Microeconomics Paper ~~1905~~29

1a $u(x_c, y) = \ln x_c + 2\ln y$

$MU_x = 1/x_c$, $MU_y = 2/y$, $MRS = -MU_x/MU_y = -y/2x_c$

$y = x_f^{2/3}$, $MRT = 2/3\, x_f^{-1/3}$

b $x_c + x_f = x_w = 14$

$x_c = 6, y = 4 \Rightarrow x_f = 14-6 = 8$, $MRS = -4/2(6) = -1/3$, $MRT = 2/3(8)^{-1/3}$
$= 1/3 \Rightarrow MRS = MRT$.

At $x_c = 6, y = 4$, $MRS = MRT$, this allocation is efficient.

c $P_y = 1$

At equilibrium, $MRS = -P_x/P_y \Rightarrow P_x = 1/3$

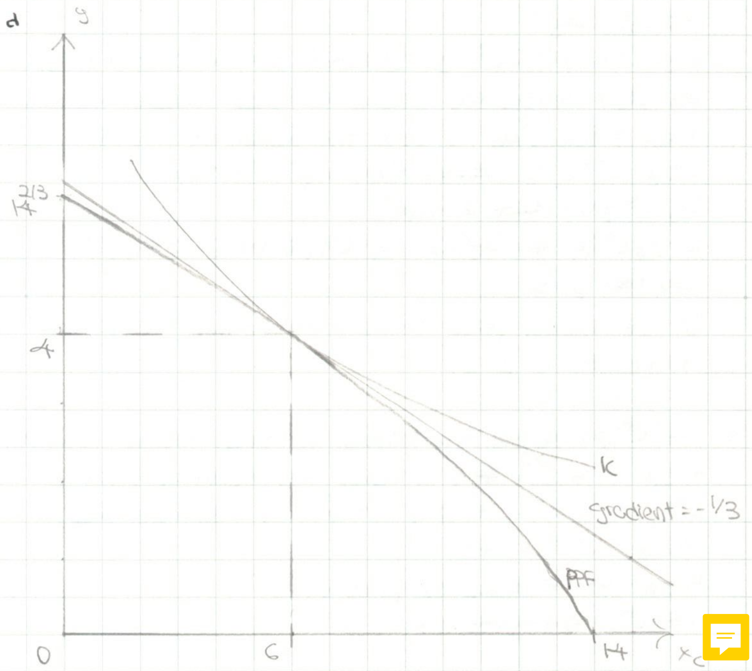
$\pi = P_y y - P_x x_f = 4 - (1/3)(8) = 4/3$

Budget constraint : $P_x x_c + P_y y \leq P_x x_w + \pi$

$\Rightarrow 1/3(6) + 1(4) \leq 1/3(14) + 4/3 \Leftrightarrow 6 \leq 6$

The consumer's budget constraint is satisfied (with equality).

d



2a Player i's strategy a_i is strictly dominant iff for all strategy profiles s_{-i} of other players, $u_i(a_i, s_{-i}) > u_i(a_i', s_{-i})$, ~~where a_i~~ for all of player i's other strategies a_i', and u_i is player i's payoff function. In other words, a_i is strictly dominant for player i iff a_i yields strictly greater payoff than any alternative strategy a_i' ~~for all~~ against all strategy profiles s_{-i} of other players.

Player i's strategy a_i is weakly dominant iff for all such s_{-i}, $u_i(a_i, s_{-i}) \geq u_i(a_i', s_{-i})$ for all a_i' and for some s_{-i}, $u_i(a_i, s_{-i}) > u_i(a_i', s_{-i})$.

If a ~~strictly~~ dominant strategy exists for each player, then the strategy profile under which each player plays his ~~strictly~~ dominant strategy is a NE since no player has strict incentive to deviate.

If a strictly dominant strategy exists for each player, then at NE, each player plays his strictly dominant strategy since at any other strategy profile, each player has strict incentive to deviate to his strictly dominant strategy.

Players need not play ~~weakly dominant strategies~~ at NE since ~~other strategies~~ could yield equal ~~payoff~~ against the Nash strategy profile.

b 1: G is strictly dominant for each player. Rationality requires each player to play G.

	G	R
G	5̲, 5̲	4, 6̲
R	6̲, 4	5̲, 5̲

Best responses underlined

	B	T
B	3̲, 3̲	2, 4
T	4, 2	5̲, 5̲

2: No player has a SD strategy, rationality does not identify a unique outcome, ISD does not eliminate any strategies, CKR does not identify a unique outcome, all strategies are rationalisable. There are two NE, it is not possible to predict the outcome by rationality, CKR, and "correct beliefs"

c Each game does not necessarily have a unique NE. NE is not guaranteed by CKR. So NE does not yield a unique prediction for the outcome of a game, and even if it did, the strong assumption that players have correct beliefs about each other player's strategy is necessary to think that the NE will result.

3a The certainty equivalent $CE(L)$ of lottery L, ~~is the~~ ~~amou~~ to the agent with preferences $\succsim$ and implied $\sim$, $\succ$, is the amount such that $[1; CE(L)] \sim L$, ~~i.e. the amount~~ i.e. such that the agent is indifferent between L and $CE(L)$ with certainty. If the agent's preferences over lotteries have utility representation U, then $CE(L)$ is such that $U([1; CE(L)]) = U(L)$. If the agent has expected utility preferences with Bernoulli utility u, then $CE(L)$ is such that $u(CE(L)) =$ $E(u(L))$.

Let L denote some lottery in final wealth levels. If for some agent $CE(L) > w$, where w is initial wealth, that agent prefers to participate. If $CE(L) = w$, the agent is indifferent, if $CE(L) < w$, the agent prefers not to participate.

b ~~$L = [1/2, 1/2; 12, 0]$~~ $L = [1/2, 1/2; 12+4, 0+4]$

$u_J(y) = 2y^{1/2}$

$U_J(L) = 1/2(2(16)^{1/2}) + 1/2(2(4)^{1/2}) = 1/2(8) + 1/2(4) = 6$

Let $L^S(p)$ denote the lottery in final wealth values that J faces if J sells the ticket at price p.

$L^S(p) = [1/2, 1/2; 4+p, 4+p] = [1, 4+p]$

$U_J(L^S(p)) = 2(4+p)^{1/2}$

$L \sim_J L^S(p) \Leftrightarrow U_J(L) = U_J(L^S(p)) \Leftrightarrow 6 = 2(4+p)^{1/2}$

$\Leftrightarrow p = 5$

The lowest price J would be willing to sell the ticket for is $p = 5$.

c Let $L^B(p)$ denote the lottery in final wealth values that S faces if S buys the ticket at price p. $L^B(p) = [1/2, 1/2; 4+12-p, 4-p]$. If S does not buy, S faces $L^0 = [1; 4]$.

~~From (a), S buys at price p iff~~
~~$CE(L^B(p)) \geq y = 4 \Leftrightarrow$~~

~~$u(L^B(p)) = 1/2($~~ $L^B(p) \sim L^0$

$U(L^B(p)) = U(L^0) \Leftrightarrow 1/2(2(16-p)^{1/2}) + 1/2(2(4-p)^{1/2}) = 2(4)^{1/2}$

$\Leftrightarrow (16-p)^{1/2} + (4-p)^{1/2} = 4 \Rightarrow$ ~~$16-p+4-p+(16-p)$~~

$16-p+4-p+2(16-p)^{1/2}(4-p)^{1/2} = 16 \Leftrightarrow$

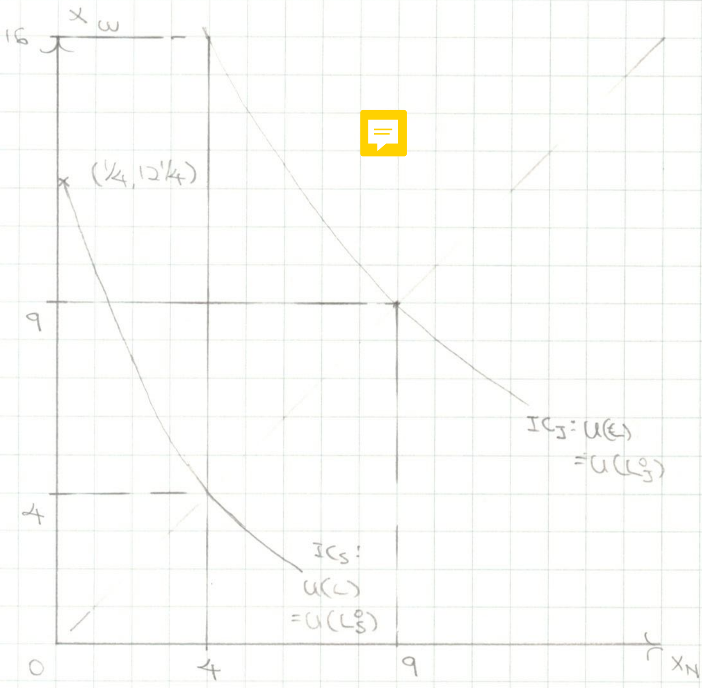
$20-2p+2(64+p^2-20p)^{1/2} = 16 \Leftrightarrow$

$2(64+p^2-20p)^{1/2} = -4+2p \Leftrightarrow$

$256+4p^2-80p = 16+4p^2-16p \Leftrightarrow$

$240-64p = 0 \Leftrightarrow p = 240/64 = 15/4$

The highest price S would be willing to buy the ticket for is $p = 15/4$.



$u(y) = 2y^{1/2}$, $u'(y) = y^{-1/2}$, $u''(y) = -\frac{1}{2}y^{-3/2}$, $A = -u''(y)/u'(y)$
$= \frac{1}{2}y^{-1}$

J and S have decreasing absolute risk aversion, J has higher initial wealth than S, hence J is less risk ~~averse~~ averse, hence J's ~~[illegible]~~ valuation of the ticket is greater than S's. Graphically, this is reflected in S's indifference curves being steeper than J's.

4a P chooses contract of the form $(w, e=1)$ such that A is just indifferent between accepting this contract and not since P's payoff is decreasing in w. Under this contract A is paid 0 if $e \neq 1$, so ~~if~~ if A accepts, $e=1$ is optimal.

$u(w, e=1) = \bar{u} \Leftrightarrow \sqrt{w} - 5(1) = 10 \Leftrightarrow \sqrt{w} = 15 \Rightarrow w = 225$
~~(e.g. ... = 5)~~

b ~~PC: $u(w_H, e=1) \geq \bar{u}$~~
~~when PC binds, $u(w_H, e=1) = \bar{u}$~~
~~$\sqrt{w_H} - 5 = 10 \Rightarrow \sqrt{w_H} = 15 \Rightarrow w_H = 225$ (reject this)~~
~~IC:~~

PC: $E(u(w(\pi), e=1) | e=1) \geq \bar{u}$
when PC binds,
$E(u(w(\pi), e=1)(e=1) = \bar{u}$
$5/8 [\sqrt{36} - 5] + 3/8 [\sqrt{w_H} - 5] = 10 \Leftrightarrow$
$5/8(1) + 3/8 [\sqrt{w_H} - 5] = 10 \Leftrightarrow$
$3/8 [\sqrt{w_H} - 5] = 10 - 5/8 = 75/8 \Leftrightarrow$
$\sqrt{w_H} - 5 = 25 \Leftrightarrow w_H = 900$

IC: $E(u(w(\pi), e=1 | e=1) \geq E(u(w(\pi), e=0 | e=0)$
$E(u(w(\pi), e=0)(e=0) = 5/6\sqrt{36} + 1/6\sqrt{900} = 5 + 5 = 10$
$\leq E(u(w(\pi), e=1)(e=1)$,
IC is satisfied with equality.

c The agency cost is the excess of the expected wage required to induce high effort in the case where effort is unobservable over the wage required to induce high effort where effort is observable.

$E(w(\pi)) =$ ~~$5/6(36) + 1/6(900) =$~~ $5/8(36) + 3/8(900)$
$= 45/2 + 675/2 = 720/2 = 360$

Agency cost $= 360 - 225 = 135$

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10a All traders engage in trade. At the initial allocation, by inspection of the common utility function, each trader has zero utility (which is the minimum, given the natural positivity constraints $x_A, x_B, x_C \geq 0$). Each trader is strictly better off if ~~he~~ trades some but not all of his endowment for some amount of each good that he is not endowed with, however small. Supposing that the goods are perfectly divisible, ~~this is~~ such trades are feasible.

By inspection, each trader has Cobb-Douglas utility. Then, each trader's demands are such that each trader spends $1/2$ of ~~his~~ the value of his endowment on A, $1/4$ on B, and $1/4$ on C.

Verify by Lagrangian optimisation. Let m denote the value of an arbitrary trader's endowment. Arbitrary trader's optimisation problem is

$\max_{x_A, x_B, x_C} u(x_A, x_B, x_C) = x_A^{1/2} x_B^{1/4} x_C^{1/4}$ s.t.

BC: $p_A x_A + p_B x_B + x_C \leq m$

$\ln$ is a monotonic transformation, so this is equivalent to

$\max_{x_A, x_B, x_C} u(x_A, x_B, x_C) = \frac{1}{2}\ln x_A + \frac{1}{4}\ln x_B + \frac{1}{4}\ln x_C$ s.t.

BC: $p_A x_A + p_B x_B + x_C \leq m$

By inspection, u is concave, BC is linear, and the constraint set is non-empty, so the FOC is necessary and sufficient for an optimum. By inspection, u is increasing in each x, hence BC binds at the optimum. The optimisation problem reduces to one with equality constraints.

~~FOC_A: $1/2x_A$~~

$\mathcal{L} = \frac{1}{2}\ln x_A + \frac{1}{4}\ln x_B + \frac{1}{4}\ln x_C - \lambda(p_A x_A + p_B x_B + x_C - m)$

FOC_A: $1/2x_A = \lambda p_A \Rightarrow x_A = 1/2\lambda p_A$

FOC_B: $1/4x_B = \lambda p_B \Rightarrow x_B = 1/4\lambda p_B$

FOC_C: $1/4x_C = \lambda \Rightarrow x_C = 1/4\lambda$

FOC_λ: $p_A x_A + p_B x_B + x_C = m$

$\Rightarrow \lambda = 1/m \Rightarrow x_A = m/2p_A,\ x_B = m/4p_B,\ x_C = m/4$

Then

$x_A^1 = 10p_A/2p_A = 5,\ x_B^1 = 10p_A/4p_B,\ x_C^1 = 10p_A/4$

$x_A^2 = 5p_B/2p_A,\ x_B^2 = 5p_B/4p_B = 5/4,\ x_C^2 = 5p_B/4$

$x_A^3 = 20/2p_A = 10/p_A,\ x_B^3 = 20/4p_B = 5/p_B,\ x_C^3 = 20/4 = 5$

b ~~$z_A = x_A^1$~~ Excess demands

$z_A = 10x_A^1 + 5x_A^2 + 5x_A^3 - 10\omega_A^1 = 50 + 25p_B/2p_A + 50/p_A - 100$

$= -50 + 25p_B/2p_A + 50/p_A$

$z_B = 10x_B^1 + 5x_B^2 + 5x_B^3 - 5\omega_B^2 = 25p_A/p_B + 25/4 + 25/p_B - 25$

$= -75/4 + 25p_A/p_B + 25/p_B$

$z_A = 0 \Leftrightarrow -50p_A + \frac{25}{2}p_B + 50 = 0$ (1)

$z_B = 0 \Leftrightarrow -\frac{75}{4}p_B + 25p_A + 25 = 0$ (2)

~~Solving~~ Solve simultaneously by elimination

(1)+2(2) $\frac{25}{2}p_B + 50 - \frac{75}{2}p_B + 50 = 0 \Leftrightarrow 25p_B = 100$

$\Leftrightarrow p_B = 4$ (3)

Sub (3) into (1)

$-50p_A + 50 + 50 = 0 \Leftrightarrow p_A = 2$

$x_A^1 = 5,\ x_B^1 = 5/4,\ x_C^1 = 5$

$x_A^2 = 5,\ x_B^2 = 5/4,\ x_C^2 = 5$

$x_A^3 = 5,\ x_B^3 = 5/4,\ x_C^3 = 5$

The competitive equilibrium is the above allocation and the price vector $(p_A, p_B, p_C) = (2, 4, 1)$

By Walras's Law if the markets for A and B clear, i.e. $z_A = 0, z_B = 0$, then the market for C clears, i.e. $z_C = 0$. This follows from the binding budget constraints.

c The allocation is Pareto efficient. ~~At the allocation each trader consumes each good the three goods in~~

$MU_A = 1/2x_A,\ MU_B = 1/4x_B,\ MU_C = 1/4x_C$

Then, the marginal rate of substitution of any type of trader between any two goods depends only on the ratio of the two goods. Then at the given allocation, any two traders have equal MRS for any two goods, and there is no mutually profitable trade, so the given allocation is Pareto optimal.

From (a), at any equilibrium, each trader spends $1/2$ of his "income" on A, $1/4$ on B and $1/4$ on C. So the price vector ($p_A = 2, p_B = 4, p_C = 1$) (or any uniformly scaled price vector), together with the given allocation, constitutes a competitive equilibrium.

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