3E Problem set 4

1 $Y_i = \beta_0 + \beta_1 X_{1i} + \dots + \beta_K X_{Ki} + u_i$

$u_i \perp\!\!\!\perp (X_{1i}, \dots, X_{Ki})$

$E(u_i) = 0$

$E(u_i | X_{1i}, \dots, X_{Ki})$

$= E(u_i)$

since $u_i \perp\!\!\!\perp (X_{1i}, \dots, X_{Ki})$

$= 0$

$E(u_i^2 | X_{1i}, \dots, X_{Ki})$

$= E(u_i^2)$

since $u_i \perp\!\!\!\perp (X_{1i}, \dots, X_{Ki})$

Hence u_i is homoskedastic

2a $Y = \beta_0 + \beta X + u$

where $E(u) = 0$ and $E(Xu) = 0$

$H_0: \beta_1 = 0$, $H_1: \beta_1 \cancel{\neq} > 0$

under H_0,

~~$n^{1/2}(\hat\beta_1 - \beta_1) \xrightarrow{d} N(0, \omega^2_{\beta_1})$~~

~~$n^{1/2}(\hat\beta_1 - \beta_1)/\hat\omega_{\beta_1}$~~

$se(\hat\beta_1) = n^{-1/2}\hat\omega_{\beta_1} = n^{-1/2} \hat{sd}(\hat{\tilde{X}}_i \hat u)/\hat{sd}(\hat{\tilde{X}}_i)$

t-statistic

$t = (\hat\beta_1 - \beta_1)/se(\hat\beta_1) = n^{1/2}(\hat\beta_1 - \beta_1)/\hat\omega_{\beta_1} \xrightarrow{d} N(0,1)$ where $\hat\omega_{\beta_1} = \hat{sd}(\hat{\tilde{X}}_i \hat u)/\hat{var}(\hat{\tilde{X}}_i)$

For one-sided test at level of significance $\alpha = 10\%$,

Reject H_0 if $t > c_\alpha$

$\alpha = 0.10 = \Phi(-c_\alpha)$, $c_\alpha = 1.2816$

bi when $\beta_1 = 0.01$, sampling distribution of $\hat\beta_1$ is closer to the sampling distribution under the null than when $\beta_1 = 100$. Observed value of the t-statistic is likely to be less positive. Given the decision rule from (a), we are less likely to reject the false null. The test is less powerful when $\beta_1 = 0.01$ than when $\beta_1 = 100$.

ii when $\beta_1 = -1$, sampling distribution of $\hat\beta_1$ has mean -1. Observed value of the t-statistic is likely to be less positive than when $\beta_1 = 0.01$, 1, or 100. Given the decision rule from (a), we are less likely to reject the false null. The test is less powerful when $\beta_1 = -1$ than when $\beta_1 \in [0, +\infty)$

ci $\hat\beta_1$ is approximately normally distributed with mean β_1. The closer the mean of the sampling distribution to the hypothesised value of β_1 under the null, the greater the probability mass of the sampling distribution ~~of~~ around this hypothesised value, hence the lower the probability of observing a t-statistic with a large absolute value, and of rejecting the null, under a two-sided test. For all $\beta_1 \neq 0$, as $|\beta_1|$ increases, so does the power of the two sided test.

3a $\cancel{\text{Restricted model: } Y = \beta_0 + u}$
$\cancel{\text{where } E(u) = 0}$
$\cancel{Y_i = \beta_0 + u_i}$
$\cancel{E(Y) = E(\beta_0) + E(u) = \beta_0}$
$\cancel{Y_i = \beta_0 + u_i}$
$\cancel{\text{where } \bar{u} = 0}$
$\cancel{\bar{Y} = \beta_0 + \bar{u} = \beta_0}$
$\cancel{SSR_{rs} = \sum_{i=1}^{n} u_i^2}$
$\cancel{= \sum_{i=1}^{n} (Y_i - \beta_0)^2}$
$\cancel{= \sum_{i=1}^{n} (Y_i - \bar{Y})^2}$
$\cancel{= TSS_{rs}}$

Restricted model:
$\cancel{Y = \beta_0 + u_{rs}}$
$Y = \beta_{0,rs} + u_{rs}$ where $E(u_{rs}) = 0$
$Y_i = \beta_{0,rs} + u_{i,rs}$ where $\bar{u}_{rs} = 0$
$\bar{Y} = \beta_{0,rs} + \bar{u}_{rs} = \beta_{0,rs}$
$SSR_{rs} = \sum_{i=1}^{n} u_{i,rs}^2$
$= \sum_{i=1}^{n} (Y_i - \beta_{0,rs})^2$
$= \sum_{i=1}^{n} (Y_i - \bar{Y})^2$
$= TSS$

b $\cancel{\text{Unrestricted model:}}$
$\cancel{Y = \beta_0 + \beta_1 X + u_{un} \text{ where } E(u_{un}) = 0 \text{ and } E(Xu_{un}) = 0}$
$\cancel{Y_i = \hat{\beta}_0 + \hat{\beta}_1 X_i + \hat{u}_{un,i} \text{ where } \bar{u}_{un} = 0 \text{ and } \hat{cov}(X, \hat{u}_{un}) = 0}$
$\cancel{\text{From } \hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X},\ \hat{\beta}_1 = \hat{cov}(Y,X)/\hat{var}}$

Unrestricted model
$\cancel{Y = \beta_0 + \beta_1 X}$
$Y = \beta_{0,un} + \beta_1 X + u_{un}$
where $E(u_{un}) = 0$ and $E(Xu_{un}) = 0$
$Y_i = \hat{\beta}_{0,un} + \hat{\beta}_1 X_i + \hat{u}_{un,i}$
where $\bar{\hat{u}}_{un} = 0$ and $\hat{cov}(X, \hat{u}_{un}) = 0$
$\hat{\beta}_1 = \hat{cov}(Y,X)/\hat{var}(X)$, $\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$
$\cancel{SSR_{un} = \sum_{i=1}^{n} \hat{u}_{un,i}^2}$
$\cancel{= \sum_{i=1}^{n} (Y_i - \hat{\beta}_{0,un} - \hat{\beta}_1 X_i)^2}$
$\cancel{= \sum_{i=1}^{n} (Y_i - \bar{Y} - \hat{\beta}_1 (X_i - \bar{X}))^2}$
$\cancel{= \sum_{i=1}^{n} (Y_i - \bar{Y})^2 + \hat{\beta}_1^2 \sum_{i=1}^{n} (X_i - \bar{X})^2}$
$\cancel{- 2\hat{\beta}_1 \sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}$

$SSR_{un} = \sum_{i=1}^{n} \hat{u}_{un,i}^2$
$= \sum_{i=1}^{n} (Y_i - \hat{\beta}_{0,un} - \hat{\beta}_1 X_i)^2$
$= \sum_{i=1}^{n} (Y_i - \bar{Y} - \hat{\beta}_1 (X_i - \bar{X}))^2$
$= \sum_{i=1}^{n} (Y_i - \bar{Y})^2 + \hat{\beta}_1^2 \sum_{i=1}^{n} (X_i - \bar{X})^2$
$- 2\hat{\beta}_1 \sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})$

$= SSR_{rs} + \hat{\beta}_1^2 \sum_{i=1}^{n} (X_i - \bar{X})^2$
only if $\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y}) = 0$ or $\hat{\beta}_1 = 0$
$\downarrow$ implies
$\hat{cov}(Y, X) = 0$
$\downarrow$ implies
$\hat{\beta}_1 = 0$

SSR_{rs}
$= TSS$
$= \sum_{i=1}^{n} (Y_i - \bar{Y})^2$
$= \sum_{i=1}^{n} (\hat{Y}_i + \hat{u}_i - \bar{Y})^2$
$= \sum_{i=1}^{n} (\hat{Y}_i - \bar{Y})^2 + \sum_{i=1}^{n} \hat{u}_i^2 - 2\sum_{i=1}^{n} \hat{u}_i (\hat{Y}_i - \bar{Y})$
$= \sum_{i=1}^{n} (\hat{Y}_i - \bar{Y})^2 + \sum_{i=1}^{n} \hat{u}_i^2$
$= \sum_{i=1}^{n} (\hat{\beta}_{0,un} + \hat{\beta}_1 X_i - \bar{Y})^2 + \sum_{i=1}^{n} \hat{u}_i^2$
$= \sum_{i=1}^{n} (\bar{Y} - \hat{\beta}_1 \bar{X} + \hat{\beta}_1 X_i - \bar{Y})^2 + \sum_{i=1}^{n} \hat{u}_i^2$
$= \sum_{i=1}^{n} (\hat{\beta}_1 (X_i - \bar{X}))^2 + \sum_{i=1}^{n} \hat{u}_i^2$
$= \hat{\beta}_1^2 \sum_{i=1}^{n} (X_i - \bar{X})^2 + \cancel{\sum_{i=1}^{n} \hat{u}_i^2}\ SSR_{un}$

b $SSR_{rs} - SSR_{un} = \hat{\beta}_1^2 \sum_{i=1}^n (x_i - \bar{x})^2$

c $F = CR/q = CR$

$= [(SSR_{rs} - SSR_{un})/q] / [SSR_{un}/(n-k-1)]$

$= \hat{\beta}_1^2 \sum_{i=1}^n (x_i - \bar{x})^2 / (\sum_{i=1}^n \hat{u}_{un,i}^2 / (n-k-1))$

$t = (\hat{\beta}_1 - 0) / se(\hat{\beta}_1)$

$\cancel{= \hat{\beta}_1 / [s_{\hat{u}} / (\sum_{i=1}^n (x_i - \bar{x})^2)]^{1/2}}$

$\cancel{= \hat{\beta}_1 / [(1/n-k-1 \sum_{i=1}^n \hat{u}_i^2)^{1/2} / \sum_{i=1}^n (x_i - \bar{x})^2]^{1/2}}$

$= \hat{\beta}_1 / \{s_{\hat{u}} / [\sum_{i=1}^n (x_i - \bar{x})^2]^{1/2}$

$= \hat{\beta}_1 [\sum_{i=1}^n (x_i - \bar{x})^2]^{1/2} / [1/n-k-1 \sum_{i=1}^n \hat{u}_i^2]^{1/2}$

$t^2 = \hat{\beta}_1^2 \sum_{i=1}^n (x_i - \bar{x})^2 / [1/n-k-1 \sum_{i=1}^n \hat{u}_i^2]$

$= F$

d ~~$R^2 = ESS/TSS$~~

4a $R^2 = ESS/TSS = 12851/95948 = 0.13394$

$\bar{R}^2 = 1 - (SSR/(n-k-1))/(TSS/n-1)$

$= 1 - (TSS-ESS/n-k-1)/(TSS/n-1)$

$= 1 - (95948-12851/6028-9-1)/(95948/6028-1)$

$= 0.13264$

$s_{\hat{u}} = \sqrt{s_{\hat{u}}^2} = \sqrt{1/n-k-1 \sum_{i=1}^n \hat{u}_i^2}$

$= \sqrt{1/n-k-1 (TSS-ESS)}$

$= \sqrt{1/6028-9-1 (95948-12851)}$

$= 3.7159$

~~H_0:~~

Let the ~~regre~~ OLS regression model be

$Y_i = \hat{\beta}_0 + \hat{\beta}_1 X_{1i} + \ldots + \hat{\beta}_9 X_{9i} + \hat{u}_i$

where $\bar{u} = 0$ and $\widehat{cov}(X_k, u) = 0$ for $k \in \{1, \ldots, 9\}$

$H_0: \beta_1 = \ldots = \beta_9 = 0$

$H_1: \beta_k \neq 0$ for some $k \in \{1, \ldots, 9\}$

F-statistic

$F = CR/q =$ ~~$[SSR_{rs} - SSR_{un}]$~~

~~$= [(SSR_{rs} - ESS$~~

$= [(SSR_{rs} - SSR_{un})/q] / [SSR_{un}/(n-k-1)]$

$= [(TSS - SSR)/q] / [SSR/(n-k-1)]$

~~$= [(95948 - 12851)/9] / [(95948-12851)/(6028-9-1)]$~~

$= [12851/9] / [(95948-12851)/(6028-9-1)]$

$= 103.41$

~~F-statistic has~~

~~F-stati~~
~~Sampling distri~~
F-statistic converges in distribution to $F_{9,\infty}$

Reject Ho if $F > c_\alpha$
$\alpha = 0.05$, $= P(F_{9,\infty} > c_\alpha)$, $c_\alpha = 1.8799$

Reject Ho

b $H_0: \beta_{teacher\_exp} = 0$
$H_1: \beta_{teacher\_exp} \neq 0$

Under the null,
t-statistic
$t = (\hat{\beta}_t - \beta_t)/se(\hat{\beta}_t)$
$= 0.04/0.02$
$= 2$
~~t-statistic~~ $t \xrightarrow{d} N(0,1)$

For two-sided test at level of significance $\alpha = 5\%$
Reject Ho if $|t| > c_\alpha$
$\alpha = 0.05$, $c_\alpha = -\Phi^{-1}(0.05/2) = 1.9600$

Reject Ho

c $H_0: \beta_{small\_class} = 0$
$H_1: \beta_{small\_class} \neq 0$

Under the null,
t-statistic
$t = (\hat{\beta}_s - \beta_s)/se(\hat{\beta}_s)$
$= 0.66/0.30$
$= 2.20$
$t \xrightarrow{d} N(0,1)$

P-value
$p = 2\Phi(-|t|)$
$= 2\Phi(-2.20)$
$= 0.0139$

Under the null, the probability of observing a t statistic at least as unfavourable to the null as that actually observed is 0.0139. Reject the null at all levels of significance $\alpha > 0.0139$.

d ~~$H_0: \beta_1 = b$~~ $H_0: \beta_{summer\_baby} = b$
~~$H_1: \beta_1$~~ $H_1: \beta_{summer\_baby} \neq b$
$\alpha = 0.01$, $c_\alpha = -|\Phi^{-1}(\alpha/2)| = 2.58$

Confidence interval
$C = \{ b \in \mathbb{R} \mid H_0: \beta_{s\_b} = b \text{ is accepted} \}$
$= \{ b \in \mathbb{R} \mid |t(b)| \leq c_\alpha \}$
$= [\hat{\beta}_{sb} - c_\alpha se(\hat{\beta}_1), \hat{\beta}_{sb} + c_\alpha se(\hat{\beta}_1)]$
$= [-0.8538, -0.2862]$

There is a 0.99 probability that the interval $[-0.8538, -0.2862]$ contains the population regression parameter β_{summer_baby}.

e Elasticity $= (\partial maths\_test / maths\_test) / (\partial teacher\_exp / teacher\_exp)$

$= \hat{\beta}_t(\overline{maths\_test} / \overline{teacher\_exp})$

$= 0.04(61.81/13.93)$

$= 0.17749$

$H_0: E = e$

$H_1: E \neq e$

$\alpha = 0.10$, $c_\alpha = -\Phi^{-1}(\alpha/2) = 1.645$

Confidence interval

$C = \{e \in \mathbb{R} \mid H_0: E = e$ is accepted$\}$

$= \{e \in \mathbb{R} \mid H_0: \hat{\beta}_t(\overline{m\_t}/\overline{t\_e}) = e$ is accepted$\}$

$= \{e \in \mathbb{R} \mid H_0: \hat{\beta}_t = e\overline{t\_e}/\overline{m\_t}$ is accepted$\}$

$= \{e \in \mathbb{R} \mid |t(e\overline{t\_e}/\overline{m\_t})| \leq c_\alpha\}$

$= [(\hat{\beta}_t - c_\alpha se(\hat{\beta}_t))(\overline{m\_t}/\overline{t\_e}),$
$(\hat{\beta}_t + c_\alpha se(\hat{\beta}_t))(\overline{m\_t}/\overline{t\_e})]$

$= [0.0315, 0.323]$

Ignoring the sampling variability of sample means, there is a ~~90~~ 0.90 probability that the interval $[0.0315, 0.323]$ contains the elasticity of math test score with respect to teacher experience.

f ~~$H_0: \beta_{black} = 0$~~
~~(there is no difference in math test scores of Black students and non-Black students)~~
~~$H_1: \beta_{black} \neq 0$~~
~~(there is some differe~~

$H_0: \beta_{black} = 0$

$H_1: \beta_{black} \neq 0$

Under the null,

t-statistic

$t = (\hat{\beta}_{black} - \beta_{black}) / se(\hat{\beta}_{black})$

$= -1.53/0.16$

$= -9.5625$

p-value

$p = 2\Phi(-|t|)$

$= 0$

Under the null, the probability of observing a t statistic as unfavourable to the null as that actually observed is effectively zero. Reject the null at all non-zero levels of significance α, there is strong evidence for differences in math test scores between Black and non-Black students.

$H_0: \beta_{other\_non\_wh} = 0$
$H_1: \beta_{other\_non\_wh} \neq 0$

under the null,
t-statistic
$t = (\hat{\beta}_{onw} - \beta_{onw}) / se(\hat{\beta}_{onw})$
$= 0.90 / 0.59$
$= 1.5254$
p-vale
$p = 2\Phi(-|t|)$
$= 0.12716$

under the null, the probability of obseriving a t-statistic at least as unfavourable to the null as that actually observed is 0.12716, reject Ho at all levels of significance greater than 0.12716. There is some evidence for difference in test scores between other non white students and Black and white students.

g The coefficient of free_lunch in the regression model gives a reliable estimate of the effect of free_lunch on math_test only if the other determinents of math_test excluded from the model are uncorrelated with free_lunch. This is unlikely.

math_test is likely to be affected by parents' income and hence access to educational resources. Free school lunches are (presumably) part of a programme to support underprivileged children, whose parents' income is likely to be lower. free_lunch is likely correlated with parents' income, which is a determinant of math_test excluded from the model.

Abolishing the provision of free school lunches could have no effect or even negative effect on math test scores if the relationship between free_lunch and math_test is entirely accounted for by parents' income, since parents' income remains unchanged ~~as~~ when free school lunches are abolished.

5a $C_i + T_i + R_i = 1$
C_i, T_i, and R_i are perfectly multicollinear

bi On average, a one-year increase in years of experience is associated with a $\beta_X = \gamma_X$ increase in hourly wages, all other factors being equal.

ii On average, an individual who lives in the city has an hourly wage β_C higher than an individual who lives in a rural area, and γ_C higher than an individual who lives in a town.

c $W_i = \beta_0 + \beta_X X_i + \beta_C C_i + \beta_T T_i + u_i$
where $E(u)=0$, $E(Xu)=0$, $E(Cu)=0$, $E(Tu)=0$
$W_i = \gamma_0 + \gamma_X X_i + \gamma_C C_i + \gamma_R R_i + v_i$
where $E(v)=0$, $E(Xv)=0$, $E(Cv)=0$, $E(Rv)=0$

$W_i = \beta_0 + \beta_X X_i + \beta_C C_i + \beta_T T_i + u_i$
$= \beta_0 + \beta_X X_i + \beta_C C_i + \beta_T(1 - C_i - R_i) + u_i$
$= \beta_0 + \beta_X X_i + (\beta_C - \beta_T) C_i - \beta_T R_i + u_i$
Since R_i is a linear function of C_i and T_i, and $E(Cu)=0$, $E(Tu)=0$, $E(Ru)=0$
$W_i = \beta_0 + \beta_X X_i + (\beta_C - \beta_T) C_i - \beta_T R_i + u_i$
where $E(u)=0$, $E(Xu)=0$, $E(Cu)=0$, $E(Ru)=0$
$\beta_0 = \gamma_0$, $\beta_X = \gamma_X$, $\beta_C - \beta_T = \gamma_C$, $-\beta_T = \gamma_R$, $u_i = v_i$ Since the solution to the regression model is unique

$-\beta_T = \gamma_R$
~~γ_R gives the~~ An individual living in a rural area, on average, has an hourly wage γ_R higher than an individual living in a town, all other factors being equal. γ_R is equal to $-\beta_T$, where ~~β_T is the diff~~ on average, an individual living in a town has an hourly wage β_T higher than an individual living in a rural area, all other factors being equal.

d $H_0: \beta_C = \beta_T = 0$
$H_1: \beta_C \neq 0$ or $\beta_T \neq 0$

Restricted model: $W_i = \beta_0 + \beta_X X_i + u_i$
Unrestricted model: $W_i = \beta_0 + \beta_X X_i + \beta_C C_i + \beta_T T_i + u_i$

F-statistic
$F = [(SSR_{rs} - SSR_{un})/q] / [SSR_{un}/(n-k-1)]$
$F \overset{d}{\to} F_{2,\infty}$

Reject H_0 if $F > c_\alpha$
where $\alpha = P(F_{2,\infty} > c_\alpha)$

Under H_0, on average, an individual living in a city and an individual living in a town both have hourly wage no higher or lower than an individual living in a rural area, holding years of experience equal.

e Fit
$W_i = \beta_0 + \beta_X X_i + \beta_C C_i + \beta_T T_i + \beta_{CX} C_i X_i + \beta_{TX} T_i X_i + u_i$

Not sure how to explain (formally) why this works

F-test
$H_0: \beta_{CX} = \beta_{TX} = 0$
$H_1: \beta_{CX} \neq 0$ or $\beta_{TX} \neq 0$

6a ~~$Y_i = A_i K_i^{\alpha} L_i^{\beta} e^{\varepsilon_i}$~~

~~$= A K_i^{\alpha} L_i^{\beta} e^{\varepsilon_i}$~~

~~$\ln Y_i = \ln A + \alpha \ln K_i + \beta \ln L_i$~~

$Y_i = A_i L_i^{\alpha} K_i^{\beta} e^{\varepsilon_i}$

$= A L_i^{\alpha} K_i^{\beta} e^{\varepsilon_i}$

$\ln Y_i = \ln A + \alpha \ln L_i + \beta \ln K_i + \varepsilon_i$

Let k be such that $E(\varepsilon_i + k) = 0$

$\ln Y_i = (\ln A - k) + \alpha \ln L_i + \beta \ln K_i + (\varepsilon_i + k)$

Given that $\varepsilon_i \perp\!\!\!\perp L_i, K_i$,

$(\varepsilon_i + k) \perp\!\!\!\perp \ln L_i, \ln K_i$

Since $+k$ and $\ln$ are one-to-one — Is this right?

Let Y_i^*, γ_0^*, L_i^*, K_i^*, ε_i^* be

$\ln Y_i$, $(\ln A - k)$, $\ln L_i$, $\ln K_i$, $(\varepsilon_i + k)$ respectively

$Y_i^* = \gamma_0^* + \alpha L_i^* + \beta K_i^* + \varepsilon_i^*$

where $E(\varepsilon_i^*) = 0$ and $\varepsilon_i^* \perp\!\!\!\perp L_i^*, K_i^*$

$Y_i^* = \gamma_0^* + \alpha L_i^* + \beta K_i^* + \varepsilon_i^*$

is a population linear regression model of Y_i^* on L_i^* and K_i^*

$\hat{\alpha} = \hat{cov}(Y_i^*, L_i^*)/\hat{var}(L_i^*)$

$\hat{\beta} = \hat{cov}(Y_i^*, K_i^*)/\hat{var}(K_i^*)$

consistently estimate α and β

b The production function exhibits constant returns to scale iff $\alpha + \beta = 1$

H_0: ~~#~~ $\alpha + \beta = 1$

H_1: $\alpha + \beta \neq 1$

Under H_0:

$Y_i^* = \gamma_0^* + \alpha L_i^* + (1-\alpha)\cancel{\beta} K_i^* + \varepsilon_i^*$

$= \gamma_0^* + \alpha(L_i^* - K_i^*) + K_i^* + \varepsilon_i^*$

~~$Y_i^* - K_i^*$~~

$Y_i^* - K_i^* = \gamma_0^* + \alpha(L_i^* - K_i^*) + \varepsilon_i^*$

Let Y_i^{**}, ~~γ_0^{**}~~ X_i^{**} be $Y_i^* - K_i^*$ and $L_i^* - K_i^*$ respectively

~~$Y_i^{**} = \gamma_0^* + \alpha X_i^{**} + \varepsilon_i^*$~~

~~Restricted model population~~

Restricted model

$Y_i^{**} = \gamma_0^* + \alpha X_i^{**} + \varepsilon_i^*$

Compute

SSR_{un}, ~~SSR~~

SSR_{rs}

I want to write the formulae for these but the same symbols $\hat{\gamma}_0^*$ and $\hat{\alpha}$ appear in both, but refer to different estimators, is there any way to avoid this notational difficulty?

$F = [(SSR_{rs} - SSR_{un})/q]/[SSR_{un}/(n-k-1)]$

where $q = 1$, $k = 2$

At level of significance α

Reject H_0 if $F > c_\alpha$

where $\alpha = P(F_{1,\infty} > c_\alpha)$

c Yes.

$\hat{\alpha}$ and $\hat{\beta}$ would then estimate both the direct effect of K_i and L_i on Y_i and the respective indirect effects through A_i.

This seems unsatisfactory but I dont know how to make it more precise d $\cancel{Y_{i,2015}/Y_{i,2014} = A_{i,2015} L_{i,2015}^{\alpha} K_{i,2015}^{\beta},}$

$$Y_{i,2015}/Y_{i,2014} = L_{i,2015}^{\alpha} K_{i,2015}^{\beta} e^{\varepsilon_{i,2015}} L_{i,2014}^{-\alpha} K_{i,2014}^{-\beta} e^{-\varepsilon_{i,2014}}$$
$$= (L_{i,2015}/L_{i,2014})^{\alpha} (K_{i,2015}/K_{i,2014})^{\beta} e^{\varepsilon_{i,2015} - \varepsilon_{i,2014}}$$

$\cancel{\ln(Y_{i,2015}/Y_{i,2014})}$
$\cancel{= \alpha(\ln Y_{i,2015} - \ln Y_{i,2014}) + \beta(K_{i,2015}/K_{i,2014})}$
$\cancel{+\beta(\ln K_{i,2015}}$

$$\ln(Y_{i,2015}) - \ln(Y_{i,2014})$$
$$= \alpha(\ln L_{i,2015} - \ln L_{i,2014})$$
$$+ \beta(\ln K_{i,2015} - \ln K_{i,2014})$$
$$+ (\varepsilon_{i,2015} - \varepsilon_{i,2014})$$

let k be such that $E(\varepsilon_{i,2015} - \varepsilon_{i,2014} - k) = 0$

$$\ln Y_{i,2015} - \ln Y_{i,2014}$$
$$= \alpha(\ln L_{i,2015} - \ln L_{i,2014})$$
$$+ \beta(\ln K_{i,2015} - \ln K_{i,2014})$$
$$+ (\varepsilon_{i,2015} - \varepsilon_{i,2014} - k)$$
$$+ k$$

Let Y_i^*, L_i^*, K_i^*, ε_i^* be
$\ln Y_{i,2015} - \ln Y_{i,2014}$,
$\ln L_{i,2015} - \ln L_{i,2014}$,
$\ln K_{i,2015} - \ln K_{i,2015}$,
$\varepsilon_{i,2015} - \varepsilon_{i,2014} - k$
respectively

$$Y_i^* = k + \alpha L_i^* + \beta K_i^* + \varepsilon_i^*$$
where $E(\varepsilon_i^*) = 0$, $\varepsilon_i^* \perp\!\!\!\perp L_i^*, K_i^*$ How can I justify this?

$Y_i^* = k + \alpha L_i^* + \beta K_i^* + \varepsilon_i^*$
is a population linear regression model of Y_i^* on L_i^* and K_i^*

$$\hat{\alpha} = \widehat{cov}(Y^*, L^*)/\widehat{var}(L^*)$$
$$\hat{\beta} = \widehat{cov}(Y^*, K^*)/\widehat{var}(K^*)$$
consistently estimate α and β

7a $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_1^2 + \beta_3 X_2 + \beta_4 X_1 X_2 + u$ ①

$Y = \beta_0 + \beta_1 \ln X_1 + \beta_2 X_2 + \beta_3 X_2 \ln X_1 + u$ ②

①

Let $X_1^*, X_2^*, X_3^*, X_4^*$

be $X_1, X_1^2, X_2, X_1 X_2$ respectively

$Y = \beta_0 + \beta_1 X_1^* + \beta_2 X_2^* + \beta_3 X_3^* + \beta_4 X_4^* + u$

① is linear in the parameters

②

Let X_1^*, X_2^*, X_3^*

be ~~X_1~~ $\ln X_1, X_2, X_2 \ln X_1$ respectively

$Y = \beta_0 + \beta_1 X_1^* + \beta_2 X_2^* + \beta_3 X_3^* + u$

② is linear in the parameters

bi ~~①~~

~~$E(u) = 0$, $E(X_i^* u) = 0$ for $i \in \{1, \dots, 4\}$~~

~~②~~

~~$E(u) = 0$, $E(X_i^* u) = 0$ for $i \in \{1, \dots, 3\}$~~

~~ii $E(u \mid X_1^*, \dots, X_4^*) = 0$~~

~~$E(u \mid X_1^*, \dots, X_3^*) = 0$~~

① $E(u) = E(X_1 u) = E(X_1^2 u) = E(X_2 u) = E(X_1 X_2 u) = 0$

② $E(u) = E(u \ln X_1) = E(X_2 u) = E(X_2 u \ln X_1) = 0$

ii ① $E(u \mid X_1, X_2) = 0$

② $E(u \mid X_1, X_2) = 0$