2 $x+3y-2z=2$ ①
$x+2y+z=1$ ②
$x+5y+\alpha z=\beta$ ③

From ①
$x=2-3y+2z$ ④
Sub ④ into ②
$2-3y+2z+2y+z=1$
$1-y+3z=0$
$y=1+3z$ ⑤
Sub ⑤ into ④
$x=2-3(1+3z)+2z=-1-7z$ ⑥
Sub ⑤ and ⑥ into ③
$(-1-7z)+5(1+3z)+\alpha z=\beta$
$4+8z+\alpha z=\beta$ ⑦
$\cancel{(8+\alpha)z=\beta}$
$(\alpha+8)z=\beta-4$
If $\alpha\neq-8$,
$z=(\beta-4)/(\alpha+8)$ ⑧
Sub ⑧ into ⑥ and ⑤
$x=-1-7(\beta-4)/(\alpha+8)$ $\cancel{⑧}$ ⑨
$y=1+3(\beta-4)/(\alpha+8)$ $\cancel{⑨}$ ⑩
If $\alpha=-8$,
then if $\beta=4$,
~~there are infinitely many solutions, or~~
⑦ reduces to $0=0$
~~then~~ if instead $\beta\neq4$
⑦ has no solution.

So if $\alpha=-8$ and $\beta=4$, the linear system has infinitely many solutions given by $x=-1-7z$, $y=1+3z$, $z\in\mathbb{R}$. If $\alpha=-8$ and $\beta\neq4$, the linear system has no solutions. If $\alpha\neq-8$, the linear system has a unique solution $x=-1-7(\beta-4)/(\alpha+8)$, $y=1+3(\beta-4)/(\alpha+8)$, $z=(\beta-4)/(\alpha+8)$

The linear system can be written as $A\cdot\vec{x}=\vec{b}$ where
$A=\begin{pmatrix}1&3&-2\\1&2&1\\1&5&\alpha\end{pmatrix}$, $\vec{x}=\begin{pmatrix}x\\y\\z\end{pmatrix}$ and $\vec{b}=\begin{pmatrix}2\\1\\\beta\end{pmatrix}$

$\begin{pmatrix}1\\1\\1\end{pmatrix},\begin{pmatrix}3\\2\\5\end{pmatrix},\begin{pmatrix}-2\\1\\\alpha\end{pmatrix}$ span $\vec{b}$ iff $A\cdot\vec{x}=\vec{b}$ has at least one solution iff $\alpha\neq-8$ or $\beta=4$.