Philosophical Logic Problem Set 5

1a. ~~Prove by induct~~
Consider arbitrary PC model $M = \langle D, I \rangle$

Prove by induction that for all PC-wff ϕ, if variable assignments g and h for M agree on the free variables in ϕ, then $V_{M,g}(\phi) = V_{M,h}(\phi)$.

Base case

Consider arbitrary PC-wff ϕ such that ~~each~~ complexity, i.e. number of connectives $C(\phi)=0$. Then ϕ is a basic wff $\pi\alpha_1 \cdots \alpha_n$. $|\pi|_{M,g} = I(\pi) = |\pi|_{M,h}$. $|\alpha|_{M,g} = I(\alpha)$ if α is a constant, $g(\alpha)$ if α is a variable ~~$I(\alpha)$ if~~. ~~α is free~~ each of $\alpha_1, \ldots, \alpha_n$ is free in ϕ, so ~~$g(\alpha)$~~ for each of these, if it is a variable, $g(\alpha) = h(\alpha)$, so ~~$|\alpha|_M$~~ $|\alpha|_{M,g} = |\alpha|_{M,h}$ for $\alpha \in \{\alpha_1, \ldots, \alpha_n\}$. $V_{M,g}(\phi) = 1$ iff $\langle |\alpha_1|_{M,g}, \ldots, |\alpha_n|_{M,g} \rangle \in |\pi|_{M,g}$ iff $\langle |\alpha_1|_{M,h}, \ldots, |\alpha_n|_{M,h} \rangle \in |\pi|_{M,h}$ iff ~~$V_{M,}$~~ $V_{M,h}(\phi)=1$. so for such ϕ, $V_{M,g}(\phi) = V_{M,h}(\phi)$.

Induction Hypothesis

Given n, for all $m<n$, ~~if~~ for all ϕ such that $C(\phi)=m$, if g and h agree on free variables in ϕ, then $V_{M,g}(\phi) = V_{M,h}(\phi)$.

Induction Step

Consider arbitrary PC-wff ϕ such that $C(\phi)=n$. Then $\phi = \sim\psi$, $\psi \to \chi$ or $\forall\alpha\psi$.

Suppose $\phi = \sim\psi$. Then $V_{M,g}(\phi)=1$ iff $V_{M,g}(\psi)=0$ iff by IH, noting ~~given~~ that all free variables in ~~ϕ~~ ψ are free in ϕ so g and h agree on free variables in ψ, ~~$V_{M,g}(\psi)$~~ $V_{M,h}(\psi)=0$ iff ~~$V_{M,g}$~~ $V_{M,h}(\phi)=1$, so $V_{M,g}(\phi) = V_{M,h}(\phi)$.

Suppose $\phi = \psi \to \chi$. Then $V_{M,g}(\phi)=1$ iff $V_{M,g}(\psi)=0$ or $V_{M,g}(\chi)=1$ iff by IH noting ~~given~~ that all free variables in ψ ~~are free in~~ and all free variables in χ are free variables in ϕ, and that ~~$C(\psi)$~~ $C(\psi), C(\chi) < n$, $V_{M,h}(\psi)=0$ or $V_{M,h}(\chi)=1$ iff $V_{M,h}(\phi)=1$, so $V_{M,g}(\phi) = V_{M,h}(\phi)$

Suppose $\phi = \forall\alpha\psi$. Then $V_{M,g}(\phi)=1$ iff $\forall u \in D$: $V_{M,g^\alpha_u}(\psi)=1$. ~~All eg~~ ~~are~~ Every free variable ~~in~~ in ψ except α is a free variable in ϕ, so g and h agree on these. Then g^α_u and h^α_u agree on these. By construction, g^α_u and h^α_u agree on α. So g^α_u and h^α_u agree on the free variables in ~~ψ~~ ψ. $C(\psi)=n-1$. by IH $V_{M,g}(\phi)=1$ iff $\forall u \in D$: $V_{M,g^\alpha_u}(\psi)=1$ iff $\forall u \in D$: $V_{M,h^\alpha_u}(\psi)=1$ iff $V_{M,h}(\phi)=1$, so $V_{M,g}(\phi) = V_{M,h}(\phi)$.

By cases, for ϕ such that $C(\phi)=n$, if g and h agree on the free variables in ϕ, then $V_{M,g}(\phi) = V_{M,h}(\phi)$.

By induction, for all PC-wff ϕ, if g and h agree on the free variables, then $V_{M,g}(\phi) = V_{M,h}(\phi)$.

ii Prove the given claim by strong induction over the complexity $C(\phi)$ of ϕ.

~~Base Case.~~
~~$C(\phi)=1$. ϕ is some basic wff $\pi\alpha_1 \cdots \alpha_n$.~~
~~$|\pi|_{M,g} = I(\pi) = |\pi|_{M,h}$. For $\alpha \in \{\alpha_1, \ldots, \alpha_n\}$ if~~
~~α is a constant, then $|\alpha|_{M,g}$~~ ~~it is given~~
~~that $|\alpha|_{M,g} = |\beta|_{M,g}$. $V_{M,h}(\phi)=$~~ $V_{M,g}(\phi)=1$ iff
$\langle |\alpha_1|_{M,g}, \ldots, |\alpha_n|_{M,g} \rangle \in |\pi|_{M,g}$ iff
~~$\langle |\beta_1|_{M,g}, \ldots, |\beta_n|_{M,g} \rangle$~~ ~~$\langle |\alpha_1|_{M,g}, \ldots, |\alpha_n|_{M,g} \rangle (\beta/\alpha)$~~
~~ϕ~~ $= \langle |\alpha_1|_{M,g}$

Date