Game Theory Paper 170605

1a

	L	C	R
T	$\underline{15}$ / 10	11 / 16	12 / $\underline{8}$
M	2 / $\underline{12}$	1 / $\underline{20}$	$\underline{5}$ / 5
B	7 / 11	10 / 19	$\underline{11}$ / 4

Best responses underlined.

$M \succ_1 B$, then $L \succ_2 C$ (and $R \succ_2 C$), then no remaining strategy ~~do~~ strictly dominates any other. T, M, L, R survive ISD.

b A player's pure strategy is rationalisable iff it is a best response (i.e. weakly maximises expected payoff) against some potentially correlated mix of other players' rationalisable strategies. ~~Equivalently~~ A player's pure strategy is not rationalisable if it is not a best response against any potentially correlated mix of other players' strategies. Then, an equivalent definition of rationalisability is that a ~~players~~ pure strategy is rationalisable iff it survives iterated elimination of unrationalisable strategies.

Pearce's Lemma: in a two player finite game, a player's strategy is not dominated by any pure or mixed strategy iff it is a best response against some pure or mixed strategy of the other player.

By Pearce's Lemma, all and only strategies that survive ISD (in a two player finite game) are rationalisable. Then, in the given game, only T, M, L, R, are rationalisable.

c By ISD, the given game reduces to

	L	R
T	$\underline{15}$ / 10	12 / $\underline{8}$
M	2 / $\underline{12}$	$\underline{5}$ / 5

Best responses underlined. By inspection, there are no pure NE where players play mutual pure best responses. By inspection, best responses are strict, so there are no hybrid NE.

Suppose that P1 mixes T and M with probability p and 1-p, ~~then~~ at NE. Then, by definition of NE, P1 has no profitable deviation.

$\Rightarrow \pi_1(T, \sigma_2^*) = \pi_1(M, \sigma_2^*)$, where σ_2^* is P2's strategy at this NE

$\Leftrightarrow 10q + 8(1-q) = 12q + 5(1-q)$, where q is the probability σ_2^* assigns to L.

$\Leftrightarrow 3 - 3q = 2q \Leftrightarrow q = 3/5$

$\Rightarrow$ P2 mixes

$\Rightarrow \pi_2(L, \sigma_1^*) = \pi_2(R, \sigma_1^*)$ where σ_1^* is P1's strategy at this NE

$\Leftrightarrow 15p + 2(1-p) = 12p + 5(1-p) \Leftrightarrow 3p = 3(1-p) \Leftrightarrow p = 1/2$

The unique NE is the mixed NE $(\frac{1}{2}T + \frac{1}{2}M, \frac{3}{5}L + \frac{2}{5}R)$

A full characterisation of best responses is given in the payoff matrix in a.

The above NE is unique because only a mixed NE exists and if P1 mixes, P1 and P2 mix with exactly the probabilities above.

d Let * denote the above NE.

$\pi_1^* = \pi_1(T, \sigma_2^*) = 10q + 8(1-q) = 30/5 + 16/5 = 46/5$

$\pi_2^* = \pi_2(L, \sigma_1^*) = 15p + 2(1-p) = 17/2$

~~Suppose that the required correlated equilibrium can be constructed from only the strategies T, M, L, R. Then the required correlated equilibrium satisfies can be characterised by the probabilities $p_{TL}, p_{TR}, p_{ML}, p_{MR}$ in the intuitive way. The required correlated equilibrium satisfies payoff equivalence (PE) and incentive constraints (IC).~~

PE_1: $10p_{TL} + 8p_{TR} + 12p_{ML} + 5p_{MR} = 46/5$

PE_2: $15p_{TL} + 12p_{TR} + 2p_{ML} + 5p_{MR} = 17/2$

IC_{1T}: $10p_{TL} + 8p_{TR} \geq 12p_{TL} + 5p_{TR} \Leftrightarrow 3p_{TR} \geq 2p_{TL}$

IC_{1M}: $12p_{ML} + 5p_{MR} \geq 10p_{ML} + 8p_{MR} \Leftrightarrow 2p_{ML}$

IC_{2L}: $15p_{TL} + 2p_{ML} \geq 12p_{TL} + 5p_{ML} \Leftrightarrow p_{TL} \geq p_{ML}$

IC_{2R}: $12p_{TR} + 5p_{MR} \geq 15p_{TR} + 5p_{MR} \Leftrightarrow p_{ML} \geq p_{TL}$

~~Suppose that the required correlated equilibrium can be constructed from only the strategies M, B, C, R. Then the required correlated equilibrium can be characterised~~

At the required correlated equilibrium, ~~only~~ strictly dominated strategies are played with probability zero since these strategies are not incentive compatible. Then, only rationalisable strategies T, M, L, R, are played with non-zero probability. The required correlated equilibrium can be characterised by the probabilities ~~p_{TL}, p_{ML}, p~~ $p_{TL}, p_{TR}, p_{ML}, p_{MR}$ in the intuitive way. It satisfies payoff equivalence (PE) and the incentive constraints (IC).

$PE_1: 10 P_{TC} + 8 P_{TR} + 12 P_{mC} + 5 P_{mR} = 46/5$

$PE_2: 15 P_{TC} + 12 P_{TR} + 2 P_{mC} + 5 P_{mR} = 17/2$

$IC_T: 10 P_{TC} + 8 P_{TR} \geq 12 P_{TC} + 5 P_{TR} \Leftrightarrow 3 P_{TR} \geq 2 P_{TC}$

$IC_m: 12 P_{mC} + 5 P_{mR} \geq 10 P_{mC} + 8 P_{mR} \Leftrightarrow 2 P_{mC} \geq 3 P_{mR}$

$IC_C: 15 P_{TC} + 2 P_{mC} \geq 12 P_{TC} + 5 P_{mC} \Leftrightarrow P_{TC} \geq P_{mC}$

$IC_R: 12 P_{TR} + 5 P_{mR} \geq 15 P_{TR} + 2 P_{mR} \Leftrightarrow P_{mR} \geq P_{TR}$

$\Rightarrow P_{TR} \leq P_{mR} \leq \frac{2}{3} P_{mC} \leq \frac{2}{3} P_{TC} \leq P_{TR}$

$\rightarrow P_{TR} = P_{mR} = \frac{2}{3} P_{mC} = \frac{2}{3} P_{TC}$

$\Rightarrow P_{TR} = P_{mR} = 1/5, \; P_{mC} = P_{TC} = 3/10$

The required correlated equilibrium is

$\Omega = \{TC, TR, mC, mR\}$

$P(\omega = TC) = P(\omega = mC) = 3/10$

$P(\omega = TR) = P(\omega = mR) = 2/10$

$P_1 = \{\{TC, TR\}, \{mC, mR\}\}, \; P_2 = \{\{TC, mC\}, \{TR, mR\}\}$

$\sigma_1(TC) = \sigma_1(TR) = T, \; \sigma_1(mC) = \sigma_1(mR) = M$

$\sigma_2(TC) = \sigma_2(mC) = C, \; \sigma_2(TR) = \sigma_2(mR) = R$

The required correlated equilibrium is unique and extensionally identical to the above unique mixed NE, the two are only "intensionally" different in the sense that one is a correlated equilibrium while the other is a mixed NE.

2a
~~3a~~. Formulate as a Bayesian game.

Players : $N = \{1, 2\}$ $(n=2)$

Actions : $A_i = [0,1]$ for $i \in N$

States : $\Omega = \{EE, EI, IE, II\}$

denoting the states ~~where~~ $(F_1 = F_2 = 0)$, $(F_1 = 0, F_2 = 1)$ $(F_1 = 1, F_2 = 0)$, $(F_1 = F_2 = 1)$ respectively

Signals : ~~$t_1(\omega = EE$ $T_1(\omega = EE) = T_1(\omega$~~

$t_1 = \tau_1(\omega) = 0$ if $\omega = EE$ or EI, 1 otherwise

$t_2 = \tau_2(\omega) = 0$ if $\omega = EE$ or IE, 1 otherwise

Beliefs : $P_1(\omega = EE \mid t_1) = P_1(\omega = IE \mid t_1) = 1-\gamma$

$P_1(\omega = EI \mid t_1) = P_1(\omega = II \mid t_1) = \gamma$

$P_2(\omega = EE \mid t_2) = P_2(\omega = EI \mid t_2) = 1-\gamma$

$P_2(\omega = IE \mid t_2) = P_2(\omega = II \mid t_2) = \gamma$

Payoffs : $u_i(a_i, a_{-i} \mid t_i, t_{-i})$

$$= \begin{cases} -c(a_i) & \text{if } a_i < a_{-i} \\ 0.5 - c(a_i) & \text{if } a_i = a_{-i} \\ 1 - c(a_i) & \text{if } a_i = a_{-i} \end{cases}$$

where $c(a_i) = 0$ if $a_i = 0$, $a_i + t_i$ otherwise

A pure strategy of player i ~~is~~ in this BNE is a signal contingent pure ~~BE~~ strategy that is fully characterised by the pair (a_i^E, a_i^I).

Suppose for reductio that a symmetric pure BNE σ^* exists. Then $\sigma^* = (\sigma_1^*, \sigma_2^*) = ((a_1^E, a_1^I), (a_2^E, a_2^I))$ $= ((a^E, a^I), (a^E, a^I))$. ~~By inspection, $a_i = 0$ is weakly strictly dominant in interim payoffs for I types~~. This candidate BNE fails to deviation by either player i to $(a_i^{E\prime} = a^E + \varepsilon, a_i^I = a^I)$ for sufficiently small ε since the deviating player thereby expends ε on the occasions that it is E type (with probability γ) and captures revenue 1 instead of 0.5 on the occasions that both are E type (with probability γ), and for sufficiently small ε the outcomes in other cases are unchanged and $0.5\gamma^2 > \varepsilon\gamma$.

b Given : BNE is symmetric, type E players mix over $[0, \bar{x}]$ according to a continuous distribution characterised by G.

~~Given that typ Type I players face type E players with probability γ and given type E players mix as above, type E players play 0 with zero probability. Then,~~ the ~~I~~ interim payoff to type I players ~~of p~~ from playing 0 is ≥ 0 and from playing any other strategy ~~is~~ x^I is no greater than ~~1~~ $-x^I < 0$. Then 0 is strictly dominant in interim payoffs for type I players, and at ~~BE~~ (any) the required BNE, type I players play 0 with certainty.

Interim expected payoff to type E ~~of~~ from playing x^E
~~$E(u(x^E$~~
$= \gamma(1-x^E) + (1-\gamma)[\,G(x^E)(1-x^E) + (1-G(x^E))(-x^E)\,]$

Given that 0 is in the support of type E players mixed strategy, and at BNE players mix over actions with equal payoff, the above is equal to the interim payoff to type E player from action 0.

$= \cancel{E(u(x^E))}\ \gamma$

Noting that, given G is continuous, i.e. there are no atoms in type E's mixed strategy, ties occur with probability zero.

$$\gamma(1-x^E) + (1-\gamma)[G(x^E)(1-x^E) + (1-G(x^E))(-x^E)] = \gamma$$

$$\Leftrightarrow \gamma x^E = (1-\gamma)[\,G(x^E) - x^E\,]$$

$$\Leftrightarrow \gamma x^E = G(x^E) - x^E - \gamma G(x^E) + \gamma x^E$$

$$\Leftrightarrow x^E = (1-\gamma)\,G(x^E)$$

$$\Leftrightarrow G(x^E) = x^E/1-\gamma$$

Given $G(x^E) \leq 1$, $x^E \leq 1-\gamma$

The candidate required BNE is the strategy profile under which each player plays 0 if type I and mixes over $[0, \cancel{x}\ 1-\gamma]$ according to cdf $G(x) = x/1-\gamma$ if type E.

From earlier argument, type I player has no profitable deviation since 0 is strictly dominant in interim payoffs. ~~From above.~~

$$\gamma(1-x^E) + (1-\gamma)[\,\tfrac{x^E}{1-\gamma}(1-x^E) + (1-\tfrac{x^E}{1-\gamma})(-x^E)\,]$$
$$= \gamma(1-x^E) + (1-\gamma)[-x^E + \tfrac{x^E}{1-\gamma}]$$
$$= \gamma(1-x^E) + x^E - x^E + \gamma x^E$$
$$= \gamma$$

For all $x^E \in [0, \cancel{\bar{x}}\ 1-\gamma]$, the ~~expect~~ interim expected payoff to type E is equal, type E cannot profitably deviate by redistributing probability mass within $[0, 1-\gamma]$.

For $x^E > 1-\gamma$, interim expected payoff is $1 - x^E < \gamma$, then type E cannot profitably deviate by ~~allo~~ reallocating ~~problem~~ probability mass from $[0, 1-\gamma]$ to $[1-\gamma, 1]$

Then no player-type has a profitable deviation and the candidate BNE is in fact a BNE.

c As γ increases, the range $[0, \bar{x} = 1-\gamma]$ over which type E uniformly mixes contracts, the lower bound remains at 0 while the upper bound decreases.

Intuitively, the greater the probability that ~~a~~ type E faces type I, the less incentive type E has to "bid" higher. ~~The type~~ Analytically, the upper bound is such that type E ~~is indifferent between attaining with certainty~~ has equal payoff from this ~~amount~~ "bidding" this amount and from bidding just above zero. The greater γ,

the greater the payoff from bidding just above 0
hence the lower the maximum rational bid.

Date

3a Suppose for reductio that there is a BNE σ^* such that both players abstain, i.e. $\sigma_1^*(s_1) = 0$, $\sigma_2^*(s_2) = 0$ where 0 denotes the ~~action~~ abstain action. Then ex-ante payoffs are $E(u_i(\sigma^*, \omega)) = 1/2$ for $i \in \{1,2\}$. This candidate BNE fails to deviation of player 1 to $\sigma_1'(s_1) = a$ if $s_1 = A$, b if $s_1 = B$ (also to deviation of P2 to $\sigma_2'(s_2) = a$ if $s_2 = A$, b if $s_2 = B$). At this deviation, ex-ante payoffs are $E(u_i(\sigma_1', \omega)) = r_1 > 1$ for $i \in \{1,2\}$ (where $\sigma^{*\prime} = (\sigma_1', \sigma_2^*)$). P1 has profitable deviation from the candidate BNE σ^*, it is not in fact a BNE. By reductio, there is no BNE σ^* such that both players abstain.

b Consider the candidate BNE $\sigma^* = (\sigma_1^*, \sigma_2^*)$ where P2 abstains, i.e. $\sigma_2^*(s_2) = 0$ and P1 votes according to his signal, i.e. $\sigma_1^*(s_1) = a$ if $s_1 = A$, b if $s_1 = B$.

~~interim expected payoff~~

interim expected payoffs

$E_1(u_1(\sigma_1^*, \sigma_2^*; \omega) \mid s_1) = r_1$

$E_2(u_2(\sigma_2^*, \sigma_1^*; \omega) \mid s_2) = r_1$

It is trivial that P1 has no profitable deviation. Given s_1, P1's expected payoff from playing in accord with s_1 (i.e. a if $s_1 = A$, b if $s_1 = B$) is r_1, from playing contrary to s_1 (i.e. a if $s_1 = B$, b if $s_1 = A$) is $1 - r_1 < r_1$, and from abstaining is $1/2$. Then playing in accord with s_1 is ~~strictly~~ the only rational strategy for P1 at the interim stage, given P2 plays σ_2^* (i.e. abstains).

Suppose P2 plays in accord with s_2, then with probability $r_1 r_2$, ~~$s_1 = s_2$, and the ... P2 has's ex-post~~ ~~ex-post payoff is~~ $s_1 = s_2 = \omega$ and P2 has ex-post payoff 1. With probability $(1-r_1)(1-r_2)$ $s_1 = s_2 \neq \omega$ and P2 has ex-post payoff 0. These ~~outcomes are identical to the case wh~~ payoffs are equal to those under σ^*. With probability $r_1(1-r_2)$, ~~P2 has payoff~~ $s_2 \neq s_1 = \omega$, and P2 has ex-post payoff $1/2$. With probability $r_2(1-r_1)$, $s_1 \neq s_2 = \omega$ and P2 has ex-post payoff $1/2$. Under σ^*, the corresponding payoffs are 1 and 0. Given $r_1 \geq r_2$, $r_1(1-r_2) \geq r_2(1-r_1)$, then P2's interim payoff from playing in accord with s_2 is weakly lower than from σ_2^*.

Given $r_2 > 1/2$, it is trivial that P2's interim expected payoff from playing ~~against~~ contrary to s_2 is lower than that from abstaining. Then, abstaining is ~~the only rational strategy for P2~~ rational for P2 given P1 plays σ_1^*.

By definition of BNE, since each player has no profitable deviation, the candidate BNE is in fact a BNE.

c Let σ^* denote the required BNE.

It is trivial that $\sigma_1^*(s_1) = a$. a strictly dominates each of 0 and b in ex-post payoffs for A hence is strictly dominant in interim and ex-ante payoffs for P1. Then, at any strategy profile such that P1 does not play a, P1 can profitably deviate to A. By definition of BNE, P1 plays a at BNE.

~~For given s_2, P2's interim payoff from playing in accord with s_2 is~~

s_2	P2 action	interim payoff to P2
A	a	r_2
A	b	$1/2$
A	0	r_2
B	a	$1-r_2$
B	b	$1/2$
B	0	$1-r_2$

By inspection of the above table, P2 best responds to P1's dominant strategy by playing $\sigma_2^* = \alpha a + (1-\alpha)0, b$, i.e. any ~~mix of~~ (potentially degenerate) mix of a and 0 if $s_2 = A$ and b if $s_2 = B$.

The BNE are

$\sigma^* = (\sigma_1^*, \sigma_2^*)$

where $\sigma_1^*(s_1) = a$ for all s_1

and $\sigma_2^*(s_2) = \begin{cases} \alpha a + (1-\alpha)0 & \text{for } \alpha \in [0,1], \text{ if } s_2 = A \\ b & \text{if } s_2 = B \end{cases}$

Date

4a Denote the uninformed type u, the informed type that knows the good is valuable v, and the informed type that knows the good is not valuable L (for lemon). Let A abbreviate B's action of accepting the offer and R abbreviate rejection. Let $b \in \{A, R\}$ denote B's action.

Suppose for reductio that there exists PBE (σ^*, μ^*) such that ~~$p_v = 10$, b~~ $p^v = 10$, ~~$b^u = b^v$~~ $b^u(p=10) = A$, $b^u(p=10) = A$ and $p^v = 10$ is perfectly informative. By Bayes's rule, $p^v = 10$ is perfectly informative only if $p^u, p^L \neq 10$. ~~It is sequ~~ Given $b^u(p=10) = A$, ~~it is~~ $p^L = 10$ is sequentially rational. S^L never sells to B^L and never faces B^v, hence maximises payoff by ~~selling~~ maximising payoff from selling to B^u. ~~B^u never~~ By sequential rationality, $b^u(p>10) = R$, then if $b^u(p=10) = A$, S^L maximises payoff by choosing $p^L = 10$. By reductio, no such SPE exists.

b Given: at PBE, $p^u = p^v = p^L = 15/2$

By Bayes's rule, B^u infers nothing from p, and ~~$b^u(p=$~~ $b^u(p = 15/2) = A$ is (just) sequentially rational. ~~B^v~~ $b^v(p = 15/2) = A$ is sequentially rational, as is $b^L(p = 15/2) = R$.

Set B^u's beliefs off the equilibrium path (i.e. for all $p \neq 15/2$) such that B^u believes $S = S^L$ ~~for~~. All other beliefs ~~(including~~ ~~B^v and~~ concern only other players' information, not the value of the good, hence not payoffs and can be freely set. ~~Then~~

Then, for any other $p \leq 10$, $b^u = R$, $b^v = A$, $b^L = R$. Then $p^u = p^v = p^L = 15/2$ is sequentially rational. ~~At any other price $p \leq 10$, expected payoff is $3/8\,p < 30/8 = 15/4$~~

At any other price, S^u sells only to B^v and has payoff $3/8\,p$ ~~te~~ which is less than $105/16$ from $p = 15/2$. S^L sells ~~only~~ to ~~B^v and has~~ no B and has payoff 0 rather than $15/4$ from selling to B^u at $p = 15/2$. S^v sells only to B^v and has payoff $p/2$ rather than $105/16$ from selling to B^u and B^v at $p = 15/2$.

c Given: at PBE, $p^u = p^v = 10$, $p^L = 0$

By sequential rationality $b^v(p=10) = A$. Only B^v buys at $p = 10$.

Set B^u's beliefs off the equilibrium path such that B^u believes only S^L chooses any $p \neq 10$. Then by sequential rationality, $b^u(p) = R$ for all $p \neq 0$, then $p^u = p^v = 10$ is sequentially rational. B^v always buys at this price, which maximises ~~profit~~ ~~to~~ ~~pay~~ profit to S from B^v and S cannot profitably deviate since B^u and B^L never buy (at $p \neq 0$).

By sequential rationality, $b^v(p \leq 10) = A$, $b^L(p) = R$ for all p.

The required PBE is such that $p^u = p^v = 10$, $p^L = 0$,

~~$b^v(p \leq 10) = A$~~

$b^v(p) = A$ if $p \leq 10$, R otherwise

$b^u(p) = R$, $b^L(p) = R$ for all p,

B^u believes ~~$S = S^L$~~ $S = S^L$ for all $p \neq 0$

d Pooling equilibrium

B^u and B^v buy at $p = 15/2$, B^L does not buy

$E(\pi) = 7/8\,(15/2) = 105/16$

Separating equilibrium

B^v buys at $p = 10$, B^u and B^v do not buy

$E(\pi) = 3/8\,(10) = 15/4$

The pooling equilibrium is more profitable for S in ex ante expectations.

Date

5a In a two player symmetric game, strategy α^* is an ESS iff (α^*, α^*) is an NE and there is no strategy $\alpha' \neq \alpha^*$ such that α' is a best response to α^* and $u(\alpha', \alpha') \geq u(\alpha^*, \alpha')$.

The concept is motivated by the phenomenon of evolution in biology. The above definition captures the idea that α^* is evolutionarily stable iff ~~it cannot be successfully invaded~~ ~~&~~ no member of a population of α^* players has "evolutionary pressure" to evolve ~~the no~~ and ~~so~~ the population cannot be successfully invaded by mutant α' players who fare as well as α^* against α^* and better than α^* against α', and so fare better on average.

b

	A		B	
A		-3		$\underline{0}$
	-3		$\underline{2}$	
B		$\underline{2}$		1
	$\underline{0}$		1	

Best responses underlined. By inspection, there are no pure NE where players play ~~the~~ mutual pure best responses. By inspection, each player has a strict best response to every pure strategy, then there are no hybrid NE.

Let $\sigma^* = (\sigma^*_1, \sigma^*_2)$ be ~~some~~ some mixed NE. Then, by definition of NE, neither player has a profitable deviation. $\pi_1(A, \sigma^*_2) = \pi_1(B, \sigma^*_2) \Leftrightarrow -3q + 2(1-q) = 1-q$, where q is the probability σ^*_2 assigns to A. Then $1-q = 3q$, $q = 1/4$. By symmetry, the probability σ^*_1 assigns to A $p = 1/4$. The unique NE is $(1/4A + 3/4B,\ 1/4A + 3/4B)$. This is the only candidate ESS.

Let $\alpha^* = (1/4A + 3/4B)$. Let α' be some arbitrary strategy strategy ~~(pA+(1-p))~~ $pA + (1-p)B)$. Given that (α^*, α^*) is ~~an~~ a NE, α^* is a best response to α^*, then each of A and B (that α^* mixes over) is a best response to α^*, then any mix of A and B, α' is a best response to α^*.

$$\pi(\alpha', \alpha') = -3p^2 + 2p(1-p) + 1(1-p)^2 = -3p^2 + 2p - 2p^2 + p^2 - 2p + 1$$
$$= -4p^2 + 1$$
$$\pi(\alpha^*, \alpha') = p[1/4(-3) + 3/4(0)] + (1-p)[1/4(2) + 3/4(1)]$$
$$= -3/4p + 5/4(1-p) = 5/4 - 2p$$
$$\pi(\alpha', \alpha') \geq \pi(\alpha^*, \alpha') \Leftrightarrow -4p^2 + 1 \geq 5/4 - 2p$$
$$\Leftrightarrow -4p^2 + 2p - 1/4 \geq 0 \Leftrightarrow 4p^2 - 2p + 1/4 \leq 0 \Rightarrow p = 1/4 \Rightarrow \alpha' = \alpha^*$$

No $\alpha' \neq \alpha^*$ is such that $\pi(\alpha', \alpha') \geq \pi(\alpha^*, \alpha')$. Then α^* is in fact an ESS and it is unique.

c Replicator equation

$\dot{p}_X = p_X(1-p_X)([Up]_X - [Up]_Y)$ for $X \neq Y \in \{A, B\}$

where U is the Row payoff matrix, $p = (p_A, p_B)^T$, i.e. the proportions of A-players and B-players in the population, ~~and~~ hence $[Up]_X$ is the average payoff to X-players.

$$\dot{p}_A = p_A(1-p_A)(-3p_A + 2(1-p_A) - [1(1-p_A)])$$
$$= p_A(1-p_A)(1 - 4p_A)$$
$$\dot{p}_A = 0 \Leftrightarrow p_A = 0, 1, 1/4$$
$$p_A \in (0, 1/4) \Rightarrow \dot{p}_A > 0$$
$$p_A \in (1/4, 1) \Rightarrow \dot{p}_A < 0$$

The absorbing states are $p_A = 0$, $p_A = 1/4$, $p_A = 1$. For all $p_A \in (0, 1/4)$, $\dot{p}_A > 0$, ~~hence such st~~ and for all $p_A \in (1/4, 1)$, $\dot{p}_A < 0$, hence all such states evolve to $p_A = 1/4$. This is the ESS.

d

	C		D		E	
C		0		-1		$\underline{1}$
	0		$\underline{1}$		-1	
D		$\underline{1}$		0		-1
	-1		0		$\underline{1}$	
E		-1		$\underline{1}$		0
	$\underline{1}$		-1		0	

Best responses underlined. The game is rock-paper-scissors. By inspection there are no pure NE and best responses to pure actions are strict, so there are no hybrid NE.

Suppose P1 plays C with zero probability at NE, then D strictly dominates E for P2, so P2 plays E with zero probability, then C strictly dominates D for P1, so P1 plays D with zero probability, then P1 plays E with certainty and the NE is pure or hybrid. By reductio, P1 plays C with non-zero probability. By symmetry, each player plays every action with non-zero probability.

At NE σ^*, given each player mixes over every strategy, $\pi_1(C, \sigma^*_2) = \pi_1(D, \sigma^*_2) = \pi_1(E, \sigma^*_2) \Leftrightarrow q_D - q_E = q_E - q_C = q_C - q_D \Rightarrow q_C = q_D = q_E = 1/3$ where q_C, q_D, q_E are the probabilities assigned by σ^*_2. By symmetry, $p_C = p_D = p_E = 1/3$ where p_C, p_D, p_E are the probabilities assigned by σ^*_1.

The unique NE is the symmetric NE $(1/3C + 1/3D + 1/3E,\ 1/3C + 1/3D + 1/3E)$. Then the only candidate ESS is $\alpha^* = 1/3C + 1/3D + 1/3E$. ~~From the above~~.

From the above, $\pi(C, \alpha^*) = \pi(\alpha^*, \alpha^*)$, then C is a best response to α^*. $\pi(C, C) = 0 = \pi(\alpha^*, C)$. Then α^* is not an ESS. α^* is "susceptible to invasion by mutant C".

e Replicator equation
$\dot{p}_x = p_x([Up]_x - p^T U p)$ for $x \in \{C, D, E\}$.
where U ~~and~~ and p defined as before.

By inspection, the game is zero-sum, then $p^T U p = 0$ for all p. The above reduces to

$$\dot{p}_x = p_x [Up]_x$$

By substitution,
$\dot{p}_C = p_C(p_D - p_E)$, $\dot{p}_D = p_D(p_E - p_C)$, $\dot{p}_E = p_E(p_C - p_D)$.

$$\begin{aligned}
\frac{d}{dt} p_C p_D p_E &= p_C \frac{d}{dt} p_D p_E + p_D p_E \frac{d}{dt} p_C \\
&= p_C \left[p_D \frac{d}{dt} p_E + p_E \frac{d}{dt} p_D \right] + p_D p_E \frac{d}{dt} p_C \\
&= p_C p_D \frac{d}{dt} p_E + p_C p_E \frac{d}{dt} p_D + p_D p_E \frac{d}{dt} p_C \\
&= p_C p_D \dot{p}_E + p_C \dot{p}_D p_E + \dot{p}_C p_D p_E \\
&= p_C p_D p_E \left[(p_D - p_E) + (p_E - p_C) + (p_C - p_D) \right] \\
&= 0
\end{aligned}$$

$p_C p_D p_E$ remains is static. Then the state does not ~~co~~ converge to the NE (given that $p_C p_D p_E$ is not a constant).

5a In a two-player symmetric game, a strategy α^* is an ~~evolutionarily~~ ESS iff (α^*, α^*) is a ~~NE~~ NE, and there is no ~~stra~~ other strategy $\alpha' \neq \alpha^*$ such that ~~$u(\alpha', \alpha^*) > u(\alpha^*, \alpha^*)$, and~~ α' is a best response against α^* and $u(\alpha', \alpha') > u(\alpha^*, \alpha')$.

The concept of an ESS is motivated by the ~~phe~~ phenomenon of evolution in biology. Informally, the above definition captures the intuition that the strategy of α^* is evolutionarily stable iff a population of α^* players cannot be successfully invaded by mutant α' players who fare at least as well as α^* players against α^* players and better than α^* players against α' players and so fare better on average. ~~and a population of α^* players~~ ~~As α^* player has strict incentive to~~

b

G	A	B
A	-3 -3	0 2
B	5 0	1 1

Construct G' as the two player symmetric Bayesian game under which each player with ~~½~~ probability ½ plays as Row in G and with probability ½ plays as Column in G, where each player's "role" is publicly ~~k~~ known. ~~Then, each player's pure strate and each player's payoffs are the ex ante~~ Then each player's strategy is some role-contingent plan of action. Let G'' be the two player symmetric ~~ga~~ strategic form game ~~where each player's actions~~ with the same players as G', where each player's ~~a~~ actions are his strategies in G', and each player's payoffs are his ex-ante expected payoffs in G'.

G''	AA	AB	BA	BB
AA	-3	$-1/2$	1	1
AB	$-3/2$	$\underline{7/2}$	-1	$3/2$
BA	$-3/2$	-1	$\underline{5/2}$	$\underline{3}$
BB	$\underline{0}$	$1/2$	$1/2$	1

~~Best r~~ Row player's payoffs. Best responses underlined. By inspection, the only symmetric NE are (AB, AB) and (BA, BA). Both are strict NE, then both are ESS.

Attempted with error in question.

Date

6a. Suppose that if the case goes to trial this happens in a third period, such that payoffs are further discounted.

Solve by backward induction.

In the second period, plaintiff P1 offers defendant ~~P~~ P2 z_2^2 (and retains z_1^2). If P2 rejects, P2's payoff is $\delta^2 y$. If P2 accepts, P2's payoff is δz_2^2. Then P2 accepts iff $z_2^2 \geq \delta y$. In the second period, P1's payoff is $\delta^2(x-k)$ if $z_2^2 < \delta y$ and $\delta z_1^2 = \delta(x+y-z_2^2)$ if $z_2^2 \geq \delta y$. If P1 finds it optimal to induce P2 to accept, P1 chooses $z_2^2 = \delta y$ and has payoff $\delta(x+y-\delta y)$. $\delta(x+y-\delta y) > \delta x > \delta^2 x > \delta^2(x-k)$. Then P1 finds it optimal to induce P2 to accept. P1 offers $z_2^2 = \delta y$ ~~and~~, $z_1^2 = x+y-\delta y$, P2 then accepts, and their payoffs are $\delta(x+y-\delta y)$ and $\delta^2 y$ respectively.

In the first period, P1 offers z_2^1 (and retains z_1^1). If P2 rejects, from the above, P2's payoff is $\delta^2 y$. If P2 accepts, P2's payoff is z_2^1. P2 accepts any $z_2^1 \geq \delta^2 y$ and rejects any $z_2^1 < \delta^2 y$. P1's payoff is $\delta(x+y-\delta y)$ if P2 rejects z_2^1 and $z_1^1 = x+y-z_2^1$ if P2 accepts. If P1 finds it optimal to induce P2 to accept, P1 optimally chooses $z_2^1 = \delta^2 y$ and has payoff $x+y-\delta^2 y > \delta(x+y-\delta y)$. Then P1 finds it optimal to induce P2 to accept. P1 offers $z_2^1 = \delta^2 y$, $z_1^1 = x+y-\delta^2 y$, P2 accepts, and their payoffs are z_1^1 and z_2^1 respectively.

The unique SPE is such that P1 offers ($z_1^1 = x+y-\delta^2 y \Rightarrow z_2^1 = \delta^2 y$, $z_1^2 = x+y-\delta y \Rightarrow z_2^2 = \delta y$) and P2 in period 1 accepts z_2^1 iff $z_2^1 \geq \delta^2 y$ and in period 2 accepts z_2^2 iff $z_2^2 \geq \delta y$.

P2's outside option is made less attractive by discounting. P1 has all the bargaining power since only P1 is in the position to make ~~take it or leave it of~~ offers. So at SPE P2's payoff is $\delta^2 y$ and P1 captures the remainder of $x+y$. Penalty k is irrelevant to P1 because P1 always finds it optimal to induce P2 to accept.

b Solve by backward induction.

In the second period if P2 rejects (R), $u_2 = \delta^2 y = 0$ if P2 accepts (A) $u_2 = \delta z_2^2$. P2 accepts z_2^2 iff $z_2^2 \geq \delta y$ and rejects otherwise. $u_1 = \delta(x+y-z_2^2)$ iff $z_2^2 \geq \delta y$ and $\delta^2(x-k)$ otherwise. P1 ~~finds it~~ optimally induces A by choosing $z_2^2 = \delta y$, then $u_1 = \delta(x+y-\delta y)$. P1 finds it optimal to induce A iff $\delta(x+y-\delta y) \geq \delta^2(x-k)$ ~~$\Leftrightarrow (1-\delta)(x+y) \geq -\delta k$~~ $\Leftrightarrow_1 -\delta^2 y \geq \delta^2 x - \delta^2 k \Leftrightarrow_2$ ~~$x+y$~~ $0 \leq k$ where $\Leftrightarrow_1$ and $\Leftrightarrow_2$ follow from $y=-x$. So P1 finds it optimal to induce A

~~In the first period, if P2 rejects, P2's payoff is~~ P1 offers $z_1^2 = -\delta y$, $z_2^2 = \delta y$ and P2 accepts in the second period subgame.

In the first period if P2 rejects, $u_2 = \delta^2 y$. If P2 accepts, $u_2 = z_2^1$. Then P2 accepts iff $z_2^1 \geq \delta^2 y$. $u_1 = (x+y) - z_2^1 = -z_2^1$ if P2 accepts, $\delta(x+y-\delta y) = -\delta^2 y$ otherwise. P1 optimally induces A by choosing $z_2^1 = \delta^2 y$. Then P1 is indifferent between doing so and doing otherwise.

~~The~~ Each ~~SPE~~ some SPE is such that P1 offers $z_2^1 \leq \delta^2 y$ in period 1, P2 accepts iff $z_2^1 \geq \delta^2 y$ in period 1, P1 offers $z_2^2 = \delta y$ in period 2, P2 accepts iff $z_2^2 \geq \delta y$.

When $x+y=0$, P1 is indifferent between receiving this payoff (and paying off P2) in period 1 and in period 2. P1 maintains all bargaining power and offers P2 an amount of equal value to P2's (discounted) outside option, but is indifferent between terminating negotiations in period 1 and terminating in period 2 since deferring termination does not diminish the value of the eventual payoff (which is $x+y=0$).

c If P2 rejects z_2^2, $u_2 = \delta y$. If P2 accepts, $u_2 = z_2^2$. P2 accepts iff $z_2^2 \geq \delta y$.

$\pi_1(z_2^2) =$ ~~$P(z_2^2 \geq \delta y)(x+y-z_2^2) + P(z_2^2 < \delta y)\delta E(x-k)$~~

~~$= P(-z_2^2/\delta \leq x)(-z_2^2) + P(-z_2^2/\delta > x)\delta(x-k)$~~

~~$= (1+z_2^2/\delta)(-z_2^2) + (-z_2^2/\delta)\delta(x-k)$~~

$= P(z_2^2 \geq \delta y)(-z_2^2) + P(z_2^2 < \delta y)\delta E(x-k \mid z_2^2 < \delta y)$

$= P(-z_2^2/\delta \leq x)(-z_2^2) + P(-z_2^2/\delta > x)\delta E(x-k \mid -z_2^2/\delta > x)$

$= P(x > -z_2^2/\delta)(-z_2^2) + P(x < -z_2^2/\delta)\delta(E(x \mid x < -z_2^2/\delta) - k)$

$= (1 - -z_2^2/\delta)(-z_2^2) + (-z_2^2/\delta)\delta(-z_2^2/2\delta - k)$

$= (1 + z_2^2/\delta)(-z_2^2) + (z_2^2/\delta)\delta(z_2^2/2\delta + k)$

Let z abbreviate z_2^2

$= -z - z^2/\delta + z(z/2\delta + k)$

$= -z - z^2/\delta + z^2/2\delta + zk$

$= (k-1)z - z^2/2\delta$

FOC: $k - 1 - 2z/2\delta = 0 \Rightarrow z = \delta(k-1)$

SOC: $-1/\delta < 0$

P1 maximises π_1 by offering $z_2^2 = \delta(k-1)$ (and retaining $z_1^2 =$ ~~$\delta(1$~~ $\delta(1-k)$). P2 accepts iff $z_2^2 \geq \delta y$ and rejects otherwise. This is the unique BNE.

The equilibrium offer is independent of the magnitudes of x and y and depends only on k and δ.

~~Where only the defendant knows the amount of damages, the plaintiff~~

d

Date

7a Suppose for simplicity that in the event of a tie, buyer 1 (P1) loses. This supposition (or some ~~like it op~~ similar supposition) appears necessary given that if P1 wins ~~in the event of a tie~~ with some positive probability in the event of a tie then the expected payoff to P2 from $\{0,0\}$ is less than x_2, which is implied by the given formula.

Given b, $P(X_1 > b) = 1 - P(X_1 < b) = 1 - b^2$
In the event that $X_1 > b$, P2 wins one unit. P2 has valuation x_2 for this unit and pays the highest losing bid b. P2 has payoff $x_2 - b$ in this event.

Suppose $x_1 < b$, then ~~B~~ P2 wins two units and pays x_1 per unit. P2 has valuation $x_2 + Y_2$ for the two units won. P2 has payoff $x_2 + Y_2 - 2x_1$ in this event. In this event, P2 has expected payoff $E[x_2 + Y_2 - 2X_1 \mid X_1 < b]$
$= \frac{1}{P(X_1<b)} \int_0^b (x_2 + Y_2 - 2x) f(x)\, dx = \frac{1}{b^2} \int_0^b (x_2 + Y_2 - 2x) 2x\, dx$

P2's expected payoff from $\{a, b\}$ is
$\pi_2(a,b; x_1, 0) = P(X_1 > b)(x_2 - b) + P(X_1 < b) E[x_2 + Y_2 - 2X_1 \mid X_1 < b]$
$= (1 - b^2)(x_2 - b) + \int_0^b (x_2 + Y_2 - 2x) 2x\, dx$

b By inspection, ~~It is~~ $\pi_2(a,b; x_1, 0)$ is independent of a.

$\partial/\partial b\ \pi_2(a,b; x_1, 0) = -2b(x_2 - b) + (1 - b^2)(-1) + (x_2 + Y_2 - 2b)2b$
$= -2bx_2 + 2b^2 - 1 + b^2 + 2bx_2 + 2bY_2 - 4b^2$
$= -b^2 - 1 + 2bY_2$
≤ 0 (given $Y_2 \leq 1$)

$\pi_2(a,b; x_1, 0)$ is decreasing in b and independent of a, then $\pi_2(x_2, 0; x_1, 0)$ is a maximum and $\{x_2, 0\}$ is a best response against $\{x_1, 0\}$, (in interim expected payoffs). By symmetry, $\{x_1, 0\}$ is a best response against $\{x_2, 0\}$. Then, the strategy profile where each player i plays $\{x_i, 0\}$ is a BNE.

c No. Suppose that $x_1 \geq Y_1 > x_2 \geq Y_2$. Then the efficient outcome is such that P1 receives both units. ~~Under~~ At the above BNE, P1 and P2 each receive 1 unit.

d Under a K-unit Vickrey auction, each bidder i submits a list of valuations $(v_i^1, v_i^2, \ldots, v_i^K)$, for each unit up to the K^{th} unit. Then, the first unit is assigned to the player with the highest valuation, (of all valuations submitted), the second unit is assigned to the player with the second highest valuation, and so on. ~~Then each player~~ ~~For each player i let V_{-i} denote the total valuation of each winning~~ Each player then pays the sum of bids that would have won if not for his participation. ~~Then, each player's payoff~~

Under this mechanism, each player's payoff is equal to (1) his valuation ~~#~~ of the units won

A sequential second price auction is such that truthful bidding is ~~ap~~ weakly dominant ~~and~~ (and efficient). Under this mechanism, in the first K rounds, each bidder i submits one bid x_i^k and the first unit is allocated to the player with the highest bid. This player pays the second highest bid (submitted in this round). This is repeated for each of the K units available.

Truthful bidding is ~~ef~~ weakly dominant in each round. Suppose buyer i in round k has valuation v_i^k for the marginal unit. ~~If he bids $x_i^k > v_i^k$, with non-zero probability he wins the bid and pays $\hat{x}$~~ Suppose v_i^k is ~~not~~ less than the highest other bid $\hat{x}$. Then any bid x_i^k including v_i^k up to $\hat{x}$ yields zero payoff. x_i^k equal to $\hat{x}$ yields $\frac{1}{2}(v_i^k - \hat{x}) < 0$ and any higher bid yields $v_i^k - \hat{x} < 0$. Suppose $v_i^k > \hat{x}$. Then any $x_i^k > \hat{x}$ including v_i^k yields $v_i^k - \hat{x}$. Any lower bid yields lower payoff. Suppose $v_i^k = \hat{x}$. Then any ~~higher bid yields negative payoff, any~~ $x_i^k = v_i^k$ yields 0, any higher bid yields negative payoff and any lower bid yields 0. So ~~*~~ $x_i^k = v_i^k$ is weakly dominant.

Truthful bidding is weakly dominant in the sequential second price auction.

Date

3a

	L	C	R
U	$\underline{3}$, 3	0, $\underline{4}$	$\underline{2}$, 2
D	-1, $\underline{1}$	$\underline{2}$, $\underline{1}$	0, -2

Best responses underlined. By inspection the unique pure NE is ~~the~~ (D,C), where players play mutual pure best responses.

By inspection, each of L,C strictly dominates R for ~~player~~ P2, then P2 plays R with zero probability at NE.

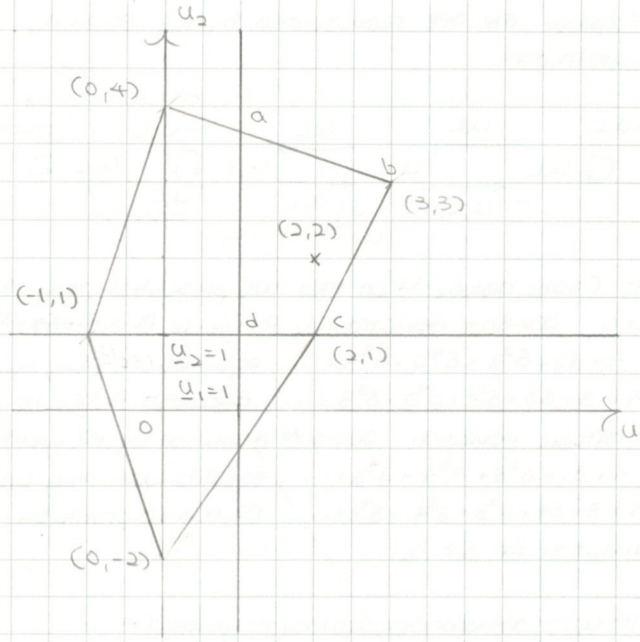
Suppose P1 mixes at NE σ^*, then by definition of NE, P1 has no profitable deviation, then $\pi_1(U, \sigma_2^*) = \pi_1(D, \sigma_2^*)$ $\Leftrightarrow 3q = -1q + 2(1-q) \Leftrightarrow 4q = 2-2q \Leftrightarrow 6q = 2 \Leftrightarrow q = 1/3$, where q is the probability σ_2^* assigns to L. Then P2 mixes at NE. $\Rightarrow \pi_2(L, \sigma_1^*) = \pi_2(C, \sigma_1^*) \Leftrightarrow$ $3p + 1(1-p) = 4p + 1(1-p) \Rightarrow p = 0 \Rightarrow$ P1 does not mix at NE. By reductio, P1 does not mix at NE.

If P2 mixes, P2 mixes only L, C, and P1 plays D with certainty. This candidate equilibrium fails to deviation by P1 to U. iff $q_L > 1/3$. Any $(D, q_L L + (1-q_L)C)$ for $q_L \leq 1/3$ is a mixed NE.

b ~~$\underline{v}_2 = \min_{a_1} \max_{a_2} \pi_2(a_1, a_2)$~~
~~Given $a_1 = U$, $\max_{a_2} \pi_2(a_1, a_2) = 4$~~
~~Given $a_1 = D$, $\max_{a_2} \pi_2(a_1, a_2) = 1$~~
~~Then P1 min~~

By inspection, P2 weakly maximises payoff by playing C. ~~Given~~ Then P1 min maxes P2 by playing D. If P2 plays C, P2's payoff is no lower than 1. If P1 plays D, P2's payoff is no higher than 1. $\underline{v}_2 = 1$.

$\pi_1(U, \sigma_2) = 3q_L + 2(1-q_L-q_C) = q_L - 2q_C + 2$
$\pi_1(D, \sigma_2) = -q_L + 2q_C = 2 - \pi(U, \sigma_2)$
P2 minmaxes P1 by choosing σ_2 such that
$\pi_1(U, \sigma_2) = \pi_1(D, \sigma_2) = 2 - \pi_1(U, \sigma_2) \Rightarrow \pi(U, \sigma_2) = \pi_1(D, \sigma_2) = 1$
for all ~~other~~ $\pi_1(U, \sigma_2) \neq 1$, $\max\{\pi_1(U, \sigma_2), \pi_1(D, \sigma_2)\} > 1$.
Then $\underline{v}_1 = 1$



The set of feasible and individually rational payoff pairs is represented by area abcd.

c The required strategy profile is such that each player plays his part of (U,L) in each period iff no player previously deviated and plays his part of (D,R) otherwise.

Suppose that P2 plays the prescribed strategy. ~~the~~ Then if P1 plays as prescribed, the present value PV of P1's payoffs is $3/1-\delta$. If P1 optimally deviates, ~~in period t,~~ P1 chooses D ~~then~~ in one period and U in each subsequent period which yields PV $-1 + 2\delta/1-\delta < 3/1-\delta$. P1 has no profitable deviation.

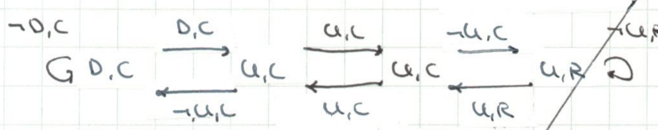
Suppose that P1 plays as prescribed. Then if P2 plays as prescribed, this yields PV $3/1-\delta$. Optimal deviation to to C then C or C in each subsequent period yields PV $4 + \delta/1-\delta$. $3/1-\delta \gtreqless 4 + \delta/1-\delta \Leftrightarrow$ $3 \gtreqless 4 - 4\delta + \delta = 4 - 3\delta \Leftrightarrow \delta \gtreqless 1/3$. P2 has no profitable deviation given $\delta = 1/3$.

This strategy profile is a NE.

At any history such that there has been a previous deviation, ~~as~~ P1 has profitable one-shot deviation to U (and P2 has such deviation to L, C). By the one-shot deviation principle, this strategy profile is not an SPE.

d Given $\delta = 1/4$, any candidate SPE such that on the equilibrium path (U,L) is played in every period fails to deviation by P2 to "C in ~~per~~ all periods". This deviation yields PV $\geq 4 + \delta/1-\delta$ ~~as~~. Equilibrium play yields $3/1-\delta$. $4 + \delta/1-\delta \geq 3/1-\delta \Leftrightarrow \delta \leq 1/3$. Given $\delta = 1/4$, this deviation is profitable. The candidate SPE is not an NE hence not a SPE.

e Consider the SPE represented by the following automaton.



At (some history such that the prescribed play is) U,C, one-shot deviation by P1 yields PV = ~~-1+δ2+δ^2 3+δ^3~~ $= -1+\delta 2+\delta^2 3+\delta^4 0+\delta^5 3+\dots$, equilibrium play yields PV $= 3+\delta 0+\delta^2 3+\delta^4 0+\delta^5 3+\dots$. ~~one-shot~~ P1 has no profitable deviation. One-shot deviation by P2 yields PV $= 4+\delta 1+\delta^2 3+\delta^3 4+\delta^4 3+\dots$, equilibrium play yields PV $= 3+\delta 4+\delta^2 3+\delta^3 4+\delta^4 3+\dots$. P2 has no profitable deviation for $\delta \geq 1/3$.

~~At U,C, one-shot deviation by P1 yields PV = 2+δ0+δ^2 3+δ^3 0+..., equilibrium play yields PV = 0+δ3+δ^2~~

~~At U,C, one-shot deviation by P1 yields~~

At U,C, one-shot deviation by P1 yields PV $= 2+\delta 2+\delta^2 0+\delta^3 3+\delta^4 0+\dots$, equilibrium play yields PV $= 0+\delta 3+\delta^2 0+\delta^3 3+\delta^4 0+\dots$. P1 has no profitable deviation for $\delta \geq 2/3$. One-shot deviation by P2 yields PV $= 3+\delta 0+\delta^2 4+\delta^3 0+\delta^4 4+\dots$, equilibrium play yields PV $= 4+$

Consider the ~~SPE represented by the following automaton~~ such that in the first period ~~players~~ each player plays his part of (U,C), then play alternates between (U,C) and (U,C) iff no player previously deviated. ~~and~~ otherwise revert to the mixed Nash with $q_L = 1/3$ in perpetuity.

It is trivial that there is no profitable deviation in the punishment phase.

Equilibrium payoffs in PV

At U,C to P1: $3+\delta 0+\delta^2 3+\dots = 3/1-\delta^2$

At U,C to P1: $0+\delta 3+\delta^2 0+\dots = 3\delta/1-\delta^2$

At U,C to P2: $3+\delta 4+\delta^2 3+\dots = 3+4\delta/1-\delta^2$

At U,C to P2: $4+\delta 3+\delta^2 4+\dots = 4+3\delta/1-\delta^2$

One-shot ~~payoffs in PV~~ deviation payoffs in PV

U,L P1 : $-1+\delta 1+\delta^2 1+\dots = -1+\delta/1-\delta$

U,C P1 : $2+\delta 1+\delta^2 1+\dots = 2+\delta/1-\delta$

U,C P2 : $4+\delta 1+\delta^2 1+\dots = 4+\delta/1-\delta$

U,C P2 : $3+\delta 1+\delta^2 1+\dots = 3+\delta/1-\delta$

By inspection, no player has a profitable one-shot deviation for δ sufficiently close to 1.