Modal Propositional Logic Philosophical Rough Notes

Modality	Reasonable Constraints on Accessibility Restriction	Controversial Constraints on Accessibility Relation	Unreasonable Constraints on Accessibility Relation	Comments
Deontic; Rwu iff everything obligatory in w is true in u.	**Seriality**		**Reflexivity**	**(1)** Seriality is not sufficient for the logical truth of the utopia principle, that it ought to be that the things that ought to be are true, formalised as $\Box(\Box\phi \to \phi)$. **(2)** Neither $\Box(K \to Q)$ nor $K \to \Box Q$ satisfactorily formalise "if you kill the victim you ought to kill him quickly".
Epistemic; Rwu iff everything known (by some fixed agent) in w is true in u.	**Reflexivity**	**Transitivity** (which validates positive introspection), **Euclideanness** (which validates negative introspection)		Under K, $\phi \to \psi$ logically implies $\Box\phi \to \Box\phi$, which seems implausible under an epistemic interpretation.
Doxastic; Rwu iff everything believed (by some fixed agent) in w is true in u.		**Transitivity** (which validates positive introspection), **Euclideanness** (which validates negative introspection)	**Reflexivity**	
Weak Temporal; $t \le t'$ iff t is at least as early as t'	**Reflexivity, Strong Connectivity, Transitivity, Antisymmetry**	**Density, Eternality**	**Symmetry**	
Strict Temporal; $t < t'$ iff t is strictly earlier than t'	**Irreflexivity, Transitivity, Weak Connectivity, Transitivity, Antisymmetry**	**Density, Eternality**	**Symmetry**	

Metaphysical Modality

- Salmon argues against the appropriateness of S4 (hence also S5) for metaphysical modality. **Salmon's basic argument refers to the example of some particular wooden table Woody**, that actually originates from matter m^*. Given that Woody originates from m^*, it is metaphysically possible that Woody originates from some other matter m' but not some yet other matter m''. Suppose that m' and m'' are chosen with sufficient care, such that m' is a limiting case and m'' is metaphysically "close" to m' but "distant" from m^*. Then, if Woody had originated from m', then Woody could have originated from m''. So Woody's originating from m'' is impossible but possibly possible, and Woody's not originating from m'' is necessary but not necessarily necessary.
- **Salmon's account of possible worlds here is critical.** Salmon remains close to convention in defining metaphysical necessity as truth in every possible world and analogously for metaphysical possibility. **Where Salmon deviates from convention is in considering some worlds impossible. A world, according to Salmon, is some qualifiedly maximal entity according to which facts or states of affairs do or do not obtain. A possible world is a metaphysically possible world. In contrast, it is convention to refer to the former as "possible worlds", which obscures the (logical) possibility of metaphysically impossible worlds.** This logical possibility is made apparent by two considerations. **First, there is no metaphysically possible world where Nathan Salmon is Henry Kissinger, but there certainly is some qualifiedly maximal entity according to which that obtains. Second, there are**

mathematically possible but metaphysically impossible worlds, such as the world where Nathan Salmon is a fish. So there is indeed a world (this is conventionally referred to as a possible world) in which Woody originates from m'' but this is not a metaphysically possible world (because of metaphysical considerations of which the relation between worlds is downstream). The friend of S4 and/or S5 does not recognise this because the conventional notation of "possible world" obscures it, so such philosophers think that metaphysical modality is the "limiting case", such that metaphysical necessity is truth in every "possible world" in the conventional sense.

- **The opponent of Salmon's view reverses the direction of analysis. Metaphysical modality is not to be understood in terms of metaphysically possible worlds, rather metaphysically possible worlds are to be understood in terms of metaphysical modality. Salmon's treatment of metaphysical modality is no different from the treatment of deontic, epistemic, or temporal modality.** It is our understanding of morality, knowledge, or time, that informs our construction of deontic, epistemic, or temporal models of propositional logic, not the reverse.
- **S5 represents the "ostrich approach"** to metaphysical necessity, **under which, roughly, only directly metaphysically accessible worlds "matter". This approach misconstrues "necessary" to mean "actually necessary".**
- Another upshot of Salmon's argument is that some states of affairs are contingently possible or contingently necessary.
- Salmon goes further to attack B as a system of metaphysical modality. **Even if it were true or (metaphysically) necessarily true that it is necessary that the actual world is possible, this is not logically true.** So B is appropriate as a system of metaphysical modality only as premised on a theorem of metaphysics.

Deontic Modality

- For deontic modality, world u is accessible from world w iff the things that obtain in w are permissible under the binding moral norms of w. Then, seriality is reasonable but reflexivity is not, so the strongest appropriate system for deontic modality is D. This implies that for every world there is some world in which everything that is morally obligatory (necessary) obtains. This implies also that some thing's being morally obligatory implies its being morally permissible (the characteristic validity of D is $\Box\phi \to \Diamond\phi$). Indeed this seems to be a logical implication.
- D seems inadequate for deontic modality because it seems to be a logical truth that it ought to be the case that everything that ought to be true is true, this is the utopia principle, but $\Box(\Box\phi \to \phi)$ is not a logically valid schema in D.
- Another difficulty in treating D as a system for deontic modality is the following. "If you kill the victim, you should kill the victim quickly" seems to be best formalised by either (1) $K \to \Box Q$ or (2) $\Box(K \to Q)$. (1) seems implausible because we do not think that if you in fact kill the victim, you had an obligation to kill him quickly. (2) seems implausible because "if you kill the victim, you should kill the victim slowly and painfully" would be similarly formalised as $\Box(K \to S)$ which is a logical implication of $\Box\neg K$, but we do not think that "if you kill the victim, you should kill the victim slowly and painfully" is a logical implication of "you should not kill the victim".

Epistemic Modality

- For epistemic modality, world u is accessible from world w iff everything known in w is known in u. Reflexivity is reasonable because it is reasonable to think that knowledge implies truth. The characteristic S4 axiom implies that everything known is known to be known, this is the principle of positive introspection. The characteristic S5 axiom implies that everything not known is known to be not known, this is the principle of negative introspection. Whether positive introspection and negative introspection are correct is controversial.
- Even K seems to be inappropriate for epistemic modality. Under K (which imposes no restrictions on the accessibility relation), $\Box\phi \to \Box\psi$ is a logical consequence of $\phi \to \psi$. Under an epistemic interpretation, this means that some thing ϕ's being a logical consequence of some other thing ψ, logically implies that our knowledge of ϕ (materially) implies our knowledge of ψ. But this seems implausible. Some intricate truths are materially implied by the Peano axioms, but it seems we can know the Peano axioms without knowing such truths.

Doxastic Modality

- For doxastic modality, world u is accessible from world w iff everything believed (by some fixed agent) in w is true in u. Reflexivity is unrealistic because believe is not factive, i.e. we can and do believe falsehoods. Positive and negative introspection are controversial. If positive introspection is correct, then the appropriate accessibility relation is transitive. If negative introspection is correct, then the appropriate accessibility relation is euclidean
$\forall w, u, v \in W : (Rwu \wedge Rwv \to Ruv)$.
- Positive introspection is controversial because a caveman could believe that sticking his hand in a fire would hurt, but not believe that he believes this because he is epistemologically unsophisticated and does not have a conception of belief. It also seems possible to have a first-order belief without having a thousandth-order belief. The caveman example applies

also to negative introspection. A plausible logic for doxastic modality should validate either both or neither positive introspection and negative introspection.

Temporal Modality (Propositional Tense Logic)

- For temporal modality, the accessibility relation is a temporal ordering relation "at least as early as" (or "strictly earlier than"). Reflexivity (or irreflexivity, under the latter interpretation) is reasonable. Transitivity is reasonable. Weak (or strong, under the latter interpretation) connectivity is reasonable.
    - Note that weak connectivity here is not used in the usual sense of $\forall x \neq y : (Rxy \vee Ryx)$ but rather the sense $\forall x, y, y' : [(Rxy \wedge Rxy' \rightarrow Ryy' \vee Ry'y) \wedge (Ryx \wedge Ry'x \rightarrow Ryy' \vee Ry'y)]$, which reads as "if there is some time prior to both y and y' or subsequent to both y and y', then either $y \leq y'$ or $y' \leq y$". This disallows branching but allows distinct timelines wholly disconnected from one another.
    - Strong connectivity disallows branching, and secures that any two times are part of a single non-branching structure.
- Antisymmetry is reasonable. Density (between any two times there is another time) is also reasonable. Eternality (there is neither a first time nor a last time) is controversial. Symmetry is entirely unreasonable.