Microeconomics Paper 140530

1a $Y = 2AL^{1/2}$

$\pi = pY - wL$

The firm's profit maximisation problem is

$\max_{Y,L} \; pY - wL \quad \text{s.t. } Y = 2AL^{1/2}$

This reduces to

$\max_L \; p(2AL^{1/2}) - wL$

FOC: $p(AL^{-1/2}) - w = 0$

$\Rightarrow pAL^{-1/2} = w \Rightarrow L^{-1/2} = w/pA \Rightarrow L = (pA/w)^2$

~~$\Rightarrow Y = 2AL^{1/2} = 2A(pA/w) =$~~

SOC: $p(-\tfrac{1}{2}AL^{-3/2}) < 0$

(supposing $p, A > 0$)

$Y = 2AL^{1/2} = 2A(pA/w) = 2A^2 p/w$

$\pi = pY - wL = 2A^2 p^2/w - A^2p^2/w = A^2p^2/w$

b The household's utility maximisation problem is

$\max_{c,l} \; \ln c + (1-l) \quad \text{s.t. } pc \le wl + m$

At the optimum, budget constraint $pc \le wl + m$ binds. Every candidate optimum such that this does not bind fails to deviation by increasing c by sufficiently small amount ε such that the budget constraint remains satisfied. So the utility maximisation problem reduces to

$\max_c \; \ln c + (1 - pc - m/w)$

FOC: $1/c - p/w = 0$

$\Rightarrow c = w/p$

SOC: $-c^{-2} < 0$

$l = pc - m/w = w - m/w = 1 - m/w$

c $D = M(w/p)$

$S = N(2A^2 p/w)$

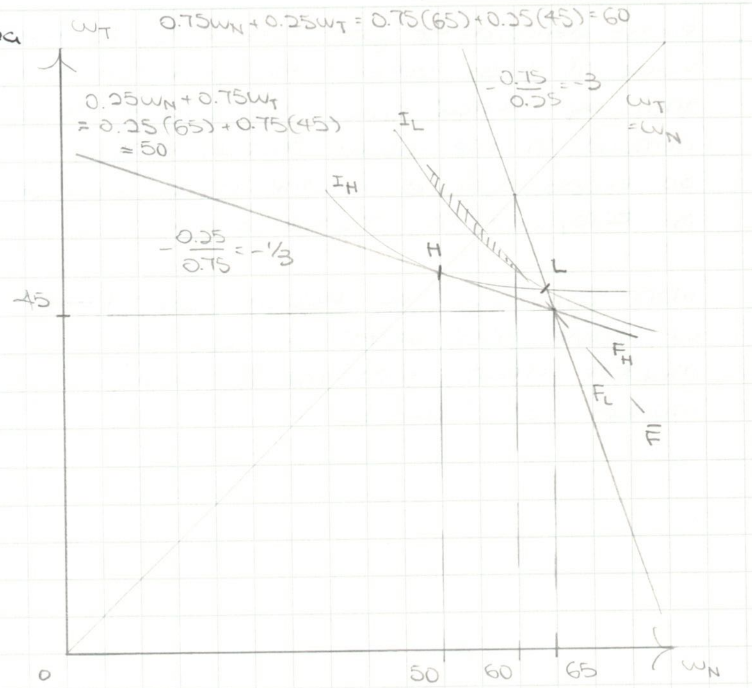
$D = S \Leftrightarrow M(w/p) = 2NA^2(p/w) \Leftrightarrow (w/p)^2 = 2NA^2/M \Leftrightarrow$

$w/p = A\sqrt{2N/M}$

d As the number of firms relative to the number of households, N/M increases, the real wage w/p increases. This is because the marginal productivity of labour increases.

e The price ratio in (c) clears the ~~market.~~ labour market. This follows from Walras' Law, which states that at any price vector, the value of excess demand in each market sums to zero. Given that the price ratio in (c) clears the market for good Y, excess demand for good Y is zero, so the value of excess demand for good Y is zero, then by Walras' Law, the value of excess demand in the labour market is zero, so excess demand in the labour market is zero, and this market clears.

More generally, by Walras' Law, if ~~the~~ at some price vector, all but one market clears, then the remaining market also clears.



In the above diagram, w_N denotes final wealth in the event of no theft, w_T denotes final wealth in the event of theft, H denotes the equilibrium insurance contract offered to High risk types, ~~&~~ I_H ~~de~~ is the indifference curve of H types through this point, F_H ~~is the~~ represents the actuarially fair contracts for H, and similarly for L.

In a competitive insurance market, supposing further that insurers are risk-neutral, only actuarially fair contracts are offered in equilibrium, because each ~~firm h~~ insurer has zero expected profit in equilibrium.

There is no pooling eqm because any candidate ~~eqm for~~ pooling eqm fails to unilateral deviation ~~by~~ by one insurer to offering some contract that is actuarially unfair to L types but strictly preferred to the pooling contract by only L types. The existence of such a contract is guaranteed by the single crossing property, which is the property of ~~the~~ H type and L type ~~preference~~ preferences that through any point, the indifference curve~~s~~ of the L type is steeper.

At ~~the~~ the separating equilibrium, H are offered full insurance. Any candidate eqm where H are offered only partial insurance fails to deviation by offering H full but (marginally) actuarially unfair insurance.

Then, the maximum insurance that L can be offered is given by the point L, which is such that the incentive compatibility constraint of H types bind, i.e. L is ~~sufficie~~ ~~just~~ ~~suf~~ just sufficiently unattractive to H types that they prefer H.

At this equilibrium, ~~the~~ both types are offered actuarially fair insurance, neither H nor L types have incentive to deviate to the other contract (since it lies at least weakly below the relevant indifference curve), and insurers have no profitable deviation (given that the proportion x of L types is sufficiently low such that ~~some~~ an equilibrium exists).

It is sufficient to restrict attention to equilibria of this form by the Revelation Principle.

b From the above, H types buy full actuarially fair insurance.

Coverage $q_H = K_H = 20$, premium (rate) $\pi_H = p_H = 0.75$
premium (amount) $\pi_H q_H = 15$

c $u(w) = 100 - 100/w$

At equilibrium, given that H types fully insure with the above contract, they face lottery

$L_H^* = [1; 50]$

$u(L_H^*) = 1u(50) = 100 - 100/50 = 98$

d The equilibrium contract for L is actuarially fair and just satisfies the incentive compatibility constraint for H types.

$\pi_L = 0.25 \Leftrightarrow P/C = 0.25$

$\cancel{u(L_H^*) = u(L_L^*)}$

$\cancel{98 = 0.75(65 \quad 0.25(65-P) + 0.75(45-P+C)}$

$\cancel{= 50 - P + 0.75C}$

$\cancel{\Rightarrow 98 = 50 + 0.5C \Rightarrow C =}$

$\cancel{CE(L_H^*) = 50 = 0.25(65-P) + 0.75(45-P+C)}$

$98 = 0.25[100 - 100/(65-P)] + 0.75[100 - 100/(45-P+C)]$
$= [25 - 25/(65-P)] + [75 - 75/(45+3P)]$
$= 100 - 25/(65-P) - 75/(45+3P) \Leftrightarrow$
$25/(65-P) + 75/(45+3P) = 2 \Leftrightarrow$
$1125 + 75P + 4875 - 75P = 2(65-P)(45+3P) \Leftrightarrow$
$-6P^2 + 300P + 5850 - 6000 \Leftrightarrow$
$P^2 - 50P + 25 = 0 \Leftrightarrow$
$\cancel{P = 5 \Rightarrow C = 20}$ $P = 25 - 10\sqrt{6} \Rightarrow C = 100 - 40\sqrt{6} \approx 2$

e when λ is ~~so~~ ~~large.~~ sufficiently large, some pooling contract that is optimal for both H and L types exists that lies above both ~~I_L~~ I_H and I_L and ~~below~~ below the line $\bar{F}$ which is actuarially fair in expectation. These contracts are represented by the shaded region. Then, deviation from the above separating equilibrium by any insurer to some such contract is strictly profitable, so the above separating equilibrium does not exist. The above separating equilibrium is the only candidate separating equilibrium, and there are no pooling equilibria. So if the above is not an equilibrium, none exist.

3a

	Y	N
Y	$-1, -1$	$\underline{3}, \underline{0}$
N	$\underline{0}, \underline{3}$	$2, 2$

Best responses underlined. By inspection, there are two pure NE where players play ~~pp~~ pure mutual best responses.

By inspection, each player has a unique best response to ~~the~~ each pure action of the other, so no hybrid NE exists.

~~Suppose~~ Consider ~~mixed~~ candidate mixed NE $\sigma^* = (pY + (1-p)N,\ qY + (1-q)N)$. By definition of NE, P1 has no profitable deviation, so P1 is indifferent between Y and N. $\pi_1(Y, \sigma_2^*) = \pi_1(N, \sigma_2^*) \Leftrightarrow -1q + 3(1-q) = 0q + 2(1-q) \Leftrightarrow 1-q = q \Leftrightarrow q = 1/2$. Then P2 mixes, P2 has no profitable deviation, P2 is indifferent, $\pi_2(Y, \sigma_1^*) = \pi_2(N, \sigma_1^*) \Leftrightarrow -1p + 3(1-p) = 0p + 2(1-p) \Leftrightarrow p = 1/2$. $\pi_1(\sigma_1^*, \sigma_2^*) = \pi_1(Y, \sigma_2^*) = -1q + 3(1-q) = -1(1/2) + 3(1/2) = 1$. By symmetry, $\pi_2(\sigma_2^*, \sigma_1^*) = 1$.

The NE are (Y, N), (N, Y), and $(\tfrac{1}{2}Y + \tfrac{1}{2}N, \tfrac{1}{2}Y + \tfrac{1}{2}N)$ which yield payoffs $(3,0)$, ~~(0,3)~~ $(0,3)$, and $(1,1)$ respectively.

b Each country is best off if only it has the new weapon because it then has a large military advantage over the other. ~~Each is worst off if both have the new weapon becau~~ Each is worst off if both have the new weapon because the risk of war and the costliness of war are both elevated. So it is preferable to be in the disadvantaged position of not having the weapon (while the other does) because the risk of war is ~~ss~~ lower. ~~Each is rea~~ But it is even more preferable for neither to have the weapon because then both the risk and costliness of war are low.

c A subgame perfect equilibrium is a NE that induces a NE in every subgame, ~~this~~ including those off the equilibrium path. This eliminates time-inconsistent strategies which involve non-credible threats to play non - ~~NE~~ NE strategies off the equilibrium path.

In a repeated game, each player's strategy is some history-contingent plan of ~~action~~ (stage game) actions.

A SPE is some strategy profile of such strategies that is an NE and induces an NE in every subgame.

In a ~~pe~~ repeated game, a subgame is any continuation of the repeated game from any history.

d Consider the ~~followin~~ strategy profile represented by the following automaton

$$\underset{(N,N)}{G(N,N)} \xrightarrow[\neg(N,N)]{} (\tfrac{1}{2}Y + \tfrac{1}{2}N,\ \tfrac{1}{2}Y + \tfrac{1}{2}N) \circlearrowleft$$

In words, each player plays the grim trigger strategy under which he plays ~~&~~ N iff in every prior period, every player played N, and $\tfrac{1}{2}Y + \tfrac{1}{2}N$ otherwise.

Verify that this is a SPE by the one shot deviation principle.

In the punishment phase (any subgame where there has been a prior deviation), players play a stage game NE in every (subsequent) period, so there is no (strictly) profitable one shot deviation.

In the cooperation ~~t~~ phase (any subgame with no prior deviation), ~~each~~ optimal one shot deviation by either player yields 3 then 1 indefinitely. Eqm play yields 2 indefinitely. Compare PVs: $2 + \frac{2\delta}{1-\delta} \geq 3 + \frac{\delta}{1-\delta} \Leftrightarrow \frac{\delta}{1-\delta} \geq 1 \Leftrightarrow \delta \geq 1-\delta \Leftrightarrow \delta \geq 1/2$.

For $\delta \geq 1/2$, there is no profitable one shot deviation (for any player, at any history), so the above is a SPE, and (N,N) indefinitely is sustainable in eqm.

4a The lottery in final wealth values associated with the investment is

$L^I = [1/2, 1/2; 20, 1]$

The lottery associated with non-participation is

$L^0 = [1; 10]$

$U(L^I) = 1/2\, u(20) + 1/2\, u(1) = 1/2 \ln 20 + 1/2 \ln 1 = 1/2 \ln 20$

$U(L^0) = 1 u(10) = \ln 10 = 1/2 \ln 100$ ~~$= 1/2$~~

$U(L^I) < U(L^0) \Rightarrow L^I \precsim L^0$

S prefers not to participate in the investment.

This can equivalently be demonstrated by ~~comp~~ computing the certainty equivalent of L^I and finding that it is less than final wealth in the event of non-participation.

~~$CE(L^I) =$~~

$[1; CE(L^I)] \sim L^I \Leftrightarrow$

$u(CE(L^I)) = U(L^I) \Leftrightarrow$

$CE(L^I) = u^{-1}(U(L^I)) = e^{1/2 \ln 20} = 20^{1/2} = 4.4721$

$CE(L^I) < 10$

b The lottery in final wealth values associated with the shared investment, when it is shared with n (total) participants is

$L^S(n) = [1/2, 1/2; 10 + 10/n, 10 - 9/n]$

$U(L^S(n)) = 1/2\, u(10 + 10/n) + 1/2\, u(10 - 9/n)$

$= 1/2 \ln(10 + 10/n) + 1/2 \ln(10 - 9/n)$

$= 1/2 \ln(10 + 10/n)(10 - 9/n)$

$U(L^0) = 1/2 \ln 100$

$L^S(n) \succsim L^0 \Leftrightarrow$

$U(L^S(n)) \geq U(L^0) \Leftrightarrow$

$1/2 \ln(10 + 10/n)(10 - 9/n) \geq 1/2 \ln 100 \Leftrightarrow$

$(10 + 10/n)(10 - 9/n) \geq 100 \Leftrightarrow$

$(10n + 10)(10n - 9) \geq 100n^2 \Leftrightarrow$

$100n^2 + 10n - 90 \geq 100n^2 \Leftrightarrow$

~~$10n^2 - 9n - 9 \geq 0 \Leftrightarrow$~~ $10n - 90 \geq 0 \Leftrightarrow$

~~$(n-9)(10n+1)$~~ $n \geq 9$

The shared lottery is (weakly) preferred to non-participation iff $n \geq 9$.

Intuitively, sharing the lottery reduces its ~~ex~~ expected value but also its variance (riskiness), hence its risk premium. The expected value of the lottery is proportionate to $1/n$, the risk premium of the lottery is approximately proportionate to its variance which in turn is proportionate to $1/n^2$. So initially, risk premium decreases more rapidly with increasing n, and the certainty equivalent of the shared lottery increases, such that it is greater than 10, and participation is profitable.

It is always profitable for a risk averse expected utility maximiser to take a non-zero stake in a favourable gamble, and this gamble is favourable because it has positive expected value, so for some sufficiently large number of participants, participation is optimal for ~~S~~ S.

c $\max_n U(L^S(n))$

FOC: $1/2 \cdot 1/(10 + 10/n)\,(-10n^{-2}) + 1/2 \cdot 1/(10 - 9/n)\,(9n^{-2}) = 0$

$\Rightarrow -10n^{-2}/(10n + 10/n) + 9n^{-2}/(10n - 9/n) = 0$

$\Rightarrow -10n/(10n + 10) + 9n/(10n - 9) = 0$

$\Rightarrow 100n^2 - 90n = 90n^2 + 90n$

$\Rightarrow 10n^2 = 180n$

$\Rightarrow 10n = 180$

$\Rightarrow n = 18$

The optimal number of participants is $n = 18$

As n increases, expected value of the lottery decreases ~~at a $1/n$ rate~~ and is proportionate to $1/n$, and ~~variance of~~ risk premium decreases, and is approximately proportionate to $1/n^2$. For small n, ~~the~~ risk premium decreases more ~~less~~ rapidly than expected value, so certainty equivalent (which is ~~monotonic~~ a monotonic function of expected utility, and is equal to expected value minus risk premium) decreases. For large n, expected value decreases more rapidly than risk premium, so certainty equivalent hence expected value decreases.

5a A public good is a good that is non-rival and non-excludable. A good is non-rival iff one consumer's consumption of ~~that~~ that good does not reduce the amount or quality of that good available for consumption by others. A good is non-excludable iff it is impossible or prohibitively costly to prevent non-paying consumers from consuming that good.

The free rider problem is the problem of underprovision of public goods, when the good is privately provided. Because a public good is non rival, provision of a unit of the public good has a large external benefit enjoyed by ~~the~~ other consumers who do not fund its provision. This positive externality is not ~~accoun~~ accounted for in private decisions to fund the public good. Because public goods are non-excludable, all consumers except the one with the highest valuations and largest demands has no incentive to fund the public good.

b ~~$MB_i = v_i'(Q)$~~

$v_i'(Q) = a - Q/\cancel{100}\,50$

The marginal private benefit to consumer i ~~of~~ from ~~consumption of~~ the public good is $v_i'(Q)$.

The Samuelson condition is

$$\sum_{i \in \{1,\dots,n\}} v_i'(Q) = c$$

$$\Leftrightarrow n(a - Q/100) = c$$

$$\Leftrightarrow na - nQ/\cancel{100}\,50 = c$$

$$\Leftrightarrow nQ/100 = na - c$$

$$\Leftrightarrow Q = \cancel{100}\,50\,na - \cancel{100}\,50\,c/n = 50(a - c/n)$$

As the number of citizens n increases, the optimal quantity increases. Intuitively, this is because the cost of provision remains ~~consta~~ constant while the ~~benefit~~ marginal social benefit from the public good increases ~~due to~~ because private benefits are enjoyed by one additional consumer. As n becomes large, ~~Q~~ optimal quantity does not increase indefinitely but converges to ~~100a~~ 50a. This is because ~~for such~~ when $Q = \cancel{100a}\,50a$ (and for larger Q), marginal private benefit is zero (or negative) so ~~the~~ marginal social benefit is zero (or negative), and provision of $Q \geq \cancel{100a}\,50a$ is ~~subo~~ never optimal.

c P1 has the following ~~FOC~~ utility maximisation problem

$$\max_{q_1} u_1 = y_1 + aQ - 1/100\,Q^2 \quad \text{s.t. } y_1 + cq_1 \leq m_1$$

where m_1 is P1's income.

At the optimum, the budget constraint binds. ~~At any~~ Every candidate optimum such that the budget constraint does not bind fails to deviation by increasing y_i. So the maximisation problem reduces to

$$\max_{q_i} m - cq_i + aQ - 1/100\,Q^2$$

$$\text{FOC: } -c + a - 1/100\,(2Q) = 0$$

$$\Rightarrow 1/50\,(Q_{-1} + q_1) = a - c$$

$$\Rightarrow q_1 = 50(a - c - Q_{-1}/50) = 50(a-c) - Q_{-1}$$

$$\text{SOC: } -1/50 < 0$$

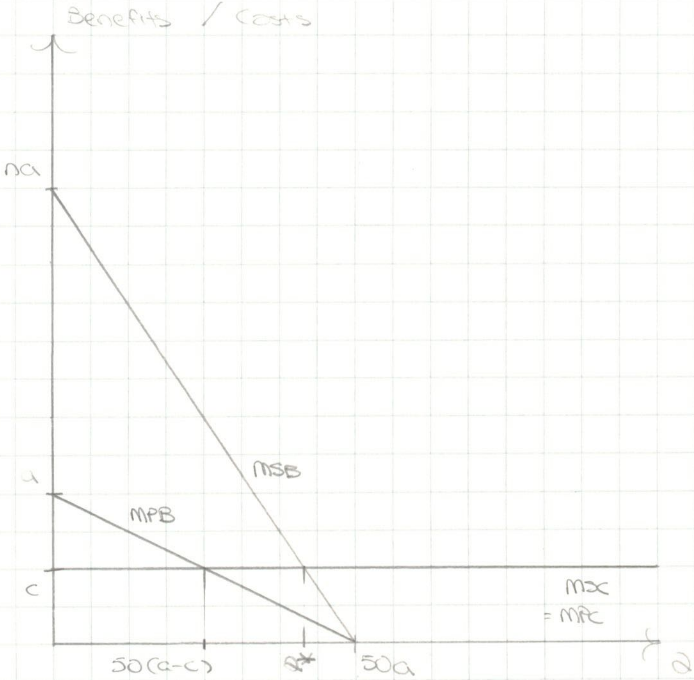
P1's best response ~~give~~ against Q_{-1} is

$B_1(Q_{-1}) = 50(a-c) - Q_{-1}$ (for $Q_{-1} \leq 50(a-c)$)

P1 ~~supp~~ funds as many ~~quan~~ units of the public good as is required such that the total quantity $Q = 50(a-c)$, ~~which is P1's satiation point.~~

d At any NE, the total amount provided is $50(a-c)$ ~~and ea~~. At every candidate NE such that fewer units are provided, each player's best response is to contribute more units. At every candidate NE where more units are provided, ~~at least~~ every player who ~~provi~~ contributes some amount has strictly profitable deviation to a less generous contribution. At the symmetric NE, each player contributes $50(a-c)/n$.

The total contribution is independent of the number of players. Analytically, this is because each player's best response is to ~~ac~~ contribute such that some privately optimal ~~quanti~~ total quantity is achieved. ~~So any "additional" player~~ And this privately optimal quantity is independent of the number of players (and common to all players). So at any NE, this is the total amount provided. Intuitively, this is because players have identical valuations, ~~so marginal private benefit~~ and ~~provision~~ contribution of the public good is a private decision, so all and only units for which marginal private benefit exceeds marginal private (social) cost c are provided, and this is independent of the number of players.



~~There is~~ The public good, when privately provided, is underprovisioned because each consumer ~~max~~'s utility is a maximum when MPB = MPC. When this quantity is achieved, no consumer has incentive to provide any further units of the good since for such units MPB<MPC. ~~But~~ But for some such units, namely up to Q*, MSB > MSC, so it is socially optimal for such units to be provided. Each consumer decides what amount ~~*~~ of the public good to contribute without consideration for the positive externality of contribution, represented by the vertical distance between MPB and MSB ~~for any~~ at any given quantity. This is positive for all quantities up to 50a.

Date