Date

Philosophical Logic Paper 150609

1a Prove the given claim by ~~±~~ strong induction over the number of connectives in PL-wff $\phi$.

Base case

Consider arbitrary PL-wff $\phi$ such that ~~comple~~ complexity $C(\phi)$ (number of connectives $\sim, \to$) $= 0$. Suppose for conditional proof that for arbitrary trivalent interpretation I, $KV_I(\phi) = 1$. Given that $C(\phi) = 0$, $\phi$ is some sentence letter $\alpha$. By definition of Kleene valuation, ~~KI~~ $KV_I(\phi) = I(\alpha) = 1$. ~~By definition of Kleene valu~~ By definition of refinement, $I^+(\alpha) = 1$. Then by definition of ~~Klee~~ Kleene valuation, $KV_{I^+}(\phi) = I^+(\alpha) = 1$. By conditional proof, if $KV_I(\phi) = 1$ then $KV_{I^+}(\phi) = 1$. Suppose for conditional proof that $KV_I(\phi) = 0$, then $KV_I(\phi) = I(\alpha) = 0$, then $I^+(\alpha) = I(\alpha) = 0$, then $KV_{I^+}(\phi) = I^+(\alpha) = 0$. By conditional proof, if $KV_I(\phi) = 0$ then $KV_{I^+}(\phi) = 0$.

Induction Hypothesis for all PL-wff $\phi$ with $C(\phi) = m$

Given n, for all $m < n$, if $KV_I(\phi) = 1$ then $KV_{I^+}(\phi) = 1$ and if $KV_I(\phi) = 0$ then $KV_{I^+}(\phi) = 0$.

Induction step

Consider arbitrary PL-wff ~~such tt~~ $\phi$ such that $C(\phi) = n$. $\phi = \sim\psi$ or $\phi = \psi \to \psi$

Suppose $\phi = \sim\psi$. Suppose for conditional proof that $KV_I(\phi) = 1$, then by definition of Kleene valuation, $KV_I(\psi) = 0$. $C(\psi) = C(\phi) - 1 = n-1 < n$. By IH, $KV_{I^+}(\psi) = 0$. Then by definition of Kleene valuation, $KV_{I^+}(\phi) = 1$. By conditional proof, if $KV_I(\phi) = 1$ then $KV_{I^+}(\phi) = 1$. Suppose for conditional proof that $KV_I(\phi) = 0$. Then $KV_I(\psi) = 1$, then by IH $KV_{I^+}(\psi) = 1$, then $KV_{I^+}(\phi) = 0$. By conditional proof, if $KV_I(\phi) = 0$ then $KV_{I^+}(\phi) = 0$. So, if $KV_I(\phi) = 1$ then $KV_{I^+}(\phi) = 1$ and if $KV_I(\phi) = 0$, then $KV_{I^+}(\phi) = 0$.

Suppose $\phi = \psi \to \chi$. Suppose for conditional proof that $KV_I(\phi) = 1$. Then, by definition of Kleene valuation, $KV_I(\psi) = 0$ or $KV_I(\chi) = 1$. $C(\psi) + C(\chi) + 1 = C(\phi)$, so $C(\psi), C(\chi) < C(\phi) = n$. By IH, $KV_{I^+}(\psi) = 0$ or $KV_{I^+}(\chi) = 1$. Then by definition of Kleene valuation, $KV_{I^+}(\phi) = 1$. By conditional proof, if $KV_I(\phi) = 1$ then $KV_{I^+}(\phi) = 1$. Suppose for conditional proof that $KV_I(\phi) = 0$. Then $KV_I(\psi) = 1$ and $KV_I(\chi) = 0$. Then, by IH, $KV_{I^+}(\psi) = 1$ and $KV_{I^+}(\chi) = 0$. Then, by definition of Kleene valuation, $KV_{I^+}(\phi) = 0$. By conditional proof, if $KV_I(\phi) = 0$ then $KV_{I^+}(\phi) = 0$.

By cases, if $KV_I(\phi) = 1$ then $KV_{I^+}(\phi) = 1$, and if $KV_I(\phi) = 0$, then $KV_{I^+}(\phi) = 0$.

By strong induction over $C(\phi)$, for arbitrary PL-wff $\phi$ ~~such that~~, if $KV_I(\phi) = 1$ then $KV_{I^+}(\phi) = 1$, and if $KV_I(\phi) = 0$ then $KV_{I^+}(\phi) = 0$. By ~~generalisatn~~ generalisation, this holds for all $\phi$.

bi Suppose that $\vDash_{PL} \phi$. Then, by definition of PL-validity, for all bivalent interpretations I, $V_I(\phi) = 1$. Suppose for reductio that for some trivalent interpretation I', $KV_{I'}(\phi) = 0$. Then, by the result in (a), there exists bivalent ~~refin~~ refinement $I^+$ of I' such that $KV_{I^+}(\phi) = 0$. Then, by the given result, $V_{I^+}(\phi) = 0$. By reductio, $KV_{I'}(\phi) \neq 0$. By generalisation, $KV_{I'}(\phi) \in \{1, \#\}$ for all trivalent I'. Then by definition of $\vDash_{D_1}$, $\vDash_{D_1} \phi$.

Suppose that ~~$KV_{I'}(\phi)$~~ $\vDash_{D_1} \phi$. Then by definition, $KV_{I'}(\phi) \neq 0$ for all trivalent I', then ~~by~~ $KV_I(\phi) \neq 0$ for all bivalent I (given that ~~the set~~ every bivalent interpretation is a (degenerate) trivalent interpretation). Then, by the given result, $V_I(\phi) \neq 0$ for all bivalent I, so $V_I(\phi) = 1$ for all bivalent I. Then, by definition of PL-validity, $\vDash_{PL} \phi$.

By biconditional proof, for $D_1 = \{1, \#\}$, $\vDash_{PL} \phi$ iff $\vDash_{D_1} \phi$.

ii $D_1 = \{1, \#\}$, $D_2 = \{1\}$.

That (α) is satisfied follows from the result in (bi).

Suppose $\vDash_{D_1} \sim\phi$. Then for all trivalent I, $KV_I(\phi) =$ 1 or #. Then by $\sim$ clause, for all trivalent I, $KV_I(\sim\phi) =$ 0 or #, then $KV_I(\sim\phi) \notin \{1\}$ so $\phi \vDash_{D_2}$. Suppose $\phi \vDash_{D_2}$, then for all I, $KV_I(\phi)$ = 0 or #, so for all I, $KV_I(\sim\phi)$ ~~for~~ $\in \{1, \#\}$, so $\vDash_{D_1} \sim\phi$. By biconditional proof, (β) holds.

iii ~~If (α) holds, $0 \notin D_1$. Suppose for reductio that (α) holds and $0 \in D_1$. Then $\vDash_{D_1} \not\sim (P \to P)$~~

~~Suppose that (β) and (γ) hold. By $\sim$ clause $D_1 = D_2 = \{\#\}$. Suppose for reductio that $D_1$. Then $\vDash_{D_1} \phi$ iff $KV_I(\sim\phi) =$~~

(α) is satisfied iff $D_1 = \{1, \#\}$. This follows from the argument in (bi). Suppose further that (γ) holds, then $D_2 = \{1, \#\}$. ~~Suppose for reductio that (β) holds. Then $KV_I(\sim\phi) \in \{1, \#\}$ for all trivalent I. Then $KV_I(\phi) = 0$ for~~

consider $\phi = \sim(P \to P)$. $KV_I(\phi) \in \{1, \#\}$ for all trivalent $I$, so $\models_{D_1} \sim\phi$. $KV_I(\phi) = \#$ for trivalent $I$ such that $I(P) = \#$, so $\phi \not\models D_2$. So $(\beta)$ does not hold. If $(\alpha)$ and $(\gamma)$ hold $(\beta)$ does not, so no $D_1, D_2$ satisfy $(\alpha)$, $(\beta)$ and $(\gamma)$.

c It seems to be a virtue of Kleene's semantics that it preserves determinate truth and falsehood in refinements. For example, we think that when vagueness about "Henry is tall" is dispelled and we ~~know~~ either know that Henry is tall or that he is not, things are determinately ~~known~~ true such as "there is some tall person in the world" remains determinately ~~known~~ true and similarly for determinate falsehoods.

That no common set of truth values ~~that~~ is ~~sati~~ ~~coin~~ such that Kleene validity coincides with PC-validity, and some statement ~~is~~ 's negation is logically true iff that statement is false under every interpretation seems troubling.

Plausibly, we should not maintain $(\alpha)$ as a desiderata. Kleene semantics are "richer" than PC, so we ~~..~~ should be satisfied if PC-logical truths are also Kleene-logical truths, but not expect Kleene-logical truths to be PC-logical truths. We would not have the same expectation, for example for PC.

Kleene semantics will struggle with ~~pe~~ penumbral connection.

But we might worry about obscure ~~co~~ counterexamples. The truth of "there are vague propositions" can be inverted by the "removal" of vagueness.

3a The triple $M = \langle W, R, I\rangle$ is a MPL model iff $W$ is some non-empty set, the set of possible worlds, $R$ is some binary relation on $W$, the ~~accessi~~ accessibility relation, and $I$ is some function from sentence letters $\alpha$ and worlds $w$ to truth values $\{0, 1\}$.

No further conditions are necessary for a K model. Every MPL model is a K-model. A MPL model is a D model iff ~~⇒~~, in addition to the above, $R$ is serial on $W$, i.e. for all $w \in W$, there exists $u \in W$ such that $Ruu$. A MPL model is a T-model iff, additionally, $R$ is reflexive (hence serial) on $W$, i.e. for all $w \in W$, $Rww$.

i

$$\begin{array}{ccccccccc} & & \text{T} & & & & & & \\ (\Diamond & P & \to & \Box & Q) & \to & \Box(P & \to & Q) \\ & 1 & \underline{\underline{1}} & 0 & & 0 & 0 & & \end{array}$$

$*$

$\downarrow$

$$\begin{array}{ccc} P & \to & Q \\ 1 & 0 & 0 \end{array}$$

~~Neither~~ ⊭ D-valid and ~~nor~~ T-valid.

Consider arbitrary D-model $M = \langle W, R, I\rangle$. Consider arbitrary world $w \in W$. Suppose for reductio

(1) $V_M((\Diamond P \to \Box Q) \to \Box(P \to Q), w) = 0$

(1), $\to \Rightarrow$

(2) $V_M(\Diamond P \to \Box Q, w) = 1$

(3) $V_M(\Box(P \to Q), w) = 0$

(3), $\Box \Rightarrow$

(4) $\exists u \in W$: $Rwu$ & $V_M(P \to Q, u) = 0$

(4), $\to \Rightarrow$

(5) $\exists u \in W$: $Rwu$ & $V_M(P, u) = 1$ & $V_M(Q, u) = 0$

~~(6) $\exists u \in W$: $Rwu$ & $V_M(Q$~~

(5), $\Diamond \Rightarrow$

(6) $V_M(\Diamond P, w) = 1$

(5), $\Box \Rightarrow$

(7) $V_M(\Box Q, w) = 0$

(6), (7), $\to \Rightarrow$

(8) $V_M(\Diamond P \to \Box Q, w) = 0$

(2), (8), reductio $\Rightarrow$

(9) $V_M((\Diamond P \to \Box Q) \to \Box(P \to Q), w) = 1$

(9), generalization, definition of D-validity

(10) $\vDash_D (\Diamond P \to \Box Q) \to \Box(P \to Q)$

From (a), (by definition of D, T models), every T-model is a D-model, so D-validity, truth in every D-model, implies T-validity, truth in every T-model, so $\vDash_T (\Diamond P \to \Box Q) \to \Box(P \to Q)$.

ii ~~$\Box\Box P \to \Diamond\Diamond\Diamond P$~~

~~Not D-v T-valid; hence~~ not D-valid.

Consider the following $\nvDash_D$-countermodel.

$M = \langle W, R, I\rangle$,

$W = \{w_0, w_1, w_2, w_3\}$

$R = \{\langle w_0, w_1\rangle, \langle w_1, w_2\rangle, \langle w_2, w_3\rangle, \langle w_3, w_3\rangle\}$

$I(P, w_2) = 1$, $I(\alpha, w) = 0$ for all other sentence letter-world pairs $(\alpha, w)$.

T-valid.

Consider arbitrary T-model $M = \langle W, R, I\rangle$ and arbitrary world $w \in W$. Suppose for reductio

(1) $V_M(\Box\Box P \to \Diamond\Diamond\Diamond P, w) = 0$

(1), $\to \Rightarrow$

(2) $V_M(\Box\Box P, w) = 1$

(3) $V_M(\Diamond\Diamond\Diamond P, w) = 0$

(2) $\Box \Rightarrow$

(4) $\forall u \in W$: if $Rwu$ then $V_M(\Box P, u) = 1$

(4), $\Box \Rightarrow$

(5) $\forall u \in W$: if $Rwu$ then $\forall v \in W$: if $Ruv$ $V_M(P, v) = 1$

(3), $\Diamond \Rightarrow$

(6) $\forall u \in W$: if $Rwu$ then $V_M(\Diamond\Diamond P, u) = 0$

(6), $\Diamond \Rightarrow$

(7) $\forall u \in W$: if $Rwu$ then $\forall v \in W$: if $Ruv$ then $V_M(\Diamond P, v) = 0$

(7), $\Diamond \Rightarrow$

(8) $\forall u \in W$: if $Rwu$ then $\forall v \in W$: if $Ruv$ then $\forall u' \in W$: ~~if $V_M(P, u') = 0$~~ if $Rvu'$ then $V_M(P, u') = 0$

(8), reflexivity of $R$ on $W \Rightarrow$

(9) $\forall u \in W$: if $Rwu$ then $\forall v \in W$: if $Ruv$ then $V_M(P, v) = 0$

(5), (9), reductio $\Rightarrow$

(10) $V_M(\Box\Box P \to \Diamond\Diamond\Diamond P, w) = 1$

(10), generalisation, definition of T-validity,

(11) $\vDash_T \Box\Box P \to \Diamond\Diamond\Diamond P$

bi (1) $(P \to \sim Q) \to (Q \to \sim P)$

(2) $\Box(P \to \sim Q) \to \Box(Q \to \sim P)$ (1), Becker

(3) $\Box(Q \to \sim P) \to (\Box Q \to \Box\sim P)$ K

~~(4) $\Box(P \to \sim Q) \to (\Box Q$~~

(4) $(\Box Q \to \Box\sim P) \to (\sim\Box\sim P \to \sim\Box Q)$ PC, contraposition

(5) $\Box(P \to \sim Q) \to (\sim\Box\sim P \to \sim\Box Q)$ (2), (3), (4), PC

(6) $\sim(\sim\Box\sim P \to \sim\Box Q) \to \sim\Box(P \to \sim Q)$ (5), PC

$(\overset{7}{\cancel{8}})\ \sim(\sim\Box\sim P \to \sim\Box Q) \to \sim\Box\sim\sim(P \to \sim Q)$ (6), PC

$= \sim(\Diamond P \to \sim\Box Q) \to \Diamond\sim(P \to \sim Q)$

$= (\Diamond P \wedge \Box Q) \to \Diamond(P \wedge Q)$

ii (1) $\Box\sim P \to \sim P$ T

(2) $\cancel{\sim\Box\sim P}\ P \to \sim\Box\sim P$ (1), PC

(3) $(P \to \sim\Box Q) \to (\Diamond P \to \sim\Box Q)$ $(\overset{2}{\cancel{3}})$, PC

(4) $\Box(P \to \sim\Box Q) \to \Box(\Diamond P \to \sim\Box Q)$ (3), Becker

(5) $\Box(\Diamond P \to \sim\Box Q) \to (\Diamond P \to \sim\Box Q)$ T

(6) $\Box(P \to \sim\Box Q) \to (\Diamond P \to \sim\Box Q)$ (4),(5), PC

(7) $\sim(\Diamond P \to \sim\Box Q) \to \sim\Box\sim\sim(P \to \sim\Box Q)$ (6), PC

$= (\Diamond P \wedge \Box Q) \to \Diamond(P \wedge \Box Q)$

iii ~~(1) (P → □P) → (∼□P → ∼P) PL~~

~~(2) (∼□P → ∼P) → (□∼□P → □∼P) K~~

$\vdash_K (\Diamond P \wedge \Box Q) \to \Diamond(P \wedge Q)$

(1) $Q \to P \to (P \wedge Q)$

Becker

K$\Diamond$

PC syl

PC imp

$\vdash_{S4} (\Diamond P \wedge \Box Q) \to \Diamond(P \wedge \Box Q)$

S4 $\Box Q \to \Box\Box Q$

S4$\Diamond$ $\Diamond\Diamond Q \to \Diamond Q$

$\Box Q \to P \to (P \wedge \Box Q)$

$\Box Q \to \Box\Box Q$

$\Box\Box Q \to \Diamond P \to \Diamond(P \wedge \Box Q)$

$\cancel{\phi}$

T $\Box\phi \to \phi$

$\cancel{B}$ T$\Diamond$ $\phi \to \Diamond\phi$

$\cancel{B}$

D $\Box\phi \to \Diamond\phi$

$\Box Q \to \Diamond P \to \Diamond(P \wedge \Box Q)$

$(\Diamond P \wedge \Box Q) \to \Diamond(P \wedge \Box Q)$

$\vdash_B \Box\Box(P \to \Box P) \to \Box(\Diamond P \to P)$

B $\Diamond\Box\phi \to \phi$

B$\Diamond$ $\phi \to \Box\Diamond\phi$

~~□(P → □P) → (◇P → P)~~

$\Box(P \to \Box P) \to (\Diamond P \to \Diamond\Box P)$ ~~K◇~~ Becker$\Diamond$

$\Diamond\Box P \to P$ B

$\Box(P \to \Box P) \to (\Diamond P \to P)$ PC syl

$\Box\Box(P \to \Box P) \to \Box(\Diamond P \to \Diamond\Box P)$

4a) i A PC= model $M$ is some ordered pair $\langle D, I\rangle$, where $D$, the domain, is some non-empty set, and $I$, the interpretation function, is some function that assigns to each constant (the constants are $a, a_1, a_2, \dots, b, b_1, b_2, \dots$) some element of $D$ and to each $n$-place predicate (the $n$-place predicates are $F, F_1, F_2, \dots, G, G_1, G_2, \dots$) some $n$-place relation over $D$.

ii A variable assignment $g$, given some PC= model $M = \langle D, I\rangle$ is some function that assigns to each variable (the variables are $x, x_1, x_2, \dots, y, y_1, y_2, \dots$) some element of $D$.

iii The PC= valuation function $V_{M,g}$, given some PC= model $M = \langle D, I\rangle$ and some variable ~~assi~~ assignment $g$ for $M$, is the unique function from PC= wffs to truth values (~~the~~ 1, 0) such that

(1) $V_{M,g}(\pi\alpha_1 \dots \alpha_n) = 1$ iff $\langle [\alpha_1]_{M,g}, \dots, [\alpha_n]_{M,g}\rangle \in [\pi]_{M,g}$, where $\pi$ is a $n$-place predicate, $[\pi]_{M,g} = I(\pi)$, each of $\alpha_1, \dots, \alpha_n$ is a term (i.e. either a ~~pt~~ constant or a variable), and $[\alpha]_{M,g} = I(\alpha)$ if $\alpha$ is a constant, $g(\alpha)$ if $\alpha$ is a variable.

(2) $V_{M,g}(\sim\phi) = 1$ iff $V_{M,g}(\phi) = 0$

(3) $V_{M,g}(\phi \to \psi) = 1$ iff $V_{M,g}(\phi) = 0$ or $V_{M,g}(\psi) = 1$

(4) $V_{M,g}(\forall\alpha\phi) = 1$ iff for all $u \in D$, $V_{M,g^{\alpha}_{u}}(\phi) = 1$, where $\alpha$ is some variable, $\phi$ is some PC= wff, and $g^{\alpha}_{u}$ is the variant assignment that differs from $g$ only in assigning $u$ to $\alpha$.

iv ~~The definition of~~

(5) $V_{M,g}(\alpha = \beta) = 1$ iff $[\alpha]_{M,g} = [\beta]_{M,g}$, where each of $\alpha, \beta$ is a term, and term denotations $[\alpha]_{M,g}$ and $[\beta]_{M,g}$ are defined as above.

iv The definition of a SOL= model is identical to that of a PC= model.

~~A SOL= variable assignment differs fr~~

The definition of a SOL= variable assignment differs from that of a PC= variable assignment only in including the additional clause for predicate variables ($X, X_1, X_2, \dots, Y, Y_1, Y_2, \dots$). A SOL= variable assignment assigns to each $n$-place predicate variable some $n$-place relation over $D$.

The definition of a SOL= valuation function given some $M, g$ differs from that of a PC= valuation function given some $M, g$ only in modifying (1), ~~(3), and (5)~~, and adding (4'), (5').

The modified clauses are as follows.

(1') $V_{M,g}(\pi\alpha_1 \dots \alpha_n) = 1$ iff $\langle [\alpha_1]_{M,g}, \dots, [\alpha_n]_{M,g}\rangle \in [\pi]_{M,g}$ where $\pi$ is some $n$-place predicate or predicate variable, $[\pi]_{M,g} = I(\pi)$ if $\pi$ is a predicate and $g(\pi)$ if $\pi$ is a predicate variable, and ~~$\alpha_1, \dots, \alpha_n$~~ each of $\alpha_1, \dots, \alpha_n$ is a ~~pt~~ term, with $[\alpha]_{M,g}$ defined as before.

(4') $V_{M,g}(\forall\pi\phi) = 1$ iff $V_{M,g^{\pi}_{U}}(\phi) = 1$ for all $n$-place relations $U$ over $D$, where $\pi$ is some $n$-place predicate variable and $g^{\pi}_{U}$ is the variant assignment that differs from $g$ only in assigning $U$ to $\pi$.

(5') $V_{M,g}(\pi = \rho) = 1$ iff $[\pi]_{M,g} = [\rho]_{M,g}$

bi ~~False~~ Not valid.

~~Cons~~ Consider the following counter-model.

$M = \langle D, I\rangle$

$D = \{0\}$

$I(c) = 0$ for all constants $c$

~~g~~ $g(z) = 0$ for all variables $z$

ii Valid.

Consider arbitrary SOL= model $M = \langle D, I\rangle$ and arbitrary variable assignment for $M$ ~~for M~~, $g$. Suppose for reductio that

(1) $V_{M,g}(\forall x\forall y(x = y \leftrightarrow \forall X(Xx \to Xy))) = 0$

(1), $\forall \Rightarrow$

(2) ~~$V_{M,g}$~~ $\exists u, v \in D: V_{M,g^{x\,y}_{u\,v}}(x = y \leftrightarrow \forall X(Xx \to Xy)) = 0$

(2), $\leftrightarrow \Rightarrow$

(3): (4) or (5)

~~(4): $\exists u, v \in D: V_{M,g^{x\,y}_{u\,v}}(x = y$~~

~~(4): (4a) and (4b)~~

~~(4a)~~

(4): $\exists u, v \in D: V_{M,g^{x\,y}_{u\,v}}(x = y) = 1$ ~~& $V_{M,g^{x\,y}_{u\,v}}(\forall X(x$~~
& $V_{M,g^{x\,y}_{u\,v}}(\forall X(Xx \to Xy)) = 0$

(5): $\exists u, v \in D: V_{M,g^{x\,y}_{u\,v}}(x = y) = 0$
& $V_{M,g^{x\,y}_{u\,v}}(\forall X(Xx \to Xy)) = 1$

Suppose (4)

(4), ~~$\forall \Rightarrow$~~ $=$, $\forall \Rightarrow$

(6) ~~$\exists u, v \in D: V_{M,g^{x\,y}_{u\,v}}(x = y) = 1$~~
~~$\exists u = v \in D: V_{M,g^{x\,y}_{u\,v}}(\forall X$~~
$\exists u = v \in D, U \in D^{1}: V_{M,g^{x\,y\,X}_{u\,v\,U}}(Xx \to Xy) = 0$

(6), $\to$, ~~pt~~ basic wff $\Rightarrow$

(7) $\exists u = v \in D, U \in D^{1}: u \in U$ & $v \notin U$

(7), reductio $\Rightarrow$

(8) $V_{M,g}(\forall x\forall y(x = y \leftrightarrow \forall X(Xx \to Xy))) = 1$

Suppose (5)

(5), $=$, $\forall \Rightarrow$

(9) ~~$\exists u$~~ ~~$\exists u \neq v \in D$~~ $\exists u \neq v \in D: \forall U \in D^{1}: V_{M,g^{x\,y\,X}_{u\,v\,U}}(Xx \to Xy) = 1$

(9), $\to$, basic wff

(10) ~~$\forall u \in D'$~~:

$\exists u \neq v \in D: \forall U \in D^1$: if $u \in D^1$ then $v \in D^1$

~~(10) $\Rightarrow$~~

~~(11)~~ (10), reductio, $\Rightarrow$

(11) $V_{M,g}(\forall x \forall y (x=y \leftrightarrow \forall X (Xx \to Xy))) = 1$

(8), (11), cases $\Rightarrow$

(12) $V_{M,g}(\forall x \forall y (x=y \leftrightarrow \forall X (Xx \to Xy))) = 1$

(12), generalization, definition of SOL= validity,

(13) $\vDash_{SOL=} \forall x \forall y (x=y \leftrightarrow \forall X (Xx \to Xy)) = 1$

Is this adequate? How can the contradiction be made clear?

Can use words here

iii ~~Not~~ valid.

How would you phrase this in English to make it intuitive?

Consider arbitrary SOL= model $M = \langle D, I \rangle$ and arbitrary variable assignment g for M, g.

Suppose for reductio that

(1) $V_{M,g}(\forall R \sim \forall X \exists x \forall y (Rxy \leftrightarrow Xy)) = 1$

(1), $\forall$ $\Rightarrow$

(2) ~~$\forall M, g$~~ $\forall U \in D^2: V_{M,g^{R}_{U}}(\sim \forall X \exists x \forall y (Rxy \leftrightarrow Xy)) = 1$

(2), $\sim$ $\Rightarrow$

(3) $\forall U \in D^2: V_{M,g^{R}_{U}}(\forall X \exists x \forall y (Rxy \leftrightarrow Xy)) = 0$

(3), $\forall$ $\Rightarrow$

(4) $\forall U \in D^2: \forall V \in D^1: V_{M,g^{R\,X}_{U\,V}}(\exists x \forall y (Rxy \leftrightarrow Xy)) = 0$

(4), $\exists$, $\forall$ $\Rightarrow$

(5) ~~$\forall U \in D^2: \forall V \in D^1: \exists u \in D: \forall v \in D: V_{M,g^{R\,X\,x\,y}_{U\,V\,u\,v}} (Rxy \leftrightarrow Xy) = 0$~~

$\forall U \in D^2: \forall V \in D^1: \forall u \in D: V_{M,g^{R\,X\,x}_{U\,V\,u}}(\forall y (Rxy \leftrightarrow Xy)) = 0$

(5), $\forall$ $\Rightarrow$

(6) $\forall U \in D^2$, $\forall V \in D^1$, $\forall u \in D$, ~~$\forall v \in D$~~ $\exists v \in D$:

~~$\forall M, g$~~ $V_{M,g^{R\,X\,x\,y}_{U\,V\,u\,v}}(Rxy \leftrightarrow Xy) = 0$

What's the proof?

Credit for shifting negation, testing odd cases like $\emptyset$, universal relation, reflexive, etc.