Microeconomic Analysis Problem Set 3

1a $x, y \in \{1,2,3,\ldots\}: x \succsim y \Leftrightarrow x \leq y+1$

Suppose $\succsim$ is not connected. Then by definition of connectedness, $\exists x, y \in \{1,2,3,\ldots\}: x \not\succsim y$ and $y \not\succsim x$, then $x \not\leq y+1$, $x > y+1$, $y < x-1$, $y \leq x+1$, $y \succsim x$. By reductio, $\succsim$ is connected.

Suppose $\succsim$ is not reflexive. Then by definition of reflexivity, $\exists x \in \{1,2,3,\ldots\}: x \not\succsim x$, then $x \not\leq \cancel{y}\, x+1$, $x > x+1$. By reductio, $\succsim$ is reflexive.

Suppose $\succsim$ is not complete. Then by definition of completeness, $\exists x, y \in \{1,2,3,\ldots\}: x \not\succsim y$ and $y \not\succsim x$, then $x \not\leq y+1$, $x > y+1$, $y < x-1$, $y \leq x+1$, $y \succsim x$. By reductio, $\succsim$ is complete.

connectedness and reflexivity imply completeness.

~~Suppose $\succsim$ is not transitive. Then by definition of transitivity, $\exists x, y, z \in \{1,2,3,\ldots\}: x \succsim y$ and $y \succsim z$ and $x \not\succsim z$, then $x \leq y+1$, $y \leq z+1$, $x \leq y+1$~~

$\succsim$ is not transitive since $x \succsim y$ and $y \succsim z$ but $x \not\succsim z$ for $x=3, y=2, z=1$, by definition of transitivity

sufficient to give counterexample

Since $\succsim$ is reflexive, it is not asymmetric. Suppose that $\succsim$ is asymmetric. Since $\succsim$ is reflexive, and $\succsim \neq \emptyset$, $\exists x \in \{1,2,3,\ldots\}: x \succsim x$. Supposing that $\succsim$ is asymmetric, by definition of asymmetry, $x \not\succsim x$. By reductio, $\succsim$ is not asymmetric.

$\succsim$ is not antisymmetric since $x \succsim y$ and $y \succsim x$ but $x \neq y$ for $x=2, y=1$, by definition of antisymmetry.

b Ann's preferences are rational iff they are complete and transitive. Since Ann's preferences are not transitive, they are not rational.

This is sufficient

$\succsim$ can be represented by a utility index only if $\succsim$ is rational, so $\succsim$ cannot be represented by a utility index.

Real numbers are fully ordered, real numbers are completely and transitively ordered by the ~~relati~~ $\geq$ relation

Apply the definition of utility representation

~~It is gi~~

By construction, $(x,y) \in R \leftrightarrow (y,x) \notin P$ ①

Suppose R is complete, then by definition of ~~Complete~~ completeness, since R is a binary relation on X, $\forall x,y \in X$: xRy or yRx, then by ① ~~$(y,x) \notin P$ or $(x,y) \notin P$~~ $\forall x,y \in X$: $(y,x) \notin P$ or $(x,y) \notin P$, i.e. $\forall x,y \in X$: not both $(x,y) \in P$ and $(y,x) \in P$, by definition of ~~transitivity~~, P is ~~not transitive (on X)~~. asymmetric (on X).

asymmetry

Suppose R is not complete, then by definition of completeness, since R is a binary relation on X, $\exists x,y \in X$: neither xRy nor yRx, then by ① $\exists x,y \in X$: yPx and xPy, by definition of asymmetry, P is not asymmetric.

Important to show ~~iff~~ bidirectional proof

Equivalent to prove with iff-chain.

So R is complete iff P is asymmetric.

Suppose that R is transitive, then by definition of transitivity, $\forall x,y,z \in X$: if xRy and yRz then xRz. Then by ①, $\forall x,y,z \in X$: if not yPx and not zPy then not zPx, by definition of negative transitivity, P is negative transitive.

Suppose that R is not transitive, then by definition of transitivity, $\exists x,y,z \in X$: xRy and yRz but not xRz. Then by ①, $\exists x,y,z \in X$: not yPx and not zPy but zPx, by definition of negative transitivity, P is not negative transitive

So R is transitive iff P is negative transitive.

3a Let $u(x_1,x_2) = x_1^3 x_2$

Then $\forall \vec{x},\vec{y} \in \mathbb{R}^2_{++}$: if $\vec{x} \succsim \vec{y}$, then ~~by definitio~~ ~~by definition~~ given $\vec{x} \succsim \vec{y} \Leftrightarrow x_1^3x_2 \geq y_1^3 y_2$, $x_1^3x_2 \geq y_1^3 y_2$, then by construction of $u(\cdot)$, $u(\vec{x}) \geq u(\vec{y})$. Then, by definition of utility representation, $\succsim$ is represented by $u(\cdot)$. So $\succsim$ can be represented by a (continuous) utility function.

b Let $u'(x_1,x_2) = x_1^3 x_2$. From (a), $\succsim$ is represented by u'.

For all ~~$f:\mathbb{R}\to\mathbb{R}$, but~~ strictly increasing $f:\mathbb{R}\to\mathbb{R}$, then $v = f\circ u'$ is a utility function that represents $\succsim$.

~~$u(x_1,x_2) = x_1^{3/4} x_2^{1/4} = f\circ u'(x_1,x_2)$~~

By definition of representation by a utility function, $\forall \vec{x},\vec{y} \in \mathbb{R}^2_{++}$: if $\vec{x} \succsim \vec{y}$ then $u'(\vec{x}) \geq u'(\vec{y})$, then ~~by~~ since f is ~~a~~ strictly increasing, $f\circ u'(\vec{x}) \geq f\circ u'(\vec{y})$, so $f\circ u'$ is a utility function that represents $\succsim$.

$u = f_u \circ u'$ where $f_u(x) = x^{1/4}$

$v = f_v \circ u'$ where $f_v(x) = \ln x$

$w = f_w \circ u'$ where $f_w(x) = x^{-1}$

f_u and f_v are strictly increasing in the range of u', so u and v represent $\succsim$.

f_w is not strictly increasing (and is in fact strictly decreasing in the range of u'; so w does not represent $\succsim$

c ~~Let u be the uniform~~

Let $u(x_1,x_2) = x_1^3 x_2$ and v be the uniform consumption equivalent utility representation, i.e. $v(x_1,x_2) = x$ s.t. $(x_1,x_2) \sim (x,x)$

Given that $(x_1,x_2) \succsim (y_1,y_2) \Leftrightarrow x_1^3x_2 \geq y_1^3y_2$, by ~~de~~ definition of $\sim$, $\vec{x} \sim \vec{y}$ iff $\vec{x} \succsim \vec{y}$ and $\vec{y} \succsim \vec{x}$, then $\vec{x} \sim \vec{y}$ iff $x_1^3x_2 = y_1^3y_2$. ~~$x_1^3 x$~~ $(x_1,x_2) \sim (x,x)$ iff $x_1^3x_2 = x^4$, $x = x_1^{3/4}x_2^{1/4}$, $v(x_1,x_2) = x_1^{3/4}x_2^{1/4}$.

The uniform consumption equivalent utility representation is ~~the Cobb-Douglas funct corresponding Cobb~~ Cobb-Douglas.

Alternatively, apply Debreu's theorem. "complete, transitive, continuous"

"lexicographic preferences are pretty much the only counterexample" presumably to rational preferences $\Rightarrow \exists$ utility representation

This should be a bidirectional proof

Have to argue complete, transitive & continuous. Difficult to argue continuity.

Monotonicity is not sufficient

1a Let $\vec{x} = (x_1, x_2, x_3)$ denote the observed student's choices where x_1 is her choice on Monday, x_2 on Tuesday, and x_3 on Wednesday. ~~Let $\succsim$ denote the observed student's preferences.~~ Let $\succ$ denote the observed student's rational strict preference relation.

can argue that F must be chosen otherwise unable to place F

Suppose $\vec{x} = (F, P, M)$, then ~~$F \succ P$~~ $F \succsim_c M$, $F \succsim_c P$, $P \succsim_c V$ and $M \succsim_c V$. We cannot infer whether $M \succ P$ or $P \succ M$, so the observed student did not choose (F, P, M)

Suppose $\vec{x} = (F, P, V)$, then $F \succsim_c M$, $F \succsim_c P$, $P \succsim_c V$ and $V \succsim_c M$, then $F \succsim_c P \succsim_c V \succsim_c M$, i.e. we can infer $\succ$, so the observed student could have chosen (F, P, V).

By symmetry, the observed student could have chosen (F, V, M).

Suppose $\vec{x} = (F, V, V)$, then we cannot infer whether $P \succ M$ or $M \succ P$, so the observed student did not choose (F, V, V)

Suppose $\vec{x} = (M, P, M)$, then we cannot infer whether $F \succ V$ or $V \succ F$, so $\vec{x} \neq (M, P, M)$. Suppose $\vec{x} = (M, P, V)$, then ~~we cannot infer whether~~ $M \succsim_c P$, $P \succsim_c V$, and $V \succsim_c M$, so $\succ$ is not transitive hence not rational. By reductio, $\vec{x} \neq (M, P, V)$. Suppose $\vec{x} = (M, V, M)$, then we cannot infer whether $F \succ P$ or $P \succ F$, so $\vec{x} \neq (M, V, M)$. Suppose $\vec{x} =$ ~~(M, V, P)~~ (M, V, V), then we cannot infer whether $F \succ P$ or $P \succ F$, so $\vec{x} \neq (M, V, V)$. So $x_1 \neq M$. By symmetry, $x_1 \neq P$.

$\vec{x} = (F, P, V) \text{ or } (F, V, M)$

b Let $X_i' = A_i / \{x_i\}$ for $i \in \{1, 2, 3\}$
$\underbrace{c(A_i, \succ)}_{=x_i}$ directly reveals $x_i \succsim_c x_i'$ for all $x_i' \in X_i'$, i.e. each choice reveals that the chosen ~~item is~~ element is preferred by the observed student to each ~~eleme~~ other element in that day's menu.
All other inferred preferences are revealed indirectly by the transitivity of rational preferences, from the directly revealed preferences.

Suppose $\vec{x} = (F, P, V)$, then the following preferences are revealed directly: $F \succsim_c M$, $F \succsim_c P$, $P \succsim_c V$, $V \succsim_c M$, and the following preferences are revealed indirectly: $P \succsim_c M$, $F \succsim_c V$.

Suppose $\vec{x} = (F, V, M)$, then $F \succsim_c M$, $F \succsim_c P$, $V \succsim_c P$, and $M \succsim_c V$ are revealed directly and $F \succsim_c V$ and $M \succsim_c P$ are revealed indirectly.

c $\vec{x} = (P, V, V) \Rightarrow P \succsim_c F$, $P \succsim_c M$, $V \succsim_c P$, $V \succsim_c M$. (directly)
$\Rightarrow V \succsim_c F$ (indirectly)
So whether $M \succ F$ or $F \succ M$ cannot be inferred from ~~$c(A_i, \succ) =$~~ $\vec{x} = (P, V, V)$. So $\succ$ is fully mapped iff $A_4 = \{F, M\}$.

"explain a bit why nothing else can be on menu"
If sth else is on the menu, then it gets picked, so whether $F \succ M$ or the reverse is not revealed.

d $\vec{x} = \cancel{(P, M, V)}\ (P, V, M) \Rightarrow P \succ_c F, P \succ_c M, M \succ_c V, V \succ_c P$ (directly)
$\Rightarrow V \succ_c F, V \succ_c M, V \succ_c V$ (indirectly). We conclude that the observed student's preferences are irrational since it is irrational to strictly prefer some element to itself.

can show intransitivity

e Denote the Monday Meat dish MM. Then
$\Delta_1 = \{F, MM, P\}$, $\Delta_2 = \{P, V\}$, $\Delta_3 = \{M, V\}$, $\Delta_4 = \{M, F, V, P\}$
$c(\Delta_1, \succ) = P$, $c(\Delta_2, \succ) = V$, $c(\Delta_3, \succ) = M \Rightarrow$
$P \succ_c F, P \succ_c MM, V \succ_c P, M \succ_c V$ (directly) $\Rightarrow$ (by transitivity)
$V \succ_c F, V \succ_c MM, M \succ_c P, M \succ_c F, M \succ_c MM$ (indirectly) $\Rightarrow$
$c(\Delta_4, \succ) = M$

"explain your reasoning a little bit more"
Maybe ~~#~~ break down the answer

a By inspection, $U(L)$ is a function of $\vec{p}$ and N, and is independent of $\vec{x}$. So if $\exists u: \mathbb{R} \to \mathbb{R}$ such that $U(L) = \sum_{i=1}^{N} p_i u(x_i)$, $u(x_i) = c$ for all x_i, where c is some constant. Then $U(L) = \sum_{i=1}^{N} c p_i = c \sum p_i = c$, then $U(L)$ is ~~neither a func~~ not a function of $\vec{p}$ and N. By reductio, $\nexists u: \mathbb{R} \to \mathbb{R}$ such that $U(L) = \sum_{i=1}^{N} p_i u(x_i)$, i.e. $U(L)$ is not consistent with the EU hypothesis.

b By an analogous argument to that in (a), since $U(L)$ is ~~an~~ independent of $\vec{x}$, ~~$U(L) = c$, where c is some~~ if the EU hypothesis holds, $U(L) = c$ where c is some constant, and $U(L)$ is not a function of $\vec{p}$. By reductio, ~~the EU hypothesis does~~ $U(L)$ is not consistent with the EU hypothesis.

violates independence axiom since $[1, x_1]$ ~~and~~ $\sim$ $[1, x_2]$ $\succ$ $[\frac{1}{2}, \frac{1}{2}; L_1, L_2]$

c ~~$U(L) = \sum_{i \geq G} p_i = \sum_{i=1}^{N} p_i u(x_i, \vec{x})$, where $u(x_i, \vec{x}) = \begin{cases} 1 & \text{iff } x_i \geq x_G \\ 0 & \text{otherwise.} \end{cases}$~~

Let $L' = [p_2, \dots, p_N, p_1; x_2, \dots, x_N, x_{N+1}]$ where $x_{N+1} > x_N$. $U(L) = p_G + \dots + p_N$, $U(L') = p_{G+1} + \dots + p_1 = U(L) - p_G + p_1$. Suppose that $U(L)$ is consistent with the EU hypothesis, then $\exists u: \mathbb{R} \to \mathbb{R}$ such that $U(L) = \sum_{i=1}^{N} p_i u(x_i)$, then $U(L') = U(L) - p_1 u(x_1) + p_1 u(x_{N+1})$, then $p_1 - p_G = p_1 u(x_{N+1}) - p_1 u(x_1)$. Let ~~$L'' = [p_2, \dots, p'_{G-1}, p'_G$~~
$L'' = [p_2, \dots, p'_{G-1} = p_{G-1} - \varepsilon, p'_G = p_G + \varepsilon, \dots, p_N, p_1; x_2, \dots x_N, x_{N+1}]$
By an analogous argument, $p_1 - p'_G = p_1 u(x_{N+1}) - p_1 u(x_1)$, ~~By reductio, since~~ then $p_1 - p'_G = p_1 - p_G$. By reductio, since $p_G \neq p'_G$, $U(L)$ is not consistent with the EU hypothesis.

d Suppose that $U(L)$ is consistent with the EU hypothesis, then $\exists u': \mathbb{R} \to \mathbb{R}$ such that $U(L) = \sum_{i=1}^{N} p_i u'(x_i)$. ~~Let i^* denote $\operatorname{argmin}_i u(x_i)$ such that $p_i > 0$. Then $u'(x_{i^*})$.~~ By inspection, ~~$U(L)$ is independent of p_i for all $i \neq i^*$, then $u'(x_i) = 0$ for all $i \neq i^*$. Let L' be the lottery identical to L except in reallocating all probability from x_{i^*} to some other result with non-zero probability in L. Then $U(L') = 0$. By inspect~~ Consider L such that $\forall i: p_i > 0$. ~~The~~ By inspection, $U(L)$ is independent of $\vec{p}$, so $u'(x_i) = 0$ for all i, so $U(L) = 0$ for all such L, so $u'(x_i) = 0$ for all x_i, so $U(L) = 0$ for all L. $U(L)$ is consistent with the EU hypothesis only if $u(x_i) = 0$ for all x_i. It is trivially true that ~~EU~~ $U(L)$ is consistent with the EU hypothesis if $u(x_i) = 0$ for all x_i.

Argue utility function is not continuous by giving some sequence

~~continuous lottery giet~~
~~sequence $\{\frac{1}{2}$~~

e ~~consider~~ $k =$ ~~argmax~~ $\max k$ st ~~$\forall$~~ $\forall i: p_k > p_i$
Let L' be the lottery obtained by reallocating probability mass from some x_i where $i \neq k$ to x_k. Suppose that $U(L)$ is consistent with the EU hypothesis. By construction of $U(L)$, $U(L) = U(L')$. $U(L) = \sum_{i=1}^{N} p_i u(x_i)$, ~~then since~~ then $p_i u(x_i) = p_i u(x_k)$. Since i and k are chosen arbitrarily, $u(x_i) = u(x_j)$ for all x_i, x_j, $u(x)$ is a constant, c, $U(L) = c$ for all L. By reductio, $U(L)$ is not consistent with the EU hypothesis.

Sequence $\{\frac{1}{2} + \frac{1}{n}, \frac{1}{2} - \frac{1}{n}; 1, 2\}_{n=1}^{\infty}$ ~~tt~~ converges to $\{\frac{1}{2}, \frac{1}{2}; 1, 2\}$, but $u(\{\frac{1}{2} + \frac{1}{n}, \frac{1}{2} - \frac{1}{n}; 1, 2\})$

6 In the first arrangement, each agent has compound lottery

$L_1 = [1/N, (N-1)/N; w+L, w]$

which reduces to

$\cancel{L_1^R = [1/N, \tfrac{N-1}{N}p_1, \tfrac{N-1}{N}p_2, \ldots \tfrac{N-1}{N}p_k; w}$

$L_1^R = [p_1/N, p_2/N, \ldots, p_k/N, (N-1)/N; w+x_1, w+x_2, \ldots, w+x_k, w]$

In the second arrangement, each agent has ~~compound~~ lottery

$L_2 = [p_1, p_2, \ldots p_k; w+x_1/N, w+x_2/N, \ldots, w+x_k/N]$

Each agent has strictly risk averse EU preferences

$U(L) = \cancel{\sum_{i=1}^k} \sum_i p_i u(w_i)$, where u is strictly concave

$U(L_1) = U(L_1^R)$

$= p_1/N\, u(w+x_1) + p_2/N\, u(w+x_2) + \ldots + p_k/N\, u(w+x_k) + (N-1)/N\, u(w)$

$= 1/N\,[p_1 u(w+x_1) + p_2 u(w+x_2) + \ldots p_k u(w+x_k) + (N-1)u(w)]$

$= 1/N \sum_{i=1}^k p_i u(w+x_i) + (N-1)/N\, u(w)$

$= \sum_{i=1}^k p_i u(1/N(w+x_i) + (N-1)/N\, w)$

By strict concavity $u(1/N(w+x_i) + (N-1)/N\, w) > 1/N\, u(w+x_i) + (N-1)/N\, u(w)$

$U(L_2) = \sum_{i=1}^k p_i u(w + x_i/N)$

$\cancel{= u(w) + \sum_{i=1}^k p_i [u(w+x_i/k) - u(w)]}$

$= u(w) + \sum_{i=1}^k p_i [u(w+x_i/N) - u(w)]$

$U(L_1) = u(w) + \sum_{i=1}^k p_i/N\,[u(w+x_i) - u(w)]$

since u is strictly concave, $u(w+x_i/N) - u(w) > 1/N\,[u(w+x_i) - u(w)]$, so $U(L_2) > U(L_1)$.

Risk averse agents prefer the second arrangement under which risk is shared across agents.

Date