year increase in educ, holding sibs and feduc constant.

Acceptable to use "educ", "meduc" etc? Or better to give what they mean in English?

The coefficient of meduc in the given regression gives a reliable estimate ~~of~~ for how a mother's educational attainment (causally) influences that of her son only if the other causal determinants of the latter are not systematically related to the former. This is unlikely.

The son's educational attainment is likely to be affected by the country/region/city he grows up in, and the quality of ~~educational~~ the education system there. Likewise for the mother. The country/region/city that mother and son grow up in are likely to be related:

The son's level of educational attainment is also likely to be ~~de~~ affected by the level of household wealth. Likewise for the mother. If the mother grows up in a wealthy household, the same is likely to be true for the son.

c $E(educ_A) = 10.36 - 0.094(0) + 0.131(12) + 0.210(12)$
$= 14.452$
$E(educ_B) = 10.36 - 0.094(0) + 0.131(16) + 0.210(16)$
$= 15.816$
$E(educ_B - educ_A) = 15.816 - 14.452 = 1.364$

The predicted difference of years of education between B and A is 1.364 years.

d Household income, ~~quality of education syste system~~ country / region / city (and hence quality of household system), household wealth, might affect educ and are all likely to be correlated with sibs, meduc, and feduc.

2a Age is likely to effect earnings because older workers ~~are more experienced,~~ tend to be more experienced, and to occupy more senior positions, hence to command a higher pay.

On average, ~~an increase in age~~ a one-year increase in age is associated with a \$0.51 increase in the hourly wage rate

The coefficient of age consistently estimates a causal effect only if the other causal determinants of wage, captured in $\hat{u}_i$, are not systematically related to age. This is unlikely.

Wage is likely to be affected by the time at which a worker enters the labour force. Workers who enter the labour force during a boom are likely to have higher wages than workers who enter the labour force during a slump, even later on in their careers. A workers age is likely to be related to the time at which he enters the workforce.

b The coefficient on female is negative and relatively large. ~~In the sam~~

On average, a female worker's hourly wage is \$3.81 less than a male worker's hourly wage.

c The estimated coefficient on degree gives a ~~consistent~~ reliable estimate of the causal effect of having a university degree on earnings only if the other determinants of earnings, captured in u_i ~~are~~ are not systematically related to degree. This is unlikely.

A worker's earnings are likely to be affected by his cognitive ability, conscientiousness, country / region / city (and hence economic opportunity). These factors are likely to be related to whether or not a worker has a university degree.

To obtain a better estimate of this causal effect, data should be collected on these other factors, and ~~a~~ a multivariate regression model should be fit on the expanded dataset.

The coefficient of degree in the resulting model gives the relationship between hourly wage and the part of degree that is uncorrelated with female, age, cognitive ability, conscientiousness, and country / region / city. Given that there are no other confounding factors, this coefficient reliably estimates the causal effect of degree on wage.

~~3a~~ No.
3b

Given that total hours recorded over the four activities must add up to 168 hours, the four regressors are perfectly multicollinear. Study is perfectly predicted by sleep, work and leisure. The component of study not predicted by sleep, work, and leisure is a constant 0, that has 0 covariance with score and 0 variance. β_1 is undefined as there is no unique solution to the OLS minimisation problem.

a No.

Since total hours must add up to 168 hours, it is not possible to vary ~~sleep~~ study without varying at least one of sleep, work, and leisure.

c Score = $\beta_0 + \beta_1$study + $\beta_2$sleep + $\beta_3$work + u

β_0 gives the expected score of a student who spends all 168 hours a week in leisure.

~~β_1, β_2, and β_3 give the~~ On average, an ~~th~~ increase in study ~~t~~ (sleep, or work) by one hour and a corresponding decrease in leisure is associated with a β_1 (β_2 or β_3) ~~there~~ change in score.

Since study, ~~work, and~~ sleep, and work are not perfectly multicollinear, there is a unique solution to the OLS minimization problem.

4 Let $C = E(Y - b_0 - b_1X_1 - b_2X_2)^2$

$\partial C/\partial b_0 = 2E(Y - b_0 - b_1X_1 - b_2X_2)(-1)$

$= -2E(Y - b_0 - b_1X_1 - b_2X_2)$

$\partial C/\partial b_1 = 2E(Y - b_0 - b_1X_1 - b_2X_2)(-X_1)$

$= -2E(Y - b_0 - b_1X_1 - b_2X_2)(X_1)$

~~∂~~ $\partial C/\partial b_2 = -2E(Y - b_0 - b_1X_1 - b_2X_2)(X_2)$

~~FOCs:~~

~~$E(Y - \beta_0 - \beta_1X_1 - \beta_2X_2) = 0$~~

~~$\beta_0 = E(Y) - \beta_1E(X_1) - \beta_2E(X_2)$~~

~~$E(Y - b_0^* - b_1^*X_1 - b_2^*X_2) = 0$~~

~~$E(b_0^*) = b_0^* = E(Y) -$~~

$Y = \beta_0 + \beta_1X_1 + \beta_2X_2 + u$

Since $E(u) = 0$,

~~$E(Y) = \beta_0 + \beta_1E(X_1) + \beta_2E(X_2)$~~

$E(Y - \beta_0 - \beta_1X_1 - \beta_2X_2) = 0$

$-2E(Y - \beta_0 - \beta_1X_1 - \beta_2X_2) = 0$

~~$\partial C/\partial b_0 = 0$~~

~~$\text{cov}(Y, X_1) = E($~~ $Y - \beta_0 - \beta_1X_1 - \beta_2X_2 = u$

$\text{cov}(u, X_1) = E(X_1u) - E(X_1)E(u)$

Since $E(u) = 0$,

$\text{cov}(u, X_1) = E(X_1u)$ ~~$= E$~~ $= 0$

$= E(Y - \beta_0 - \beta_1X_1 - \beta_2X_2)(X_1)$

$= 0$

$-2E(Y - \beta_0 - \beta_1X_1 - \beta_2X_2)(X_1) = 0$

~~$\partial C/\partial \beta_1 = 0$~~

By symmetry,

$-2E(Y - \beta_0 - \beta_1X_1 - \beta_2X_2)(X_2) = 0$

$(\beta_0, \beta_1, \beta_2)$ satisfy FOCs: $\partial C/\partial b_0 = \partial C/\partial b_1 = \partial C/\partial b_2 = 0$, hence solve the population linear regression problem.

5a $E(u|X_1, X_2) = 0$

~~$E(u) = E(E(u|X_1, X_2)) = E(0) = 0$~~

~~By LIE~~

By LIE,

$E(u) = E(E(u|X_1, X_2)) = E(0) = 0$

$cov(X_1, u) = E(X_1 u) - E(X_1)E(u)$

~~Since~~ $= E(X_1 u)$ (since $E(u) = 0$)

$= E(E(X_1 u | X_1))$

$= E(E(X_1|X_1)E(u|X_1))$

$= E(0)$

$= 0$

By symmetry, $cov(X_2, u) = 0$

b No.

Let the population linear regression of Y on X_1 alone be $Y = \beta_0' + \beta_1' X_1 + u'$

$\beta_0', \beta_1' = \text{argmin}_{b_0', b_1'} E(Y - b_0' - b_1' X_1)^2$

$\beta_0, \beta_1, \beta_2 = \text{argmin}_{b_0, b_1, b_2} E(Y - b_0 - b_1 X_1 - b_2 X_2)^2$

Let the population linear regression of X_1 on X_2 be $X_1 = \pi_0 + \pi_1 X_2 + \tilde{X}_1$,

$\tilde{X}_1 = X_1 - \pi_0 - \pi_1 X_2$

~~$\beta_1' = co$~~ It can be shown that

$\beta_1' = cov(Y, X_1) / var(X_1)$

$\beta_1 = cov(Y, \tilde{X}_1) / var(\tilde{X}_1)$

$= (cov(Y, X_1) - \pi_1^2 cov(Y, X_2)) /$

$(var(X_1) - \pi_1^2 var(X_2) - 2\pi_1 cov(X_1, X_2))$

It is not necessarily the case that $\beta_1 = \beta_1'$

β_1 captures only the relationship between the component of X_1 uncorrelated with X_2 and Y, while β_1' captures the entire relationship between X_1 and Y. This doesn't feel quite precise enough

6a $E(X_i u_i | X_i) = E(X_i|X_i)E(u_i|X_i)$ (since given X_i, X_i is ~~a con~~ fixed)

$= 0$ (since $E(u_i|X_i) = 0$)

$E[\sum_{i=1}^n X_i u_i | X_1, \dots, X_n]$

$= \sum_{i=1}^n E(X_i u_i | X_1, \dots X_n)$

~~$= \sum_{i=1}^n 0$~~

~~$= 0$~~

$= \sum_{i=1}^n X_i E(u_i | X_1, \dots, X_n)$ (since given X_i, X_i is fixed)

c $E[\sum_{i=1}^n (X_i - \bar{X})(u_i - \bar{u}) | X_1, \dots, X_n]$

$= \sum_{i=1}^n (X_i - \bar{X}) E(u_i - \bar{u} | X_1, \dots, X_n)$

$= \sum_{i=1}^n (X_i - \bar{X})(E(u_i | X_1, \dots, X_n) - \bar{u})$

$= \sum_{i=1}^n -(X_i - \bar{X})\bar{u}$

$= -\bar{u}\,(\sum_{i=1}^{n} X_i - n\bar{X})$
$= 0$

c $\hat{cov}(X,u) = \cancel{\sum_{i=1}^{n} X_i u_i}$
$= 1/n \sum_{i=1}^{n} X_i u_i - (1/n \sum_{i=1}^{n} X_i)(1/n \sum_{i=1}^{n} u_i)$
$\hat{cov}(X,Y) = 1/n \sum_{i=1}^{n} X_i Y_i - (1/n \sum_{i=1}^{n} X_i)(1/n \sum_{i=1}^{n} Y_i)$

RTP $\beta_1 = (\hat{cov}(X,Y) - \hat{cov}(X,u)) / \hat{var}(X)$

$\cancel{cov(X,Y) = cov(X, \beta_0 + \beta_1 X + u)}$
$\cancel{= \beta_1 var(X) + cov(X,u)}$
$\cancel{\beta_1 = -cov(X,u)/var(X)}$
$\hat{var}(X) = (1/n)\sum_{i=1}^{n} X_i^2 - (1/n \sum_{i=1}^{n} X_i)^2$
$\cancel{\hat{cov}(X,u)/\hat{var}(X)}$
$\cancel{= [1/n \sum_{i=1}^{n} X_i u_i - (1/n \sum_{i=1}^{n} X_i)(1/n \sum_{i=1}^{n} u_i)] / [1/n \sum_{i=1}^{n} X_i^2 - (1/n \sum_{i=1}^{n} X_i)^2]}$
$\hat{cov}(X,Y) - \hat{cov}(X,u)$
$= 1/n \sum_{i=1}^{n} X_i(Y_i - u_i) - (1/n \sum_{i=1}^{n} X_i)(1/n \sum_{i=1}^{n}(Y_i - u_i))$
$= 1/n \sum_{i=1}^{n} X_i(Y_i - u_i) - \bar{X}(\bar{Y} - \bar{u})$
$= 1/n \sum_{i=1}^{n} X_i(\beta_0 + \beta_1 X_i) - \bar{X}(\beta_0 + \beta_1 \bar{X})$
$= 1/n \sum_{i=1}^{n} \beta_0 X_i + 1/n \sum_{i=1}^{n} \beta_1 X_i^2 - \beta_0 \bar{X} - \beta_1 \bar{X}^2$
$= 1/n \sum_{i=1}^{n} \beta_1 X_i^2 - \beta_1 \bar{X}^2$
$= \beta_1 (1/n \sum_{i=1}^{n} X_i^2 - \bar{X}^2)$
$= \cancel{\beta_1}\ \beta_1 \hat{var}(X)$
$\cancel{\hat{cov}(X,Y)/var}$
$\hat{\beta}_1 = \hat{cov}(X,Y)/\hat{var}(X) = \beta_1 + \hat{cov}(X,u)/\hat{var}(X)$

d $E(\hat{\beta}_1) = E(\beta_1) + E(\hat{cov}(X,u)/\hat{var}(X))$
$\cancel{= \beta_1 +}$
$= \beta_1 + E[\,E(\sum_{i=1}^{n}(X_i - \bar{X})(u_i - \bar{u}) \mid X_1, \dots, X_n)/\hat{var}(X)]$ is this step legitimate?
$= \beta_1 + E[0/\hat{var}(X)]$
$= \beta_1$

7a $\hat{\beta}_1 = \hat{cov}(X,Y)/\hat{var}(X)$
$\hat{\beta}_1^* = \hat{cov}(X^*, Y^*)/\hat{var}(X^*)$
$= \hat{cov}(bX, aY + c)/\hat{var}(bX)$
$= ab\,\hat{cov}(X,Y)/b^2\hat{var}(X)$
$= (a/b)\,\hat{cov}(X,Y)/\hat{var}(X)$
$= (a/b)\,\hat{\beta}_1$

$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$
$\hat{\beta}_0^* = \bar{Y}^* - \hat{\beta}_1^* \bar{X}^*$
$= a\bar{Y} + c - (a/b)\hat{\beta}_1(b\bar{X})$
$= a\bar{Y} - a\hat{\beta}_1\bar{X} + c$
$= a\hat{\beta}_0 + c$

b $s_{\hat{u}} = [\frac{1}{n-k-1} \sum_{i=1}^{n}(Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i)^2]^{1/2}$
$s_{\hat{u}}^* = [\frac{1}{n-k-1} \sum_{i=1}^{n}(Y_i^* - \hat{\beta}_0^* - \hat{\beta}_1^* X_i^*)^2]^{1/2}$
$= [\frac{1}{n-k-1} \sum_{i=1}^{n}(aY_i + c - (a\hat{\beta}_0 + c) - (a/b)\hat{\beta}_1(bX_i))^2]^{1/2}$
$= [\frac{1}{n-k-1} \sum_{i=1}^{n}(aY_i - a\hat{\beta}_0 - a\hat{\beta}_1 X_i)^2]^{1/2}$
$= a[\frac{1}{n-k-1} \sum_{i=1}^{n}(Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i)^2]^{1/2}$
$= a\,s_{\hat{u}}$

SER increases from the $\{Y_i, X_i\}$ sample to the $\{Y_i^*, X_i^*\}$ sample by ~~a~~ factor a, equivalent to the scaling factor of Y. Translation of Y by c and scaling of x by factor b have no effect on the error $Y^* - \hat{\beta}_0^* - \hat{\beta}_1^* x^*$ since the former is exactly offset by an increase from $\hat{\beta}_0$ to $\hat{\beta}_0^*$ and the latter is exactly offset by a ~~scaling of~~ ~~deff~~ change from $\hat{\beta}_1^*$ to $\hat{\beta}_1^*$.

b Total sum of squares

$TSS = \sum_{i=1}^{n} (Y_i - \bar{Y})^2$

$$
\begin{aligned}
TSS^* &= \sum_{i=1}^{n} (Y_i^* - \bar{Y}^*)^2 \\
&= \sum_{i=1}^{n} (aY_i + c - (a\bar{Y}^* + c))^2 \\
&= \sum_{i=1}^{n} (aY_i - a\bar{Y})^2 \\
&= a^2 \sum_{i=1}^{n} (Y_i - \bar{Y})^2 \\
&= a^2 TSS
\end{aligned}
$$

Sum of squared residuals

$SSR = ((n-k-1)(s_{\hat{u}}))^2$

$$
\begin{aligned}
SSR^* &= ((n-k-1)(s_{\hat{u}}^*))^2 \\
&= ((n-k-1)(a s_{\hat{u}}))^2 \\
&= a^2 SSR
\end{aligned}
$$

$R^2 = 1 - SSR/TSS$

$$
\begin{aligned}
R^{2*} &= 1 - SSR^*/TSS^* \\
&= 1 - a^2 SSR / a^2 TSS \\
&= 1 - SSR/TSS \\
&= R^2
\end{aligned}
$$

Since R^2 is a relative measure of the variability of Y (Y^*) around $\bar{Y}$ ($\bar{Y}^*$), it is not affected by ~~any of the transf linear transfo~~ the scaling of Y to Y^*.

8 $\cancel{Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i}$

$\cancel{X}$ $Y_i = \cancel{\hat{\beta}_0 + \hat{\beta}_1 X_{1i} + \hat{\beta}_2 X_{2i} + \hat{u}_i}$

$\cancel{X_{1i} = \pi_0 + \pi_1 X_{2i} + \tilde{X}_{1i}}$

$\cancel{\hat{\tilde{X}}_{1i} = \quad X_{1i} = \hat{\pi}_0 + \hat{\pi}_1 X_{2i} + \hat{\tilde{X}}_{1i}}$

$\cancel{\hat{\tilde{X}}_{1i} = X_{1i} - \hat{\pi}_0 - \hat{\pi}_1 X_{2i}}$

$\cancel{\hat{\pi}_0 = \hat{\pi}_1 = \widehat{cov}(X_1, X_2) / \widehat{var}(X_2)}$

$\cancel{\hat{\pi}_0 = \bar{X}_1 - \hat{\pi}_1 \bar{X}_2}$

$\cancel{Y_i = \hat{\beta}_0 + \hat{\beta}_1(\hat{\pi}_0 + \hat{\pi}_1 X_{2i} + \hat{\tilde{X}}_{1i}) + \hat{\beta}_2 X_{2i} + \hat{u}_i}$

$\cancel{= \hat{\beta}_0 + \hat{\beta}_1 \hat{\pi}_0 + \hat{\beta}_1 \hat{\pi}_1 X_{2i} + \hat{\beta}_1 \hat{\tilde{X}}_{1i} + \hat{\beta}_2 X_{2i} + \hat{u}_i}$

$\cancel{= \hat{\gamma}_0 + \hat{\beta}_1 \tilde{X}_{1i} + \hat{\gamma}_2 X_{2i} + \hat{u}_i}$

~~where~~ $\cancel{\hat{\gamma}_0 = \hat{\beta}_0 + \hat{\beta}_1 \hat{\pi}_0, \; \hat{\gamma}_2 = \hat{\beta}_2 + \hat{\beta}_1 \hat{\pi}_1}$

$\cancel{cov(Y_i, \hat{\tilde{X}}_{1i}) = cov(\hat{\gamma}_0 + \hat{\beta}_1 \hat{\tilde{X}}_{1i} + \hat{\gamma}_2 X_{2i} + \hat{u}_i, \hat{\tilde{X}}_{1i})}$

$\cancel{= \hat{\beta}_1 \widehat{cov}(\hat{\tilde{X}}_{1i}, \hat{\tilde{X}}_{1i})}$

~~by collin bilinearity of~~ $\cancel{\widehat{cov}}$~~, since~~ $\cancel{\hat{\gamma}_0}$ ~~is a constant~~

~~constant, cov~~

$Y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 X_{2i} + u_i$

$Y_i = \hat{\beta}_0 + \hat{\beta}_1 X_{1i} + \hat{\beta}_2 X_{2i} + \hat{u}_i$

$x_{1i} = \pi_0 + \pi_2 X_{2i} + \tilde{x}_{1i}$

$x_{1i} = \hat{\pi}_0 + \hat{\pi}_2 X_{2i} + \hat{\tilde{x}}_{1i}$

where

$\hat{\pi}_2 = \cancel{\widehat{cov}(x_{1i}} \; \widehat{cov}(x_1, x_2) / \widehat{var}(x_2)$

$\hat{\pi}_0 = \bar{x}_1 - \hat{\pi}_2 \bar{x}_2$

$\hat{\tilde{x}}_{1i} = x_{1i} - \hat{\pi}_0 - \hat{\pi}_2 x_{2i}$

$= x_{1i} - (\bar{x}_1 - \hat{\pi}_2 \bar{x}_2) - \hat{\pi}_2 \tilde{x}_{2i}$

$= (x_{1i} - \bar{x}_1) - \hat{\pi}_2 (x_{2i} - \bar{x}_2)$

$Y_i = \hat{\beta}_0 + \hat{\beta}_1 (\hat{\pi}_0 + \hat{\pi}_2 x_{2i} + \hat{\tilde{x}}_{1i}) + \hat{\beta}_2 x_{2i} + \hat{u}_i$

$= (\hat{\beta}_0 + \hat{\beta}_1 \hat{\pi}_0) + \hat{\beta}_1 \hat{\tilde{x}}_{1i} + (\hat{\beta}_2 + \hat{\beta}_1 \hat{\pi}_2) x_{2i} + \hat{u}_i$

$= \gamma_0 + \hat{\beta}_1 \hat{\tilde{x}}_{1i} + \gamma_2 x_{2i} + \hat{u}_i$

$\widehat{cov}(Y, \hat{\tilde{x}}_1)$

$= \widehat{cov}(\gamma_0 + \hat{\beta}_1 \hat{\tilde{x}}_1 + \gamma_2 x_2 + \hat{u}, \hat{\tilde{x}}_1)$

$= \sum_{i=1}^{n} ((\gamma_0 + \hat{\beta}_1 \hat{\tilde{x}}_{1i} + \gamma_2 x_{2i} + \hat{u}_i)(\hat{\tilde{x}}_{1i}))$

$- \sum_{i=1}^{n} (\gamma_0 + \hat{\beta}_1 \hat{\tilde{x}}_{1i} + \gamma_2 x_{2i} + \hat{u}_i) \sum_{i=1}^{n} \hat{\tilde{x}}_{1i}$

$\cancel{= \sum_{i=1}^{n} (\gamma_0 \hat{\tilde{x}}_{1i} + \hat{\beta}_1 (\hat{\tilde{x}}_{1i})^2 + \gamma_2 x_{2i} \hat{\tilde{x}}_{1i} + \hat{u}_i \hat{\tilde{x}}_{1i}})$

$\cancel{= [n\gamma_0 + n\gamma_2 \bar{x}_2 + n\bar{u} + \hat{\beta}_1 \sum_{i=1}^{n} \hat{\tilde{x}}_{1i}] \sum \hat{\tilde{x}}_{1i}}$

$= \cancel{\gamma_0 \sum_{i=1}^{n} \hat{\tilde{x}}_{1i} + \hat{\beta}_1 \sum_{i=1}^{n} (\hat{\tilde{x}}_{1i})^2 + \gamma_2 \bar{x}_2 \sum_{i=1}^{n} \hat{\tilde{x}}_{1i} + \bar{u} \sum_{i=1}^{n} \hat{\tilde{x}}_{1i}}$

$\cancel{= \sum_{i=1}^{n} (\gamma_0 \hat{\tilde{x}}_{1i}}$

$= \sum_{i=1}^{n} (\gamma_0 \hat{\tilde{x}}_{1i} + \hat{\beta}_1 (\hat{\tilde{x}}_{1i})^2 + \gamma_2 x_{2i} \hat{\tilde{x}}_{1i} + \hat{u}_i \hat{\tilde{x}}_{1i})$

$= \gamma_0 \sum_{i=1}^{n} \hat{\tilde{x}}_{1i} + \hat{\beta}_1 \sum_{i=1}^{n} (\hat{\tilde{x}}_{1i})^2 + \gamma_2 \sum_{i=1}^{n} x_{2i} \hat{\tilde{x}}_{1i} + \sum_{i=1}^{n} \hat{u}_i \hat{\tilde{x}}_{1i}$

$= \hat{\beta}_1 \sum_{i=1}^{n} (\hat{\tilde{x}}_{1i})^2$

$= \hat{\beta}_1 \widehat{var}(\hat{\tilde{x}}_1)$

$\hat{\beta}_1 = \widehat{cov}(Y, \hat{\tilde{x}}_1) / \widehat{var}(\hat{\tilde{x}}_1)$