73 overall

Microeconomics Paper 220525

1ai A chooses x_A, y_A to maximise $u_A(x_A, y_A) = x_A^2 y_A^3$ subject to $x_A + py_A \leq$ ~~$\omega_A + p\omega$~~ $(1,p)\cdot\omega_A = 5+p$.

B chooses x_B, y_B to maximise $u_B(x_B, y_B) = \min\{2x_B, 3y_B\}$ subject to $x_B + py_B \leq (1,p)\cdot\omega_B = 1+5p$.

$MU_A^x = 2x_A y_A^3$, $MU_A^y = 3y_A^2 x_A^2$, $MRS_A = MU_A^x / MU_A^y = 2y_A/3x_A$

comment that A has Cobb-Douglas preferences, B's preferences are such that x, y are perfect complements. B has Leontief preferences

~~Solving for A's demands by Lagrangian optimisation,~~

~~$\mathcal{L} = x_A^2 y_A^3 - \lambda(x_A + py_A - (5+p))$~~

~~$\mathcal{L}_x = 2x_A y_A^3 - \lambda = 0$, $x_A = \lambda/2y_A^3$~~

~~$\mathcal{L}_y = 3y_A^2 x_A^2 - p\lambda = 0$, $y_A = \sqrt{p\lambda/3x_A^2}$~~

~~$\mathcal{L}_\lambda = -(x_A + py_A - (5+p)) = 0$~~

Since ln is a monotonic transformation, A maximises u_A iff A maximises $\ln(u_A) = 2\ln x_A + 3\ln y_A$. Solving for A's demands by Lagrangian optimisation,

Alternative: solve by MRS

$\mathcal{L} = 2\ln x_A + 3\ln y_A - \lambda(x_A + py_A - (5+p))$

$\mathcal{L}_x = 2/x_A - \lambda = 0$, $x_A = 2/\lambda$

$\mathcal{L}_y = 3/y_A - p\lambda = 0$, $y_A = 3/p\lambda$

$\mathcal{L}_\lambda = -(x_A + py_A - (5+p)) = 0$, $x_A + py_A = 5+p$,

By substitution,

$2/\lambda + 3/\lambda = 5/\lambda = 5+p$, $\lambda = 5/5+p$

$x_A = 10+2p/5$, $y_A = 15+3p/5p$ ✓

can check by Cobb-Douglas, A spends 2/5 of income on x, 3/5 on y

This is correct, and the correct method

B maximises ~~utility~~ u_B only if $2x_B = 3y_B$ since if $2x_B > 3y_B$, B can increase utility by trading some x for y, and the reverse if $2x_B < 3y_B$. So $x_B = 3/2\, y_B$. By substitution into B's budget constraint,

$3/2\, y_B + py_B = 1+5p$, $y_B = 1+5p/3/2+p = 2+10p/3+2p$, $x_B = 6+30p/6+4p = 3+15p/3+2p$.

$x = x_A + x_B = 3+15p/3+2p + 10+2p/5$

$z^x = x - \omega^x = x - 6 = 3+15p/3+2p + 10+2p/5 - 6$ (verified) ✓

6

ii ~~$\partial z^x/\partial p = (3+15p)(-1)($~~ → argue more carefully

By inspection, z^x increases with increasing p. x_A increases with increasing p because A substitutes away from the (relatively) more expensive y to the (relatively) less expensive x. x_B increases with increasing p because an increase in p ~~cost~~ causes an increase in ~~the~~ the value of B's endowment, hence has a positive income effect on B's consumption.

: Argue more carefully

Differentiate wrt p

"How does ... vary": give when p=0, p is large and $\partial z_x/\partial p$

3

Important to argue that $z_x = 0$ for some p

p ... 2

Used to argue existence

3 used to argue existence

~~bi $p=1$, $z_x = 18/5 + 12/5 - 6 = 0$~~

~~The market for x clears.~~

A competitive equilibrium exists iff $z_x = 0$ since then the market for x clears, and by Walras's Law, the market for y also clears.

Correct, sufficient to state "By Walras's Law", $z_x = 0 \Rightarrow z_y = 0$

bi $p=1$, $z^x = 18/5 + 12/5 - 6 = 0$

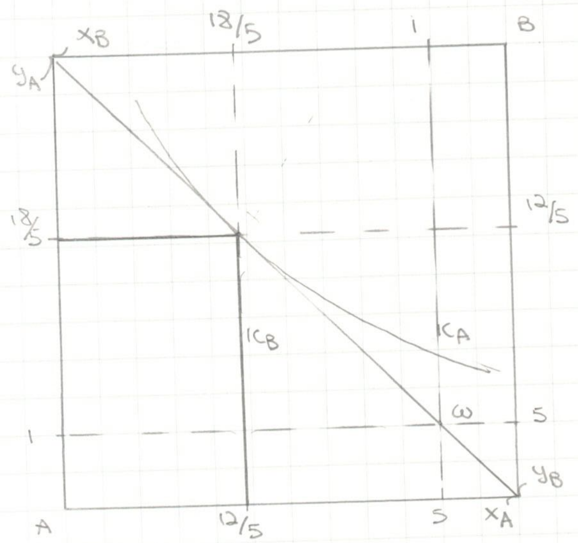
This price corresponds to an equilibrium.

$x_A = 12/5$, $y_A = 18/5$, $x_B = 18/5$, $y_B = 12/5$, $2x_B = 3y_B$

At this equilibrium, $MRS_B = \infty$, there is no exchange that B is willing to accept, since A's utility is

MRS not well-def for Leontief

: MRS not well defined for Leontief preferences increasing in x_A and in y_A, there is no ~~exchange that leaves~~ transfer that increases both players' utilities. So the equilibrium is Pareto efficient.



1st welfare Th : 1st welfare theorem

3

Is the contract curve a straight line from B ~~to~~ through the eqm? Yes, but something tricky happens at the corner

Label budget line : Label budget line

Can draw a few more indiffce cves

5

17/25

2ai

	~~X~~ X	Y
X	a, a	c, b
Y	b, c	d, d

$a>c$, $b>d$, best responses underlined
By inspection, the unique pure strategy NE is (X,X), where players play mutual best responses.
If $a=b$, then $(X, qX+(1-q)Y)$ and $(pX+(1-p)Y, X)$ give hybrid equilibria for p ~~and q~~ such that $pa+(1-p)c \geq pb+(1-p)d$ and likewise for q. ✓

~~The game is a Prisoner's Dilemma only if the Pareto dominant outcome is symmetric and not sustainable in equilibrium, i.e. $d>a$, $b\geq d$, and $b\geq d$.~~
The game resembles a Prisoner's Dilemma because each player has a strictly dominant pure strategy, namely X. The game is a Prisoner's Dilemma only if (Y,Y) Pareto dominates (X,X), ~~i.e.~~ iff $d>a$ and unilateral deviation from (Y,Y) leaves the other player worse off, iff ~~c<~~ $d>c$. ✓ 6

ii

	X	Y
X	a, a	c, b
Y	b, c	d, d

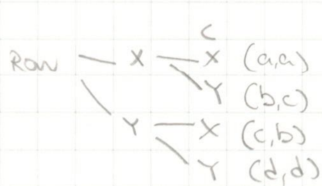
$a>c$, $b<d$, best responses underlined.
By inspection, the only pure NE are (X,X) and (Y,Y) where players play mutual best responses. ✓
Suppose there is some mixed NE σ^*, then Row mixes X and Y, and has no profitable deviation, so
$\pi_{Row}(X, \sigma^*_{Column}) = \pi_{Row}(Y, \sigma^*_{Column})$, ✓
$aq+b(1-q) = cq+d(1-q)$,
$q = \frac{b-d}{-a+b+c-d} = \frac{b-d}{(c-a)+(b-d)} \in (0,1)$ since each of $b-d$ and $c-a$ is strictly negative, where q is the probability σ^*_C assigns to X. Good ✓ to check $p,q \in (0,1)$.
By symmetry, $p = \frac{b-d}{(c-a)+(b-d)}$
The unique mixed NE $\sigma^* = (pX+(1-p)Y, qX+(1-q)Y)$ where $p=q=\frac{b-d}{(b-d)+(c-a)}$. ~~8~~ 8

bi The equilibrium concept that best applies is subgame perfect equilibrium. A SPE is an NE that induces an NE in each subgame ✓ (Say a bit more) 4 : say a bit more

SPE is strategy profile st each p's strategy is a complete contingent plan of action. "rules out non-credible threats, ~~tim~~ i.e. time inconsistent strategies" This is the ~~diff~~ key diff from NE

ii

	X	Y
X	a, a	c, b
Y	b, c	d, d



Draw tree : Draw tree

Denote Column's ~~strategy~~ pure strategies as $S_c \in \{XX, XY, YX, YY\}$, where the first letter gives column's action if Row plays X and the second if Row plays Y.
If $b>c$, the unique ~~PSP~~ SPE is (X, YX) if $c>b$ ~~B~~, the unique SPE is (Y, YX). if $b=c$, any $(pX+(1-p)Y, YX)$ is a SPE for ~~xe~~ $p\in[0,1]$.

This explanation is impt ~~4~~ 5 you are told (by) ; you are told this
Solve by backward induction or give the expanded payoff table

23/25

ai $X = [~~\frac{1}{2}~~ \frac{1}{2}, \frac{1}{2}; x_1, x_2]$, $Y = [\frac{1}{2}, \frac{1}{2}; y_1, y_2]$

Not true.

A risk averse EU maximiser prefers X to Y iff $u(X) > u(Y)$ iff $\frac{1}{2}u(x_1) + \frac{1}{2}u(x_2) > \frac{1}{2}u(y_1) + \frac{1}{2}u(y_2)$, where u is a strictly concave function.

< Bernoulli utility function

$y_1 < x_1 < x_2 < y_2$ and strict concavity of u do not together imply $\frac{1}{2}u(x_1) + \frac{1}{2}u(x_2) > \frac{1}{2}u(y_1) + \frac{1}{2}u(y_2)$ since assuming well-behaved preferences, u is strictly increasing, and there is no upper bound on y_2, so the inequality is falsified by sufficiently large y_2

(congive numerical exam : Can give numerical example

5

ii True ← don't know this

$E(X) = E(Y)$, $y_1 < x_1 < x_2 < y_2$ and strict concavity of u ~~imply~~ together imply $\frac{1}{2}u(x_1) + \frac{1}{2}u(x_2) > \frac{1}{2}u(y_1) + \frac{1}{2}u(y_2)$ then $u(X) > u(Y)$, so any risk averse EU maximiser prefers X.

Given only $E(x) > E(y)$ not $E(x) = E(y)$ is a more detailed proof / explanation required?

use SOSD : "use SOSD

3 Draw cdf, at least mention SOSD

bi $X \succ Y$

$X' = [\frac{1}{2}, \frac{1}{2}; 0, X]$

X' reduces to $X'^R = [\frac{1}{2}, \frac{1}{4}, \frac{1}{4}; 0, x_1, x_2]$

$Y' = [\frac{1}{2}, \frac{1}{2}; 0, Y]$

Y' reduces to $Y'^R = [\frac{1}{2}, \frac{1}{4}, \frac{1}{4}; 0, y_1, y_2]$

True.

Given $X \succ Y$ and the agent is an EU maximiser,

$\frac{1}{2}u(x_1) + \frac{1}{2}u(x_2) > \frac{1}{2}u(y_1) + \frac{1}{2}u(y_2)$. Then

$\frac{1}{4}u(x_1) + \frac{1}{4}u(x_2) > \frac{1}{4}u(y_1) + \frac{1}{4}u(y_2)$

$\frac{1}{2}u(0) + \frac{1}{4}u(x_1) + \frac{1}{4}u(x_2) > \frac{1}{2}u(0) + \frac{1}{4}u(y_1) + \frac{1}{4}u(y_2)$

~~$u(X'^R)$~~ $u(X') = u(X'^R) > u(Y') = u(Y'^R)$, so $X' \succ Y'$.

"This is an instance of the independence axiom"

6

ii Not true.

Consider the following counterexample.

this is the right approach

$x_1 = 10$, $x_2 = 11$, $y_1 = 0$, $y_2 = 2$,

$u(L) = -(\max_k x_k - \min_k x_k)$ where $L = [p_1, \ldots, p_n; x_1, \ldots, x_n]$

i.e. the agent's utility ~~is given~~ increases as the range of outcomes expands in absolute terms, then the agent prefers lotteries with ~~less extreme extre~~ closer extreme outcomes.

Then $u(X) = -1$, $u(Y) = -2$ so $X \succ Y$ and $u(X') = -11$, $u(Y') = -2$ so $Y' \succ X'$.

6

20/21

4a Given that insurers operate in a competitive market, ~~and~~ are risk neutral, and have no administrative cost, premiums are actuarially fair. $P_L = 1/4$, $P_H = 1/2$.

Households find both regulations equivalent because, under each regulation, ~~each household~~ given that each household is risk averse, each household fully insures since then its ~~expected~~ utility is equal to the utility of the expected value, which is greater than expected utility at any lower (or higher) level of insurance.

Add: zero profit in equilibrium.

3

Given risk neutral, payoff is equal to exp profit. then given perfect competition exp. profit is zero

Expand

3

Draw graph
Be explicit that they max ~~exp~~ EU
Can be shown mathematically

bi Given that all households fully insure, and that half of households are low risk and ~~that~~ half are high risk, the average cost of insurance to the insurer is $3/8\,K$. Given that ~~insurers operate in a~~ premiums are actuarially fair, $p = 3/8$.

Give ~~show~~ $3/8 = 1/2 \times 1/2 + 1/2 \times 1/4$

Give me detail

3

ii Suppose that there is a unique equilibrium price p^* which can be written as a weighted average of 1/2 and 1/4, at which insurers break even. Then $p^* \in [1/4, 1/2]$. At any such p^*, since households are risk averse and high risk households have probability 1/2 of incurring the loss, all high risk households fully insure.

High risk fully insure is correct

Expected profit per household at this price, $E(\pi) =$
$\cancel{1/2 p^* K + 1/2}$ $1/2[p^*K - 1/2 K] + 1/2[p^* q(p^*) - 1/4\, K] = 0$,
$\cancel{p^*K - 1/2 K = p^* q(p^*) - 1/4 K}$,
$\cancel{p^*[K - q(p^*)] = 1/4 K}$

$q(p^*)$?

Supposing that insurers maximise expected profits,
$\partial/\partial p\; E(\pi)|_{p=p^*} = K/2 + 1/2\, q(p^*) + 1/2\, p^* q'(p^*) = 0$,
$K = -q(p^*) - p^* q'(p^*)$

: $q(p^*)$ too

5

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~~By substitution,~~
~~$E(\pi) = 1/2[(p^* - 1/2)(-q(p^*) - p^* q'(p^*))] + 1/2[p^* q(p^*)$~~
~~$+ 1/2[p^* q(p^*) - 1/4($~~

In eqm: break even

Ans: $p^* = (1/4\, q(p) + 1/2\, K)/(q(p)+K)$

$p^*[K - q(p^*)] = 1/4\, K$
By substitution
$p^*[- 2q(p^*) - p^* q'(p^*)] = 1/4[q(p^*) - p^* q'(p^*)]$
~~$p^* = q(p^*)/4(-2q(p^*) - p^* q'(p^*)) - p^* q'(p^*)/4$~~

$$p^* = \frac{-q(p^*)}{4(-2q(p^*) - p^* q'(p^*))} + \frac{-p^* q'(p^*)}{4(-2q(p^*) - p^* q'(p^*))}$$

$$\cancel{= \frac{1}{4}\left[\frac{2q(p^*)}{2q(p^*) + p^* q'(p^*)} - \frac{q(p^*) + p^* q'(p^*)}{2q(p^*) + p^* q'(p^*)}\right]}$$

$$\cancel{= \frac{1}{2}\left(\frac{q(p^*)}{2q(p^*) + p^* q'(p^*)}\right)}$$

$\cancel{= 1/4[}$

$$= \frac{1}{4}\left[\frac{2q(p^*)}{2q(p^*) + p^* q'(p^*)} - \frac{q(p^*) + p^* q'(p^*)}{2q(p^*) + p^* q'(p^*)}\right]$$

$$= \frac{1}{2}\left[\frac{q(p^*)}{2q(p^*) + p^* q'(p^*)}\right] + \frac{1}{4}\left[1 - \frac{q(p^*)}{2q(p^*) + p^* q'(p^*)}\right]$$

Assuming $q(p^*) > 0$ and $q'(p^*) > -1$, p^* can be expressed as a weighted average of 1/2 and 1/4

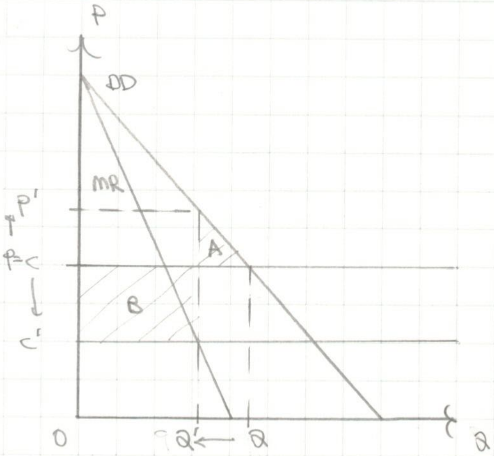
7 Competition authorities are typically concerned with the effect of a proposed merger on consumer surplus and aggregate welfare. The effect of merger from duopoly to monopoly and also the effect of merger from n-poly to (n-1)-poly depends on whether competition is well-described by Bertrand competition or Cournot competition, and ~~whethe~~ the synergies realised in merger. This essay outlines the application of oligopoly to the evaluation of such proposed mergers

continued on next page

Suppose that the two firms in duopoly compete in prices, i.e. the industry is characterised by Bertrand competition. ~~Formally, two firms N=i,~~ ~~N={1,2} each choose price~~ Formally, each firm $i \in N = \{1,2\}$ chooses price p_i given ~~&~~ constant marginal cost c_i to maximise profit $\pi_i = (p_i - c_i)q_i$, where q_i is firm i's quantity, $q_i = 0$ if $p_i > p_j$, $q_i = Q(p_i)$ if $p_i < p_j$ and $q_i = \frac{1}{2}Q(p_i)$ if $p_i = p_j$. In other words, consumers buy only from the lowest price firm(s). Suppose for simplicity that $c_1 = c_2 = c$. Let p^m ~~be~~ $= \arg\max_p (p-c)Q(p)$ denote the profit-maximising monopoly price.

Pre-merger, ~~each firm i chooses $p_i = c_i = c$ at equilibrium~~ the unique NE is st each firm i chooses $p_i = c$, ~~if firm i deviat~~ and enjoys $\pi_i = (p_i - c)\frac{1}{2}Q(p_i) = 0$. If firm i deviates by choosing greater p_i, it has 0 demand hence 0 profit. If firm i deviates by choosing lower p_i, it has negative ~~a~~ margin and positive demand hence negative profit. So each firm has no incentive to deviate from this strategy profile. At this NE, $p = c$, so ~~the marginal~~ consumer's valuation of the marginal unit, ~~given b~~ equal to price at equilibrium, is equal to the marginal cost of production, and ~~so~~ social surplus is maximised. The outcome is allocatively efficient. Since $p = c$, ~~producer~~ firms have zero margin, producer surplus is zero, so ~~consu~~ social surplus is captured entirely by consumers, and consumer surplus is maximised.

Post-merger, supposing that merger realises some synergies such that the merged firm produces at marginal cost $c' < c$ and that these synergies are non-drastic, i.e. $p^M(c') > c$, the merged firm acts as a monopolist and chooses higher price $p^M(c') \equiv \arg\max_p (p-c')Q(p) > p = c$, and enjoys positive profit $\pi^M(c') \equiv \max_p (p-c')Q(p) > 0$. Merger has two effects on aggregate welfare. Consumer surplus decreases because of both the increase in price and the corresponding decrease in quantity consumed. ~~Consumers pay more~~ Producer surplus increases because of both the increase in price and the decrease in cost. The increase in price constitutes a transfer of surplus from consumers to producers, so the effect on aggregate welfare is given by the increase in aggregate welfare due to the decrease in cost less the decrease in aggregate welfare due to the decrease in quantity. Graphically, this is given by area ~~A~~ B less area A. Competition authorities should favour mergers which realise large cost synergies. ~~in industries~~



Supposing instead that there are other firms in the industry, ~~the merger has no~~ the merger has no effect on price hence no effect on quantity and consumer surplus since the merged firm's optimal strategy is to price ~~just~~ arbitrarily below rivals marginal cost ~~(assuming synergies)~~ if there are cost synergies, and to price at the original marginal cost if there are no synergies. Merger increases ~~profit~~ insiders' profit hence aggregate welfare only because of synergies. This outcome is productively efficient because all units are produced at minimum marginal cost.

[if symmetric?]

Regulators should evaluate the relative impact of merger to monopoly on quantity and cost and favour mergers that yield large synergies in the case of duopoly. Regulators need not be too concerned with mergers short of monopoly in Bertrand competition since firms do not thereby escape the Bertrand trap. Such mergers ~~do~~ weakly increase consumer surplus and aggregate welfare. it may be difficult for regulators to estimate the magnitude of synergies.

That competition is well described by Bertrand is unrealistic since products are likely to be differentiated in reality. Then, merger short of monopoly ~~so~~ increases market power, ~~pri~~ hence prices. The magnitude of this can be ~~too~~ estimated by techniques such as the Farrell and Shapiro (2010) upward pricing pressure screen, a version of which was used by the UK CMA to evaluate the proposed Sainsbury's - Asda merger. Even in the homogenous products case, regulators may be wary of merger short of monopoly because this facilitates collusion.

expand

Suppose instead that the industry is well described by Cournot competition. Formally, each firm $i \in N = \{1,2\}$ chooses quantity q_i given marginal cost c_i to ~~max~~ maximise profit $\pi_i = [P(q_i+q_j)-c_i]q_i$ where $P(\cdot)$ is the ~~inverse~~ industry inverse demand function. Suppose for simplicity that $P(Q) = 1-Q$ and $c_1 = c_2 = c$. ~~then~~

Pre-merger, each firm i's best response $BR_i(q_j)$ ~~is~~ can be found by taking FOCs. $\partial\pi_i/\partial q_i = [P(Q)-c] + q_i P'(Q) = 0$, $q_i = -P(Q)-c\,/\,P'(Q) = -1-Q-c/-1 = 1-Q-c = 1-q_i-q_j-c$, $q_i = 1-q_j-c/2$. $BR_i(q_j) = 1-q_j-c/2$. ~~By symmetry,~~ Supposing that equilibrium is symmetric, $q_i = q_j$, $q_i = 1-q_i-c/2$, $q_i = 1-c/3$, $Q = 2(1-c)/3$, $P = 1+2c/3$, $\pi_i = (P-c)q_i = (1-c/3)(1-c/3) = (1-c)^2/9$, $\Pi = 2(1-c)^2/9$. $CS = Q^2/2 = 2(1-c)^2/9$, $W = \Pi + CS = 4(1-c)^2/9$

Post-merger, the merged firm acts as a monopolist and chooses q' to maximise $\pi' = [P(q')-c]q'$. By taking FOCs, $q' = 1-q'-c$, $q' = 1-c/2$, $P' = 1+c/2$, $\Pi' = (1-c)^2/4$, $CS' = q'^2/2 = (1-c)^2/8$, $W' = \Pi' + CS'$ $= 3(1-c)^2/8$

Price increases because the merged firm faces less price elastic demand than either of the pre merger insiders since consumers can no longer switch to a different producer in response to an increase in price. So the merged firm alienates fewer consumers by a given increase in price, and has greater incentive to increase price. Since demand is downward-sloping, total quantity falls, hence aggregate welfare falls because there are units for which consumers' valuation exceeds marginal cost that are no longer produced and consumed. The increase in price and decrease in quantity also cause a decrease in consumer surplus. So regulators should be inclined to block such a merger.

Suppose instead that there are $n > 2$ firms. Pre-merger, by taking FOCs: $q_i = -P(Q)-c/P'(Q) = P(Q)-c = 1-q_i-Q_{-i}-c$, ~~$q_i = 1-Q_{-i}-c/2$. Imposing symmetry, $q_i = 1-(n-1)q_i-c/2 = 1-c/2 - (n-1)q_i/2$~~ Imposing symmetry, $q_i = 1-nq_i-c$, $(n+1)q_i = 1-c$, $Q = n+1/n\,(1-c)$.

Merger in the absence of synergies is equivalent to the closure of 1 firm, i.e. a decrease in n by 1. ~~this results in a decrease in total output~~ By inspection, this results in an decrease in total quantity hence a ~~de~~crease in price, given downward sloping demand, hence ~~an increase in~~ a decrease in aggregate welfare and consumer surplus. Intuitively, ~~merger is~~ merger is equivalent to the closure of one insider because the merged firm ~~faces~~ has the same cost and the same product as each insider. A decrease in the number of firms results in a decrease in the competition each active firm faces. ~~Hence active firms have inc~~ So competition authorities should scrutinise mergers in Cournot since ~~these ~~ ~~dear~~ such mergers decrease aggregate welfare and consumer surplus (and also facilitate collusion).

These results do not necessarily obtain if merger realises synergies. Merger in Cournot that realises synergies may benefit consumers if the synergies are sufficiently large such that the merged firm has incentive to ~~lower pr~~ increase output because it has a very large margin. Farrell and Shapiro (1990) find that this is the case where the gross margin of the merged firm is larger than the sum of insiders' pre-merger margin, evaluated at pre-merger prices.

continued on next page

In Bertrand competition, merger to monopoly always hurts consumers if synergies are non-drastic since consumers face a higher price, but may increase aggregate welfare if the increase in profit due to synergies exceeds the decrease in consumer surplus due to the higher price. Merger short of monopoly has no effect on price hence no effect on consumer surplus since firms ~~are~~ still face perfectly elastic demand and the merged firm still prices no higher than rivals. Where products are differentiated, more sophisticated analyses are required. In general, ~~the~~ larger insiders' margin, ~~the~~ greater the diversion ~~and~~ ratio and ~~the~~ smaller ~~the~~ synergies, ~~the~~ imply higher prices and lower consumer surplus. In Cournot competition, in the absence of synergies, merger is equivalent to the closure of all but one insider, so decreases competition, increases prices, decreases quantities, hurts consumers and decreases aggregate welfare. Where there are synergies, the effect of merger on consumer surplus and aggregate welfare depends on the magnitude of these synergies. Much of what regulators in practice are concerned with is not well described by oligopoly. ~~Such con~~ Dynamic considerations, ~~or~~ environmental impacts, and concerns about the political power of firms also feed into antitrust evaluations of proposed mergers.

5: Arrows Theorem, strategy-proofness

8,9: Book work

73

Good essay
Could also consider collusion

10ai C1 maximises $u_1 = \frac{1}{3}\ln B + \frac{2}{3}\ln x_1 = \frac{1}{3}\ln(b_1+b_2) + \frac{2}{3}\ln x_1$ subject to $p_x x_1 + p_b b_1 = x_1 + b_1 \leq p_x \omega_1^x + p_b \omega_1^b = 600$.

Normally in pub goods, assume preference are quasi linear.

Solving by ~~Le~~ Lagrangian optimisation,

$\mathcal{L} = \frac{1}{3}\ln(b_1+b_2) + \frac{2}{3}\ln x_1 - \lambda(x_1+b_1-600)$

can be simplified by $x_1 = 600 - b_1$

$\mathcal{L}_{x_1} = \frac{2}{3x_1} - \lambda = 0$, $x_1 = \frac{2}{3\lambda}$

Check SOC for extra marks

$\mathcal{L}_{b_1} = \frac{1}{3b_1+3b_2} - \lambda = 0$, $b_1+b_2 = \frac{1}{3\lambda}$, $b_1 = \frac{1}{3\lambda} - b_2$

$\mathcal{L}_{\lambda} = -(x_1+b_1-600) = 0$, $x_1+b_1-600 = 0$

Concave function of b, x + ~~qt~~ linear constraint $\Rightarrow$ ~~MCT~~ ~~local max~~ stationary point is maximum

By substitution,

$\frac{2}{3\lambda} + \frac{1}{3\lambda} - b_2 - 600 = 0$, $\frac{1}{\lambda} = b_2 + 600$, $\lambda = \frac{1}{b_2+600}$

$x_1 = \frac{2}{3\lambda} = \frac{2b_2+1200}{3}$

$b_1 = \frac{1}{3\lambda} - b_2 = \frac{b_2+600}{3} - b_2 = \frac{600-2b_2}{3} = 200 - \frac{2b_2}{3}$ for $b_2 < 300$

ii By symmetry, C2 maximises $u_2 = \frac{1}{3}\ln B + \frac{2}{3}\ln x_1 = \frac{1}{3}\ln(b_1+b_2) + \frac{2}{3}\ln x_2$ subject to $p_x x_2 + p_b b_2 = x_2 + b_2 \leq p_x \omega_2^x + p_b \omega_2^b = 600$ by choosing $b_2 = 200 - \frac{2b_1}{3}$ for $b_1 < 300$

~~By substitution~~, Solving for the Nash equilibrium by substitution, $b_1 = 200 - \frac{2}{3}(200 - \frac{2}{3}b_1)$

– This is the right approach (rather than assuming $b_1 = b_2$)

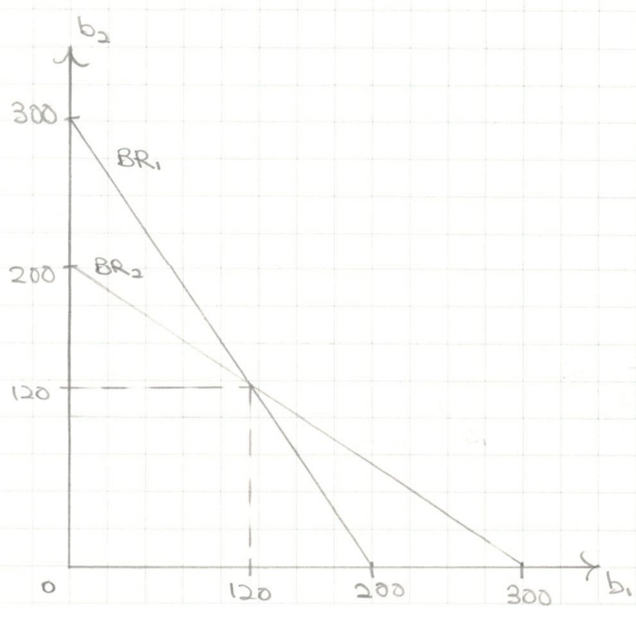
$= 200 - \frac{400}{3} + \frac{4}{9}b_1$.

$\frac{5}{9}b_1 = \frac{200}{3}$, $b_1 = \frac{1800}{15} = 120$.

$b_2 = 200 - \frac{2}{3}(120) = 120$.

Each college's best response ~~is decreasing in~~ decreases ~~with increasing~~ as the other college's choice increases. Intuitively, this is because each college enjoys diminishing marginal utility from books, so as the number of books contributed by the other college increases, each college has less incentive to buy books

At the Nash equilibrium, each college contributes 120 books. No college has a profitable deviation. The outcome will likely be different if colleges ~~de~~ chose their contributions sequentially rather than "simultaneously".



30

bi ~~AA~~ A provision of books (b_1^*, b_2^*) is Pareto-optimal iff ~~neither C1 nor C2 could be made better~~ there is no alternative provision (b_1', b_2') such that both C1 and C2 are better off.

Let $U = u_1 + u_2$, then a provision of books (b_1, b_2) is Pareto optimal if $b_1, b_2 = \arg\max_{b_1, b_2} U(b_1, b_2)$, ~~suppo~~ supposing that b_1 and b_2 fix x_1 and x_2, since then ~~at (b_1, b_2), it is not possi~~ there is no (b_1', b_2') that yields a higher U, which is necessary for Pareto improvement.

Technically, a Pareto optimum need not maximise U, should this trouble us?
Technically: maximise any weighted sum $U = \alpha u_1 + (1-\alpha) u_2$
Yes, the correct answer is the Samuelson condition, so this answer is not entirely correct.

ii $U = u_1 + u_2 = \frac{1}{3}\ln B + \frac{2}{3}\ln x_1 + \frac{1}{3}\ln B + \frac{2}{3}\ln x_2 = \frac{2}{3}[\ln B + \ln x_1 + \ln x_2]$
consider the maximisation problem ~~$\max U(b_1, b_2, x_1, x_2)$~~
$\max U(B, x_1, x_2) = \frac{2}{3}[\ln B + \ln x_1 + \ln x_2]$ subject to
$p_x x_1 + p_b b_1 = x_1 + b_1 \leq p_x \omega_1^x + p_b \omega_1^b = 600$ and $x_2 + b_2 \leq 600$.
Since multiplication by $3/2$ is a monotonic transformation, this maximisation problem coincides with the maximisation problem $\max \ln B + \ln x_1 + \ln x_2$ subject to $x_1 + b_1 \leq 600$ and $x_2 + b_2 \leq 600$

Mention Samuelson condition. : Samuelson condition $\Sigma MB = MC$ for quasilinear utilities

Solving by Lagrangian optimisation,
$\mathcal{L} = \ln B + \ln x_1 + \ln x_2 - \lambda_1(x_1 + b_1 - 600) - \lambda_2(x_2 + b_2 - 600)$
$\mathcal{L}_{b_1} = \frac{1}{b_1 + b_2} - \lambda_1 = 0$, $b_1 = \frac{1}{\lambda_1} - b_2$
$\mathcal{L}_{x_1} = \frac{1}{x_1} - \lambda_1 = 0$, $x_1 = \frac{1}{\lambda_1}$
$\mathcal{L}_{\lambda_1} = -(x_1 + b_1 - 600) = 0$, $\frac{2}{\lambda_1} - b_2 = 600$, $\lambda_1 = \frac{2}{600 + b_2}$
By substitution,
$b_1 = 300 - \frac{b_2}{2}$, $x_1 = 300 + \frac{b_2}{2}$
By symmetry,
$b_2 = 300 - \frac{b_1}{2}$, $x_2 = 300 + \frac{b_1}{2}$
Solving simultaneously,
$b_1 = 300 - \frac{1}{2}(300 - \frac{1}{2}b_1) = 150 + \frac{1}{4}b_1$, $\frac{3}{4}b_1 = 150$, $b_1 = 200$
$b_2 = 300 - \frac{1}{2}(200) = 200$
$x_1 = 300 + \frac{200}{2} = 400$, $x_2 = 300 + \frac{200}{2} = 400$ ✓

Here: $\Sigma MRS = MRT$
Intuitively: how much both Cs would pay for a marginal book is equal to the price of ~~#~~ a marginal book

$\Rightarrow x_1 + x_2 = 2B$
constraint: $x_1 + x_2 = 1200 - B$
$\Rightarrow B = 400$

2.5

No implication for x_1 and x_2 without knowing welfare weights

The optimal number of books ~~is~~ $B = b_1 + b_2 = 400$, greater than the equilibrium number ~~$B = b_1$~~ 240 (from a). This is because the provision of ~~one c~~ books by one college has a positive externality for the other, that each college fails to account for at equilibrium, so at equilibrium, each college provides books ~~at~~ until the point where the marginal private utility from an additional book is equal to the marginal private disutility from spending an additional £ on books. At this point, marginal social benefit exceeds marginal private benefit which is equal to marginal private cost which is equal to marginal social cost, so total welfare can be increased by each college's providing additional books.

"Colleges free ride on one another" ✓

The positive externality of C1 providing an additional book is given by $\partial u_2 / \partial b_1 = \frac{1}{3b_1 + 3b_2}$

ci under Lindahl pricing, the university supplies the optimal number of books and each college pays the university ~~a~~ the share of the total cost directly proportionate to its marginal utility (of books) at the optimum.

5 What is the numbers

MRS, not numbers

: what is the equilibrium (in numbers)

Each college supposes that its payment determines the supply of books, i.e. imagines other colleges contribute nothing, ~~each~~ i.e. each college max $u(x,b)$ st ~~$b=$~~ $x = 600 - pb$

ii A strategy-proof mechanism makes truthful reporting, in this case, of marginal utilities, a dominant strategy for each player. In other words, a pricing mechanism is strategy-proof iff no agent is better off reporting false preferences than reporting true preferences. Formally, let $\mathcal{F}$ denote some social choice function and $\succsim$ denote preferences, then $\mathcal{F}$ is strategy-proof iff $\mathcal{F}(\succsim_i, \succsim_{-i}) \succsim_i \mathcal{F}(\succsim_i', \succsim_{-i})$.

Strategy-proofness is relevant since if a ~~p~~ social choice function is not strategy-proof, each agent has incentive to ~~falsely~~ report false preferences, then the social choice function fails to yield the socially optimal outcome.

5 Relate to Lindahl

: Relate to Lindahl

ciii

Solve as though colleges treated price as Lindahl price

find NE as in (a ii)

$\frac{1}{3b_1+b_2} - \frac{2}{3}\frac{p}{600-pb_1} = 0$ . . .

Solve gen[?] case ~~eten~~ of (i) when consumers[?] ...

iii The magnitude of the per unit subsidy is equal to the magnitude of the positive externality evaluated at the social optimum, then each ~~firm~~ agent's marginal private cost of ~~supp~~ providing an additional book is equal to its marginal private benefit from an additional book, so each ~~firm~~ agent finds it optimal to provide its share of the optimal number of books.

Marginal private benefit from an additional book is given by ~~$\partial u_i/\partial b_i = 1/3b_i+3b_j$ for $i \neq j \in \{1,2\}$. At the optimum found in (b),~~ the marginal rate of substitution of money for books

$MRS_i = MU_i^b / MU_i^x = \partial u_i/\partial b_i \,/\, \partial u_i/\partial x_i = \frac{1}{3b_i+3b_j} \Big/ \frac{2}{3x_i}$

$= x_i / 2b_i + 2b_j$ for $i \neq j \in \{1,2\}$

At the optimum, $MRS_i = \frac{400}{2(200)+2(200)} = \frac{1}{2}$.

5

X (this is what you need) ~~(this is rel for i)~~ (i)

Here

The university must subsidise books by £1/2 per book. This mechanism is strategy-proof ~~since each~~ in the ~~college has~~ sense that it does not require ~~col~~ colleges to truthfully report their preferences, and each college finds it ~~op~~ optimal to play its part in the social optimum. This mechanism is costly to the university, so it maximises ~~welfa~~ surplus but involves a transfer of surplus from the university to the colleges. This cost can be recouped without distorting the equilibrium with a lump sum "tax".

12 overall