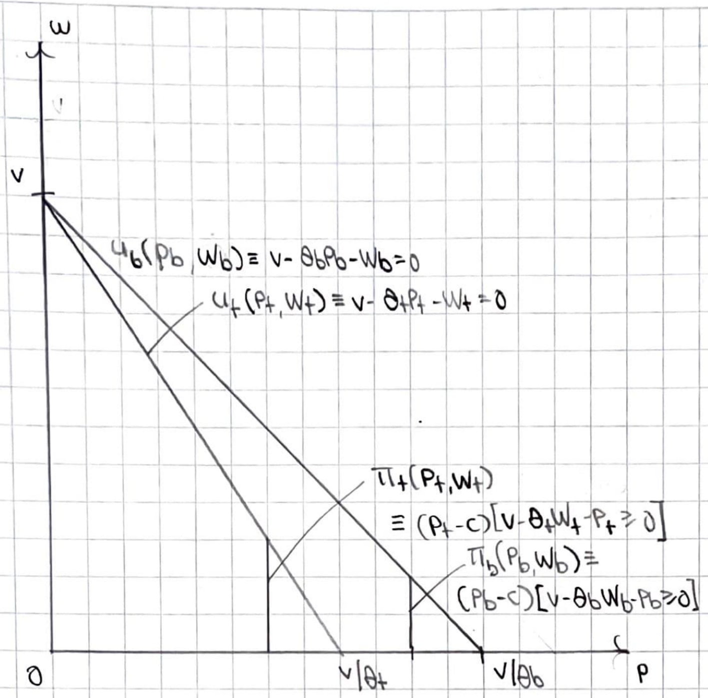


The firm's optimisation problem is

$\max_{p_b, w_b, p_t, w_t} (1-\lambda)(p_b - c) + \lambda(p_t - c)$ subject to

$PC_b: v - \theta_b p_b - w_b \geq 0$

$PC_t: v - \theta_t p_t - w_t \geq 0$

$IC_b: v - \theta_b p_b - w_b \geq v - \theta_b p_t - w_t$

$IC_t: v - \theta_t p_t - w_t \geq v - \theta_t p_b - w_b$

$p_b^+: p_b \geq 0$, $p_t^+: p_t \geq 0$, $w_b^+: w_b \geq 0$, $w_t^+: w_t \geq 0$

supposing without loss of generality that consumers have reservation utility 0

b ~~Fact~~

$v - \theta_b p_b - w_b \geq_1 v - \theta_b p_t - w_t >_2 v - \theta_t p_t - w_t \geq_3 0$

$\geq_1$ follows from IC_b, $>_2$ from $\theta_b < \theta_t$ (supposing $p_t \neq 0$)

$\geq_3$ from PC_t

$\Rightarrow v - \theta_b p_b - w_b > v - \theta_t p_t - w_t$, i.e. PC_b does not bind.

Suppose for reductio that at the optimum $\hat{p}_b, \hat{w}_b, \hat{p}_t, \hat{w}_t$, PC_t does not bind, i.e. $v - \theta_t \hat{p}_t - \hat{w}_t > 0$. Then, for sufficiently small ε (such that PC_b and PC_t continues to hold), $p_b' = \hat{p}_b + \varepsilon$, $p_t' = \hat{p}_t + \varepsilon$ is such that all constraints hold and $\pi' > \hat{\pi}$. So $\hat{p}_b, \hat{w}_b, \hat{p}_t, \hat{w}_t$ is not an optimum. By reductio, PC_t binds at the optimum, and tourists are indifferent between travelling and not travelling.

c Suppose for reductio that at the optimum, IC_b does not bind, i.e. $v - \theta_b \hat{p}_b - \hat{w}_b > v - \theta_b \hat{p}_t - \hat{w}_t$. Then, for sufficiently small ε (such that PC_b and IC_b continue to hold), $p_b' = \hat{p}_b + \varepsilon$ is such that all constraints hold and $\pi' > \hat{\pi}$ ~~So the candidate~~ By reductio, IC_b binds at the optimum, and business men are indifferent between buying at w_b and at w_t.

~~Suppose for reductio that $\hat{w}_b \neq 0$.~~ IC_b binds, i.e.

$v - \theta_b \hat{p}_b - \hat{w}_b = v - \theta_b \hat{p}_t - \hat{w}_t \Leftrightarrow \theta_b \hat{p}_b + \hat{w}_b = \theta_b \hat{p}_t + \hat{w}_t \Leftrightarrow$
$\theta_b(\hat{p}_b - \hat{p}_t) = \hat{w}_t - \hat{w}_b \Rightarrow \theta_t(\hat{p}_b - \hat{p}_t) > \hat{w}_t - \hat{w}_b \Rightarrow$
$v - \theta_t \hat{p}_b - \hat{w}_b < v - \theta_t \hat{p}_t - \hat{w}_t$, i.e. IC_t does not bind.

Suppose for reductio that $\hat{w}_b \neq 0$. Then for sufficiently small ε (such that IC_t continues to hold), $w_b' = \hat{w}_b - \varepsilon$, $p_b' = \hat{p}_b + \varepsilon/\theta_b$ is such that ~~$V-\theta_b\hat{w}_b$~~

$V - \theta_b \hat{p}_b - \hat{w}_b = V - \theta_b p_b' - w_b'$, so it is trivial that PC_b, PC_t, and IC_b continue to hold (and IC_t holds by construction of ε), and $\pi' > \hat{\pi}$. By reductio, $\hat{w}_b = 0$.

Businessmen buy at $\hat{w}_b = 0$ and are indifferent ~~to~~ between buying at this time and buying when tourists do at $\hat{w}_t$.

d

How could we know this is the right method rather than Lagrangian?

d Given that PC_t binds, $v-\theta_t\hat{p}_t-\hat{w}_t=0$, $(\hat{p}_t,\hat{w}_t)$ lies on the indifference curve $u_t(p_t,w_t)=0$. Given that IC_b binds, $v-\theta_b\hat{p}_b-\hat{w}_b=v-\theta_b\hat{p}_t-\hat{w}_t$, $(\hat{p}_b,\hat{w}_b)$ lies on the indifference $u_b(p_b,w_b)=u_b(\hat{p}_t,\hat{w}_t)$, i.e. the indifference curve that crosses $(\hat{p}_t,\hat{w}_t)$. Given that $\hat{w}_b=0$, $(\hat{p}_b,\hat{w}_b)$ lies on the intersection of that indifference curve with the p axis.

$\hat{w}_t=v-\theta_t\hat{p}_t$

$v-\theta_b\hat{p}_b-\hat{w}_b=v-\theta_b\hat{p}_t-\hat{w}_t \Rightarrow \theta_b\hat{p}_b+\hat{w}_b=\theta_b\hat{p}_t+\hat{w}_t$

$\Rightarrow \theta_b\hat{p}_b=\theta_b\hat{p}_t+v-\theta_t\hat{p}_t \Rightarrow \hat{p}_b=\hat{p}_t+v/\theta_b-\theta_t/\theta_b\,\hat{p}_t$

$\hat{\pi}=(1-\lambda)\hat{\pi}_b+\lambda\hat{\pi}_t$

$\quad=(1-\lambda)(\hat{p}_t+v/\theta_b-\theta_t/\theta_b\,\hat{p}_t-c)+\lambda(\hat{p}_t-c)$

~~At the optimum, the following FOC holds~~

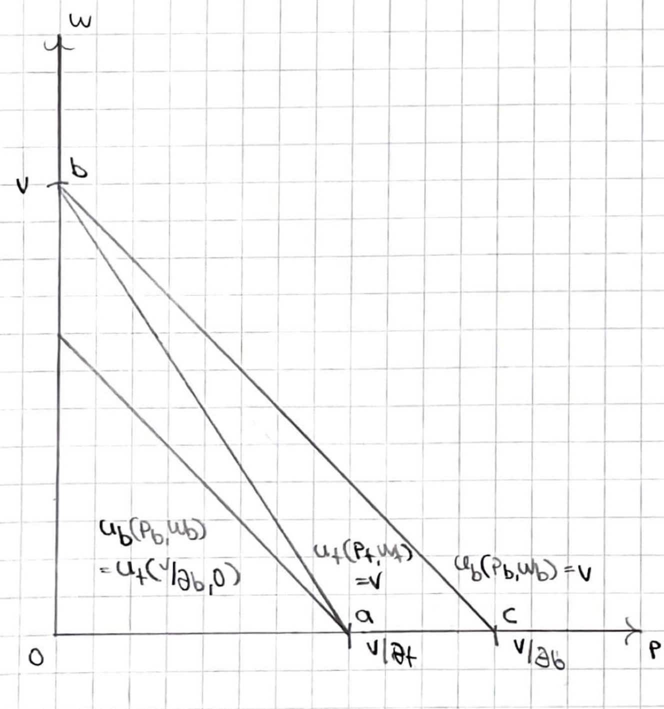
$d\hat{\pi}/d\hat{p}_t=(1-\lambda)(1-\theta_t/\theta_b)+\lambda=0 \Rightarrow$

$1-(1-\lambda)(\theta_t/\theta_b)=0 \Rightarrow$

$1-\lambda=\theta_b/\theta_t \Rightarrow \lambda=1-\theta_b/\theta_t$

Suppose $\lambda>1-\theta_b/\theta_t$, then $d\hat{\pi}/d\hat{p}_t>0$, the firm maximises profit by choosing the maximum feasible (i.e. subject to the above constraints) $\hat{p}_t$, namely v/θ_t. Then $\hat{w}_t=0$, $\hat{p}_b=\hat{p}_t=v/\theta_t$, $\hat{w}_b=0$. This is the pooling equilibrium. $\hat{\pi}=(v/\theta_t-c)$

Suppose $\lambda<1-\theta_b/\theta_t$, then $d\hat{\pi}/d\hat{p}_t>0$, the firm maximises profit by choosing the minimum feasible $\hat{p}_t$, namely 0. Then $\hat{w}_t=v$, $\hat{p}_b=v/\theta_b$, $\hat{w}_b=0$. This is the separating equilibrium. Suppose that indifferent tourists do not buy. $\hat{\pi}=(1-\lambda)(v/\theta_b-c)$



The pooling eqm is a, the separating eqm is points b and c.

e If $c>v/\theta_b$, then $c>v/\theta_t$, $\hat{\pi}<0$ in either eqm, the firm should choose high p_b, p_t, w_b, w_t such that no consumers buy.

If $v/\theta_t < c < v/\theta_b$, then only the ~~p~~ separating eqm is profitable, the firm should choose $p_b = v/\theta_b$, $w_b = 0$ and high p_t, w_t such that no tourists buy.

If $c < v/\theta_t < v/\theta_b$ the airline does not serve tourists iff

$(1-\lambda)(v/\theta_b - c) > (v/\theta_t - c) \Leftrightarrow$

$(1-\lambda) v/\theta_b - v/\theta_t > -c + (1-\lambda) c \Leftrightarrow$

$(1-\lambda) v/\theta_b - v/\theta_t > -\lambda c \quad \Leftrightarrow$

$c > \cancel{1-\lambda} \; \lambda - 1/\lambda \; v/\theta_b + v/\lambda\theta_t \quad \Leftrightarrow$

$c > \lambda - 1/\lambda \; v/\theta_b + 1/\lambda \; v/\theta_t$

Marginal cost exceeds ~~the~~ some weighted average of the maximum price each type is willing and able to pay