Microeconomic Analysis Problem Set 6

1a ~~The~~ Given that it is optimal to induce $e=1$, the principal P's optimisation problem is
min w_1, w_2, w_3 $E(w|e=1)$ subject to
Individual Rationality: $E(\sqrt{w}-e|e=1) \geq \bar{u}$ and
Incentive compatibility: $E(\sqrt{w}-e|e=1) \geq E(\sqrt{w}-e|e=0)$
This is equivalent to
min w_1, w_2, w_3 $w_1/4 + w_2/2 + w_3/4$ subject to
$\frac{1}{4}\sqrt{w_1} + \frac{1}{2}\sqrt{w_2} + \frac{1}{4}\sqrt{w_3} - 1 \geq 10$ and
$\frac{1}{4}\sqrt{w_1} + \frac{1}{2}\sqrt{w_2} + \frac{1}{4}\sqrt{w_3} - 1 \geq \frac{1}{3}(\sqrt{w_1}+\sqrt{w_2}+\sqrt{w_3})$

Linear obj fn $\Rightarrow$ convex obj fn
Concave constraint $\Rightarrow$ convex problem
$\Rightarrow$ KT FOC sufficient
Non-empty interior constraint set $\Rightarrow$
KT-FOC necessary
Participation constraint

Solving by Lagrangian optimisation,
$L = w_1/4 + w_2/2 + w_3/4 - \lambda_1(\sqrt{w_1}/4 + \sqrt{w_2}/2 + \sqrt{w_3}/4 - 11)$
$-\lambda_2(\sqrt{w_1}/4 + \sqrt{w_2}/2 + \sqrt{w_3}/4 - 1 - \frac{1}{3}(\sqrt{w_1}+\sqrt{w_2}+\sqrt{w_3}))$
FOC w_1: $\frac{1}{4} - \lambda_1(\frac{1}{8})w_1^{-1/2} - \lambda_2(-\frac{1}{24})w_1^{-1/2} = 0$
FOC w_2: $\frac{1}{2} - \lambda_1(\frac{1}{4})w_2^{-1/2} - \lambda_2(\frac{1}{12})w_2^{-1/2} = 0$
FOC w_3: $\frac{1}{4} - \lambda_1(\frac{1}{8})w_3^{-1/2} - \lambda_2(-\frac{1}{24})w_3^{-1/2} = 0$

Is there any argument for either IR binding or IC binding? Yes, from FOCs, not before FOCs

Do the KT FOCs include the CSs? Yes

Is it necessary to check the CSs? If not, why not? It is necessary

~~FOC w_1, FOC w_3 $\Rightarrow$ $w_1^* = w_3^*$~~
~~FOC w_1, FOC w_3 $\Rightarrow$ $w_1 = w_3$~~
~~Then, FOC w_1 is satisfied iff FOC w_3 is satisfied. Given that KT FOCs are necessary and sufficient for optimality FOC w_1 and FOC w_2 are together necessary and sufficient for optimality.~~

By inspection FOC w_1, FOC w_3 $\Rightarrow$ $w_1^* = w_3^*$
FOC w_1, w_2, w_3 $\Rightarrow$ $\lambda_2 \neq 0$ $\Rightarrow$ IC binds
Because $\lambda_2 = 0 \Rightarrow w_1 = w_2 = w_3 \Rightarrow$ IC fails

~~FOC w_1~~
CS λ_1: $\lambda_1 \geq 0$, $\sqrt{w_1}/4 + \sqrt{w_2}/2 + \sqrt{w_3}/4 - 1 \geq 10$,
$\lambda_1(\sqrt{w_1}/4 + \sqrt{w_2}/2 + \sqrt{w_3}/4 - 11) = 0$
CS λ_2: $\lambda_2 \geq 0$, $\sqrt{w_1}/4 + \sqrt{w_2}/2 + \sqrt{w_3}/4 - 1 \geq \frac{1}{3}(\sqrt{w_1} + \sqrt{w_2} + \sqrt{w_3})$
$\lambda_2(-\sqrt{w_1}/12 + \sqrt{w_2}/6 - \sqrt{w_3}/12 - 1) = 0$

IR binds. Suppose for reductio that IR does not bind, then $\sqrt{w_1}/4 + \sqrt{w_2}/2 + \sqrt{w_3}/4 - 1 > 10$. For sufficiently small ϵ, $w_1, w_2' = w_2 - \epsilon, w_3$ satisfies IR and IC and $E(w|e=1) > E(w'|e=1)$

$\lambda_1 > 0$, $\sqrt{w_1}/4 + \sqrt{w_2}/2 + \sqrt{w_3}/4 - 1 = 10$, $\sqrt{w_1}/2 + \sqrt{w_2}/2 = 11$,
$\sqrt{w_1} + \sqrt{w_2} = 22$

Suppose that IC does not bind, then $\lambda_2 = 0$.
FOC w_1 $\Rightarrow$ $\frac{1}{4} = \lambda_1(\frac{1}{8})w_1^{-1/2} \Rightarrow \sqrt{w_1} = \lambda_1/2$
FOC w_2 $\Rightarrow$ $\frac{1}{2} = \lambda_1(\frac{1}{4})w_2^{-1/2} \Rightarrow \sqrt{w_2} = \lambda_1/2$
$\Rightarrow w_1 = w_2 = w_3 \Rightarrow \sqrt{w_1}/4 + \sqrt{w_2}/2 + \sqrt{w_3}/4 - 1 < \frac{1}{3}(\sqrt{w_1}+\sqrt{w_2}+\sqrt{w_3})$
CS λ_2 fails. By reductio, IC binds.

What is the argument for IC binding

$\lambda_2 > 0$, $\sqrt{w_1}/4 + \sqrt{w_2}/2 + \sqrt{w_3}/4 - 1 = \sqrt{w_1}/2 + \sqrt{w_2}/2 - 1 = 10 =$
$\frac{1}{3}(\sqrt{w_1} + \sqrt{w_2} + \sqrt{w_3})$
$\sqrt{w_1} + \sqrt{w_2} = 22$, $\sqrt{w_1} + \sqrt{w_2} + \sqrt{w_3} = 30$, $\Rightarrow \sqrt{w_3} = 8 \Rightarrow$
$\sqrt{w_1} = 8 \Rightarrow \sqrt{w_2} = 14$
FOC w_1: $\frac{1}{4} - \lambda_1/64 + \lambda_2/192 = 0$
FOC w_2: $\frac{1}{2} - \lambda_1/56 - \lambda_2/168 = 0$
$\Rightarrow \lambda_1 = 22, \lambda_2 = 18$, CS hold

Is this check necessary?
Yes
If we find either $\lambda_1, \lambda_2 < 0$, then no soln exists

The optimal contract $(w_1^*, w_2^*, w_3^*) = (64, 196, 64)$

b) i. It is in the agent's interest to secretly destroy ~~reven~~ revenue in the outcome π_3 since wage is higher at the lower revenue π_2.

ii. It is never in the agent's interest to destroy revenue iff $w_1 \leq w_2 \leq w_3$.

Exactly correct, there are two new constraints

At the optimum, at least one of these inequalities is strict. Otherwise IC fails. If both inequalities hold with equality, the contract is equivalent to a fixed wage and it is in the agent's interest to choose $e=0$.

Suppose that $w_1 = w_2$. Then $w_1 = w_2 < w_3$, $\Rightarrow$ ~~then $\sqrt{w_1} = \sqrt{w_2}$~~
$\sqrt{w_1} = \sqrt{w_2} < \sqrt{w_3} \Rightarrow \frac{1}{4}\sqrt{w_1} + \frac{1}{2}\sqrt{w_2} + \frac{1}{4}\sqrt{w_3} = \frac{3}{4}\sqrt{w_1} + \frac{1}{4}\sqrt{w_3}$
$< \frac{1}{3}\sqrt{w_1} + \frac{1}{3}\sqrt{w_2} + \frac{1}{3}\sqrt{w_3} = \frac{2}{3}\sqrt{w_1} + \frac{1}{3}\sqrt{w_3} \Rightarrow$
$\frac{1}{4}\sqrt{w_1} + \frac{1}{2}\sqrt{w_2} + \frac{1}{4}\sqrt{w_3} - 1 < \frac{1}{3}(\sqrt{w_1} + \sqrt{w_2} + \sqrt{w_3})$, i.e. IC fails.
By reductio $w_1 < w_2$.

~~P's optimisation problem is~~
~~$\min_{w_1, w_2, w_3} w_1/4 + w_2/2 + w_3/4$ subject to~~
~~$\sqrt{w_1}/4 + \sqrt{w_2}/2 + \sqrt{w_3}/4 - 1 \geq 10$~~
~~$\sqrt{w_1}/4 + \sqrt{w_2}/2 + \sqrt{w_3}/4 - 1 \geq 1/3$~~
~~$w_1 \leq w_2$~~
~~$w_2 \leq w_3$~~

~~Solving by Lagrangian optimisation~~,

IR binds. Suppose for reductio that IR does not bind, i.e. $\sqrt{w_1}/4 + \sqrt{w_2}/4 + \sqrt{w_3}/4 - 1 > 10$. For sufficiently small ε, w_1, $w_2' = w_2 - \varepsilon$, w_3 satisfy IR, IC, ~~and~~ and $w_1 \leq w_2 \leq w_3$ given that $w_1 < w_2$, and $E(w \mid e=1) > E(w' \mid e=1)$. By reductio, IR binds.

At the optimum, $w_2 = w_3$. Otherwise some $w_2^{*\prime} > w_2$, $w_3' < w_3$ ~~satisf~~ satisfies IR, IC, ~~and~~ $w_1 \leq w_2 \leq w_3$, and ~~does~~ $E(w \mid e=1) > E(w' \mid e=1)$.

~~$w_1 < w_2 = w_3 \Rightarrow \sqrt{w_1}/4 + \sqrt{w_2}/2 + \sqrt{w_3}/4 = \sqrt{w_1}/4 + 3\sqrt{w_2}/4 > \sqrt{w_1}/3 + 2\sqrt{w_2}/3$~~

P's optimisation problem ~~reduces to~~ is
$\min_{w_1, w_2, w_3} w_1/4 + w_2/2 + w_3/4$ subject to
$\sqrt{w_1}/4 + \sqrt{w_2}/2 + \sqrt{w_3}/4 - 1 \geq 10$
$\sqrt{w_1}/4 + \sqrt{w_2}/2 + \sqrt{w_3}/4 - 1 \geq \frac{1}{3}(\sqrt{w_1} + \sqrt{w_2} + \sqrt{w_3})$
$w_1 \leq w_2$, $w_2 \leq w_3$

Solving by Lagrangian optimisation
$L = w_1/4 + w_2/2 + w_3/4 - \lambda_1(\sqrt{w_1}/4 + \sqrt{w_2}/2 + \sqrt{w_3}/4 - 11)$
$\quad - \lambda_2(-\sqrt{w_1}/12 + \sqrt{w_2}/6 - \sqrt{w_3}/12 - 1) + \mu_1(w_1 - w_2) + \mu_2(w_2 - w_3)$
FOC w_1: $1/4 - \lambda_1/8\, w_1^{-1/2} + \lambda_2/24\, w_1^{-1/2} + \mu_1 = 0$
FOC w_2: $1/2 - \lambda_1/4\, w_2^{-1/2} - \lambda_2/12\, w_2^{-1/2} - \mu_1 + \mu_2 = 0$
FOC w_3: $1/4 - \lambda_1/8\, w_3^{-1/2} + \lambda_2/24\, w_3^{-1/2} - \mu_2 = 0$

CS_{λ_1}: $\lambda_1 \geq 0$ $\sqrt{w_1}/4 + \sqrt{w_2}/2 + \sqrt{w_3}/4 - 1 \geq 10$

$\lambda_1(\sqrt{w_1}/4 + \sqrt{w_2}/2 + \sqrt{w_3}/4 - 1 - 10) = 0$

CS_{λ_2}: $\lambda_2 \geq 0$, $-\sqrt{w_1}/12 + \sqrt{w_2}/6 - \sqrt{w_3}/12 - 1 \geq 0$

$\lambda_2(-\sqrt{w_1}/12 + \sqrt{w_2}/6 - \sqrt{w_3}/12 - 1) = 0$

CS_{μ_1}: $\mu_1 \geq 0$, $w_1 \leq w_2$, $\mu_1(w_1 - w_2) = 0$

CS_{μ_2}: $\mu_2 \geq 0$, $w_2 \leq w_3$, $\mu_2(w_2 - w_3) = 0$

$w_1 < w_2 \Rightarrow \mu_1 = 0$

$w_2 = w_3 \Rightarrow \mu_2 \neq 0$

IR binds $\Rightarrow \lambda_1 > 0$, $\sqrt{w_1}/4 + \sqrt{w_2}/2 + \sqrt{w_3}/4 = 11$

suppose that IC binds, then $\lambda_2 > 0$,

$\sqrt{w_1}/4 + \sqrt{w_2}/2 + \sqrt{w_3}/4 - 1 = 1/3(\sqrt{w_1} + \sqrt{w_2} + \sqrt{w_3}) = 10$

$\sqrt{w_1}/4 + 3\sqrt{w_2}/4 = 11 \Rightarrow \sqrt{w_1} + 3\sqrt{w_2} = 44$

$\sqrt{w_1}/3 + 2\sqrt{w_2}/3 = 10 \Rightarrow \sqrt{w_1} + 2\sqrt{w_2} = 30$

$\Rightarrow \sqrt{w_2} = 14$, $\sqrt{w_1} = 2$, $\sqrt{w_3} = 14$

FOC_{w_1}: $1/4 - \lambda_1/16 + \lambda_2/48 = 0$

FOC_{w_2}: $1/2 - \lambda_1/56 - \lambda_2/168 + \mu_2 = 0$

~~FOC_{w_3}: $1/4 - \lambda_1/16 + \lambda_2/48 - \mu_2 = 0$~~

FOC_{w_3}: $1/4 - \lambda_1/112 + \lambda_2/336 - \mu_2 = 0$

$\lambda_1 = 22$, $\lambda_2 = 54$, $\mu_2 = 3/14$

Is this check necessary? Yes

The checks are especially necessary if the binding-ness of the constraints has not been established by argument but merely conjectured (as in Ines' solution, not here)

If some λ/μ fails the corresponding check, the case where that constraint does not hold

~~Given that th~~

Suppose that IC does not bind, then $\lambda_2 = 0$

FOC_{w_1}: $1/4 - \lambda_1/8 \, w_1^{-1/2} = 0$

FOC_{w_2}: $1/2 - \lambda_1/4 \, w_2^{-1/2} + \mu_2 = 0$

FOC_{w_3}: $1/4 - \lambda_1/8 \, w_3^{-1/2} - \mu_2 = 0 \Rightarrow 1/4 - \lambda_1/8 \, w_2^{-1/2} - \mu_2 = 0$

$\Rightarrow 1/2 - \lambda_1/4 \, w_2^{-1/2} - 2\mu_2 = 0$

$\Rightarrow \mu_2 = 0$

~~By reductio~~ There is no optimum such that IC does not bind

The optimal contract is $(w_1^* = 4, w_2^* = 196, w_3^* = 196)$

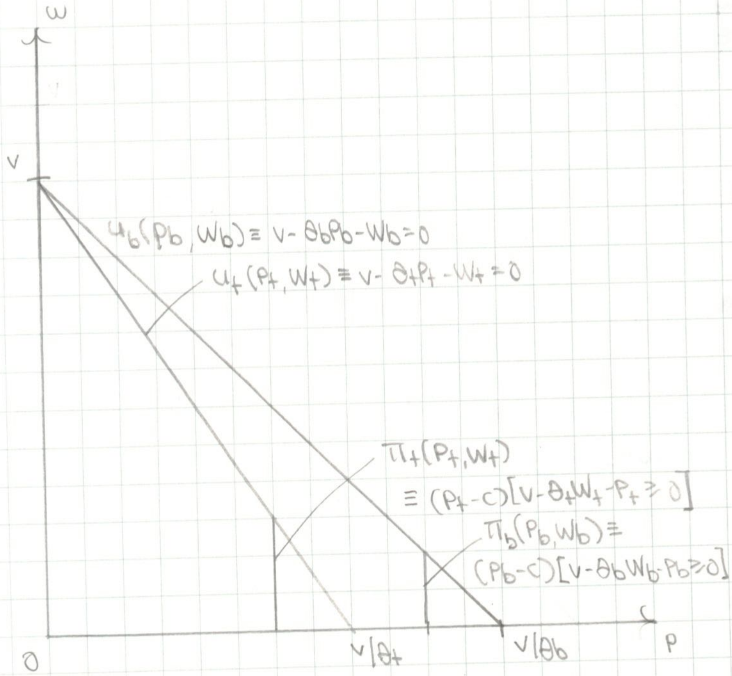
c The wage required to induce $e=1$ when effort is observable, w^* is such that $\sqrt{w^*} - 1 = \bar{u}$, $w^* = 122$

In (a), $E(w) = 1/4(64) + 1/2(196) + 1/4(64) = 130$, $AC = 8$

In (b), $E(w) = 1/4(4) + 1/2(196) + 1/4(196) = 148$, $AC = 26$

Agency cost is greater if the agent can secretly destroy revenue. This is because, where secret revenue destruction is possible, the principal must increase the wage in the event of high revenue to disincentivise revenue destruction. Then, the principal has incentive to decrease wage in the low-revenue outcome until IR and IC again bind. The new wage schedule is more risky, ~~so the cost of~~ and ~~compensating~~ the principal must compensate the agent for greater risk bearing, so agency cost increases.

Date



Think about ICs by supposing zero utility and writing W in terms of P

~~ICs~~ indiff curves give IC constraints

State that indiff curves are parallel

Isoprofits ignore constraints because we care only about "one side of the market" and dont care about feasibility

The firm's optimisation problem is
$\max_{p_b, w_b, p_t, w_t} (1-\lambda)(p_b - c) + \lambda(p_t - c)$ subject to

Correct formulation

$PC_b: v - \theta_b p_b - w_b \geq 0$
$PC_t: v - \theta_t p_t - w_t \geq 0$
$IC_b: v - \theta_b p_b - w_b \geq v - \theta_b p_t - w_t$
$IC_t: v - \theta_t p_t - w_t \geq v - \theta_t p_b - w_b$
$p_b^+: p_b \geq 0,\ p_t^+: p_t \geq 0,\ w_b^+: w_b \geq 0,\ w_t^+: w_t \geq 0$
supposing without loss of generality that consumers have reservation utility 0

Important assumption

b ~~Tour:~~
$v - \theta_b p_b - w_b \geq_1 v - \theta_b p_t - w_t \geq_2 v - \theta_t p_t - w_t \geq_3 0$,
$\geq_1$ follows from IC_b, $\geq_2$ from $\theta_b < \theta_t$ (supposing $p_t > 0$)
$\geq_3$ from PC_t
$\Rightarrow v - \theta_b p_b - w_b > v - \theta_t p_t - w_t$, i.e. PC_b does not bind.

This is a standard result

Also allows us to ignore PC_b

Suppose for reductio that at the optimum $\hat{p}_b, \hat{w}_b, \hat{p}_t, \hat{w}_t$, PC_t does not bind, i.e. $v - \theta_t \hat{p}_t - \hat{w}_t > 0$. Then, for sufficiently small ε (such that PC_b and PC_t continues to hold), $p_b' = \hat{p}_b + \varepsilon$, $p_t' = \hat{p}_t + \varepsilon$ is such that all constraints hold and $\pi' > \hat{\pi}$. So $\hat{p}_b, \hat{w}_b, \hat{p}_t, \hat{w}_t$ is not an optimum. By reductio, PC_t binds at the optimum, and tourists are indifferent between travelling and not travelling.

Because increase is the same, ICs are satisfied.

c Suppose for reductio that at the optimum, IC_b does not bind, i.e. $v - \theta_b \hat{p}_b - \hat{w}_b > v - \theta_b \hat{p}_t - \hat{w}_t$. Then, for sufficiently small ε (such that PC_b and IC_b continue to hold), $p_b' = \hat{p}_b + \varepsilon$ is such that all constraints hold and $\pi' > \hat{\pi}$ ~~So ## the candidate~~ By reductio, IC_b binds at the optimum, and business men are indifferent between buying at W_b and at W_t.

General result: only one IC binds

~~Suppose for reductio that $\hat{w}_b \neq 0$.~~ IC_b binds, i.e.
$v - \theta_b \hat{p}_b - \hat{w}_b = v - \theta_b \hat{p}_t - \hat{w}_t \Leftrightarrow \theta_b \hat{p}_b + \hat{w}_b = \theta_b \hat{p}_t + \hat{w}_t \Leftrightarrow$
$\theta_b(\hat{p}_b - \hat{p}_t) = \hat{w}_t - \hat{w}_b \Rightarrow \theta_t(\hat{p}_b - \hat{p}_t) > \hat{w}_t - \hat{w}_b \Rightarrow$
$v - \theta_t \hat{p}_b - \hat{w}_b < v - \theta_t \hat{p}_t - \hat{w}_t$, i.e. IC_t does not bind.

Suppose for reductio that $\hat{w}_b \neq 0$. Then for sufficiently small ε (such that IC_t continues to hold), $w_b' = \hat{w}_b - \varepsilon$, $p_b' = \hat{p}_b + \varepsilon/\partial_b$ is such that ~~$v-\partial_b w_b$~~
$v - \partial_b \hat{p}_b - \hat{w}_b = v - \partial_b p_b' - w_b'$, so it is trivial that PC_b, PC_t, and IC_b continue to hold (and IC_t holds by construction of ε), and $\pi' > \hat{\pi}$. By reductio, $\hat{w}_b = 0$.

Businessmen buy at $\hat{w}_b = 0$ and are indifferent ~~to~~ between buying at this time and buying when tourists do at $\hat{w}_t$.

d

d Given that PC_t binds, $v-\theta_t\hat{P}_t-\hat{W}_t=0$, $(\hat{P}_t,\hat{W}_t)$ lies on the indifference curve $u_t(P_t,W_t)=0$. Given that IC_b binds, $v-\theta_b\hat{P}_b-\hat{W}_b=v-\theta_b\hat{P}_t-\hat{W}_t$, $(\hat{P}_b,\hat{W}_b)$ lies on the indifference $u_b(P_b,W_b)=u_b(\hat{P}_t,\hat{W}_t)$, i.e. the indifference curve that crosses $(\hat{P}_t,\hat{W}_t)$. Given that $\hat{W}_b=0$, $(\hat{P}_b,\hat{W}_b)$ lies on the intersection of that indifference curve with the P axis.

How could we know this is the right method rather than Lagrangian?

$\hat{W}_b=0$

$\hat{W}_t=v-\theta_t\hat{P}_t$

$v-\theta_b\hat{P}_b-\hat{W}_b=v-\theta_b\hat{P}_t-\hat{W}_t \Rightarrow \theta_b\hat{P}_b+\hat{W}_b=\theta_b\hat{P}_t+\hat{W}_t$

$\Rightarrow \theta_b\hat{P}_b=\theta_b\hat{P}_t+v-\theta_t\hat{P}_t \Rightarrow \hat{P}_b=\hat{P}_t+v/\theta_b-\theta_t/\theta_b\,\hat{P}_t$

$\hat{\pi}=(1-\lambda)\hat{\pi}_b+\lambda\hat{\pi}_t$

$=(1-\lambda)(\hat{P}_t+v/\theta_b-\theta_t/\theta_b\,\hat{P}_t-c)+\lambda(\hat{P}_t-c)$

~~At the optimum, the following FOC holds~~

$d\hat{\pi}/d\hat{P}_t=(1-\lambda)(1-\theta_t/\theta_b)+\lambda \gtrless 0$

$1-(1-\lambda)(\theta_t/\theta_b)=0 \Rightarrow$

$1-\lambda=\theta_b/\theta_t \Rightarrow \lambda=1-\theta_b/\theta_t$

Strategy here is to express all variables in terms of $\hat{P}_t$.

Is it necessary to check CS of the original Lagrangian? Why or why not?

"We want to find the sign of the derivative to know ~~whether it maxi~~ whether the optimum $\hat{P}_t$ is the maximum feasible or the min feasible.

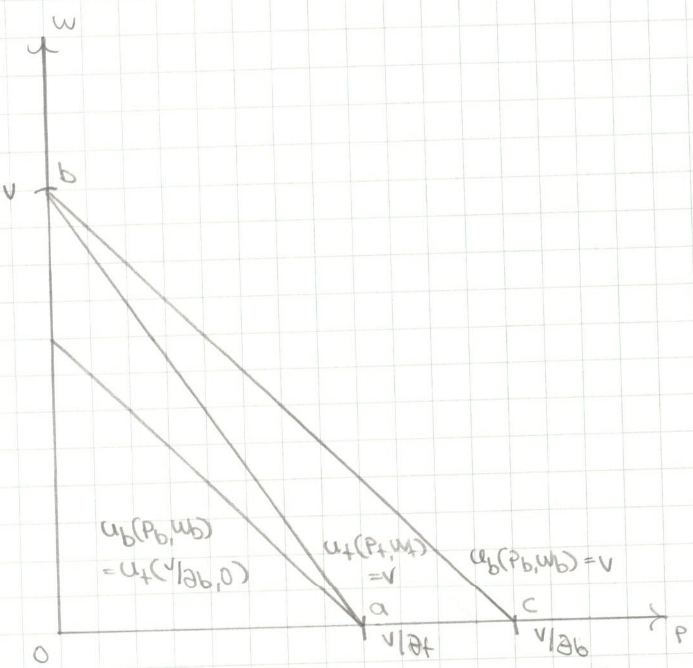
suppose $\lambda>1-\theta_b/\theta_t$, then $d\hat{\pi}/d\hat{P}_t>0$, the firm maximises profit by choosing the maximum feasible (i.e. subject to the above constraints) $\hat{P}_t$, namely v/θ_t. Then $\hat{W}_t=0$, $\hat{P}_b=\hat{P}_t=v/\theta_t$, $\hat{W}_b=0$. This is the pooling equilibrium. $\hat{\pi}=(v/\theta_t-c)$

Suppose $\lambda<1-\theta_b/\theta_t$, then $d\hat{\pi}/d\hat{P}_t>0$, the firm maximises profit by choosing the minimum feasible $\hat{P}_t$, namely 0. Then $\hat{W}_t=v$, $\hat{P}_b=v/\theta_b$, $\hat{W}_b=0$. This is the separating equilibrium. Suppose that indifferent tourists do not buy. $\hat{\pi}=(1-\lambda)(v/\theta_b-c)$

~~If we assume both buy, wouldnt it~~

We assume both buy so costs are effectively fixed/sunk, max $\pi \Leftrightarrow$ max revenue, and this solution maximises revenue

This assumption comes in the formulation of the problem in a where the PC constraints are included



The pooling eqm is a, the separating eqm is points b and c.

e If $c>v/\theta_b$, then $c>v/\theta_t$, $\hat{\pi}<0$ in either eqm, the firm should choose high P_b, P_t, W_b, W_t such that no consumers buy.

If $v/\theta_t < c < v/\theta_b$, then only the separating eqm is profitable, the firm should choose $p_b = v/\theta_b$, $w_b = 0$ and high p_t, w_t such that no tourists buy.

If $c < v/\theta_t < v/\theta_b$ the airline does not serve tourists iff

$(1-\lambda)(v/\theta_b - c) > (v/\theta_t - c) \Leftrightarrow$

$(1-\lambda)v/\theta_b - v/\theta_t > -c + (1-\lambda)c \Leftrightarrow$

$(1-\lambda)v/\theta_b - v/\theta_t > -\lambda c \Leftrightarrow$

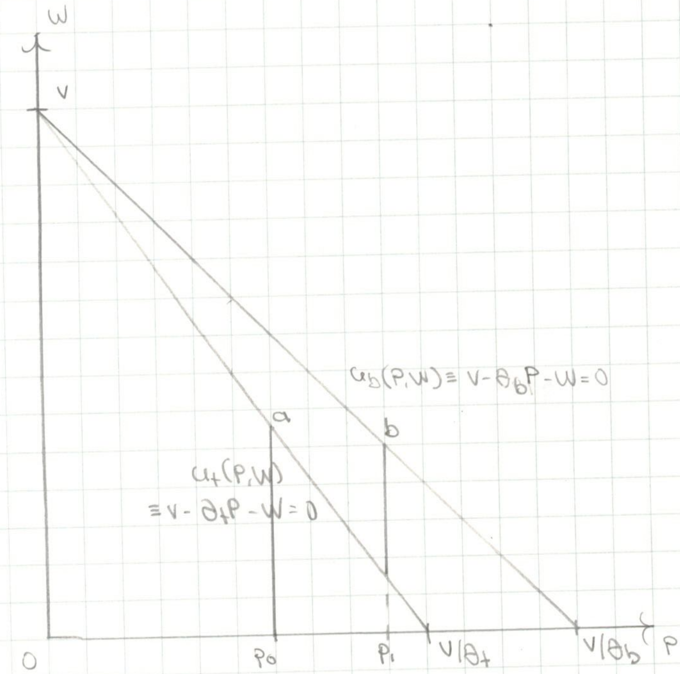
$c > \frac{\lambda-1}{\lambda} v/\theta_b + v/\lambda\theta_t \Leftrightarrow$

$c > \frac{\lambda-1}{\lambda} v/\theta_b + \frac{1}{\lambda} v/\theta_t$

marginal cost exceeds ~~the~~ some weighted average of the maximum price each type is willing and able to pay

For such c either only serve b or have pooling eqm, separating eqm is not profitable.

2a



The two lines p_0a and p_1b constitute an isoprofit. Along p_0a, the firm sells to ~~tour~~ both tourists and businessmen and has profit (p_0-c). Along p_1b, the firm sells only to businessmen and has profit $(1-\lambda)(p_1-c)$, which is equal to ~~p_0 for which~~ (p_0-c) when $p_1 = c + \frac{p_0-c}{1-\lambda}$

The firm's optimisation problem is
$\max_{p_b, w_b, p_t, w_t}$ ~~$(1-\lambda)$~~ $(1-\lambda)(p_b-c)+\lambda(p_t-c)$ subject to

$PC_b: v-\theta_b p_b - w_b \geq 0$

$PC_t: v-\theta_t p_t - w_t \geq 0$

$IC_b: v-\theta_b p_b - w_b \geq v-\theta_b p_t - w_t$

$IC_t: v-\theta_t p_t - w_t \geq v-\theta_t p_b - w_b$

$p_b^+: p_b \geq 0$, $p_t^+: p_t \geq 0$

$w_b^+: w_b \geq 0$, $w_t^+: w_t \geq 0$

b Tourists are indifferent between travelling and not travelling $\Leftrightarrow$ PC_t binds. ~~Suppose for reductio that PC_t does not bind, i.e. $v-\theta_t p_t - w_t > 0$. Either PC_b binds or PC_b does not bind. Suppose that PC_b binds, i.e. $v-\theta_b p_b - w_b = 0$, then~~
$v-\theta_b p_b - w_b \geq_1 v-\theta_b p_t - w_t >_2 v-\theta_t p_t - w_t \geq_3 0$, where $\geq_1$ follows from IC_b, $>_2$ follows from $\theta_b<\theta_t$, supposing that $p_t \neq 0$, and $\geq_3$ follows from PC_t. Then $v-\theta_b p_b - w_b > 0$, i.e. PC_b does not bind. Suppose for reductio that PC_t ~~binds, i.e. $v-\theta_t p_t -$~~ does not bind, i.e. $v-\theta_t p_t - w_t > 0$. Then, ~~$p_b' =$~~ for sufficiently small ε, $p_b' = p_b+\varepsilon$ and $p_t' = p_t+\varepsilon$ satisfy PC_b, PC_t, IC_b, IC_t, p_b^+, p_t^+, w_b^+, ~~and~~ w_t^+, and yields greater profit. So p_b, w_b, p_t, w_t such that PC_t does not bind is not an optimum. ~~By~~ reductio, PC_t binds, and tourists are indifferent between travelling and not travelling.

c Businessmen never buy their tickets in advance and are indifferent between doing this and buying when tourists buy $\iff W_b = 0$, ~~$W_t \neq 0$~~, IC_b binds.

From an argument in (b), PC_b does not bind. Suppose for reductio that IC_b does not bind, i.e. $v - \theta_b P_b - W_b > v - \theta_b P_t - W_t$. Then, for sufficiently small ε, $P_b' = P_b + \varepsilon$ satisfies all constraints and yields greater profit. By reductio, IC_b ~~does not bind at an optimum.~~ binds at the optimum.

~~Suppose for reductio that $W_b \neq 0$. Then for sufficiently small consider P_b', W_b' such that $W_b' = 0$ and $v - \theta_b P_b' - W_b' = v - \theta_b P_b - W_b \iff \theta_b(P_b' - P_b) = W_b$, i.e. an increase in price such that businessmen are indifferent between buying at $W_b' = 0$ at the new price and buying at $W_b > 0$ at the old price. It is trivial that PC_b, PC_t, IC_b, and the positivity constraints continue to hold.~~

~~$\theta_b > \theta_t \Rightarrow$ From above~~, IC_b binds, i.e. ~~$v - \theta_b P_b - W_b = v$~~
$v - \theta_b P_b - W_b = v - \theta_b P_t - W_t \iff -\theta_b(P_b - P_t) = W_b - W_t$. ~~Given $\theta_b > \theta_t$~~ $\Rightarrow -\theta_t(P_b - P_t) < W_b - W_t \Rightarrow$
$v - \theta_t P_t - W_b > v - \theta_t P_b - W_b$, i.e. IC_t does not bind.

Suppose for reductio that $W_b \neq 0$. Then for sufficiently small ε (such that IC_t continues to hold), $W_b' = W_b - \varepsilon$ and P_b' is such that $v - \theta_b P_b' - W' = v - \theta_b P_b - W_b$, it is trivial that PC_b, PC_t, IC_b, and the positivity constraints are satisfied. P_b', W_b' yield greater profit than P_b, W_b. By reductio, $W_b = 0$.

~~Suppose for reductio that $W_t \neq 0$. Then, by the result in (b), that PC_t binds, $v - \theta_t P_t - W_t = 0$, $P_t = v/\theta_t$. By the result above, that $W_b = 0$, and by IC_b, IC_t, $P_t = P_b$~~

d Given PC_t binds, $W_b = 0$, IC_b binds, the firm's optimisation problem reduces to

$\max_{P_b, P_t, W_t} (1-\lambda)(P_b - c) + \lambda(P_t - c)$ subject to

PC_b: $v - \theta_b P_b > 0$

PC_t: $v - \theta_t P_t - W_t = 0 \Rightarrow v = \theta_t P_t + W_t$

IC_b: $v - \theta_b P_b = v - \theta_b P_t - W_t \Rightarrow \theta_b P_b = \theta_b P_t + W_t$

IC_t: $v - \theta_t P_t - W_t > v - \theta_t P_b \Rightarrow \theta_t P_t + W_t < \theta_t P_b$

$P_b^+: P_b \geq 0$, $P_t^+: P_t \geq 0$, $W_t^+: W_t \geq 0$

Neglecting the positivity constraints

$L = (1-\lambda)(P_b - c) + \lambda(P_t - c) + \gamma_1(v - \theta_b P_b) - \gamma_2(\theta_t P_t + W_t - v)$
$\quad - \mu_1(\theta_b P_b - \theta_b P_t - W_t) - \mu_2(\theta_t P_t + W_t - \theta_t P_b)$

FOC P_b: $1 - \lambda - \gamma_1 \theta_b - \mu_1 \theta_b + \mu_2 \theta_t = 0$

FOC P_t: $\lambda - \gamma_2 \theta_t + \mu_1 \theta_b - \mu_2 \theta_t = 0$

FOC W_t: $-\gamma_2 + \mu_1 - \mu_2 = 0$

CS γ_1: $\gamma_1 \geq 0$, $v - \theta_b P_b \geq 0$, $\gamma_1(v - \theta_b P_b) = 0$

FOC γ_2: $\theta_t P_t + W_t = 0$

FOC μ_1: $\theta_b P_b - \theta_b P_t - W_t = 0$

CS μ_2: $\mu_2 \geq 0$, $\theta_t P_t + W_t \geq \theta_t P_b$,
$\mu_2(\theta_t P_t + W_t - \theta_t P_b) = 0$

d Given PC_t binds, $W_t = v - \theta_t P_t$.

The firm's optimisation problem reduces to
$\max_{P_b, P_t} \lambda(P_t - c) + (1-\lambda)(P_b - c)$ subject to

$PC_b: v - \theta_b P_b > 0$

$IC_b: \theta_b P_b = \cancel{\theta_t P_t + (v - \theta_t P_t) = v}\ \theta_b P_t + v - \theta_t P_t$

$IC_t: \cancel{\theta_t P_t + W_t}\ v < \theta_t P_b$

$\max_{P_b, P_t} \lambda(P_t - c) + (1-\lambda)(P_b - c)$ subject to

$\cancel{\theta_b P_t + \theta_t P_t - 2\theta_b P_b > 0}$ $\qquad \theta_t P_t > \theta_b P_t$

$\cancel{\theta_b P_b = \theta_b P_t}$

$PC_t: v - \theta_t P_t - W_t = 0 \Rightarrow W_t = v - \theta_t P_t$

$IC_b: v - \theta_b P_b - W_b = v - \theta_b P_t - W_t \Rightarrow \theta_b P_b + W_b = \theta_b P_t + W_t$

$\Rightarrow \theta_b P_b = \theta_b P_t + W_t \Rightarrow W_t = \theta_b P_b - \theta_b P_t$

$\Rightarrow \cancel{\#}\ \theta_b P_b - \theta_b P_t = v - \theta_t P_t \Rightarrow v = \theta_b P_b - \theta_b P_t + \theta_t P_t$

$PC_b: v - \theta_b P_b - W_b = -\theta_b P_t + \theta_t P_t > 0$ ~~(always holds)~~ $\Leftrightarrow \theta_t > \theta_b$ (given)

$IC_t: v - \theta_t P_t - W_t > v - \theta_t P_b - W_b \Rightarrow \theta_t P_t - W_t > -\theta_t P_b$

$\Leftrightarrow \theta_t P_t - \theta_b P_b + \theta_b P_t > -\theta_t P_b$

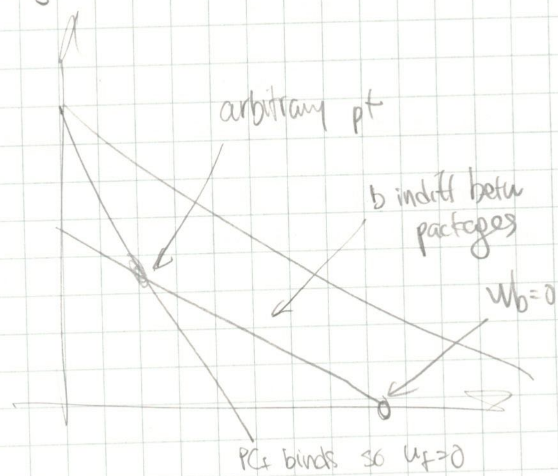
$\Leftrightarrow \theta_t (P_t - P_b) > \theta_b (P_t - P_b)$

$\Leftrightarrow \theta_t > \theta_b$ (given)

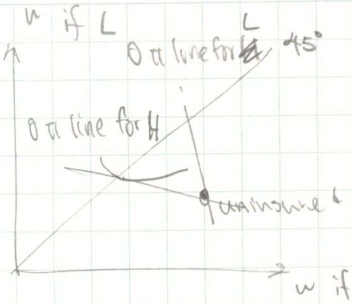
$W_t = v - \theta_t P_t$, $W_t = \theta_b P_t - \theta_b P_b$

$v = \theta_b P_t - \theta_b P_b + \theta_t P_t$

Maybe Lagrangian opt is not required, solve by considering constraints.



#3



Rothschild-Stiglitz : competitive mkt → zero profit → $\pi = p$ that crosses non-insurance line

Monopolist = sell at intersection of IC_1 and 45° since principals are risk-averse