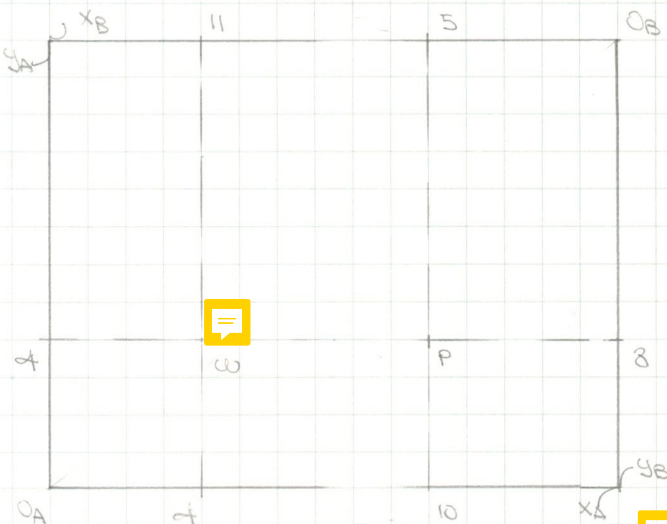
Microeconomics Paper 200527

1a Given: $u^A(x^A, y^A) = 2\ln x^A + \ln y^A$, $\omega^A = (x^A_\omega = 4, y^A_\omega = 4)$,
$u^B(x^B, y^B) = \ln x^B + 2\ln y^B$, $\omega^B = (x^B_\omega = 11, y^B_\omega = 8)$.

$\cancel{u^A(5}$ $u^A(x^A_\omega, y^A_\omega) = 2\ln 4 + \ln 4 = \ln 64$
$u^A(5,3) = 2\ln 5 + \ln 3 = \ln 75 > u^A(x^A_\omega, y^A_\omega)$
$u^B(x^B_\omega, y^B_\omega) = \ln 11 + 2\ln 8 = \cancel{704}\ \ln 704$
$u^B(10,9) = \ln 10 + 2\ln 9 = \ln 810 > u^B(x^B_\omega, y^B_\omega)$

The allocation $(x^A, y^A) = (5,3)$, $(x^B, y^B) = (10,9)$ is feasible (since $x^A + x^B \leq x^A_\omega + x^B_\omega$ and $y^A + y^B \leq y^A_\omega + y^B_\omega$) and each consumer is strictly better off under this allocation, ~~then~~ than under the given endowment. This allocation Pareto-dominates the given endowment. The given endowment is not Pareto-optimal.



b $MU^A_x = 2/x^A$, $MU^A_y = 1/y^A$, $MRS^A = -MU^A_x / MU^A_y = -2y^A/x^A$
$MU^B_x = 1/x^B$, $MU^B_y = 2/y^B$, $MRS^B = -MU^B_x / MU^B_y = -y^B/2x^B$

Suppose $y^A_\omega = 4$, $y^B_\omega = 8$, then
$MRS^A = MRS^B \Leftrightarrow -8/x^A = -8/2x^B \Leftrightarrow x^A = 2x^B$
Given $x^A + x^B = x^A_\omega + x^B_\omega = 15$, we have $x^A = 10$, $x^B = 5$

~~The allocation~~ Every allocation such that $MRS^A = MRS^B$ is Pareto-efficient, then the allocation $(x^A, y^A) = (4$ $)$, $(x^B, y^B) = (\cancel{4}, 11, 5$ $)$ is Pareto-efficient. This is achieved by a transfer of $7/2$ units of y from B to A, and is represented by P in the above diagram.

c No. By inspection, utility functions are monotonic. Then, a transfer of one good from B to A leaves B strictly worse off and A strictly better off. Neither allocation Pareto-dominates the other.

d The social planner's optimisation problem is
$\max_{x^A, y^A, x^B, y^B} u(x^A, y^A, x^B, y^B) = u^A(x^A, y^A) + u^B(x^B, y^B)$
s.t. $x^A + x^B = x^A_\omega + x^B_\omega = 15$, $y^A + y^B = y^A_\omega + y^B_\omega = 12$
which reduces to
$\max_{x^A, y^A}\ 2\ln x^A + \ln y^A + \ln(15 - x^A) + 2\ln(12 - y^A)$
s.t. $x^A \in [0, 15]$, $y^A \in [0, 12]$
~~$FOC_{x^A}: 2/x^A - 1/1-x^A = 0 \Rightarrow 2 - 2x^A = x^A \Rightarrow$~~
$FOC_{x^A}: 2/x^A - 1/15 - x^A = 0 \Rightarrow 30 - 2x^A = x^A \rightarrow x^A = 10$
$FOC_{y^A}: 1/y^A - 2/12 - y^A = 0 \Rightarrow 12 - y^A = 2y^A \Rightarrow y^A = 4$
$\Rightarrow x^B = 5, y^B = 8$

This optimum coincides with the earlier optimum.

$u^A(10,4) = 2\ln 10 + \ln 4 = \ln 400$
$u^B(5,8) = \cancel{2\ln 8 + \ln} \ \ln 5 + 2\ln 8 = \ln 320 < u^A(10,4)$
$\min\{u^A, u^B\}$ would be increased by a small transfer of either good from A to B. This allocation is not optimal for a Rawlsian social planner.

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2a The NE of a game is the strategy profile such that each player, given each other player's strategy under this profile, has no strict incentive to deviate, i.e. players play mutual best responses.

	M	I
M	0, 0	$\underline{-1}$, $\underline{2}$
I	$\underline{2}$, $\underline{-1}$	-4, -4

By inspection, there are two pure NE, ~~[illegible]~~ (M, I) and (I, M).

Suppose there is an NE σ^* such that P1 (Row) mixes, then by definition of NE, P1 has no profitable deviation then $\pi_1(M, \sigma_2^*) = \pi_1(I, \sigma_2^*) \Rightarrow \cancel{-1(1-q) =}$ $-1(1-q) = 2q + 4(1-q)$
$\Rightarrow -1 + q = 4 - 2q \Rightarrow 3q = 5 \Rightarrow q = 3/5$, where ~~$q = P(a_2 = M)$~~ q is the probability σ_2^* assigns to M. Then P2 mixes and by an analogous argument $p = 3/5$, where p is the probability σ_1^* assigns to M. There are no hybrid NE. The only mixed NE is $(3/5 M + 2/5 I, 3/5 M + 2/5 I)$.

Expected payoffs at each pure NE are given by the payoff table. At the mixed NE, $\pi_1(\sigma_1^*, \sigma_2^*) = \pi_1(M, \sigma_2^*) = -1(1-q) = -2/5$. By symmetry, $\pi_2(\sigma^*) = -2/5$

b An SPE is an NE that induces an NE in each subgame (including subgames off the equilibrium path). SPE excludes time inconsistent NEs that involve non-credible threats.

c Consider the grim trigger strategy under which the player (whose strategy it is), in each period, plays M if no player previously deviates, plays M if the first deviation was by the other, plays I if the first deviation was by himself, and "ignores" simultaneous deviations.

Consider the strategy profile ~~where~~ under which each player plays this strategy. Suppose some player deviates in (only) period t. This player receives payoff 2 for one period and payoff -1 in each subsequent period hence total payoff $2 - \delta/(1-\delta) < 0$ (the equilibrium ~~payoff~~ continuation payoff) for ~~[illegible]~~ δ sufficiently close to 1. By the one shot deviation principle, this is an SPE.

3a The certainty equivalent $CE(L)$ of lottery L ~~is the amount~~ to the agent with preferences $\succsim$ (and implied $\sim$, $\succ$), ~~or the agent~~ is the amount such that $[1; CE(L)] \sim L$, i.e. the agent is indifferent between ~~receivi~~ lottery L and receiving $CE(L)$ with certainty. If the agent's preferences over lotteries can be represented by a utility function U, then $CE(L)$ is such that $U([1; CE(L)]) = U(L)$. If the agent has expected utility preferences and Bernoulli utility u, then $CE(L)$ is such that $u(CE(L)) = U(L)$

Given: J has EU preferences, monotonic Bernoulli utility u_J, CARA

$L_X = [1/2, 1/2; 6, 30]$, $L_Y = [1/2, 1/2; 12, 20]$ ~~$Y \succ_J X$~~
$L_Y \succ_J L_X$

b The given argument is valid. L_Z FOSDs L_Y since L_Z is obtained from L_Y by reallocating probability mass from lower outcomes (12 and 20) to higher outcomes (14 and 22 respectively). Given J ~~has~~ has EU preferences and (strictly) monotonic u_J $U_J(L_Y) = 1/2 u_J(12) + 1/2 u_J(20) < 1/2 u_J(14) + 1/2 u_J(22) = U_J(L_Z) \Rightarrow L_Z \succ_J L_Y$. By inspection, L_X is a mean-preserving spread of L_Z, ~~then~~ then if J is risk-neutral or risk-loving, by the weak convexity of u_J, $U_J(L_X) = 1/2 u_J(6) + 1/2 u_J(30) \geq 1/2 u_J(14) + 1/2 u_J(22) = U_J(L_Z) \Rightarrow L_X \succsim_J L_Z$. Given J has EU preferences, $\succ_J$ and $\succsim_J$ are transitive, then if J is risk-loving or risk-neutral, $L_Y \prec_J L_Z \precsim_J L_X \Rightarrow L_Y \prec_J L_X$. By reductio, J is risk-averse.

c Let c denote the cost ~~of~~ increase and L'_Y denote the ~~updated~~ lottery associated with the updated project Y.

$L'_Y = [1/2, 1/2; 12-c, 20-c]$

$L_X \sim_J L'_Y \Leftrightarrow U_J(L_X) = U_J(L'_Y) \Leftrightarrow 1/2 \ln(6) + 1/2 \ln(30) = 1/2 \ln(12-c) + 1/2 \ln(20-c) \Leftrightarrow 180 = (12-c)(20-c) \Leftrightarrow$
$c =$ ~~12~~ 2 or 30 (reject)

~~For $c \neq 2$~~ The largest increase J finds acceptable is $c=2$. For $c>2$, $U_J(L_X) > U_J(L'_Y) \Rightarrow L_X \succ_J L'_Y$.

4a Type H workers would like to signal their productivity. Given that firms are competitive, each firm pays each worker wage w equal to that worker's expected productivity such that the expected payoff to employing a worker is zero (supposing without loss of generality that each firm's payoff if it does not hire a worker is zero). Then, suppose a type H worker cannot signal his productivity. Then expected (by the firm) productivity is $\bar{\theta}$, the weighted average of θ_L and θ_H, ~~hence~~ $\bar{\theta} < \theta_H$, hence $w = \bar{\theta}$. If type H worker can credibly signal his productivity, $w = \theta_H > \bar{\theta}$.

A signal of high productivity is credible iff it would be irrational for type L workers to imitate

b When no signals are available, by the above argument from competition between firms, each firm offers $w = \bar{\theta} = 3/5(500) + 2/5(400) = 460$ and each worker accepts (given no outside option).

c ~~Payof~~ Let e ~~de~~ be a dummy variable that has value 1 iff a worker acquired an education, and zero otherwise.

At the given outcome, $w(e=1) = \theta_H = 500$, $w(e=0) = \theta_L = 400$
Payoff to type H worker, $\pi_H = 500 - 60 = 440$.
Payoff to type L worker, $\pi_L = 400$

The ~~outc~~ given outcome is not an equilibrium because ~~it~~ it fails to deviation by type L workers to $e=1$, which yields $\pi_L = w(e=1) - 80 = 420 > 400$.

Low productivity workers have strict incentive to imitate high productivity workers.

The pooling outcome where neither type acquires an education is an equilibrium. ~~If~~ Formally at this outcome, each firm chooses $w(e=1) = \bar{\theta} = 460$, $w(e=0) = \bar{\theta} = \cancel{40}\ 460$, then no worker has incentive to deviate to $e=1$ since this incurs some cost and does not yield ~~an~~ a higher wage.

This equilibrium is efficient since there is zero "unproductive" expenditure on education.

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10a Firm i's profit $\pi_i = P(q_i+q_{-i})q_i - 15q_i = (75-q_i-q_{-i})q_i - 15q_i$

Firm i's optimisation problem (taking q_{-i} as given)

$\max_{q_i} \pi_i = (75-q_i-q_{-i})q_i - 15q_i$ s.t. $q_i \geq 0$

FOC: $(75-q_i-q_{-i}) - q_i - 15 = 0 \Rightarrow q_i = {60-q_{-i}}/{2} = 30 - {q_{-i}}/{2}$

Firm i's best response $BR_i(q_{-i}) = 30 - {q_{-i}}/{2}$

At NE, firms choose mutual best responses

$q_1^* = BR_1(q_2^*) = 30 - {q_2^*}/{2}$, $q_2^* = BR_2(q_1^*) = 30 - {q_1^*}/{2}$

$\Rightarrow q_1^* = 30 - (30 - {q_1^*}/{2})/2 = 15 + {q_1^*}/{4} \Rightarrow {3q_1^*}/{4} = 15 \Rightarrow q_1^* = 20 \Rightarrow q_2^* = 20$

$P(Q^*) = 75-20-20 = 35$, $\pi_1^* = (35-15)(20) = 400$, $\pi_2^* = \pi_1^* = 400$

The result of the cournot game is symmetric and each firm makes positive profit.

b ~~Let ' denote the even values given firm 1 invests~~.

$\pi_1^* = P(q_1+q_2)q_1 - 9q_1 = (75-q_1-q_2)q_1 - 9q_1$

$\max_{q_1} (75-q_1-q_2)q_1 - 9q_1$ s.t. $q_1 \geq 0$

FOC: $(75-q_1-q_2) - q_1 - 9 = 0 \Rightarrow q_1 = {66-q_2}/{2} = 33 - {q_2}/{2}$

$BR_1^*(q_2) = 33 - {q_2}/{2}$

BR_2 is unchanged from (a)

~~At NE~~ $q_1^* = BR_1(q_2^*)$, $q_2^* = BR_2(q_1^*) \Rightarrow$

$q_1^* = 33 - \frac{1}{2}(30 - \frac{1}{2}q_1^*) = 18 + \frac{1}{4}q_1^* \Rightarrow {3q_1^*}/{4} = 18 \Rightarrow q_1^* = 24$

$\Rightarrow q_2^* = 18$

$P(Q^*) = 75-24-18 = 33$, $\pi_1^* = (33-9)(24) = 576$,

$\pi_2^* = (33-15)(18) = 324$

Firm 1 earns more because the cost reduction has a positive direct and a positive strategic effect on profit. The direct effect consists in the cost reduction directly causing an increase in margin hence an increase in profit. The strategic effect consists in the cost reduction causing an increase in margin hence an increase in firm 1's incentive to increase quantity (it makes firm 1 "tough") which in turn causes a decrease in price hence a decrease in firm 2's margin hence a decrease in firm 2's incentive to increase output which in turn causes firm 2 to choose lower output, (compete less aggressively) which benefits firm 1 since their products are substitutes.

Firm 2 earns less because firm 1's investment makes firm 1 "tough", increases firm 1's incentive to choose high output (compete aggressively) which shrinks residual demand for firm 2.

c Firm 1's payoff is its net (of investment cost) profit. From (a) and (b), if firm 1 does not invest, its ~~total~~ payoff is 400, if firm 1 invests, its payoff is $576 - K$. Then firm 1 is indifferent iff $K = 176$ and invests for all lower K. ~~Supposing that entry is costless hence that firm 2 always enters~~).

~~At SPE, firm 1's strategy is: invest, then produce output $q_1 = 24$. Firm 2's strategy is: if firm 1 invested choose $q_2 = 18$, if not choose $q_2 = 20$.~~

At SPE, firm 1's strategy is: invest, then choose $q_1 = 24$ if firm 1 previously invested and if not, ~~else~~ choose $q_1 = 20$. Firm 2's strategy is: choose ~~$q_2=20$~~ $q_2 = 18$ if firm 1 invested, choose $q_2 = 20$ if not.

A strategy fully describes each player's action in any event where he may be called upon to act, it is a complete contingent plan and ~~includes contingencies~~ includes a plan for contingencies even those that involve prior deviations or "mistakes".

d Suppose firm 1 invests and firm 2 produces $q_2 = 20$ as in (a).

$\pi_1 = (75-20-9-q_1)q_1 = (46-q_1)q_1$

$\max_{q_1} \pi_1$ s.t. $q_1 \geq 0$

FOC: $46 - 2q_1 = 0 \Rightarrow q_1 = 23$

$P(Q) = (75-20-23) = 32$

$\pi_1 = (32-9)(23) = 529$

Net (of investment cost) profit $= 529 - 150 = 377 < 400$

(believes)

If firm 1 believes firm 2, then it is strictly better off if it does not invest, firm 1 does not invest.

Firm 1 should invest. Firm 2's announcement is not credible. The threat is not time consistent. Suppose that firm 1 invests, then ~~firm 2 finds it optim~~ if firm 2 chooses $q_2 = 20$ as in (a) and firm 1 optimally responds by choosing $q_1 = 23$ as above, then firm 2 would have been better off had it chosen a lower output, so the above outcome is not a NE in the subgame. At the subgame NE (from (b)) $q_1 = 24$, $q_2 = 18$ and π_1 is sufficiently large to justify investment.

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