REM

QE Problem Set 8

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1a $\Delta X_t = \beta_0 + \alpha t + \delta X_{t-1} + \sum_{i=1}^{p\Delta} \gamma_i X_{t-i} + u_t$
where $\Delta X_t := X_t - X_{t-1}$, ~~E(ut) E(ut)~~ $E(u_t) = 0$, $cov(u_t, t) = cov(u_t, X_{t-1}) = cov(u_t, \Delta X_{t-i})$ for $i \in \{1, \ldots, p\Delta\}$ by construction

Estimate the parameters of the regression by OLS,

$H_0: \delta = 0$
$H_1:$ ~~δ≠0~~ $\delta < 0$

calculate the t-statistic
$t(0) = \hat{\delta} / \hat{se}(\hat{\delta})$

under the null, $t(0) \xrightarrow{d} DF_{tr}$.
Reject the null if $t(0) <$ critical value c_α, where α is the level of significance, and c_α is such that ~~st~~ $\alpha = P(DF_{tr} < c_\alpha)$.

If we fail to reject the null, $\{X_t\}$ is a unit root autoregressive process, and the first difference of X_t can be described by an $AR(p\Delta)$ model. Assuming that this model is stationary, $X_t \sim I(1)$.
If we reject the null, $\{X_t\}$ is a (trend) stationary process, hence $X_t \sim I(0)$.

b Many of the candidate series for unit roots exhibit a deterministic trend as well as a stochastic trend. If a ~~deterministic trend~~ linear trend is not included in the regression, the case where the series exhibits a deterministic trend is included in the null ~~but not~~ and not the alternative. Thus, it is possible that we fail to reject the null of a unit root because of the presence of a deterministic drift in a series that is 'otherwise' stationary.

c Since the sampling distributions of $\hat{\gamma}_i$s are approximately normal in large samples, ~~the lag order of ADF regression can be deter~~ the familiar methods of statistical inference on $\hat{\gamma}_i$s are valid, and the lag order of ADF regression can be determined by a stepwise testing down procedure or the information criteria.

→ The disadvantage is that you have to use more negative critical values, giving the test a lower power.

d From table T, ~~that~~ the optimal lag order of the ADF regression, by the AIC, is 2. For this model, the test statistic for the hypothesis of a unit root, t-adf is −1.697. Assuming that the ADF regression includes a constant and linear trend, fail to reject the null hypothesis that Y_t is ~~a unit root autoregressive~~ a unit root autoregressive process at $\alpha = 5\%$, an autoregressive model is appropriate for ΔY_t.

From table ΔY, the optimal lag order of the ADF regression, by the AIC is 1. For this model, the test statistic for the hypothesis of a unit root, t-adf is -7.119. Assuming that the ADF regression includes a constant and linear trend, reject the null hypothesis that ΔY_t is a unit root autoregressive process at the 5% level of significance. ΔY_t is (trend) stationary. $Y_t \sim I(1)$.

From table C, the optimal lag order of the ADF regression, by the AIC is 3. For this model, the test statistic for the hypothesis of a unit root, t-adf is -1.859. Assuming that the ADF ~~reg~~ regression includes a constant and linear trend, fail to reject the null hypothesis that C_t is a unit root autoregressive process at the 5% level of significance. An autoregressive model is appropriate for ΔC_t.

From table ΔC, the optimal lag order of the ADF regression, by the AIC is 2. For this model, the test statistic for the hypothesis of a unit root, t-adf is -5.075. Assuming that the ADF regression includes a constant and linear trend, reject the null hypothesis that ΔC_t is a unit root autoregressive process at the 5% level of significance. ΔC_t is (trend stationary). $C_t \sim I(1)$.

e) i Compute $\varepsilon_t = Y_t - C_t$ and conduct an ADF test for a unit root in ε_t. If we reject the null, we conclude that $\varepsilon_t \sim I(0)$ and Y_t and C_t are cointegrated with cointegrating coefficient $\theta = 1$

ii Estimate θ by OLS, and compute $\hat{\varepsilon}_t = Y_t - \hat{\theta} C_t$ and conduct an ADF test for a unit root in $\hat{\varepsilon}_t$ using adjusted (Engle-Granger) critical values. If we reject the null, we conclude that $\varepsilon_t \sim I(0)$ and Y_t and C_t are cointegrated with ~~est~~ cointegrating coefficient $\theta = \hat{\theta}$.

What OLS regression is this? It seems qu complicated / obscure in the notes. Do need to know the details of this regre

No, this is what you need to know.

②

2a $H_0: \beta_1 = 0$
$H_1: \beta_1 \neq 0$
where β_1 is the coefficient ~~of~~ on π_{t-1} in the given regression.

t-statistic
$t(0) = \hat{\beta}_1 / \hat{se}(\hat{\beta}_1) = 0.84/0.04 = 21$

At level of significance $\alpha = 0.05$, critical value c_α, assuming that $t(0) \xrightarrow{d} N(0,1)$ under the null, is such that
$\alpha = 2\Phi(Z < -c_\alpha)$
$c_\alpha = 1.96$

$t(0) > c_\alpha$, the probability of observing a test-statistic as unfavourable to the null, under the null, as that actually observed, is vanishingly slim. Reject the null hypothesis that lagged inflation is useful in forecasting future inflation at the 5% level of significance.

It is appropriate to use critical values taken from the N(0,1) distribution if the test statistic ~~is~~ converges in distribution to N(0,1) under the null, which is the case if ~~\#~~ π_t is stationary and CLT holds.

b ~~\#~~ $\pi_{2017Q2|2017Q1}$ ~~= 0.68 + 0.84(0.02)~~ =
= 0.68 + 0.84(2) = 2.36%

Not sure about the biases

We might be concerned by the downward bias in the OLS estimate of β_1, which becomes particularly pronounced when β_1 is near unity.

c ~~St~~ Let δ_1 be the coefficient on π_{t-1} in the ADF regression (2).

~~t-stats~~ $H_0: \delta_1 = 0$
H_1: ~~$\delta_1 \neq 0$~~ $\delta_1 < 0$

t-statistic
$t(0) = \hat{\delta}_1 / \hat{se}(\hat{\delta}_1) = -0.107 / 0.041 = -2.6098$

Under the null, $t(0) \xrightarrow{d} DF_{cn}$. ~~Fail to reject, at the 5% level of significance~~ At the 5% level of significance, critical value c_α is such that ~~\#~~ $\alpha = P(DF_{cn} < c_\alpha)$, $c_\alpha = -2.86$.

Fail to reject, at the 5% level of significance, the null hypothesis that π_t is a unit root autoregressive process, hence that π_t is stationary. ~~Estimated~~ If π_t is a unit root autoregressive process, estimates of regression coefficients in (1) are biased. We thus have reason to think that (2) will yield more accurate ~~regression~~ forecasts of inflation.

d $H_0: \gamma_3 = 0$

$H_1: \gamma_3 \neq 0$

where γ_3 is the coefficient on $\Delta\pi_{t-3}$ in (2).

t-statistic

$t(0) = \hat{\gamma}_3 / \hat{se}(\hat{\gamma}_3) = 0.196 / 0.075 = 2.6133$

under the null, $t(0) \xrightarrow{d} N(0,1)$

At the level of significance $\alpha = 5\%$, critical value ~~c~~ c_α is such that

$\alpha = 2\Phi(\cancel{-|t|} - c_\alpha)$

$c_\alpha = -\Phi^{-1}(\alpha/2) = 1.96$

$t(0) > c_\alpha$. ~~Re~~ Reject the null hypothesis that $\Delta\pi_{t-3}$ ~~does not~~ is not useful in predicting $\Delta\pi_t$ at the 5% level of significance. $\Delta\pi_t$ does not follow an AR(3) model, which excludes $\Delta\pi_{t-3}$.

e The given claim suggests that the distributions of inflation in the sample period vary with time, in particular, that the distributions of inflation ~~to~~ during Volcker's tenure differs from the distribution before and the distribution after. The claim suggests two breakpoints in the series, one around the beginning of Volcker's tenure, and one around the end.

We can investigate the ~~claim~~ suggestion of a breakpoint around the beginning of Volcker's tenure by identifying some interval within which this breakpoint is plausible, computing the QLR using the Chow breakpoint statistics for the ~~times~~ periods in this interval, ~~to test the hypothesis of~~ to test the hypothesis that there is a breakpoint in this interval.

~~AA G~~ The hypothesis that there is a breakpoint around the end of Volcker's tenure can be tested by ~~stm~~ a similar approach.

What level of detail is appropriate for this question?

We set an AR(3) model for $\Delta\pi_t$, allowing for a break at any date in the range 1973Q3 - 1987Q2 (So we test for a structural break in inflation, that would happen as a result of a change in policy)

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3a $\{Y_t\}$ has a unit root iff $\beta_1+\beta_2=1$

b ΔY_t

$= Y_t - Y_{t-1}$

$= \beta_0+\beta_1 Y_{t-1}+\beta_2 Y_{t-2}+u_t - Y_t$

$= \beta_0+(\beta_1-1)Y_{t-1}+\beta_2 Y_{t-2}+u_t$

$= \beta_0 - \beta_2 Y_{t-1}+\beta_2 Y_{t-2}+u_t$

since $\beta_1+\beta_2=1$

$= \beta_0-\beta_2\Delta Y_{t-1}+u_t$

$\gamma_0=\beta_0,\ \gamma_1=-\beta_2$

c ~~Given that $\{\Delta Y_t\}$ is stationary, the order of integration of $\{Y_t\}$ is 1. The order of integration of a process $\{Y_t\}$ is defined as the smallest $d\in\{1,2,\ldots\}$ such that $\Delta^d Y_t$ is stationary~~

Given that $\{Y_t\}$ has a unit root $\{Y_t\}$ is non stationary. Given further that $\{\Delta Y_t\}$ is stationary, the order of integration of $\{Y_t\}$ is 1. The order of integration of a process $\{Y_t\}$ is defined as the smallest $d\in\{1,2,\ldots\}$ such that $\Delta^d Y_t$ is stationary.

So, y_t is not stationary but Δy_t is, so we can conclude that $y_t \sim I(1)$

d Given that ΔY_t is stationary, $E(\Delta Y_t)$ is time-invariant.

Let $\mu := E(\Delta Y_t)$ and $v_t := \Delta Y_t - \mu$.

Since ΔY_t is stationary and μ is a constant, v_t is stationary.

$E(v_t) = E(\Delta Y_t - \mu) = E(\Delta Y_t - E(\Delta Y_t)) = E(\Delta Y_t) - E(\Delta Y_t) = 0$

~~$Y_t = Y_{t-1}+\Delta Y_t$~~

$= Y_{t-1}+\Delta Y_t$

$= Y_{t-2}+\Delta Y_{t-1}+\Delta Y_t$

...

$= Y_0+\sum_{i=1}^{t}\Delta Y_i$

$= Y_0+\sum_{i=1}^{t}\mu+v_i$

$= Y_0+\mu t+\sum_{i=1}^{t}v_i$

where v_t is a mean-zero stationary process

μ

$= E(\Delta Y_t)$

$= E(\beta_0-\beta_2\Delta Y_{t-1}+u_t)$

$= \beta_0-\beta_2 E(\Delta Y_{t-1})$

since $E(u_t)=E(E(u_t|Y_{t-1},Y_{t-2},\ldots))=E(0)=0$

by linearity of expectations

$= \beta_0-\beta_2\mu$

since $\mu := E(\Delta Y_t)$ is time-invariant

$(1+\beta_2)\mu=\beta_0$

$\mu=\beta_0/(1+\beta_2)$

$v_t = \Delta Y_t - E(\Delta Y_t)$

d ~~$v_t = \Delta Y_t$~~

~~$= \Delta Y_t - \mu$~~

~~$= \beta_0 - \beta_2 \Delta Y_{t-1} + u_t - \beta_0/(1+\beta_2)$~~

?

~~Δv_t~~

~~$= v_t - v_{t-1}$~~

~~$= -\beta_2(\Delta Y_{t-1} - \Delta Y_{t-2}) + u_t - u_{t-1}$~~

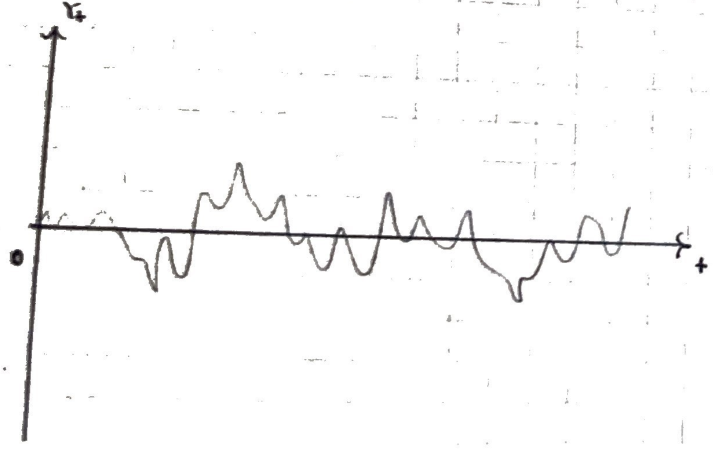
~~$= -\beta_2(Y_t - Y_{t-1} - (Y_{t-1}) - Y_{t-2}$~~

e $\sum_{s=1}^{t} v_s$ is a random walk iff v_t is a sequence of stationary uncorrelated random variables.

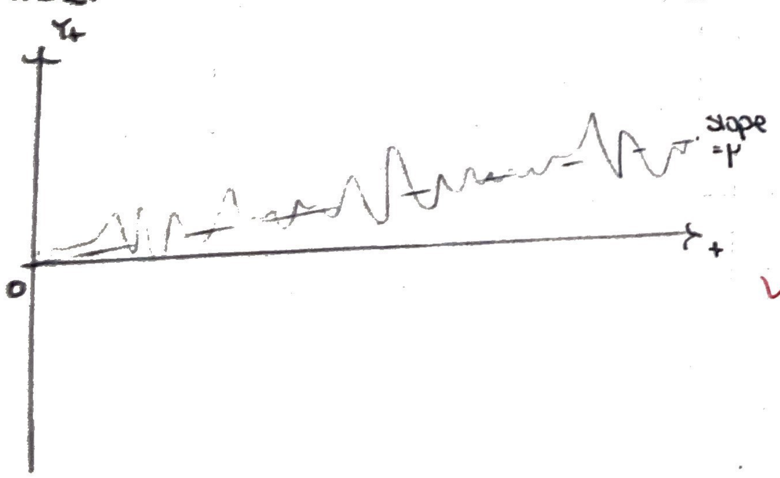
Given that ΔY_t is stationary, $E(\Delta Y_t)$ is time-invariant, hence $v_t = \Delta Y_t - E(\Delta Y_t)$ is stationary.

v_t is serially uncorrelated iff ΔY_t is serially uncorrelated, which is the case iff $\beta_2 = 0$. ✓

f If $\mu = 0$, Y_t is decomposed into an initial value and a stochastic trend, which is the partial sum of ~~a stationary process~~ mean-zero stationary process.



If $\mu \neq 0$, Y_t is decomposed into an initial value, a linear deterministic trend, and a stochastic trend.



✓

$$Y_t = \sum_{s=1}^{t} \Delta Y_s + Y_0$$

We are given that ΔY_t is a stationary process with mean:

$$\mu := E\,\Delta y_t = \frac{\gamma_0}{1-\gamma_1} = \frac{\beta_0}{2-\beta_1} = \frac{\beta_0}{1+\beta_2}$$

Setting $v_t := \Delta y_t - \mu$

gives:

$$y_t = \mu t + \sum_{s=1}^{t} (\Delta y_s - \mu) + y_0 = \mu t + \sum_{s=1}^{t} v_s + y_0$$

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②

a At the $\alpha = 5\%$ level of significance, the ADF critical values are -2.86 for the DF_μ distribution and -3.41 for the DF_τ distribution.

~~Reject the hypotheses that LUSUK$_t$ is stationary at t~~

At the 5% level of significance, fail to reject the null hypotheses that $LUSUK_t$ has a unit root and that ~~LUSE~~ $LUSEU_t$ has a unit root.

At the 5% level of significance, reject the null hypotheses that $\Delta LUSUK_t$ has a unit root and that $LUSEU_t$ has a unit root.

~~The~~ $LUSUK_t \sim I(1)$, $LUSEU_t \sim I(1)$

b At the 5% level of significance, fail to reject the null hypothesis that $LUSUK_t - LUSEU_t$ has a unit root, but reject the null hypothesis that $\Delta LUSUK_t - \Delta LUSEU_t$ has a unit root.
$(LUSUK_t - LUSEU_t) \sim I(1)$

c Since $LUSUK_t \sim I(1)$ and $LUSEU_t \sim I(1)$, ~~because~~ there is a tendency to find statistically significant regression relationships between the two series even when they are entirely independent because I(1) series have a stochastic trend, and thus a tendency to exhibit long swings of increase or decrease such that movements in one series appear to align with movements in the other. As a result, estimates of regression coefficients are not consistent for zero, t-statistic t(0) diverges in magnitude as the sample size grows and does not settle to any distribution, and R^2 does not converge to zero and will be high with a non-negligible probability. Because of spurious correlation between I(1) series, the high t-statistics on the regressors and high R^2 values do not indicate a real ~~descript~~ relationship between $LUSUK_t$ and $LUSEU_t$.

d Test for cointegration between $LUSEU_t$ and $LUSUK$ at the three horizons by estimating cointegrating coefficient θ by OLS, computing the estimated equilibrium error $\hat{\varepsilon}_t = LUSEU_t - \hat{\theta}\, LUSUK_{t-i}$ for $i \in \{1, 7, 21\}$, performing an ADF test of the null that $\hat{\varepsilon}_t$ has a unit root against the alternative that $\hat{\varepsilon}_t$ is stationary using adjusted Engle-Granger critical values. If the null is rejected, we conclude that ~~LUSUK$_t$ and the~~ $LUSEU_t$ and $LUSUK_{t-1}$ are cointegrating, the observed relationship is 'genuine', and $LUSUK_{t-i}$ is able to forecast $LUSEU_t$.

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