Microeconomics Paper 180526

1a Each firm has revenue $pX = pAL^{\alpha}/\alpha$ and cost wL hence profit $\pi = pAL^{\alpha}/\alpha - wL$ and profit maximization problem

$\max_L \ pAL^{\alpha}/\alpha - wL$

FOC: $pAL^{\alpha-1} - w = 0$

SOC: $(\alpha-1)pAL^{\alpha-2} < 0$

(supposing that $p, A, L > 0$, and given that $0 < \alpha < 1$)

FOC $\Rightarrow pAL^{\alpha-1} = w \Rightarrow L^{\alpha-1} = w/pA \Rightarrow L = (w/pA)^{1/\alpha-1}$

$L = (w/pA)^{1/\alpha-1} = w^{1/\alpha-1} p^{-1/\alpha-1} A^{-1/\alpha-1}$

~~$X = AL^{\alpha}/\alpha = A(w/pA)^{\alpha/\alpha-1}$~~

$X = AL^{\alpha}/\alpha = A(w/pA)^{\alpha/\alpha-1}/\alpha = A^{-1/\alpha-1} w^{\alpha/\alpha-1} p^{-\alpha/\alpha-1} \alpha^{-1}$

$\pi = pX - wL = A^{-1/\alpha-1} w^{\alpha/\alpha-1} p^{-1/\alpha-1} \alpha^{-1} - w^{\alpha/\alpha-1} p^{-1/\alpha-1} A^{-1/\alpha-1}$

$= A^{-1/\alpha-1} p^{-1/\alpha-1} w^{\alpha/\alpha-1} (\alpha^{-1} - 1)$

Each firm has labour demand function

$L = A^{-1/\alpha-1} w^{1/\alpha-1} p^{-1/\alpha-1}$

Output supply function

$X = A^{-1/\alpha-1} w^{\alpha/\alpha-1} p^{-\alpha/\alpha-1} \alpha^{-1}$

Profit function

$\pi = A^{-1/\alpha-1} w^{\alpha/\alpha-1} p^{-1/\alpha-1} (\alpha^{-1} - 1)$

b Value of output for each firm

$pX = A^{-1/\alpha-1} w^{\alpha/\alpha-1} p^{-1/\alpha-1} \alpha^{-1}$

~~$1/\alpha^{-1} = \alpha$ of~~

~~wL/pX~~

$\pi/pX = \alpha^{-1} - 1/\alpha^{-1} = 1-\alpha/1 = 1-\alpha$

$wL/pX = 1 - \pi/pX = \cancel{\alpha/1} \ \alpha$

The share of value of output that goes to ~~wages~~ wages is α, the remaining share $1-\alpha$ goes to profit. The share of value captured by labour increases with the labour-intensity of production.

c Suppose that each worker inelastically supplies one unit of labour. It is given that $w=1$. At equilibrium, N units of labour are inelastically supplied and this is equal to labour demand at the eqm price ratio p. $L = N$.

$N = MA^{-1/\alpha-1} p^{-1/\alpha-1} \Leftrightarrow$

~~$p^{-1/\alpha-1} = NM^{-1}A^{1/\alpha-1} \Leftrightarrow$~~

~~$p = (N/M)^{1/\alpha-1}$~~

~~$p^{-1/\alpha-1} = NM^{-1}A^{1/\alpha-1} \Leftrightarrow$~~

~~$p = (N/M)^{1-\alpha} / A$~~

$(pA)^{-1/\alpha-1} = N/M \Leftrightarrow$

$(pA)^{1/1-\alpha} = N/M$

d N and M are exogenous and fix the RHS of the above equation, which holds at eqm, so $\uparrow A \Rightarrow \downarrow p$ at eqm. Benefits are distributed in the same ratio as before between capitalists and workers, with a $1-\alpha$ share to the former and an α share to the latter.

w is unchanged, so $\downarrow p \Rightarrow \uparrow$ real w. Ap is unchanged, so nominal profit is unchanged and $\downarrow p \Rightarrow \uparrow$ real π.

2a ~~some~~ A NE of a game is a strategy profile for that game such that, ~~taking~~ for each player, taking the strategies of the other players as given, there is no strictly profitable deviation, i.e. ~~at~~ all players play mutual best responses. NE play is implied by common knowledge of rationality and correct beliefs in eqm.

Existence of a NE does not necessarily yield a good prediction of the outcome of a game because there is not necessarily a unique NE, so ~~the~~ NE does not necessarily yield a unique prediction, and players could have wrong beliefs (for example, resulting in non-NE play due to miscoordination in Battle of the Sexes).

	L	R
U	t	$\underline{3}$
	0	$\underline{3}$
D	0	$\underline{5}$
	$\underline{5}$	0

$t<3$. Best responses underlined. By inspection, R is strictly dominant for P2, U is ~~a strict~~ the unique best response for P2 against R. ~~B~~ only (U,R) survives iterated strict dominance. Only strategies that survive ISD are played at NE because ~~players do not play~~ strictly dominated strategies are not best responses. The unique strategy that survives ISD, (U, ~~B~~ R) is the unique NE. There are no mixed or hybrid NE.

b

	L	R
U	$\underline{t}$	3
	0	$\underline{3}$
D	0	$\underline{5}$
	$\underline{5}$	0

Best responses underlined. By inspection, there are no pure NE where players play pure mutual best responses.

~~Suppo~~ Consider candidate NE σ^* such that P1 mixes. Then P1 has no profitable deviation, and P1 is indifferent between U and D.

$\pi_1(U, \sigma_2^*) = \pi_1(D, \sigma_2^*) \iff$

$3(1-q) = 5q \iff$

$8q = 3 \iff$

$q = 3/8$,

where $\sigma_2^* = qL + (1-q)R$

Then P2 mixes, P2 is indifferent

$\pi_2(L, \sigma_1^*) = \pi_2(R, \sigma_1^*) \iff$

~~$5(1-p) =$~~ ~~tp~~ $tp = 3p + 5(1-p) \iff$

$(t+2)p = 5 \iff$

$p = 5/(t+2) = 5/6$,

~~$\sigma^* = ($~~ where $\sigma_1^* = ~~\pounds~~ pU + (1-p)D$

σ^* ~~where~~ ~~π~~ $= (pU + (1-p)D, qL + (1-q)R)$ is a NE.

If P1 mixes ~~s~~ at NE, so does P2, and vice versa, so there are no hybrid NE.

c

	L	R
U	$\underline{3}$	$\underline{3}$
	0	$\underline{3}$
D	0	$\underline{5}$
	$\underline{5}$	0

Best responses underlined. By inspection, (U,R) is the unique pure NE where players play pure mutual best responses.

There is no mixed NE. Suppose that P2 mixes then P2 is indifferent, then P1 plays U with certainty (because if P1 plays D with non-zero probability, then P2 strictly prefers R), U with certainty is rational for P1 iff

~~$\pi_1(U, R) \geq \pi$~~

$\pi_1(U, \sigma_2^*) \geq \pi_1(D, \sigma_2^*) \iff$

~~3~~ $3(1-q) \geq 5q \iff$

$3 \geq 8q \iff$

$q \leq 3/8$

~~Any~~ $\sigma^* = (U,$ ~~$qL +$~~ $qL + (1-q)R)$ for $q \leq 3/8$ is a hybrid NE.

There is no ~~mixed~~ hybrid NE where P1 mixes because ~~at~~ at any such candidate NE, P2 strictly prefers R, then P1 strictly prefers U.

3a Expected net return

$EV(L^n) = \frac{1}{2}(26) + \frac{1}{2}(-24) = 1$

The ~~CE~~ lottery in final wealth values associated with the project is

$L = [\frac{1}{2}, \frac{1}{2}; 64+26, 64-24]$

$= [\frac{1}{2}, \frac{1}{2}; 90, 40]$

Expected utility

$U(L) = \frac{1}{2}u(90) + \frac{1}{2}u(40)$

$= \frac{1}{2}\ln 90 + \frac{1}{2}\ln 40$

$= \ln 3600^{1/2}$

$= \ln 60$

certainty equivalent CE(L) is such that ~~CE(L) is such that~~ $[1; CE(L)] \sim_J L$, i.e. J is indifferent between receiving CE(L) with certainty (as final wealth) and participating in the lottery.

~~CE(L)~~ $u(CE(L)) = U(L) \Leftrightarrow$

$CE(L) = 60$

Expected value (in final wealth)

$EV(L) = \frac{1}{2}(90) + \frac{1}{2}(40)$

$= 65$

Risk premium

$RP(L) = EV(L) - CE(L)$

$= 65 - 60$

$= 5.$

The ~~tot~~ associated lottery has positive risk premium for J because it is risky and J is risk averse (Bernoulli utility u is concave).

$CE(L) = 60 < 64$. Given that J's preferences are strictly monotonic, by definition of CE(L), $[1; 64] \succ_J$ ~~$[1; 60]$~~ $[1; CE(L)] \sim_J L$, so J strictly prefers certain final wealth of 64 ~~at~~ to participation in the project and does not participate.

b Let L^S denote the shared ~~lott~~ between S and J lottery in final wealth.

$L^S = [\frac{1}{2}, \frac{1}{2}; 64+13, 64-12]$

$= [\frac{1}{2}, \frac{1}{2}; 77, 52]$

Expected net return

$EV(L^S) - 64 = \frac{1}{2}(77) + \frac{1}{2}(52) - 64 = \frac{1}{2}$

~~$U(L^S)$~~

Expected utility

$U(L^S) = \frac{1}{2}\ln 77 + \frac{1}{2}\ln 52$

$= \ln\sqrt{77 \times 52}$

$= \ln 63.277$

certainty equivalent

$CE(L^S) = u^{-1}(U(L^S))$

$= 63.277 < 64$

S and J, by an argument exactly analogous to that given above, each strictly prefers not to participate in the project.

Let $L^S(n)$ denote the lottery associated with the shared project when it is shared among n participants.

$L^S(n) = [\frac{1}{2}, \frac{1}{2}; 64 + \cancel{24}\, ^{26}/_n, 64 - ^{24}/_n]$

Expected utility

$U(L^S(n)) = \frac{1}{2}\ln(64 + ^{26}/_n) + \frac{1}{2}(64 - ^{24}/_n)$

~~certainty equi~~

$= \ln\sqrt{(64 + ^{26}/_n)(64 - ^{24}/_n)}$

certainty equivalent

$CE(L^S(n)) = u^{-1}(U(L^S(n)))$

$= \sqrt{(64 + ^{26}/_n)(64 - ^{24}/_n)}$

$CE(L^S(n)) \geq 64 \Leftrightarrow$

$\sqrt{(64 + ^{26}/_n)(64 - ^{24}/_n)} \geq 64 \Leftrightarrow$

$4096 + ^{128}/_n - ^{624}/_{n^2} \geq 64^2 \Leftrightarrow$

~~$4032 + ^{128}/_n - ^{624}/_{n^2} \geq 0 \Leftrightarrow$~~

~~$4032n^2 + 128n - 64 \geq 0 \Leftrightarrow$~~

~~$63n^2 + 2n - 1 \geq 0 \Leftrightarrow$~~

$^{128}/_n - ^{624}/_{n^2} \geq 0 \Leftrightarrow$

~~62~~ $128n \geq 624 \Leftrightarrow$

$n \geq 4.875$

The shared lottery is acceptable with at least $n=5$ participants.

Sharing the lottery reduces both the expected value and the risk premium. The expected value decreases at a $^1/_n$ rate, the risk premium for a small lottery is approximately proportionate to the variance, so it decreases at a $^1/_{n^2}$ rate. Risk premium of the lottery initially falls faster than expected value, so certainty equivalent increases and the lottery becomes more acceptable.

c $\max_n U(L^S(n))$

$\equiv \max_n \frac{1}{2}\ln(64 + ^{26}/_n)(64 - ^{24}/_n)$

$\equiv \max_n (64 + ^{26}/_n)(64 - ^{24}/_n)$

FOC: $-624(2n) + 128 = 0 \Rightarrow n = 9.75$

SOC: $-1248 < 0$

Expected utility of each identical participant is maximised at $n = 9.75$. By concavity, the maximum subject to the integer constraint is either $n=9$ or $n=10$.

$U(L^S(9)) = \frac{1}{2}\ln(4102.518)$

$U(L^S(10)) = \frac{1}{2}\ln(4235.76)$

$n=10$ maximises expected utility of each particip

As n increases beyond 10, risk premium of the lottery decreases less rapidly than expected value ~~certainty equivalent~~, ~~≠~~ certainty equivalent, hence expected utility decreases.

4a ~~At~~ At the efficient outcome, each bicycle is owned by the ~~consu~~ type of consumer with the highest valuation for that bicycle. So High (H) quality bicycles are all owned by Buyers (B), M are all owned by B, L are all owned by S.

b For $p<45$, ~~as~~ supply of each quality is zero, supposing that Bs assume only a L quality will be sold at $p<45$, demand is zero, this is an eqm.

For $p=45$, ~~all and only~~ there is an eqm where no qualities are supplied, Bs ~~assume~~ believe only L types will be supplied, demand is zero.

For $45<p<60$, ~~all~~ all L types are supplied, ~~the valu cons~~ Bs have valuation $80<p$, demand is zero, there is no eqm.

For ~~60~~ $p=60$, ~~there is an eqm where 1/3 of~~ even if all M qualities are supplied, Bs have expected valuation 47.5, and ~~de~~ demand is zero, there is no eqm. Likewise for ~~p<60~~ $60<p<75$.

For $p=75$, ~~a~~ even if all ~~ty~~ qualities are supplied, Bs expected valuation is 65, demand is zero, there is no eqm. Likewise for $p>75$.

~~The~~ At eqm, $p\leq 45$, supply and demand are both zero.

c B valuations increase by the amount of the expected payout, S valuations ~~decrease~~ increase by the same amount.

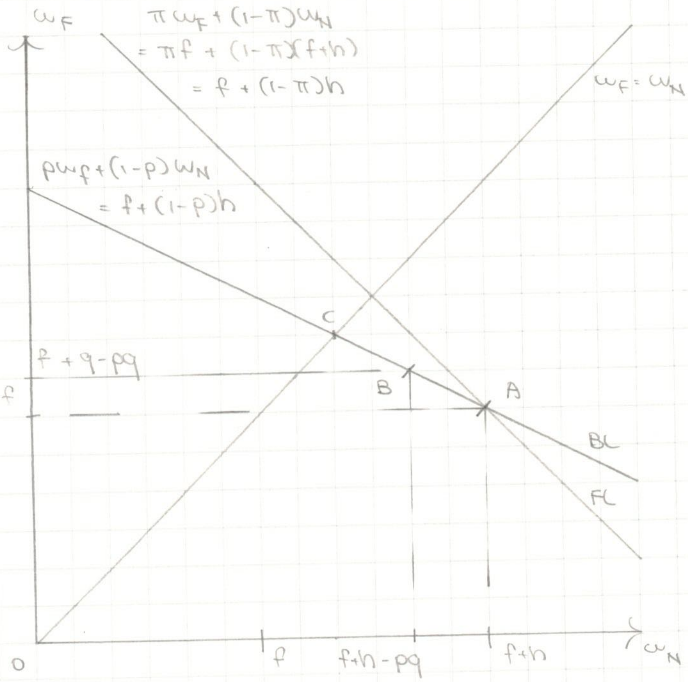
	B	S
H	100	75
M	90	~~35~~ 85
L	80	~~5~~ 95

For ~~85<p<95~~, $85<p<95$, all and only H and M types are supplied. ~~B~~ B's expected valuation is ~~100+9~~ $100+90/2=95$. ~~Dem~~ Each B demands one bicycle. Supposing that there are as many H and M types as there are Bs, any ~~85<p<75~~ is an eqm. If there are more Bs, price is bid up and there is no eqm at such price.

For $p=85$, all H types are supplied, and any number of M types may be supplied. Demand is for all H and M types. ~~Either~~ Supposing that there are weakly more buyers than H and M types, demand equals supply and this is an eqm.

d There are inefficient eqm where $75\leq p<85$ such that only H types are supplied and demand is non-zero. These are eqm only if there are fewer Bs than Hs.

10a



A represents Alex's initial position. The line BL is the budget line given actuarially unfair $(p > \pi)$ insurance. The line FL is the budget line with counterfactual actuarially fair insurance, on this line, expected ~~wealth~~ final wealth is ~~equal to~~ ~~initial wealth~~ constant. If A buys q coverage, A's position is given by B.

b Yes. Concavity of Bernoulli utility u for an expected utility maximiser implies risk aversion. A has positive Arrow-Pratt measure of risk aversion $A(x) = -u''(x)/u'(x) > 0$ for all x. ~~A prefers~~ ~~[t~~ By concavity of u, for all $x, x', t \in (0,1)$, ~~tu(~~ $tu(x) + (1-t)u(x') < u(tx + (1-t)x') \Longrightarrow u([1; tx + (1-t)x']) > u([t, 1-t; x, x']) \Longrightarrow [1; tx + (1-t)x'] \succ [t, 1-t; x, x']$, i.e. for all lotteries over two outcomes, A prefers ~~the~~ ~~expecte~~ to receive the expected value with certainty than to participate in the lottery, ~~i.e. A~~ so A has positive risk premium. This result generalises.

c Let $L(q)$ denote the lottery in final wealth A faces if A buys coverage q.

$L(q) = [\pi, 1-\pi \,;\, f+q-pq \,,\, f+h-pq]$

Expected utility

$u(L(q)) = \pi u(f+q-pq) + (1-\pi)u(f+h-pq)$

$\partial/\partial q\; u(L(q)) \,|_{q=h}$

$\cancel{= \pi u'(f+h-ph)(}$

$= [\pi u'(f+q-pq)(1-p) + (1-\pi)u'(f+h-pq)(-p)]_{q=h}$

$\cancel{= [-p}$

$= \pi u'(f+h-ph)(1-p) + (1-\pi)u'(f+h-ph)(-p)$

$= u'(f+h-ph)\,[\pi(1-p) + (1-\pi) - p]$

$= u'(f+h-ph)\,[\pi - p] < 0$

given that $u'(x) > 0$ for all x and $p > \pi$

Evaluated at full insurance $q = h$, ~~marginal~~ expected utility is decreasing in coverage q, so A maximizes utility by choosing some $q < h$.

Intuitively, any risk-averse expected utility maximiser takes a ~~f~~ non-zero stake in a favourable gamble. When insurance is actuarially unfair, under insurance is a ~~fa~~ favourable gamble.

Any risk averse expected utility maximiser takes a non-zero share in a positive gamble because the expected value of such a gamble increases in direct proportion to the share of ~~such a~~ the gamble and the risk premium is approximately ~~equal~~ proportionate to the variance of ~~such a~~ the gamble, which increases in proportion to the square of the share. So initially expected value increases more rapidly and certainty equivalent increases.

c $u(L(q)) = \pi \ln(f+q-pq) + (1-\pi)\ln(f+h-pq)$

$\max_q u(L(q))$

FOC: $\cancel{\pi(1-q} \;\; \pi(1-p)/(f+q-pq) + (1-\pi)(-p)/(f+h-pq) = 0 \Rightarrow$

$\pi(1-p)/(f+q-pq) = p(1-\pi)/(f+h-pq) \Rightarrow$

$\pi(1-p)[f+h-pq] = p(1-\pi)[f+q-pq] \Rightarrow$

$\cancel{\pi(1-p)[f+h)}$

$\pi(1-p)[f+h] - p(1-\pi)[f] = p(1-\pi)(q-pq) - \pi(1-p)(-pq)$

$\Longrightarrow$

$\pi(1-p)(f+h) - p(1-\pi)f = p(1-\pi)(1-p)q + \pi(1-p)pq \Rightarrow$

$\pi(1-p)(f+h) - p(1-\pi)f = p(1-p)q \rightarrow$

$q^* = \pi/p\,(f+h) - (1-\pi)/(1-p)\, f$

$\cancel{= (1-p)\pi(f+h) - p(1-\pi)f / p(1-p)}$

$= (1-p)\pi(f+h) - p(1-\pi)f / p(1-p)$

$q^* = 0 \Leftrightarrow$

$(1-p)\pi(f+h) \leq p(1-\pi)f$ ~~tta~~ $\Leftrightarrow$

$\cancel{\pi/(1-\pi)\,(f+h) \leq p/(1-p)\, f}$

$(1-\pi)/\pi\; f \geq (1-p)/p\; f+h$

A ~~b~~ optimally buys zero insurance iff A's indifference curve at A is steeper than ~~BL such~~ BL such that no point on BL along AC (where A buys positive insurance) lies above the IC through A. This is if insurance is sufficiently actuarially unfair. In ~~this~~ such cases, insurance is so actuarially unfair that betting ~~against~~ ~~the~~ against the house is preferable.

di $\partial/\partial h\; q^* = \pi/p > 0$

$\partial/\partial f\; q^* = \pi/p^{\wedge} - (1-\pi)/(1-p)^{\vee} < 0$

As h increases, A optimally buys more insurance because, intuitively, the riskiness of the initial position has increased.

As f increases, optimal insurance decreases because A has decreasing absolute risk aversion, so ~~the risk premium~~ becomes less risk averse as "initial" wealth increases. A's indifference curves become gentler in the positive w_N, w_F direction, so the point of tangency between the budget line and the highest attainable indifference curve lies closer to the initial position.

$A = -u''(x)/u'(x) = -(-x^{-2})/x^{-1} = x^{-1}$
$\partial A/\partial x = -x^{-2} < 0$

Arrow-Pratt measure of risk aversion is decreasing in wealth x. A in fact has constant relative risk aversion.

Wealthier A has lower Arrow-Pratt metric, lower risk premium, so ~~with de~~ as coverage decreases, risk premium increases less quickly than expected value, so CE increases over a longer stretch. A optimally underinsures by a larger amount.