Microeconomics Paper MOE26

1a Consumer C maximises utility subject to production constraint.

$\max_{x,c} u(x,c)$ s.t. $x = 2c^{1/2}$

$\equiv \max_c 2c^{1/2} - \frac{1}{2}c^2$

FOC: $c^{-1/2} - c = 0 \Rightarrow$

$c = 1$

SOC: $-\frac{1}{2}c^{-3/2} - 1 < 0$

The unique solution to C's ~~optim~~ utility maximisation problem is $c = 1 \Rightarrow x = 2c^{1/2} = 2$

b A general competitive equilibrium is an allocation of goods and a price vector such that firms maximise profit, ~~consum~~ (taking prices as given), consumers maximise utility (taking prices as given), ~~and~~ profits are distributed to consumers, and markets clear, i.e. aggregate excess demand for each good/input is zero.

c The firm has revenue px and cost wc hence profit ~~for~~ $px - wc$ and profit maximisation problem

$\max_{x,c} px - wc$ s.t. $x = 2c^{1/2}$

$\equiv \max_c 2pc^{1/2} - wc$

FOC: $pc^{-1/2} - w = 0 \Rightarrow$

$c^{-1/2} = w/p \Rightarrow$

$c = (w/p)^{-2}$

SOC: $-\frac{1}{2}pc^{-3/2} < 0$

$\Rightarrow x = 2(w/p)^{-1}$

$c = (w/p)^{-2}$, $x = 2(w/p)^{-1}$ uniquely solves the firm's profit maximisation problem. This yields profit

$\pi = px - wc = 2p^2/w - p^2/w = p^2/w$

C has utility maximisation problem

$\max_{x,c} x - \frac{1}{2}c^2$ s.t. $px \leq wc + \pi$

$\equiv \max_c (wc + p^2/w)/p - \frac{1}{2}c^2$

FOC: $w/p - c = 0 \Rightarrow$

$c = w/p$

SOC: $-1 < 0$

$\Rightarrow x = (w^2/p + p^2/w)/p = (w/p)^2 + (w/p)^{-1}$

$c = w/p$, $x = (w/p)^2 + (w/p)^{-1}$ uniquely solves C's utility maximisation problem.

At ~~the~~ eqm, labour supply equals labour demand,

$w/p = (w/p)^{-2} \Rightarrow w/p = 1$

Eqm price-wage ratio $p/w = 1$

$(w/p) = 1 \Rightarrow c = w/p = 1$, $x = (w/p)^2 + (w/p)^{-1} = 2$

Eqm level of employment and output are 1 and 2 respectively.

The competitive equilibrium allocation coincides with the utility-maximising allocation found in (a). This follows from 1FWT, which says that any competitive equilibrium allocation is Pareto efficient: in this case, there is a single agent, C, so Pareto efficiency requires maximising C's utility.

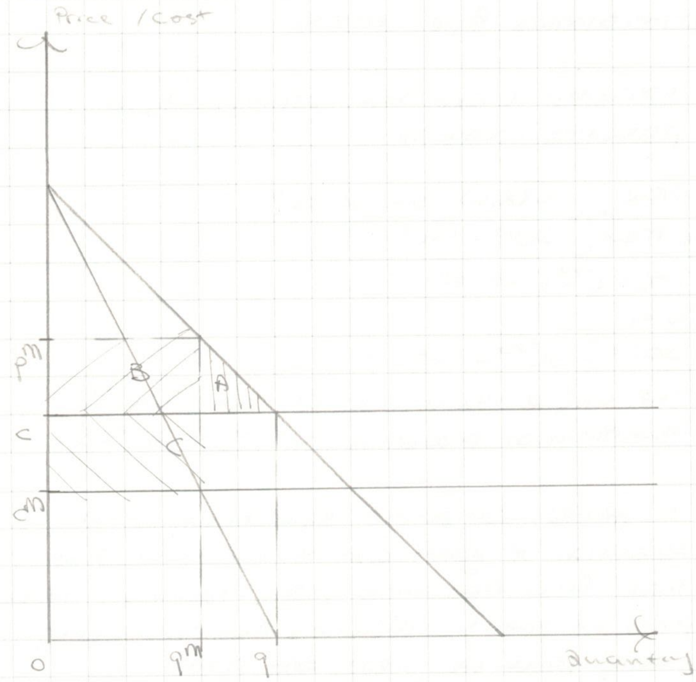
If there is a tax on wage income, real wage w/p faced by the firm is higher than real wage received by the consumer. ~~Let~~ Denote the after-tax quantities by superscript a.

$w^a/p^a = (w/p)^{-2} = c$

$2(w/p)^{-1} = (w^a/p^a)^2 + (w^a/p^a)^{-1} = x$

Labour supply and output both fall.

2 A merger to monopoly from Bertrand duopoly potentially increases aggregate welfare, but only if and because such merger realises large synergies. ~~so~~ This is the Williamson trade off.



Suppose that firms have MC c pre-merger and merger realises non-drastic synergies such that the merged firm has MC c^M. Pre-merger, by the familiar Bertrand result, the unique NE is such that both firms choose price equal to MC. Deviation to higher price is not strictly profitable ~~and no~~ because no consumers buy (given homogenous products) deviation to lower price is strictly unprofitable because then margin ~~is negat~~ hence profit is negative. Post-merger, ~~firms~~ the merged firm chooses price given by the demand curve at the intersection of MR and MC $= c^M$. ~~#~~ By supposition of non-drastic synergies, this price is greater than c.

Merger causes a decrease in consumer surplus due to the increase in price and the associated decrease in quantity. This is represented by area A+B. Merger causes an increase in profit due to ~~a~~ the increase in price and the decrease in cost. This is represented by area B+C. The change in price constitutes a transfer of surplus from consumers to ~~producers~~ producers, so the net effect on welfare is C-A. Merger has a positive effect on welfare iff ~~the~~ synergies are large and the reduction in quantity is small. In this case, consumers are strictly worse off.

Where merger realises drastic synergies, i.e. sufficiently large decrease in price such that post-merger monopoly ~~profit~~ price is lower than pre-merger ~~as~~ MC (hence competitive price) consumers benefit from lower price and higher quantity. Otherwise, consumers are strictly worse off.

3a $u(w) = 2w^{1/2}$

~~$A(x) = -u''(x)/u'(x)$~~

$u'(w) = w^{-1/2}$

$u''(w) = -\frac{1}{2}w^{-3/2}$

$A(w) = -u''(w)/u'(w)$

$= \cancel{-1/w}$

~~$\partial/\partial w\, A(w) = --w^{-2} = w^{-2}$~~

$= \frac{1}{2}w^{-1}$

$\partial/\partial w\, A(w) = -\frac{1}{2}w^{-2} < 0$

$R(w) = A(w)w$

$= \frac{1}{2}$

$\partial/\partial w\, R(w) = 0$

Absolute risk aversion $A(w)$ is decreasing in initial wealth w, relative risk aversion ~~to is~~ $R(w)$ is constant. J has CRRA and DARA preferences.

b The lottery in final wealth values is
$L = [\frac{1}{2}, \frac{1}{2}; 64, 144]$

Expected utility

$u(L) = \frac{1}{2}u(64) + \frac{1}{2}u(144)$

$= \frac{1}{2}(2\sqrt{64}) + \frac{1}{2}(2\sqrt{144})$

$= 20$

Certainty equivalent $CE(L)$ is such that $[1; CE(L)] \sim_J L$, i.e. J is indifferent between receiving $CE(L)$ with certainty and participating in L.

$u(CE(L)) = u(L) \Leftrightarrow$

$2(CE(L))^{1/2} = 20 \Leftrightarrow$

$CE(L)^{1/2} = 10 \Leftrightarrow$

$CE(L) = 100$

Expected value

$EV(L) = \frac{1}{2}(64) + \frac{1}{2}(144)$

$= 100$

Risk premium

$RP(L) = EV(L) - CE(L) \cancel{= 104 - 100}$

$= 104 - 100$

$= 4$

c The risk premium of a mean-zero lottery is approximately equal to $\frac{1}{2}A(w)\sigma^2$, where $A(w)$ is the agent's Arrow-Pratt measure of risk aversion, and σ^2 is the variance of the lottery. J's lottery, L^S has ~~that~~ 1/4 the variance of J's, so the risk premium for L^S (given identical preferences and initial wealth), is ~~approximately~~ approximately 1/4 that of L.

4a H types would like to signal their type. If H types credibly signal their type, given that firms are competitive and risk neutral, they pay wage ~~equal to prod ex Ap~~ equal to expected productivity, so H types receive wage $w_H = \theta_H > \bar{w} = \lambda\theta_H + (1-\lambda)\theta_L$. H types have ~~inc~~ incentive to signal productivity if the cost of this signal to H types is less than the increase in wage. The signal is credible iff it is too expensive for L types to ~~ee~~ profitably imitate.

b $\lambda = 1/3$, where λ is the proportion of H types. When no signal is available, competitive risk neutral firms ~~pays~~ pay each worker, regardless of type, wage equal to expected productivity $\bar{w} = \lambda\theta_H + (1-\lambda)\theta_L = 1/3(240) + 2/3(180) = 200$. Payoff to risk neutral firms is equal to expected productivity less wage. ~~At~~ At the equilibrium, given competitive firms, each firm has payoff equal to ~~outside opt~~ reservation payoff (0) from outside option (~~cessation~~ hiring no worker).

c H types signal, ~~receive w~~ L types don't. By Bayesian beliefs, the signal is perfectly informative. ~~So~~ By the argument above, firms pay ~~$w_H = \theta_H$, $w_L = \theta_L$~~ θ_H to each signalling worker, θ_L to each non-signalling worker. H types have payoff $240 - 45 = 195$. L types have payoff 180. L types have profitable deviation to signalling, which yields wage 240 and costs 55 hence yields payoff $185 > 180$. This is not a ~~separati~~ eqm.

The eqm outcome is such that neither type signals, and ~~A~~ firms pay wage $\bar{w} = 200$, the expected productivity to all non-signalling ~~players~~ workers, so each worker has payoff 200 in eqm. Firm's off eqm path beliefs are irrelevant. Regardless of these, firms pay no more than 240 to a signalling worker because no worker has productivity higher than 240. Deviation by H types yields $240 - 45 = 195$. Deviation by L types yields $240 - 55 = 185$, so neither has profitable deviation regardless of ~~beliefs~~ firm's off eqm path beliefs.

This eqm is efficient. Costly, unproductive signalling is minimised.

10a Suppose that principal P intends to induce e_L, then P maximises expected net profit ~~subject~~ given $e=1$, subject to ~~A~~ agent A's participation constraint, which is the condition that ~~parti~~ participation in the contract is weakly preferred to the outside option. Given $e=1$, expected gross (of wage) profit is fixed, so maximisation of expected net profit consists in minimisation of expected wage. P offers a fixed wage where effort is observable because a variable ~~wage~~ (profit-contingent) wage is not necessary to induce any ~~desired~~ level of effort. At the optimum, PC binds. Any candidate optimum such that PC does not bind fails to deviation by reducing w by sufficiently small amount ε such that PC remains satisfied.

$u(w_L, e_L) = \sqrt{w_L} - 1 = 0 \Rightarrow w_L = 1$

$u(w_m, e_m) = \sqrt{w_m} - 2 = 0 \Rightarrow w_m = 4$

$u(w_H, e_H) = \sqrt{w_H} - 3 = 0 \Rightarrow w_H = 9$

$\cancel{E[\pi - w_L | e_L]}$

$E[\pi(e_L) - w_L | e_L] = 10 - 1 = 9$

$E[\pi(e_m) - w_m | e_m] = \frac{4}{5}(10) + \frac{1}{5}(50) - 4 = 14$

$E[\pi(e_H) - w_H | e_H] = \frac{1}{5}(10) + \frac{4}{5}(50) - 9 = 33$

The optimal wages to induce e_L, e_m, e_H ~~#~~ respectively are $w_L = 1$, $w_m = 4$, $w_H = 9$. Given these, the contract $(w = w_H = 9, e = e_H = 3)$ maximises expected net profit for P. P chooses this contract.

b A prefers e_m to e_H iff

$\cancel{u(w_m, e_m) \geq u(w_H, e_H) \Leftrightarrow}$

$\cancel{\sqrt{w_m} - e_m \geq \sqrt{w_H} - e_H \Leftrightarrow}$

$E[u(w, e_m)|e_m] \geq E[u(w, e_H)|e_H] \Leftrightarrow$

$\frac{4}{5}u(w_1, e_m) + \frac{1}{5}u(w_2, e_m) \geq \frac{1}{5}u(w_1, e_H) + \frac{4}{5}u(w_2, e_H) \Leftrightarrow$

$\frac{4}{5}v_1 + \frac{1}{5}v_2 - e_m \geq \frac{1}{5}v_1 + \frac{4}{5}v_2 - e_H \Leftrightarrow$

$\cancel{\frac{3}{5}v_1 - \frac{3}{5}v_2 \geq}$

$\frac{3}{5}v_2 - \frac{3}{5}v_1 \leq e_H - e_m \Leftrightarrow$

$v_2 - v_1 \leq \frac{5}{3}(e_H - e_m) = \frac{5}{3}$

A prefers e_m to e_L iff

$E[u(w, e_m)|e_m] \geq E[u(w, e_L)|e_L] \Leftrightarrow$

$\frac{4}{5}u(w_1, e_m) + \frac{1}{5}u(w_2, e_m) \geq u(w_1, e_L) \Leftrightarrow$

$\frac{4}{5}v_1 + \frac{1}{5}v_2 - e_m \geq v_1 - e_L \Leftrightarrow$

$\frac{1}{5}v_2 - \frac{1}{5}v_1 \geq e_m - e_L \Leftrightarrow$

$v_2 - v_1 \geq 5(e_m - e_L) = 5$

It is incentive compatible for A to choose e_m iff A prefers e_m to e_H and A prefers e_m to e_L. This is iff, from the above, $v_2 - v_1 \leq 5/3$ and $v_2 - v_1 \geq 5$, which is a contradiction. So it is never incentive compatible for A to choose e_m.

The result that intermediate levels of effort are never incentive compatible does not generally hold. ~~because~~ In this case, the result obtains because under any variable wage contract, ~~either~~ an extreme level of effort is preferable.

c: In general, P optimally induces e_L by offering fixed wage contract $w_1 = w_2 = w_L$, which is the optimal wage to induce low effort where effort is observable, found above. This ~~con~~ satisfies A's participation constraint and trivially satisfies the incentive constraints because there is disutility of effort, so under a fixed wage contract, A has strict incentive to choose the minimum feasible effort.

P optimally induces e_H by offering variable wage contract that satisfies PC and IC with equality. ~~#~~ Any candidate optimum where PC is strictly satisfied fails to deviation by decreasing ~~w₁ an~~ ~~w₁ and w₂ each by ε~~ v_1 and v_2 each by sufficiently small amounts such that PC remains satisfied. IC remains satisfied because both LHS and RHS decrease by equal amounts. This deviation ~~#~~ decreases expected wage. Any candidate optimum where IC is strictly satisfied fails to the deviation consisting in a small mean preserving contraction of ~~w~~ w_1, w_2 and a small increase in each of the two. The former relaxes ~~the~~ PC because A is risk-averse, so the latter does not result in a violation of PC, but ~~increase~~ decreases expected wage.

PC: $\frac{4}{5}\sqrt{w_2} + \frac{1}{5}\sqrt{w_1} - 3 \geq 0$

IC: $\frac{4}{5}\sqrt{w_2} + \frac{1}{5}\sqrt{w_1} - 3 \geq \sqrt{w_1} - 1$

$\Rightarrow$

$\cancel{\frac{4}{5}v_1 + \frac{1}{5}v_2 = 3}$, $\quad \frac{4}{5}v_2 + \frac{1}{5}v_1 = 3$

$\cancel{\frac{4}{5}v_1 + \frac{1}{5}v_2 - 3 = v_1 - 1}$ $\quad \frac{4}{5}v_2 + \frac{1}{5}v_1 - 3 = v_1 - 1$

Solve simultaneously,

$\cancel{4v_1 + v_2 = 15 \Rightarrow v_2 = 15 - 4v_1}$

$\cancel{4v_1 + v_2 - 15 = 5v_1 - 5 \Rightarrow -v_1 + v_2 - 10 = 0}$

$\cancel{\Rightarrow}$

$\cancel{-v_1 + (15 - 4v_1) - 10 = 5 - 5v_1 = 0 \Rightarrow v_1 = 1}$

$\cancel{\Rightarrow v_2 = 11}$

$4v_2 + v_1 = 15$, $\quad 4v_2 + v_1 - 15 = 5v_1 - 5$

$v_1 = 15 - 4v_2$, $\quad 4v_2 + (15 - 4v_2) - 15 = 0 = 5v_1 - 5 \Rightarrow v_1 = 1$

$v_2 = 15 - v_1 / 4 = 14/4 = 7/2$

The optimal contract to induce e_H is $(w_1, w_2) = (1, (7/2)^2) = (1, 49/4)$

Expected profit from this contract is $4/5(50 - 49/4) + 1/5(10 - 1) = 32$

Expected profit from the ~~cost~~ optimal contract that induces low effort is unchanged from the observable effort case, this is ~~8~~ 9.

It is optimal to induce high effort (it is not possible to induce medium effort), so the optimal contract is $(w_1, w_2) = (1, (7/2)^2)$.

d Agency cost is equal to the difference in the expected net profit (equivalently the difference in expected wage) ~~between the~~ in inducing ~~effort~~ high effort between the observable effort case and the unobservable effort case, this is equal to $33 - 32 = 1$.

P incurs ~~so~~ an agency cost because a variable wage scheme is necessary to induce high effort, i.e. to make e_H incentive compatible. ~~This~~ Under a variable wage scheme, A bears some risk (because outcomes are uncertain even with high effort). A is risk averse (utility is concave in w), so A has positive risk premium under this variable wage scheme. Then, expected wage increases such that the contract continues to satisfy PC, i.e. participation remains individually rational. The agency cost is equal to this risk premium.

Agency cost is zero ~~off~~ if P finds it optimal to induce low effort in both observable and unobservable cases, or if A is risk neutral.