# Product Differentiation Notes

## Chamberlin (1929) Monopolistic Competition Model

- Parameters
  - Consider the simplified Chamberlin model of monopolistic competition. Suppose that a large number of firms each produce a differentiated good that is a substitute for each of the goods produced by other firms, and compete in prices. Each firm faces downward-sloping demand and each firm's price has only a negligible effect on the demand faced by any other firm. Entry and exit are costless such that at (the long run) equilibrium, each active firm enjoys zero profit. Suppose that each firm has decreasing average cost, i.e. enjoys economies of scale, up to some minimum efficient scale.
- Analysis
  - The unique equilibrium is such that each firm produces quantity given by the point of tangency between its downward-sloping residual demand curve and its downward-sloping convex average cost curve. If the two curves do not intersect, then there is no profitable quantity and the firm exits the market. If the two curves intersect at multiple points, then there is some price (and corresponding quantity) which yields positive profit, the firm chooses this price and enjoys positive profit, entry occurs and residual demand of active firms decreases until the zero profit condition is satisfied. If the two curves meet at exactly one point (the point of tangency), the firm maximises profit by choosing the corresponding price, and enjoys zero profit.
- Result (Apparent Over-provision of Diversity)
  - Since demand is downward-sloping, this point of tangency necessarily fails to coincide with the minimum efficient scale (where average cost is neither increasing nor decreasing). Each firm fails to exhaust economies of scale. Chamberlin concludes that product diversity is over-provisioned, and that it would be socially optimal to have fewer firms collectively producing fewer unique goods, and each producing a greater quantity at lower average cost.
- Evaluation (Assumptions)
  - The Chamberlin model of monopolistic competition assumes that there is a large number of firms, and each firm competes equally with all other firms, hence each firm treats its choices as having a negligible effect on each other firm's choices and thus as having a negligible strategic effect on its own profit. Each firm treats itself as a monopolist facing downward-sloping (residual) demand, with no need to consider the potential reaction of competitors. This assumption is not entirely realistic. In reality, even in industries with a very large number of firms, each firm tends to have a small number of close competitors. Each firm's choices has a non-negligible effect on each of its close competitors, and the former must thus account for their likely reactions. For example, in the restaurant industry, even though there is a very large number of restaurants in any market, each restaurant has a small number of close competitors which are physically nearby and/or offer similar dishes.
- Discussion
  - Economies of scale may exist because, for example, firms incur some fixed set up cost, such that the average fixed cost decreases with increasing quantity.

### Dixit and Stiglitz (1977) Criticism

- Discussion
  - The Chamberlinian argument that monopolistic competition yields over-provision of product diversity due to under-exploitation of economies of scale is inadequate because it fails to "consider the desirability of variety in an explicit form". (Dixit and Stiglitz, 1977) "With scale economies, resources can be saved by producing fewer goods and larger quantities of each. However, this leaves less variety, which entails some welfare loss." This welfare loss is neglected in the Chamberlinian argument. The Hotelling and Salop models of product differentiation treat the welfare loss due to a decrease in product diversity explicitly by modelling a "transport cost" incurred by consumers in consuming a product that is "distant" from them in "product space", i.e. a product whose characteristics do not perfectly match a consumer's preferences.

## Hotelling (1929) Linear Model

### Fixed Locations

- Parameters

- Consider the Hotelling linear model of product differentiation with fixed locations. Two firms, $A$ and $B$ located at $x_A = 0$ and $x_B = 1$ respectively in product space $x \in [0,1]$ each produces a good at common constant marginal cost $c$ and chooses price $p_X$ to maximise profit $\pi_X = (p_X - c)q_X$ where $q_X$ is the quantity of firm $X$ and $X \in \{A, B\}$. A unit mass of consumers is uniformly distributed in product space $x \in [0,1]$, and each consumer has unit demand and sufficiently high valuation $v$ such that he buys exactly one unit. Each consumer $i$ buys one unit at the lowest available total cost, where the total cost to consumer $i$ of buying from firm $X$ is given by $p_X + t|x_i - x_X|$, where $t$ is the unit transport cost.

- Analysis
  - The consumer $i$ at $x_i$ is indifferent between buying from firm $A$ and buying from firm $B$ iff $p_A + tx_i = p_B + t(1-x_i)$, which is iff $x_i = \frac{p_B - p_A + t}{2t}$. For each consumer $j$ at $x_j \in [0, x_i)$, $p_A + tx_i < p_B + t(1-x_i)$. These consumers buy from firm $A$. Given that consumers are uniformly distributed in product space, and there is a unit mass of consumers, $q_A = x_i = \frac{p_B - p_A + t}{2t}$. Given that consumers have sufficiently high valuations such that each consumer buys one unit, and there is a unit mass of consumers, $q_B = 1 - q_A = \frac{p_A - p_B + t}{2t}$. By substitution, $\pi_A = (p_A - c)\frac{p_B - p_A + t}{2t}$ and $\pi_A = (p_B - c)\frac{p_A - p_B + t}{2t}$. At the Nash equilibrium, each firm $X$ chooses price $p_X$ to maximise profit $\pi_X$, hence the first-order conditions for $p_X$ hold. $\frac{\partial \pi_A}{\partial p_A} = -\frac{p_A - c}{2t} + \frac{p_B - p_A + t}{2t} = 0$, $p_B - 2p_A + c + t = 0$. At Nash equilibrium, by symmetry, $p_A = p_B$, hence $p_A = p_B = c + t$. $q_A = q_B = \frac{1}{2}$, $\pi_A = \pi_B = \frac{t}{2}$.
- Result (Bertrand Paradox)
  - Price-setting firms with differentiated products escape the Bertrand trap. Because products are differentiated, an increase in price alienates only those marginal consumers who gain more from switching to a cheaper competitor than they lose from consuming a product that more poorly matches their preferences. Demand faced by each firm is not perfectly price-elastic, hence each firm can increase price to capture some share of consumer surplus as profit.
- Result (Transport Cost and Price)
  - Price increases with transport cost because an increase in transport cost causes consumers to be more sensitive to each firm's location in product space (relative to price) results in a decrease in the price-elasticity of demand faced by each firm. Each firm thus has greater incentive to increase price since fewer consumers are thereby alienated.
- Discussion
  - The Hotelling linear model of product differentiation can be understood as describing heterogenous consumer preferences which lie along a continuum. Consumers incur some disutility from the consumption of a product that does not perfectly satisfy their preferences, which increases with the difference between the product and the consumer's preference.
  - For example, a consumer's location in the product space could be understood as a consumer's preference over the brightness of colour of a car, such that each consumer incurs some disutility from buying a car that is either too brightly-coloured or too darkly-coloured.

## Fixed Prices

- Parameters
  - Consider the Hotelling linear model of product differentiation with fixed prices. Two firms $A$ and $B$ each produces a good at common constant marginal cost $c$ and sells at exogenous price $p$. Each firm $X$ chooses location $x_X$ in product space $x \in [0,1]$ to maximise profit $\pi_X = (p-c)q_X$, where $q_X$ is the quantity of firm $X$. A unit mass of consumers is uniformly distributed in product space $x \in [0,1]$, and each consumer has unit demand and sufficiently high valuation $v$ such that he buys exactly one unit. Each consumer $i$ buys one unit from the nearest firm. Suppose, without loss of generality, that $x_A \leq x_B$.
- Analysis
  - The unique Nash equilibrium is such that both firms choose $x_A = x_B = \frac{1}{2}$ and each firm captures half of all consumers and enjoys profit given by $\frac{p-c}{2}$. Suppose that firm $A$ chooses location $x_A$ less than $\frac{1}{2}$, then it captures all consumers $i$ located at $x_i \in [0, x_A]$ and half the consumers $j$ located at $x_j \in [x_A, x_B = \frac{1}{2}]$, hence $q_A < \frac{1}{2}$ and $\pi_A < \frac{p-c}{2}$. Firm $A$ has no incentive to choose location $x_A < \frac{1}{2}$. By symmetry, no firm $X$ has incentive to choose location $x_X \neq \frac{1}{2}$.
- Result (Under-provision of Product Diversity)
  - Product diversity is under-provided where prices are given exogenously. Where prices are given exogenously, choosing a location closer to the center does not have the indirect strategic effect of softening price competition, and only has the positive direct "market share" effect. Firms choose locations as close to the center as possible to maximise the share of demand captured.

## Continuous Two-Stage

- Parameters
  - Consider the Hotelling linear model of product differentiation. In the first of two stages, two firms $A$ and $B$ each choose location $x_A$ and $x_B$ respectively in product space $x \in [0, 1]$. In the second stage, each firm $X$ chooses price $p_X$ to maximise profit $\pi_X = (p_X - c)q_X$ given common knowledge of both firms' locations, where $c$ is the common constant marginal cost and $q_X$ is the quantity of firm $X$. A unit mass of consumers is uniformly distributed in product space $x \in [0, 1]$, and each consumer has unit demand and sufficiently high valuation $v$ such that he buys exactly one unit. Each consumer $i$ buys one unit at the lowest available total cost, where the total cost to consumer $i$ of buying from firm $X$ is given by $p_X + t(x_i - x_X)^2$, where $t$ is the unit transport cost. Quadratic transport costs are assumed to eliminate discontinuities in each firm's residual demand function. Suppose, without loss of generality, that $x_A \leq x_B$.
- Analysis
  - The unique subgame-perfect equilibrium is such that $x_A = 0$, $x_B = 1$ and $p_A = p_B = c + t$.
  - The marginal effect of firm $A$'s choosing, in the first stage, location closer to the center (and closer to $B$) on firm $A$'s profit can be decomposed into a direct effect and a strategic effect. By the envelope theorem, $\frac{d\pi_A}{dx_A} = \frac{\partial \pi_A}{\partial x_A} + \frac{\partial \pi_A}{\partial p_A}\frac{dp_A}{dx_A} + \frac{\partial \pi_A}{\partial x_B}\frac{dx_B}{dx_A} + \frac{\partial \pi_A}{\partial p_B}\frac{dp_B}{dx_A} = \frac{\partial \pi_A}{\partial x_A} + \frac{\partial \pi_A}{\partial p_B}\frac{dp_B}{dx_A}$. Since $x_A$ and $x_B$ are chosen simultaneously hence $\frac{dx_B}{dx_A} = 0$ and in equilibrium, first-order condition $\frac{\partial \pi_A}{\partial p_A} = 0$ holds. Hence $\frac{d\pi_A}{dx_A} = \frac{\partial \pi_A}{\partial x_A} + \frac{\partial \pi_A}{\partial p_B}\frac{dp_B}{dx_A}$. Firm $A$'s choice of location affects firm $B$'s optimal choice of price, which in turn affects firm $A$'s profit.
  - Under the given strategy profile, if firm $A$ chooses location closer to the center (and closer to $B$), it captures a greater share of consumers hence enjoys greater profit, ceteris paribus. So choosing has a positive direct effect on firm $A$'s profit. If firm $A$ chooses location closer to the center, firm $B$ faces more price-elastic demand since the transport cost incurred by consumers in buying from firm $A$ decreases, and consumers are more willing to switch to firm $A$ in response to an increase in price by firm $B$, firm $B$ decreases price, some consumers switch from firm $A$ to firm $B$ and firm $A$ decreases price, hence firm $A$'s profit decreases. So choosing has a negative strategic effect on firm $A$'s profit. Given quadratic transport costs, it can be shown that the strategic effect dominates the direct effect, hence each firm has incentive to choose location as far from the other as possible.
  - Given that $x_A = 0$ and $x_B = 1$, it can be shown that each firm maximises profit by choosing $p = c + t$.
- Evaluation (Result)
  - In reality, firms tend not to maximally differentiate. For example, retail outlets are concentrated in malls and city centres, and professional services firms are concentrated in business districts, often within walking distance of each other. The concentration of firms can be explained within the Hotelling linear model of product differentiation as the result of concentrated demand, an absence of price competition, and/or positive externalities between firms. Positive externalities include cost externalities and demand externalities. Geographical concentration of firms increases incentive for private and public provision of common installations and trade centers. Geographic concentration of firms also reduces search costs for consumers, hence increases aggregate demand and demand for each firm.
- Discussion
  - The Hotelling linear model of product differentiation can be understood as describing heterogenous consumer preferences which lie along a continuum. Consumers incur some disutility from the consumption of a product that does not perfectly satisfy their preferences, which increases with the difference between the product and the consumer's preference.
  - For example, a consumer's location in the product space could be understood as a consumer's preference over the brightness of colour of a car, such that each consumer incurs some disutility from buying a car that is either too brightly-coloured or too darkly-coloured.

## Continuous Two-Stage, Concentrated Demand

- Parameters
  - Consider the Hotelling linear model of product differentiation given above. Suppose instead that the product space is $x \in [-1, 2]$. Continue to suppose that a unit mass of consumers is uniformly distributed in the interval $x \in [0, 1]$, and no consumers exist except within this interval.
- Analysis
  - Since no consumers exist beyond the initial product space $x \in [0, 1]$, the extension of the product space is not relevant to firms' decisions, hence firms choose locations $x_A = 0$ and $x_B$ =1, and prices $p_A = p_B = c + t$ as in the initial continuous two-stage Hotelling linear model.
- Result (Concentration)
  - Firms do not maximally differentiate in product space, but concentrate around the region of high demand. Suppose that demand becomes more concentrated in the center and firms are initially located at opposite extremes. The density of consumers at the margin (the location of the marginal consumer indifferent between buying from one firm

and buying from the other) is greater than when consumers are uniformly distributed. By choosing location closer to the center, a firm captures more consumers, such a move has a larger "direct" effect on that firm's profit, hence firms have greater incentive to choose location closer to the center. The trivial example given above is an extreme case of concentrated demand, but the reasoning is analogous for more plausible distributions.

- Evaluation (Anecdotes)
  - Anecdotally, the concentration of demand around city centres seems to be a key factor in the concentration of supermarkets around city centres.

## Discrete Two-Stage

- Parameters
  - Consider the Hotelling linear model of product differentiation. In the first of two stages, two firms $A$ and $B$ each choose location $x_A$ and $x_B$ respectively in product space $x \in \{0,1\}$. In the second stage, each firm $X$ chooses price $p_X$ to maximise profit $\pi_X = (p_X - c)q_X$ given common knowledge of both firms' locations, where $c$ is the common constant marginal cost and $q_X$ is the quantity of firm $X$. A unit mass of consumers is uniformly distributed in product space $x \in [0,1]$, and each consumer has unit demand and sufficiently high valuation $v$ such that he buys exactly one unit. Each consumer $i$ buys one unit at the lowest available total cost, where the total cost to consumer $i$ of buying from firm $X$ is given by $p_X + t(x_i - x_X)^2$, where $t$ is the unit transport cost. Quadratic transport costs are assumed to eliminate discontinuities in each firm's residual demand function. Suppose, without loss of generality, that $x_A \leq x_B$.
- Analysis
  - The unique subgame-perfect equilibrium is such that $x_A = 0$, $x_B = 1$ and $p_A = p_B = c + t$.
  - If, in the first stage, firms $A$ and $B$ choose $x_A = x_B$, then both firms' products are entirely undifferentiated, hence the Bertrand paradox holds and each firm chooses price $p_A = p_B = c$ and enjoys zero profit. If, in the first stage, firms $A$ and $B$ choose $x_A \neq x_B$, then $x_A = 0$ and $x_B = 1$.
  - The consumer $i$ at $x_i$ is indifferent between buying from firm $A$ and buying from firm $B$ iff $p_A + tx_i = p_B + t(1 - x_i)$, which is iff $x_i = \frac{p_B - p_A + t}{2t}$. For all consumers $j$ at $x_j \in [0, x_i)$, $p_A + tx_i < p_B + t(1 - x_i)$. These consumers buy from firm $A$. Given that consumers are uniformly distributed in product space, and there is a unit mass of consumers, $q_A = x_i = \frac{p_B - p_A + t}{2t}$. Given that consumers have sufficiently high valuations such that each consumer buys one unit, and there is a unit mass of consumers, $q_B = 1 - q_A = \frac{p_A - p_B + t}{2t}$. By substitution, $\pi_A = (p_A - c)\frac{p_B - p_A + t}{2t}$ and $\pi_A = (p_B - c)\frac{p_A - p_B + t}{2t}$. At the Nash equilibrium, each firm $X$ chooses price $p_X$ to maximise profit $\pi_X$, hence the first-order conditions for $p_X$ hold. $\frac{\partial \pi_A}{\partial p_A} = -\frac{p_A - c}{2t} + \frac{p_B - p_A + t}{2t} = 0$, $p_B - 2p_A + c + t = 0$. At Nash equilibrium, by symmetry, $p_A = p_B$, hence $p_A = p_B = c + t$. $q_A = q_B = \frac{1}{2}$, $\pi_A = \pi_B = \frac{t}{2}$.

# Non-Entry

- Parameters
  - Consider the Bertrand free-entry model. In the first stage, a sufficiently large number of potential entrants choose whether to enter the market at fixed set-up cost $F$. In the second stage, the active firms each produce a homogenous good at common constant marginal cost $c = 0$ and compete in prices. Demand is downward-sloping. Suppose for simplicity that the demand function is $Q(p) = 1 - p$ hence inverse demand is $P(Q) = 1 - Q$. Each active firm's payoff is given by its net (of fixed set-up cost) profit, and the payoff of non-entrants is normalised to zero.
- Analysis
  - For intermediate levels of $F$, the unique subgame-perfect equilibrium is such that no firm enters the market. In the second stage, if there is only one active firm, it chooses price $p$ to maximise profit $\pi = (p - c)Q(p) = p(1 - p)$. At equilibrium, the active firm chooses monopoly price $p^M = \frac{1}{2}$ and enjoys monopoly profit $\pi^M = \frac{1}{4}$. Social surplus is given by the excess of consumers' valuations over marginal cost for all units produced and consumed. Social surplus $W = \int_{q=0}^{q^M=\frac{1}{2}} (P(q) - c)dq = \frac{3}{8}$. Suppose that $\frac{1}{4} < F < \frac{3}{8}$, then the payoff of this active firm is negative, and this firm would be better off if it did not enter the market.
- Result (Under-provision of Product Diversity)
  - In equilibrium, there is under-provision of product diversity, since no products are produced and sold where it would be socially optimal for at least one product to be produced and sold. No firm enters the market but it is socially optimal for at least one firm to enter because of the non-appropriability of social surplus, i.e. some positive share of the social surplus resulting from entry accrues to consumers.
- Discussion
  - The business stealing effect is not relevant in this case because the equilibrium number of active firms is zero, there are no competitors from which an entrant would steal business. In general, the non-appropriability of social surplus results from the inability of firms to perfectly price-discriminate.

# Salop (1979) Free Entry Model

- Parameters
  - Consider the Salop circular model of free entry with differentiated products. In the first of two stages, a sufficiently large number of potential entrants choose whether to enter a market at fixed set-up cost $F$. In the second stage, the $n$ active firms are evenly distributed around a circle of diameter $1$ and each active firm $i$ produces a product at common constant marginal cost $c$ and chooses price $p_i$ to maximise profit $\pi_i$. A unit mass of consumers is uniformly distributed on the circumference of the circle. Each consumer has unit demands, has sufficiently high valuation such that he always buys one unit of the product, and always buys from the firm for which the total cost is lowest. The total cost to the consumer located at $x$ of buying one unit of product from firm $i$ located at $x_i$ is equal to $p_i + t\min\{|x-x_i|, 1-|x-x_i|\}$, where $t$ is the unit transport cost. The payoff of each active firm is given by its net (of fixed set-up cost) profit, and the payoff of each non-entrant is normalised to $0$.
- Analysis
  - In the second stage subgame, if there are $n$ active firms, the consumer $x$ units clockwise of firm $i$ is indifferent between buying from firm $i$ and firm $i+1$ iff $p_i + tx = p_{i+1} + t(\frac{1}{n} - x)$, $x = \frac{p_{i+1}-p_i+\frac{t}{n}}{2t}$. By symmetry, the consumer $y$ units (of distance) anticlockwise of firm $i$ is indifferent between buying from firm $i$ and firm $i-1$ iff $y = \frac{p_{i-1}-p_i+\frac{t}{n}}{2t}$. Quantity of firm $i$, $q_i = D_i(p_{i-1}, p_i, p_{i+1}) = \frac{p_{i+1}-p_i+\frac{t}{n}}{2t} + \frac{p_{i-1}-p_i+\frac{t}{n}}{2t}$. Profit of firm $i$, $\pi_i = (p_i - c)q_i$. By taking first-order conditions, then by symmetry, it can be shown that at the subgame Nash equilibrium is such that each firm $i$ chooses $p_i = c + \frac{t}{n}$, has quantity $q_i = \frac{1}{n}$ and enjoys profit $\pi_i = \frac{t}{n^2}$.
  - In the first stage, disregarding the integer constraint on the number of active firms, the subgame-perfect equilibrium is such that the equilibrium number of active firms $n^e$ satisfies the free-entry condition $\frac{t}{(n^e)^2} - F = 0$, i.e. net profit is equal to zero. If net profit is greater than zero, then each non-entrant would be better off if it had entered the market. If net profit is less than zero, than each active firm would be better off if it had not entered the market. $n^e = \sqrt{\frac{t}{F}}$. By substitution, subgame-perfect equilibrium price, $p_e = c + \sqrt{tF}$.
- Result (Transport Cost)
  - Equilibrium price and number of firms increases with increasing transport cost. An increase in transport costs (which represents consumers' "product-sensitivity") causes each consumer to be more sensitive to the distance in product space between himself and each firm (which represents the closeness of each firm's product to this consumer's preference), relative to price. Demand for each firm is thus less price-elastic, and firms are able to increase price and thereby extract a greater share of consumer surplus as profit. Gross profit increase, net profit increases, hence incentive to enter increases and entry occurs.
- Result (Set-up Cost)
  - Equilibrium price increases with increasing set-up cost, and equilibrium number of firms decreases with increasing set-up cost. Since firms' products are mutual substitutes, a decrease in the number of active firms results in an increase in gross (of set-up cost) profit. Since, at equilibrium, each firm enjoys zero net (of set-up cost) profit, an increase in set-up cost is matched by an increase in gross profit at equilibrium, hence a decrease in the equilibrium number of firms. A decrease in the number of firms causes a decrease in the price-elasticity of each active firm since adjacent firms are, on average, further from each consumer, hence each consumer is less able and willing to switch from one firm to another in response to an increase in the former's price. Firms optimally respond to the decrease in price-elasticity by increasing price since a smaller quantity is thereby forgone.
- Result (Welfare)
  - Social welfare $W$ is equal to the sum of consumer surplus $CS$ and producer surplus $PS$. Consumer surplus is given by the excess of consumers' valuation over total cost (of consumption, equal to the sum of price and transport cost) for all units consumed. Producer surplus is given by net profit, which is the excess of price over marginal cost for all units produced, less fixed set-up cost. Since consumers' valuations and marginal cost are given exogenously, maximisation of social welfare consists in the minimisation of the sum of transport costs and entry costs. It can be shown that the socially optimal number of firms $n^o = \frac{1}{2}\sqrt{\frac{t}{F}} = \frac{n^e}{2}$. In equilibrium, there are twice as many firms as is socially optimal. Under the Salop circular model of product differentiation, the business stealing effect which results in a bias toward over-provision of product diversity dominates the non-appropriability of social surplus effect which results in a bias toward under-provision of product diversity. Entry occurs beyond the socially optimal level because each firm imposes a negative externality on all other firms which it does not account for in deciding whether to enter, hence entry is (just) profitable to the marginal entrant but decreases social welfare.
- Result (Market Size)
  - Suppose that there is a mass of consumers $S$. In the second stage, the subgame Nash equilibrium price is independent of market size, since the effect of market size is modeled as a uniform scaling of demand. By symmetry,

subgame Nash equilibrium quantity of each firm (taking the number of active firms as exogenous) is directly proportionate to market size. Since price hence price-cost margin is independent of market size, and quantity is directly proportionate to market size, an increase in market size (directly) causes a directly proportionate increase in gross (of set-up cost) profit.
- Since, in the subgame-perfect equilibrium, gross profit is equal to set-up cost, such that the marginal entrant is indifferent between entry and non-entry, the subgame-perfect equilibrium is such that the number of active firms increases with market size such that the increase in profit due to the increase in market size is exactly offset by the decrease in profit due to the increase in competition. The number of active firms increases less than proportionately to the increase in market size firm size increases with market size.

# Schmalensee (1978) Product Proliferation Model

- Parameters
  - Consider the Schmalensee model of entry deterrence by product proliferation. An incumbent monopolist $I$ chooses to launch $n_I$ brands at fixed set-up cost $F$ per brand, which are evenly distributed along the circumference of a Salop circle. Then, a potential entrant $E$ chooses whether to enter the industry at fixed set-up cost $F$ and at the same price as the incumbent (for example, because the incumbent always matches a lower price and a higher price fails to maximise the entrant's post-entry profit).
- Analysis
  - If the potential entrant enters the market, it locates between two incumbent brands, and, by symmetry, captures half the demand of consumers between the two adjacent incumbent brands. Pre-entry, the incumbent's net profit per brand $\pi_I = \frac{p-c}{n_I} - F$. Launching $n_I$ brands is profitable for the incumbent iff $n_I < \frac{p-c}{F} \equiv n^C$. Post-entry, the entrant's net profit $\pi_E = \frac{p-c}{2n_I} - F$. Entry is unprofitable for the potential entrant iff $n_I < \frac{p-c}{2F} = \frac{n^C}{2}$. Launching $n_I$ brands is profitable for the incumbent and renders entry unprofitable iff $\frac{n^C}{2} < n_I < n^C$.
- Result
  - An incumbent can profitably deter entry by product proliferation to strategically crowd the product space. If an incumbent attempts to deter entry by product proliferation, product diversity is not necessarily over-provided, as in the Salop circular model of product differentiation with free entry. A "marginal entrant" (by the incumbent) enjoys positive profit because the free entry zero (net) profit condition does not bind. Then, this is potentially greater than the business stealing effect (the decrease in other brand's profit due to a decrease in output, i.e. holding their margins fixed), so the total effect on welfare is positive. There is no "output effect" on consumer welfare because consumers consume the same quantity regardless. "Price effects" are irrelevant to welfare because they constitute only a transfer of surplus.
- Evaluation (Commitment)
  - Entry deterrence by product proliferation is only possible if the incumbent can credibly commit to not re-positioning or withdrawing a brand post-entry. This assumption is realistic given that brand re-positioning is typically costly, the cost of launching a brand is typically sunk hence not recoverable upon withdrawal.
- Case Study (Ready-to-Eat Cereal)
  - The ready-to-eat breakfast cereal market in the U.S. was heavily concentrated from 1950 to 1972, with a $4$-firm concentration ratio of $85\%$ and a $6$-firm concentration ratio of $95\%$. Throughout this period, no new firm achieved a non-negligible market share. Incumbents enjoyed very high profits, that could not apparently be explained as compensation for risk-bearing, given limited fluctuation in sales and profits. Over this period the $6$ leading incumbents introduced over $80$ brands into distribution beyond test market. Minimum efficient scale in this industry is thought to be small, at $3-5\%$ market share, neither patents, ownership of raw material sources, production knowhow, advertising, nor capital costs seems to constitute a significant barrier to entry (by introducing a new brand) (especially for the potential entrants Pet and Colgate, which are large, diversified food processing firms).

# Shaked and Sutton (1982) Vertical Differentiation Model

- Parameters
  - Consider the Shaked and Sutton model of vertical differentiation. Each of two firms, indexed by $X \in \{A, B\}$ produces a good of quality $s_X$ at common constant marginal cost $c$, and chooses to sell at price $p_X$ to maximise profit $\pi_X = (p_X - c)q_X$, where $q_X$ is the quantity of firm $X$. There is a unit mass of consumers. Each consumer $i$ has unit demands, and sufficiently high valuation $v$ such that he always buys one unit of product from either firm. If consumer $i$ buys one unit from firm $X$, his utility is given by $v - p_X + \theta_i s_X$, where $\theta_j$ is this consumer's marginal utility of quality. Consumers choose which firm to buy from to maximise utility. $\theta$ is distributed uniformly on the interval $[0, 1]$ in the population. Suppose, without loss of generality, that $s_A \geq s_B$.

- Analysis
  - The consumer $i$ is indifferent between buying from firm $A$ and buying from firm $B$ iff $v - p_A + \theta_i s_A = v - p_B + \theta_i s_B$, $\theta_i = \frac{p_A - p_B}{s_A - S_B}$. For each consumer $j$ such that $\theta_j \in [\theta_i, 1]$, $v - p_A + \theta_j s_A \geq v - p_B + \theta_j s_B$, utility is maximised by buying from firm $A$. Given that $\theta$ is distributed uniformly on the interval $[0,1]$ in the population, $q_A = 1 - \theta_i = 1 - \frac{p_A - p_B}{s_A - s_B}$. Since there is a unit mass of consumers and each consumer buys exactly one unit of product, $q_B = 1 - q_A = \frac{p_A - p_B}{s_A - s_B}$. $\pi_A = (p_A - c)(1 - \frac{p_A - p_B}{s_A - s_B})$, $\pi_B = (p_B - c)\frac{p_A - p_B}{s_A - s_B}$. At the Nash equilibrium, each firm $X$ chooses $p_X$ to maximise $\pi_X$, hence the first-order conditions for $p_A$ and $p_B$ hold. $\frac{\partial \pi_A}{\partial p_A} = (1 - \frac{p_A - p_B}{s_A - s_B}) - \frac{p_A - c}{s_A - S_B} = 0$, $s_A - s_B - 2p_A + p_B + c = 0$, $\frac{\partial \pi_B}{\partial p_B} = \frac{p_A - p_B}{s_A - s_B} - \frac{p_B - c}{s_A - s_B} = 0$, $p_A - 2p_B + c = 0$. Solving simultaneously, $p_A = c + \frac{2(s_A - s_B)}{3}$, $p_B = c + \frac{s_A - s_B}{3}$. $q_A = \frac{2}{3}$, $q_B = \frac{1}{3}$, $\pi_A = \frac{4(s_A - s_B)}{9}$, $\pi_B = \frac{s_A - s_B}{9}$.
- Result (Asymmetry)
  - The high quality firm has higher price, greater quantity, and higher profit than the low quality firm. Vertical differentiation results in asymmetry, unlike horizontal differentiation, because all consumers prefer the high quality product to the low quality product, all else being equal, whereas in the case of horizontal differentiation, some consumers prefer one good and other consumers prefer another.
  - The high quality firm is always better off than the low quality firm. If firms choose quality, the "first-mover" maximises profit by choosing the highest possible quality, and always enjoys greater profit than the "second mover" that optimally responds by choosing the lowest possible quality.
- Result (Maximal Differentiation)
  - The profits of both firms are increasing with the magnitude of vertical differentiation $s_A - s_B$, hence both firms have incentive to maximally differentiate, firm $A$ has incentive to increase quality and firm $B$ has incentive to decrease quality.
  - For firm $A$, the total effect of an increase in quality can be decomposed into a direct "market share" effect and an indirect strategic "competition effect". $\frac{d\pi_A}{ds_A} = \frac{\partial \pi_A}{\partial s_A} + \frac{\partial \pi_A}{\partial p_A}\frac{dp_A}{ds_A} + \frac{\partial \pi_A}{\partial s_B}\frac{ds_B}{ds_A} + \frac{\partial \pi_A}{\partial p_B}\frac{dp_B}{ds_A} = \frac{\partial \pi_A}{\partial s_A} + \frac{\partial \pi_A}{\partial p_B}\frac{dp_B}{ds_A}$ since $s_A$ and $s_B$ are chosen simultaneously hence $\frac{ds_B}{ds_A} = 0$ and at the subgame Nash equilibrium, firm $A$ chooses $p_A$ to maximise $\pi_A$ hence the first-order condition $\frac{\partial \pi_A}{\partial p_A} = 0$. An increase in $s_A$ has a positive direct "market share" effect, $\frac{\partial \pi_A}{\partial s_A}$ since firm $A$ thereby captures greater demand and enjoys greater profit, taking price hence price-cost margin as given. An increase in $s_A$ also has a positive indirect strategic "competition" effect $\frac{\partial \pi_A}{\partial p_B}\frac{dp_B}{ds_A}$ since differentiation between products increases, firm $A$ faces less price-elastic demand, and chooses a higher price (acts less aggressively) given any price chosen by firm $B$, firm $B$ optimally responds by choosing a higher price (acting less aggressively) given that prices in this model are strategic substitutes, which benefits firm $A$. Both firms face less price-elastic demand and increase price to extract a greater share of consumer surplus as profit when differentiation increases. Firm $A$ maximally differentiates because it thereby captures a greater share of the market and also softens price competition.
  - For firm $B$, the total effect of an increase in quality can be similarly decomposed. An increase in quality has a positive market share effect since firm $B$ thereby captures greater demand, but a negative competition effect since differentiation decreases, the products are closer substitutes, price competition is tougher, and firm $A$ chooses a lower, more aggressive price, which hurts firm $B$. In the Shaked and Sutton model, the competition effect dominates, and an increase in firm $B$'s quality hurts firm $B$. Firm $B$ maximally differentiates to soften price competition.