Predicate Logic Reading

Quine, 1986

Quine, W. V. (1986) Philosophy of Logic. London, England: Harvard University Press. pp. 61-70.

- Quine first considers whether predicate logic with identity is truly logic rather than mathematics.
- Reasons for considering predicate logic with identity as mathematics rather than logic include the following.
  - "Logical truths" of predicate logic with identity (that Quine refers to as truths of identity theory) are falsifiable by substituting the identity predicate with other predicates. In contrast, we think that (truly) logical truths are not so falsifiable. For example, substituting some sentence (not sentence letter) for another in propositional logic, does not falsify the truths of propositional logic. Such a substitution involves only considering a different "instantiation" of a sentence letter. Similarly, substituting some predicate (not predicate letter) or some constant (not constant term) for another in simple predicate logic does not falsify the truths of simple predicate logic. Again, such a substitution involves only considering different "instantiations" of predicate letters and constant terms (which schematise over the predicates and constants that instantiate them).
  - The contrast between generalities that can be expressed in the object language (English, for our purposes) and generalities that "call for" semantic ascent "marks a conspicuous and tempting place at which to draw the line between the other sciences and logic". The generality "all men are mortal" in English, generalises on "Tom is mortal", "Dick is mortal", and so on. This generalisation can be expressed in English. In contrast, generalisation on "Tom is mortal or Tom is not mortal" and "Snow is white or snow is not white" requires semantic ascent. We so generalise by saying "Every sentence of the form p or not p is true", this generalisation involves talk of the truth of sentences, while the earlier generalisation does not. Among the sentences generalised on, in the former case, it is names that are changed, whereas in the latter case, it is sentences or clauses that are changed. We generalise on "Tom is Tom", "Dick is Dick", and so on in English by saying "everything is itself". If the identity predicate is "proper to" logic, then it is unique in that its generalities can be expressed in English without semantic ascent.
- Reasons for considering predicate logic with identity as logic rather than mathematics include the following.
  - Godel proved that complete proof procedures are available for predicate logic with identity, but also that complete proof procedures are not available for elementary number theory.
  - Predicate logic with identity "treats all objects impartially", whereas mathematics "privileges" numbers, and set theory "privileges" sets.
  - A "serviceable facsimile" of identity can be constructed in a language with truth functions ($\neg$, $\rightarrow$, etc.) and quantification ($\forall$, $\exists$) and a finite lexicon of predicates. The idea is to construct a facsimile of $x = y$ as a sentence that "reads" x As iff y As, and Bs iff y Bs, ..., and for all a, x Ps a iff y Ps a, ..., and so on. This "simulation" of identity "means" that x and y are indistinguishable by any sentences that can be phrased in the language. If x and y are indeed distinct, then this simulation fails to capture such distinctness, but such failure is "unobservable from within the language".
    - In predicate logic, schematic predicate letters (F, G) are used in place of actual predicates ("is red", "is the parent of"). In the same spirit, the notation $x = y$ can be viewed schematically as standing in place of the compound sentence (in the object language, English), whose counterpart is the construct above.
- Quine then considers whether set theory is proper to logic or mathematics. Quine will conclude that set theory is proper to mathematics but not to logic.
  - Quine first clarifies the "language" of set theory.
- In simple predicate logic, F in Fx stands in place of a predicate (in the object language, English), it does not refer to a predicate, or to an attribute or set. Fx "simulates" a sentence in English, it is a sentence schema. F does not name an (unspecified) predicate. When we quantify over variables, the "sentence" (in the logical "meta" language) that follows the quantification ($\forall x$, $\exists x$) has x standing in place of some name. For example, in $\exists x(\mathrm{x\ walks})$, the thing that walks is the thing named by the name that x stands in the place of. "The quantifier '$\exists F$' or '$\forall F$' says not that some or all predicates are thus and so, but that some or all entities of the sort named by predicates are thus and so."
  - Most plausibly, such entities are attributes. To speak of attributes in logic is undesirable because for some predicate variable (like X and Y in Sider's presentation) to range over an attribute, we need to make some sense of what an individual attribute is, and what makes two attributes different attributes. Different attributes may be extensionally identical, so it is not the extension of an attribute (the things that have it) that makes an attribute the individual attribute that it is. Most plausibly, two attributes are the same iff they "mean the same thing", but it is not clear that we can make satisfactory sense of "meaning the same thing".

    - This is also undesirable because predicates are neither names of attributes nor names of sets. Predicates have attributes as their intension, and sets as their extension, but are names of neither. The predicate letter F, like the sentence letter P is not a value-taking variable, but a substitution-taking schematic letter. Second-order logic treats predicate letters as value-taking variables, that take the value of sets, rather than as substitution-taking schematic letters, that stand in place of predicates.
- Quine acknowledges a tendency to speak of sets and attributes "where logic in a narrower sense would have sufficed".
    - For example, we say "the same applies to Eisenhower" to save repeating a sentence. It is tempting to think that this means "the same attribute applies to Eisenhower". But this reference to attributes is not "made use of" and "easily paraphrased away", presumably by fully spelling out the sentence that would have been repeated.
    - Similarly, we say "it is an attribute of the born seaman to never quail at the fury of the gale". But this simply means "born seamen never quail at the fury of the gale".
    - Quine discusses the tendency to speak of sets where logic in a narrower sense would have sufficed. It is not clear that this is examinable.

Boolos, 1975

Boolos, G. S. (1975) "On Second-Order Logic," The Journal of Philosophy, 72(16), pp. 509-527.

- According to Boolos, Quine's argument for rejecting quantification of predicates is that predicates are not names (of either sets or attributes) and it is predicates that predicate letters stand in place of, but the variables (things like x of the logical language) that we quantify "belong in name positions", i.e. they stand in place of a name. Boolos's response to Quine, in summary, rejects the inference from our ordinarily quantifying over variables that "belong in name positions" to the impermissibility of quantifying over variables that "belong in predicate positions".
    - Again, Quine's argument is that ordinary quantification over variables as in "$\forall x : x \text{ walks}$" means not that all things that could occupy the position that "x" does walk, but that the things named by all things that could occupy this position walk. "Extraordinarily" quantifying over predicates as in "$\forall X : \text{Aristotle is } X$" treats X as standing in place of some name of something that Aristotle is. But "X" stands in the place of a predicate, not a name for some attribute (that Aristotle could have).
    - Boolos responds that to quantify over X is to treat X as having a range, but not necessarily to treat X as being a name for something. "Extraordinarily" quantifying over predicates in "$\forall X : \text{Aristotle is } X$" does not treat X as standing in place of a name for something that Aristotle is, but treats X as standing in place of a predicate with an extension which Aristotle is in.
    - Quine hastily supposes that quantification over predicate variables is necessarily exactly analogous to quantification over ordinary variables.
    - "$\exists X$ does not have to be taken as saying that some entities of the sort named by predicates are thus and so; it can be taken to say that some of the entities (extensions) had by predicates contain thus and such."
- Quine argues that the sentences of second-order logic are better expressed in set-theoretic notation. For example, Quine argues that we should write "$\forall \alpha : a \in \alpha$" rather than "$\forall X : Xa$". Quine's primary argument is apparently that this notation makes the set-theoretic presuppositions of second-order logic explicit.
    - One reason in favour of Quine's attribution of second-order logic to set theory rather than to logic is that in first-order logic, it need not be assumed that predicate letters like F have extensions in order to construct a "materially adequate" theory of truth.
    - Boolos objects on the ground that rewriting second-order formulas as Quine suggests can result in a loss of validity and implication.
        - $\exists X \forall x : Xx$ is valid but $\exists \alpha \forall x : x \in \alpha$ is not.
        - $\forall X : (Xa \to Xb)$ implies $a = b$, but $\forall \alpha : (a \in \alpha \to b \in \alpha)$ does not.
    - Quine alleges that the assumptions of set theory, that for example, that a set of F things exists (where F stands in place of a predicate) are "cunningly hidden" in second-order logic. $\exists \alpha \forall x : (x \in \alpha \leftrightarrow Fx)$ (which is controversial or problematic in set theory) is "disguised" as $\exists X \forall x : (Xx \leftrightarrow Fx)$ which trivially follows from the triviality $\forall x : (Fx \leftrightarrow Fx)$.
- Boolos argues that second-order logic is a natural extension of first-order logic. An interpretation remains an ordered pair consisting of some domain (whose definition is entirely unchanged) and an interpretation function (whose definition is also entirely unchanged). The valuation function is extended quite naturally, and the definitions of logical validity and semantic consequence are entirely unchanged.
    - That $\exists X \forall x : (Xx \leftrightarrow \neg x \in x)$ is valid means simply that it is true in all interpretations. But this does not imply that there is a set of all non-self-membered sets unless there is some interpretation whose domain is the set of all sets. But there is no reason to think a set of all sets exists (and apparently plenty of reason from set theory to think that no

    such set exists). So second-order logic is not committed to the existence of a set of all non-self-membered sets. Second-order logic does not simply obfuscate the "ontological cost", and is properly exempt from such cost.
    - This seems to show that "it is impermissible to use the notation of second-order logic in the formalisation of discourse about certain sorts of objects [...] in case there is no set to which all objects of that sort belong".
    - First-order logic is committed to the non-emptiness of the ontology of discourse. "But there is a striking difference between the commitment to non-emptiness and the commitment to sethood: we believe that our own ontology is non-empty, but not that it forms a set!"
    - "No valid second-order sentence can assert the same thing as a theorem of set theory."
- Boolos argues positively in favour of second-order logic on the ground of its greater "expressive capacity". It is possible to form a sentence in second-order logic that is true only in interpretations with infinite domains, and also to form a sentence in second-order logic that is true only in interpretations with countable domains. This is not possible in first-order logic.
    - Similarly, notions of progression, ancestral, and identity cannot be characterised in first-order logic.
    - First-order logic cannot consider as inconsistent some apparently inconsistent infinite sets of sentences, such as { 'Smith is an ancestor of Jones', 'Smith is not a parent of Jones', 'Smith is not a grandparent of Jones', ... }. But this sort of inconsistency seems to be the same sort of inconsistency as that in the set { 'There are fewer than three stars', 'Not: there are exactly no stars', 'Not: there is exactly one star', 'Not: there are exactly two stars' }. The reason for inconsistency in both cases seems to be the kind of reason "it has always been the business of logic to give".
- Boolos responds to Quine's argument against considering second-order logic to be logic on the ground that second-order logic is not complete.
    - First-order logic is not decidable, but some fragments of first-order logic are. This means that there exists an "effective method", like constructing a truth table for determining logical truths for fragments of first-order logic, like propositional logic. Such methods require rigour but not ingenuity, terminate after a finite number of steps, and are in some sense mechanical. Some monadic fragment of second-order logic is also decidable.
    - The definition of truth in an interpretation can be extended from first-order logic to second-order logic, but presumably not between other logics.
    - It is not clear why the "partial" standard of effectiveness, completeness, should be used to delineate between logic and mathematics (rather than the more "thorough" standard of effectiveness, decidability).
- Quine does not apparently think that completeness is sufficient reason for so delineating.
    - Quine cites a "remarkable concurrence of diverse definitions of logical truth" in the case of first-order logic but not second-order logic. One definition of logical truth is the usual definition of truth under all interpretations. Another definition of logical truth is truth in a reasonably rich object language (languages like English) under all substitutions of open sentences for simple sentences.
- Boolos argues that this "concurrence" occurs only for weak concurrence and not strong concurrence. The relevance of alternative definitions of truth (that Quine finds to concur with the usual definition in the case of first-order logic) is diminished by their failure to extend over the kindred logical relation of semantic consequence.

Shapiro, 1991

Shapiro, S. (1991) Foundations Without Foundationalism: A Case for Second-Order Logic. Oxford, England: Clarendon Press. pp. 79-133.