Advertising Rough Notes

Lecture

- Structure
  - Monopoly Advertising
    - Persuasive Advertising
    - Informative Advertising
    - Bagwell Model (Advertising and Capital Investment)
  - Oligopoly Advertising
    - Butters Model (Free Entry, Homogenous Products, Informative Advertising)
    - Grossman and Shapiro Model (No Entry, Differentiated Products, Informative Advertising)
- Persuasive Advertising
  - Persuasive advertising alters consumers' preferences.
  - Consider the generic Dorfman-Steiner model of monopoly advertising, where a monopolist produces some good at constant marginal cost c, chooses price p and advertising expenditure a. Demand for this product is given by $Q = Q(p, a)$ which is decreasing in p, increasing in a and strictly concave in a, i.e. $\frac{\partial Q}{\partial p} < 0$, $\frac{\partial Q}{\partial a} > 0$, $\frac{\partial^2 Q}{\partial a^2} < 0$. The monopolist maximises $\pi = (p - c)Q - a$.
  - First-order condition for p, $\frac{\partial \pi}{\partial p} = Q + (p - c)\frac{\partial Q}{\partial p} = 0$ holds. $-Q(\frac{\partial Q}{\partial p})^{-1} = p - c$, $(-\frac{p}{Q}\frac{\partial Q}{\partial p})^{-1} = \frac{p-c}{p}$, $\frac{p-c}{p} = \frac{1}{\epsilon}$, where ϵ is the price-elasticity of demand.
  - First-order condition for a, $(p - c)\frac{\partial Q}{\partial a} - 1 = 0$, $(p - c)\frac{\partial Q}{\partial a} = 1$ holds.
  - $p(\frac{p-c}{p})(\frac{\partial Q}{\partial a}) = 1$, by substitution of result from first-order condition for p, $p(\frac{1}{\epsilon})(\frac{\partial Q}{\partial a}) = 1$, multiplying across by $\frac{a}{Q}$, $p(\frac{1}{\epsilon})(\frac{\partial Q}{\partial a}\frac{a}{Q}) = \frac{a}{Q}$, rearranging, $\frac{a}{pQ} = \frac{\alpha}{\epsilon}$, where $\alpha = \frac{\partial Q}{\partial a}\frac{a}{Q}$ is the advertising-elasticitiy of demand. Advertising expenditure as a fraction of total revenue is equal to the ratio of advertising-elasticity of demand to price-elasticity of demand, at equilibrium. The Dorfman-Steiner condition holds.
  - At equilibrium, the first-order condition for p holds, hence $\frac{p-c}{p} = \frac{1}{\epsilon}$. Since c is given exogenously, p is determined entirely by ϵ. The effect of a on p at equilibrium depends on the effect of a on ϵ.
  - Consumers' valuation under persuasive advertising is given by $g(a)v$, where $g(a)$ is an increasing function of a, i.e. $g'(a) > 0$ and $g(0) = 1$, and v is consumers' intrinsic valuation, which is distributed uniformly on $[0, 1]$. Each consumer has unit demands and buys one unit of the good iff inflated (by advertising) valuation exceeds price. The number of consumers is normalised to 1.
  - The consumer with intrinsic valuation v is indifferent between buying one unit and buying no units iff $g(a)v = p$, $v = \frac{p}{g(a)}$. All consumers with higher intrinsic valuations buy one unit, hence $Q = 1 - \frac{p}{g(a)}$ subject to $Q \leq 1$, given the distribution specified above. Price-elasticity of demand, $\epsilon = -\frac{p}{Q}\frac{\partial Q}{\partial p} = -\frac{pg(a)}{g(a)-p}\frac{-1}{g(a)} = \frac{p}{g(a)-p}$. At the margins, an increase a results in an increase in $g(a)$ hence a decrease in ϵ and an increase in p.
  - By substitution of ϵ into $\frac{p-c}{p} = \frac{1}{\epsilon}$, equilibrium price $p^* = \frac{g(a)+c}{2}$. Equilibrium price is increasing with $g(a)$ hence with a. Equilibrium advertising expenditure depends on the function $g(a)$.
  - Diagrammatically, persuasive advertising results in a postive vertical scaling of the demand curve.
  - Assuming, with Dixit and Norman (1978), that consumer surplus does not depend directly on a, aggregate consumer surplus, $S(p)$ is equal to the total valuation of all units consumed, $U(Q(p))$ less the total expenditure on all units consumed $pQ(p)$, i.e. $S(p) = U(Q(p)) - pQ(p)$. $S'(p) = U'(Q(p))Q'(p) - Q(p) - pQ'(p) = [U'(Q(p)) - p]Q'(p) - Q(p) = -Q(p)$, since at equilibrium, the valuation of the marginal unit, $U'(Q(p))$ is equal to price p.
  - Social welfare $W(a)$ is equal to the sum of consumer surplus $S(p(a))$ and monopoly profit $\pi(p(a), a)$. $W(a) = S(p(a)) + \pi(p(a), a)$, $W'(a) = S'(p(a))p'(a) + \frac{\partial \pi}{\partial p}p'(a) + \frac{\partial \pi}{\partial a} = -Q(p(a))p'(a)$ by substitution of $S'(p) = -Q(p)$ from above, since at equilibrium, first-order conditions for p and a hold, i.e. $\frac{\partial \pi}{\partial p} = \frac{\partial \pi}{\partial a} = 0$. Supposing $Q > 0$ in equilibrium, effect of advertising on equilibrium welfare depends on the effect of advertising on price. Advertising improves welfare iff it decreases price.
  - From above, persuasive advertising increases price hence decreases welfare.
  - If it is instead assumed that consumer surplus depends directly on a, and that the impact of advertising should not be measured against a fixed standard (either pre-advertising valuations or post-advertising valuations) persuasive advertising has a direct positive effect on consumer surplus, and the overall effect is ambiguous. If the increase in consumer welfare due to persuasive advertising is a result of consumers' having a better understanding of the

product or an increase in the social prestige of a product (where advertising is a complement to the product), there is less reason to ignore these direct effects.

- Informative Advertising
  - Informative advertising informs consumers of the existence, price, and characteristics of a product.
  - Under informative advertising, demand is multiplicative in a function of advertising, i.e. $Q(p,a) = N(a)q(p)$, where $N(a)$ is the number of informed consumers, and $q(p)$ is the demand of each informed consumer at price p. Under the most intuitive model, $n(a)$ is increasing in a and concave, and $q(p)$ is decreasing in p.
  - Price-elasticity of demand, $\epsilon = \frac{p}{Q}\frac{\partial Q}{\partial p} = (\frac{p}{Q})(N(a))(\frac{\partial q}{\partial p}) = (\frac{p}{N(a)q})(N(a))(\frac{\partial q}{\partial p}) = \frac{p}{q}\frac{\partial q}{\partial p}$. ϵ is independent of a, hence p is independent of a. Where demand is multiplicative in a function of advertising, market price-elasticity of demand is equal to individual price-elasticity of demand, hence elasticity and price are independent of advertising expenditure.
  - Diagrammatically, informative advertising results in a positive horizontal scaling of the demand curve.
  - Let $s(p)$ denote individual consumer surplus of an informed consumer given p. Given that p is independent of a, $W(a) = N(a)s(p) + \pi(p,a)$, $W'(a) = N'(a)s(p) + \frac{\partial \pi}{\partial a} = N'(a)s(p) > 0$. Informative advertising increases welfare and equilibrium level of advertising fails to maximise welfare.
  - The monopolist chooses level of advertising below the socially optimal level because of the non-appropriability of social surplus.
  - [Insert Diagram]
- Bagwell Model (Advertising and Capital Investment)
  - A monopolist produces some good at marginal cost $c(k)$ which is a convex decreasing function of capital k, i.e. $c'(k) < 0$ and $c''(k) > 0$. The monopolist chooses price p, advertising expenditure a, and level of capital k to maximise profit $\pi = (p - c(k))Q(p,a) - rk$, where $Q(p,a)$ is the demand for the product and r is the rental rate of capital. Demand for this product is multiplicative in a function of advertising, i.e. $Q(p,a) = n(a)q(p)$, where $n(a)$ is the number of informed consumers given a, and $q(p)$ is the demand of each informed consumer given p. Under the most intuitive model, $n(a)$ is increasing in a and concave, and $q(p)$ is decreasing in p.
  - At equilibrium, the monopolist chooses p to maximise π, hence the first-order condition $\frac{\partial \pi}{\partial p} = 0$ holds and $\frac{p-c}{p} = \frac{1}{\epsilon}$.
  - At equilibrium, the monopolist chooses k to maximise π, hence the first-order condition $\frac{\partial \pi}{\partial k} = -n(a)q(p)c'(k) - r = 0$, $-n(a)q(p)c'(k) = r$.
  - An increase in advertising increases the number of informed consumers, hence the incentive to invest in capital. An increase in capital reduces marginal cost, hence reduces price.
- Butters Model (Free Entry, Homogenous Products, Informative Advertising)
  - A large number of firms in monopolistic competition each produce a homogenous good at common constant marginal cost c, choose price p and to serve informative advertisements (that include a price quote) to m random consumers at common unit advertising cost c' to maximise profit $\pi = (p - c)q - c'm$ where q is the quantity sold by this firm.
  - Each of N consumers has unit demand and common valuation for the good v. Each consumer buys no units if served no advertisements quoting $p \leq v$, buys one unit from the advertising firm if served one advertisement quoting $p \leq v$, buys one unit from the lowest-price advertising firm if served multiple advertisements quoting $p \leq v$. Suppose that $v > c + c'$, otherwise no advertising and sale occurs in equilibrium.
  - Let M denote the total number of advertisements served by firms. Let ϕ denote the fraction of consumers served at least one advertisement. A consumer's probability of being served no advertisements, $1 - \phi = (1 - \frac{1}{N})^M \simeq e^{-\frac{M}{N}}$ for large N, hence $M = N \ln \frac{1}{1-\phi}$, aggregate advertising cost of reaching the ϕ fraction of consumers is $c'M = Nc' \ln \frac{1}{1-\phi}$.
  - At equilibrium, no $p > v$ is advertised since no consumers who are served such an advertisement buy, and no $p < c + c'$ is advertised since, even if all consumers served such an advertisement buy, the firm has negative margin on units thus sold. Butters (1977) shows that at equilibrium, any price p such that $c + c' \leq p \leq v$ is advertised.
  - Let $x(p)$ denote the probability that an advertisement quoting p results in a sale. At the free entry equilibrium, all firms receive zero profit, hence zero profit per advertisement (given no fixed costs), i.e. $(p - c)x(p) - c' = 0$.
  - At equilibrium, the probability $x(v)$, that an advertisement quoting $p = v$ results in a sale, is equal to the probability that a consumer receives no advertisements, $1 - \phi^e$, where ϕ^e is the value of ϕ in this equilibrium. From the zero-profit condition above, $(v - c)x(v) - c' = 0$, $1 - \phi^e = x(v) = \frac{c'}{v-c}$.
  - Because consumers are homogenous and have unit demands, allocation of the good between consumers is irrelevant to welfare, which is entirely a function of the quantity sold and total advertising expenditure. Total welfare W is equal to welfare from consumption and production $N\phi(v - c)$ less total advertising expenditure $Nc' \ln \frac{1}{1-\phi}$, $W = N\phi(v - c) - Nc' \ln \frac{1}{1-\phi} = N\phi(v - c) + Nc' \ln(1 - \phi)$. When total welfare is maximised, first-order condition $\frac{\partial W}{\partial \phi} = N(v - c) - \frac{Nc'}{1-\phi} = 0$ holds, $v - c = \frac{c'}{1-\phi^o}$, $1 - \phi^o = \frac{c'}{v-c}$, where ϕ^o is the socially-optimal value of ϕ.
  - $\phi^e = \phi^o$, the monopolistically competitive equilibrium level of advertising is socially optimal.

- A firm serves an advertisement quoting $p = v$ iff, given the zero profit condition under monopolistic competition, the cost of serving that advertisement c' is equal to the expected increase in gross (of advertising cost) profit from doing so $\phi(v-c)$, since the firm captures margin $v-c$ with probability ϕ. This private incentive coincides exactly with the social incentive because the increase in gross margin is exactly equal to the increase in social welfare, i.e. at the margin, social surplus is entirely appropriated by firms. Advertisements quoting $p = v$ also have no business-stealing effect. Given the zero profit condition, the private benefit to a firm of serving advertisements is independent of the price quoted, hence private and social benefits agree for all other $p \in [c+c', v]$.
- If instead consumers discover products not only through advertising but also by search, at equilibrium, there is insufficient search and excessive advertising.

- Grossman and Shapiro Model (No Entry, Differentiated Products, Informative Advertising)
  - Two firms, A and B located at $x_A = 0$ and $x_B = 1$ in linear product space $x \in [0, 1]$ each produce a good at common constant marginal cost c, choose to serve informative advertisements to m_A, m_B consumers (independent of consumers' location in product space) and prices p_A, p_B to maximise profits π_A, π_B.
  - Consumers are uniformly distributed in product space. Each consumer i at x_i buys only from firms that he has been served advertisements for and thus knows to exist, has unit demands, has sufficiently high valuation such that he always buys from one firm if he has been served advertisements by at least one firm, faces total cost $p_X + t|x_i - x_X|$ if he buys from firm X, and chooses to buy from firm X if served advertisements by both firms and doing so minimises his total cost. The number of consumers is normalised to 1.
  - $m_A m_B$ consumers are served advertisements by both firms, $m_A(1-m_B)$ are served advertisements by only firm A, $(1-m_A)m_B$ are served advertisements by only firm B and $(1-m_A)(1-m_B)$ are served no advertisements. Consumers served advertisements by only firm A buy from firm A at any price. Consumers served advertisements by both firms (who are uniformly distributed in product space) buy from firm A iff the total cost of buying from firm A is less than that of buying from firm B. By the result of the Hotelling duopoly model with fixed locations, of these consumers, a fraction $\frac{p_B - p_A + t}{2t}$ buy from firm A. Firm A captures demand $q_A = m_A(1-m_B) + m_A m_B(\frac{p_B - p_A + t}{2t})$.
  - Suppose that the cost to firm X of serving m_X advertisements is $\frac{am_X^2}{2}$ hence $\pi_X = (p_X - c)q_X - \frac{am_X^2}{2}$. Suppose further that firms choose p_A, p_B, m_A, m_B simultaneously, then by symmetry, $p_A = p_B = p^e$ and $m_A = m_B = m^e$.
  - It can be shown that, at equilibrium, $p^e = c + \sqrt{2at} = c + t\sqrt{\frac{2a}{t}}$, $m^e = \frac{2}{1+\sqrt{\frac{2a}{t}}}$, and equilibrium profit $\pi^e = \frac{2}{(1+\sqrt{\frac{2a}{t}})^2}$.
  - Equilibrium price in the Grossman and Shapiro model is higher than in the Hotelling duopoly model with fixed locations (where consumers instead have full information, hence informative advertising is unnecessary) because each firm need not compete with the other for "captive" consumers, hence each firm's demand is less price-elastic, and each firm finds it optimal to increase prices.
  - An increase in transport cost results in an increase in price because an increase in transport cost reduces price-elasticity of demand. This effect is greater than in the case of perfect information.
  - An increase in transport cost results in an increase in advertising because an increase in transport cost implies that consumers are less sensitive to price (relative to location) hence demand is less price-elastic, optimal price increases, and the incentive to reach consumers through informative advertising increases. As transport cost increases, price competition over "selective" consumers decreases, hence the negative strategic effect whereby advertising increases price competition is dampened and firms have greater incentive to advertise.
  - An increase in transport cost results in an increase in profit because an increase in transport cost softens price competition.
  - An increase in advertising cost results in an increase in profit. An increase in advertising cost has a direct negative effect on profit, holding all other factors unchanged. An increase in advertising cost has an indirect positive strategic effect on profit. An increase in advertising cost decreases competitors' advertising expenditure, hence increases the relative (to consumers served both advertisements) share of "captive" consumers served by each firm, decreases the price-elasticity of demand, increases prices and profit. The strategic effect dominates the direct effect.
  - Whether advertising is under-provided or over-provided is ambiguous, a priori. The increase in consumer surplus due to informative advertising is not entirely captured by firms as increased profits, hence firms generally have less incentive to serve informative advertising than a social planner. The non-appropriability of social surplus results in a bias toward under-provision of informative advertising. An increase in advertising by one firm decreases the profit of its competitor, and this negative externality is not accounted for when firms choose their level of advertising. The business stealing effect results in a bias toward over-provision of informative advertising.