Philosophical Logic Paper 180608

1a) Consider arbitrary B-model $M=\langle W,R,I\rangle$ and arbitrary world $w\in W$.
Suppose for reductio that

(1) $V_M(\Diamond\Box P\to\Box\Diamond P, w)=0$
  (1), $\to$ $\Rightarrow$
(2) $V_M(\Diamond\Box P, w)=1$
(3) $V_M(\Box\Diamond P, w)=0$
  (2), $\Diamond$ $\Rightarrow$
(4) $\exists u\in W, Rwu : V_M(\Box P, u)=1$
  (4), $\Box$ $\Rightarrow$
(5) $\exists u\in W, Rwu : \forall v\in W, Ruv : V_M(P,v)=1$
  (3), $\Box$ $\Rightarrow$
(6) $\exists u\in W, Rwu : V_M(\Diamond P, u)=0$
  (6), $\Diamond$ $\Rightarrow$
(7) $\exists u\in W, Rwu : \forall v\in W, Ruv : V_M(P,v)=0$
  (5), ~~refle~~ symmetry of R on W $\Rightarrow$
(8) $V_M(P,w)=1$
  (7), symmetry of R on W $\Rightarrow$
(9) $V_M(P,w)=0$
  (8), (9), reductio $\Rightarrow$
(10) $V_M(\Diamond\Box P\to\Box\Diamond P, w)=1$
  (10), generalisation, definition of $\models_B$ $\Rightarrow$
(11) $\models_B \Diamond\Box P\to\Box\Diamond P$

Consider the following S4-countermodel.

$M=\langle W,R,I\rangle$
$W=\{0,1,2\}$
$R=\{\langle 0,1\rangle,\langle 0,2\rangle,\dots\}$
The remaining pairs are those ~~of~~ implied by the reflexivity and transitivity of R on W.
$I(P,1)=1$, $I(\alpha,w)=0$ for all other sentence letters and worlds α,w.

$V_M(\Diamond\Box P\to\Box\Diamond P, 0)=0$ $\Rightarrow$
$\not\models_{S4} \Diamond\Box P\to\Box\Diamond P$.

b i No. Consider the following counterexample.

$M=\langle W,R,I\rangle$
$W=\{0\}$
$R=\varnothing$
$I(\alpha,w)=0$ for all sentence letters and worlds α,w.

M is an X model, but R is not reflexive on W, so M is not a S5-model.

ii Consider arbitrary K-model $M=\langle W,R,I\rangle$, ~~arbitr~~ arbitrary X-model $M^*=\langle W,R^*,I\rangle$ such that $R\subseteq R^*$, arbitrary world $w\in W$, and arbitrary MPL-wff ϕ. Suppose for conditional proof that $V_M(\Diamond\phi,w)=1$. Then, by $\Diamond$ clause, $\exists v\in W, Rwv$: $V_M(\phi,v)=1$. Consider arbitrary $v''\in W$ such that R^*wv''. Given that $R\subseteq R^*$, we have that ~~R^*~~ $\exists v\in W, R^*wv : V_{M^*}(\phi,v)=1$. ~~By const.~~ Denote one such v as v'. Then, by construction of R^*, $R^*v''v'$, then ~~$V_{M^*}(\phi$ $\Diamond$~~ by $\Diamond$ clause, $V_{M^*}(\Diamond\phi, v'')=1$. By generalisation, $\forall v''\in W$, $R^*wv'' : V_{M^*}(\Diamond\phi, v'')=1$, then by $\Box$ clause, $V_{M^*}(\Box\Diamond\phi, w)=1$. By conditional proof, generalisation, for any K-model $M=\langle W,R,I\rangle$, any X-model $M^*=\langle W,R^*,I\rangle$ with $R\subseteq R^*$, any MPL-wff ϕ, and any $w\in W$, if $V_M(\Diamond\phi,w)=1$ then $V_{M^*}(\Box\Diamond\phi,w)=1$.

iii Every S5-model is an X-model. Consider arbitrary S5-model $M=\langle W,R,I\rangle$. ~~Suppose for conditional proof that for $t,u,v\in W$.~~ Consider arbitrary $t,u,v\in W$. Suppose for conditional proof that $\langle t,u\rangle,\langle t,v\rangle\in R$. By symmetry of R on W, $\langle u,t\rangle\in R$. By transitivity, $\langle u,v\rangle\in R$. By conditional proof, ~~definition of an X-model~~, generalisation, ~~every~~ definition of an X-model, generalisation, every S5-model is an X-model. (If R is an equivalence relation on W then R is euclidean on W).

~~Suppose for conditional proof that $\not\models_{S5}\phi$~~
Consider arbitrary MPL-wff ϕ. Suppose for conditional proof that $\not\models_{S5}\phi$. By definition of $\models_{S5}$, there exists S5-model $M=\langle W,R,I\rangle$ ~~such that $V_M\phi$~~ and world $w\in W$ such that $V_M(\phi,w)=0$. ~~Then there exists X-model~~ Then there exists X-model $M'=\langle W',R',I'\rangle$ and world such that $V_{M'}(\phi,w')=0$ namely $M'=M$ and $w'=w$. By definition of ~~$\not\models$~~ $\models_X$, $\not\models_X\phi$. By conditional proof, generalisation, for all MPL-wff ϕ, if $\not\models_{S5}\phi$ then $\not\models_X\phi$, so if $\models_X\phi$ then $\models_{S5}\phi$.

iv No. Consider the following counterexample.

$M=\langle W,R,I\rangle$
$W=\{0,1,2\}$
$R=\{\langle 0,1\rangle,\langle 1,2\rangle\}$
$I(P,1)=1$, $I(\alpha,w)=0$ for all other sentence letters and worlds α,w.
$\phi=P$

$V_M(\Box\phi\to\Box\Box\phi, 0)=0$ $\Rightarrow$
~~$\not\models_{S4}$~~ $\not\models_X \Box\phi\to\Box\Box\phi$.

Euclideanness does not imply transitivity.

c. ~~The following~~ Belief is not factive, i.e. ~~we~~ (fixed agent) S can believe things that a[re] false, so we do not want $\Box\phi\to\phi$ to be a logical truth in a ~~logic~~ logic for doxasti[c] modality. This rules out any ~~some~~ of the conventional systems stronger than D, which are reflexive. Euclideanness (~~which property of R a~~ which is the sole property of R in an X model) does not imply reflexivity because, for example, the empty set is euclidean but not reflexive. So X satisfies this desideratum.

Positive introspection, i.e. that if S believes ϕ then S believes that S believes ϕ, is controversial. ~~We~~ It seems that we can ~~believe cer~~ believe certain things without believing that we do. For example, we can imagine a self-proclaimed amoralist who would not throw rocks into a crowd ~~b~~ because he in fact ~~thinks~~ believes this is wrong, but does not believe that he so believes. Further, if positive introspection is a logical truth, then an examiner's believing that this is an excellent answer would also believe that he believes that he believes that ... this is an excellent answer. But ~~so~~ it seems possible to have the first order belief without the 1000th order belief. So a rejection of positive introspection is potentially a desideratum for a ~~theory~~ logic of doxastic modality, and X satisfies this.

Negative introspection ~~is also con~~, i.e. if S does not believe ϕ then S believes that S does not believe ϕ, is also controversial. But ~~the~~ ~~the~~ ~~$\neg\Box\phi \to$~~ $\neg\Box\phi \to \Box\neg\Box\phi$ is a logical truth of X. We think the self-proclaimed amoralist does not believe throwing rocks into a crowd is permissible but it is not clear that he believes he does not so believe. While negative introspection does not commit X to the logical truth of ~~ϕ~~ first order belief implying 1000th order belief, it is not clear what reason we have to accept negative ~~intro s~~ introspection but not positive introspection.

It is a virtue of X that $\Box\phi \to \Box\psi$ is not a logical consequence of $\phi \to \psi$, which is the case for any conventional system stronger than K. We ~~do~~ think we could fail to believe some of the obscure implications of our beliefs. For example, we could fail to believe "~~the~~ grass is green ~~and~~ or Socrates was born on Christmas Day" even when we believe "grass is green".

X seems attractive because it abandons some of the most problematic ~~pr~~ commitments of other systems for doxastic modality. But it seems X has not abandoned enough. ~~Aban~~ Abandoning ~~even more th~~ euclideanness ~~could leave~~ would leave an "empty" and "powerless" logic. But perhaps belief ~~is~~ is in fact so unregulated or poorly behaved that this is the best we can do.

2ai $\pi\alpha_1 \ldots \alpha_n$ is a PC-wff, where π is some n-place predicate $(F, G, F_1, G_1, \ldots)$ and each of $\alpha_1, \ldots, \alpha_n$ is a term, i.e. a constant $(a, b, a_1, b_1, \ldots)$ or a variable $(x, y, x_1, y_1, \ldots)$.

If ϕ ~~is a PC~~ and ψ are PC-wffs, then so are $\sim\phi$ and $\phi \to \psi$.

If α is a variable and ϕ is a PC-wff, then so is $\forall\alpha\phi$.

All and only strings that can be shown to be PC-wffs by the above clauses are PC-wffs.

~~Ea~~ The definition of ~~SOL~~ a SOL-wff extends the above clauses in the natural way. The extended clauses are as follows.

$\pi\alpha_1 \ldots \alpha_n$ is a SOL-wff, where π is some n-place predicate or n-place predicate variable $(X, Y, X_1, Y_1, \ldots)$ and ...

If α is a variable or a predicate variable, and ...

The clause for $\sim$ and $\to$ is unchanged.

ii A PC-model is some ordered pair $\langle D, I\rangle$ where D, the domain, is some non-empty set and I, the interpretation function, is some function that assigns to each ~~constan~~ constant some element of D and to each n-place predicate some n-ary relation over D.

~~this definition is~~ the definition of a SOL-model is identical.

iii Given PC-model $M = \langle D, I\rangle$, a variable assignment g for M is some function that assigns to each variable some element of D.

~~A SOL~~ Given SOL-model $M = \langle D, I\rangle$ a variable assignment for M is some function that assigns to each variable some element of D and to each n-place predicate some element of D^n, where D^n is the set of n-ary relations over D.

iv Given PC-model M and variable assignment g for M, the PC-valuation function $V_{M,g}$ is the unique function from PC-wffs to truth values $\{0,1\}$ ~~such that~~ that satisfies the following clauses.

$V_{M,g}(\pi\alpha_1 \ldots \alpha_n) = 1$ ~~iff~~ where π is a n-place predicate and each of $\alpha_1, \ldots, \alpha_n$ is a term iff $\langle |\alpha_1|_{M,g}, \ldots, |\alpha_n|_{M,g}\rangle \in |\pi|_{M,g}$, where $|\alpha|_{M,g} = I(\alpha)$ if α is a constant and $g(\alpha)$ if α is a variable, and $|\pi|_{M,g} = I(\pi)$.

$V_{M,g}(\sim\phi) = 1$ iff $V_{M,g}(\phi) = 0$

$V_{M,g}(\phi \to \psi) = 1$ iff $V_{M,g}(\phi) = 0$ or $V_{M,g}(\psi) = 1$

$V_{M,g}(\forall\alpha\phi) = 1$ where α is a variable, ϕ is a wff iff $\forall u \in D: V_{M,g^{\alpha}_{u}}(\phi) = 1$, where g^{α}_{u} is the variant assignment that differs from g only in assigning u to α.

The definition of a SOL-valuation function is identical except in modifying the ~~first and final cla~~ first clause and the final clause as follows.

$V_{M,g}(\pi\alpha_1 \cdots \alpha_n) = 1$ ~~iff~~ ~~and $|\pi$~~ where π is a n-place predicate or a n-place predicate variable ... and $|\pi|_{M,g} = I(\pi)$ if π is a predicate, and $g(\pi)$ if π is a predicate variable.

$V_{M,g}(\forall\alpha\phi) = 1$ where α is a variable or a n-place predicate variable iff $\forall u \in D^n: V_{M,g^{\alpha}_{u}}(\phi) = 1$, where $n = 0$ if α is a variable, and $D^n = D$, and g^{α}_{u} is ... .

b $\gamma^0_0 = Rab$

$\gamma^0_1 = Rax_1 \wedge \gamma^0_0[x_1/a]$

$= Rax_1 \wedge Rx_1b$

$\gamma^0_2 = Rax_2 \wedge \gamma^0_1[x_2/a]$

$= Rax_2 \wedge Rx_2x_1 \wedge Rx_1b$

&

$\gamma^1_1 = \exists x_1 (Rax_1 \wedge Rx_1b)$

$\gamma^1_2 = \exists x_1 \gamma^0_2$

$= \exists x_1 (Rax_2 \wedge Rx_2x_1 \wedge Rx_1b)$

$\gamma^2_2 = \exists x_2 \exists x_1 (Rax_2 \wedge Rx_2x_1 \wedge Rx_1b)$

$\gamma^n_n = \exists x_n \cdots \exists x_1 (Rax_n \wedge \cdots \wedge Rx_1b)$

$\Gamma = \{\sim\gamma^0_0\} \cup \{\sim\gamma^n_n : n \in \mathbb{N}\}$

$\sim\forall X [\forall x(Rax \to Xx) \wedge \forall x\forall y(Xx \wedge Rxy \to Xy) \to Xb]$

This reads as "it is not the case that for all unary predicates X, if all things that a bears R to have X, and all things that some X thin[g] bears R to have X, then b has X"

Consider some arbitrary SOL-model $M = \langle D, I\rangle$ and some arbitrary SOL-variable assignment g for M. Suppose for conditional proof that

(1) $V_{M,g}(\sim\gamma^0_0) = 1$

(2) $\forall n \in \mathbb{N}: V_{M,g}(\sim \gamma^{n}_{n}) = 1$

Suppose for reductio that

(3) $V_{M,g}(\sim \forall X[\cdots]) = 0$

  (3), $\sim \Longrightarrow$

(4) $V_{M,g}(\forall X[\cdots]) = 1$

  (4), $\forall \Longrightarrow$

(5) $\forall u \in D': V_{M,g^{X}_{u}}([\cdots]) = 1$

Let V be the following recursively defined subset of D. For all $d \in D$, if $\langle |a|_{M,g}, d \rangle \in |R|_{M,g}$, then $d \in V$. For all $d \in D$, if exists $v \in V$ such that $\langle v, d \rangle \in |R|_{M,g}$, then $d \in V$. No other $d \in V$.

  (5) $\Longrightarrow$

(6) $V_{M,g^{X}_{V}}([\cdots]) = 1$

  By construction of V, $\rightarrow \Longrightarrow$

(7) $\forall u \in D: V_{M,g^{X}_{V}{}^{x}_{u}}(Rax \rightarrow Xx) = 1$

  (7), $\forall \Longrightarrow$

(8) $V_{M,g^{X}_{V}}(\forall x(Rax \rightarrow Xx)) = 1$

  By construction of V, $\wedge, \rightarrow \Longrightarrow$

(9) $\forall u,v \in D: V_{M,g^{X}_{V}{}^{x}_{u}{}^{y}_{v}}(Xx \wedge Rxy \rightarrow Xx) = 1$

  (9), $\forall \Longrightarrow$

(10) $V_{M,g^{X}_{V}}(\forall x \forall y(Xx \wedge Rxy \rightarrow Xx)) = 1$

  (6), (8), (10), $\wedge, \rightarrow \Longrightarrow$

(11) $V_{M,g^{X}_{V}}(Xb) = 1$

  (1), (2), construction of V $\Longrightarrow$

(12) $V_{M,g^{X}_{V}}(Xb) = 0$

  (11), (12), reductio $\Longrightarrow$

(13) $V_{M,g}(\sim \forall X[\cdots]) = 1$

  (13), conditional proof, generalization, definition of $\vDash_{SOC} \Longrightarrow$

(14) $\Gamma \vDash_{SOC} \sim \forall X[\cdots]$

Each premise n reads as "b is not a $n+1^{th}$ order R-descendant of a", and the conclusion reads as "b is not a R-descendant of a".

c SOC has significantly greater expressive resources than PC or first order logic. For ~~exam~~ example, ~~not of infinitude and a~~ sentences that use notions of infinitude or ancestral can be formalised in SOC but not PC. The ~~a~~ English "b is not a R-descendant of a" can be formalised in SOC (as in the conclusion above) but not in PC. This enables SOC but not PC to recognise "palpably" logical truths, consequences, and inconsistencies to do with ~~fin~~ infinitude and ancestral. ~~The~~ For example, SOC can recognise ~~the log~~ "Halbach is not a descendant of Williamson" as the logical consequence of "Halbach is not a son of Williamson", "Halbach is not a grandson of Williamson" and so on. If we think one function of logic is to systematically recognise ~~such~~ such logical truths, consequences, and inconsistencies, ~~that a~~ the greater expressive power of SOC would ~~to~~ seem to be a virtue.

But SOC seems to have significant set-theoretic commitments. ~~For example,~~ In PC, to quantify over a variable is to treat that variable as substitution-taking, i.e. as standing in ~~a~~ place of ~~where or simulating~~ the name of some thing ~~would~~, i.e. as simulating a name. But in SOC, to quantify over a predicate variable is not to treat a predicate variable as substitution taking but as value-taking, ~~i.e. it a to treat the predicate variable not as standing in place of something named by a predicate because predicates are not names for things, but predicates that~~ to quantify over predicate variables is not to say that all things named by predicates are so and such. For example to say ~~that~~ $\exists X$ (Aristotle X) is not to say that Aristotle is or has something named by a predicate, because it is not clear what such a thing could be, or that a predicate is a name for something. So what must be meant by $\exists X$ (Aristotle X) is that ~~Aristotle is in~~ the extension had by predicates ~~had~~ X stands in the place of contains Aristotle. ~~So~~ And these extensions are sets, so SOC is doing "set theory in sheeps clothing". It is not appropriate to think that X ranges over attributes because attributes are inadequately individuated. Then the SOC-validity of $\exists X \forall x Xx$ seems to be a SOC-commitment to the existence of the universal set. Similarly $\exists X \forall x \sim Xx$ ~~see~~ is SOC-valid, and would seem to say that the empty set exists. This violates the topic-neutrality of logic. We think logic should be silent on the existence of some sets as it is on the existence of dinosaurs.

But the logical validity of $\exists X \forall x Xx$ should not be interpreted in this way. This logical validity simply means that for all domains (which are sets) there is some ~~set~~ set, namely the domain itself, that contains all elements of the domain. The ~~log~~ SOC-validity of $\exists X \forall x Xx$ is not a SOC-commitment to the existence of the universal set, but rather a product of the set-theoretic ingredients in the definition of a SOC-model, SOC-validity, and the like. These set-theoretic "commitments" are at best modest because SOC is not committed to ~~a two-elem~~ the existence of a two-membered set, or the universal power set, or the set of all non-self-membered sets (unless a set of all sets exists, which there is good reason to doubt. So the ontological commitments of SOL are not costly at all. ~~They ar~~ They also do not violate the topic-neutrality of logic.

SOL naturally extends PC (evident from (a)) ~~so this is~~ and this is reason to accept it as logic rather than as something extra-logical like set theory.

Date

4ai Ordered triple $M = \langle W, \preceq, I \rangle$ is an SC-model iff W is some non-empty set, the set of possible worlds, $\preceq$ is some ternary relation that "encodes" some binary relation for each $w \in W$, i.e. $\preceq_w = \{\langle u, v\rangle : \langle u, v, w\rangle \in \preceq\}$, such that for all $w \in W$, $\preceq_w$ is a linear order on W, i.e. is reflexive, transitive, antisymmetric, and connected on W, and satisfies the base assumption, i.e. for all $u \in W$, $w \preceq_w u$, and I is some two-place function, the interpretation function, from sentence letters and worlds α, w to truth values $(0, 1)$ that satisfies the limit assumption, i.e. for all ~~#~~ SC-wff ϕ, if there exists $u \in W$ such that $V_M(\phi, u) = 1$, then for all $w \in W$, there exists some w-closest ϕ world, i.e. some world v such that $V_M(\phi, v) = 1$ and for all v' such that $V_M(\phi, v') = 1$, $v' \preceq_w v$.

ii

ϕ	ψ	$\sim$	ϕ	ϕ	$\rightarrow$	ψ
1	1	0	1	1	1	1
1	0	0	1	1	0	0
1	#	0	1	1	#	#
0	1	1	0	0	1	1
0	0	1	0	0	1	0
0	#	1	0	0	1	#
#	1	#	#	#	1	1
#	0	#	#	#	#	0
#	#	#	#	#	#	#

bi Given the definition of a KC-model ~~and~~, precisification of a KC-model, every precisification of a KC-model is a SC-model. Then, if $\nvDash_{KC} \phi$, there exists some KC-~~mod~~ model M, ~~and~~ some precisification M' of M, and some world $w \in W$, such that $V_{M'}(\phi, w) = 0$. M' is a ~~#~~ SC-model and the KC-~~valuat~~ valuation function agrees with the SC-valuation function for bivalent I, so there exists SC-model, namely M' ~~such~~ and world namely w such that $V_{M'}(\phi, w) = 0$, so $\nvDash_{SC} \phi$. Every SC-model is a precisification of some KC-model, so by analogous argument, if $\nvDash_{SC} \phi$ then $\nvDash_{KC} \phi$. So $\vDash_{KC} \phi$ iff $\vDash_{SC} \phi$ for all wff ϕ.

ii SC-valid hence KC-valid.

Consider arbitrary SC-model $M = \langle W, \preceq, I \rangle$ and arbitrary world $w \in W$.
Suppose for reductio that
(1) $V^{SC}_M((P \Box\!\!\rightarrow Q) \vee (P \Box\!\!\rightarrow \sim Q), w) = 0$
(1), $\vee \Rightarrow$
(2) $V^{SC}_M(P \Box\!\!\rightarrow Q, w) = 0$
(3) $V^{SC}_M(P \Box\!\!\rightarrow \sim Q, w) = 0$
(2), $\Box\!\!\rightarrow \Rightarrow$
(4) $V^{SC}_M(Q, u) = 0$, where u is the w-closest ϕ-world.
(3), $\Box\!\!\rightarrow \Rightarrow$
(5) $V^{SC}_M(\sim Q, u) = 0$, where u is the w-closest ϕ-world. (By definition of SC-model and $\Box\!\!\rightarrow$ clause, a unique such world exists).
(5), $\sim \Rightarrow$
(6) $V^{SC}_M(Q, u) = 1$
(4), (6), reductio $\Rightarrow$
(7) $V^{SC}_M((P \Box\!\!\rightarrow Q) \vee (P \Box\!\!\rightarrow \sim Q), w) = 1$
(7), generalisation, definition of $\vDash_{SC} \Rightarrow$
(8) $\vDash_{SC} (P \Box\!\!\rightarrow Q) \vee (P \Box\!\!\rightarrow \sim Q)$
(8), result in (bi) $\Rightarrow$
(9) $\vDash_{KC} (P \Box\!\!\rightarrow Q) \vee (P \Box\!\!\rightarrow \sim Q)$

Not CC-valid.

Consider the following countermodel.

$M = \langle W, \preceq, I \rangle$
$W = \{0, 1, 2\}$
$\preceq_0 = \{\langle 1, 2\rangle, \langle 2, 1\rangle, \dots\}$.
The remaining pairs are ~~to~~ those implied by reflexivity, transitivity, connectivity, and the base assumption.
$\preceq_1 = \{\langle 2, 0\rangle, \dots\}$
$\preceq_2 = \{\langle 1, 0\rangle, \dots\}$
$I(P, 1) = I(P, 2) = I(Q, 1) = 1$
$I(\alpha, w) = 0$ for all other sentence letters and worlds α, w.

$V^{CC}_M((P \Box\!\!\rightarrow Q) \vee (P \Box\!\!\rightarrow \sim Q), 0) = 0 \Rightarrow$
$\nvDash_{CC} (P \Box\!\!\rightarrow Q) \vee (P \Box\!\!\rightarrow \sim Q)$.

c According to Stalnaker, that SC semantics validate CEM is a virtue. $(\phi \Box\!\!\rightarrow \psi) \vee (\phi \Box\!\!\rightarrow \sim \psi)$ is ~~equ~~ semantically equivalent to $\sim(\phi \Box\!\!\rightarrow \psi) \rightarrow (\phi \Box\!\!\rightarrow \sim\psi)$. The converse is a logical validity under both SC and CC semantics, and is uncontroversial. So under SC semantics but not CC semantics, $\sim(\phi \Box\!\!\rightarrow \psi) \leftrightarrow (\phi \Box\!\!\rightarrow \sim\psi)$, is a logical validity. This seems to be ~~a~~ virtue of SC because in English, we do not distinguish between, for example ~~"it is not the case that if I do well on this exam"~~ "it is not the case that if I commit murder, I will be arrested" and "if I commit murder, I will not be arrested". We seem to take the two as semantically equivalent.

According to Lewis, it is a fault of SC semantics that it validates CEM. Consider Lewis' example ~~of~~ "If Bizet and Verdi were compatriots then Bizet would be Italian or if Bizet and Verdi were compatriots then Bizet would not be Italian." According to Lewis, this sentence is not true let alone a logical validity. The former disjunct is apparently not flatly true because we think if Bizet and Verdi were compatriots, then Bizet might be Italian, but not that he flatly would be Italian. Similarly, the latter disjunct seems also not flatly true. So, according to Lewis, the Bizet and Verdi sentence in English is ~~not ta be~~ not true let alone logically valid.

What, then, seems to go wrong in ~~the~~ SC-semantics, is the antisymmetry assumption. There seem to be two equally close ~~to~~ (to the actual world) possible compatriot worlds, one in which Bizet is Italian, and one in which Bizet is French, and so it simply not the case that in every closest compatriot world, Bizet is Italian, or that in every such world, Bizet is French. This is the ~~strateg~~ strategy of the counterexample in (bii). SC rules out ~~cases of~~ ~~ties in~~ ties in closeness and so cannot adequately handle Bizet and Verdi type cases.

Stalnaker rejects that the antisymmetry assumption is inappropriate. According to Stalnaker, the antisymmetry assumption is an idealising assumption in the same way that the assumption of two truth values is an idealising assumption in PL and that the assumption of sharply defined domains and extensions are idealising assumptions in PC. The assumption is not unmotivated, but derives from Stalnaker's identification of a selection function (that selects an antecedent-world to check the truth of the consequent in in evaluating the truth of a counterfactual conditional) ~~which is far more explicit~~ as primitive, which is far more explicit in Stalnaker's presentation of SC than Sider's. We might think that a selection function of this sort is primitive because when we evaluate English counterfactual conditionals, what we seem to do is select some antecedent-world and check the truth of the consequent in that world.

So in the Bizet and Verdi case, it is ambiguous, vague, or otherwise indeterminate whether Bizet would have been Italian or not, so each disjunct is in some sense indeterminate. This seems plausible so far. We would be just as hesitant to affirm either English disjunct as we would be to deny it. So the problem in the Bizet and Verdi case is a problem of indeterminacy or an instance of the "pervasive semantic underdetermination of natural language" that leaves ~~at~~ the selected world ~~sea~~ somewhat indeterminate. Then, ~~some~~ what is required is not a different set of semantics for counterfactual conditionals, but some method for dealing with this indeterminacy. Stalnaker's preferred method is a sort of supervaluation, like that in KC, ~~which is~~ whose set of logical validities is extensionally identical to that of SC.

KC leaves open the possibility of ~~that~~ indeterminate ~~tru~~ truth values and relative nearness in each model but defines truth in a model as supertruth, ~~~~ to handle this indeterminacy.

So at least on CEM, SC seems to be vindicated.

5ai ~~funct~~ Given some trivalent interpretation, the t-valuation function tV_I is the unique function from PC-wffs to truth values (1, 0, #) that satisfies the following clauses.

$tV_I(\alpha) = I(\alpha)$ for all sentence letters α.

$tV_I(\sim\phi) = \begin{cases} 1 \text{ iff } tV_I(\phi)=0 \\ 0 \text{ iff } tV_I(\phi)=1 \\ \# \text{ otherwise} \end{cases}$

$tV_I(\phi \to \psi) = \begin{cases} 1 \text{ iff } tV_I(\phi)=0 \text{ or } tV_I(\psi)=1 \text{ or } tV_I(\phi)=tV_I(\psi)=\# \\ 0 \text{ iff } tV_I(\phi)=1 \text{ and } tV_I(\psi)=0 \\ \# \text{ otherwise} \end{cases}$

$tV_I(\phi \wedge \psi) = \begin{cases} 1 \text{ iff } tV_I(\phi)=tV_I(\psi)=1 \\ 0 \text{ iff } tV_I(\phi)=0 \text{ or } tV_I(\psi)=0 \\ \# \text{ otherwise} \end{cases}$

$tV_I(\phi \vee \psi) = \begin{cases} 1 \text{ iff } tV_I(\phi)=1 \text{ or } tV_I(\psi)=1 \\ 0 \text{ iff } tV_I(\phi)=tV_I(\psi)=0 \\ \# \text{ otherwise} \end{cases}$

$\vDash_t \phi$ iff for all trivalent interpretations I, $tV_I(\phi)=1$

$\Gamma \vDash_t \phi$ iff for all trivalent interpretations I, if for all $\gamma \in \Gamma$ $tV_I(\gamma)=1$, then $tV_I(\phi)=1$.

ii $\Delta\phi = \sim(\phi \to \sim\phi)$

consider trivalent I such that $I(P) = \#$.
$tV_I(\Delta P) = 0$

Prove by induction that for all PC-wff ϕ containing only sentence letter P and connectives $\sim, \wedge, \vee$, $tV_I(\phi) = \#$ (for the I such that $I(P)=\#$ considered above)

Base case
Consider arbitrary PC-wff ϕ containing only ... such that complexity $c(\phi)=0$. Then $\phi = P$. Then $tV_I(P) = I(P) = \#$. By generalisation, for all such ϕ, $tV_I(\phi)=\#$.

Induction Hypothesis
~~Consider arbitrary~~ Given n, for all $m < n$, for all PC-wff ϕ containing only ... such that $c(\phi) = m$, $tV_I(\phi) = \#$.

Induction Step
Consider arbitrary PC-wff ϕ containing only ... such that $c(\phi) = n$. Then $\phi = \sim\psi$, $\psi \wedge \chi$ or $\psi \wedge \chi$. ~~So~~

Suppose $\phi = \sim\psi$. Then $c(\psi) = n-1$. ~~$tV_I(\phi)$ = By~~ IH, $tV_I(\psi) = \#$, by $\sim$ clause, $tV_I(\phi) = \#$.

Suppose $\phi = \psi \wedge \chi$, then $c(\psi) + c(\chi) = n-1$, $c(\psi), c(\chi) < n$, then by IH, $tV_I(\psi) = tV_I(\chi) = \#$ then by $\wedge$ clause, $tV_I(\phi) = \#$.

The argument is exactly analogous for the remaining case.

By cases, generalisation, for all such ϕ, $tV_I(\phi) = \#$.

By induction over complexity, for all ϕ PC-wff ϕ containing only ---, $tV_I(\phi) = \#$ so for no such ϕ is $tV_I(\phi) = tV_I(\Delta P)$ for each trivalent I.

bi $\nabla\phi = \sim\Delta\sim\phi$

ϕ	$\sim(\phi \to \sim\phi)$	$\Delta\phi$
1	1 1 0 0 1	1
0	0 0 1 1 0	0
#	0 # 1 # #	0

ϕ	$\sim\Delta\sim\phi$
1	1 0 0 1
0	0 1 1 0
#	1 0 # #

$\Delta\phi$ is true iff ϕ is ~~definitely~~ determinately true. $\nabla\phi$ is true iff ϕ is ~~poss~~ not determinately false.

bi $\phi \vDash_t \Delta\phi$

Consider arbitrary trivalent interpretation I.
Suppose for conditional proof that
~~$tV_I(\phi$~~
(1) $tV_I(\phi) = 1$
Suppose for reductio that
(2) $tV_I(\Delta\phi = \sim(\phi \to \sim\phi)) \neq 1$
(1), $\sim \Rightarrow$
(3) $tV_I(\sim\phi) = 0$
(1), (3), $\to \Rightarrow$
(4) $tV_I(\phi \to \sim\phi) = 0$
(4), $\sim \Rightarrow$
(5) $tV_I(\sim(\phi \to \sim\phi)) = 1$
(2), (5), reductio
(6) $tV_I(\Delta\phi) = 1$
(6), conditional proof, generalisation, definition of $\vDash_t$
(7) $\phi \vDash_t \Delta\phi$.

ii $\nvDash_t \Delta\phi \vee \Delta\sim\phi$

Consider the following counterexample
$\phi = P$
$I(P) = \#$, $I(\alpha) = 0$ for all other sentence letters α

$tV_I(\Delta\phi \vee \Delta\sim\phi) = 0 \implies$
$\not\models_t \Delta\phi \vee \Delta\sim\phi$.

iii $\models_t \Delta\phi \vee \sim\Delta\phi$

Consider arbitrary trivalent interpretation I
Suppose for reductio that
(1) $tV_I(\Delta\phi \vee \sim\Delta\phi) \neq 1$
Truth tables above $\implies$
(2) $tV_I(\Delta\phi) = 1$ or 0
(3) $tV_I(\sim\Delta\phi) = 1$ or 0
(1), (2), (3), $\vee \implies$
(4) $tV_I(\Delta\phi) = 0$
(5) $tV_I(\sim\Delta\phi) = 0$
(5), $\sim \implies$
(6) $tV_I(\Delta\phi) = 1$
(4), (6), reductio
(7) $tV_I(\Delta\phi \vee \sim\Delta\phi) = 1$
(7), generalisation, definition of $\models_t \implies$
(8) $\models_t \Delta\phi \vee \sim\Delta\phi$

iv $\{\nabla\phi, \nabla\psi\} \models_t \nabla(\phi \wedge \psi)$

Consider arbitrary trivalent interpretation I.
Suppose for conditional proof that
(1) $tV_I(\nabla\phi) = 1$
(2) $tV_I(\nabla\psi) = 1$
Suppose for reductio that
(3) $tV_I(\nabla(\phi \wedge \psi)) \neq 1$
(3), truth tables for $\nabla \implies$
(4) $tV_I(\phi \wedge \psi) = 0$
~~(4), $\wedge \implies$~~
~~(5): (6) or (7)~~
~~(6) $tV_I(\phi) = 0$~~
~~(7) $tV_I(\psi)$~~
(1), truth tables for $\nabla \implies$
(5) $tV_I(\phi) = 1$ or $\#$
(2), $\nabla \implies$
(6) $tV_I(\psi) = 1$ or $\#$
(5), (6), $\wedge \implies$
(7) $tV_I(\phi \wedge \psi) = 1$ or $\#$
(4), (7), reductio $\implies$
(8) $tV_I(\nabla(\phi \wedge \psi)) = 1$
(8), conditional proof, generalisation, definition of $\models_t \implies$
(9) $\{\nabla\phi, \nabla\psi\} \models_t \nabla(\phi \wedge \psi)$.

c The most natural interpretation of Δ and ∇ is as "definitely" and "possibly" respectively. On this interpretation, the # truth value is interpreted as denoting future-contingent indeterminacy. So, for example, we would formalise "it might rain" as ∇R ~~and~~ "it will definitely rain" as ΔR and "it will rain" as R.

Intuitively, ~~that~~ "it will definitely rain is a logical consequence of it will rain". This is result (i). It is not logically true that it ~~fails will rain or it will not rain~~ definitely will rain or definitely will not rain ~~because it~~ if and because whether it rains is genuinely chancey. ~~The~~ Then, it might or might not rain. ~~and will not defini~~ This is result (ii). It is logically true that it either will definitely rains or will not definitely rain. This is result (iii). And if it might rain and I might not have my umbrella, it seems to logically follow that it might be that it rains and I ~~forget my umbrella~~ do not have my umbrella. This is result (iv).

So t ~~seems~~ semantics seem appropriate for reasoning about future contingents.

Result (iv) seems suspicious. Suppose we formalise "This coin might land heads. This coin might land tails. So this coin might land heads ~~or~~ and land tails." as ∇H, ∇T, therefore $\nabla(H \wedge T)$. The English argument is intuitively ~~valid but the~~ invalid, but the conclusion $\nabla(H \wedge T)$, according to t semantics, is a genuine logical consequence of the premises.

What goes wrong here is not in the t-semantics but in the formalisation. When we say "the coin lands heads", we do not take this and "the coin lands tails" to be in some sense independent. We think that if the coin lands heads it does not land tails and vice versa. So when we say "the coin lands heads", we really mean "the coin lands heads and does not land tails", or we take it as understood that "the ~~coin~~ coin does not both land heads and land tails". So the above ~~argum~~ English argument is best formalised as either $\nabla(H \wedge \sim T), \nabla(T \wedge \sim H) \implies \nabla(T \wedge H)$ or $\nabla H, \nabla T, \sim(T \wedge H) \implies \nabla(T \wedge H)$, then t semantics will recognise it as invalid, aligning with our intuitions. The above formalisation has left out semantically significant content of the premises.