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Logic Collection

1a: Suppose for reductio that

$\nvDash_B \Diamond\Box P \to \Box\Diamond P$ ①

From ①, by definition of B-validity,

there is some B-model $M = \langle W, R, I \rangle$ and some world $w \in W$ such that

$V_M(\Diamond\Box P \to \Box\Diamond P, w) = 0$ ②

From ②, by → clause

$V_M(\Diamond\Box P, w) = 1$ ③

$V_M(\Box\Diamond P, w) = 0$ ④

From ③, by derived ◇ clause,

$V_M(\Box P, v) = 1$ for some $v \in W$ such that Rwv ⑤

From ⑤, by □ clause

$V_M(P, u) = 1$ for all $u \in W$ such that Rvu ⑥

From ④, by □ clause

$V_M(\Diamond P, v') = 0$ for some $v' \in W$ such that Rwv' ⑦

From ⑦, by derived ◇ clause

$V_M(P) = 0$ for all $u' \in W$ such that $Rv'u'$ ⑧

~~For all B-Models M, R is symmet~~ since M is a B-model,

R is ~~symmetric on~~ reflexive and symmetric on W ⑨

From ⑥, ⑨,

$V_M(P, w) = 1$ ⑩ (All $v \in W$ such that Rwv are such that Rvw)

From ⑧, ⑨

$V_M(P, w) = 0$ ⑪

⑩, ⑪ contradict, reject ①

$\vDash_B \Diamond\Box P \to \Box\Diamond P$. (by reductio)

Q1: 75
Q5: 98
Q2: 54
Overall: 76% V good

~~i.~~ $\Diamond \Box P \to \Box \Diamond P$ r

1 0 1 0 0 ~~0 0~~ 1 1

$\Box P$ a $\Diamond P$ b

1 1   0 0

The S4-model $M = \langle W, R, I \rangle$ is a countermodel

$W = \{r, a, b\}$

$R = \{\langle r,a \rangle, \langle r,b \rangle, \ldots\}$

$I(P,r) = I(P,a) = 1$

$I(\alpha, w) = 0$ for all other sentence letter - world pairs α, w

$V_M(\Diamond\Box P \to \Box\Diamond P, r) = 0$ (by definition of S4-valuation)

need to spell this out a bit more

$\nvDash_{S4} \Diamond\Box P \to \Box\Diamond P$ (by definition of S4-validity)

ii Suppose for reductio that

$\nvDash_{S4} (\sim\Box P \to \Box Q) \to \Box(\sim\Box P \to \Box Q)$ ①

From ①, by definition of S4-validity,

There is some S4-model $M = \langle W, R, I \rangle$ and some world $w \in W$ such that

$V_M((\sim\Box P \to \Box Q) \to \Box(\sim\Box P \to \Box Q), w) = 0$ ②

From ②, by $\to$ clause

$V_M(\sim\Box P \to \Box Q, w) = 1$ ③

$V_M(\Box(\sim\Box P \to \Box Q), w) = 0$ ④

From ④, by $\Box$ clause

$V_M(\sim\Box P \to \Box Q, v) = 0$ for some $v \in W$ such that Rwv ⑤

From ⑤, by $\to$ clause

$V_M(\sim\Box P, v) = 1$ ⑥

$V_M(\Box Q, v) = 0$ ⑦

From ⑥, by $\sim$ clause

$V_M(\Box P, v) = 0$ ⑧

From ⑦ by $\Box$ clause,

$V_M(Q, v') = 0$ for some $v' \in W$ such that Rvv' (9)

From (8), by □ clause

$V_M(P, v'') = 0$ for some $v'' \in W$ such that Rvv'' (10)

Since M is an S4-model, ??

R is reflexive and transitive on W (11)

From (9), (11)

$V_M(Q, v') = 0$ for some $v' \in W$ such that Rwv' (12)

From (10), (11)

$V_M(P, v'') = 0$ for some $v'' \in W$ such that Rwv'' (13)

From (12), by □ clause

$V_M(\Box Q, w) = 0$ (14)

From (13), by □ clause

$V_M(\Box P, w) = 0$ (15)

From (15), by ~ clause

$V_M(\sim\Box P, w) = 1$ (16)

From (14), (16), by → clause

$V_M(\sim\Box P \to \Box Q, w) = 0$ (17)

(3), (17) contradict, reject (1)

$\vdash_{S4} (\sim\Box P \to \Box Q) \to \Box(\sim\Box P \to \Box Q)$ (by reductio)

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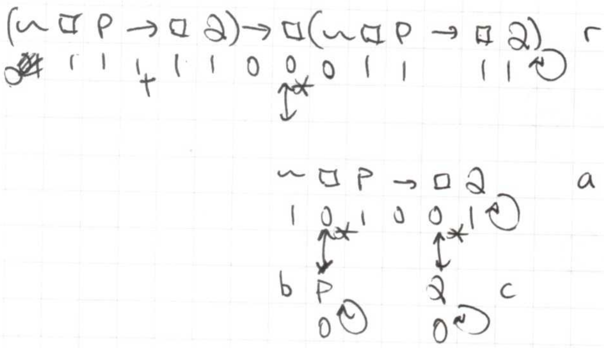
~~~□P~~



try to put the diagram and reasoning on the same page
~~~

The B-model $\mathcal{M} = \langle W, R, I \rangle$ is a counter model

$W = \{r, a, b, c\}$ need to fill in more, to show it's symmetric

$R = \{\langle r, a\rangle, \langle a, b\rangle, \langle a, c\rangle, \ldots\}$

$I(P, r) = I(P, a) = I(Q, r) = I(Q, a) = 1$

$I(\alpha, w) = 0$ for all other sentence letter - world pairs α, w

$V_{\mathcal{M}}((\sim\Box P \to \Box Q) \to \Box(\sim\Box P \to \Box Q), r) = 0$ (by definition of B-valuation)

$\nvDash_B (\sim\Box P \to \Box Q) \to \Box(\sim\Box P \to \Box Q)$ (by definition of B-validity).

b: i Not every X-model is an S5-model since ~~not~~ it is not necessary that R is reflexive on W in an X-model $\mathcal{M} = \langle W, R, I \rangle$, and R is reflexive on W for all S5-models.

Consider the X-model $\mathcal{M} = \langle W, R, I \rangle$ need to explain why this is an X-model

$W = \{r\}$, $R = \emptyset$

R is not reflexive on W, hence $\mathcal{M}$ is not an S5-model.

ii Suppose ~~for reductio that~~ that

$V_{\mathcal{M}}(\Diamond\phi, u) = 1$ ①

Suppose further for reductio that

$V_{\mathcal{M}^*}(\Box\Diamond\phi, u) = 0$ ②

From ①, by derived $\Diamond$ clause

$V_{\mathcal{M}}(\phi, v) = 1$ for some $v \in W$ such that Ruv ③

From ② by $\Box$ clause

$V_{\mathcal{M}^*}(\Diamond\phi, v') = 0$ for some $v' \in W$ such that R^*uv'. ④

From ④, by derived $\Diamond$ clause

$V_{\mathcal{M}^*}(\phi, u') = 0$ for all $u' \in W$ such that $R^*v'u'$ ⑤

From ③, since $R \subseteq R^*$

$V_{\mathcal{M}^*}(\phi, v) = 1$ for some $v \in W$ such that R^*uv ⑥

From ④, ⑥, by definition of X-model, yes, but what you write afterwards doesn't make sense

$V_M^*(\phi, v) = 1$ for some $v \in W$ such that $Rv'v$ ⑦

but already have

From ⑤

$V_M^*(\phi, v) = 0$ ⑧

⑦, ⑧ contradict, reject ②

If $V_M(\Diamond\phi, u) = 1$, then $V_M^*(\Box\Diamond\phi, u) = 1$

iii ~~For all S5-model~~

Suppose that $M = \langle W, R, I \rangle$ is an S5-model ①

Suppose further for reductio that M is not an X-model ②

From ②, ~~there is S1~~

there is some t, v, u $\in W$ such that ~~$\langle t,v \rangle, \langle t,u \rangle \in R$ and $\langle u,v \rangle$~~

$\langle t, v \rangle \in R$ ③

$\langle t, u \rangle \in R$ ④

$\langle u, v \rangle \notin R$ ⑤

From 4, by symmetry of R on W in S5-model M,

$\langle u, t \rangle \in R$ ⑥

From ③, ⑥, by transitivity of R on W is S5-model M,

$\langle u, v \rangle \in R$ ⑦

⑤, ⑦ contradict, reject ②

If M is an S5-model, it is an X-model.

If $\models_X \phi$ then ~~$V_M(\phi, w)$~~ ϕ is true in all X-models. Since all S5-models are X-models

ϕ is true in all S5-models, hence $\models_{S5} \phi$.

iv S4: $\Box\phi \to \Box\Box\phi$

Axiom schema is not valid.

The X-model $M = \langle W, R, I \rangle$ is a countermodel

$W = \{r, a, b\}$

$R = \{\langle r, a \rangle, \langle a, b \rangle, \langle b, b \rangle\}$

not an X-model, e.g.
have $\langle r, a \rangle \in R$ but not $\langle a, a \rangle \in R$
$\langle r, a \rangle \in R$

$I(P, a) = 1$

$I(\alpha, w) = 0$ for all other sentence letter-world pairs α, w

$V_M(\Box P \to \Box\Box P, r) = 0$

~~$\vDash_{S4} \Box P \to \Box\Box$~~ $\nvDash_X \Box P \to \Box\Box P$

S4 axiom schema is valid only if R is transitive on W, which is not necessarily the case in X-models.

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c We might think that belief is transparent: if P believes that X, then P believes that P believes that X, just as we might think that knowledge is transparent. If this is the case, then $\Box\phi$ implies $\Box\Box\phi$, and X appears inappropriate ~~for~~ for ~~as~~ ~~an i~~ the given interpretation.

But it is controversial whether or not belief is transparent in this way. ~~A~~ For example, a caveman might believe that if he sticks his hand in a fire, it will hurt (and therefore refrain from doing so) but fail to believe that he believes this (perhaps because he is epistemologically unsophisticated and does not possess a notion of belief * ~~or because he~~). ~~In which if belief * that~~

If the caveman's belief about fire does not imply that he has beliefs about what he believes, then ~~#~~ it does not count against X as an appropriate logic for the given interpretation, since we would not demand that $\vDash_X \Box\phi \to \Box\Box\phi$.

A good pt, but need to say a bit more

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75%

~~Sai $KV_I(\phi)$ - $I(\alpha)$ iff if ϕ~~

For any trivalent interpretation I

KV_I is the function that assigns to each wff exactly one of $1, 0,$ or $\#$ such that for any wffs ϕ and ψ,

$KV_I(\phi) = I(\phi)$ if ϕ is a sentence letter

$KV_I(\sim\phi) = 1$ ~~iff~~ if $KV_I(\phi) = 0$ ~~and~~
$\quad 0$ if $KV_I(\phi) = 1$
$\quad \#$ otherwise

$KV_I(\phi \to \psi) = 1$ if $KV_I(\phi) = 0$ or $KV_I(\psi) = 1$ or $KV_I(\phi) = KV_I(\psi) = \#$
$\quad 0$ if $KV_I(\phi) = 1$ and $KV_I(\psi) = 0$
$\quad \#$ otherwise

$KV_I(\phi \wedge \psi) = 1$ if $KV_I(\phi) = KV_I(\psi) = 1$
$\quad 0$ if $KV_I(\phi) = 0$ or $KV_I(\psi) = 0$
$\quad \#$ otherwise

$KV_I(\phi \vee \psi) = 1$ if $KV_I(\phi) = 1$ or $KV_I(\psi) = 1$
$\quad 0$ if $KV_I(\phi) = KV_I(\psi) = 0$
$\quad \#$ otherwise

??

ϕ is ~~tautos~~ K-valid iff $KV_I(\phi) = 1$ for all trivalent interpretations I

ϕ is a K-semantic consequence of Γ iff $KV_I(\phi) = 1$ for all trivalent interpretations I such that $KV_I(\gamma) = 1$ for all $\gamma \in \Gamma$.

ii

P	ΔP	$\sim(P \to \sim P)$				
1	1	1	1	0	0	1
0	0	0	0	1	1	0
#	0	0	#	1	#	#

Let $I_{P\#}$ be ~~the~~ some trivalent interpretation such that ~~$I(P) = \#$~~ $I_{P\#}(P) = \#$.

Let $C(\phi)$ ~~be #~~ $= n$, where n is the number of connectives, $\wedge$, $\vee$, and $\sim$ in PL-wff ϕ which contains only these connectives and sentence letter P.

Let $A(n)$ be the proposition that for all ϕ containg only the connectives $\wedge, \vee,$ and $\sim$, and sentence letter P $KV_{IP\#}(\phi) = \#$.

Base case

RTP $A(0)$

The only PL-wff containing no connectives and only sentence letter P is P.

$KV_{I_{P\#}}(P) = I_{P\#}(P) = \#$ (by definition of K valuation.

Inductive hypothesis

Assume that for given $n \in \mathbb{N}$, $\forall k\, A(k)$ where $k \leq n$

Inductive step

RTP, given IH, $A(n+1)$

For all ϕ ~~such that~~ (only $\wedge \vee \sim P$), such that $C(\phi) = n+1$, ϕ has the form (A) $\sim \psi$, (B) $\psi \wedge \chi$, (C) $\psi \vee \chi$.

(A)

Since $C(\phi) = n+1$, $C(\psi) = n$. By IH, $KV_{I_{P\#}}(\psi) = \#$

$KV_{I_{P\#}}(\phi) = \#$ (by ~~defi~~ definition of K-valuation. $\sim$ clause)

(B)

Since $C(\phi) = n+1$, $C(\psi) + C(\chi) = n$, $C(\psi) \leq n$, $C(\chi) \leq n$, By IH $KV_{I_{P\#}}(\psi) = KV_{I_{P\#}}(\chi) = \#$

$KV_{I_{P\#}}(\phi) = \#$ (by definition of K-valuation, $\wedge$ clause)

(C)

Since $C(\phi) = n+1$, $C(\psi) + C(\chi) = n$, $C(\psi), C(\chi) \leq n$, By IH, $KV_{I_{P\#}}(\psi) = KV_{I_{P\#}}(\chi) = \#$

$KV_{I_{P\#}}(\phi) = \#$ (by definition of K-valuation, $\vee$ clause).

By induction over the number of connectives in ϕ, $KV_{I_{P\#}}(\phi) = \#$ for all ϕ containing only connectives $\sim \wedge \vee$ and sentence letter P.

$KV_{I_{P\#}}(\Delta P) = 0$ (by inspection of earlier truth table).

Therefore, for no such ϕ is $KV_I(\phi) = KV_I(\Delta P)$ for each trivalent interpretation I.

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b) i Suppose ~~that~~ for reductio that

$\phi \nvDash_K \Delta\phi$ ①

From ①, by definition of K-semantic consequence

There is some trivalent interpretation I such that

$KV_I(\phi) = 1$ ②

$KV_I(\Delta\phi) = 0$ or # ③

ϕ	$\sim(\phi \to \sim\phi)$
1	1 1 0 0 1
0	0 0 1 1 0
#	0 # 1 # #

From ②, by ~~the~~ truth table above,

$KV_I(\Delta\phi) = 1$ ④

③, ④ contradict, reject ①

$\phi \vDash_K \Delta\phi$

ii Consider the countermodel I

such that $I(P) = \#$ ~~...~~

~~$\phi = P$~~ ~~...~~ $KV_I(\Delta P \vee \Delta\sim P) = \#$ (by definition of K-valuation)

$\nvDash_K \Delta P \vee \Delta\sim P$ (by definition of K validity)

break it down a bit more

iii Suppose for reductio that

$\nvDash_K$ ~~ϕ~~ $\Delta\phi \vee \sim\Delta\phi$ ①

From ①, by definition of K-validity,

There is some trivalent interpretation I such that

~~#~~ $KV_I(\Delta\phi \vee \sim\Delta\phi) = 0$ or # ②

From ②, by $\vee$ clause

$KV_I(\Delta\phi) = 0$ or # ③

$KV_I(\sim\Delta\phi) = 0$ or # ④

~~From ③, by truth table for Δ given earlier,~~

~~$KV_I(\phi) = 0$ or # ⑤~~

~~From ④, by truth table for Δ given earlier,~~

From (4), by ~ clause

$KV_I(\Delta\phi) = 1$ or # (5)

From (5), by truth table for Δ given earlier,

$KV_I(\phi) = 0$ or # (6)

From (5), by truth table for Δ given earlier,

$KV_I(\phi) = 1$ (7)

(6), (7) contradict, reject (1)

$\vDash_K \Delta\phi \vee \sim\Delta\phi$

ii)

ϕ	$\sim$	∇	$\sim$	ϕ
1	1	0	0	1
0	0	1	1	0
#	1	0	#	#

Suppose for reductio that

$\nabla\phi, \nabla\psi \nvDash_K \nabla(\phi \wedge \psi)$ (1)

From (1), by definition of K-semantic consequence

There is some trivalent interpretation I such that

$KV_I(\nabla\phi) = 1$ (2)

$KV_I(\nabla\psi) = 1$ (3)

$KV_I(\nabla(\phi \wedge \psi)) = 0$ or # (4)

From (4), by truth table for ∇ given above

$KV_I(\phi \wedge \psi) = 0$ (5)

From (2), by truth table for ∇ given above

$KV_I(\phi) = 1$ or # (6)

From (3), by truth table for ∇ given above

$KV_I(\psi) = 1$ or # (7)

From (6) and (7), by $\wedge$ clause

$KV_I(\phi \wedge \psi) = 1$ or # (8)

(5), (8) contradict, reject (1)

$\nabla\phi, \nabla\psi \vDash_K \nabla(\phi \wedge \psi)$.

say it's at the start of the Q

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c We ~~might~~ reason about the future with statements such as
" It will rain"
" It will definitely rain"
" It might rain"
and we might attempt to formalise these statements in Ł's three valued logic as
R
ΔR
∇R
Respectively. We might also interpret the three truth values as ~~expressing beliefs a~~ ~~1: it will happen~~ $\mathrm{ŁV}_I(R)=1$ if it will rain, $\mathrm{ŁV}_I(R)=0$ if it will not rain, and $\mathrm{ŁV}_I(R)=\#$ if whether or not it rains is indeterminate or genuinely chancey.

~~Under this sort of interpretation~~, We think the following inferences true, intuitively.
~~If~~ : ~~From~~ If it will rain, then it will definitely rain (i)
Either it will definitely rain or it is not the case that it will definitely rain (iii)
If it might rain and I might forget my umbrella, then it might be the case that it rains and I forget my umbrella (iv)
We also intuitively reject that ~~following inference~~
Either it definitely rains or definitely does not rain (ii) because we think that it is sometimes indeterminate whether or not it will rain.
If we accept these intuitive judgements, then Ł appears ~~to~~ appropriate for reasoning about the future, since the inferences (i), (iii) and (iv) are valid (shown in b) while the inference (ii) is not (also shown in (b)).

we might reject the intuitive ~~judg~~ force behind (iv)
Consider the inference
If this coin might land ~~tai~~ heads and this coin might land tails, then this coin might land both heads and tails.
The inference is intuitively implausible - we do not think a coin could land both heads and tails.
K is inappropriate or the formalisation as in (iv) is inadequate because it fails to capture the ~~semantica signi~~ ~~semantic significance~~ semantically significant content of landing heads and landing tails - that these are mutually exclusive. It is not clear that this counterexample should count against the appropriateness of K rather than against the adequacy of the formulation. If we include the implicit ~~$(\phi \vee$~~ $\sim(\phi \wedge \psi)$ in the set of premises, we can no longer infer $\nabla(\phi \wedge \psi)$.

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98% excellent!

2aii PC

a PC-term is ~~either~~ a constant, a, b... or a variable x, y...

PC-wff: if π is a n-place predicate and each of $\alpha_1 \dots \alpha_n$ is a term, then

$\pi\alpha_1 \dots \alpha_n$ is a PC-wff

if ϕ and ψ are PC-wff, then so are

$\sim\phi$, $\phi \to \psi$, $\phi \wedge \psi$, and $\phi \vee \psi$

If ϕ is a PC-wff and α is a variable, then

$\forall\alpha\phi$ is a PC-wff

only strings that can be shown to be PC-wffs by the above clauses are PC-wffs.

SOL: ~~If π~~ (Additional clauses)

If π is an n-place predicate variable, ~~then~~ and each of $\alpha_1 \dots \alpha_n$ is a term, then $\pi\alpha_1 \dots \alpha_n$ is a ~~PC~~ SOL-wff.

If π is an n-place predicate variable and ϕ is a ~~PC~~ SOL-wff, then $\forall\pi\phi$ is a ~~PC~~ SOL-wff.

In the case of second order logic, primitive wffs can be formed using predicate variables in addition to just predicates, and the quantifier $\forall$ can quantify over predicate variables in addition to ordinary variables.

ii A PC-model ~~is~~ is an ordered pair $\langle D, I \rangle$ ~~set~~ such that

D is some non-empty set (the domain)

and I is a function ~~from every constant into the D~~, which assigns each constant exactly one element of D. ~~(the interpretation function)~~

~~An SOL-model is an ordered pair $\langle D, I \rangle$ such that~~

~~D is some non empty set (the domain)~~

~~and I is a function from that assigns~~

~~every consta to every~~

and each n-place predicate an n-place relation over elements of D

~~An SOL-model is defined~~

the definition of an SOL model is identical to that of a PC-model.

iii A PC variable assignment g over PC-model $M = \langle D, I \rangle$ is some function such that $g(\alpha) \in D$ for all ~~sentence l~~ variables α ~~where D is th~~

An SOL variable assignment over model $M = \langle D, I \rangle$, is some function g ~~such~~ which assigns

each variable some element of D

each n-place ~~pred~~ predicate variable some n-place relation over elements of D.

~~Hence~~ SOL variable assignments assign semantic values to ~~of~~ predicate variables in addition to ordinary variables

iv The valuation function $V_{M,g}$ for a PC-model $M = \langle D, I \rangle$ and variable assignment g over M is the unique function that assigns 0 or 1 to each wff such that

$V_{M,g}(\pi \alpha_1 \ldots \alpha_n) = 1$ iff $\langle [\alpha_1]_{M,g}, \ldots, [\alpha_n]_{M,g} \rangle \in I(\pi)$ for terms $\alpha_1 \ldots \alpha_n$ and n-ary predicate π.

$V_{M,g}(\sim\phi) = 1$ iff $V_{M,g}(\phi) = 0$ for PC-wff ϕ

$V_{M,g}(\phi \to \psi) = 1$ iff $V_{M,g}(\phi) = 0$ or $V_{M,g}(\psi) = 1$ for PC-wffs ϕ and ψ

$V_{M,g}(\forall \alpha \phi) = 1$ iff $V_{M,g^{\alpha}_{u}}(\phi) = 1$ for all $u \in D$, where g^{α}_{u} is the variant assignment that differs from g only in assigning u to α.

SOL (additional clauses for predicate variables)

$V_{M,g}(\pi \alpha_1 \ldots \alpha_n) = 1$ iff $\langle [\alpha_1]_{M,g} \ldots [\alpha_n]_{M,g} \rangle \in g(\pi)$

$V_{M,g}(\forall \pi \phi) = 1$ iff $V_{M,g^{\pi}_{u}}(\phi) = 1$ for all n-place relations over D. ?

?

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b $\gamma_0^0 = Rab$

$\gamma_1^0 = (Rax_1 \wedge Rx_1b)$

$\gamma_2^0 = (Rax_2 \wedge (Rx_2x_1 \wedge Rx_1b))$

$\gamma_3^0 = (Rax_3 \wedge (Rx_3x_2 \wedge (Rx_2x_1 \wedge Rx_1b))$

$\gamma_0^1 = \exists x_1 Rab$

$\gamma_0^2 = \exists x_2 \exists x_1 Rab$

$\gamma_0^3 = \exists x_3 \exists x_2 \exists x_1 Rab$

$\gamma_1^1 = \exists x_1 (Rax_1 \wedge Rx_1b)$

$\gamma_2^2 = \exists x_2 \exists x_1 (Rax_2 \wedge (Rx_2x_1 \wedge Rx_1b))$

$\Gamma = \{\neg\gamma_n^n \text{ for } n>0\}$ and $\neg\gamma_0^0$

Suppose for reductio that

$\Gamma \nvDash \neg\forall X[(\forall x(Rax \to Xx) \wedge \forall x \forall y(Xx \wedge Rxy \to Xy)) \to Xb]$ ①

From ①, by definition of SOL-semantic consequence,

There is some ~~FO~~ SOL-model $M = \langle D, I \rangle$, and some variable assignment g such that

$V_{M,g}(\gamma) = 1$ for all $\gamma \in \Gamma$ ②

$V_{M,g}(\neg\forall X[(\forall x(Rax \to Xx) \wedge \forall x\forall y(Xx \wedge Rxy \to Xy)) \to Xb]) = 0$ ③

From ③, by $\neg$ clause

$V_{M,g}(\forall X[(\forall x(Rax \to Xx) \wedge \forall x\forall y(Xx \wedge Rxy \to Xy)) \to Xb]) = 1$ ④

From ④, by $\forall$ clause (predicate variables)

$V_{M,g^X_u}((\forall x(Rax \to Xx) \wedge \forall x\forall y(Xx \wedge Rxy \to Xy)) \to Xb) = 1$ for all n-place relations over D, (

⑤

ran out of time - bad luck

17/30

54%