# Horizontal Merger Notes

## Salant et al. (1983) Merger Incentive

- Parameters
  - Consider a merger in a standard static Cournot oligopoly. Pre-merger, each of $n$ firms indexed by $i \in \{1, 2, \dots, n\}$ produces a homogenous good at common constant marginal cost $c$ and chooses quantity $q_i$ to maximise profit $\pi_i = [P(q_i + Q_{-i}) - c]q_i$ where $Q_{-i}$ is the total quantity of all other firms and $P(Q)$ is the inverse demand function. Suppose that demand and marginal revenue are downward-sloping, i.e. $P'(Q) < 0$ and $P(Q) + QP''(Q) < 0$. $m < n$ "insiders" merge. Let $I$ denote the set of insiders and $O$ denote the set of $n - m$ "outsiders". Suppose that merger realises no synergies, i.e. the merged firm has constant marginal cost $c$.
- Analysis
  - See graphical analysis in Smith (2022, p. 5).
  - Since merger realises no synergies, the merged firm operates as a single firm with constant marginal cost $c$. Merger is equivalent to the closure of $m - 1$ insiders.
- Result
  - Whether insiders' joint profit increases post-merger depends on the number of insiders. Merger has two effects on insiders' joint profit, a positive direct effect due to the internalisation of negative externalities between insiders, and a negative strategic effect due to outsiders' competing more aggressively.
  - Pre-merger, each insider imposes a negative externality on each other insider in choosing a high quantity since an increase in one firm's quantity causes a decrease in price hence a decrease in each other firm's profit. Post-merger, the negative externalities between insiders are internalised and the merged firm has incentive to decrease quantity since a decrease in quantity causes an increase in price hence an increase in margin on the merged firm's large quantity and a large increase in profit.
  - Post-merger, compared to pre-merger, given that insiders have incentive to decrease output because of the internalisation of negative externalities between insiders, they compete more "soft", i.e. choose a lower output given any joint outsider output. Given that demand and marginal revenue are downward-sloping, outputs are strategic substitutes, outsiders optimally respond to insiders' competing more soft by competing more aggressively, i.e. choosing higher output. Outsider's choosing higher output causes a decrease in price hence a decrease in insiders' joint profit.
  - Salant et al. (1983) show that more generally, under symmetric Cournot competition and where merger realises no synergies, joint profit of insiders increases only if merger involves more than $80\%$ of the firms in the industry.
- Discussion
  - The Cournot merger paradox is the result that in Cournot oligopoly, merger does not increase insiders' joint profit unless insiders' (joint) market share is very large. This result is a paradox because merger has a first-order positive direct effect on insiders' profit due to the internalisation of negative externalities between insiders. The negative strategic effect on insiders' profit due to outsiders' competing more aggressively is only a second-order effect. It is therefore surprising that the second-order effect dominates the first-order effect unless insiders' market share is very large.
  - The Cournot merger paradox suggests that managers could have non-profit motivations in pursuing merger. For example, it could be that managers prefer to manage large firms, because managing a large firm confers a certain sort of status.
- Evaluation
  - That a merger of $m$ firms in Cournot oligopoly is analytically equivalent to the closure of $m - 1$ firms holds only because products are homogenous and costs are symmetric. The more general result, that the effects of merger in Cournot oligopoly on insiders' joint profit is ambiguous holds even if these assumptions are relaxed since merger in Cournot oligopoly necessarily results in the elimination of negative externalities between insiders and more aggressive competition by outsiders.
  - That the effects of merger on insiders' joint profit is ambiguous is unlikely to hold where firms compete in prices. Where firms compete in prices, merger continues to result in the elimination of negative externalities between insiders who then compete more "soft". But since prices are often strategic complements, outsiders respond to "softer" insiders by competing less aggressively, which increases insiders' joint profit.
  - Stigler (1950) argued that even if merger increases insiders' joint profit, merger need not occur. This is because merger causes insiders to compete less aggressively because of the internalisation of negative externalities between

insiders, hence increases outsiders' profit. If each firm finds it optimal to be an outsider rather than an insider post-merger, each firm has no incentive to participate in the merger.

## Cournot Merger Incentive with Synergies

- Parameters
  - Suppose instead that merger realises synergies such that the marginal cost of the merged firm is $c' < c$.
- Analysis
  - The decrease in marginal cost due to the realisation of synergies causes an increase in the merged firm's margin hence an increase in the merged firm's incentive to increase output. Hence, post-merger, compared to where merger realises no synergies, the merged firm competes more "tough", i.e. chooses higher output given any (joint) outsiders' output. Given that marginal revenue is downward-sloping, quantities are strategic substitutes. Post-merger, compared to where merger realises no synergies, outsiders compete less aggressively, i.e. choose lower (joint) output, which causes an increase in the merged firm's profit.
- Result
  - The realisation of synergies has two effects on the merged firm's profit, a positive direct effect due to the increase in margin, and a positive strategic effect due to outsiders' competing less aggressively.

## Stigler (1950) Merger to Monopoly Incentive

- Analysis
  - Consider a merger to monopoly. Let $\Pi_I^P(n,m)$ denote insiders' joint profit post-merger where there are $n$ firms pre-merger and $m$ firms merge. Let $\pi_O^P(n,m)$ denote each outsider's profit post-merger where there are $n$ firms pre-merger and $m$ firms merge. Suppose that merger occurs even if $m = n-1$.
  - Each firm $i$ has incentive to participate in a merger to monopoly iff its share of the merged firm's profit is greater than the profit it would enjoy if it remained an outsider post-merger, i.e. $s_i\Pi_I^P(n,n) > \pi_O^P(n,n-1)$, where $s_i$ is firm $i$'s share of the merged firm's profit.
  - Merger to monopoly occurs iff each firm $i$ has incentive to participate, i.e. $s_i\Pi_I^P(n,n) > \pi_O^P(n,n-1)$ for all $i$, iff $\min_i s_i\Pi_I^P(n,n) > \pi_O^P(n,n-1)$, only if $\frac{\Pi_I^P(n,n)}{n} > \pi_O^P(n,n-1)$.
- Result
  - Even if merger to monopoly maximises insiders' joint profit, merger to monopoly does not occur if each firm finds it optimal to be an outsider rather than an insider.
- Evaluation
  - In practice, the above necessary condition for merger to monopoly is unlikely to hold because (1) the number of firms is large or (2) merger to monopoly does not realise sufficiently large synergies such that $\Pi_I^P(n,n)$ is large.

## Williamson (1968) Merger to Monopoly Aggregate Welfare

- Parameters
  - Consider the Williamson (1968) model of merger to monopoly. Pre-merger, a large number of firms produce a homogenous good at common constant marginal cost $c$ and the industry is perfectly competitive. Merger to monopoly occurs. Merger realises synergies such that post-merger, the monopolist produces the same homogenous good at constant marginal cost $c' < c$. Suppose that the synergies realised are non-drastic, i.e. monopoly price given constant marginal cost $c'$ $p^M(c')$ is greater than $c$. Demand is downward-sloping.
- Analysis
  - See graphical analysis in Smith (2022, p. 15)
  - Pre-merger, given that the industry is perfectly competitive, price $p^A$ is equal to marginal cost $c$. Post-merger, the monopolist chooses price $p^P$ equal to monopoly price $p^M(c')$. By supposition, $p^P > p^A$. Given that demand is downward-sloping, post-merger quantity $q^P$ is less than pre-merger quantity $q^A$.
  - Post-merger, compared to pre-merger, consumer surplus decreases because of (1) the decrease in quantity and (2) the increase in price. Aggregate profit increases because of (1) the increase in price and (2) the decrease in marginal cost. The increase in price constitutes a transfer of surplus from consumers to producers. Hence, merger has two effects on aggregate welfare, a positive effect due to the realisation of synergies, and a negative effect due to the decrease in quantity.
- Result
  - Aggregate welfare increases iff the increase in aggregate profit due to the realisation of synergies is greater than the decrease in consumer surplus due to the decrease in quantity.

- Discussion
  - The trade-off, from the perspective of antitrust authorities, in evaluating a horizontal merger (by an aggregate welfare standard), is between the realisation of synergies that benefits firms and the increase in market power that hurts consumers.
  - In practice, competition authorities, at least in the U.S. the U.K. and Europe tend to evaluate potential mergers by a consumer surplus standard rather than by an aggregate welfare standard, which considers also the impact of merger on insiders' and outsiders' profit. While this preference is no doubt in part explained by the fact that estimating the impact of merger on profit is empirically challenging, this preference may also reflect a normative judgement about what the "socially desirable" consists in. Competition authorities could, for example, be motivated by distributive concerns in attaching greater weight to the welfare of consumers than firms.
- Case Study (Superior Propane/ICG Propane)
  - In 1998, Superior Propane Inc. and ICG Propane Inc., the two largest distributors of propane in Canada contested the Canadian Commissioner of Competition's application to block a proposed merger on the basis of a substantial decrease in competition by presenting evidence that the merger would realise synergies that reduce cost by CAD29m per year. (Belleflamme and Peitz, 2010, pp. 381-382)

# Farrell and Shapiro (1990) Cournot Aggregate Welfare

- Parameters
  - Consider the Farrell and Shapiro (1990) model of merger in Cournot oligopoly. Pre-merger, each of $n$ firms indexed by $i \in \{1, 2, \ldots, n\}$ produces a homogenous good at common constant marginal cost $c$ and chooses quantity $q_i$ to maximise profit $\pi_i = [P(q_i + Q_{-i}) - c]q_i$ where $Q_{-i}$ is the total quantity of all other firms and $P(Q)$ is the inverse demand function. Suppose for simplicity that $P(Q) = 1 - Q$. $m$ insiders merge. Let $I$ denote the set of insiders and $O$ denote the set of outsiders.
- Analysis
  - Given $P(Q) = 1 - Q$, consumer surplus $CS = \frac{Q^2}{2}$. Outsiders' joint profit $\Pi_O = \sum_{i \in O} [P(Q) - c]q_i$. External (excluding insiders) welfare $E = CS + \Pi_O$.
  - It is natural to suppose that all proposed mergers increase insiders' joint profit. Suppose further that merger is price-increasing (since price-decreasing mergers straightforwardly benefit consumers and need not attract significant antitrust scrutiny).
  - Given $P(Q) = 1 - Q$, $Q = 1 - P$, hence $\frac{dQ}{dP} = -1$.
  - At equilibrium, outsider $i$ chooses quantity $q_i$ to maximise $\pi_i$ hence the first-order condition for $q_i$ $\frac{d\pi_i}{dq_i} = P(Q) + q_i P'(Q) - c = 0$ holds. $q_i = -\frac{P(Q)-c}{P'(Q)} = P(Q) - c = 1 - Q - c$ hence $\frac{\partial q_i}{\partial Q} = -1$.
  - $\frac{dE}{dQ} = \frac{dCS}{dQ} + \frac{d\Pi_O}{dQ} = Q + \sum_{i \in O}(q_i P'(Q) + [P(Q) - c]\frac{dq_i}{dQ})$ $\frac{dE}{dQ} = Q - \sum_{i \in O} q_i - \sum_{i \in O} q_i = Q_I - Q_O$, where $Q_I$ is insiders' joint output and $Q_O$ is outsiders' joint output.
- Result
  - A proposed price-increasing (and privately profitable) merger increases aggregate welfare if it increases external welfare iff $\frac{dE}{dP} = \frac{dE}{dQ}\frac{dQ}{dP} > 0$ iff $\frac{dE}{dQ} < 0$ iff $Q_I < Q_O$ iff $s_I < \frac{1}{2}$, where $s_I$ is the output share of insiders. Intuitively, (price-increasing) merger between firms with small market share is more likely to increase external welfare because (1) the increase in outsiders' margin due to insiders' competing more "soft" is enjoyed over large outsiders' quantity and hence results in a large increase in outsiders' profit and (2) the increase in price and decrease in quantity due to insiders' competing more "soft" is small hence the decrease in consumer surplus is small.
- Discussion
  - The Farrell and Shapiro (1990) aggregate welfare result above suggests that antitrust authorities (if they evaluate proposed mergers against an aggregate welfare standard) should permit mergers between firms with (joint) market share less than $50\%$ and block mergers between firms with greater market share.
- Discussion (Only Pre-Merger Data)
  - This merger screen is relatively undemanding in the sense that it requires only pre-merger data (although market definition is not straightforward).
- Evaluation (Privately Profitable Mergers)
  - The Farrell and Shapiro (1990) aggregate welfare result above relies on the assumption that only privately profitable mergers are proposed. This assumption is questionable because some empirical studies suggest that managers have non-profit motivations for merger. For example, it could be the case that managers are rewarded (financially or otherwise) for managing larger firms.

# Farrell and Shapiro (1990) Cournot Consumer Surplus

- Parameters
  - Consider the Farrell and Shapiro (1990) model of merger in Cournot oligopoly. Pre-merger, each of $n$ firms indexed by $i \in \{1, 2, \dots, n\}$ produces a homogenous good at constant marginal cost $c_i$ and chooses quantity $q_i$ to maximise profit $\pi_i = [P(q_i + Q_{-i}) - c_i]q_i$ where $Q_{-i}$ is the total quantity of all other firms and $P(Q)$ is the inverse demand function. Suppose that demand is downward sloping. Firm $1$ and firm $2$ merge. Suppose, without loss of generality, that $c_1 \leq c_2$.
- Analysis
  - Let $q_1^*$ and $q_2^*$ denote the pre-merger equilibrium quantities of firm $1$ and firm $2$ respectively. Let $Q^*$ denote the pre-merger equilibrium total output.
  - Pre-merger, at equilibrium, the first-order conditions for $q_1$ and $q_2$ hold. $\frac{\partial \pi_1}{\partial q_1}|_{q_1=q_1^*} = P(Q^*) - q_1^*P'(Q^*) - c_1 = 0$ and $\frac{\partial \pi_2}{\partial q_2}|_{q_2=q_2^*} = P(Q^*) - q_2^*P'(Q^*) - c_2 = 0$. Summing the two first-order conditions and rearranging, $[P(Q^*) - c_1] + [P(Q^*) - c_2] = P'(Q^*)[q_1^* + q_2^*]$.
  - Post-merger, compared to pre-merger, consumer surplus increases iff output increases since price necessarily decreases given downward-sloping demand iff insiders' joint output increases iff the merged firm's marginal profit is positive evaluated at pre-merger quantities, i.e. $\frac{\partial \pi_M}{\partial q_M}|_{q_M=q_1^*+q_2^*} = P(Q^*) - (q_1^* + q_2^*)P'(Q^*) - c_M > 0$. Rearranging, $P(Q^*) - c_M > (q_1^* + q_2^*)P'(Q^*)$. By substitution, $P(Q^*) - c_M > [P(Q^*) - c_1] + [P(Q^*) - c_2]$.
- Result
  - Post-merger, compared to pre-merger, consumer surplus increases iff the merged firm enjoys sufficiently large synergies such that, evaluated at the pre-merger equilibrium price, post-merger margin is greater than the sum of insiders' pre-merger margins. Farrell and Shapiro (1990) show that this result generalises to mergers between three or more firms. Intuitively, merger has two (first-order) effects on insiders' joint output, internalisation of negative externalities between insiders constitutes incentive to decrease output, cost-reducing synergies increase margin hence constitutes incentive to increase output. Therefore, insiders' joint output increases only if merger realises sufficiently large synergies such that the latter effect outweighs the former. In Cournot competition, outsiders respond to an increase in insiders' joint output by a smaller decrease in output, hence total output increases if insiders' joint output increases. Given downward-sloping demand, price decreases iff total output increases, hence consumer surplus increases if insiders' joint output increases.
- Discussion (Synergy Estimation Challenging)
  - In practice, antitrust authorities find it difficult to accurately estimate synergies realised in merger.
- Discussion (Large Synergies Unrealistic)
  - The condition that merger realises such large synergies is very demanding since this is unlikely in reality.

# Farrell and Shapiro (2010) Bertrand Consumer Surplus

- Parameters
  - Consider the Farrell and Shapiro (2010) model of merger in price-setting oligopoly. Pre-merger, each of $n$ firms indexed by $i \in \{1, 2, \dots, n\}$ produces a differentiated product at constant marginal cost $c_i$ and chooses price $p_i$ to maximise profit $\pi_i = (p_i - c_i)q_i$ where $q_i$ is firm $i$'s quantity. Demand for each product is downward sloping and products are mutual substitutes, i.e. $q_i$ increases with decreasing $p_i$ and with increasing $p_j$. Firm $1$ and firm $2$ merge.
- Analysis
  - Suppose that post-merger, each insider $i$ is made to internalise the negative externality imposed on the other by an internal Pigouvian tax of $T_i$. $T_1 = -\frac{\partial \pi_2}{\partial q_1}$ and $T_2 = -\frac{\partial \pi_1}{\partial q_2}$. The internal Pigouvian tax constitutes an increase in the effective marginal cost for each insider. An increase in effective marginal cost causes a decrease in effective margin hence constitutes incentive to increase price since the corresponding decrease in quantity results in a smaller decrease in profit. The internal Pigouvian tax can therefore be interpreted as the upward pricing pressure on each insider due to merger.
  - Evaluated at pre-merger prices, $T_1 = |\frac{\partial \pi_2}{\partial q_1}| = |\frac{\partial \pi_2}{\partial q_2}\frac{\partial q_2}{\partial q_1} + \frac{\partial \pi_2}{\partial p_2}\frac{\partial p_2}{\partial q_1}| = |\frac{\partial \pi_2}{\partial q_2}\frac{\partial q_2}{\partial q_1}| = (p_2 - c_2)D_{12}$ where $D_{12} = |\frac{\frac{\partial q_2}{\partial p_1}}{\frac{\partial q_1}{\partial p_1}}| = |\frac{\partial q_2}{\partial q_1}|$ is the diversion ratio from $1$ to $2$, since at the pre-merger equilibrium, the first-order condition for $p_2$ $\frac{\partial \pi_2}{\partial p_2} = 0$ holds. $D_{12}$ is the fraction of demand for product $1$ alienated by an increase in $p_1$ that is captured by product $2$ rather than lost to other products or lost entirely. By symmetry, $T_2 = (p_1 - c_1)D_{21}$.
  - Suppose that merger realises synergies such that the marginal cost of insider $i$ decreases by $E_i$, then the total effect of merger on insider $i$'s effective marginal cost is $T_i - E_i$, which can be interpreted as the net upward pricing pressure on each insider due to merger.
- Result

    - Whether some merger in a price-setting oligopoly is price-increasing (hence decreases consumer surplus) is ambiguous a priori, and depends on the relative magnitudes of upward pricing pressure and synergies. Upward pricing pressure is likely to be large if other insiders have large margin or if insiders' products are close substitutes such that the diversion ratio is high. Intuitively, the greater each insiders' margin and the greater the relevant diversion ratios, the greater the negative externalities between insiders hence the greater each insiders' incentive to decrease quantity and increase price post-merger.
- Discussion (Empirically Estimable)
    - The Farrell and Shapiro (2010) aggregate welfare result above suggests that antitrust authorities (if they evaluate proposed mergers against a consumer surplus standard) should evaluate proposed mergers by estimating upwards pricing pressure and synergies. The diversion ratio $D_{12}$ can be estimated as the fraction of consumers who switch to good $2$ of the consumers who stop buying good $1$ in response to an increase in the price of good $1$, which can be (to some extent) measured empirically. This screen is relatively undemanding in the sense that it does not require the antitrust authority to define the industry.
- Discussion (Partial Equilibrium Analysis)
    - Upward pricing pressure analysis is not a full equilibrium analysis because upward pricing pressure is evaluated at pre-merger prices (since only then is empirical estimation feasible). Upward pricing pressure analysis therefore does not account for the strategic effect of an increase in insiders' effective marginal costs. In price-setting oligopoly, assuming that prices are strategic complements, upward pricing pressure is likely to underestimate the increase in equilibrium price since outsiders respond to an increase in insiders' prices by increasing price, which further increases insiders' incentive to increase price.
- Case Study (Sainsbury's/Asda Merger)
    - In 2019, the U.K. CMA blocked the Sainsbury's/Asda merger on the basis of upward pricing pressure analysis. The CMA computed a "Gross (of synergies) Upward Pricing Pressure Index (GUPPI)" by estimating relevant margins and diversion ratios. Diversion ratios were estimated using surveys conducted in person at stores, where consumers were asked where they would have shopped instead.

# Collusion

- Result
    - Horizontal merger results in a decrease in the number of firms, hence a decrease in the critical discount factor, making collusion easier to sustain.
- Case Study (Nestle/Perrier)
    - In 1992, Nestle and Perrier, two of the three largest European bottled water producers, with $17\%$ and $36\%$ market share respectively proposed to merge. This merger was blocked by the European Commission because the merged firm would have enjoyed excessive market power. Nestle and Perrier then proposed to (additionally) sell Volvic, a Perrier subsidiary with $15\%$ market share to the primary outsider, BSN, which had $23\%$ market share, hence yielding an approximately balanced duopoly. This proposal was rejected by the European Commission because it feared this would facilitate tacit collusion. (Belleflamme and Peitz, 2010, pp. 392-393)

## Symmetry-Increasing Merger

- Parameters
    - Consider the dynamic Bertrand oligopoly. Suppose that each firm $i$ has some "market share" $s_i \in [0,1]$ such that if all firms choose the same price $p$, firm $i$'s quantity $q_i = s_i Q(p)$, where $\sum_i s_i = 1$ and $s_j < s_k$ for some $j$, $k$.
- Analysis
    - Consider the grim trigger strategy under which each firm chooses monopoly price iff no firm previously chose otherwise and price equal to marginal cost otherwise.
    - Suppose that in period $t$ no firm previously chose price not equal to monopoly price. If firm $i$ optimally deviates, it captures all demand at price arbitrarily below monopoly price, hence enjoys profit arbitrarily below monopoly profit. In all subsequent periods, all other firms choose price equal to marginal cost and firm $i$ makes at most zero profit. $\Pi_i = \pi^M$.
    - If firm $i$ does not deviate, then it captures $s_i Q(p^M)$ and enjoys its corresponding share of monopoly profit (which is unchanged since demand parameters and marginal cost are unchanged) in period $t$ and each subsequent period. $\Pi_i = s_i \pi^M + s_i \delta \pi^M + s_i \delta^2 \pi^M + \ldots = s_i \frac{\pi^M}{1-\delta}$.
    - No firm $i$ has incentive to deviate from the given strategy profile iff $\frac{s_i}{1-\delta} \geq 1$ iff $\delta \geq 1 - s_i$.
- Result

- Collusion is most likely to be sustainable when firms have similar market shares such that $\max_i 1 - s_i$ is minimised. Intuitively, small firms have greater incentive to deviate from collusive arrangements. This is because, where firms compete, small firms impose large externalities on other firms (collectively) since a decrease in price or increase in quantity by the small firm causes a decrease in margin over other firms' large (joint) output hence a large decrease in other firms' (joint) profit. Under collusion, small firms are made to account for these large externalities, and thus face significant restrictions. Hence, the benefit to small firms from deviation is greater than that to large firms.