Price Discrimination Notes

Price Discrimination

- Definition
  - Price discrimination describes the practice under which a firm sells the same good at different prices, or, more generally, "when two or more similar goods are sold at prices that are in different ratios to marginal costs". (Varian, 1989)
- Necessary Conditions
  - Price discrimination is practicable only if (1) arbitrage is not possible, (2) the firm has some degree of market power and can thus choose prices above marginal costs, and (3) the firm has some ability to "sort or classify customers".
- Incentive
  - Firms have incentive to price discriminate since they thereby capture a greater share of consumer surplus as profit. Where a firm price discriminates, compared to where it does not, it additionally captures (1) the surplus of consumers with valuations above the non-discriminatory price by choosing a higher price for such consumers and (2) the surplus of consumers with valuations below the non-discriminatory price but above marginal cost who otherwise do not buy by choosing a lower price for such consumers.

Monopoly First-degree Price Discrimination

- Definition
  - First-degree price discrimination describes the practice under which a firm sells a good to each consumer at a price equal to that consumer's valuation.
- Necessary Conditions
  - First-degree price discrimination is practicable only if (1) arbitrage between consumers is not possible, (2) the firm has complete market power, (3) the firm has perfect knowledge of each consumer's valuations, and (4) the firm has complete bargaining power (for example, where the firm can make a take-it-or-leave-it offer to each consumer).
- Analysis
  - Under first-degree price discrimination, marginal revenue of the price discriminating firm is equal to consumers' valuation of the marginal unit since this unit is bought by a consumer at a price equal to his valuation of this unit and this sale has no effect on the price of infra-marginal units. The firm sells all and only units for which consumers' valuation exceeds marginal cost each at price equal to consumers' valuation of that unit.
- Result (Welfare)
  - Under first-degree price discrimination, social surplus is maximised since all and only units for which consumers' valuation exceeds marginal cost are produced and consumed. Social surplus is entirely captured by the firm since it sells each unit at price equal to consumers' valuation of that unit hence captures all surplus from the production and consumption of that unit.
- Evaluation
  - In reality, the necessary conditions for first-degree price discrimination are unlikely to hold. In particular, firms are unlikely to have perfect knowledge of each consumer's valuations since each consumer has no incentive to truthfully disclose his valuation to firms. Each consumer is better off if he reports a valuation below his true valuation and then buys at a lower price. Also, firms are unlikely to have complete bargaining power since firms have incentive to renegotiate after a consumer rejects an initial offer.

Monopoly Second-degree Price Discrimination

- Definition
  - Second-degree price discrimination describes the practice under which a firm offers each consumer a common menu of non-linear price-quantity (or price-quality) packages such that consumers with different valuations choose different packages. The firm "sorts or classifies consumers" by incentivising each consumer to "self-select" rather than by an exogenous signal.
- Necessary Conditions
  - Second-degree price discrimination is practicable only if (1) arbitrage between consumers is not possible, (2) the firm has some degree of market power, and (3) the firm has some knowledge of consumers' valuations.

- Parameters
  - Consider the following simple model of a monopolist practicing second-degree price discrimination. A monopolist produces some good at zero marginal cost and sells to consumers. Half of consumers are "large" and have high valuations and large demand, and half of consumers are "small" and have low valuations and small demand. The monopolist chooses a menu of price-quantity packages to maximise profit. Each consumer then chooses a price-quantity package from this menu to maximise his surplus.
- Analysis
  - See Cowan, 2022, p. 24 for graphical analysis.
  - Under second-degree price discrimination, the monopolist's choice of the menu of price-quantity packages is subject to the constraint of incentive compatibility, i.e. the consumer that some package is targeted at must find it optimal to buy that package. If the constraint does not hold, some consumers practice personal arbitrage, and buy a package targeted at other consumers. For example, the analogue of first-degree price discrimination is not incentive-compatible since large consumers then find it optimal to buy the small package. The monopolist is so constrained ultimately because it is unable to distinguish between different types of consumers by an exogenous signal and must allow consumers to self-select.
  - At equilibrium, the monopolist distorts the quantity of the small package, i.e. quantity of the small package is less than socially optimal while the quantity of the large package is socially optimal. This is because a decrease in the quantity of the small package causes a decrease in each large consumer's surplus if he buys the small package hence an increase in the price each large consumer is willing and able to pay for the large package (when the incentive compatibility constraint is binding) and an increase in the profit from each large consumer. At the undistorted price, a marginal decrease in the quantity of the small package has no effect on each small consumer's valuation of the package since each small consumer's valuation of the marginal unit is zero, hence no effect on profit from each small consumer. The monopolist therefore has incentive to distort the quantity of the small package.
- Result (Welfare)
  - Under second-degree price discrimination, small consumers enjoy zero surplus and consume less than the socially optimal quantity, large consumers enjoy positive surplus and consume the socially optimal quantity, and the monopolist enjoys positive profit. Compared to where price discrimination is not practicable, the monopolist is (weakly) better off since it price discriminates only if doing so increases profit, small consumers are (weakly) worse off because they enjoy zero surplus, and whether large consumers are better or worse off is ambiguous.
- Discussion (Quality)
  - By re-labelling variables (namely by re-labelling "quality" as "quantity"), the model, analysis, and results above can be interpreted as describing second-degree price discrimination where a monopolist offers each consumer a common menu of price-quality packages.
- Case Study (Passenger Rail)
  - Passengers in third class on passenger rail in the 1800s sat on wooden benches in open carriages while passengers in second class enjoyed upholstered seats in closed carriages.
  - The conditions in third class were not primarily a cost-saving measure, and were instead aimed at making third class travel sufficiently unattractive for passengers who could afford to travel in second class. Rail companies "hit the poor, not because they want to hurt them, but to frighten the rich".
- Case Study (IBM Printers)
  - IBM sold a printer in two versions, a primary version which printed twelve pages per minute, and a secondary version that was handicapped such as to only print eight pages per minute. The two printers were otherwise identical.
  - The objective of this handicapping, presumably, is to enable IBM to sell to mid-market customers at a lower price without also having to sell to more affluent customers at the lower price. By handicapping the printer, IBM made the low-end model sufficiently unattractive for affluent customers.

Two-Part Tariff

- Definition
  - A two-part tariff describes the practice under which a firm offers a menu of price-quantity packages where the price of each package $T = A + pq$, where A is a fixed amount, p is the unit price, and q is the quantity.

Bulow (1982) Durable Goods Model

- Motivation
  - The conventional model of second-degree price discrimination does not easily translate to the intertemporal case where a monopolist sells a durable good at different prices in different periods, i.e. chooses a menu of price-time

packages, because the monopolist generally cannot credibly commit to offering the price-time packages it initially announces.

- Parameters
  - Consider the simplified Bulow model of a durable goods monopolist. In the first period of a two period game, a monopolist produces a durable good at zero marginal cost and chooses price p_1 at which to sell. In the second period, the monopolist produces the same durable good at the same marginal cost and chooses price p_2 at which to sell. In each period, consumers choose whether to buy. There is a unit mass of consumers with valuation v distributed uniformly on $[0,1]$. Each consumer has unit demands, and enjoys surplus $2v-p_1$ if he buys in the first period and $v-p_2$ if he buys in the second period and 0 if he does not buy. Each consumer maximises his surplus and the firm maximises the undiscounted sum of profit across the two periods $\Pi=\pi_1+\pi_2$ where $\pi_i=p_iq_i$ and q_i denotes quantity in period i.
- Analysis
  - Suppose that price discrimination is not practicable, i.e. the monopolist can choose only $p_2=p_1$, and this is common knowledge. Let $p_2=p_1=p$. A consumer buys in period 2 iff $v-p\geq 0$ and $v-p\geq 2v-p$. Given the distribution of v specified above, there are $q_2=0$ such consumers, hence each consumer buys only in period 1. A consumer buys in period 1 iff $2v-p\geq 0$. Given the distribution of v specified above, there are $q_1=1-\frac{p}{2}$ such consumers. $\Pi=\pi_1+\pi_2=p_1q_1+p_2q_2=p(1-\frac{p}{2})$. At equilibrium, the first-order condition $\frac{d\Pi}{dp}=0$ holds. $p=1$, $q_1=\frac{1}{2}$, $q_2=0$, $\pi_1=\frac{1}{2}$, $\pi_2=0$, $\Pi=\frac{1}{2}$.
  - Suppose instead that price discrimination is practicable, i.e. the monopolist can choose any p_1,p_2, and this is common knowledge. In period 2, supposing that q_1 consumers bought in period 1, given that consumers have unit demands, and that these q_1 consumers have the highest valuations, $1-q_1$ consumers with valuations uniformly distributed on $[0,1-q_1]$ remain in the market. A remaining consumer with valuation v buys in period 2 iff $v-p_2\geq 0$. Given the distribution of v among remaining consumers, there are $q_2=1-q_1-p_2$ such consumers. Given that the monopolist's choice of p_2 has no effect on π_1, the monopolist chooses $p_2=\frac{1-q_1}{2}$ to maximise $\pi_2=p_2q_2=p_2(1-q_1-p_2)$. In period 1, given common knowledge of rationality and incentives, consumers know $p_2=\frac{1-q_1}{2}$ A consumer buys in period 1 iff $2v-p_1\geq 0$ and $2v-p_1\geq v-p_2$ iff $2v-p_1\geq 0$ and $v\geq p_1-p_2$. Given the distribution of v specified above, there are $q_1=1-(p_1-p_2)$ such consumers. $q_1=1-p_1+\frac{1-q_1}{2}$. $\frac{3q_1}{2}=1-p_1+\frac{1}{2}$, $q_1=1-\frac{2p_1}{3}$. $p_2=\frac{1-q_1}{2}=\frac{p_1}{3}$, $q_2=1-q_1-p_2=\frac{p_1}{3}$. $\Pi=\pi_1+\pi_2=p_1q_1+p_2q_2=p_1(1-\frac{2p_1}{3})+\frac{p_1}{3}(\frac{p_1}{3})=p_1-\frac{5p_1^2}{9}$. In equilibrium, the first-order condition $\frac{d\Pi}{dp_1}=1-\frac{10p_1}{9}=0$ holds. $p_1=\frac{9}{10}$, $q_1=\frac{4}{10}$, $p_2=\frac{3}{10}$, $q_2=\frac{3}{10}$, $\pi_1=0.36$, $\pi_2=0.09$, $\Pi=0.45$.
- Result (Profit)
  - Where price discrimination is practicable, compared to where it is not, price in each period decreases and profit decreases. This is because in the former case, the firm cannot credibly commit to maintaining a high price in the second period. In the second period, the firm has incentive to choose a low price in order to capture profit from consumers who did not buy in the first period. Given common knowledge of rationality and incentives, consumers anticipate the decrease in price, and consumers with intermediate valuations defer consumption. The monopolist then sells to these consumers at a lower (second period) price and generates less profit from each of these consumers. The monopolist also finds it optimal to partially offset consumers' deferring consumption by choosing a lower price in the first period. Bulow (1982), Stokey (1981), and Gul et al. (1986) show that in the extreme, with an infinite time horizon, monopoly power is entirely lost, and the durable goods monopolist chooses price close to marginal cost in each period.

Monopoly Pure Bundling

- Introduction
  - Bundling can be considered a form of second-degree price discrimination since the price of the bundle is not equal to the linear sum of the prices of its components.
- Parameters
  - Consider the following simple model of a multiproduct monopolist practicing pure bundling. A multiproduct monopolist produces two goods A and B, each at constant marginal cost c, and sells to a unit mass of consumers. Each consumer has unit demands for each good and valuations v_A and v_B respectively.
  - Under separate selling, the monopolist chooses price of A p_A and price of B p_B to maximise profit $\Pi=\pi_A+\pi_B=(p_A-c)q_A+(p_B-c)q_B$ where q_A and q_B denote the quantity of the respective good. Each consumer chooses whether to buy each good to maximise his surplus given by v_A-p_A if he buys only A, v_B-p_B if he buys only B $v_{A+B}-p_A-p_B$ if he buys both A and B and 0 otherwise.
  - Under pure bundling, the monopolist chooses bundle price p_{A+B} to maximise profit $\Pi=(p_{A+B}-2c)q_{A+B}$ where q_{A+B} denotes the quantity of the bundle. Each consumer chooses whether to buy the bundle to maximise his surplus given

by $v_{A+B} - p_{A+B}$ if he buys and 0 otherwise, where v_{A+B} denotes his valuation of the bundle $A + B$.
    - Suppose that $c = 0$, v_A and v_B are independently distributed, each of v_A and v_B is uniformly distributed on the interval $[0, 1]$, and A and B are independently valued, i.e. $v_{A+B} = v_A + v_B$.
- Analysis
    - Under separate selling, a consumer with valuation v_X for good X buys X iff $v_X \geq p_X$. Given the distribution of v_A and v_B specified above, there are $q_X = 1 - p_X$ such consumers. $\pi_X = p_X(1 - p_X)$.
    $\Pi = \pi_A + \pi_B = p_A(1 - p_A) + p_B(1 - p_B)$. At equilibrium, the first-order conditions $\frac{\partial \Pi}{\partial p_A} = 1 - 2p_A = 0$ and $\frac{\partial \Pi}{\partial p_B} = 1 - 2p_B = 0$ hold, hence $p_A = p_B = \frac{1}{2}$. $q_A = q_B = \frac{1}{2}$, $\pi_A = \pi_B = \frac{1}{4}$, $\Pi = \frac{1}{2}$.
    - Under pure bundling, a consumer with valuation v_X for $X \in \{A, B, A + B\}$ buys $A + B$ iff $v_{A+B} = v_A + v_B \geq p_{A+B}$. Given the distribution of v_A and v_B specified above, there are $q_{A+B} = 1 - \frac{p_{A+B}^2}{2}$ such consumers. $\Pi = p_{A+B}(1 - \frac{p_{A+B}^2}{2})$. At equilibrium, the first-order condition $\frac{d\Pi}{dp_{A+B}} = (1 - \frac{p_{A+B}^2}{2}) - p_{A+B}^2 = 0$ holds, hence $p_{A+B} = \sqrt{\frac{2}{3}}$. $q_{A+B} = \frac{2}{3}$, $\Pi = (\frac{2}{3})^{\frac{3}{2}} > \frac{1}{2}$.
- Result (Profit)
    - Under pure bundling, compared to under separate selling, profit increases. This is because consumers' valuations of the bundle are more concentrated around the mean than are consumers' valuations of each separately sold good. Then, a change in price yields a greater change in quantity, i.e. demand is more price-elastic, at intermediate prices, hence the firm can capture a greater share of consumer surplus as profit by choosing a lower price. Informally, the firm enjoys greater profit because it "forces" consumers who would otherwise buy only one good to buy both.
- Result (Welfare)
    - Under pure bundling, compared to under separate selling, given the assumptions above, consumer surplus increases hence social surplus increases. Note that the monopolist finds it optimal to offer the bundle at a discount, i.e. $p_{A+B} < p_A + p_B$. Consumers who buy both goods in either case are better off directly because of the discount. Some consumers who buy both goods only under pure bundling are better off because they enjoy positive surplus from the additional good(s) bought at a discount. Other consumers with high valuation for one good and low valuation for another are worse off because they are "forced" to buy an additional good which decreases their surplus.
    - Pure bundling has two effects on social surplus, a negative misallocation effect and a positive output effect. Under pure bundling, compared to under separate selling, goods are misallocated. Some consumers have high valuation for one good and low valuation for the other and buy the bundle. Other consumers have intermediate valuation for each good and do not buy the bundle. Consumer surplus and social surplus would increase if the latter good were transferred from the former consumer to the latter. Under pure bundling, compared to under separate selling, output increases since demand is more price-elastic at intermediate prices, hence the firm has greater incentive to decrease price and increase quantity.

Mixed Bundling

- Parameters
    - Under mixed bundling, the monopolist chooses price of A p_A, price of B p_B and price of the bundle p_{A+B} to maximise profit $\Pi = \pi_A + \pi_B + \pi_{A+B} = (p_A - c)q_A + (p_B - c)q_B + (p_{A+B} - 2c)q_{A+B}$, where q_A, q_B and q_{A+B} denote the respective quantities. Each consumer chooses whether to buy A, B, $A + B$ or none to maximise his surplus given by $v_A - p_A$ if he buys only A, $v_B - p_B$ if he buys only B, $v_{A+B} - p_{A+B}$ if he buys the bundle, and 0 otherwise.
- Result (Price)
    - Under mixed bundling, compared to under pure bundling, the price of the bundle increases. In the former case, an increase in the price of the bundle causes some consumers to buy only one good instead of the bundle. In the latter case, consumers alienated by an increase in the price of the bundle do not buy at all. The monopolist has positive margin on each separately sold good, hence has greater incentive to increase the price of the bundle in the former case.
    - Under mixed bundling, compared to under separate selling, the price of each separately sold good increases. In the former case, an increase in the price of each separately sold good causes some consumers to buy the bundle instead. In the latter case, consumers alienated by an increase in the price of one separately sold good do not instead buy both separately sold goods. The monopolist has positive margin on the bundle, hence has greater incentive to increase the price of each separately sold good in the former case.
- Result (Profit)
    - Under mixed bundling, compared to under pure bundling, profit increases. Analytically, this is because the pure bundling result is achievable under mixed bundling, where the monopolist chooses a prohibitively high price for each of the separately sold goods. Intuitively, this is because the monopolist enjoys a larger margin on the bundle and also enjoys positive profit from consumers who buy only one good.

- Result (Welfare)
  - It can be shown that given the assumptions above, under mixed bundling, compared to under pure bundling, consumer surplus decreases, and compared to under separate selling, consumer surplus increases.

Bundling with Interrelated Products

- Parameters
  - Suppose instead that A and B are interrelated, i.e. $v_{A+B} \neq v_A + v_B$. If the two goods are complements, then $v_{A+B} > v_A + v_B$. If the two goods are substitutes, then $v_{A+B} < v_A + v_B$.
- Result
  - An increase in the complementarity (equivalently, a decrease in the substitutability) of the two goods corresponds to an increase in the valuation of the bundle relative to the valuation of each separately sold good hence an increase in the number of consumers who would prefer to consume either both goods or neither good than to consume only one good and a diminution of the difference between separate selling and bundling. In the extreme, where the two goods are perfect complements such that the valuation of each separately sold good is zero, all consumers buy either both goods or neither good, hence separate selling and bundling are equivalent. The reverse is true for an increase in substitutability (equivalently, a decrease in complementarity).

Bundling with Correlated Valuations

- Parameters
  - Suppose instead that v_A and v_B are not independently distributed, i.e. $cov(v_A, v_B) \neq 0$.
- Result
  - If valuations of the two goods are perfectly positively correlated, price, quantity, profit, and consumer surplus are unchanged under bundling compared to under separate selling. A decrease in the correlation between valuations of the two goods results in an increase in profit and a decrease in consumer surplus.
  - Under pure bundling, compared to under separate selling, profit increases only if the correlation between valuations of the two goods is negative or sufficiently small if positive. If valuations of the two goods are perfectly positively correlated, under separate selling, since by symmetry prices are equal, each consumer buys either both goods or neither good. Then, bundling is equivalent to separate selling. A decrease in the correlation between valuations of the two goods causes an increase in concentration of valuations of the bundle around the mean, hence an increase in price-elasticity of demand at intermediate prices (since a change in price yields a greater change in quantity). Then, the monopolist is able to capture a greater share of consumer surplus as profit by choosing a lower price. In the extreme, if valuations of the two goods are perfectly negatively correlated, under pure bundling, demand for the bundle is perfectly elastic at some price, the monopolist chooses this price and all consumers buy the bundle. Under separate selling, each consumer buys only one good. Pure bundling doubles profit.

Bundling with Many Goods

- Parameters
  - Suppose instead that the multiproduct monopolist produces $n > 2$ goods.
- Result
  - By the law of large numbers, an increase in the number of goods in the bundle causes an increase in the concentration of the distribution of valuations of the bundle around the mean hence an increase in the price-elasticity of demand for the bundle at intermediate prices (since a change in price yields a greater change in quantity). Then, the monopolist is able to capture a greater share of consumer surplus as profit by choosing a lower price. Since quantity increases, deadweight loss decreases.
  - This result holds only if marginal costs are sufficiently close to zero. Otherwise, the decrease in profit due to the increase in the marginal cost of the bundle could be greater than the increase in profit due to the increase in revenue from bundling.

Monopoly Third-degree Price Discrimination

- Definition
  - Third-degree price discrimination describes the practice under which a firm offers different consumers different linear price schedules on the basis of some exogenous signal of each consumer's valuations. The price-discriminating firm sorts consumers into different sub-markets on the basis of this exogenous signal.
- Necessary Conditions

- Third-degree price discrimination is practicable only if (1) arbitrage between different sub-markets is not possible, (2) the firm has some degree of market power, and (3) the firm observes some exogenous signal of consumers' valuations.

- Parameters
  - Consider the following simple model of a monopolist practicing third-degree price discrimination. A monopolist produces a product at zero marginal cost and sells to consumers. Half of consumers are "large" and (collectively) have high valuations and price-inelastic demand, and half of consumers are "small" and (collectively) have low valuations and price-elastic demand. Large and small consumers can be distinguished by some exogenous signal observable to the monopolist. The monopolist chooses price p_A for large consumers and price p_B for small consumers to maximise total profit $\Pi = \pi_A + \pi_B = p_A q_A + p_B q_B$, where π_X and q_X denote profit and quantity in sub-market $X \in \{A, B\}$ respectively.
- Analysis
  - See Cowan, 2022, p. 16 for graphical analysis.
  - Under third-degree price discrimination, the monopolist chooses a higher price in the low-elasticity sub-market. Given that arbitrage is not possible between sub-markets, demand in each sub-market is independent of price in the other sub-market, and total profit is maximised iff profit in each sub-market is maximised. By taking first-order conditions for quantity in each sub-market, it can be shown that, at equilibrium, price is higher in the low-elasticity sub-market.
- Result (Profit)
  - Where third-degree price discrimination is practicable, compared to where it is not, profit increases if demand is not equally price-elastic in each sub-market. If demand is equally price-elastic in each sub-market, then by the analysis above, the monopolist chooses a common price in each sub-market, hence the outcome in the former case is equivalent to that in the latter case. If demand is not equally price-elastic in each sub-market, profit increases because the firm chooses a more optimal price in each sub-market.
- Result (Welfare)
  - Where third-degree price discrimination is practicable, compared to where it is not, whether welfare increases or decreases is ambiguous a priori. In the former case, the monopolist is (weakly) better off since it price discriminates only if doing so increases profit. Large consumers are worse off since they face a higher price (and consume lower quantity) while small consumers are better off since they face a lower price (and consume a higher quantity).
  - Third-degree price discrimination has two effects on welfare, a negative misallocation effect, and a positive output effect. Under third-degree price discrimination, output is inefficiently allocated across sub-markets since the valuation of the marginal unit by consumers in the high-elasticity market (equal to price in this market) is lower than that in the low-elasticity market, hence welfare would be increased if some quantity were transferred from the former consumers to the latter. Under third-degree price discrimination, if there is an increase in output, given that the firm never chooses price below marginal cost and consumers never buy at price above their valuation, valuation of these additional units exceeds marginal cost, hence production and consumption of these additional units increases welfare.
  - Where all sub-markets are comparably-sized and demand is linear in each sub-market, it can be shown that third-degree price discrimination has no effect on output, and therefore no positive "output effect". Third-degree price discrimination in this case decreases social welfare.
  - More generally, price discrimination increases social welfare if demand in the high-elasticity low-price market is more convex (in the sense given by Aguirre, Cowan, and Vickers, 2016) than in the low-elasticity high-price market and the price difference with discrimination is small. An increase in convexity of demand in the high-elasticity low-price sub-market corresponds to (1) a decrease in the steepness of the marginal revenue curve in this sub-market hence an increase in the output in this sub-market at the discriminatory price and (2) an increase in the marginal social value of additional units of output. Both (1) and (2) (analytically) cause an increase in the increase in social welfare in this sub-market due to price discrimination. An increase in concavity of demand in the low-elasticity high-price sub-market corresponds to (1) an increase in the steepness of the marginal revenue curve in this sub-market hence a decrease in the output in this sub-market at the discriminatory price and (2) a decrease in the marginal social value of infra-marginal units of output. Both (1) and (2) (analytically) cause a decrease in the decrease in social welfare in this sub-market due to price discrimination.
  - Where third-degree price discrimination "opens a new market", (supposing for simplicity that there are only two sub-markets), compared to where price discrimination is not practiced, social welfare increases. Third-degree price discrimination "opens a new market" iff the non-discriminatory price exceeds the reservation price in the high-elasticity sub-market such that no consumers in this sub-market buy at the non-discriminatory price. If price discrimination opens a new market, where price discrimination is practiced, compared to where it is not, price faced by consumers in the low-elasticity sub-market is unchanged hence welfare of these consumers is unchanged. This is

because in the former case, the firm treats this sub-market as independent of the other and in the latter case, the firm does not serve the other sub-market, hence the firm chooses price to maximise profit in this sub-market in both cases. Additionally, in the former case, consumers in the high-elasticity low-price sub-market are better off because they consume greater (non-zero) quantity hence enjoy greater (non-zero) surplus. The firm is better off because it enjoys additional profit in this sub-market.

- Discussion (Distributive Effects)
  - Third-degree price discrimination has ambiguous distributive effects. Surplus is transferred from consumers to the price-discriminating firm, which is generally undesirable. Surplus is transferred from low-elasticity (generally high income) consumers to high-elasticity (generally low income) consumers, which is generally desirable.
- Discussion (Intertemporal Third-degree Price Discrimination)
  - Third-degree price discrimination, where the observable exogenous signal of consumers' valuations is the time of purchase, constitutes an instance of intertemporal price discrimination.

Katz (1987) Intermediate Goods Model

- Motivation
  - The conventional model of final goods third-degree price discrimination does not easily translate to the case of intermediate goods because (1) in final goods markets, each buyer's demand is typically independent of the quantity consumed by any other buyer whereas in intermediate goods markets, each buyer's demand is generally dependent on the quantity consumed by all other buyers and (2) buyers in final goods markets are not able to integrate backward whereas buyers in intermediate goods markets may be able to do so.
- Parameters
  - Consider the simplified Katz (1987) model of third-degree price discrimination in intermediate goods markets. An upstream monopolist produces some intermediate good at constant marginal cost and sells to one chain store and a large number of local stores. In a large number of downstream markets, one local store competes with one branch of the chain store in Cournot. In the first stage of a three-stage game, the upstream monopolist chooses price at which to sell to the chain store and price at which to sell to each of the local stores. In the second stage, the chain store decides whether or not to integrate backward at some fixed set-up cost. If the chain store integrates backward, it produces the intermediate good at some constant marginal cost greater than that of the upstream incumbent in the third stage. In the third stage, in each downstream market, the chain store branch and local store compete in Cournot.
- Result (Welfare)
  - Suppose that the upstream monopolist finds it optimal to induce (or, not to deter) backward integration regardless of whether price discrimination is practicable. Then, regardless of whether price discrimination is practicable, the upstream incumbent serves only local stores in the third stage, and the practicability of price discrimination (between the chain store and local stores) is irrelevant.
  - Suppose instead that the upstream monopolist finds it optimal to deter backward integration only if price discrimination is practicable. Then, price discrimination could increase social welfare if and because it prevents socially inefficient backward integration. Backward integration could be socially inefficient if and because the chain store incurs a fixed set-up cost that is not otherwise incurred and imposes a negative externality on the local stores since it then enjoys lower input costs and competes more aggressively and on the upstream incumbent since the upstream incumbent then faces less demand.
  - Suppose instead that the monopolist finds it optimal to deter backward integration regardless of whether price discrimination is practicable. Then, price discrimination decreases social welfare and the decrease in social welfare is potentially dramatic. Suppose that, in the case where price discrimination is not practicable, the upstream monopolist is constrained by the threat of backward integration, and thus chooses a sufficiently low non-discriminatory price such that the chain store (only just) has no incentive to integrate. Then, in the case where price discrimination is practicable, the upstream monopolist, in choosing its price for local stores, is not so constrained, and finds it optimal to choose a higher price. The higher input cost for local stores causes local stores to compete less aggressively and increases profit of the chain store. The upstream monopolist then has incentive to increase price for the chain store until the integration constraint is again binding. An increase in input cost for both the chain store and each local store causes a decrease in quantity of the final good hence a decrease in social surplus.

Oligopoly Third-degree Price Discrimination

- Parameters

- Consider the simplified Thisse and Vives (1988) model of oligopoly price discrimination. Two firms, A and B, located at $x_A = 0$ and $x_B = 1$ respectively in product space $x \in [0, 1]$ each produce a good at constant marginal cost c and sell to a unit mass of consumers distributed uniformly in product space. Each firm X chooses price $p_X(x)$ for the consumer at x to maximise profit π_X. Each consumer has unit demands and sufficiently high valuation such that he always consumes one unit. There are linear transport costs such that the total cost to consumer x of buying from firm X is given by $p_X(x) + t|x_X - x|$ where t is the unit transport cost. Each consumer chooses which firm to buy from to minimise total cost.

- Analysis
  - Consumer i located at x_i such that $x_i \in [x_A = 0, \frac{1}{2})$ is indifferent between buying from A and buying from B iff $p_A(x_i) + tx_i = p_B(x_i) + t(1 - x_i)$. At Nash equilibrium, $p_A(x_i) = c + t(1 - 2x_i)$ and $p_B(x_i) = c$ and the consumer buys from A. If A chooses a higher price, the consumer buys from B and A enjoys zero profit from this consumer. If A chooses a lower price, A enjoys less profit from this consumer. For consumer j located at x_j such that $x_j \in (\frac{1}{2}, x_B = 1]$, at Nash equilibrium, $p_A(x_j) = c$, $p_B(x_j) = c + t(2x_j - 1)$ and the consumer buys from B.
  - Suppose instead that price discrimination is not practiced, i.e. each firm X chooses non-discriminatory price p_X for all consumers, then by the result of the Hotelling model, at equilibrium, each firm X chooses $p_X = c + t$.

- Result
  - Where price discrimination is practiced, compared to where it is not, each consumer faces a (weakly) lower price, and consumers located at $x \neq 0, 1$ face a strictly lower price. Since, by supposition, consumer valuations and marginal cost are unchanged, and quantity is unchanged, welfare is unchanged. The decrease in price constitutes a transfer of surplus from producers to consumers. Profit of each firm decreases and consumer surplus increases. Intuitively, firms compete more aggressively where price discrimination is practiced. Where price discrimination is not practiced, if a firm were to decrease its price to capture greater quantity, its margin on infra-marginal units decreases, hence its profit decreases. Where price discrimination is practiced, a firm can capture greater quantity by offering a lower price to the marginal consumer without decreasing price for infra-marginal consumers. Firms have greater incentive to decrease prices where price discrimination is practiced.