Phua Ren Ping

78/1d overall. Excellent

Prof Alan Beggs

Microeconomics Collection

1a $u_a(x_a, y_a) = x_a y_a^3$, $u_b(x_b, y_b) = 16 x_b y_b$. $\omega_a = (8,8)$, $\omega_b = (0,0)$. $P_y = 1$, $P_x = p$

$MU_x^a = \partial u_a / \partial x_a = y_a^3$, $MU_y^a = \partial u_a / \partial y_a = 3 x_a y_a^2$

$MU_x^b = \partial u_b / \partial x_b = 16 y_b$, $MU_y^b = \partial u_b / \partial y_b = 16 x_b$

Under the allocation $(x_a, y_a) = (2,4)$, $(x_b, y_b) = (6,4)$:

Don't calculate MU's

In fact FOC not valid here

$MU_x^a = 4^3 = 64$, $MU_x^b = 16(4) = 64$, $MU_x^a = MU_x^b$

$MU_y^a = 3(2)(4)^2 = 96$, $MU_y^b = 16(6) = 96$, $MU_y^a = MU_y^b$

Since $MU_x^a = MU_x^b$, transfer of a marginal unit of x from a to b increases b's utility by the same amount as the transfer decreases a's utility, the reverse is true of a transfer from b to a. Total utility cannot be increased by any transfer of x. Similarly, since $MU_y^a = MU_y^b$, total utility cannot be increased by any transfer of y.

Total utility is maximised under the given allocation, the allocation is best according to the utilitarian definition.

Since this allocation maximises total utility, the utility of one agent cannot be increased without decreasing the utility of the other, the allocation is Pareto-efficient.

~~$MRS_a = MU_x^a / MU_y^a$~~

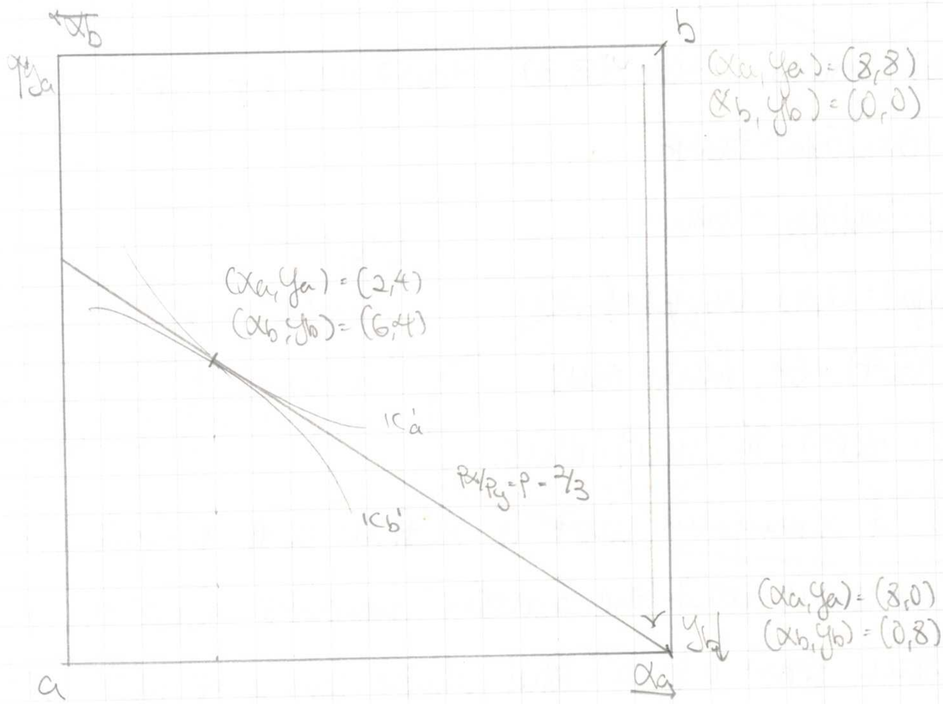
Under the given allocation:

$MRS_a = MU_x^a / MU_y^a = 64/96 = 2/3$, $MRS_b = MU_x^b / MU_y^b = 64/96 = 2/3$, $MRS_a = MRS_b$

The two agents' indifference curves are tangential. Given monotonic, continuous, convex preferences, there is no allocation on a higher indifference curve for one agent that is not on a lower indifference curve for the other. ~~The given~~

The given allocation is Pareto efficient.

b



The social planner must transfer 8 units of y from a to b.
At the price ratio $P_x/P_y = p = 2/3$, ~~trading from the post-transfer allocat~~, given the post-transfer allocation $(x_a, y_a) = (8,0)$, $(x_b, y_b) = (0,8)$, both agents maximise utility subject to their budget constraint at the allocation $(x_a, y_a) = (\cancel{4}\ 2,4)$, $(x_b, y_b) = (6,4)$

ci Preferences of both agents are well-behaved: monotonic, continuous, and convex.
Preferences are monotonic: utility is strictly increasing in each good. Given monotonic and continuous preferences, it is always possible to increase one agents' utility ~~by a marginal amount at~~ at the expense of a marginal amount of the other agent's utility by some transfer of some good from the latter to the former. If utility of both agents are not equal, utility of the worse-off agent can be increased at the expense of a marginal amount of the better off agent's utility by some such transfer. The worse off agent would be better off under the latter allocation than under the former, the former allocation is not optimal under the Rawlsian criterion.

ii ~~a~~ If $MRS_a \neq MRS_b$, ~~then~~ indifference curves are not tangential, there are allocations which lie on higher indifference curves for both agents. At these allocations the worse-off agent would be better off than the worse off agent ~~in~~ at the original allocation, the original allocation would not be optimal under the Rawlsian Criterion.

There are no other necessary conditions | exhaust budget

22/23

2a $u_a(y, m_a) = 42y - y^2 + m_a$, $u_b(y, m_b) = 1/4\, y^2 + 4y + m_b$

$y = 2h$, $p_h = 20$, $c(y) = 10y$] explain more fully

$m_a = m_a^* - c(y)$

$u_a(y) = 42y - y^2 + m_a^* - c(y) = 42y - y^2 + m_a^* - 10y = 32y - y^2 + m_a^*$

College a chooses y^e to maximise $u_a(y) = 32y - y^2 + m_a^*$

FOC: $\partial u_a/\partial y = 32 - 2y = 0$, $y = 16$

SOC: $\partial^2 u_a/\partial y^2 = -2 < 0$

College a maximises $u_a(y)$ by choosing $y^e = 16$

$y^e = 2h^e$, $h^e = y^e/2 = 16/2 = 8$

College a contracts 8 hours of gardening work

$MB = MU_y^a + MU_y^b = \partial u_a/\partial y + \partial u_b/\partial y = 42 - 2y + 1/2\, y + 4 = 46 - 3/2\, y$

$MC = c'(y) = 10$

Under the Pareto-optimal allocation,

$MB = MC$, $46 - 3/2\, y = 10$, $36 = 3/2\, y$, $y = 24$

Total utility is maximised by the level of output $y^o = 24$, this level of output is Pareto-efficient.

Under the equilibrium level of output y^e:

$u_a(y^e) = \cancel{42y^e - y^{e2} + m_a^* - 10y^e}\ 32y^e - y^{e2} + m_a^* = 256 + m_a^*$

$u_b(y^e, m_b^*) = 1/4\, y^{e2} + 4y^e + m_b^* = 128 + m_b^*$

Under the Pareto-optimal level of output y^o:

$u_a(y^o) = 32y^o - y^{o2} + m_a^* = 196 + m_a^*$ 192

$u_b(y^o, m_b^*) = 1/4\, y^{o2} + 4y^o + m_b^* = 240 + m_a^*$

Neither level of output is ~~Pareto-superior to the other~~ Pareto-dominates the other (given that a pays for the trees and there are no further transfers) since a is better-off under y^e ~~and b is better off~~ than y^o while b is better off under y^o than y^e.

Given no further transfers of m, ~~if~~ no level of output y Pareto-dominates y^e since y^e is utility maximising for a.

b Let the amount of the transfer from b to a be t.

The agreement is acceptable to a ~~only if~~ iff

$u_a(y^o, m_a^* - c(y^o) + t) \geq u_a(y^e, m_a^* - c(y^e))$

$196 + m_a^* + t \geq 256 + m_a^*$

$t \geq 60$ (1) 64

The agreement is acceptable to b iff

$u_b(y^o, m_b^* - t) \geq u_b(y^e, m_b^*)$

$240 + m_b^* - t \geq 128 + m_b^*$

$t \leq 112$

The agreement is ~~of~~ acceptable to both parties iff $60 \leq t \leq 112$.

c Utility functions of both colleges are quasi-linear in money m.

+ There is an externality

~~that~~ Therefore $MU_m^a = MU_m^b = 1$, $MRS_a = MU_y^a$, $MRS_b = MU_y^b$

Because utility is quasi-linear in money, there is only one Pareto-efficient level of output y^o, when $MRS_a = MRS_b$, $MU_y^a = MU_y^b$.

Also externals. Could mention concavity, convexity

18/25

3a $p(q) = \{ \alpha - q$ if $q \le \alpha$; 0 otherwise , $\alpha > 0$

$\alpha > c_1 > c_2 > 0$

$c_1(q_1) = c_1 q_1$, $c_2(q_2) = c_2(q_2)$

Profits of ~~both~~ firm 1

$\pi_1(q_1) = p(q_1+q_2)q_1 - c_1(q_1) = (\alpha - q_1 - q_2)q_1 - c_1 q_1 = (\alpha - q_2 - c_1)q_1 - q_1^2$

Firm 1 chooses q_1 to maximise $\pi_1(q_1)$ ~~st $q_1 \ge 0$~~ st $q_1 \ge 0$

Put in a few more words

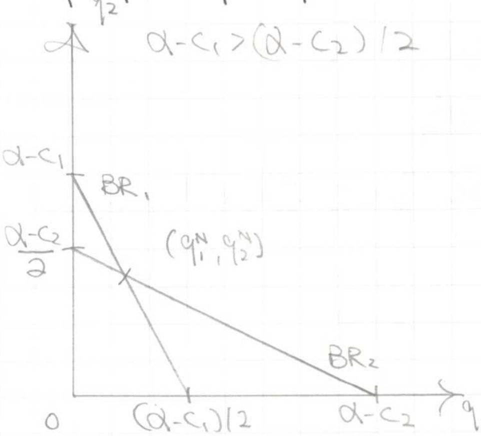
FOC: $\pi_1'(q_1) = (\alpha - q_2 - c_1) - 2q_1 = 0$, $q_1 = (\alpha - q_2 - c_1)/2$

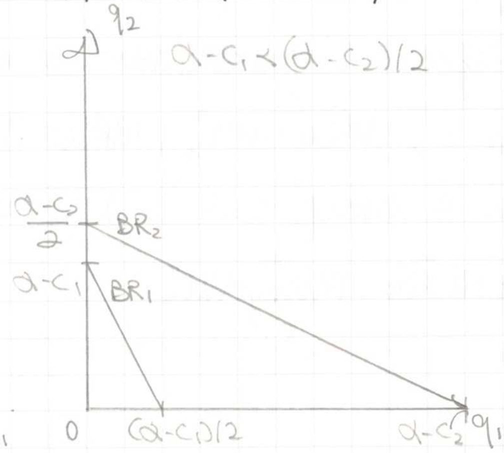
SOC: $\pi_1''(q_1) = -2 < 0$

Firm 1 maximises profits by choosing $q_1^* = (\alpha - q_2 - c_1)/2$ if $q_2 \le \alpha - c_1$; 0 otherwise

since $q_1 \ge 0$

By symmetry, $q_2^* = (\alpha - q_1 - c_2)/2$ if $q_1 \le \alpha - c_2$; 0 otherwise





BR extends indefinitely
BRs defined for all competitor moves

~~b $\alpha - c_1 > (\alpha - c_2)/2$:~~

b $\alpha - c_1 > (\alpha - c_2)/2$, the constraints $q_1 \ge 0$ and $q_2 \ge 0$ are not binding

~~$q_1^* = (\alpha - q_2^*$~~ $q_1^N = (\alpha - q_2^N - c_1)/2$, ~~$q_2^N =$~~ Sub in $q_2^N = (\alpha - q_1^N - c_2)/2$

$q_1^N = (\alpha - (\alpha - q_1^N - c_2)/2 - c_1)/2 = (\alpha - \alpha/2 + q_1^N/2 + c_2/2 - c_1)/2 = \alpha/4 + q_1^N/4 + c_2/4 - c_1/2$

$3q_1^N/4 = \alpha/4 + c_2/4 - c_1/2$, $q_1^N = \alpha/3 + c_2/3 - 2c_1/3 = (\alpha - 2c_1 + c_2)/3$

By symmetry, $q_2^N = (\alpha - 2c_2 + c_1)/3$

Each firm's Cournot Nash output is increasing in demand parameter α and competitor's cost, and decreasing in its own cost.

Since $c_1 > c_2$, $q_2^N > q_1^N$. the more efficient firm produces ~~the~~ more output.

$\alpha - c_1 < (\alpha - c_2)/2$ the constraint $q_1 \ge 0$ is binding for firm 1

$q_1^N = 0$, $q_2^N =$ ~~$q_2^*(0)$~~ $= q_2^*(q_1^N = 0) = (\alpha - c_2)/2$

Firm 1's costs are so high that it is better off producing zero output given that firm 2 produces its ~~Nash~~ Cournot Nash output $\frac{q^N}{2}$.

~~F~~ The more efficient firm 2 produces more output in either scenario ~~since~~ because marginal revenue is greater than firm 2's marginal cost over a larger number of units than it is for firm 1.

23/25

4a $u(w,e) = \sqrt{w} - e$, $\bar{w} = 36$, $\bar{e} = 2$

$\pi_L = 30$, $\pi_H = 70$, $P(\pi = \pi_H | e=0) = 1/4$, $P(\pi = \pi_H | e=1) = 1/2$

The reservation utility $\bar{u}$ of the worker has to do with his outside option. The reservation utility is the minimum utility of a contract offered to the worker such that the worker would accept, ~~this that~~ the utility of this contract must be weakly greater than the utility of his outside option.

$\bar{u} = u(\bar{w}, \bar{e}) = \sqrt{36} - 2 = 4$ ✓ So $\bar{u}$ is . . .

In order to induce $e=0$

> offer w_0 such that $u(w_0, e=0) \geq \bar{u}$

$u(w_0, e=0) = \sqrt{w_0} - 0 = \sqrt{w_0} = \bar{u} = 4$, $w_0 = 16$ ✓ explain

In order to induce $e=1$ ✓

$u(w_1, e=1) = \sqrt{w_1} - 1 = \bar{u} = 4$, $\sqrt{w_1} = 5$, $w_1 = 25$

~~Expected g~~ If the principal induces $e=0$, ✓

expected gross profit $E(\pi) = 1/4\,\pi_H + 3/4\,\pi_L = 40$

expected net profit $E(\pi - w_0)$, $E(\pi) - w_0 = 40 - 16 = 24$ ✓

If the principal induces $e=1$

expected gross profit $E(\pi) = 1/2\,\pi_H + 1/2\,\pi_L = 50$

expected net profit $E(\pi - w_1) = E(\pi) - w_1 = 50 - 25 = 25$

The risk-neutral principal maximises expected net profit by choosing ✓ to induce $e=1$, he offers the contract $(w = w_1 = 25, e = 1)$

b Let ~~$w_L = w(\pi_L)$~~ $L = w(\pi_L)$, and $H = w(\pi_H)$ ✓

The optimal way to induce low effort is to offer ~~$(L=16, H=16)$~~ $(L = w_0 = 16, H = w_0 = 16)$

Any other contract that the worker will accept is sub-optimal ~~to~~ for the principal.

The contract to induce high effort must satisfy individual rationality and incentive compatibility.

individual rationality:

expected utility to the agent of the contract must exceed reservation utility.

$E(u | e=1) \geq \bar{u}$ ✓

$P(\pi = \pi_H | e=1)\, u(H, e=1) + P(\pi = \pi_L | e=1)\, u(L, e=1) \geq \bar{u}$

~~(1/2)~~ $(1/2)(\sqrt{H}-1)+(1/2)(\sqrt{L}-1) \geq 4$

$1/2\sqrt{H} + 1/2\sqrt{L} \geq 5$

$\sqrt{H} + \sqrt{L} \geq 10$

~~Incentive~~ Incentive compatibility:

The expected utility maximizing choice for the agent must be to choose $e=1$

$E(u|e=1) \geq E(u|e=0)$

$P(\pi=\pi_H|e=1)u(H,e=1) + P(\pi=\pi_L|e=1)u(L,e=1) \geq P(\pi=\pi_H|e=0)u(H,e=0) + P(\pi=\pi_L|e=0)u(L,e=0)$

$(1/2)(\sqrt{H}-1) + (1/2)(\sqrt{L}-1) \geq (1/4)(\sqrt{H}-0) + (3/4)(\sqrt{L}-0)$

$2\sqrt{H}-2+2\sqrt{L}-2 \geq \sqrt{H} + \cancel{3\sqrt{H}}\ 3\sqrt{L}$

Note can assume =

$\sqrt{H} \geq \sqrt{L}+4$

At the margins:

$\sqrt{H}+\sqrt{L}=10,\ \sqrt{H}=\sqrt{L}+4,\ \sqrt{H}=7,\ \sqrt{L}=3,\ H=49,\ L=9,\ w(\pi_H)=49,\ w(\pi_L)=9$

the optimal contract to induce high effort is $(H=49, L=9)$

Expected wage $\cancel{E(w)}\ E(w(\pi)|e=1) = P(\pi=\pi_H|e=1)w(\pi_H) + P(\pi=\pi_L|e=1)w(\pi_L) = 1/2\,49 + 1/2\,9 = 29$

Agency cost $\cancel{E(w)}\ E(w(\pi)|e=1) - w_1 = 29-25 = 4$

The principal incurs an additional cost to induce high effort in a risk-averse agent.

Expected net profit of inducing high effort $E(\pi-w(\pi)|e=1) = E(\pi|e=1) - E(w(\pi)|e=1) = 50-29=21$

Expected net profit of inducing low effort $E(\pi-w(\pi)|e=0) = 40-16=24 > 21$

The optimal contract is the contract which induces low effort $(H=16, L=16)$

$\frac{22}{25}$

5 = Arrows ~~vot~~ Theorem
+
Incentives for truthful voting (revelation

6 = Rivalry

Excludable

Funding Mechanisms + Limitations

8 = Risk aversion = Concavity
a more risk averse than b iff ~~u~~ u_a is a concave transform of u_b
OR measurements : $u''(x)/u'(x)$
OR size of risk premium

second part: if offered actuarially fair insurance,
however risk-averse, fully insure
+ diagram

High risks + high income for pro athletes

9 Low ability prefer pooling always
High ability ambiguous → depending on signal costliness

7 This essay begins by defining the concepts of a best response and Nash equm. Nash eqm does not generate strong and robust predictions. Rationality is not sufficient for predicting the outcome of a game with NE. Even common knowledge of rationality is insufficient. Nash eqm predicts the outcome of games only if it is given that each player believes all other players will play their NE strategy. NE is nonetheless useful as a solution concept because it can ~~generat~~ predict the outcome of repeated games given certain assumptions about beliefs, as in Cournot-Nash eqm, and can help describe games where there is no dominant strategy equm.

Iterated deletion of dominated strategies
↳ robust prediction given R & CKR
↳ if prediction unique → NE
+ any NE survives iterated del, not vice versa
Nash adds refinement: correct beliefs abt each other

+ multiple equilibria
+ (learning (iterated best responses) → equilibriate at NE

A player's best response given the strategies of other players is the strategy of this player that weakly maximises his payoff given the strategies ~~prof~~ of other players. The strategy s_i of player i is a best response to the strategy profile that specifies the strategies of all other players s_{-i} if $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$ for all other strategies of player i s_i'. A Nash equilibrium is ~~the~~ a strategy profile s^N such that each player's strategy in this strategy profile is a best response given the strategies of all other players. Nash equilibria are strategy profiles where there is best response correspondence. At the Nash equilibrium, no player has incentive to change his strategy since his strategy ~~is his~~ is a best response and maximises his payoff.

Nash equilibrium does not generate a strong prediction of the outcome of a game because a game can have multiple Nash equilibria. Consider the following game.

Best responses underlined

		Column Left (A)	Right (B)
Row	Top (A)	1, 1	0, 0
	Bottom (B)	0, 0	1, 1

The game is a coordination game: if one player chooses A, it is rational for the other player to do the same, if one player chooses B, it is rational for the other player to choose the same. There are two Nash equilibria in this game: (A, A) and (B, B). At either ~~of~~ Nash eqm, neither player has incentive to change his strategy: ~~to~~ this would result in miscoordination and a lower payoff to both players. The outcome of this game cannot be predicted by the concept of Nash eqm also because each player's ~~decision~~ strategy depends on that player's beliefs about what the other player will choose - ~~both~~ both players want to choose the same strategy as the other. Rationality is not sufficient for predicting the outcome of this game. Nash equilibrium does not generate a robust prediction of the outcome of the game

Nash equilibrium ~~is nonetheless useful~~ does not generate strong and robust predictions even ~~under~~ if it is given that players are rational and there is common knowledge of rationality (CKR). Given CKR, Row knows that Column will want to play the same strategy as Row and vice versa. ~~This does not give Row any more reason to think~~ What Row thinks Column will play depends on what Column thinks Row will play. ~~Even~~ Even given CKR, neither player has any reason to choose one option over the other without knowledge of ~~at least one~~ the other player's beliefs.

The concept of Nash eqm is useful because it describes an outcome from which no player has incentive to deviate. ~~Consider the above game sequential move game where each players take turns to choose~~ It ~~pre~~ is capable of predicting outcomes in games given some information about ~~how~~ player's beliefs. Consider the Cournot duopoly when two firms produce a homogeneous good with inverse demand function

$p(q) = a - q$

Firms have identical cost functions $c_i(q_i) = cq_i$

Profit of firm i $\pi_i(q_i) = p(q_i + q_{-i})q_i - c(q_i) = (a - q_i - q_{-i})q_i - cq_i = (a - c - q_{-i})q_i - q_i^2$

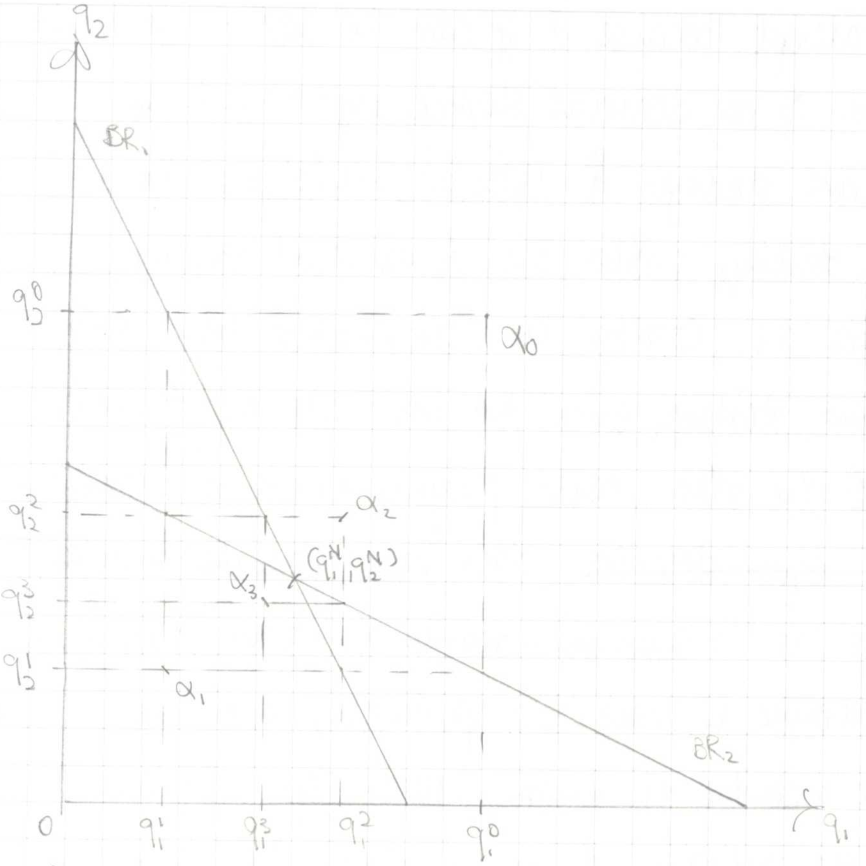
Each firm chooses q_i to maximise $\pi_i(q_i)$

FOC: $\partial\pi_i/\partial q_i = (a - c - q_{-i}) - 2q_i = 0$, $q_i = (a - c - q_{-i})/2$

SOC: $\partial^2\pi_i/\partial q_i^2 = -2 < 0$

$q_i^* = (a - c - q_{-i})/2$

~~Each fi~~ Firm i's best response function $q_i^*(q_{-i}) = (a - c - q_{-i})/2$



If each firm believes that the other firm will produce in the next period what it did in the previous period, output of both firms converge to the Cournot Nash output. Consider the arbitrary initial strategy profile ~~*~~ given by point x_0. Given each firm's beliefs about the other firm's output in the next period, both firms produce q_1^1 and q_2^1 respectively in the next period. In the subsequent period, both firms produce q_1^2 and q_2^2 respectively. The process continues until the Nash equilibrium is reached and neither firm has any incentive to change output in subsequent periods given their belief that the other firm will continue producing the same output. In a repeated game where each player believes that the others will play in subsequent rounds what they did in previous rounds, ~~#~~ if strategies converge to an eqm, the strategy profile at eqm is a Nash equilibrium.

(But learning not approp justification when playing a new game?)

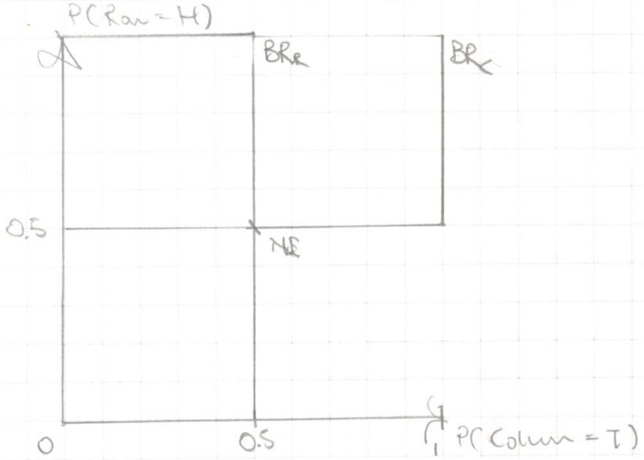
68 This essay makes sensible points but you need to mention iterated deletion of strictly dominated strats – ~~if common knowledge of rationality then can argue~~ can argue this cap needs common knowledge of rationality

Nash eqm is a useful solution concept because it ~~g~~ can be used to generate predictions for games where there is no dominant strategy equm. ~~A do~~ Player i's strategy s_i is a dominant strategy iff $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$ for all alternative strategies ~~s_i~~ s_i' for all strategy profiles s_{-i}. A dominant strategy is a best response to any strategy profile s_{-i}. Consider the coordination game given earlier: no player has a dominant strategy since the best response of each player depends on the choice of the other; each player wants to choose the same strategy as the other. ~~Nash eqm describes~~ Nash eqm ~~generate a non-unique and non-robust predictions~~ allows us to ~~say mo~~ ~~describe the game~~ say more about the game than we would otherwise be able to: specifically about the outcomes from which no agents have incentive to deviate. Nash eqm may even generate unique predictions about the outcome of games when there is no DSE. Consider the Matching Pennies game

Best responses underlined

		Column Left (H)	Column Right (T)
Row	Top (H)	-1 1	1 -1
	Bottom (T)	1 -1	-1 1

No player has a dominant strategy since each player's best ~~respond~~ response depends on the choice of the other player. Row would prefer to choose the same ~~as~~ strategy as Column, Column would ~~like~~ prefer to choose a different strategy from Row. There is a unique mixed strategy Nash eqm in this game where each player chooses H wp 50% and T wp 50%. The expected payoff to each player given that the other plays this strategy is zero, regardless of what the former chooses, ~~the fo~~ each player has no reason to deviate from the Nash eqm.



Nash equilibrium NE is the point of best response correspondence.

Nash eqm is a reasonable solution concept if ~~quat~~ appropriately qualified: NE does not ~~p~~ generate strong and robust predictions of outcomes. NE is only predicted as an outcome if all players believe all other players will play their NE strategies. ~~NE is more useful at predicting the outcome of repea~~ NE enables us to describe a game in greater detail than if we relied solely on the concept of DSE.

2a $L_w = [0.4, 0.3, 0.3 ; 36, 36, 25]$

~~$L_r = [0.4, 0.3$~~

$L_r = [0.4, 0.3, 0.3 ; 64, 36, 9]$

$L_s = [0.4, 0.3, 0.3 ; 100, 36, 0]$

$EV(L_w) = 0.4 \times 36 + 0.3 \times 36 + 0.3 \times \overset{25}{\cancel{36}} = \cancel{36}\ 32.7$

$EV(L_r) = 0.4 \times 64 + 0.3 \times 36 + 0.3 \times 9 = 39.1$

$EV(L_s) = 0.4 \times 100 + 0.3 \times 36 + 0.3 \times 0 = 50.8$

Soybeans have the highest expected return (shown)

$EU(L_w) = 0.4\,u(36) + 0.3\,u(36) + 0.3\,u(\overset{25}{\cancel{36}}) = u(\cancel{36}) = \cancel{\sqrt{36} = 6}\ \ 0.7\sqrt{36} + 0.3\sqrt{25} = 0.7(6) + 0.3 \times 5 = 5.7$

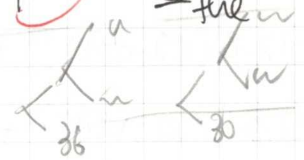
$EU(L_r) = 0.4\,u(64) + 0.3\,u(36) + 0.3\,u(9) = 0.4\sqrt{64} + 0.3\sqrt{36} + 0.3\sqrt{9} = 0.4 \times 8 + 0.3 \times 6 + 0.3 \times 3 = \cancel{6.7}\ 5.9$

$EU(L_s) = 0.4\,u(100) + 0.3\,u(36) + 0.3\,u(0) = 0.4\sqrt{100} + 0.3\sqrt{36} + 0.3\sqrt{0} = 0.4 \times 10 + 0.3 \times 6 = 5.8$

An expected utility maximiser with utility function $u(x) = \sqrt{x}$ maximises expected utility by choosing L_r since $EU(L_r) > EU(L_w) > EU(L_s)$.

i.e. lower spread outcomes

Because the utility function $u(x) = \sqrt{x}$ has the property of decreasing marginal utility of money, the agent is risk-averse. Rice is a less risky crop than Soybeans since it has a return of 9 even in the Bad case, as opposed to 0 for Soybeans. Because ~~of~~ the agent has decreasing marginal utility, ~~additional~~ wealth in the Bad outcome increases expected utility more than ~~an equivalent~~ the ~~an~~ increase in equivalent decrease in wealth in the ~~a~~ the Good outcome decreases expected utility.



prefer depends only on this

Independence axiom

b Yes

The preferences of expected utility maximisers have the property of independence:

$L_1 \succsim L_2$ iff $[p, 1-p ; x, L_1] \succsim [p, 1-p ; x, L_2]$ for all $p \in [0,1]$, $x \in \mathbb{R}$

Let $p = 0.3$, $L_1 = [4/7, 3/7 ; 64, 9]$, $L_2 = [4/7, 3/7 ; 100, 0]$, $L_3 = [4/7, 3/7 ; 36, 25]$

$L_w = [p, 1-p ; 36, L_3]$, $L_r = [p, 1-p ; 36, L_1]$, $L_s = [p, 1-p ; 36, L_2]$

~~so~~ $L_r \succsim L_s \xrightarrow{\text{by independence}} L_1 \succsim L_2 \xrightarrow{\text{by independence}} [p, 1-p ; 30, L_1] \succsim [p, 1-p ; 30, L_2]$

$\rightarrow [0.4, 0.3, 0.3 ; 64, 30, 9] \succsim [0.4, 0.3, 0.3 ; 100, 30, 0]$

~~Sil~~ Analogously for $L_r \succsim L_w \rightarrow [0.4, 0.3, 0.3 ; 64, 30, 9] \succsim [0.4, 0.3, 0.3 ; 36, 30, 25]$

the expected utility maximiser would nonetheless prefer to grow rice over the other two crops.

c $L_i = [0.4, 0.3, 0.3;\ 100-21,\ \cancel{57}^{36}-21,\ 0-21+36] = [0.4, 0.3, 0.3;\ 79,\ \cancel{43}^{15},\ 15]$

$EU(L_i) = 0.4u(79) + 0.3u(\cancel{43}^{15}) + 0.3u(15) = 0.4\sqrt{79} + 0.3\sqrt{\cancel{43}^{15}} + 0.3\sqrt{15} = 5.8791 < EU(L_r)$

Expected utility of growing soybeans under the insurance contract is lower than expected utility of growing rice uninsured. G maximises expected utility by growing rice uninsured, and will not contract the plan.

d The ~~shared lott~~ lottery each farmer faces if they share returns in this way

$L_{us} = [0.4, 0.3, 0.3;\ \frac{100+36}{2},\ \frac{36+36}{2},\ \frac{25+0}{2}] = [0.4, 0.3, 0.3;\ 68, 36, 12.5]$

$EU(L_{us}) = 0.4\sqrt{68} + 0.3\sqrt{36} + 0.3\sqrt{12.5} = 6.1591$

$EU(L_{us}) > EU(L_r)$, $L_{us} \succsim L_r$

Returns under any outcome are weakly better in L_{us} than in L_r, L_{us} FOSDs L_r, hence G and R ~~can~~ are better off under L_{us} than under L_r.

~~Good~~ Good

regardless of a fa given risk-aversion

e the lottery each farmer faces

$L_\lambda = [0.4, 0.3, 0.3;\ 36\lambda + 100(1-\lambda),\ 36\lambda + 36(1-\lambda),\ 25\lambda + 0(1-\lambda)]$

$= [0.4, 0.3, 0.3;\ 100-64\lambda,\ 36,\ 25\lambda]$

$EU(L_\lambda) = 0.4\sqrt{100-64\lambda} + 0.3\sqrt{36} + 0.3\sqrt{25\lambda}$

Maximisation problem $\max_{\lambda \in [0,1]} EU(L_\lambda) = 0.4\sqrt{100-64\lambda} + 0.3\sqrt{36} + 0.3\sqrt{25\lambda}$

FOC: $\partial EU(L_\lambda)/\partial\lambda = 0.4(1/2)(100-64\lambda)^{-1/2}(-64) + 0.3(1/2)(25\lambda)^{-1/2}(25)$

$= -12.8(100-64\lambda)^{-1/2} + 0.3(1/2)(25)^{-1/2}(25)(\lambda)^{-1/2}$

$= -12.8(100-64\lambda)^{-1/2} + 0.75\lambda^{-1/2} = 0,$

$12.8(100-64\lambda)^{-1/2} = 0.75\lambda^{-1/2}$

$6.4(100-64\lambda)^{-3/2}(-64) - 3/8\,\lambda^{-3/2}$
$-\sim(\sim)^{-3/2} - \sim(\sim)^{-3/2} < 0$

$12.8^{-2}(100-64\lambda) = 0.75^{-2}\lambda$

$12.8^{-2} \times 100 = (0.75^{-2} + 12.8^{-2} \times 64)\lambda$

$\lambda = (12.8^{-2} \times 100)/(0.75^{-2} + 12.8^{-2} \times 64) = 0.28148$

SOC?

~~$\lambda^* = 0.2814$~~ The farmers maximise each farmer's expected utility by choosing $\lambda^* = 0.28148$

f If all farmers grow wheat, $\lambda = 1$, $L_\lambda = [0.4, 0.3, 0.3;\ 36, 36, 25] = L_w$ since the outcome of each farmer's crop is perfectly correlated with that of all other farms in the region.

G and M sharing profits would benefit both risk-averse agents because this allows them to pool their risks. Expected returns are unchanged but both agents face a less risky lottery because the ~~poole~~ unpooled lottery is a mean-preserving spread of the pooled lottery, the former SOSDs the latter. Any risk-averse agent would prefer the former to the latter.

~~Good~~

Show in detail

↳ show the numbers

"greater probability density around the mean than around the tails"

+ Plot CDFs

8.5
10