Microeconomic Analysis Paper 170601

1a i Represent the system of linear equations as an augmented matrix. Solve by Gauss-Jordan elimination.

$$\left(\begin{array}{cccc|c} -1 & 3 & -2 & -6 & 2 \\ 2 & 1 & 4 & 5 & 3 \\ 1 & 2 & 2 & 1 & t \end{array}\right) \longrightarrow \begin{array}{l} R_3 \leftarrow R_1 + R_3 \\ R_2 \leftarrow 2R_1 + R_2 \end{array}$$

$$\left(\begin{array}{cccc|c} -1 & 3 & -2 & -6 & 2 \\ 0 & 7 & 0 & -7 & 7 \\ 0 & 5 & 0 & -5 & t+2 \end{array}\right) \longrightarrow \begin{array}{l} R_3 \leftarrow R_3 - \frac{5}{7}R_2 \\ R_2 \leftarrow R_2/7 \end{array}$$

$$\left(\begin{array}{cccc|c} -1 & 3 & -2 & -6 & 2 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & t-3 \end{array}\right) \longrightarrow R_1 \leftarrow R_1 - 3R_2$$

$$\left(\begin{array}{cccc|c} -1 & 0 & -2 & -3 & -1 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & t-3 \end{array}\right)$$

For $t \neq 3$, there are no solutions and the system is inconsistent. For $t=3$, there are infinitely many solutions that satisfy $-x_1 - 2x_3 - 3x_4 = -1$, $x_2 - x_4 = 1$, so the solutions are $x_1 = 1 - 2x_3 - 3x_4$, $x_2 = 1 + x_4$, $x_3 = x_3$, $x_4 = x_4$.

ii Compute the rank by Gauss-Jordan elimination.

$$\begin{pmatrix} -1 & 3 & -2 & -6 & 2 \\ 2 & 1 & 4 & 5 & 3 \\ 1 & 2 & 2 & 1 & t \end{pmatrix} \longrightarrow$$

$$\begin{pmatrix} -1 & 0 & -2 & -3 & -1 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & t-3 \end{pmatrix}$$

For $t \neq 3$, there are three non-zero rows, so ~~there~~ the collection of vectors has rank 3. For $t=3$, there are two non-zero rows, so the collection of vectors has rank 2.

The set of vectors spans $\mathbb{R}^3$ iff it contains three linearly independent vectors, which is iff it ~~spans~~ has rank 3, which is iff $t \neq 3$.

The set does not form a basis of $\mathbb{R}^3$ because even if it did span $\mathbb{R}^3$ it is not a minimal collection of vectors that spans $\mathbb{R}^3$, it contains two more vectors than is necessary for that.

A subset of the vectors ~~spans $\mathbb{R}^3$ iff that~~ is a basis of $\mathbb{R}^3$ iff it contains ~~three~~ 3 linearly independent vectors. $t \neq 3$ is necessary for this. The first and third vector are linearly dependent. One subset that is a basis of $\mathbb{R}^3$ is $\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ t \end{pmatrix}$ where $t \neq 3$.

iii No two vectors in the set can be extended to form a basis simply by adding ~~some~~ any vector that is not in the set because for any two such vectors $\vec{v}_1, \vec{v}_2$, there is some linear combination of the two, $\vec{u} = \alpha\vec{v}_1 + \beta\vec{v}_2$, that is not in the set, for example $\vec{u} = 100\vec{v}_1$. Then $\{\vec{v}_1, \vec{v}_2, \vec{u}\}$ is not a set of linearly independent vectors, which is necessary for a basis of $\mathbb{R}^3$.

b i $F(x,y,z) = x^3 + 3y^2 + 2xz^3 - z^3y - 1$

~~$F(1,0,z$~~

$F(1,0,z) = 0 \Rightarrow 1 + 2z^3 - 1 = 0 \Rightarrow z = 0$

$F_x(1,0,z=0) = 3x^2 + 2z^3 |_{1,0,0} = 3$

$F_y(1,0,z=0) = 6y - z^3 |_{1,0,0} = 0$

$F_z(1,0,z=0) = 6xz^2 - 3yz^2 |_{1,0,0} = 0$

F is a polynomial so it is C^1 (hence C^1 in an open ball around $(1,0,0)$).

~~$F_y(1,0,z=0)$~~ $F_z(1,0,z=0) = 0$, so IFT is not applicable to define z as a function of x and y in a neighbourhood of $(1,0,0)$.

ii $F(-1,-1,z) = 0 \Rightarrow -1 + 3 - 2z^3 + z^3 - 1 = 0 \Rightarrow 1 - z^3 = 0$

$\Rightarrow z = 1$

$F_x(-1,-1,z=1) = 3x^2 + 2z^3 |_{-1,-1,1} = 5$

$F_y(-1,-1,z=1) = 6y - z^3 |_{-1,-1,1} = -7$

$F_z(-1,-1,z=1) = 6xz^2 - 3yz^2 |_{-1,-1,1} = -3$

F is C^1 in an open ball around $(-1,-1,1)$, $F(-1,-1,1) = 0$, and $F_z(-1,-1,1) \neq 0$, so IFT is applicable to define z as an implicit function of x and y in a neighbourhood of $(-1,-1,1)$.

$\partial/\partial x \; z(x,y) = -F_x(-1,-1,1)/F_z(-1,-1,1) = 5/3$

$\partial/\partial y \; z(x,y) = -F_y(-1,-1,1)/F_z(-1,-1,1) = -7/3$

Date

2ai Real-valued function $f : \mathbb{R}^n \to \mathbb{R}$ is concave iff for all $\vec{x}, \vec{x}' \in \mathbb{R}^n$ and for all $t \in (0,1)$, $f(t\vec{x} + (1-t)\vec{x}') \geq tf(\vec{x}) + (1-t)f(\vec{x}')$. The property is strict if the inequality ~~=~~ holds strictly and the definition of convexity reverses the direction of the inequality.

~~A~~ A concave ~~maximisation~~ optimisation problem has the following form.
$\max_{\vec{x}} f(\vec{x})$ s.t. $\vec{g}(\vec{x}) \leq \vec{b}$
where f is concave and each ~~of $g_1, \dots$ is convex~~. of $g_1, \dots, g_m$ is convex.

If an ~~co~~ optimisation problem is concave, then the KT-FOCs are sufficient for a maximum. If, in addition, the constraint set has non-empty interior, or the constraint qualification is satisfied, then the KT-FOCs are also necessary.

A convex optimisation problem has the following form.
$\min_{\vec{x}} f(\vec{x})$ s.t. $\vec{g}(\vec{x}) \geq \vec{b}$
where f is convex and each of $g_1, \dots, g_m$ is concave.

If an optimisation problem is convex, the same applies for a minimum.

ii f is ~~convex ⇒~~ concave ⇒
$f(t\vec{x} + (1-t)\vec{x}') \geq tf(\vec{x}) + (1-t)f(\vec{x}')$ for all $\vec{x}, \vec{x}' \in \mathbb{R}^n$, $t \in (0,1) \Rightarrow$
$f(t\vec{x} + (1-t)\vec{x}') \geq t\min\{f(\vec{x}), g(\vec{x})\} + (1-t)\min\{f(\vec{x}'), g(\vec{x}')\}$
g is concave ⇒
$g(t\vec{x} + (1-t)\vec{x}') \geq tg(\vec{x}) + (1-t)g(\vec{x}') \Rightarrow$
$g(t\vec{x} + (1-t)\vec{x}') \geq t\min\{f(\vec{x}), g(\vec{x})\} + (1-t)\min\{f(\vec{x}'), g(\vec{x}')\}$
⇒
$\min\{f(t\vec{x} + (1-t)\vec{x}'), g(t\vec{x} + (1-t)\vec{x}')\} \geq t\min\{f(\vec{x}), g(\vec{x})\} + (1-t)\min\{f(\vec{x}'), g(\vec{x}')\}$
⇒
$\min\{f, g\}$ is concave.

bi $f(x,y) = \ln x + 3\ln y$ (with $x>0, y>0$)
$Df(x,y) = (1/x, \; 3/y)$
$D^2f(x,y) = \begin{pmatrix} -x^{-2} & 0 \\ 0 & -3y^{-2} \end{pmatrix}$
$\operatorname{tr} D^2f(x,y) = -x^{-2} - 3y^{-2} < 0$
$\det D^2f(x,y) = 3x^{-2}y^{-2} > 0$
All eigenvalues of $D^2f(x,y)$ are strictly negative, $D^2f(x,y)$ is negative definite, f is strictly concave.

ii $g(x,y) = -x^4 + 8xy - y^4$
$Dg(x,y) = (-4x^3 + 8y \quad 8x - 4y^3)$
$D^2g(x,y) = \begin{pmatrix} -12x^2 & 8 \\ 8 & -12y^2 \end{pmatrix}$
tr ~~det~~ $D^2g(x,y) = -12(x^2+y^2) \leq 0$
$\det D^2g(x,y) = 144x^2y^2 - 64$
~~For x, y x = y = 0, det~~
$\det$ ~~$D^2g(x,y)$~~ $D^2g(0,0) = -64 < 0$
~~The eige~~ One eigenvalue of $D^2g(0,0)$ is strictly negative, the other is strictly positive, so $D^2g(0,0)$ is indefinite, g is not concave.

iii $h(x,y) = xy/(x+y)$ (with $x>0, y>0$)
$Dh(x,y) =$ ~~$(x(-1)(x+y)^{-2} + (x+y)^{-1} \quad y(-1)(x+y)^{-2} + (x+y)^{-1})$~~
~~$= (y/(x+y)^2 \quad x/(x+y)^2)$~~
$= (xy(-1)(x+y)^{-2} + y(x+y)^{-1} \quad xy(-1)(x+y)^{-2} + x(x+y)^{-1})$
$= (y^2/(x+y)^2 \quad x^2/(x+y)^2)$
$D^2h(x,y) = \begin{pmatrix} y^2(-2)(x+y)^{-3} & x^2(-2)(x+y)^{-3} + 2x(x+y)^{-2} \\ y^2(-2)(x+y)^{-3} + 2y(x+y)^{-2} & x^2(-2)(x+y)^{-3} \end{pmatrix}$
$= (x+y)^{-2} \begin{pmatrix} -2y^2/x+y & -2x^2/x+y + 2x \\ -2y^2/x+y + 2y & -2x^2/x+y \end{pmatrix}$
$= (x+y)^{-2} \begin{pmatrix} -2y^2/x+y & 2xy/x+y \\ 2xy/x+y & -2x^2/x+y \end{pmatrix}$
$= 2(x+y)^{-3} \begin{pmatrix} -y^2 & xy \\ xy & -x^2 \end{pmatrix}$

$\operatorname{tr} D^2h(x,y) = 2(x+y)^{-3}(-y^2 - x^2) < 0$
$\det D^2h(x,y) = 2(x+y)^{-3}(x^2y^2 - x^2y^2) = 0$
One eigenvalue is zero, the other is negative, $D^2h(x,y)$ is negative semi-definite, h is weakly concave.

ci $MU_x = \partial u/\partial x = 1/x$, $MU_y = \partial u/\partial y = 3/y$
Marginal utilities tend to infinity as the quantities tend to zero, so positivity constraints will not bind.

ii $\max_{x,y} \ln x + 3\ln y$ s.t.
BC: $x + 2y \leq 24$
TC: $2x + y \leq 24$
$\mathcal{L} = \ln x + 3\ln y - \lambda_B(x+2y-24) - \lambda_T(2x+y-24)$
FOC_x: $1/x - \lambda_B - 2\lambda_T = 0$
FOC_y: $3/y - 2\lambda_B - \lambda_T = 0$
CS_B: $\lambda_B \geq 0$, $x+2y \leq 24$, $\lambda_B(x+2y-24) = 0$
CS_T: $\lambda_T \geq 0$, $2x+y \leq 24$, $\lambda_T(2x+y-24) = 0$

Suppose $\lambda_B > 0$, $\lambda_T = 0$, then from FOC_x, FOC_y,
$\lambda_B = 1/x = 3/2y \Rightarrow x = 2/3\,y$. From CS_B,
$x + 2y - 24 = 0 \Rightarrow 8/3\,y = 24 \Rightarrow y = 9 \Rightarrow x = 6 \Rightarrow$
$\lambda_B =$ ~~1/4~~ $1/6$, $2x + y = 21 \leq 24$, all KT-FOC are satisfied.

Suppose $\lambda_B = 0$, $\lambda_T > 0$, then from FOC_x, FOC_y,
$\lambda_T = 1/2x = 3/y \Rightarrow x = y/6$. From CS_T,
$2x + y = 24 \Rightarrow 4/3\,y = 24 \Rightarrow y = 18 \Rightarrow x = 3 \Rightarrow$

$2y + x = 39 > 24$, CSB is violated. There is no solution such that $\lambda_B = 0$, $\lambda_T > 0$.

Suppose that $\lambda_B = \lambda_T = 0$, then by FOCx, $1/x = 0$, so x is undefined, FOCx cannot be satisfied. There is no solution such that $\lambda_B = \lambda_T = 0$.

Suppose that $\lambda_B, \lambda_T > 0$. Then by CSB, CST, $x + 2y = 2x + y = 24 \Rightarrow x = y = 8$, then by FOCx, FOCy, $1/8 = \lambda_B + 2\lambda_T$, $3/8 = \cancel{3/4}\ 2\lambda_B + \lambda_T \Rightarrow \lambda_B - \lambda_T = 2/8 \Rightarrow 3\lambda_T = -1/8 \Rightarrow \lambda_T = -1/24 < 0$, CST is violated. There is no solution such that $\lambda_B, \lambda_T > 0$.

The unique solution to the KT-FOCs is $(x, y) = (6, 9)$.

iv From (bi), the objective function is concave. The constraints are linear hence (weakly) convex. So the optimisation problem is concave. The constraint set has non-empty interior. So KT-FOCs are necessary and sufficient for a maximum. They have a unique solution, so the solution is the unique global maximum.

v At the optimum, $\lambda_B > 0$, $\lambda_T = 0$, only BC binds. ~~the~~ Marginal utility from relaxation of BC is positive, that from relaxation of TC is zero. The consumer would rather have more time.

4ai Consider two lotteries L_F, L_G with cumulative distribution functions F and G respectively. ~~$L_F \succ_{SOSD} L_G$ iff for all x, $\int_{-\infty}^{x} F(u)\,du \le \int_{-\infty}^{x} G(u)\,du$.~~ If expected value $EV(L_F) = EV(L_G)$, i.e. $\int_{-\infty}^{\infty} xF'(x)\,dx = \int_{-\infty}^{\infty} xG'(x)\,dx$, then $L_F \succ_{SOSD} L_G$ iff for all x, $\int_{-\infty}^{x} F(u)\,du \le \int_{-\infty}^{x} G(u)\,du$.

This is iff L_F is a mean-preserving contraction of L_G and implies that any risk-averse expected utility maximiser prefers L_F to L_G.

The common condition is that the plot of F(x) against x crosses the plot of G(x) against x exactly once from below. This condition is sufficient but not necessary.

ii $L_1 = (4/20, 4/20, 4/20, 4/20, 4/20)$
$L_2 = (3/20, 6/20, 0/20, 10/20, 1/20)$
L_1 is a mean-preserving spread of L_2: obtained by reallocating 1/20 probability mass from -1 to each of -2 and 0 and reallocating 3/20 probability mass from +1 to each of 0 and +2.

$L_2 \succ_{SOSD} L_1$, so any risk-averse expected utility maximiser strictly prefers L_2 to L_1. L_1 and L_2 have equal expected value but L_2 is less risky and has lower risk premium hence higher certainty equivalent.

bi The lottery that agent A faces ~~in the~~ initially ~~state~~ is $L_0 = [\frac12, \frac12; 125, 80]$

The lottery that A faces if A buys the lottery at price p is $L_1(p) = [\frac12, \frac12; 125 + 15 - p, 80 + 105 - p] = [\frac12, \frac12; 140 - p, 185 - p]$

The maximum price $\bar{p}$ that A is willing to pay is such that A is indifferent.

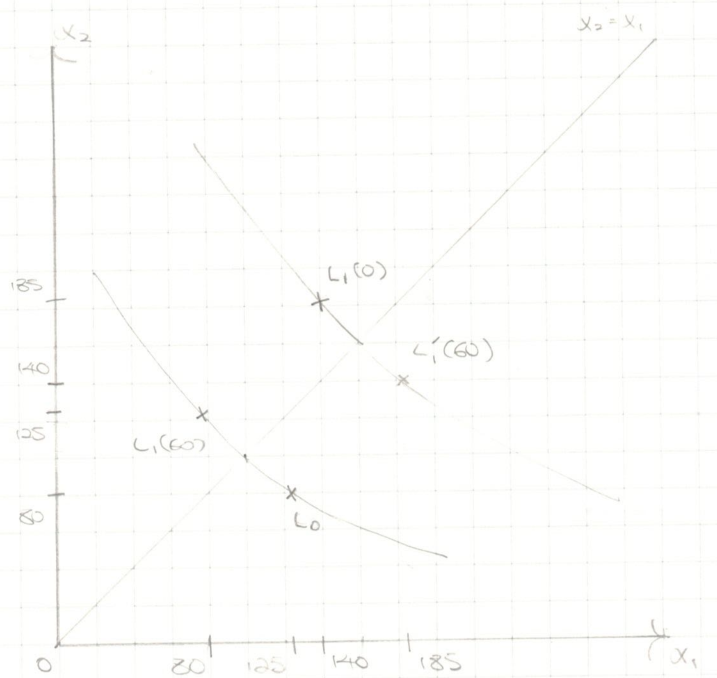
$L_0 \sim_A L_1(\bar{p}) \Leftrightarrow$
$U(L_0) = U(L_1(\bar{p})) \Leftrightarrow$
$\frac12 \ln 125 + \frac12 \ln 80 = \frac12 \ln(140-\bar{p}) + \frac12 \ln(185-\bar{p}) \Leftrightarrow$
$\frac12 \ln 10000 = \frac12 \ln(140-\bar{p})(185-\bar{p}) \Leftrightarrow$
$10000 = (140-\bar{p})(185-\bar{p}) \Leftrightarrow$
$\bar{p} = 60$ or 265 (reject)

ii ~~In the~~ initially ~~state~~, A faces
$L_0' = [\frac12, \frac12; 140, 185]$
If A sells the lottery at price p, A faces $L_1'(p) = [\frac12, \frac12; 125+p, 80+p]$

The minimum price $\underline{p}$ at which A would be willing to sell is such that A is indifferent.

$L_0' \sim_A L_1'(\underline{p}) \Leftrightarrow$
$U(L_0') = U(L_1'(\underline{p})) \Leftrightarrow$
$\frac12 \ln 140 + \frac12 \ln 185 = \frac12 \ln(125+\underline{p}) + \frac12 \ln(80+\underline{p}) \Leftrightarrow$
$(140)(185) = (125+\underline{p})(80+\underline{p}) \Leftrightarrow$
$\underline{p} = 60$ or -265 (reject)

The two prices are equal. This is in some sense a coincidence, or an artifact of the numerical values involved. It does not depend on the particular functional form of the utility function.



Buying the ticket at the maximum acceptable price $\bar{p} = 60$ involves a shift from L_0 to $L_1(0)$ and then ~~being~~ ~~when~~ in receiving the ticket and a shift from $L_1(0)$ to $L_1(60)$ in the 45° direction back to the original indifference curve. Selling the ticket at the minimum acceptable price $\underline{p} = 60$ involves a shift from $L_1(0)$ to $L(0)$ ~~when~~ in losing the ticket and a shift from L_0 to $L_1'(60)$ in the 45° direction back to the original indifference curve. ~~The two shifts are $L_0 \to L_1(0)$ and $L_1(0) \to$~~ The prices are the same because $L'(0)$, $L_1(60)$ are mirror images of L_0 and $L_1'(60)$ in the 45° line.

$u(x) = \ln x$, $u'(x) = 1/x$, $u''(x) = -x^{-2}$,
$A(x) = -u''(x)/u'(x) = -1/x$, so A has decreasing absolute risk aversion. So for other lotteries, if, for example, the

~~initia~~ endowment is risk free and the lottery is risky, the buying price would be lower than the selling price because the selling A is more wealthy so less risk averse and has a lower risk ~~premi~~ premium.

5a. If the monopolist M can perfectly price discriminate it sells to each type of consumer at ~~the~~ price equal to that consumer's valuation. Then, M sells to high valuation ~~ty~~ H types first ~~t~~.

~~Demand~~ Inverse demand is equal to marginal revenue.

$$MR = \begin{cases} 5 & \text{for } q \leq 10 \\ 4 & \text{for } 10 < q \leq 20 \end{cases}$$

Marginal cost

$$MC = \cancel{2q} \; {}^{2q}/_{10} = {}^{q}/_{5}$$

$$MR = MC \iff q = 20.$$

The monopolist sells 20 units in total, 10 to each of H and L, at price $p_H = 5$ and $p_H = 4$. ~~The~~

$$\pi = \cancel{20^2} \; 5(10) + 4(10) - {}^{20^2}/_{10} = 50$$

b. M optimally sells to H and L at $p = 4$. ~~Fo~~ For all $p < 4$, demand is perfectly inelastic, so it is strictly profitable to increase price. For $p > 4$, L types do not buy. Similarly, M optimally sells to only H at $p = 5$.

$p = 5 \Rightarrow q = 10$ (all and only H types buy), $C = {}^{q^2}/_{10} = 10 \Rightarrow \pi = 5(10) - 10 = 40$

$p = 4 \Rightarrow$ ~~$q = 10$ (all an~~ $q = 20$ (both types buy), $C = {}^{q^2}/_{10} = 40 \Rightarrow \pi = 4(20) - 40 = 40$

M is indifferent between the two options and has profit $\pi = 40$ in either case.

c. Maximum profit under a two part tariff is no lower than profit under a linear price schedule because any linear price schedule p can be replicated under a two part tariff with ~~F=0~~ $T = 0 + qp$.

~~At the opt~~ ~~Suppose $T > 0$. Then for $p > 4$, at most H types buy because $T + qp > qv_L$ i.e. total tariff is ~~q always~~ greater than total valuation for L types. Similarly, for $p > 5$, no consumers buy because $T + qp > qv_H > qv_L$, i.e. total tariff is greater than total valuation for any type.~~

~~Suppose $4 < p \leq 5$, then all and only H types buy. $qv_H = T + pq$ ~~$\iff q =$~~ $q \leq 10$, so $MC \leq 2 < p$. So inducing $q = 10$ is optimal. ~~Then~~ $q = 10$ is optimally induced by any $T, p \geq 0$ such that $T + 10q = 50$. This is equivalent to selling 10 units to H at unit price $p = 5$.~~

~~Suppose $p \leq 4$. Then $q \leq 20$, so $MC \leq 4$, so it is optimal to induce $q = 20$ (because $p \geq MC$ for all units up to $q = 20$) This requires~~

Suppose $p \leq 4$, then surplus of L types ~~$\neq$~~ $qv_L - (F + p\cancel{q}) =$ ~~$\neq$~~ $(v_L - p)q - F$ is increasing in q, so if it is optimal to buy any units, it is optimal to buy 10 units. Suppose $p > 4$ then for all $F \geq 0$, ~~qv_L~~ for all q, ~~$\neq$~~ $qv_L \overset{<}{\cancel{\geq}} F + qp$, so it is optimal to buy 0 units, so L types buy either 0 units or 10 units. A similar argument applies for H types.

Then H type buys 10 units if $F + 10p \geq 50$ and 0 otherwise, L type buys 10 units if $F + 10p \geq 40$ and 0 otherwise. So ~~a~~ any outcome of a two part tariff can be replicated with a linear price $p' = {}^{1}/_{10}(F + 10p)$.

d. ~~The monopolist~~ M can offer the following set of contracts to replicate the outcome of any two part tariff $F + qp$.

$\{(q, T) : q \in [0, 10], T = F + qp\}$.

The optimal set of contracts, if M screens consumers, satisfies the participation constraint (~~PC~~ PC) and incentive constraint ~~of each type~~ (IC) of each type.

PC_L: ~~$\neq$~~ $v_L q_L - T_L \geq 0$

PC_H: $v_H q_H - T_H \geq 0$

IC_L: $v_L q_L - T_L \geq v_L q_H - T_H$

IC_H: $v_H q_H - T_H \geq v_H q_L - T_L$

$0 \leq_1 v_L q_L - T_L <_2 v_H q_L - T_L \leq_3 v_H q_H - T_H$

$\leq_1$ by PC_L, $<_2$ ~~by~~ given $v_H > v_L$, $\leq_3$ by IC_H, so PC_H is redundant.

At ~~any~~ the optimum, PC_L binds. Any candidate optimum such that PC_L does not bind fails to deviation ~~by~~ by increasing both T_L, T_H by small amount ε. For small ε, PC_L and PC_H remain satisfied. LHS and RHS of each IC ~~changes~~ change by equal amounts. So each IC remains satisfied. Revenue hence profit increases.

At the optimum, IC_H binds. Any candidate optimum such that IC_H does not bind fails to deviation by increasing T_H by small amount ε. ~~PC_H and IC_H rem.~~ For small ε, PC_H and IC_H remain satisfied. Increase in T_H "loosens" ~~PC~~ IC_L and has no effect on PC_L. Revenue hence profit increases.

$PC_L: 4q_L = T_L$

$IC_H: 5q_H - T_H = 5q_L - T_L = q_L$

$\pi = T_L + T_H - (q_L+q_H)^2/10$

$\quad = 4q_L + (5q_H - q_L) - (q_L+q_H)^2/10$

$\quad = 3q_L + 5q_H - (q_L+q_H)^2/10$

~~Neglecting IC_L~~,

$\cancel{FOC\, q_L: 3 - 2(q_L+q_H)/10 = 0}$

$\cancel{FOC\, q_H: 5 - 2(q_L+q_H)/10 = 0}$

$\partial\pi/\partial q_H = 5 - 2(q_L+q_H)/10 = 5 - Q/5 > 0$ for $Q \leq 20$

So M, given q_L, optimally chooses $q_H = 10$, i.e the upper bound on q_H is binding.

$\pi = 3q_L + 50 * - (q_L+10)^2/10$

$\cancel{\partial} \partial\pi/\partial q_L = 3 - 2(q_L+10)/10 = 0 \Leftrightarrow$

$q_L+10/5 = 3 \Leftrightarrow$

$q_L = 5 \Rightarrow$

$T_L = 20,\ T_H = 45 \Rightarrow$

$4q_L - T_L = 0 \geq 4q_H - T_H = -5$, so IC_L is satisfied.

The optimal screening contract is $(\hat{q}_L, \hat{T}_L), (\hat{q}_H, \hat{T}_H)$
$= (5, 20), (10, 45)$.

$\hat{\pi} = 20 + 45 - (10+5)^2/10 = 42.5$