4a Let $\succsim$ denote A's preferences over lotteries. Given that A has EU preferences, there is some function U that represents $\succsim$ such that $U(L) = \sum_{i=1}^{n} p_i u(x_i)$ for some u, where $L = [p_1, \dots, p_n; x_1, \dots, x_n]$. Let $L(x)$ denote the lottery given x.

$\cancel{L_x}\overset{L(x)}{} = [\alpha, 1-\alpha; w+2x, w-x]$

$U(L(x)) = \alpha u(w+2x) + (1-\alpha)u(w-x)$

A's maximisation problem is $\max_x U(L(x))$

FOC: $\partial U(L(x))/\partial x|_{x=x^*} = 2\alpha u'(w+2x^*) - (1-\alpha)u'(w-x^*) = 0$

~~By implicit differentiation wrt α~~

$\cancel{2u'(w+2x^*)}$

By implicit differentiation wrt α

$2u'(w+2x^*) + 4\alpha u''(w+2x^*)\,\partial x^*/\partial\alpha$

$+u''(w-x^*)\,\partial x^*/\partial\alpha$

$+u'(w-x^*) - \alpha u''(w-x^*)\,\partial x^*/\partial\alpha$

$= 0$

Rearranging,

$[-4\alpha u''(w+2x^*) + (1-\alpha)u''(w-x^*)]\,\partial x^*/\partial\alpha$

$= -2u'(w+2x^*) - u'(w-x^*)$

Supposing that A prefers more wealth to less, u is strictly increasing, i.e. ~~$u'(x)>0$~~ $\forall x: u'(x) > 0$. Given that A is risk averse, ~~it~~ $\forall x: u''(x) < 0$. $\alpha \in [0,1]$

Then $\partial x^*/\partial\alpha > 0$

SOC: $\partial^2 U(L(x))/\partial x^2|_{x=x^*} = 4\alpha u''(w+2x^*) + (1-\alpha)u''(w-x^*) < 0$

~~The FOC is satisfied at~~ The FOC is satisfied at a local maximum.

$x^* = \arg\max_x U(L(x))$ increases with increasing α, the higher the value of α, the higher the value x A optimally chooses.

b $\forall r \in (0,1)$: CRRA $(r) \Rightarrow u(x) = x^{1-r}$

$u'(x) = (1-r)x^{-r}$

FOC: $\partial U(L(x))/\partial x|_{x=x^*} = 2\alpha(1-r)(w+2x^*)^{-r} - (1-\alpha)(1-r)(w-x^*)^{-r} = 0$

$2\alpha(w+2x^*)^{-r} = (1-\alpha)(w-x^*)^{-r}$

$(2\alpha)^{-1/r}(w+2x^*) = (1-\alpha)^{-1/r}(w-x^*)$

$[2(2\alpha)^{-1/r} + (1-\alpha)^{-1/r}]x^* = [-(2\alpha)^{-1/r} + (1-\alpha)^{-1/r}]w$

$x^* = [(1-\alpha)^{-1/r} - (2\alpha)^{-1/r}]w \,/\, [(1-\alpha)^{-1/r} + 2(2\alpha)^{-1/r}]$

$\cancel{\uparrow r \Rightarrow \downarrow 1/r \Rightarrow \uparrow -1/r \Rightarrow \uparrow (1-\alpha)^{-1/r}, \uparrow (2\alpha)^{-1/r}}$

$u''(x) = -r(1-r)x^{-1-r} < 0$, so the SOC holds and the FOC is satisfied at a local maximum.

For $\alpha \leq 1/3$, $(w+2x^*)^{-r} \geq (w-x^*)^{-r}$, $w+2x^* \leq w-x^*$, $x^* \leq 0$.

Given that $x \in [0, w]$, for $\alpha < 1/3$, A's maximisation problem has a corner solution, $x^* = 0$. For such ~~x~~ α, x^* (weakly) ~~decreases~~ decreases with increasing r.

For $\alpha > 1/3$, $\cancel{(w+2x^*)^{-r} < (w-x^*)^{-r}}$

$(2\alpha)^{-1/r}(w+2x^*) = (1-\alpha)^{-1/r}(w-x^*)$,

$w+2x^*/w-x^* = \cancel{(2\alpha/1-\alpha)^{-1/r}}\ (1-\alpha/2\alpha)^{-1/r}$

$\cancel{2\alpha/1-\alpha > 1}$, $1-\alpha/2\alpha < 1$

$\cancel{\text{so } \uparrow r \Rightarrow \downarrow 1/r \Rightarrow \uparrow -1/r \Rightarrow \uparrow (2\alpha/1-\alpha)^{-1/r} \Rightarrow \uparrow w+2x^*/w-x^* \Rightarrow \uparrow x^*}$

$\cancel{x^*}$

so $\uparrow r \Rightarrow \downarrow 1/r \Rightarrow \uparrow -1/r \Rightarrow \downarrow (1-\alpha/2\alpha)^{-1/r} \Rightarrow \downarrow w+2x^*/w-x^* \rightarrow \downarrow x^*$

x^* strictly decreases with increasing r.

CRRA(1) $\Rightarrow u(x) = \ln x$

$u'(x) = \cancel{\#\#}\, 1/x,\quad u''(x) = -x^{-2} < 0$ for all x

FOC: $\cancel{\partial(((}\ \partial u(c(x))/\partial x^* \big|_{x=x^*} = 2\alpha/(w+2x^*) - (1-\alpha)/(w-x^*) = 0$

$2\alpha(w-x^*) = (1-\alpha)(w+2x^*)$

$\cancel{-2\alpha x^* + 2\alpha x^* = -2\alpha w + (1-\alpha)w}$

$-2\alpha x^* - (1-\alpha)2x^* = -2\alpha w + (1-\alpha)w$

$-2x^* = (1-3\alpha)w$

$x^* = 3\alpha - 1/2\ w$

As $r \to 1$ from 0, $-1/r \to -1$ from $-\infty$

$x^* = [(1-\alpha)^{-1/r} - (2\alpha)^{-1/r}]w / [(1-\alpha)^{-1/r} + 2(2\alpha)^{-1/r}]$

$\to [(1-\alpha)^{-1} - (2\alpha)^{-1}]w / [(1-\alpha)^{-1} + 2(2\alpha)^{-1}]$

$= [2\alpha \pm (1-\alpha)/2\alpha(1-\alpha)]w / [2\alpha + 2(1-\alpha)/2\alpha(1-\alpha)]$

$= 3\alpha - 1/2\ w$

~~So x^* decreases with increasing r for~~

So $x^*(r)$ is continuous ~~for $r \in (0,1]$.~~ over the domain $r \in (0,1]$, and x^* decreases with increasing r in this domain.