Microeconomics Problem Set 1

5a The farmer has wage income wl. Because the farmer derives zero utility from "consumption" of the field, the field (all of the single unit of it) is rented out in ~~equilibrium~~ any candidate eqm, so the farmer also has rental income r. So the farmer has total income $wl+r$ which can be spent on turnips ~~tt~~ which have price 1. So the farmer's budget constraint is $t \le wl+r$.

b $\max_{t,l} \ln(t) + \ln(1-l)$ s.t.

BC: $t \le wl + r \iff t - wl \le r \iff t + w(1-l) \le r + w$

The farmer's preferences are Cobb-Douglas over turnips t and leisure $1-l$ with coefficients $\frac{1}{2}$ and $\frac{1}{2}$. At the consumption optimum, the farmer "expends" $\frac{1}{2}$ of the "~~budget~~ income" $r+w$ on each good. ~~which~~ ~~that~~ At the optimum, $t = \frac{r+w}{2}$, $1-l = \frac{r+w}{2w} \Rightarrow l = 1 - \frac{r+w}{2w}$ $= \frac{w-r}{2w}$.

At the optimum, BC binds. Any candidate optimum such that BC does not bind fails to deviation ~~to a bundle with~~ by increasing each of t and $1-l$ by small amount ε. The optimization problem reduces to

$\max_t \ln t + \ln(1 - \frac{t-r}{w})$

FOC: $\frac{1}{t} + \frac{w}{w+r-t}(-\frac{1}{w}) = 0 \Rightarrow$

$\frac{1}{t} = \frac{1}{w+r-t} \Rightarrow$

$2t = w + r \Rightarrow$

$t = \frac{w+r}{2} \Rightarrow$

$l = 1 - \frac{w+r}{2w}$

Labour supply is positive

$l > 0 \iff$

$1 - \frac{w+r}{2w} > 0 \iff$

$r < w$

c $C(L,F) = wL + rF$

d The cost minimisation problem is

$\min_{L,F} wL + rF$ s.t.

$y = L^{1/2}F^{1/2}$

$\mathcal{L} = wL + rF - \lambda(L^{1/2}F^{1/2} - y)$

FOC_L: $w - \lambda F^{1/2}(\frac{1}{2}L^{-1/2}) = 0$

FOC_F: $r - \lambda L^{1/2}(\frac{1}{2}F^{-1/2}) = 0$

$FOC_L \Rightarrow \frac{1}{2}\lambda F^{1/2}L^{-1/2} = w$

$FOC_F \Rightarrow \frac{1}{2}\lambda L^{1/2}F^{-1/2} = r$

$\Rightarrow \frac{\lambda^2}{4} = wr \Rightarrow \lambda = \sqrt{4wr}$.

$\Rightarrow F^{1/2}L^{-1/2} = \frac{2w}{\lambda} = \frac{w}{\sqrt{wr}} = \sqrt{\frac{w}{r}}$

$\Rightarrow F^{-1/2}L^{1/2} = \sqrt{\frac{r}{w}}$

$\Rightarrow \frac{F}{L} = \frac{w}{r}$

$\Rightarrow L^{1/2}(\frac{wL}{r})^{1/2} = y \Rightarrow L(\frac{w}{r})^{1/2} = y \Rightarrow L = y(\frac{r}{w})^{1/2}$

$\Rightarrow F = y(\frac{w}{r})^{1/2}$

$c(y) = w(y(\frac{r}{w})^{1/2}) + r(y(\frac{w}{r})^{1/2})$

$= y[wr^{1/2} + wr^{1/2}]$

$= 2y(wr)^{1/2}$

λ is the marginal cost of producing an additional turnip, evaluated at ~~the cost-minimising~~ ~~produ~~ output y, produced at minimum cost.

e The three market-clearing conditions are

$t = y$, $l = L$, $1 = F$, $\iff$

~~$\frac{w+r}{2} = L^{1/2}F^{1/2}$ $\frac{w-r}{2w} =$~~

$\frac{w+r}{2} = L^{1/2}F^{1/2}$, $\frac{w-r}{2w} = y(\frac{r}{w})^{1/2}$, $1 = y(\frac{w}{r})^{1/2} \iff$

$\frac{w+r}{2} = L^{1/2}$, $\frac{w-r}{2w} = y(\frac{r}{w})^{1/2}$, ~~$(\frac{w}{r})^{1/2}$~~ $\frac{w}{r} = y^{-2}$

Date