Philosophical Logic Paper 170603

1a i Consider arbitrary trivalent interpretation $I$. $KV_I(\phi)=1$, ~~1~~ 0, or #. ~~By ~ clause, $KV_I(\sim\phi)$~~ Suppose $KV_I(\phi)=1$, then by ~ clause, $KV_I(\sim\phi)=0$, then it is not the case that $KV_I(\phi)=KV_I(\sim\phi)=1$. Suppose $KV_I(\phi)=0$, then it is not the case that $KV_I(\phi)=KV_I(\sim\phi)=1$. Suppose $KV_I(\phi)=\#$, then it is not the case that $KV_I(\phi)=KV_I(\sim\phi)=1$. By cases, generalisation, for all trivalent ~~the~~ interpretations $I$, it is not the case that $KV_I(\phi)=KV_I(\sim\phi)=1$. Then, vacuously, for all trivalent interpretations $I$ such that $KV_I(\phi)=KV_I(\sim\phi)=1$, $KV_I(\psi)=1$. By definition of $\models_1$, $\phi, \sim\phi \models_1 \psi$ (by generalisation, for all PL-wffs $\phi$ and $\psi$).

ii Consider ~~$\phi=P$~~ the following counterexample.

$\phi=P$, $\psi=Q$
$I(P)=\#$, $I(Q)=1$,
$I(\alpha)=0$ for all other sentence letters $\alpha$.

$KV_I(\psi)=I(\psi)=I(Q)=1$
$KV_I(\sim(\phi\wedge\sim\phi))=\#$
$\Rightarrow \psi \not\models_1 \sim(\phi\wedge\sim\phi)$

iii consider the following example

$\phi = P\wedge\sim P$, $\psi = Q$

Consider arbitrary trivalent interpretation $I$.
Suppose for conditional proof that
(1) $KV_I(P\wedge\sim P)=1$
(1), $\wedge \Rightarrow$
(2) $KV_I(P)=1$
(3) $KV_I(\sim P)=1$
(3), $\sim \Rightarrow$
(4) $KV_I(P)=0$
(2), (4), reductio $\Rightarrow$
(5) $KV_I(Q)=1$
(5), generalisation, definition of $\models_1 \Rightarrow$
(6) $P\wedge\sim P \models_1 Q$

Consider the following counter-interpretation.

$I(P)=\#$, $I(Q)=0$
$I(\alpha)=0$ for all other sentence letters $\alpha$

$KV_I(\sim\psi)=1$
$KV_I(\sim\phi)=\#$
$\Rightarrow$ ~~#~~ $\sim\psi \not\models_1 \sim\phi$

b i Consider the following counterexample.

$\phi=P$, $\psi=Q$
$I(P)=\#$, ~~$I(\phi)=$~~ $I(Q)=0$
$I(\alpha)=0$ for all other sentence letters $\alpha$

$KV_I(\phi)=\#$, $KV_I(\sim\phi)=\#$, $KV_I(Q)=0$
$\Rightarrow \{\phi, \sim\phi\} \not\models_{1\#} \psi$

ii Consider arbitrary trivalent interpretation $I$, arbitrary PL-wffs ~~&~~ $\phi, \psi$.
Suppose for conditional proof that
(1) $KV_I(\psi) = 1$ or #
Suppose for reductio that
(2) $KV_I(\sim(\phi\wedge\sim\phi)) \neq 1$ or #
$\Leftrightarrow KV_I(\sim(\phi\wedge\sim\phi))=0$
(2), $\sim \Rightarrow$
(3) $KV_I(\phi\wedge\sim\phi)=1$
(3), $\wedge \Rightarrow$
(4) $KV_I(\phi)=1$
(5) $KV_I(\sim\phi)=1$
(5), $\sim \Rightarrow$
(6) $KV_I(\phi)=0$
(4), (6), reductio $\Rightarrow$
(7) $KV_I(\sim(\phi\wedge\sim\phi)) = 1$ or #
(7), conditional proof, generalisation, definition of $\models_{1\#}$, generalisation $\Rightarrow$
(8) for all PL-wffs $\phi, \psi$, $\psi \models_{1\#} \sim(\phi\wedge\sim\phi)$

iii consider the following example.

$\phi=P$, $\psi=\sim(Q\wedge\sim Q)$

From (ii), $\phi \models_{1\#} \psi$.

Consider the following counter-interpretation

$I(P)=1$, $I(Q)=\#$

$KV_I(\sim\sim(Q\wedge\sim Q))=\#$, $KV_I(\sim P)=0$
$\Rightarrow \sim\psi \not\models_{1\#} \sim\phi$

c The two logical systems are Kleene's and Priest's. Priest's, prima facie, seems to be an odd candidate for ~~the~~ a logic for vague language because Priest is motivated ~~by~~ primarily by paradoxes such as the liar sentence "this sentence is false" which would seem to be both true and false.

The ~~result of~~ (i)-result is ex-falso quodlibet, the (ii)-result is the logical truth of the law of excluded middle. The (iii) result is the failure of a form of contraposition.

Date

Even for a vague language, it seems we would want EF2 to be a logical inference. Consider for example "Henry is tall" and "Henry is not tall". We think that there is a logical contradiction in this even if ~~Henry's~~ is a borderline case, ~~and Henry~~ so ~~whether~~ "Henry is tall" is ~~true or not~~ is vague. Then, ~~if both~~ we should infer from the two sentences ~~that~~ any other proposition (insofar as we think that EF2 is a valid logical inference in ordinary, non-vague cases). This is an advantage to $\models_1$.

Even for a vague language, we want the law of excluded middle to be a logical truth (logical consequence of any sentence). For example, we think "it is not the case that ~~both~~ Henry is both tall and not tall", seems to be a logical truth, for much the same reason that "Henry is tall" "Henry is not tall" seem to be logically inconsistent. This is an advantage to $\models_{1\#}$.

* law of non-contradiction

We think also that logical consequence should obey contraposition, ~~For exa~~ even in a vague language. For example, "~~#~~ John is tall" is a logical consequence of "Henry is tall, and if Henry is tall, so is John". And we think also that "not: Henry is tall, and if Henry is tall so is John" is a logical ~~con~~ consequence of "John is not tall". So neither $\models_1$ nor $\models_{1\#}$ fares well here.

It seems no notion of logical consequence based on Kleene truth tables could do better because it would make no sense to include 0 in the designated values, so $D_1 = \{1\}$ and $D_2 = \{1, \#\}$ are the only two sensible options.

Supervaluationist semantic consequence would do better. This obeys EF2, and validates CEM. LEM fails under $\models_1$ because of the problem of penumbral connections and the truth-functionality of Kleene valuation. ~~Kleene~~ Truth-functional Kleene valuations are blind to the logical relationship between "Henry is tall" ~~and~~ and "Henry is not tall" where Henry is a borderline case. Supervaluationist semantics would correctly identify the conjunction of these as false. ~~Supervaluationist semantic consequence obeys contraposition.~~ It is not obvious that supervaluationist semantic consequence obeys contraposition, but it fares better at least on EF2 and LEM.

2a i A SOL-model is defined as an ordered pair $\langle D, I \rangle$ where $D$ is some non-empty set, the domain, and $I$ is some function that assigns to each constant some member of $D$ and to each n-place predicate constant some n-ary relation over $D$.

This is identical to the definition of a PC-model

ii A SOL variable assignment $g$ for a given SOL-model $M$ ~~is some fun.~~ $= \langle D, I \rangle$ is some function that assigns to each variable some element of $D$ and to each n-place predicate variable ~~som~~ some n-ary relation over $D$.

This extends the definition of a PC-variable assignment by assigning extensions also to n-place predicate variables.

iii A SOL-valuation function, given SOL-model $M$ and variable assignment $g$ is the unique function from SOL-wffs to truth values (0,1) that satisfies the following clauses.

$V_{M,g}(\pi \alpha_1 \cdots \alpha_n) = 1 \stackrel{\text{df}}{=}$, where $\pi$ is a n-place predicate or a n-place predicate variable and each of $\alpha_1, \cdots, \alpha_n$ is a ~~constant~~ term, i.e. a constant or a variable, iff $\langle |\alpha_1|_{M,g}, \ldots, |\alpha_n|_{M,g} \rangle \in |\pi|_{M,g}$, where for each $\alpha$ of $\alpha_1, \ldots, \alpha_n$, $|\alpha|_{M,g} = I(\alpha)$ if $\alpha$ is a constant, $g(\alpha)$ if $\alpha$ is a variable and likewise $|\pi|_{M,g} = I(\pi)$ if $\pi$ is a predicate constant and $g(\pi)$ if $\pi$ is a predicate variable.

$V_{M,g}(\neg\phi) = 1$ iff $V_{M,g}(\phi) = 0$

$V_{M,g}(\phi \to \psi) = 1$ iff $V_{M,g}(\phi) = 0$ or $V_{M,g}(\psi) = 1$

$V_{M,g}(\forall\alpha\phi) = 1$ iff for all $u \in D$, $V_{M,g^{\alpha}_{u}}(\phi) = 1$, where $g^{\alpha}_{u}$ is the variable assignment that differs from $g$ only in assigning $u$ to $\alpha$ (a variable).

$V_{M,g}(\forall\pi\phi) = 1$ iff for all $U \in D^n$, $V_{M,g^{\pi}_{U}}(\phi) = 1$, where $g^{\pi}_{U}$ is the variable assignment that differs from $g$ only in assigning $U$ to $\pi$ (a n-place predicate variable), and $D^n$ is the set of n-ary ~~set~~ relations over $D$.

b P1: $\forall X \forall x [Xx \land \forall y_1 \forall y_2 (Xy_1 \land Ry_1y_2 \to Xy_2) \to \forall y (Dxy \to Xy)]$

P2: $Daz \land Ga$

P3: $\forall y_1 \forall y_2 (Gy_1 \land Ry_1y_2 \to By_2) \land \forall y_1 \forall y_2 (By_1 \land Ry_1y_2 \to Gy_2)$

C: $Gz \lor Bz$

R: ...2 is an offspring of ...1

D: ...2 is a descendant of ...1

G: ... has green eyes

B: ... has blue eyes

a: Adam

z: Zena

Consider arbitrary ~~\$~~ SOL-model $M = \langle D, I \rangle$ and arbitrary variable assignment $g$ for $M$.

Suppose for conditional proof that

(1) $V_{M,g}(P1) = 1$

(2) $V_{M,g}(P2) = 1$

(3) $V_{M,g}(P3) = 1$

~~Suppose for reductio that~~

~~(4) $V_{M,g}(C) = 0$~~

~~(3), $\land \Rightarrow$~~

~~(5) $V_{M,g}(\forall x \forall y (Gx \land Rxy \to By)) = 1$~~

~~(6) $V_{M,g}(\forall x \forall y (Bx \land Rxy \to Gy)) = 1$~~

~~(5), $\forall \Rightarrow$~~

~~(7) $\forall u,v \in D$: $V_{M,g^{x}_{u}{}^{y}_{v}}(Gx \land Rxy \to By) = 1$~~

~~(7), $\to$, $\lor \Rightarrow$~~

~~(8) $\forall u,v \in D$: $V_{M,g^{x}_{u}{}^{y}_{v}}(Gx \land Rxy \to (Gy \lor By)) = 1$~~

~~(6), $\forall$, $\to$, $\lor \Rightarrow$~~

~~(9) $\forall u,v \in D$: $V_{M,g^{x}_{u}{}^{y}_{v}}(Bx \land Rxy \to (Gy \lor By)) = 1$~~

~~(8), (9), $\to$, $\land$, $\lor \Rightarrow$~~

~~(10) $\forall u,v \in D$: $V_{M,g^{x}_{u}{}^{y}_{v}}(((Gx \lor Bx) \land Rxy \to (Gy \lor By)) = 1$~~

~~(10), $\forall \Rightarrow$~~

~~(11) $\forall$ $V_{M,g}(\forall x \forall y ((Gx \lor Bx) \land Rxy \to (Gy \lor By))) = 1$~~

~~(2), $\land$, $\lor \Rightarrow$~~

~~(12) $V_{M,g}((Ga \lor Ba)) = 1$~~

~~Let~~

~~(13) $U = I(G) \cup I(B)$~~

~~(10), (13), atomics, $\lor \Rightarrow$~~

~~(14) $\forall u,v \in D$: $V_{M,g^{X}_{U}{}^{x}_{u}{}^{y}_{v}}(Xx \land Rxy \to Xy) = 1$~~

(3), $\land \Rightarrow$

(5) $V_{M,g}(\forall y_1 \forall y_2 (Gy_1 \land Ry_1y_2 \to By_2)) = 1$

(6) $V_{M,g}(\forall y_1 \forall y_2 (By_1 \land Ry_1y_2 \to Gy_2)) = 1$

(5), $\forall \Rightarrow$

(7) $\forall u,v \in D$: $V_{M,g^{y_1}_{u}{}^{y_2}_{v}}(Gy_1 \land Ry_1y_2 \to By_2) = 1$

(7), $\to$, $\land$, $\lor \Rightarrow$

(8) $\forall u,v \in D$: $V_{M,g^{y_1}_{u}{}^{y_2}_{v}}(Gy_1 \land Ry_1y_2 \to (Gy_2 \lor By_2)) = 1$

(6), $\forall$, $\to$, $\land$, $\lor \Rightarrow$

(9) $\forall u,v \in D$: $V_{M,g^{y_1}_{u}{}^{y_2}_{v}}(By_1 \land Ry_1y_2 \to (Gy_2 \lor By_2)) = 1$

(8), (9), $\to$, $\land$, $\lor \Rightarrow$

(10) $\forall u,v \in D$: $V_{M,g^{y_1}_{u}{}^{y_2}_{v}}(((Gy_1 \lor By_1) \land Ry_1y_2 \to (Gy_2 \lor By_2)) = 1$

Let

(11) $U = I(G) \cup I(B)$

(10), (11), atomics, $\lor \Rightarrow$

(12) $\forall u,v \in D$: $V_{M,g^{X}_{U}{}^{y_1}_{u}{}^{y_2}_{v}}(Xy_1 \land Ry_1y_2 \to Xy_2) = 1$

(12), $\forall \Rightarrow$

(13) $V_{M,g^{X}_{U}}(\forall y_1 \forall y_2 (Xy_1 \land Ry_1y_2 \to Xy_2)) = 1$

(2), $\land$, $\lor \Rightarrow$

(14) $V_{M,g}(Ga \lor Ba) = 1$

(10), (14), atomics, $\lor \Rightarrow$

(15) $V_{M,g^{X}_{U}}(Xa) = 1$

let

(16) $u = I(a)$

(13), (15), (16), $\wedge \Rightarrow$

(17) $V_{M, g^{X\ x}_{u\ u}}(Xx \wedge \forall y_1 \forall y_2 (Xy_1 \wedge Ry_1y_2 \rightarrow Xy_2)) = 1$

(1), $\forall \Rightarrow$

(18) $V_{M, g^{X\ x}_{u\ u}}((Xx \wedge \forall y_1 \forall y_2 (Xy_1 \wedge Ry_1y_2 \rightarrow Xy_2)) \rightarrow \forall y(Dxy \rightarrow Xy)) = 1$

(17), (~~##~~16), $\rightarrow \Rightarrow$

(19) $V_{M, g^{X\ x}_{u\ u}}(\forall y(Dxy \rightarrow Xy)) = 1$

let

(20) $v = I(z)$

(19), (20), $\forall \Rightarrow$

(21) $V_{M, g^{X\ x\ y}_{u\ u\ v}}(Dxy \rightarrow Xy) = 1$

(2), $\wedge$, atomics $\Rightarrow$

(22) $V_{M, g^{X\ x\ y}_{u\ u\ v}}(Dxy) = 1$

(21), (22), $\rightarrow \Rightarrow$

(23) $V_{M, g^{X\ x\ y}_{u\ u\ v}}(Xy) = 1$

(23), atomics, $\vee \Rightarrow$

(24) $V_{M, g}(Gz \vee Bz) = 1$

(24), conditional proof, generalisation, definition of $\models_{SOL} \Rightarrow$

(25) $P1, P2, P3 \models_{SOL} C$.

c Quine argues that second order logic is set theory in ~~the~~ sheep's clothing. The ~~argument~~ argument proceeds in several steps. First, Quine notes that quantification over predicate variables is not entirely analogous to quantification over variables. For example $\exists x$ (x walks) means that some thing named by the name that x stands in place of in "x walks" walks. In other words, x has a name position in "x walks" and is a substitution-taking variable. But ~~A~~ $\exists X$ (Aristotle X) does not mean that Aristotle is or has some thing named by a predicate that X stands in the place of in "Aristotle X", not least because predicates do not seem to be names of any thing. What Aristotle is is a member of some extension of some predicate and what Aristotle has is some attribute associated with some predicate. So $\exists X$ (Aristotle • X) really means either (1) there is some predicate whose extension contains Aristotle or (2) there is some predicate associated with an attribute of Aristotle. Then, ~~that~~ quantification over predicate variables is in fact either quantification over sets or over attributes. But quantification over attributes is problematic because attributes are ~~inady~~ inadequately individuated, and it is evident from the semantics of SOL that SOL quantifies over (some sorts of sets).

~~If~~ understood in this way, SOL appears to have costly set-theoretical commitments. For example $\exists X \forall x Xx$ and ~~$\exists t$~~ $\exists X \forall x \neg Xx$ are ~~$\exists$~~ SOL-validities, so SOL appears committed to the existence of the universal set and the empty set. But this violates the topic neutrality of logic. We think that logic should be topic-neutral, i.e. that it should remain silent on the existence of, for example, dinosaurs, and ~~so~~ likewise should remain silent on the existence of the universal set and the empty set. So SOL is not really logic, but set theory in sheep's clothing.

~~But for~~ Quine interprets SOL too strongly. The logical validity of $\exists X \forall x Xx$ is simply the result that for all domains (which are sets), there is some set that contains all elements of the domain (namely the domain itself). So, for example, SOL is not committed to ~~#~~ the existence of ~~problem~~ the ~~non~~ set of all non-self-membered sets or even the universal power set. Similarly, ~~$\exists X \forall x \neg Xx$~~ the SOL validity of $\exists X \forall x \neg Xx$ simply means that for any domain, there is some ~~set that~~ subset that contains none of its elements. These are both ~~weak~~ very weak claims.

SOL is not committed to basic set-theoretic truths e.g. the existence of the two-membered set, because ~~$\exists x \exists y \exists y$~~ $\exists X \exists x_1 \exists x_2 (Xx_1 \wedge Xx_2 \wedge x_1 \neq x_2)$ is false in a single-membered domain.

So the ~~so~~ set-theoretic commitments of SOL are at best, very weak. This is not good reason to think that SOL is in fact set theory and not logic.

We have good reason to think SOL is logic and not set theory. First, SOL is a natural extension of PC, as evident from the definitions in (a). Second, SOL ~~can~~ can recognise "palpably" logical ~~vali~~ truths, consequences, and consistencies to do with infinity and ancestrals, that PC cannot, such as those in (b). So if we think that one of the primary functions of a logic is to identify such things, we should think SOL is logic and not set theory.

Date

4a) i Consider the following SC-countermodel.

$M = \langle W, \preccurlyeq, I\rangle$

$W = \{0, 1, 2, 3\}$

$\preccurlyeq_0 = \{\langle 1,2\rangle, \langle 1,3\rangle, \langle 2,3\rangle, \ldots\}$

The remaining pairs are those implied by reflexivity, transitivity, antisymmetry, ~~conn~~ connectivity, and the base assumption.

$\preccurlyeq_1 = \{\langle 0,2\rangle, \langle 0,3\rangle, \langle 2,3\rangle, \ldots\}$

$\preccurlyeq_2 = \{\langle 1,0\rangle, \langle 1,3\rangle, \langle 3,0\rangle, \ldots\}$

$\preccurlyeq_3 = \{\langle 2,1\rangle, \langle 2,0\rangle, \langle 1,0\rangle, \ldots\}$

$I(P,1)=1$, $I(Q,2)=1$, $I(R,2)=1$, $I(P,3)=1$, $I(P,2)=1$, $I(R,3)=1$, $I(\alpha,\omega)=0$ for all other sentence letters and worlds $\alpha, \omega$.

$V_M((P \,\Box\!\!\rightarrow (Q \,\Box\!\!\rightarrow R)) \rightarrow ((P\wedge Q) \,\Box\!\!\rightarrow R), 0) = 0$

ii $(P ⥽ (Q ⥽ R)) \rightarrow ((P\wedge Q) ⥽ R)$

$= \Box(P \rightarrow \Box(Q\rightarrow R)) \rightarrow \Box((P\wedge Q)\rightarrow R)$

~~$(1)\ \Box(Q\rightarrow R) \rightarrow (\Box Q \rightarrow \Box R)$ (K)~~

~~$(2)\ (P \rightarrow \Box(Q\rightarrow R)) \rightarrow (P \rightarrow (\Box Q \rightarrow \Box R))$ (1, PL)~~

~~$(3)\ \Box(P \rightarrow \Box(Q\rightarrow R)) \rightarrow \Box(P \rightarrow (\Box Q \rightarrow \Box R))$ (2, NEC, K, MP)~~

~~(4)~~

~~$(1)\ \Box(P \rightarrow \Box(Q\rightarrow R)) \rightarrow (\Box P \rightarrow \Box\Box(Q\rightarrow R))$ (K)~~

~~$(2)\ \Box\Box(Q\rightarrow R) \rightarrow \Box(Q\rightarrow R)$ (T)~~

~~$(3)\ \Box(P \rightarrow \Box(Q\rightarrow R)) \rightarrow (\Box P \rightarrow \Box(Q\rightarrow R))$ (1, 2, PL)~~

~~$(4)\ \Box(Q\rightarrow R) \rightarrow (\Box Q \rightarrow \Box R)$ (K)~~

~~$(5)\ \Box(P \rightarrow \Box(Q\rightarrow R)) \rightarrow (\Box P \rightarrow (\Box Q \rightarrow \Box R))$ (3, 4, PL)~~

~~$(6)\ (\Box P \rightarrow (\Box Q \rightarrow \Box R)) \rightarrow ((\Box P\wedge \Box Q) \rightarrow \Box R)$ (PL importation)~~

~~$(7)\ \Box(P \rightarrow \Box(Q\rightarrow R)) \rightarrow ((\Box P\wedge \Box Q) \rightarrow \Box R)$ (5, 6, PL syllogism)~~

~~$(8)\ \Box R \rightarrow R$ (T)~~

~~$(9)\ \Box(P \rightarrow \Box(Q\rightarrow R)) \rightarrow ((\Box P\wedge \Box Q) \rightarrow R)$ (7, 8, PL)~~

~~$\vdash_{SS} \Box(P \rightarrow \Box(Q\rightarrow R)) \rightarrow$~~

$(1)\ \Box(Q\rightarrow R) \rightarrow (Q \rightarrow R)$ (T)

$(2)\ (P \rightarrow \Box(Q\rightarrow R)) \rightarrow (P \rightarrow (Q\rightarrow R))$ (1, PL)

$(3)\ (P \rightarrow (Q\rightarrow R)) \rightarrow ((P\wedge Q) \rightarrow R)$ (PL importation)

$(4)\ (P \rightarrow \Box(Q\rightarrow R)) \rightarrow ((P\wedge Q) \rightarrow R)$ (2, 3, PL syllogism)

$(5)\ \Box(P \rightarrow \Box(Q\rightarrow R)) \rightarrow \Box((P\wedge Q) \rightarrow R)$ (4, NEC, K, MP)

$\vdash_{SS} (P ⥽ (Q ⥽ R)) \rightarrow ((P\wedge Q) ⥽ R)$

b. P1: $(M\vee B) \,\Box\!\!\rightarrow (S \wedge (B_1 \vee B_2))$

P2: $\Diamond M$

P3: $M \,\Box\!\!\rightarrow {\sim}B_1$

C: $M \,\Box\!\!\rightarrow B_2$

M: Mark leaves

B: Brix leaves

S: The band splits up

$B_1$: Brix goes solo

$B_2$: Brix returns to the states.

English

P1 involves a disjunctive antecedent that SC and CC cannot straightforwardly accommodate. We formalise this here in the naive way. In formalising English P2 as $\Diamond M$, we take the sort of modality in English P2 to be the ~~sa~~ same ~~meta~~ sort as in the other English premises.

~~$P1, P2, P3 \vDash_{SC} C$~~

~~Consider arbitrary SC-model $M=\langle W, \preccurlyeq, I\rangle$ and~~

~~an arbitrary world $w \in W$.~~

~~Suppose for conditional proof that~~

~~$(1)\ V_M(P1, w)=1$~~

~~$(2)\ V_M(P2, w)=1$~~

~~$(3)\ V_M(P3, w)=1$~~

~~Suppose for reductio that~~

~~$(4)\ V_M(C, w)=0$~~

~~$(2), \Diamond \Rightarrow$~~

~~$(5)\ \exists v\in W: V_M(M, v)=1$~~

~~$(5), \vee \Rightarrow$~~

~~$(6)\ \exists v\in W: V_M(M\vee B, v)=1$~~

~~(6), limit $\Rightarrow$~~

~~$(7)\ \exists u\in W: V_M(M\vee B, u)=1$ and $\forall u'\in W$ such that $V_M(M\vee B, u')=1$, $u \preccurlyeq_w u'$. Denote this world $u$.~~

~~$(1), (7), \Box\!\!\rightarrow \Rightarrow$~~

~~$(8)\ V_M(S\wedge(B_1\vee B_2), u)=1$~~

~~$(8), \wedge \Rightarrow$~~

~~$(9)\ V_M(B_1 \vee B_2, u)=1$~~

~~$(3), (8) \Rightarrow$~~

~~$(10)\ V_M({\sim}B_1, u)=1$~~

~~$(10), {\sim} \Rightarrow$~~

~~$(11)\ V_M(B_1, u)=0$~~

~~$(9), (11), \vee \Rightarrow$~~

~~$(12)\ V_M(B_2, u)=1$~~

~~$(12), \Box\!\!\rightarrow \Rightarrow$~~

~~$(13)\ V_M($~~

$P1, P2, P3 \nvDash_{SC} C$ , $P1, P2, P3 \nvDash_{CC} C$

Consider the following countermodel for both SC and CC.

$M = \langle W, \preccurlyeq, I\rangle$

$W = \{0, 1, 2\}$

$\preccurlyeq_0 = \{\langle 1,2\rangle, \ldots\}$

The remaining pairs are those implied by reflexivity, transitivity, antisymmetry, connectivity, and SC's base assumption.

$\preccurlyeq_1 = \{\langle 0,2\rangle, \ldots\}$

$\preccurlyeq_2 = \{\langle 0,1\rangle, \ldots\}$

$I(B,1) = I(S,1) = I(B_1,1) = I(M,2) = 1$

$I(Q, w) = 0$ for all other sentence letters and worlds $Q, w$.

Each premise evaluates (under either semantics) as true at world 0, but the conclusion evaluates as false in that world.

c We think intuitively that the English argument in (b) is valid. But neither SC nor LC semantics yields this result. This failure is apparently to do with the disjunctive antecedent in P1. Plausibly, the naive ~~for~~ formalisation of the English P1 is to blame. If ~~P1 is~~ instead formalised as $(M \Box\!\!\rightarrow (S \wedge (B_1 \vee B_2))) \wedge (B \Box\!\!\rightarrow (S \wedge (B_1 \vee B_2)))$, the conclusion is an SC-semantic consequence of the premises ~~and also for~~ (but not a LC semantic consequence). LC fails here because of the disjunctive ~~consequent~~ consequent in either the naive or the revised P1. A countermodel where there are two equally 0-close worlds, one in which $B_1$ is true and $B_2$ false and another in which $B_2$ is true and $B_1$ false ~~invalidates C at~~ is such that the premises are true at 0 but the conclusion is false. So SC appears to have an advantage here.

The revised formalisation of P1 seems suspicious. It abandons the form of the English premise and "replaces" a disjunction with a conjunction. ~~This is not a~~ It is suspicious, but not entirely idiosyncratic nor unmotivated. Naive formalisations are not always appropriate. For example, it would be inappropriate to formalise "there aint no cake" as $\neg\neg C$ and to think that it ~~is a logical con~~ logically implies "there is cake". For a closer example, consider "you are free to stay or go". This does not simply mean "you are free to do the action: stay or go" because that would be a logical consequence of "you are free to do the action stay" But we think "you are free to stay or go" false when spoken to an inmate who is free to stay but not to go. So what is meant by "you are free to stay or go" is "you are free to stay and you are free to go". This is apparently the case for the English P1, which means "if M then ... and also if B then ...". The revised formalisation ~~reverses close to~~ strays from the English syntax but is arguably closer to the English semantics.

This strategy will not work to rescue LC from the problem of disjunctive consequents because there is no reason to think that something different is meant by counterfactual conditionals with disjunctive consequents. Where LC seems to go wrong is in its failure to tie the truth of a counterfactual conditional to the truth of the consequent in some selected world as SC does (more explicitly in Stalnaker's presentation than in Sider's). This seems to be how we in fact evaluate counterfactual conditionals, namely by identifying some counterfactual world where the antecedent is true and evaluating the truth of the consequent there.

~~Treating~~ Formalisation with a strict conditional yields the intuitive result (regardless of whether we take the naive or the revised approach). But the strict conditional has the fault or defect of obeying importation. We do not think English counterfactual conditionals obey importation. For example. We do not think "if Bill had married Hilary and Bill had married Laura he would have a happy marriage" ~~is a be~~ is implied by "if Bill had married Laura ~~and~~ then if Bill had married Hilary ~~be~~ then he would have had a happy marriage" because Bill's marriage in the first case would be troubled by disputes between his two wives.

But a rejection of importation seems just as odd. ~~Consider~~ We think "if I had forgotten my umbrella then if it rains, then I would get soaked" implies "if I had forgotten my umbrella ~~then if~~ and it rains, then I would get soaked." What seems to go wrong with both ~~SC and~~ SC $\Box\!\!\rightarrow$ and $\rightarrow$ is that both are context insensitive, whereas English counterfactuals are context sensitive. In the Bill case, the English internal counterfactual conditional does not hold Bill's being married to Laura fixed, whereas in the rain case, the internal conditional holds my forgetting the umbrella fixed. No semantics (among those considered) can do both.

5a i consider arbitrary D-model $M = \langle W, R, I \rangle$, and arbitrary world $w \in W$.
Suppose for reductio
(1) $V_M(\Diamond(P \to P), w) = 0$
(1), $\Diamond$ $\Rightarrow$
(2) $\nexists u \in W, Rwu : V_M(P \to P, u) = 1$
(2) $\Rightarrow$
(3) $\forall u \in W, Rwu : V_M(P \to P, u) = 0$
seriality of $R$ on $W$ $\Rightarrow$
(4) $\exists u \in W : Rwu$. Denote this world $u$
(3), (4) $\Rightarrow$
(5) $V_M(P \to P, u) = 0$
(5), $\to$ $\Rightarrow$
(6) $V_M(P, u) = 1$
(7) $V_M(P, u) = 0$
(6), (7), reductio $\Rightarrow$
(8) $V_M(\Diamond(P \to P), w) = 1$
(8), generalisation, definition of $\models_D$ $\Rightarrow$
(9) $\models_D \Diamond(P \to P)$

consider the following K-countermodel

$M = \langle W, R, I \rangle$
$W = \{0\}$
$R = \varnothing$
$I(\alpha, w) = 0$ for all sentence letters and worlds $\alpha, w$.

$V_M(\Diamond(P \to P), w) = 0 \Rightarrow$
$\not\models_K \Diamond(P \to P)$.

ii consider arbitrary T-model $M = \langle W, R, I \rangle$ and arbitrary world $w \in W$.
Suppose for reductio
(1) $V_M(\Diamond(P \to \Box P), w) = 0$
(1), $\Diamond$ $\Rightarrow$
(2) $\forall u \in W, Rwu : V_M(P \to \Box P, u) = 0$
(2), $\to$ $\Rightarrow$
(3) $\forall u \in W, Rwu : V_M(P, u) = 1, V_M(\Box P, u) = 0$
(3), $\Box$ $\Rightarrow$
(4) $V_M(\Box P, w) = 1$
(3), reflexivity of $R$ on $W$ $\Rightarrow$
(5) $V_M(\Box P, w) = 0$
(4), (5), reductio $\Rightarrow$
(6) $V_M(\Diamond(P \to \Box P), w) = 0$
(6), generalisation, definition of $\models_T$ $\Rightarrow$
(7) $\models_T$ ~~$V_M(\Diamond(P \to$~~ $\Diamond(P \to \Box P)$

consider the following D-countermodel

$M = \langle W, R, I \rangle$
$W = \{0, 1\}$
$R = \{\langle 0, 1 \rangle, \langle 1, 0 \rangle\}$
$I(P, 1) = 1$, $I(\alpha, w) = 0$ for all other sentence letters and worlds $\alpha, w$

$V_M(\Diamond(P \to \Box P), 0) = 0 \Rightarrow$
$\not\models_D \Diamond(P \to \Box P)$

b i Given that $M$ is a D model, $R$ is serial on $W$, i.e. for all $u \in W$, exists ~~$u$~~ $w \in W$ such that $Ruw$. For all $u^* \in W^*$, by construction either $u^* \in W$ hence exists $u \in W$ such that ~~that~~ $Ru^*u$ ~~hence~~ hence by construction, exists $u \in W^*$ such that $R^*u^*u$ (~~give~~ $W \subset W^*$, $R \subset R^*$), or $u^* = x$ hence exists $u \in W^*$ namely $u$ such that $Rxu$, so for all $u^* \in W^*$, exists $u \in W^*$ such that $R^*u^*u$, so $R^*$ is serial on $W^*$.

Given that $W^* = W \cup \{x\}$, $W^*$ is some non-empty set. By construction, $I^*(\alpha, u)$ is a two-place function from all sentence letters and worlds $\alpha, u$ ($u \in W^*$) to truth values.

So $W^*$ is a MPL-model and a D-model.

ii Base case
consider arbitrary MPL-wff $\phi$ such that complexity $C(\phi) = 0$, consider arbitrary world $u \in W$. $\phi$ is some sentence letter $\alpha$. $V_M(\phi, u) = I(\alpha, u) = I^*(\alpha, u) = V_{M^*}(\phi, u)$ by definition of MPL-valuation and by construction of $I^*$. By generalisation, for all such $\phi$ and $u$, $V_{M^*}(\phi, u) = V_M(\phi, u)$

Induction Hypothesis
Given $n$, for all $m \leq n$, for all MPL-wff $\phi$ such that $C(\phi) = m$, for all $u \in W$, $V_M(\phi, u) = V_{M^*}(\phi, u)$.

Induction Step
Consider arbitrary MPL-wff $\phi$ such that $C(\phi) = n$. Consider arbitrary world $u \in W$. $\phi = {\sim}\psi$, $\psi \to \chi$, or $\Box\psi$.

Suppose $\phi = \Box\psi$, then $C(\psi) = n-1$. $V_{M^*}(\phi, u) = 1$ iff for all $v \in W^*$ such that $R^*uv$, $V_{M^*}(\psi, v) = 1$ iff ~~for all~~ (given that $u \neq x$) for all $v \in W$ such that $Ruv$, $V_{M^*}(\psi, v) = 1$ iff by IH for all $v \in W$ such that $Ruv$, $V_M(\psi, v) = 1$ iff $V_M(\phi, u) = 1$

Suppose $\phi = {\sim}\psi$, then $C(\psi) = n-1$, $V_{M^*}(\phi, u) = 1$ iff $V_{M^*}(\psi, u) = 0$ iff by IH $V_{M^*}(\psi, u) = 0$ iff $V_{M^*}(\phi, u) =$

Suppose $\phi = \psi \to \chi$, then $C(\psi) + C(\chi) = n-1$, $C(\psi)$, $C(\chi) < n$. $V_{M^*}(\phi, u) = 1$ iff $V_{M^*}(\psi, u) = 0$ or $V_{M^*}(\chi, u) =$ iff by IH $V_M(\psi, u) = 0$ or $V_M(\chi, u) = 1$ iff $V_M(\phi, u) = 1$.

By cases, generalisation, for all $\phi$ such that $C(\phi) = n$, for all $u \in W$, $V_{M^*}(\phi, u) = V_M(\phi, u)$.

By induction, for all MPL-wff $\phi$, for all $u \in W$,

$V_M^*(\phi, w) = V_M(\phi, w)$.

iii ~~If suppose $V_M$~~ consider arbitrary MPL-wff $\phi$. Suppose $V_M(\phi, w) = 0$. Then by the result of (ii) $V_M^*(\phi, w) = 0$. Then, given that $x$ only bears R to $w$, ~~for~~ $\nexists u \in W^*$ ~~s.t.~~ $Rxu$: $V_M^*(\phi, u) = 1$. Then, by derived $\Diamond$ clause, $V_M^*(\Diamond\phi, x) = 0$. ~~By generalisation, for all MPL-wff $\phi$, if $V_M(\phi, w) = 0$~~
By conditional proof, generalisation, for all MPL-wff $\phi$, if $V_M^*(\phi, w) = 0$ then $V_M^*(\Diamond\phi, x) = 0$

iv Consider arbitrary MPL-wff $\phi$. Suppose that $\phi$ is not D-valid. Then by definition, there exists D-model $M = \langle W, R, I \rangle$ and world $w \in W$ such that $V_M(\phi, w) = 0$. Then, by the result of (iii), there exists D-model $M^*$, defined as in the question, such that $V_M^*(\phi, x) = 0$. Then by definition, $\Diamond\phi$ is not D-valid. By conditional proof, if $\phi$ is not D-valid, then $\Diamond\phi$ is not D-valid. Then if $\Diamond\phi$ is D-valid, $\phi$ is D-valid. By generalisation, this is true of all MPL-wff $\phi$.

v No. $\Diamond(P \rightarrow \Box P)$ is T-valid (from aii) but $P \rightarrow \Box P$ is not. Consider the following T countermodel.

$M = \langle W, R, I \rangle$
$W = \{0, 1\}$
$R = \{\langle 0, 0 \rangle, \langle 1, 1 \rangle, \langle 0, 1 \rangle\}$
$I(P, 0) = 1$, $I(\alpha, w) = 0$ for all other sentence letters and worlds $\alpha, w$.

$V_M(P \rightarrow \Box P, 0) = 0 \Rightarrow$
$\nvDash_T P \rightarrow \Box P$

c ~~The in~~ An interpretation of $\Diamond$ as "it is compatible with the law that" implies the following. First, $\Box$ is interpreted as "it is required by the law that". Second, each world $w'$ accessible from each world $w$ is such that all the things that obtain in $w'$ are compatible with the actual law of $w$.

We think it is a logical truth that, for example, if this is an excellent answer then this is an excellent answer is compatible with the law. ~~More precisely, we think that "this is an excellent answer" is false (which is what is meant by assigning truth value 0) or "this is an excellent answer" is true~~
More generally, we think that such logical truths are compatible with the law and that this compatibility is logically necessary. In other words, it would be illogical for the law to prohibit logical necessities. So this (and result aii) seems to be reason to favor D over K. But we should be careful here. When we say that it would be illogical to prohibit logical necessities, we seem to mean that ~~a person~~ such prohibitions are irrational or senseless in a non-technical sense. But there is no reason to think that such prohibitions are strictly speaking illogical. ~~The~~ It certainly is physically and practically possible to construct if not impose some "damned if you do, damned if you do not" law, so this must be logically possible. So we do not want it to be a logical truth that "if this answer is excellent then this answer is excellent" is compatible with the law. Then a less restrictive system like K should be preferred. (T also validates $\Diamond(P \rightarrow P)$)