# Vertical Relations Rough Notes

## Lecture

- Structure
    - Vertical Externality
        - Basic Model
    - Vertical Restraints
        - Franchise Fee/Two-Part Tariff
        - Retail-Price Maintenance
        - Sponsoring Downstream Competition
        - Chicago School Exclusive Contract
            - Rasmusen et al. Downstream Firms in Separate Markets
            - Aghion and Bolton Exclusive Contract with Penalty
            - Hart and Tirole Secret Contracts
    - Vertical Merger
        - Salinger (1988) Input Foreclosure
        - Luco-Marshall (2020) Multiproduct Firms
- Vertical Externality
    - Parameters
        - Consider the following simple model of vertical relations. In the first of two stages, an upstream monopolist $U$ produces an intermediate good at constant marginal cost $c_U$ and chooses wholesale price $w$ at which to sell to a downstream monopolist $D$. In the second stage, the downstream monopolist transforms each unit of the intermediate good into one unit of a final good at constant marginal cost $c_D$ and chooses retail price $p$ at which to sell to consumers. The downstream monopolist faces downward-sloping demand $Q(p)$. The downstream monopolist maximises profit $\pi_D = (p - w - c_D)Q(p)$ and the upstream monopolist maximise profit $\pi_U = (w - c_U)Q(p)$. Quantity of the upstream firm is given by demand at price $p$ chosen by the downstream firm since demand for the intermediate good is derived from demand for the final good.
    - Analysis
        - Suppose that $U$ and $D$ are vertically integrated, then the integrated firm produces the final good at marginal cost $c_U + c_D$ and chooses retail price $p_I$ to maximise profit $\pi_I$. At equilibrium, $p_I = p^M(c_U + c_D) \equiv \arg\max_p[(p - c_U - c_D)Q(p)]$ and $\pi_I = \pi^M(c_U + c_D) \equiv \max_p[(p - c_U - c_D)Q(p)] = (p^M(c_U + c_D) - c_U - c_D)Q(p^M(c_U + c_D))$.
        - Suppose instead that $U$ and $D$ are vertically separated. In the second stage sub-game, $D$ chooses $p$ given $w$ to maximise $\pi_D$. At the subgame Nash equilibrium, $p = p^M(w + c_D) \equiv \arg\max_p[(p - w - c_D)Q(p)]$. $D$ has marginal cost $w + c_D$ and chooses the corresponding monopoly price. In the first stage, $U$ chooses $w$ to maximise $\pi_U = (w - c_U)Q(p^M(w + c_D))$. At the subgame-perfect equilibrium, $w > c_U$ since $\pi_U \leq 0$ if $w \leq c_U$ and $Q(p^M(w + c_D)) > 0$ for some $w > c_U$ hence $\pi_U > 0$ for some $w > c_U$.
        - Assuming that marginal revenue is decreasing, an increase in (constant) marginal cost causes a decrease in the profit-maximising monopoly quantity (at which marginal revenue is equal to marginal cost) hence an increase in monopoly price (since demand is downward-sloping). Since $w > c_U$ and $p^M(.)$ is an increasing function, $p^M(w + c_D) > p^M(c_U + c_D)$, i.e. equilibrium retail price is higher where $U$ and $D$ are vertically separated than when $U$ and $D$ are vertically integrated.
    - Result (Vertical Externality)
        - Equilibrium retail price is higher under vertical separation than under vertical integration because of double marginalisation in the former case. Under vertical separation, where both upstream and downstream industries are not perfectly competitive, each firm chooses price above its marginal cost. Under vertical separation, each firm does not account for the negative externality inflicted on the other in choosing a high price. An increase in the wholesale price by the upstream firm directly causes an increase in cost hence a decrease in profit for the downstream firm. An increase in the retail price by the downstream firm causes a decrease in consumers' demand for the final good hence a decrease in derived demand for the intermediate good and a decrease in profit for the upstream firm. Under vertical integration, these externalities are internalised, and the integrated firm has less incentive to increase retail price. Vertical integration increases joint profit by internalisation of negative

externalities and increases consumer surplus by elimination of double marginalisation. Vertical integration is Pareto optimal.

- Franchise Fee
  - Parameters
    - Suppose that $U$ uses a two-part tariff, whereby it sells $x$ units of the intermediate good to $D$ for the total amount $F + wx$, where $F$ is a fixed franchise fee.
  - Analysis
    - The subgame-perfect equilibrium is such that $U$ chooses $(w, F) = (c_U, \pi^M(c_U + c_D))$. In the second stage subgame, $D$ has marginal cost (at any non-zero quantity) $w + c_D = c_U + c_D$, and chooses $p$ to maximise $\pi_D$. In the subgame Nash equilibrium, $D$ chooses $p = p^M(c_U + c_D)$ and enjoys gross (of franchise fee) profit equal to $\pi^M(c_U + c_D)$ hence net profit $\pi_D = \pi^M - F = 0$. If $D$ instead chooses $p \neq p^M(c_U + c_D)$, $\pi_D \leq 0$. Since $U$ sells at marginal cost to $D$, it enjoys zero gross (of franchise fee) profit hence net profit $\pi_U$ equal to franchise fee $F = \pi^M(c_U + c_D)$. $U$ enjoys net profit equal to that of a vertically integrated monopolist producing at constant marginal cost $c_U + c_D$, this is the maximum feasible joint profit in the vertical, hence the maximum feasible profit for $U$. $U$ has no incentive to choose otherwise, hence the given strategy profile is a subgame-perfect equilibrium.
  - Result
    - Where the upstream firm can use a two-part tariff, equilibrium retail price is equal to that under vertical integration and profit of the vertical structure is entirely captured by the upstream firm. Since the downstream firm enjoys zero profit if it does not buy the intermediate good, the upstream firm is able to entirely appropriate the (gross) profit of the downstream firm through the franchise fee. The upstream firm's profit is maximised iff joint profit is maximised and it entirely appropriates the downstream firm's (gross) profit through the franchise fee. The upstream firm optimally chooses wholesale price equal to marginal cost of the intermediate good such that the downstream firm's profit maximisation coincides with joint profit maximisation. Joint profit is maximised at a lower price than the equilibrium price in the absence of vertical restraints because vertical externalities are not accounted for in the latter case.
- Retail-Price Maintenance
  - Parameters
    - Suppose that $U$ sells the intermediate good to $D$ only if $D$ agrees to sell the final good at price $p$ chosen by $U$, i.e. $U$ chooses both the wholesale price $w$ and the retail price $p$ to maximise $\pi_U$ subject to $\pi_D \geq 0$ (since $D$ produces zero units of the final good and buys zero units of the intermediate good otherwise).
  - Analysis
    - The equilibrium is such that $U$ chooses $p = p^M(c_U + c_D)$ and $w = p^M(c_U + c_D) - c_D$. By definition of $p^M(.)$, $p = p^M(c_U + c_D)$ maximises joint profit $\Pi = (p - w - c_D)Q(p) + (w - c_U)Q(p) = (p - c_U - c_D)Q(p)$. At the equilibrium, $\pi_D = (p - w - c_D)Q(p) = 0$. Hence $p = p^M(c_U + c_D)$ and $w = p^M(c_U + c_D) - c_D$ maximise $\pi_U = \Pi - \pi_D$ subject to $\pi_D \geq 0$.
  - Result
    - Where the upstream firm can impose retail-price maintenance, equilibrium retail price is equal to that under vertical integration and profit of the vertical structure is entirely captured by the upstream firm. The upstream firm maximises its profit by choosing retail price to maximise joint profit and choosing wholesale price such that the downstream firm enjoys zero profit and (only just) has incentive to operate. Joint profit is maximised at a lower price than the equilibrium price in the absence of vertical restraints because vertical externalities are not accounted for in the latter case.
- Sponsoring Competition
  - Parameters
    - Suppose that the downstream industry is perfectly competitive and downstream firms compete in prices.
  - Analysis
    - In the second stage subgame, by the Bertrand result, each downstream firm chooses price equal to marginal cost, $p = w + c_D$ and enjoys zero profit. In the first stage, the subgame-perfect equilibrium is such that $U$ chooses $w = p^M(c_U + c_D) - c_D$. At the subgame-perfect equilibrium, $p = p^M(c_U + c_D)$. By definition of $p^M(.)$, $p = p^M(c_U + c_D)$ maximises joint profit $\Pi = (p - w - c_D)Q(p) + (w - c_U)Q(p) = (p - c_U - c_D)Q(p)$ hence upstream firm's profit $\pi_U = \Pi - \pi_D = \Pi$.
  - Result
    - Where the downstream industry is perfectly competitive and downstream firms compete in prices, equilibrium retail price is equal to that under vertical integration and profit of the vertical structure is entirely captured by the upstream firm. The upstream firm maximises its profit by choosing wholesale price such that each downstream firm's marginal cost is equal to the price which maximises joint profit. Each downstream firm then chooses price

equal to marginal cost, since the downstream industry is perfectly competitive. Joint profit, hence upstream firm's profit is maximised. The upstream monopolist has incentive to sponsor competition in the downstream industry. (The reverse is also true, that downstream firms have incentive to sponsor competition in the upstream industry.)

- Exclusive Contract
  - Parameters
    - Consider the following model of the Chicago view of exclusive contracts. In the first of four stages, an incumbent upstream monopolist $U$ chooses whether to offer an incumbent downstream buyer $D$ an exclusive contract with payment $t$. In the second stage, $D$ chooses whether to accept the exclusive contract. If $D$ accepts the exclusive contract, it receives payment $t$ and must buy only from $U$ in the fourth stage. If $D$ does not accept the exclusive contract, it is free to buy from any firm in the fourth stage. In the third stage, potential entrant $E$ chooses whether to enter the upstream industry at fixed set-up cost $F$. In the fourth stage, each active firm produces a homogenous intermediate good at constant marginal cost and chooses price at which to sell to $D$ to maximise profit. $D$'s demand $Q(p)$ for the intermediate good is downward-sloping, and is such that marginal revenue is downward-sloping.
    - Suppose that $E$ is more efficient than $U$, i.e. the constant marginal cost of $E$, $c_E$ is less than the constant marginal cost of $U$, $c_U$, and that the difference in efficiency is non-drastic, i.e. monopoly price of $E$, $p^M(c_E)$ is no less than $c_U$ such that if $E$ were to enter, it would optimally choose price $c_U$ to just undercut $U$. Suppose further that $(c_U - c_E)Q(c_U) > F$, such that $E$ would enter if $D$ did not accept the exclusive contract with $U$.
  - Analysis
    - If $D$ did not accept the exclusive contract with $U$, then $E$ enters, and by the result of the asymmetric Bertrand game, $U$ enjoys zero profit. If $D$ accepted the exclusive contract with $U$, then $E$ does not enter since it would have zero quantity hence zero gross (of set-up cost) profit and negative net profit, and $U$ enjoys monopoly profit $\pi^M(c_U) = \max_p(p - c_U)Q(p)$. The maximum amount $\bar{t}$ that $U$ would be willing and able to offer for an exclusive contract is thus equal to $\pi^M(c_U)$.
    - If $D$ did not accept the exclusive contract with $U$, then $E$ enters, and $D$ buys $Q(c_U)$ units at price $c_U$ from $E$. If $D$ accepted the exclusive contract with $U$, then $E$ does not enter and $D$ buys $Q(p^M(c_U))$ units at price $p^M(c_U)$ from $U$. The minimum amount $\underline{t}$ that $D$ would be willing and able to accept for an exclusive contract is given by the difference between $D$'s surplus in the former case and $D$'s surplus in the latter case.
    - Diagrammatically, it can be shown that the $\underline{t} > \bar{t}$, i.e. the minimum amount $D$ would be willing and able to accept for an exclusive contract is greater than the maximum amount $U$ would be willing and able to offer. $U$ cannot afford to compensate $D$ for not buying from $E$.
  - Result
    - The downstream firm does not accept an exclusive contract and entry occurs. The benefit to the upstream incumbent from deterring entry and selling at the higher monopoly price is less than the benefit to the downstream buyer from buying at the lower price from the upstream entrant because a greater quantity of the intermediate good is produced and consumed in the latter case.
- Buyers in Separate Markets
  - Parameters
    - Suppose instead that there are two downstream buyers, $D1$ and $D2$, each in a separate market and each with downward-sloping demand for the intermediate good $Q(p)$. If $E$ enters, by the result of the asymmetric Bertrand game, it optimally chooses price $c_U$ to just undercut $U$'s marginal cost and enjoys profit equal to $(c_U - c_E)Q(c_U)$ in each market. Suppose that entry is only profitable for $E$ if it sells to both downstream buyers but not if it sells to only one downstream buyer, i.e. $2(c_U - c_E)Q(c_U) > F > (c_U - c_E)q(C_U)$. The minimum amount $\underline{t}$ that each of $D1$ and $D2$ is willing and able to pay is given by the surplus forgone in buying the intermediate good at the higher price $p^M(c_U)$ from the upstream incumbent rather than at the lower price $c_U$ from the upstream entrant if entry occurs. Suppose that the minimum amount either $D1$ or $D2$ willing and able to accept for an exclusive contract is less than the total profit $U$ would enjoy if $E$ does not enter, i.e. $\underline{t} < 2\pi^M(c_U)$.
  - Analysis
    - Entry is deterred iff at least one of $D1$ and $D2$ accepts an exclusive contract with $U$. Given that $\underline{t} < 2\pi^M(c_U)$, and that, by the result of the asymmetric Bertrand game, $U$ enjoys zero profit if entry occurs, $U$ maximises profit by offering an exclusive contract to one of the two downstream buyers with payment $\underline{t}$ such that its net (of exclusive contract payment) profit is $2\pi^M - t > 0$. Entry is deterred, and $Q(p^M(c_U))$ units of the intermediate good are sold to each of $D1$ and $D2$ at price $p^M(c_U)$.
  - Result
    - The upstream incumbent finds it optimal to deter entry because of externalities across buyers in separate markets. In accepting the exclusive contract, the (one) buyer that accepts the contract renders entry unprofitable

for the upstream entrant, which increases the price faced by the other downstream buyer and decreases its profit. This externality is not accounted for by the contracting downstream buyer, which demands only to be compensated for the higher price it faces when entry is deterred.

  - Discussion (Sequential Move)
    - Suppose instead that $U$ offers an exclusive contract to each of $D1$ and $D2$ sequentially, and that the contracts offered are common knowledge once offered, and are not necessarily identical. In the subgame-perfect equilibrium, $U$ offers $D1$ an exclusive contract with payment $t$ equal to some arbitrarily small amount $\epsilon$, $D1$ accepts the exclusive contract, and $E$ does not enter. It is optimal for $D1$ to accept such a contract because, given common knowledge of rationality and each player's incentives, $D1$ knows that if it does not accept the contract, $U$ offers $D2$ an exclusive contract with payment $\underline{t}$, which $D2$ accepts. Then, $E$ does not enter and $D1$ faces the high monopoly price $p^M(c_U)$. $D1$ faces the high monopoly price regardless of whether it accepts the exclusive contract, hence $D1$ is better off (by arbitrarily small amount $\epsilon$) if it accepts the exclusive contract. If the upstream incumbent can offer exclusive contracts sequentially, publicly, and not necessarily identically, it can deter entry at almost zero cost since the first downstream buyer understands that entry deterrence is certain, and is better off if it enjoys some small payment by accepting the exclusive contract.
- Exclusive Contracts with Penalties
  - Parameters
    - Consider the Aghion and Bolton model of exclusive contracts with penalties. In the first of three stages, an incumbent upstream monopolist $U$ and an incumbent downstream buyer $D$ agree on an exclusive contract with penalties $(p, d)$, such that, in the third stage, $U$ offers an intermediate good to $D$ at price $p$ and $D$ pays damages $d$ to $U$ if it buys from another firm. In the second stage, potential entrant $E$ learns its constant marginal cost $c_E$ and chooses whether to enter the market at zero set-up cost. In the third stage, if $E$ chose to enter, each of $U$ and $E$ produce a homogenous intermediate good at marginal cost $c_U$ and $c_E$ respectively, $E$ chooses price $p_E$ to offer $D$, and $D$ decides whether to buy from $U$ or $E$. If $E$ chose not to enter, $U$ produces the homogenous intermediate good at marginal cost $c_U$ and sells to $D$ at price $p$ as agreed in the exclusive contract.
    - $D$ has unit demand for the intermediate good, and valuation $v$ such that its surplus is $v - p$ if it buys one unit at price $p$, and $0$ if it does not buy. Suppose for simplicity that $v = 1$, $c_U = \frac{1}{2}$ and $c_E$ is uniformly distributed in the interval $[0, 1]$.
  - Analysis
    - In the third stage subgame, if $E$ chose to enter, and if $c_E < p - d$, then $E$ chooses $p_E = p - d$ such that $D$ buys from $E$ since the total cost of buying from $E$, $p_E + d = p$ is (weakly) less than the total cost of buying from $U$, $p$. $E$ enjoys gross profit $p_E - c_E = p - d - c_E$. If $E$ chooses any higher price, it sells zero units and enjoys zero gross profit. If $E$ chooses any lower price, it does not maximise gross profit. If instead $c_E > p - d$, then $E$ chooses any price $p_E > p - d$, $D$ buys from $U$, $E$ sells zero units and enjoys zero gross profit. If $E$ chooses any other price, $D$ buys from $E$, $E$ has negative gross margin hence negative profit $p_E - c_E$. In the second stage subgame, $E$ enters if $c_E < p - d$ since then its gross profit is positive and there is zero fixed set-up cost. If $c_E < p - d$, $E$ enters, $D$ buys from $E$ and enjoys surplus $v - p_E - d$, $U$ is paid damages by $D$ and enjoys surplus $d$, hence $U$ and $D$ have joint surplus $v - p_E = 1 - p_E = 1 - (p - d)$. If $c_E \geq p - d$, $E$ does not enter (or equivalently, $E$ enters and $D$ continues to buy from $U$), $D$ enjoys surplus $v - p$ and $U$ enjoys surplus $p - c_U$, hence $U$ and $D$ have joint surplus $v - c_U = \frac{1}{2}$. Given that $c_E$ is distributed uniformly on the interval $[0, 1]$, $c_E < p - d$ with probability $p - d$. Supposing that in the first stage, $U$ and $D$ choose $(p, d)$ to maximise expected joint surplus $(p - d)(1 - (p - d)) + (1 - (p - d))\frac{1}{2}$, the subgame-perfect equilibrium is such that $p - d = \frac{1}{4}$ hence $E$ enters iff $c_E < \frac{1}{4}$, and chooses $p_E = \frac{1}{4}$ if it enters.
  - Result (Damages)
    - If entry occurs, the entrant offers the downstream buyer a price such that the downstream buyer just prefers to buy from the entrant rather than the upstream incumbent. This price is such that the total cost to the downstream buyer of buying from the entrant (equal to the sum of this price and damages) is just below the price offered by the upstream incumbent in the exclusive contract. At any higher price, the downstream buyer buys from the incumbent, and at any lower price, the entrant fails to maximise profit. Therefore, if entry occurs, the entrant's price is always just below the incumbent's price less damages, hence damages are ultimately paid by the entrant and the incumbent and downstream buyer effectively set the entrant's price.
  - Result (Partial Exclusion)
    - The entrant is partially excluded since it finds it optimal to enter only if it is significantly more efficient than the incumbent, i.e. has significantly lower marginal cost. In the absence of the exclusive contract, the entrant enters iff it has (even marginally) lower marginal cost than the incumbent and thus can enjoy positive net profit by undercutting the incumbent, since there is zero fixed set-up cost. Social surplus is not maximised if a more

efficient entrant is excluded by such a contract because the contract between incumbent and buyer imposes a negative externality on the entrant.

- Secret Contracts
  - Parameters
    - Consider the Hart and Tirole model of exclusive contracts to reduce downstream competition. In the first of three stages, an upstream monopolist $U$ simultaneously and publicly offers each of two downstream firms, indexed by $i \in \{1,2\}$, $D_i$ to sell $q_i$ units of a common intermediate good for the total amount $T_i$. In the second stage, each $D_i$ simultaneously chooses whether to accept the offer it received. $U$ produces $Q$ units of the intermediate good at total cost $C(Q)$ to meet demand. In the third stage, each $D_i$ that accepted the offer it received costlessly transforms all $q_i$ units of the intermediate good into as many units of a common final good and each unit of the final good is sold at the market clearing price given by inverse demand function $P(Q)$. Let $A_i$ be an indicator variable that takes value $1$ if downstream firm $D_i$ accepted the offer from $U$ and takes value $0$ otherwise. Each downstream firm $i$'s payoff is given by its profit $\pi_i = P(A_i q_i + A_{-i} q_{-i}) A_i q_i - A_i T_i$. $U$'s payoff is given by its profit $A_1 T_1 + A_2 T_2 - C(A_1 q_1 + A_2 q_2)$.
  - Analysis
    - The subgame-perfect equilibrium is such that $U$ offers each $D_i$ for $i \in \{1,2\}$ $(q_i, T_i) = (\frac{Q^M}{2}, \frac{p^M Q^M}{2})$ where $Q^M \equiv \arg\max_Q [P(Q) - C(Q)]Q$ denotes monopoly output and $p^M \equiv P(Q^M)$ denotes monopoly price, each $D_i$ accepts the offer, and transforms all $q_i$ units of the intermediate good into the final good. In the third stage subgame, each downstream firm $D_i$ transforms $\frac{Q^M}{2}$ units of the intermediate good into $\frac{Q^M}{2}$ units of the final good, total quantity $Q = Q^M$ and price $p = P(Q^M) = p^M$, hence each downstream firm enjoys profit $\pi_i = \frac{p^M Q^M}{2} - T_i = 0$. In the second stage subgame, each downstream firm $D_i$ is (weakly) better off if it accepts the contract. In the first stage, $U$'s profit is equal to $p^M Q^M - C(Q^M) = [P(Q^M) - C(Q^M)]Q^M$. By definition of $Q^M$ , $U$'s profit is equal to the maximum profit of the vertical structure. $U$'s maximises its payoff under the given strategy profile.
    - [Dropped] By the result of the Cournot model, since $q_i = \frac{Q^M}{2} < Q^M$, marginal revenue is positive for all units of the final good up to $q_i$ is positive. Since each unit of the final good is produced at zero marginal cost, and $D_i$ can produce no more than $q_i$ units of the final good, $D_i$ maximises profit by producing and selling $q_i$ units of the final good.
  - Result (Profit)
    - $U$ fully exerts its monopoly power to entirely capture the profit of the vertical structure.
  - Parameters
    - [Ignore This]
    - Suppose instead that in the first stage, the upstream monopolist $U$ can only make secret offers. Suppose for simplicity that $U$ has constant marginal cost $c$
  - Analysis
    - [Ignore This]
    - In choosing whether to accept its offer, each downstream firm $D_i$ must form some conjecture about its competitor $D_{-i}$'s offer. Consider the equilibrium in which each $D_i$'s conjecture is correct.
    - This equilibrium is such that $U$ offers each $D_i$ for $i \in \{1,2\}$ $(q_i, T_i) = (q^C, \pi^C)$, where $q^C$ denotes the Cournot Nash equilibrium output of each firm in a duopoly where each firm has constant marginal cost $c$ and inverse demand function is given by $P(Q)$, and $\pi^C$ denotes the Cournot Nash equilibrium profit of each firm in such a duopoly
    - This equilibrium is such that $U$ offers each $D_i$ for $i \in \{1,2\}$ $(q_i, T_i) = (q^C, \pi^C)$ (specified below), and each $D_i$ accepts its offer. In the third stage subgame, each $D_i$ enjoys profit $\pi_i = P(q_i + q_{-i}) q_i - T_i = P(2q^C) q^C - \pi^C$. In the second stage subgame, each $D_i$ accepts its offer iff its profit in the third stage is (weakly) positive. In the first stage subgame, [difficult to explain]
  - Discussion (Secret Contracts)
    - Suppose instead that contracts are secret. Suppose also, for simplicity, that $U$ has constant marginal cost $c$. In choosing whether to accept its offer, each downstream firm $D_i$ must form some conjecture about its competitor $D_{-i}$'s offer. For example, if $D_1$ believes that $U$ and $D_2$ agree on $(\frac{Q^M}{2}, \frac{p^M Q^M}{2})$, then $U$ and $D_1$ have incentive to maximise joint profit $\Pi_{U,D_1} = [P(q_1 + \frac{Q^M}{2}) - c] q_1$. By the result of the Cournot duopoly model, this joint profit is maximised by some $q_1 > \frac{Q^M}{2}$. Under any equilibrium where each $D_i$'s conjecture about $D_{-i}$'s offer is correct, $U$ and $D_2$ do not agree on $(\frac{Q^M}{2}, \frac{p^M Q^M}{2})$ since $U$ and $D_1$ agree on a different contract $(q_1, T_1)$, $D_2$ correctly believes so, and expects negative profit $\pi_2 = P(q_1 + \frac{Q^M}{2}) \frac{Q^M}{2} - \frac{p^M Q^M}{2} < 0$ (since $q_1 > \frac{Q^M}{2}$ hence $P(q_1 + \frac{Q^M}{2}) < P(Q^M) \equiv p^M$) if it accepts the contract.

- An increase in $T_i$, if the offer $(q_i, T_i)$ is accepted by $D_i$, constitutes a transfer of surplus form $D_i$ to $U$, and $D_i$ accepts any offer so long as its expected surplus is (weakly) positive, hence $U$ only offers contracts such that the value of the contract is captured entirely by $U$.
- Suppose that $D_i$ believes that $U$ and $D_{-i}$ agree on $(q_{-i}, T_{-i})$, then for any $q_i$, $D_i$ expects profit $\pi_i = P(q_i + q_{-i})q_i - T_i$, hence only (rationally) accepts offers such that $T_i \leq P(q_i + q_{-i})q_i$ for any $q_i$. $U$ chooses $q_i$ and offers $D_i$ $(q_i, P(q_i + q_{-i})q_i)$ to maximise surplus of $U$, which is equal to joint surplus of $U$ and $D_i$, $\Pi_{U,D_i} = [P(q_i + q\_i) - c]q_i$. $U$'s maximisation problem is identical to that of a firm in Cournot duopoly with constant marginal cost $c$ and where inverse demand is given by $P(Q)$, hence, $U$ chooses $q_i = R^C(q_{-i})$, where $R^C(.)$ is the reaction function in such a Cournot duopoly. By symmetry, $q_i = q_{-i}$, hence at equilibrium, where each $D_i$'s conjecture about its competitor $D_{-i}$'s offer is correct, $q_1 = q_2 = R^C(q_1) = R^C(q^2) = q^C$ where $q^C$ is the Cournot output, $p = P(q_1 + q_2) = P(2q^C) = p^C$, $\pi_1 = \pi_2 = 0$ and $\pi_U = [P(2q^C) - c]2q^C = 2\pi^C$, where $p^C$ is the Cournot price and $\pi^C$ is the Cournot (individual firm) profit.

- Result
  - Equilibrium price and quantities are as though the two downstream firms each produce at marginal cost $c$ and compete in quantities. The upstream monopolist is unable to exercise its monopoly power to restrict output, increase price, and increase profit.

- Vertical Mergers and Market Foreclosure
  - Parameters
    - [Ignore] Consider the simplified Salinger model of vertical merger resulting in input foreclosure. Pre-merger, each of two upstream firms, $U_1$ and $U_2$ produces a homogenous intermediate good at common constant marginal cost $c_U$. Each upstream firm $U_i$ chooses output $q_i$ to maximise profit $\pi_i = [W(q_i + q_{-i}) - c_U]q_i$, where $W(.)$ is the inverse demand function for intermediate goods. Each of two downstream firms, $D_1$ and $D_2$ buys some quantity $q_i'$ of the intermediate good at wholesale price $w = W(q_i + q_{-i})$ and transforms it into an equal quantity of the final good at common constant marginal cost $c_U$
    - [Ignore] Consider the simplified Salinger model of vertical merger resulting in input foreclosure. Pre-merger, each of two upstream firms, $U_1$ and $U_2$ produces a homogenous intermediate good at common constant marginal cost $c_U$. Each of two downstream firms, $D_1$ and $D_2$ buys some quantity of the intermediate good and transforms it into an equal quantity of a homogenous final good at common constant marginal cost $c_D$. Inverse demand for the final good is given by $P(Q)$, where $Q$ is the total quantity of the final good. $U_1$ and $D_1$ merge such that the merged firm.
    - Consider the simplified Salinger model of vertical merger resulting in input foreclosure. Pre-merger, there are two upstream firms, $U_1$ and $U_2$, and each upstream firm $U_i$ chooses to produce $q_i^U$ units of a homogenous intermediate good at common constant marginal cost $c_U$ to maximise profit $\pi_i^U = [w - c_U]q_i^U$, where $w$ is the wholesale price of the intermediate good. There are two downstream firms, $D_1$ and $D_2$, and each downstream firm $D_i$ buys and costlessly transforms $q_i^D$ units of the intermediate good into as many units of a homogenous final good. Demand for the final good is downward-sloping, and inverse demand is given by $P(Q^D)$, where $Q^D \equiv q_1^D + q_2^D$ is the total quantity of the final good. Each downstream firm $D_i$ chooses $q_i^D$ given $w$ to maximise profit $\pi_i^D = [P(q_i^D + q_{-i}^D) - w]q_i^D$. At equilibrium, $Q^D = Q^U \equiv q_1^U + q_2^U$. $U_1$ and $D_1$ merge. Post-merger, the merged firm maximises joint profit which is equal to the sum of profit from the sale of the final good to consumers and profit from the sale of the intermediate good to $D_2$ (if the merged firm sells to $D_2$).
  - Analysis
    - Ordover, Saloner, and Salop (1990) find that in equilibrium, the merged firm does not sell the intermediate good to the downstream outsider because this confers market power to the upstream outsider, which increases input cost for the downstream outsider, hence decreases the output of the downstream outsider, which increases profit for the merged firm.
    - Output of the merged firm is greater than the pre-merger output of the downstream merger partner because of the elimination of double marginalisation. The merged firm has lower marginal cost $c_U$ than the pre-merger downstream insider $w > c_U$, hence has incentive to produce greater output.
    - Input foreclosure causes a decrease in the downstream outsider's output. Elimination of double marginalisation causes an increase in the merged firm's output. The effect of vertical merger resulting in input foreclosure on price is ambiguous. If the former effect is greater than the latter, then total output decreases and price increases. If the latter effect is greater than the former, then total output increases and price decreases.

- Vertical Mergers and Multiproduct Firms
  - Parameters
    - Consider the simplified Luco-Marshall model of vertical integration with multiproduct downstream firms. Pre-merger, two large upstream firms each supply a differentiated intermediate good to a large number of

downstream firms, which each buy from both upstream firms to produce two corresponding differentiated final goods. Suppose that the two final goods are substitutes. One of the large upstream firms then merges with a number of downstream firms.

- Analysis
    - Post-merger, downstream insiders decrease prices of the "insider" final good (produced with the intermediate good sold by the upstream insider) and increase prices of the "outsider" final good (produced with the intermediate good sold by the upstream outsider). This is because downstream insiders enjoy a larger margin on the insider final good due to elimination of double marginalisation, and thus have incentive to divert demand from the outsider final good to the insider final good by increasing the price of the former. The total effect of vertical integration on price and consumer welfare is ambiguous because, even though vertical integration eliminates double marginalisation, prices of non-integrated products could increase post-merger.
- Result