

### Game Theory Problem Set 3

|          |                                  |          |
|----------|----------------------------------|----------|
| 10       | L                                | R        |
| T        | <u><math>3+\epsilon_1</math></u> | 0        |
| <u>L</u> | 0                                |          |
| B        | 0                                | <u>1</u> |
| O        | <u><math>3+\epsilon_1</math></u> |          |

Best responses underlined.

By inspection,  $(T, L)$  and  $(B, R)$  are the only pure strategy Nash equilibria where players play mutual best responses.

Suppose that there is some NE where P1.1 plays a mixed strategy, and plays action T with probability  $p$  and action B with probability  $1-p$ . Then,

$\pi_1(T, \sigma_2^*) = \pi_1(B, \sigma_2^*)$ ,  $q = (3+\epsilon_1)(-q)$ , where  $q$  is the probability that strategy of player 2  $\sigma_2^*$  at O\* assigns to L such that P1.1 has no profitable deviation

$q = \frac{1}{4+\epsilon_1}$ . Then P1.2 plays a mixed strategy at O\*,

so  $\pi_2(L, \sigma_1^*) = \pi_2(R, \sigma_1^*)$ ,  $(3+\epsilon_2)p = 1-p$ ,  $p = \frac{1}{4+\epsilon_2}$ .

so  $O^* = (\frac{1}{4+\epsilon_1}p \times T + (1-p) \times B, q \times L + (1-q) \times R)$  is a

mixed strategy NE where  $p = \frac{1}{4+\epsilon_2}$  and  $q = \frac{1}{4+\epsilon_1}$ .

From the above argument if P1.1 mixes, so does P1.2.

By symmetry, if P1.2 mixes, so does P1.1. Then, there are no hybrid NE.

Let  $s^*$  denote the pure BNE where P1.1 plays T iff  $\epsilon_1 < \epsilon_1^*$  and B otherwise and P1.2 plays L iff  $\epsilon_2 > \epsilon_2^*$  and R otherwise.

$$\pi_1(T, s_2^*) = P(\epsilon_2 > \epsilon_2^* | 1) + P(\epsilon_2 < \epsilon_2^* | 0)$$

$$= \frac{\epsilon_2^*}{\epsilon_2} - \frac{\epsilon_2^*}{\epsilon_2} = 0$$

$$\pi_2(s_1^*, B) = P(\epsilon_1 > \epsilon_1^* | 0) + P(\epsilon_1 < \epsilon_1^* | 3+\epsilon_1)$$

$$= \frac{\epsilon_1^*}{\epsilon_1} (3+\epsilon_1)$$

$$\pi_1(T, s_2^*) \geq \pi_1(B, s_2^*) \text{ iff}$$

$$1 - \frac{\epsilon_2^*}{\epsilon_2} = \frac{\epsilon_1^*}{\epsilon_1} (3+\epsilon_1) (3+\epsilon_1)$$

$$1 \geq \frac{\epsilon_2^*}{\epsilon_2} (4+\epsilon_1)$$

$$4+\epsilon_1 \leq \frac{\epsilon_2^*}{\epsilon_2} / \frac{\epsilon_1^*}{\epsilon_1}$$

$$\epsilon_1 \leq \frac{\epsilon_2^*}{\epsilon_2} / \frac{\epsilon_1^*}{\epsilon_1} - 4$$

Since  $s^*$  is a BNE, P1.1 has no profitable deviation

from  $s_1^*$ , so under  $s_1^*$  P1.1 plays T iff  $\pi_1(T, s_2^*) \geq \frac{1}{2}$

$\pi_1(B, s_2^*) \text{ iff } \epsilon_1 \leq \frac{\epsilon_2^*}{\epsilon_2} / \frac{\epsilon_1^*}{\epsilon_1} - 4 \text{ iff } \epsilon_1 < \epsilon_1^* / \epsilon_2^* - 4$ , so  $\epsilon_1^* \geq \frac{\epsilon_2^*}{\epsilon_2} / \frac{\epsilon_1^*}{\epsilon_1} - 4$

$$\geq \frac{\epsilon_2^*}{\epsilon_2} / \frac{\epsilon_1^*}{\epsilon_1} - 4$$

$$\pi_2(L, s_1^*) = P(\epsilon_1 < \epsilon_1^* | 3+\epsilon_1) + P(\epsilon_1 > \epsilon_1^* | 0)$$

$$= \frac{\epsilon_1^*}{\epsilon_1} (3+\epsilon_1)$$

$$\pi_2(R, s_1^*) = P(\epsilon_1 < \epsilon_1^* | 0) + P(\epsilon_1 > \epsilon_1^* | 1)$$

$$= 1 - \frac{\epsilon_1^*}{\epsilon_1}$$

$$\pi_2(L, s_1^*) \geq \pi_2(R, s_1^*) \text{ iff}$$

$$\frac{\epsilon_1^*}{\epsilon_1} (3+\epsilon_1) \geq 1 - \frac{\epsilon_1^*}{\epsilon_1}$$

$$\frac{\epsilon_1^*}{\epsilon_1} (3+\epsilon_1) \geq 1$$

$$3+\epsilon_1 \geq \frac{\epsilon_1^*}{\epsilon_1}$$

$$\epsilon_1 \geq \frac{\epsilon_1^*}{\epsilon_1} - 3$$

$$\epsilon_1^* = \frac{\epsilon_1^*}{\epsilon_1} - 3$$

By substitution,

$$\frac{\epsilon_1^*}{\epsilon_1} = \frac{\epsilon_1^*}{\epsilon_1} - \left( \frac{\epsilon_1^*}{\epsilon_1} - 3 \right) - 3$$

$$= \frac{\epsilon_1^*}{\epsilon_1} - \frac{\epsilon_1^*}{\epsilon_1} + \frac{3\epsilon_1^*}{\epsilon_1} - 3\epsilon_1^*$$

Supposing that the BNE is symmetric, i.e.  $\epsilon_1^* = \epsilon_2^*$ ,

$$\begin{aligned}\varepsilon_i^* &= \bar{\varepsilon}/\varepsilon^* - 4, \\ (\varepsilon_i^* + 4)\varepsilon_i^* &= \bar{\varepsilon} \\ (\varepsilon_i^* + 2)^2 &= \bar{\varepsilon} + 4 \\ \varepsilon_i^* = -2 + \sqrt{\bar{\varepsilon} + 4} &\text{ or } -2 - \sqrt{\bar{\varepsilon} + 4} \quad (\text{reject since } \varepsilon_i^* \in [0, \bar{\varepsilon}])\end{aligned}$$

Then  $\varepsilon_2^* = \varepsilon_1^* = -2 + \sqrt{\bar{\varepsilon} + 4}$ .  
 $\varepsilon_2^* = \bar{\varepsilon}/\varepsilon^* - 4$ , so P1.2 has no profitable deviation, and  
 $\star$  is a BNE.

c Let  $p$  and  $q$  denote the ex ante probability that P1.1 plays T and the ex ante probability that P1.2

plays L respectively.

$$\begin{aligned}p &= P(\varepsilon_1 < \varepsilon_1^*) = \frac{\varepsilon_1^*}{\bar{\varepsilon}} = \frac{-2 + \sqrt{\bar{\varepsilon} + 4}}{\bar{\varepsilon}} \\ &= \frac{(-2 + \sqrt{\bar{\varepsilon} + 4})(+2 + \sqrt{\bar{\varepsilon} + 4})}{\bar{\varepsilon}(\sqrt{\bar{\varepsilon} + 4} + 2)} \\ &= \cancel{\frac{\bar{\varepsilon}}{\bar{\varepsilon}}} \frac{1/\sqrt{\bar{\varepsilon} + 4} + 2}{1 - 1/\sqrt{\bar{\varepsilon} + 4}} \\ q &= P(\varepsilon_2 > \varepsilon_2^*) = 1 - \frac{1/\sqrt{\bar{\varepsilon} + 4}}{1 - 1/\sqrt{\bar{\varepsilon} + 4}} + 2\end{aligned}$$

$$\lim_{\bar{\varepsilon} \rightarrow 0} p = 1/4$$

$$\lim_{\bar{\varepsilon} \rightarrow 0} q = 3/4$$

The ex ante probability distributions induced by the pure NE of the perturbed game converge to the probability distribution of the mixed NE in the unperturbed game (where  $\varepsilon_1 = \varepsilon_2 = 0$ ), where from the result in (a),  $p = 1/4 + \varepsilon_2 = 1/4$ ,  $q = 3 + \varepsilon_1/4 + \varepsilon_1 = 3/4$ .

d Mixed NE are difficult to justify because at the mixed NE, each player has equal  $\varepsilon$  for each player, each of the actions he mixes yields equal expected payoff, so each player has no incentive to mix over these actions in any particular way.

Harsanyi's purification theorem is that the probability distributions induced by the pure BNE of the perturbed game in which players use threshold strategies converges to, as the perturbation vanishes, to the probability distribution of the mixed NE in the unperturbed strategic form game.

Harsanyi's purification theorem can be interpreted as justifying mixed NE in the sense that the players who mix can be understood as responding to an unobservable, vanishingly small payoff shock, i.e. a whim.

2-player:  $N = \{S_t, G\}$

Actions:  $A_{S_t} = \{L, R\}$ ,  $A_G = \{l, r\}$

States:  $S_t = \{S_tL, S_tR\}$

Signals:  $t_{S_t}(S_tL) = S_tL$ ,  $t_{S_t}(S_tR) = S_tR$ ,  $t_G(S_tL) = t_G(S_tR)$

Beliefs:  $P_{S_t}(t_{S_t} = S_tL | t_G = l) = p_L$

$$P_G(t_G = S_tL | t_G = l) = P_G(t_{S_t} = S_tL | t_G = l) = \frac{1}{2}$$

Payoffs:

|  |  | <u>l</u> | <u>r</u> |      |     | <u>l</u> | <u>r</u> |     |     |
|--|--|----------|----------|------|-----|----------|----------|-----|-----|
|  |  | L        | 0.3      | 0    | C   | 0.9      | 0.2      | R   | 0.2 |
|  |  | 0.7      | <u>L</u> | 0.6  | 0.8 |          |          | 0   | 0.3 |
|  |  | R        | 0.2      | 0.4  | R   | 0        | 0.3      | 0.8 | 0.6 |
|  |  | 0.8      | 0.6      | 1    | 0.7 |          |          | 0.2 | 0.3 |
|  |  | S_tL     |          | S_tR |     |          |          |     |     |

Best responses underlined

In a Bayesian strategic game, player  $i$ 's strategy is a type-contingent plan of action. Since  $G$  has only one type, its strategy  $\sigma_G^*$  is some action  ~~$\sigma_G \in A_G$~~  or  $\sigma_G \in A_G = \{l, r\}$  or some probability distribution over  $A_G$ .  $S_t$ 's strategy  $\sigma_{S_t}$  is some pair  $(\sigma_{S_tL}, \sigma_{S_tR})$  where  $S_t$  plays  $\sigma_{S_tL}$  if  $t_{S_t} = S_tL$  and  $\sigma_{S_tR}$  if  $t_{S_t} = S_tR$ , and each of  $\sigma_{S_tL}$  and  $\sigma_{S_tR}$  is some action  $a_{S_t} \in A_{S_t} = \{L, R\}$  or some probability distribution over  $A_{S_t}$ .

By inspection of the payoff tables above,  $S_t$ 's best response  $\sigma_{S_t}(\sigma_G) = (R, R)$  if  $\sigma_G = l$

$$B_{S_t}(\sigma_G) = (R, R) \text{ if } \sigma_G = l$$

$$(L, L) \text{ if } \sigma_G = r$$

$G$ 's best response

$$B_G(\sigma_{S_t}) = \overbrace{\text{if } \sigma_{S_t} = (R, R)}^{\rightarrow l \text{ if } \sigma_{S_t} = (L, L)}$$

$$\neq l \text{ if } \sigma_{S_t} = (L, L)$$

$$r \text{ if } \sigma_{S_t} = (R, R)$$

$$\{l, r\} \text{ if } \sigma_{S_t} = (L, L) \text{ or } (R, R)$$

The payoff table can be rewritten as

|  |  | <u>l</u> | <u>r</u> |      |  |
|--|--|----------|----------|------|--|
|  |  | L        | 0.35     | 0.1  |  |
|  |  | 0.65     | 0.9      |      |  |
|  |  | R        | 0.15     | 0.15 |  |
|  |  | 0.85     | 0.85     |      |  |
|  |  | RR       | 0.1      | 0.35 |  |
|  |  | 0.9      | 0.65     |      |  |
|  |  | RL       | 0.3      | 0.3  |  |
|  |  | 0.7      | 0.7      |      |  |

By inspection, there are no mutual best responses in pure strategies, so there is no pure BNE.

b Let  $\sigma^*$  be a hybrid BNE where  $G$  mixes  $l$  and  $r$ , then  $\sigma_G^* = q, 1-q$  let  $p, q$  be the probability  $\sigma_G^*$  assigns to  $l$  and  $1-q$  be the probability it assigns to  $r$ .

$$I_{S_t}(L, \sigma_G^*; t_{S_t} = S_tL) = 0.7q + (1-q) = 1 - 0.3q$$

$$I_{S_t}(R, \sigma_G^*; t_{S_t} = S_tL) = 0.8q + (1-q) = 0.6 + 0.2q$$

$$I_{S_t}(L, \sigma_G^*; t_{S_t} = S_tR) = 0.6q + 0.8(1-q) = 0.2 - 0.2q$$

$$I_{S_t}(R, \sigma_G^*; t_{S_t} = S_tR) = q + 0.7(1-q) = 0.7 + 0.3q$$

$\sigma^*$  is a BNE only if  $G$  has no profitable deviation

$$\text{which is iff } E[I_{S_t}(L, \sigma_G^*; t_{S_t})] = E[I_{S_t}(R, \sigma_G^*; t_{S_t})]$$

Suppose St plays  $\sigma_{St}^* = (L, L)$

$$E_G(\pi_G(l, \sigma_{St}^*; t_{St})) = 0.35 <$$

$$E_G(\pi_G(r, \sigma_{St}^*; t_{St})) = 0.1$$

so G can profitably deviate by reallocating probability mass from r to l

By symmetry, if St plays  $\sigma_{St}^* = (R, R)$ , G can profitably deviate from  $\sigma^*$  by reallocating probability mass from l to r.

Suppose St plays  $\sigma_{St}^* = (L, R)$

$$\begin{aligned} E_G(\pi_G(l, \sigma_{St}^*; t_{St})) &= \frac{1}{2}[0.2q + q(1-q)] + \frac{1}{2}[0q + 0.3(1-q)] \\ &= 0.15q + 0.15 - 0.15q \\ &= 0.15 \end{aligned}$$

$$E_G(\pi_G(r, \sigma_{St}^*; t_{St})) = \frac{1}{2}[1]$$

$$= \frac{1}{2}[0.3] + \frac{1}{2}(0.3) + \frac{1}{2}(0) = 0.15$$

$$E_G(\pi_G(r, \sigma_{St}^*; t_{St})) = \frac{1}{2}(0) + \frac{1}{2}(0.3) = 0.15$$

Then G has no incentive to deviate for any q.

$$T_{St}(\sigma_{St}^*, \sigma_G^*; t_{St} = SRL) =$$

$$T_{St}(\sigma_{St}^*, \sigma_G^*; t_{St} = SRL) = 0.7q + (1-q) = 1 - 0.3q >$$

$$T_{St}(\sigma_{St}', \sigma_G^*; t_{St} = SRL) = 0.8q + 0.6(1-q) = 0.6 + 0.2q$$

where  $\sigma_{St}'$  is a pure strategy of St that yields a different action given  $t_{St}$ , and

$$T_{St}(\sigma_{St}^*, \sigma_G^*; t_{St} = STR) = q + 0.7(1-q) = 0.7 + 0.3q >$$

$$T_{St}(\sigma_{St}', \sigma_G^*; t_{St} = STR) = 0.6q + 0.8(1-q) = 0.8 - 0.2q$$

$$1 - 0.3q > 0.6 + 0.2q, \quad 0.4 > 0.5q, \quad q < 0.8$$

$$0.7 + 0.3q > 0.8 - 0.2q, \quad 0.5q > 0.1, \quad q > 0.2$$

so  $\sigma^* = (LR, q \cdot l + (1-q) \cdot r)$  is a hybrid BNE

for all  $q \in (0.2, 0.8)$

Suppose that St plays  $\sigma_{St}^* = (R, L)$

$$E_G(\pi_G(l, \sigma_{St}^*; t_{St})) = \frac{1}{2}(0.2) + \frac{1}{2}(0.4) = 0.3$$

$$E_G(\pi_G(r, \sigma_{St}^*; t_{St})) = \frac{1}{2}(0.4) + \frac{1}{2}(0.2) = 0.3$$

Then G has no incentive to deviate for any q.

St has no incentive to deviate if

$$T_{St}(\sigma_{St}^*, \sigma_G^*; t_{St} = SRL) > T_{St}(\sigma_{St}', \sigma_G^*; t_{St} = SRL)$$

$$0.6 + 0.2q > 1 - 0.3q$$

$$0.5q > 0.4$$

$$q > 0.8$$

$$T_{St}(\sigma_{St}^*, \sigma_G^*; t_{St} = STR) > T_{St}(\sigma_{St}', \sigma_G^*; t_{St} = STR)$$

$$0.8 - 0.2q > 0.7 + 0.3q$$

$$0.1 > 0.5q$$

$$q < 0.2$$

so St always has incentive to deviate, therefore there are no hybrid BNE where St plays pure strategy RL and G mixes.

- c Suppose that some type L St mixes at equilibrium. Then, by definition of NE, this St has no profitable deviation. Then, the payoff to this St from ~~other~~  
L and the payoff from R are equal. It is possible only if G mixes l and r and assigns high probability to l. If G so mixes, then an St a type R St has ~~higher~~ higher payoff from R than from L, and so always chooses R. Then G ~~as~~ very ~~badly~~ off enjoys low payoff against St and fails to maximize his expected payoff, so it is not an equilibrium.

3. ~~Exercise~~

All efficiency profiles  $\mathbf{s}^* = (x^*, y^*)$  where  $x^* + y^* = 1$  are pure NE.  $(1, 1)$  is also a pure NE.

$$\text{u}_1(x, y) = \begin{cases} x^\alpha & \text{if } x+y \leq z \\ 0 & \text{otherwise} \end{cases}$$

$$\text{u}_2(y, x) = \begin{cases} y^\alpha & \text{if } x+y \leq z \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbb{E}_1[\text{u}_1(x, y)] = \begin{cases} 0 & \text{if } x+y \geq z > 1 \\ (P(x+y \leq z))x^\alpha & \text{otherwise} \end{cases}$$

$$= \begin{cases} 0 & \text{if } x+y > 1 \\ [1 - (x+y)^\alpha]x^\alpha & \text{otherwise} \end{cases}$$

~~By symmetry~~

$$\mathbb{E}_2[\text{u}_2(y, x)] = \begin{cases} 0 & \text{if } x+y > z \\ [1 - (x+y)^\alpha]y^\alpha & \text{otherwise} \end{cases}$$

~~Suppose~~

\* ~~Supposing that~~

~~Suppose given that  $z \in [0, 1]$ ,  $F(z) = (x+y) F(z=1) = 1$ , so~~

~~$F(z)$  the expected utility function simplifies to~~

$$\mathbb{E}_1[\text{u}_1(x, y)] = [1 - (x+y)^\alpha]x^\alpha$$

$$\text{By symmetry, } \mathbb{E}_2[\text{u}_2(y, x)] = [1 - (x+y)^\alpha]y^\alpha$$

c FOCs:

$$\frac{\partial}{\partial x} \mathbb{E}_1[\text{u}_1(x, y)] = \frac{\partial}{\partial x} [1 - (x+y)^\alpha]x^\alpha = 0$$

$$\frac{\partial}{\partial y} \mathbb{E}_1[\text{u}_1(x, y)] = \frac{\partial}{\partial y} [1 - (x+y)^\alpha]y^\alpha = 0$$

$$\cancel{[1 - (x+y)^\alpha] \alpha x^{\alpha-1} + -\alpha(x+y)^{\alpha-1} x^\alpha = 0}$$

$$\alpha [1 - (x+y)^\alpha] = \alpha x(x+y)^{\alpha-1}$$

$$\alpha [1 - (x+y)^\alpha] = \alpha y(x+y)^{\alpha-1}$$

$$2\alpha [1 - (x+y)^\alpha] = \alpha(x+y)^\alpha$$

$$2\alpha = (\alpha+1)(x+y)^\alpha$$

$$x+y = (\alpha+1)^{-1}$$

~~cancel~~

$$x=y = \alpha [1 - (x+y)^\alpha] / \alpha(x+y)^{\alpha-1}$$

$$x=y = 1/2 (2\alpha/\alpha+1)^{1/\alpha}$$

~~At optimal demands are  $x^* = y^* = 1/2 (2\alpha/\alpha+1)^{1/\alpha}$~~

d Suppose  $z = u(0, 1)$ , then  $F(z) = z$  for  $z \in [0, 1]$

$$\mathbb{E}_1[\text{u}_1(x, y)] = [1 - (x+y)^\alpha]x^\alpha$$

$$\mathbb{E}_2[\text{u}_2(y, x)] = [1 - (x+y)^\alpha]y^\alpha$$

FOCs:

$$\frac{\partial}{\partial x} \mathbb{E}_1[\text{u}_1(x, y)] = 0$$

$$\frac{\partial}{\partial y} \mathbb{E}_2[\text{u}_2(y, x)] = 0$$

$$[1-x-y] \alpha x^{\alpha-1} - x^\alpha = 0$$

$$[1-x-y] \alpha - x = 0$$

$$(\alpha+1)x = (1-y)\alpha$$

$$x = \alpha/(\alpha+1)(1-y)$$

By symmetry,

$$y = \alpha/(\alpha+1)(1-y)$$

$$x = y = (1-x-y)\alpha$$

$$x = \alpha/(\alpha+1)(1-x) \quad \text{By substitution } y=x$$

$$x = \alpha/(\alpha+1) - \alpha x/(\alpha+1)$$

$$2\alpha/(\alpha+1) x = \alpha/(\alpha+1)$$

$$x = y = \alpha/2\alpha+1$$

As players become more risk averse, the marginal utility of money decreases more rapidly,  $\alpha$  decreases.  
Each player's demand decreases.

$$\begin{aligned} \ln x^* &= \frac{1}{2\alpha} \ln \left( \frac{1}{2} \left( \frac{2d}{2d+n} \right)^\alpha \right) \\ &= \frac{1}{2\alpha} \ln \frac{1}{2} + \frac{1}{2\alpha} \ln \frac{2d}{2d+n} \rightarrow \frac{1}{2\alpha} \ln \left( \frac{2d}{2d+n} \right) \\ &= \frac{1}{2\alpha} [\ln 2d - \ln 2 - \ln(2d+n)] \end{aligned}$$

Let  $f(n) = [\ln 2d - \ln 2 - \ln(2d+n)]$  and  $g(n) = n$

$$f'(n) = -\frac{1}{2d+n}, g'(n) = 1$$

$\lim_{n \rightarrow \infty}$

$$\begin{aligned} \ln x^* &= \ln \left( \frac{1}{2} \left( \frac{2d}{2d+n} \right)^\alpha \right) \\ &= \ln \frac{1}{2} + \frac{1}{2\alpha} \ln 2d - \frac{1}{2\alpha} \ln(2d+n) \end{aligned}$$

Let  $f(n) = \ln 2d - \ln(2d+n)$  and  $g(n) = n$

$$f'(n) = -\frac{1}{2d+n}, g'(n) = 1$$

$$\lim_{n \rightarrow \infty} \frac{1}{n} [\ln 2d - \ln(2d+n)]$$

$$= \lim_{n \rightarrow \infty} \frac{f'(n)}{g'(n)} \text{ by L'Hopital's rule}$$

$$= \lim_{n \rightarrow \infty} -\frac{1}{2d+n}$$

$$= 0$$

$$\lim_{n \rightarrow \infty} \ln x^* = \lim_{n \rightarrow \infty} \ln \frac{1}{2} + \lim_{n \rightarrow \infty} \frac{1}{2\alpha} \ln 2d - \frac{1}{2\alpha} \ln(2d+n)$$

(by sum  $\frac{1}{n} \xrightarrow{n \rightarrow \infty}$ )

$$= \ln \frac{1}{2}$$

$$\lim_{n \rightarrow \infty} x^* = e^{\lim_{n \rightarrow \infty} \ln x^*} = \frac{1}{2}$$

By symmetry,  $\lim_{n \rightarrow \infty} y^* = \frac{1}{2}$ .

Are we expected to know 'L'Hopital's rule?

As  $n$  increases, the probability distribution of  $z$  narrows,  $z$  converges in probability to 1. This can be interpreted as a "perturbation" in  $z$  becoming 0. This can be interpreted as a perturbation of the original full information game becoming vanishingly small. This process identifies the risk dominant equilibrium in the original game in (a).

4a State space  $S_2 = \{K, C, B\}$

State space  $S_2 = \{K_{db}, C_{db}, B_{db}, K_{cv}, C_{cv}, B_{cv}\}$

where in each K-state, the sauce is ketchup, in each C-state the sauce is chocolate, in each B-state the sauce is Bechamel, in each db state Charlie is colourblind, and in each cv state Charlie has colour vision.

$$b P_E = (\{K_{db}, C_{db}\}, \{B_{db}\}, \{K_{cv}\}, \{C_{cv}\}, \{B_{cv}\})$$

$$P_A = (\{K_{db}, C_{db}, K_{cv}, C_{cv}\}, \{B_{db}, B_{cv}\})$$

c  $\underline{B_{db}}$  and  $\underline{B_{cv}}$  let the event that the cup contains

Bechamel sauce be  $B = \{B_{db}, B_{cv}\}$ . Player i knows

$\underline{B_{db}}$  and  $\underline{B_{cv}}$  iff in state w iff  $P_i(w) \subseteq B$ .

By inspection, C knows B in states  $B_{db}$  and  $B_{cv}$ .

$$K_C(B) = \{B_{db}, B_{cv}\}$$

d By inspection, A knows  $K_C(B)$  in states  $B_{db}$  and  $B_{cv}$ .

e By inspection, let Choc be the event that the cup contains chocolate sauce.

$$\text{By inspection, } K_A(\text{Choc}) = \{K_{cv}\}$$

f By inspection,  $K_A(K_C(\text{Choc})) = \emptyset$ , A never knows that C knows that Choc.