Innovation Rough Notes

Summary

Patent Races

• Bertrand competition, drastic innovation, each of incumbent and potential entrant chooses investment k and has probability of success $\rho(k)$. Then, for incumbent, expected profit is zero if entrant succeeds, otherwise it is $\pi^M(c_L)$ if incumbent succeeds and $\pi^M(c_H)$ if incumbent fails. For entrant, expected profit is zero if incumbent succeeds and if entrant fails. So only in the event that entrant succeeds and incumbent fails does the entrant have positive profit $\pi^M(c_L)$. So incumbent chooses k_I to maximise $(1-\rho(k_E))\rho(K_1)\pi^M(c_L)+(1-\rho(k_E))(1-\rho(k_I))\pi^M(c_H)-k_I$ and entrant chooses k_E to maximise $(1-\rho(k_I))\rho(k_E)\pi^M(c_L)$. Rearranging for the incumbent, the objective function is $(1-\rho(k_E))\rho(k_I)[\pi^M(c_L)-\pi^M(c_H)]-k_E$ Best response functions are downwards sloping, investment amounts are strategic substitutes, and the entrant is more aggressive (intuitively, because the entrant has more to gain). This is the familiar replacement effect. It is possible to solve by taking first-order conditions. The result holds even for non-drastic innovation because the replacement effect remains intact. This can be demonstrated graphically.

Killer Acquisitions

- In the first stage, the incumbent chooses whether to acquire the entrant at some endogenous price. In the second stage, the project owner chooses whether to develop the project at fixed, common cost k. The project succeeds with respective probabilities ρ_I, ρ_E . If the project succeeds, the project owner earns $\pi^S > \pi^M$. If the project fails, the incumbent continues to enjoy π^M .
- The general result is that for an intermediate range of project costs k, the incumbent has incentive to acquire and terminate the project whereas the entrant would have continued to develop it.
- Entrant develops iff $\rho_E \pi^S k \ge 0$.
- Endogenous price in first stage is $\rho_E \pi^S k$.
- Incumbent, having acquired, terminates iff $\rho_I(\pi^S \pi^M) k < 0$. (Not apparently critical).
- If incumbent acquires and terminates, incumbent has profit $\pi^M (\rho_E \pi^S k)$.
- If incumbent does not acquire, incumbent has expected profit $(1-\rho_E)\pi^M$.
- Rearranging, incumbent acquires and terminates iff $k \geq \rho_E(\pi_S \pi_M)$.

Lecture

- Structure
 - Innovation
 - Market Structure and Innovation (Arrow Argument)
 - · Entry and Innovation (Deterministic)
 - Entry and Innovation (Probabilistic, Patent Race)
 - Killer Acquisitions
 - Patent Life
 - Empirical Issues
- Innovation
 - Research and development constitutes a fixed cost for firms, hence firms must face imperfect competition, and thus
 price above marginal cost, in order to recover research and development costs.
 - Research and development produces knowledge about products and/or processes, which is non-rival, it is thus socially efficient, ex post for the new information to be freely available.
 - Innovation is intrinsically risky. Process innovation reduces cost while product innovation improves existing goods and services or creates new goods and services.
- Market Structure and Innovation (Arrow)
 - Drastic Innovation
 - Consider a monopolist initially with high constant marginal cost c^H facing downward-sloping demand, which can obtain a process innovation which lowers marginal cost to c^L . The process innovation is drastic such that monopoly price $p^M(c^L)$ given c^L , is less than c^H .

- The monopolist initially enjoys positive profit, and its profit increases if it buys the process innovation. The monopolist's valuation of the process innovation is equal to the resulting increase in profit.
- [Insert Diagram from Cowan, 2022]
- Suppose instead that there are multiple firms in Bertrand competition, each with high constant marginal cost c^H collectively face downward-sloping demand. At most one firm can obtain a process innovation which lowers marginal cost to c^L and the process innovation is drastic.
- Each firm initially enjoys zero profit, and enjoys positive profit if it buys the process innovation equal to the monopoly profit $\pi^M(c^L)$ at the lower cost, since $p^M(c^L) < c^H$ and no other firm can profitably undercut the monopoly price. Each firm's valuation of the process innovation is equal to the resulting increase in profit, which is equal to the monopoly profit at the lower cost.
- Firms under the competitive market structure have higher valuations for process innovations, and equivalently,
 higher incentive to innovate. This is because the monopolist "replaces itself" when it innovates, and its incentive
 is equal to the increase in profit, whereas firms under the competitive market structure face incentive equal to the
 resulting monopoly profit.

• Non-Drastic Innovation

- Consider a monopolist initially with high constant marginal cost c^H facing downward-sloping demand, which can obtain a process innovation which lowers marginal cost to c^L . The process innovation is non-drastic such that monopoly price $p^M(c^L)$ given c^L , is greater than c^H .
- The monopolist initially enjoys positive profit, and its profit increases if it buys the process innovation. The monopolist's valuation of the process innovation is equal to the resulting increase in profit.
- [Insert Diagram from Cowan, 2022]
- Suppose instead that there are multiple firms in Bertrand competition, each with high constant marginal cost c^H collectively face downward-sloping demand. At most one firm can obtain a process innovation which lowers marginal cost to c^L and the process innovation is non-drastic.
- Each firm initially enjoys zero profit, and enjoys positive profit if it buys the process innovation. If some firm buys the process innovation, it chooses price arbitrarily below c^H to maximise profit. At this price, this firm's gross margin is $c^H c^L$ and the quantity sold by this firm is given by the demand curve at c^H . The increase in profit enjoyed by this firm is less than the monopoly profit $\pi^M(c^L)$ given c^L but greater than the increase in profit enjoyed by the monopoly. Each firm's valuation of the process innovation is equal to this increase in profit.
- Firms under the competitive market structure again have higher valuations for process innovations because of the replacement effect.
- In both the case of drastic innovation and the case of non-drastic innovation, the increase in social surplus due to the process innovation exceeds the valuation of both the monopolist and each firm under a competitive market structure, the incentive of the social planner to innovate exceeds the private incentive of any firm.
- Entry and Innovation (Deterministic Innovation, Pre-emptive Patenting, Gilbert and Newbery)
 - Consider an incumbent monopolist initially with high constant marginal cost c^H facing downward-sloping demand, which can obtain a process innovation which lowers marginal cost to c^L . Either the incumbent monopolist obtains the innovation or a potential entrant obtains the innovation. Iff a potential entrant obtains the innovation, it enters the market.
 - Let $\pi^M(c^H)$ and $\pi^M(c^L)$ denote the monopoly profit obtained by a firm with c^H and a firm with c^L respectively. Let $\pi^D(c^H,c^L)$ and $\pi^D(c^L,c^D)$ denote profit obtained by the firm with c^H in duopoly competition with a firm with c^L and the profit obtained by the firm with c^L in duopoly competition with a firm with c^H respectively. Let q(p) be the demand function, denoting the quantity demanded at price p.
 - Suppose that the innovation is non-drastic, i.e. $p^M(c^L) > c^H$. Then under Bertrand competition, $\pi^D(c^H, c^L) = 0$ since the low cost firm chooses price arbitrarily below the high cost firm and the high cost firm captures zero demand and $\pi^D(c^L, c^H) = (c^H c^L)q(c^H)$. The incumbent monopolist enjoys increase in profit $\Delta \pi_I = \pi^M(c^L) \pi^D(c^H, c^L) = \pi^M(c^L)$ if it obtains the innovation instead of the entrant and the entrant enjoys increase in profit $\Delta \pi_E = \pi^D(c^L, c^H) 0 = \pi^D(c^L, c^H)$ if it obtains the innovation instead of the incumbent. $\pi^D(c^L, c^H) = (c^H c^L)q(c^H)$ is the profit of the low-cost firm if it chooses price $c^H \neq p^M(c^L)$ and captures all demand. By definition of $p^M(c^L)$, $\pi^M(c^L) > \pi^D(c^L, c^H)$, i.e. the low cost firm would enjoy greater profit if its price were not constrained at the upper bound by c^H .
 - Under Cournot competition, $\pi^D(c^L, c^H) > \pi^D(c^H, c^L) > 0$, and $\pi^M(c^L) > \pi^D(c^L, c^H) + \pi^D(c^H, c^L)$ since each firm in duopoly imposes a negative externality on the other hence aggregate profit is not maximised, and $\pi^M(c^L)$ is equal to the maximum aggregate profit, i.e. if the firms were to optimally collude, the high-cost firm would not produce and the low-cost firm would act as a monopoly. The incumbent monopolist enjoys increase in profit

 $\Delta \pi_I = \pi^M(c^L) - \pi^D(c^H, c^L) > \pi^D(c^L, c^H)$ if it obtains the innovation instead of the entrant and the entrant enjoys increase in profit $\Delta \pi_E = \pi^D(c^L, c^H) - 0 = \pi^D(c^L, c^H)$ if it obtains the innovation instead of the incumbent.

- Suppose instead that the innovation is drastic, i.e. $p^M(c^L) < c^H$. Then under both Bertrand and Cournot competition, the low cost firm is unconstrained by the high cost firm and acts as a monopolist, i.e. $\pi^D(c^L,c^H) = \pi^M(c^L)$ and the high cost firm is priced out of the market, i.e. $\pi^D(c^H,c^L) = 0$. The incumbent monopolist enjoys increase in profit $\Delta \pi_I = \pi^M(c^L) \pi^D(c^H,c^L) = \pi^M(c^L)$ if it obtains the innovation instead of the entrant and the entrant enjoys increase in profit $\Delta \pi_E = \pi^D(c^L,c^H) 0 = \pi^D(c^L,c^H) = \pi^M(c^L)$ if it obtains the innovation instead of the incumbent.
- In all cases (drastic or non-drastic innovation and Bertrand or Cournot competition), $\Delta \pi_I \geq \Delta \pi_E$, the incumbent monopolist always has (weakly) greater incentive to innovate than the entrant.
- This is because the incumbent monopolist more efficiently realises the gains due to the process innovation since it is
 unconstrained by the existence of a less efficient competitor (whereas the entrant, if it obtains the innovation, is
 constrained by the existence of the legacy monopolist).
- Under this model, the monopolist either "replaces itself" or is replaced by a lower-cost entrant, hence its incentive is
 equal to the difference in profit between the two cases, not the difference between its initial profit and its profit if it
 innovates successfully. The monopolist's incentive to innovate is not depressed by a "replacement effect".
- Entry and Innovation (Uncertain Innovation, Reinganum)
 - Consider an incumbent monopolist initially with high marginal cost c^H facing downward-sloping demand, that can invest research and development expenditure k_I such that with probability $\rho(k_I)$ it obtains a drastic process innovation which lowers marginal cost to c^L , where ρ is an increasing, strictly concave, and strictly less than 1. A potential entrant can similarly invest k_E such that it obtains the same innovation with probability $\rho(k_E)$. If the entrant's investment is successful, it enters the market and competes with the incumbent in prices.
 - If the entrant's investment is successful, it enters the market with marginal cost (weakly) lower than the incumbent. By the familiar Betrand result, the incumbent receives zero profit. Only the cases where (1) the entrant's investment is unsuccessful and the incumbent's investment is successful, and (2) the entrant's investment is unsuccessful and the incumbent's investment is unsuccessful are relevant to the incumbent's expected profit.

$$\begin{split} E(\pi_I) &= \rho(k_I)(1-\rho(k_E))\pi^M(c^L) + (1-\rho(k_I))(1-\rho(k_E))\pi^M(c^H) - k_I. \text{ First-order condition for } k_I: \\ \frac{\partial E(\pi_I)}{\partial k_I} &= \rho'(k_I)(1-\rho(k_E))\pi^M(c^L) - \rho'(k_I)(1-\rho(k_E))\pi^M(c^H) - 1 = \rho'(k_I)(1-\rho(k_E))(\pi^M(c^L) - \pi^M(c^H)) - 1 = 0, \\ \rho'(k_I)(1-\rho(k_E))[\pi^M(c^L) - \pi^M(c^H)] &= 1. \end{split}$$

- If the entrant's investment is unsuccessful, it does not enter the market, and its profit is normalised to zero. If both the entrant's investment and the incumbent's investment are successful, the entrant enters the market and, by the familiar Bertrand result, the entrant receives zero profit. Only the case where the entrant's investment is successful and the incumbent's investment is unsuccessful is relevant to the entrant's expected profit.
- $E(\pi_E) = \rho(k_E)(1-\rho(k_I))\pi^M(c^L) k_E. \text{ First-order condition for } k_E: \frac{\partial E(\pi_E)}{\partial k_E} = \rho'(k_E)(1-\rho(k_I))\pi^M(c^L) 1 = 0,$ $\rho'(k_E)(1-\rho(k_I))\pi^M(c^L) = 1.$ Research and development expenditures are strategic substitutes. An increase in k_E results in an increase in $\rho(k_E)$ to
- Research and development expenditures are strategic substitutes. An increase in k_E results in an increase in $\rho(k_E)$ to which the incumbent optimally responds by reducing k_I hence increasing $\rho'(k_I)$ such that the incumbent's first-order condition continues to hold, given that $\pi^M(c^L)$ and $\pi^M(c^H)$ are independent of k_I and k_E . Similarly, an increase in k_I results in an increase $\rho(k_I)$ to which the entrant optimally responds by reducing k_E hence increasing $\rho'(k_E)$ such that the entrant's first-order condition continues to hold. Each firm benefits from research and development only if the other firm's investment is unsuccessful. The greater one firm's research and development expenditure, the less likely its investment is to fail, hence the less incentive its competitor has to invest.
- From the first-order conditions, $\frac{\rho'(k_I)}{1-\rho(k_I)} \frac{1-\rho(k_E)}{\rho'(k_E)} \frac{\pi^M(c^L)-\pi^M(c^H)}{\pi^M(c^L)} = 1$, $\frac{\rho'(k_E)}{1-\rho(k_E)} \frac{1-\rho(k_I)}{\rho'(k_I)} = \frac{\pi^M(c^L)-\pi^M(c^H)}{\pi^M(c^L)} < 1$. Let $f(k) \equiv \frac{\rho'(k)}{1-\rho(k)}$, then $\frac{f(k_E)}{f(k_I)} < 1$, hence $f(k_E) < f(k_I)$ Supposing that $\rho(k) = \frac{k}{k+1}$, f(k) is strictly decreasing hence $k_E > k_I$.
- If, instead, the innovation is non-drastic, then the entrant earns $\pi^D(c^L,c^H)$ instead of $\pi^M(c^L)$ if its investment is successful and the incumbent's is not. Diagrammatically, $\pi^M(c^L) \pi^M(c^H) < \pi^D(c^L,c^H)$. $\frac{f(k_E)}{f(k_I)} = \frac{\pi^M(c^L) \pi^M(c^H)}{\pi^D(c^L,c^H)} < 1$, hence $k_E > k_I$.
- The incumbent has less incentive to invest in research and development because of the replacement effect, i.e. if the incumbent is successful (and the entrant is unsuccessful), the incumbent's profit increases by only $\pi^M(c^L) \pi^M(c^H)$ whereas if the entrant is successful (and the incumbent is unsuccessful), the entrant's profit increases by $\pi^M(c^L)$ (or $\pi^D(c^L, c^H) > \pi^M(c^L) \pi^M(c^H)$ if innovation is non-drastic).
- The Gilbert and Newbery model and the Reinganum model yield contrasting results about the incentives to innovate for incumbents and entrants because the former assumes deterministic innovation, while the latter assumes stochastic innovation. Suppose for simplicity that innovation is drastic, and the successful innovator enjoys a monopoly, such that there is no efficiency effect. Consider a marginal increase in innovation. Given stochastic innovation, the incumbent enjoys greater probability of monopolising the market but suffers diminished probability of continuing to receive a stream of baseline profit, whereas the entrant enjoys the former but does not suffer the latter.

Given deterministic innovation, in the only case where a marginal increase is relevant, a marginal increase yields (for both incumbent and entrant) a difference in profit equal to post-innovation monopoly profit. There is no replacement effect for the monopolist at the margin because innovation is certain and zero-sum.

Plausibly, the degree of cost reduction and the degree of certainty of innovation are related, hence incremental non-drastic innovations are more certain while drastic innovations are more stochastic. Incumbents would be more likely to realise incremental gains while entrants would be more likely to realise drastic gains, and leapfrog the incumbent. Anecdotally, this prediction seems to be realistic.

Killer Acquisitions

- Consider a three-stage game between an incumbent monopolist I and a potential entrant A. The potential entrant has a project to develop a new, superior product. In the first stage, I can acquire E at an endogenous price equal to the expected value of the project to E. In the second stage, the project owner (E if it was not acquired, I if it acquired E) can spend a fixed amount k>0 to complete the project. In the third stage, the project is successful with probability ρ_E if it is owned by E and E and E and it succeeds, E arms increased monopoly profit E. If the project is owned by E and it succeeds, E is forced to exit the market and E earns monopoly profit E. If the project is owned by E and it fails, E does not enter the market and E earns baseline monopoly profit E.
- Suppose that $\rho^E\pi^S-k>0$ such that expected value of the project to E, $\rho^E\pi^S$ exceeds the cost of development k, and E would develop the project in the second stage if it is not acquired in the first. Suppose further that $\rho^I(\pi^S-\pi^B)-k<0$ such that the expected value of the project to I, given that I acquired E, $\rho^I(\pi^S-\pi^B)$ is less than the cost of development k, and I would kill the project if it acquired I.
- The cost to acquire E in the first stage is equal to the expected value of the project to E, $\rho^E \pi^S k$.
- If, in the first stage, I acquires E (and kills the project), its total payoff $\pi^B (\rho^E \pi^S k)$ is equal to the baseline profit π^B less the cost of the acquisition $\rho^E \pi^S k$.
- If, in the first stage, I does not acquire E, its total (expected) payoff is $(1 \rho^E)\pi^B$ since with probability ρ^E , the entrant's project succeeds and I is forced out of the market, and with probability $1 \rho^E$, the entrant's project fails and I remains in the market and continues to earn the baseline profit.
- Given the assumptions above, if $\pi^B (\rho^E \pi^S k) > (1 \rho^E)\pi^B$ iff $\rho^E \pi^B \ge \rho^E \pi^S k$, I acquires E and kills the project.
- For intermediate k, the project is profitable for the entrant to develop but not for the incumbent, because of the replacement effect. If, additionally, the value of the project to the entrant is less than the threat the project poses to the incumbent, the incumbent finds it optimal to acquire and kill the project.
- Killer acquisitions tend to decrease social welfare because consumer surplus is greater if the high qualty product is developed, and aggregate profit is also greater ($\pi^S > \pi^B$).

Patent Life (Nordhaus)

- Consider an industry in which multiple firms in Bertrand competition each produce a homogenous good, initially at high marginal cost c^H and collectively face downward-sloping demand. At most one firm can obtain a patent for a process innovation which reduces marginal cost to $c^L(k)$ by expending k on research and development. For time T after the patent is issued, this firm produces at $c^L(k)$ while all other firms continue to produce at c^H . Thereafter, the innovation is made available to all firms who produce at $c^L(k)$.
- Suppose that future profits are discounted such that a continuous stream of one unit of profit over time T has present value $\frac{1-e^{-rT}}{r}$. Equivalently, one unit of profit at time t has present value e^{-rt} .
- The innovating firm, given $c^L(k)$ prices optimally by choosing price p arbitrarily below c^H , thereby undercutting all competitors and capturing the entire market demand, and receives profit $\pi = [c^H c^L(k)]q(c^H)$ per unit time for the duration of the patent, where $q(c^H)$ is demand given $p = c^H$. After the patent expires, all firms have common marginal cost $c^L(k)$ hence each firm enjoys zero profit, by the familiar Bertrand result.
- Given T, the innovating firm chooses k to maximise present value of profits $V = [c^H c^L(k)]q(c^H)(\frac{1-e^{-rT}}{r}) k$. First-order condition for k, $-q(c^H)(\frac{1-e^{-rT}}{r})(c^L)'(k) 1 = 0$. Given that q, c^H , and r are given exogenously, the innovating firm optimally responds to an increase in T by increasing k such that the first-order condition holds.
- An increase in patent life increases the incentive for innovation hence decreases the post-innovation cost, increases the private benefit to the firm while the patent is active and increases the social welfare once the patent expires. An increase in patent life defers the realisation of the increase in social welfare, hence decreases the present value of the increase in social welfare due to innovation. A social planner optimally chooses patent life to balance, on the one hand, the increase in profit during the patent life and the increase in (absolute) social welfare after the patent life, and on the other hand, the decrease in the present value of social welfare after the patent life due to the deferral.

· Other Policy Instruments

•	Other policy instruments to incentivise innovation include copyright, direct provision of innovation by the government, for example, through blue-skies research in universities and research institutes, research and innovation subsidies or tax cuts, research prizes, and joint ventures.