Microeconomics Paper 160529

1a Players : $N = \{1, \dots, n\}$, $n=2$

Actions : $A_i = \mathbb{R}_{\geq 0}$ for each player i

Payoffs : $\pi_i(a_i, a_{-i}) = \begin{cases} (a_i - c_i)\, q(a_i) & \text{if } a_i < a_{-i} \\ (a_i - c_i)\, q(a_i)/2 & \text{if } a_i = a_{-i} \\ 0 & \text{otherwise} \end{cases}$,

where $c_i = c$ for ~~all~~ each player i

The players are firms, their actions are prices and their payoffs are their profits, which are given by margin $(a_i - c_i)$ multiplied by quantity, which is 0 for the ~~highest price~~ higher price firm, $q(a_i)$ for the ~~lowest~~ lower price firm, or shared, $q(a_i)/2$ if they have the same price.

At NE, ~~$a_i =$~~ $a_1 = a_2 = c$ $(p_1 = p_2 = c)$, hence $\pi_1 = \pi_2 = 0$. Deviation by either firm to a higher price is not strictly profitable because ~~this~~ then demand is zero so profit remains zero. Deviation to lower price is not strictly profitable because then margin hence profit is negative.

b When firms ~~sell~~ offer differentiated products this result does not hold. Under such a model, demand ~~for each firm~~ q_i for each firm i given p_i, p_{-i} is some $\alpha - \beta p_i + \gamma p_{-i}$, where products are mutual substitutes but not perfect substitutes.

Intuitively, firms with differentiated products escape the Bertrand trap because demand faced by each firm is no longer perfectly elastic (when prices are equal), each firm can raise price above marginal cost ~~to capture~~ without alienating all consumers, hence enjoys positive profit from doing so. Only those consumers for whom the increase in price is not worth suffering for a product that better matches their preferences switch.

2. Consider arbitrary lottery ~~$L = [p_1, \dots, p_n; x_1, \dots, x_n$~~
$L = [p_1, \dots, p_n; x_1, \dots, x_n]$ and arbitrary agent with expected utility preferences and Bernoulli utility u.

Expected utility
$U(L) = \sum_{i=1}^{n} p_i u(x_i)$
Certainty equivalent $CE(L)$ is the amount such that the agent is indifferent between ~~EV CE~~ receiving $CE(L)$ with certainty and participating in ~~the~~ L.
$[1; CE(L)] \sim L \Leftrightarrow$
$u(CE(L)) = U(L) \Leftrightarrow$
$CE(L) = u^{-1}(U(L))$
Expected value
$EV(L) = \sum_{i=1}^{n} p_i x_i$
Risk premium
$RP(L) = EV(L) - CE(L)$.

Risk premium of some lottery ~~L~~ for some agent is the difference between that lottery's expected value and its certainty equivalent for that agent.

b This would imply that J is risk averse. Let L^J and L^S denote the respective lotteries.
$L^J = [1/2, 1/2; 10, 100]$
$L^S = [1/2, 1/2; 35, 65]$
Consider $L' = [1/2, 1/2; 5, 95]$
Suppose that J's preferences are strictly monotonic, i.e. J likes money.
Then $L^J \succ_J L'$ because L^J is ~~obtained by~~ equivalent to L' and receiving an additional 5 units of wealth. L' is a mean-preserving spread of L^S, so if J is risk-neutral or risk-loving, then $L' \succsim_J L^S$. Then, supposing that J has transitive preferences over lotteries, if J is risk-neutral or risk loving, $L^J \succ_J L^S$. Given instead $L^S \succsim_J L^J$, by reductio, J is risk averse.

c ~~The lott~~ Denote the lottery that J faces if they exchange tickets at price p as $L^e(p)$
$L^e(p) = [1/2, 1/2; 35-p, 65-p]$
The maximum p J is willing to pay is such that $L^J \sim_J L^e(p)$, which is iff
$u(L^e(p)) = u(L^J) \Leftrightarrow$
$\frac{1}{2}\ln(35-p) + \frac{1}{2}\ln(65-p) = \frac{1}{2}\ln 10 + \frac{1}{2}\ln 100 \Rightarrow$
$(35-p)(65-p) = 1000 \Rightarrow$
$p = 15$ or 85 (reject, since $\ln 35-85$ is undefined)

J is willing to pay no more than 15 for the exchange. $EV(L^S) = EV(L^J) + 5$, so the risk premium of L^J to J is $5+15=20$ greater than the risk premium of L^S to J.

3a If the quality of each bicycle b is commonly known, then (except in cases where the number of consumers of a given type is a binding constraint), each ~~bicycle~~ b is traded to the consumer with ~~the~~ ~~hig~~ the highest valuation for it at eqm (otherwise there is a mutually profitable trade). The candidate eqm is such that high quality H is owned by Z, M by ~~X~~ Y and C by Z. The above constraint does not bind, so this is the unique ~~optimal~~ eqm.

Eqm price of M is fully determined because at $p>60$, demand is zero and supply is one and at $p<60$, demand is at least two (from the two Ys) and supply is one, so only at $p=60$ ~~is~~ does the market for M clear.

Eqm prices of H and C are not fully determined. Any prices ~~$90 \le p_H \le$~~ $100 \le p_H \le 115$, $25 \le p_C \le 30$ and $115 - p_H = 30 - p_C$ can be sustained in eqm. At such prices ~~demand~~ supply of each is one, and demand for each is one.

b For ~~p~~ ~~$20 \le p \le 30$~~ $p=30$, there is an eqm such that C is traded from X to Z.

Suppose there is some eqm such that M is traded (and not H), then $p \geq 35$, so C is traded, so Y has expected valuation $60+20/2 = 40$, and Z has expected valuation $50+30/2 = 40$. Demand is zero, there is no such eqm.

Suppose there is some eqm such that ~~C, M~~, H ~~are~~ is traded, then $p \geq 90$, ~~so Y~~ then C, M are supplied, Y has expected valuation $100+60+20/3 = 60$, Z has expected valuation $115+50+30/3 = 65$, so demand is zero, market does not clear, there is no such eqm.

There is adverse selection. At eqm, only C is traded. Competing Z buyers bid the price up to $p=30$, demand is one, supply is one, the market clears.

c The equilibrium ~~outco~~ allocation is unchanged, there is no trade, ~~this is sustained i~~ at eqm, the price is no greater than 30, no bicycles are supplied, and demand is zero.

4a Given h, the subscriber s has utility maximisation problem
$\max_x B(x)$

FOC: $a - 2bx - p = 0 \Rightarrow \cancel{2bx = a}$

$2bx = a - p \Rightarrow$

$x = a-p/2b$

SOC: $-2b < 0$

$x^* = a-p/2b$ uniquely solves s's utility maximisation problem. The optimal number of tracks is independent of h because h is an upfront fixed cost, so ~~and a trade~~ has no effect on marginal utility, and consumers make decisions at the margin.

s has net benefit, given optimal x

$$B(x^*) = a(a-p/2b) - b(a-p/2b)^2 - h - p(a-p/2b)$$
$$= a(a-p)/2b - \tfrac{1}{2}(a-p)^2/2b - h - p(a-p)/2b$$
$$= a(a-p) - \tfrac{1}{2}(a-p)^2 - p(a-p)/2b - h$$
$$= (a-p)^2 - \tfrac{1}{2}(a-p)^2/2b - h$$
$$= (a-p)^2/4b - h$$

b A consumer subscribes iff for this consumer, net benefit given the optimal number of tracks is positive. This is iff

$(a-p)^2/4b - h \geq 0$

$h \leq (a-p)^2/4b$

Given that h is uniformly distributed on the unit interval, there are

$N^* = N(a-p)^2/4b$ such consumers

c The firm has gross profit px^* from each consumer (which, as argued above, is independent of h).

$$\pi = N^* p x^*$$
$$= N(a-p)^2/4b \; p \; (a-p)/2b$$
$$= Np \; \cancel{(a-p)^2} \; (a-p)^3/8b^2$$

$\max_p \pi$

FOC: $N(a-p)^3/8b^2 + Np\, 1/8b^2\, 3(a-p)^2(-1) = 0 \Rightarrow$

$(a-p)^3/8b^2 = \cancel{P/8b^2}\ \cancel{3(a-p)}\ 3p(a-p)^2/8b^2 \Rightarrow$

$(a-p)^3 = 3p(a-p)^2 \Rightarrow$

$(a-p) = 3p \Rightarrow$

$p = a/4$

$p^* = a/4$ uniquely solves the firm's profit maximisation problem. It is intuitive that the optimal price is increasing in consumer's ~~cost~~ valuations.

d Given that tracks are "produced" at zero marginal cost, the efficient price is $p=0$.

$N^* = \cancel{N}\ N a^2/4b$ given $p=0$.

At price equal marginal cost, marginal valuation is equal to marginal cost, social surplus is maximised

Consumers are worse off at the profit-maximising price because they pay a higher price and consume fewer tracks. The firm is better off at this price because it maximises profit. At the efficient price, the firm ~~~~ has zero profit, social surplus is maximised and entirely captured by consumers, so consumer surplus is maximised. Tracks are a club ~~go~~ good because they are excludable but non-rival.

10a Firm i has profit function

$\pi_i(q_i, q_{-i}) = P(q_i + q_{-i})q_i - C(q_i)$

$= (100 - q_i - q_{-i})q_i - 10q_i$

$= (90 - q_i - q_{-i})q_i$

Compute firm i's best response. Firm i maximises π_i, taking q_{-i} as given

$\max_{q_i} \pi_i(q_i, q_{-i})$

FOC: $(90 - q_i - q_{-i}) + -q_i = 0 \Rightarrow$

$2q_i = 90 - q_{-i} \Rightarrow$

$q_i = 90 - q_{-i}/2$

SOC: $-2 < 0$

$q_i^\dagger = 90 - q_{-i}/2$ uniquely solves firm i's profit maximisation problem given q_{-i}. Firm i's best response function is $b_i(q_{-i}) = 90 - q_{-i}/2$.

At Cournot Nash, firms play mutual best responses

$q_1^* = b_1(q_2^*)$, $q_2^* = b_2(q_1^*) \Rightarrow$

$q_1^* = 90/2 - q_2^*/2$

$= 90/2 - 1/2(90/2 - q_1^*/2)$

$= 90/4 + q_1^*/4 \Rightarrow$

$3q_1^*/4 = 90/4 \Rightarrow$

$q_1^* = 90/3 = 30 \Rightarrow$

$q_2^* = 90/2 - q_1^*/2 = 30$

~~At Cou~~ The Cournot Nash eqm is the strategy profile $(q_1^* = 30, q_2^* = 30)$. It is symmetric because firms have equal marginal costs and move simultaneously, so the game is symmetric.

$\pi_1^* = \pi_2^* = (100-30-30)30 - 10 \times 30 = 30^2 = 900$

Each firm's best response is decreasing in the other firm's output because outputs are strategic substitutes. An increase in output ~~decre~~ by one firm causes a decrease in price hence a decrease in margin and a decrease in marginal profit wrt output for the other firm.

b Firm 1's ~~best f~~ profit function at the lower cost

$\pi_1(q_1, q_2) = P(q_1 + q_2)q_1 - C(q_1)$

$= (100 - q_1 - q_2)q_1 - 4q_1$

$= (96 - q_1 - q_2)q_1$

$\max_{q_1} \pi_1(q_1, q_2)$

FOC: $96 - q_1 - q_2 - q_1 = 0 \Rightarrow$

$q_1 = 96 - q_2/2$

SOC: $-2 < 0$

$q_1^\dagger = 96 - q_2/2$ uniquely solves firm 1's profit maximisation problem given q_2. Firm 1's best response function is $b_1(q_2) = 96 - q_2/2$.

Firm 2's best response function is unchanged from before.

At Cournot NE, firms play mutual best responses.

$q_1^* = b_1(q_2^*)$, $q_2^* = b_2(q_1^*)$

$q_1^* = 96/2 - 1/2(90/2 - 1/2 q_1^*)$

$= 51/2 - 1/4 q_1^* \Rightarrow$

$3q_1^*/4 = 51/2 \Rightarrow$

$q_1^* = 204/6 = 34$

$q_2^* = 90/2 - q_1^*/2 = 28$

$\pi_1^* = \pi_1(q_1^*, q_2^*) = (100-34-28)34 - 4(34) = 1156$

$\pi_2^* = \pi_2(q_2^*, q_1^*) = (100-34-28)(28) - 10(28) = 784$

c Firm 1 earns more at the latter Cournot NE because the reduction in cost has two positive effect on profit. The first is the direct effect. A decrease in ~~marginal~~ constant marginal cost causes a decrease in total cost (all else being equal) hence an increase in profit. The second is the strategic effect. A decrease in marginal cost makes firm 1 more aggressive. This is because firm 1's margin increases, so firm 1 has greater incentive to produce high output given any firm 2 output. Then, because quantities are strategic substitutes, as explained earlier, firm 2 responds by reducing output. This benefits firm 1 because their products are substitutes. So at eqm, firm 1 plays more aggressively (higher output) which hurts firm 2, and firm 2 plays less aggressively (lower output) which benefits firm 1.

d Solve for the SPE by backward induction.

If firm 1 does not invest, the outcome of the Cournot subgame at the subgame NE is that found in (a). Firm 1 has gross (of investment cost) profit 900 and net profit 900.

If firm 1 does ~~not~~ invest, the outcome of the Cournot subgame at the subgame NE is that found in (b). Firm 1 has gross profit 1156 and net profit $1156 - K$.

In the first stage, investing is optimal for firm 1 iff $1156 - K \geq 900 \iff K \leq 256$.

Suppose $K \leq 256$, then at SPE, firm 1's strategy is Invest, then $q_1 = 34$ if Invest, $q_1 = 30$ otherwise. Firm 2's strategy is $q_2 = 28$ if (firm 1) Invest, $q_2 = 30$ otherwise. A SPE is a strategy profile that is a NE that induces a NE in each subgame, and such a strategy profile consists of strategies that ~~spec~~ are complete contingent (on past actions even off eqm path) plans of action, hence the above form.

e $K = 200$.

Consider the strategy profile such that firm 1 plays Not invest ~~and~~ then $q_1 = 30$ regardless and firm 2 plays $q_2 = 30$ regardless.

~~Suppose P1 invested and q_2~~

Consider the subgame in which P1 played invest. Suppose $q_2 = 30$, then $b_1^*(q_2) = 96/2 - q_2/2 = 48 - 15 = 33$, then $\pi_1(b_1(q_2), q_2 = 30) = (100 - 33 - 30)33 - 4(33) = 1089$. Then firm 1 has net profit $1089 - K = 889$. This is the maximum profit firm 1 can make if it invests.

At the above Not invest strategy profile, firm 1 has gross profit 900 (from a) hence net profit $900 > 889$. Firm 1 has no profitable deviation to Invest (and any quantity subsequently). Firm 1 also has no profitable deviation to Not invest and any other quantity, ~~because $q_1 = 30$ is a best~~ nor does firm 2 to any other quantity because $q_1 = q_2 = 30$ is a subgame NE given Not invest.

So neither firm has profitable deviation from this strategy profile, it is an NE.

f Firm 2's threat to play $q_2 = 30$ regardless of firm 1's action in the first stage is not credible. This strategy is not time consistent because if firm 1 does invest, and optimally responds to $q_2 = 30$ with $q_1 = 33$, then firm 2 best responds with $q_2 \neq 30$ (given that $q_1 = 33$, $q_2 = 30$ is not a NE in the invest subgame, as found in b).

Firm 2 has higher profit (900) in the non-credible NE, than in the SPE (784), so firm 2 prefers this, but it is not sustainable in eqm.

SPE rules out strategies that involve non-credible threats and are time-inconsistent. Such strategy profiles do not induce NE in off eqm path subgames, which are relevant in evaluating whether some strategy profile is a SPE.