Philosophical Logic Paper 220530

1a i: Not: for all MPL-wffs ϕ $\models_T \Box\Box\phi \leftrightarrow \Box\phi$

Consider the following counterexample.

$\phi = P$
$M = \langle W, R, I \rangle$
$W = \{0, 1, 2\}$
$R = \{\langle 0,1 \rangle, \langle 1,2 \rangle, \dots\}$
The remaining pairs are those implied by reflexivity of R on W.
$I(P, 0) = I(P, 1) = 1$
$I(\alpha, \omega) = 0$ for all other sentence letters and worlds α, ω.

$V_M(\Box\Box\phi \leftrightarrow \Box\phi, 0) = 0 \implies$
$\not\models_T \Box\Box\phi \leftrightarrow \Box\phi$

ii For all MPL-wffs ϕ $\models_{S5} \Box\Diamond\Box\phi \leftrightarrow \Box\phi$

Consider arbitrary MPL-wff ϕ. Consider arbitrary S5-model M ~~and~~ $= \langle W, R, I \rangle$ and world $w \in W$.
Suppose for reductio
(1) $V_M(\Box\Diamond\Box\phi \leftrightarrow \Box\phi, w) = 0$
(1), $\leftrightarrow \implies$
(2): (3) or (4)
(3) $V_M(\Box\Diamond\Box\phi, w) = 1$ and $V_M(\Box\phi, w) = 0$
(4) $V_M(\Box\Diamond\Box\phi, w) = 0$ and $V_M(\Box\phi, w) = 1$

Suppose (3)
(3), $\Box \implies$
(5) $\forall u \in W, Rwu: V_M(\Diamond\Box\phi, u) = 1$
(6) $\cancel{\forall} u \in W, Rwu: V_M(\phi, u) = 1 \iff$
$\exists u \in W, Rwu: V_M(\phi, u) = 0$. Denote this world u_1
(5) $\implies$
(7) $V_M(\Diamond\Box\phi, w) = 1$
(7), $\Diamond \implies$
(8) $\exists u \in W, Rwu: V_M(\Box\phi, u) = 1$
(8), $\Box \implies$
(9) $\exists u \in W, Rwu: \forall v \in W, Ruv: V_M(\phi, v) = 1$
(9), symmetry $\implies$
(10) $V_M(\phi, u_1) = 1$
(6), (9), reductio $\implies$
(11) $V_M(\Box\Diamond\Box\phi \leftrightarrow \Box\phi, w) = 1$

Suppose (4)
~~(4), □ ⟹~~
~~(12) ∀u∈W: V_M(◇□φ, u) = 1 ⟺~~
~~∃u∈W: V_M(◇□φ, u) = 0~~
~~(13)~~
~~symmetry ⟹~~
~~(12) Rww and V_M(□φ, w) = 1~~
~~(12), ◇ ⟹~~
~~(13) V_M(◇□φ, w) = 1~~

(4), $\Box \implies$
(12) $\cancel{\forall} u \in W, Rwu: V_M(\Diamond\Box\phi, u) = 1 \iff$
$\exists u \in W, Rwu: V_M(\Diamond\Box\phi, u) = 0$. Denote this u_2
(13) $\forall u \in W, Rwu: V_M(\phi, u) = 1$
14 (12), $\Diamond \implies$
(14) $\forall u \in W, Ru_2u: V_M(\Box\phi, u) = 0$
(13), transitivity
(15) $\forall u \in W, Rwu: \forall v \in W, Ruv: \forall u' \in W: Rvu': V_M(\phi, u') = 1$
(15), $\Box \implies$
(16) $\forall u \in W, Rwu: \forall v \in W, Ruv: V_M(\Box\phi, v) = 1$
(16) $\implies$
(17) $\forall v \in W, Rwv: V_M(\Box\phi, v) = 1$
(14), (17), reductio $\implies$
(18) $V_M(\Box\Diamond\Box\phi \leftrightarrow \Box\phi, w) = 1$

(11), (18), cases
(19) $V_M(\Box\Diamond\Box\phi \leftrightarrow \Box\phi, w) = 1$
(19), generalisation, definition of $\models_{S5}$
(20) $\models_{S5} \Box\Diamond\Box\phi \leftrightarrow \Box\phi$

iii Not: for all MPL-wffs ϕ $\models_{S4} \Box\Diamond\Box\phi \leftrightarrow \Box\phi$

Consider the following counter~~model~~example.

~~A~~ $\phi = P$
$M = \langle W, R, I \rangle$
$W = \{0, 1\}$
$R = \{\langle 0,1 \rangle, \dots\}$
The remaining pairs are those ~~required~~ implied by reflexivity and transitivity
$I(P, 1) = 1$, $I(\alpha, \omega) = 0$ for all other sentence letters and worlds α, ω.

$V_M(\Box\Diamond\Box\phi \leftrightarrow \Box\phi, 0) = 0$ (because the LHS evaluates as true while the RHS evaluates as false)
$\implies \not\models_{S4} \Box\Diamond\Box\phi \leftrightarrow \Box\phi$

b: Base Case
$n = 0$. Suppose for conditional proof that $\models_K \phi \to \psi$. Then $\models_K \Diamond^0\phi \to \Diamond^0\psi$. ($= \phi \to \psi$). By conditional proof ~~for~~ if $\models_K \phi \to \psi$ then $\models_K \Diamond^0\phi \to \Diamond^0\psi$.

Induction Hypothesis
Given n, for all $m < n$, if $\models_K \phi \to \psi$ then $\models_K \Diamond^m\phi \to \Diamond^m\psi$

Induction Step.
Suppose for conditional proof that $\models_K \phi \to \psi$. By IH, $\models_K \Diamond^{n-1}\phi \to \Diamond^{n-1}\psi$. By definition of $\models_K$, for all K-models $M = \langle W, R, I \rangle$ and all worlds $w \in W$, $V_M(\Diamond^{n-1}\phi \to \Diamond^{n-1}\psi, w) = 1$. Consider arbitrary K-model $M = \langle W, R, I \rangle$ and world $w \in W$. Suppose for reductio that $V_M(\Diamond^n\phi \to \Diamond^n\psi, w) = 0$
Suppose for reductio

Then by $\rightarrow$, $V_M(\Diamond^n \phi, w)=1$ and $V_M(\Diamond^n \psi, w)=0$. By $\Diamond$, $\exists u \in W, Rwu : V_M(\Diamond^{n-1} \phi, u)=1$, denote this world u_1. Then ~~$V_M(\Diamond^n$~~ by $\rightarrow$ $V_M(\Diamond^{n-1} \psi, u_1)=1$. Then by $\Diamond$, $V_M(\Diamond^n \psi, w)=1$. By reductio, $V_M(\Diamond^n \phi \rightarrow \Diamond^n \psi, w)=1$. By generalisation, definition of $\vDash_K$, $\vDash_K \Diamond^n \phi \rightarrow \Diamond^n \psi$. By conditional proof, this holds if $\vDash_K \phi \rightarrow \psi$.

By induction, for all n, ~~$\vDash_K \phi$~~ if $\vDash_K \phi \rightarrow \psi$ then $\vDash_K \Diamond^n \phi \rightarrow \Diamond^n \psi$.

ii Base case
$n=0$. Consider arbitrary B-model $M=\langle W, R, I\rangle$ and world $w \in W$. Suppose for reductio that $V_M(\phi \rightarrow \phi, w)=0$. By $\rightarrow$, $V_M(\phi, w)=1$ and $V_M(\phi, w)=0$. By reductio, generalisation, definition of $\vDash_B$, $\vDash_B \phi \rightarrow \phi = \Diamond^0 \Box^0 \phi \rightarrow \phi$.

Induction Hypothesis
Given n, for all $m<n$, $\vDash_B \Diamond^m \Box^m \phi \rightarrow \phi$.

Induction Step.
Consider arbitrary B-model $M=\langle W, R, I\rangle$ and world w. By IH, definition of $\vDash_B$, ~~$V_M(\Diamond$~~ $V_M(\Diamond^{n-1} \Box^{n-1} \phi \rightarrow \phi, w)=1$ and for all $u \in W$, $V_M(\Diamond^{n-1} \Box^{n-1} \phi \rightarrow \phi, u)=1$, ~~then~~ and $V_M(\Diamond^{n-1} \Box^{n-1} \Box \phi \rightarrow \Box \phi, u)=1$. Suppose for reductio that $V_M(\Diamond^n \Box^n \phi \rightarrow \phi, w)=0$, then by $\rightarrow$, $V_M(\Diamond^n \Box^n \phi, w)=1$ and $V_M(\phi, w)=0$. By $\Diamond$, $\exists u \in W, Rwu : V_M(\Diamond^{n-1} \Box^n \phi, u)=1$. Denote this world u_1. Then by above ~~it~~ lemma, $V_M(\Box \phi, u_1)=1$. By symmetry of R on W, Ru_1w, then by $\Box$, $V_M(\phi, w)=1$. By reductio, ~~defin~~ generalisation, definition of $\vDash_B$, ~~$V_M(\Diamond^n$~~ $\vDash_B \Diamond^n \Box^n \phi \rightarrow \phi$.

By induction, $\vDash_B \Diamond^n \Box^n \phi \rightarrow \phi$ for all n.

c There are different ~~sorts~~ modalities that we could be interested in modelling ~~in a~~ with a MPL model. ~~#~~ These include Metaphysical modality, epistemic modality, doxastic modality, ~~and~~ deontic modality and temporal modality. It is ~~reason~~ reasonable to doubt the existence of a real metaphysical accessibility relation. ~~A meta~~ This would be a relation that holds between metaphysically possible worlds if some world is a metaphysical possibility in another. But we can be skeptical about the existence of metaphysically possible worlds or about the notion of some such world being metaphysically possible.

Room for skepticism diminishes if we are clear about what a possible world ~~in MPL~~ is (as MPL treats it). A possible ~~world, are we~~ under the possible worlds interpretation of MPL, is some ~~=~~ qualifiedly maximal entity according to which propositions or states of affairs obtain or do not obtain. It is an abstract sort of entity, ~~so to posit the existence of possible worlds is not to~~ and can be, to some extent, mentally constructed. So the possible worlds interpretation of MPL semantics does not posit the existence of metaphysically dubious entities.

Room for skepticism diminishes further if we consider other ~~forms of~~ modality. For temporal modality, the possible worlds are times and the accessibility relation is some temporal ordering. Admittedly, the existence of discrete times and a temporal ordering of them is philosophically controversial, but ~~within~~ within ~~the~~ this interpretation, there are some sorts of accessibility relations that are clearly appropriate and others that are clearly inappropriate. For example, that the accessibility relation should be antisymmetric is reasonable, that it should be euclidean is not. Similarly for deontic modality, it would be inappropriate to impose reflexivity (because we think wrongs are logically possible) but it is reasonable to impose serialness. (because we think morality is in some sense coherent or satisfiable).

This should extend naturally to metaphysical modality. The reason if any that a ~~the~~ "correct" or plausible metaphysical accessibility relation seems elusive is that the notions of metaphysical necessity and metaphysical possibility ~~often~~ are elusive and controversial.

2a $\phi \mathrel{\Diamond\!\!\rightarrow} \psi = \sim(\phi \mathrel{\Box\!\!\rightarrow} \sim\psi)$

Given LC model $M = \langle W, \preceq, I\rangle$,

$LV_M(\phi \mathrel{\Box\!\!\rightarrow} \psi, w) = 1$ iff ~~∄~~ (1) $\nexists u \in W: LV_M(\phi,u)=1$, or (2) $\exists u\in W: LV_M(\phi,u)=1$ and $\forall u' \in W, u' \preceq_w u$: $LV_M(\phi\to\psi, u')=1$.

~~This is~~ Then $LV_M(\phi\mathrel{\Box\!\!\rightarrow}\sim\psi, w)=1$ iff (1) $\nexists u\in W: LV_M(\phi,u)=1$ or (2) $\exists u \in W: LV_M(\phi,u)=1$ and $\forall u'\in W, u'\preceq_w u: LV_M(\phi\to\sim\psi,u')=1$.

Then $LV_M(\sim(\phi\mathrel{\Box\!\!\rightarrow}\sim\psi),w)=1$ iff (1) $\exists u\in W$: $LV_M(\phi,u)=1$ and (2) $\nexists u\in W: LV_M(\phi,u)=1$ and $\forall u'\in W, u'\preceq_w u: LV_M(\phi\to\sim\psi,u')=1$.

This is iff $LV_M(\Diamond\phi,w)=1$ and $\forall u\in W$: if $LV_M(\phi,u)=1$ then $\not\forall u'\in W, u'\preceq_w u: LV_M(\phi\to\sim\psi,u')=1$.

this is iff $LV_M(\Diamond\phi,w)=1$ and $\forall u \in W, LV_M(\phi,u)=1$: $\exists u'\in W, u'\preceq_w u: LV_M(\phi\to\sim\psi,u')=0$.

This is iff $LV_M(\Diamond\phi,w)=1$ and $\forall u\in W, LV_M(\phi,u)=1$: $\exists v\in W, v\preceq_w u: LV_M(\phi\wedge\psi,v)=1$. This is the ~~re~~ required result.

b i ~~SC~~ $P \mathrel{\Diamond\!\!\rightarrow} Q \vDash_{SC} P\mathrel{\Box\!\!\rightarrow} Q$

Consider arbitrary SC-model $M=\langle W,\preceq,I\rangle$, and world $w\in W$.
Suppose for conditional proof that
(1) $V_M(P\mathrel{\Diamond\!\!\rightarrow} Q, w)=1$
Suppose for reductio that
(2) $V_M(P\mathrel{\Box\!\!\rightarrow} Q, w)=0$
 (1), ~ ⇒
(3) $V_M(P\mathrel{\Box\!\!\rightarrow}\sim Q,w)=0$
 (2), □→ ⇒
(4) $V_M(Q,u)=0$, where u is the w-closest P-world, ~~which~~ i.e. the unique $u\in W$ such that $V_M(P,u)=1$ and for all $u'\in W$ such that $V_M(P,u')=1$, $u'\preceq_w u$.
 (3), □→ ⇒
(5) $V_M(\cancel{Q} \sim Q, u)=0$
 (5), ~ ⇒
(6) $V_M(Q,u)=1$
 (4), (6), reductio ⇒
(7) $V_M(P\mathrel{\Box\!\!\rightarrow} Q,w)=1$
 (7), conditional proof, generalisation, definition of $\vDash_{SC}$ ⇒
(8) $P\mathrel{\Diamond\!\!\rightarrow} Q \vDash_{SC} P\mathrel{\Box\!\!\rightarrow} Q$

ii $P\mathrel{\Diamond\!\!\rightarrow} Q \nvDash_{LC} P\mathrel{\Box\!\!\rightarrow} Q$

Consider the following SC-countermodel.

$W=\{0,1,2\}$
$\preceq_0 = \{\langle1,2\rangle,\langle2,1\rangle,\ldots\}$
The remaining pairs are those implied by reflexivity, transitivity, connectivity, and the LC base assumption.
$\preceq_1=\{\langle2,0\rangle,\ldots\}$
$\preceq_2=\{\langle1,0\rangle,\ldots\}$
~~I(P)~~ $I(P,1)=I(P,2)=I(Q,1)=1$
$I(\alpha,w)=0$ for all other sentence letters and worlds α, w.

$V^{LC}_M(\phi\mathrel{\Diamond\!\!\rightarrow}\psi,0)=1$, $V^{LC}_M(\phi\mathrel{\Box\!\!\rightarrow}\psi,0)=0$ ⇒ ~~⊭~~ $\phi\mathrel{\Diamond\!\!\rightarrow}\psi\nvDash_{LC}\phi\mathrel{\Box\!\!\rightarrow}\psi$.

ii $P\mathrel{\Box\!\!\rightarrow} Q, P\mathrel{\Diamond\!\!\rightarrow} R \vDash_{SC} (P\wedge R)\mathrel{\Box\!\!\rightarrow} Q$

Consider arbitrary SC model $M=\langle W,R,I\rangle$ and world $w\in W$.
Suppose for conditional proof that
(1) $V_M(P\mathrel{\Box\!\!\rightarrow} Q,w)=1$
(2) $V_M(P\mathrel{\Diamond\!\!\rightarrow} R,w)=1$
~~Suppose for reductio that~~
~~(3) $V_M((P\wedge R)\Box\!\!\rightarrow Q,w)=0$~~
 (2), ~ ⇒
(4) $V_M(P\mathrel{\Box\!\!\rightarrow}\sim R,w)=0$
 (4), □→ ⇒
(5) $V_M(\sim R,u)=0$, where u is the w-closest P world, ~~whose existence is~~ ~~that exists, by~~ that is, by □→, known to exist.
 (1), (5), □→ ⇒
(6) $V_M(Q,u)=1$, where u is the same as before
 (5), ~ ⇒
(7) $V_M(R,u)=1$
 (7), (8) ⇒
(9) u is also the w-closest $(P\wedge R)$ world
 (6), (9), □→ ⇒
(10) $V_M((P\wedge R)\mathrel{\Box\!\!\rightarrow} Q,w)=1$
 (10), conditional proof, generalisation, definition of $\vDash_{SC}$ ⇒
(11) ~~⊭~~ $P\mathrel{\Box\!\!\rightarrow} Q, P\mathrel{\Diamond\!\!\rightarrow} R\vDash_{SC}(P\wedge R)\mathrel{\Box\!\!\rightarrow} Q$

$P\mathrel{\Box\!\!\rightarrow} Q, P\mathrel{\Diamond\!\!\rightarrow} R\vDash_{LC}(P\wedge R)\mathrel{\Box\!\!\rightarrow} Q$

Consider arbitrary LC-model $M=\langle W,R,I\rangle$ ~~countermod~~ and world $w\in W$.
Suppose for conditional proof that
(1) $V_M(P\mathrel{\Box\!\!\rightarrow} Q,w)=1$
(2) $V_M(P\mathrel{\Diamond\!\!\rightarrow} R,W)=1$
~~Suppose for reductio that~~
~~(3) $V_M((P\wedge R)\Box\!\!\rightarrow Q,w)=0$~~
 (2), derived ◇→ ai ⇒
(4) $V_M(\Diamond P,w)=1$
(5) $\forall u\in W, V_M(P,u)=1$: $\exists v\in W, v\preceq_w u: V_M(P\wedge R,v)$
 (4), ◇ ⇒
(6) $\exists u\in W: V_M(P,u)=1$

(1), (6), $\Box\!\!\rightarrow$ $\Rightarrow$

(7) $\exists u \in W$: $V_m(P, u) = 1$ and $\forall v \in W, v \preceq_w u$: ~~$V_m$~~ $V_m(P \rightarrow Q, v) = 1$. Denote this world u_1

~~(7), (5) $\Rightarrow$~~

~~(8) $\forall v \in W$: $v \preceq_w u_1$: $V_m(P \rightarrow Q, v) = 1$ and $\exists v \in W$: $v \preceq_w u_1$: $V_m(P \wedge R, v) = 1$ Denote this world v_1~~

~~(8) $\Rightarrow$~~

~~(9) $V_m(P \wedge R, v_1) = 1$ and $V_m(P \rightarrow Q, v_1) = 1$~~

~~(9), $\wedge$, $\rightarrow$ $\Rightarrow$~~

~~(10) $V_m((P \wedge R) \rightarrow Q, v_1) = 1$~~

~~(3), $\Box\!\!\rightarrow$ $\Rightarrow$~~

~~(11)~~

(7) $\Rightarrow$

(8) $V_m(P, u_1) = 1$ and $\forall v \in W, v \preceq_w u_1$: $V_m(P \rightarrow Q, v) = 1$

(5), (8) $\Rightarrow$

(9) $\exists v \in W$: $v \preceq_w u_1$: $V_m(P \wedge R, v) = 1$. Denote this world v_1

(8), (9), transitivity of $\preceq_w$, $\rightarrow$

(10) $V_m(P \wedge R, v_1) = 1$ and $\forall v \in W, v \preceq_w v_1$: $V_m((P \wedge R) \rightarrow Q, v) = 1$

(10), $\Box\!\!\rightarrow$ $\Rightarrow$

(11) $V_m((P \wedge R) \Box\!\!\rightarrow Q, w) = 1$

(11), conditional proof, ~~definition of $\models$ generalisation~~ generalisation, definition of $\models_{LC}$ $\Rightarrow$

(12) $P \Box\!\!\rightarrow Q$, $P \Diamond\!\!\rightarrow R \models_{LC} (P \wedge R) \Box\!\!\rightarrow Q$

c Stalnaker's semantics for the might counterfactual appear inappropriate. Consider "if this is an excellent answer, then I might graduate with first class honours". This would be formalised as $E \Diamond\!\!\rightarrow F$. But according to SC, $E \Box\!\!\rightarrow F$ is a logical consequence of $E \Diamond\!\!\rightarrow F$. In fact, the two are semantically identical. But we do not think that "if this is an excellent answer, then I will graduate with first class honours" is a logical consequence of the English might counterfactual above. ~~#~~ It might be the case that I do terribly on the remaining exams. ~~So~~ $E \Box\!\!\rightarrow F$ is not a semantic consequence of $E \Diamond\!\!\rightarrow F$ on Lewis's semantics, so LC seems to do better here.

SC seems to fare worse here because of the antisymmetry assumption. It cannot accomodate cases where there are equally close, for example, excellent answer worlds (to the actual world), some in which I graduate with first class honours and some in which I dont.

Other candidate formalisations of the might counterfactual on SC fare no better. ~~#~~ These are $\Diamond(\phi \wedge \psi)$, $\Diamond(\phi \Box\!\!\rightarrow \psi)$, $\phi \Box\!\!\rightarrow \Diamond\psi$, and $\phi \Box\!\!\rightarrow \Diamond(\phi \wedge \psi)$ in every case, the formalisation ~~eve~~ gets the penny case wrong. Suppose I do not look in my pocket and there is no penny to be found. Intuitively "if I look in my pocket, I might find a penny" is true, but each candidate formalisation evaluates as false.

Stalnaker rejects Lewis's treatment of the English might counterfactual. When we say "if I had looked in my pocket, I might have found a penny", what we mean is "if I had looked in my pocket, I might, for all I know, have found a penny". The ~~sort of~~ modality of "might" is epistemic. So this is best formalised as $\Diamond^E(\phi \Box\!\!\rightarrow \psi)$, which presumably would yield the correct result.

This formalisation seems suspiciously distant from the English syntax, but we should be reassured because it is not entirely idiosyncratic. For example, we uncontroversially formalise "if he is ~~unmarried~~ a bachelor, then he must be unmarried" as $\Box(B \rightarrow U)$ rather than $B \rightarrow \Box U$ because we do not think a man's ~~happening to~~ being a bachelor means this is necessarily so. The English might counterfactual seems to work the same way. We seem to mean "it might, for all I know, be the case that if I look in my pocket I find a penny".

Stalnaker does not have to assume that "might" in such ~~cor~~ English counterfactuals has epistemic modality, he only requires that the modality of "might" is different from the modality of the counterfactual ~~or~~ conditional. Indeed the context sensitivity of English counterfactual conditionals gives us reason to think that the two could have different modalities. ~~In the exam case, for example, the "might" could have a sort of future-contingent modality whereas the counterfactual conditional has a metaphysical modality.~~ ~~The~~ In the exam case, the sorts of nearness in the might counterfactual that are relevant seem to be different than the sorts of nearness relevant that are relevant to the would counterfactual.

3ci $\phi \models_{S^*} \Delta\phi$

Consider arbitrary trivalent interpretation I. Consider arbitrary precisification C of I. Suppose for conditional proof that

(1) $SV^*_I(\phi)=1$

~~Suppose for reductio that~~

~~(2) $SV^*_I(\Delta\phi)=0$~~

(1), definition of SV^*_I ⇒

(3) ∀ precisifications C of I: $Vm_I(\phi, C)=1$

(3), ⇒

(4) ∀ C of I : ∀ C' of I, RCC': $Vm_I(\phi, C')=1$

(4), Δ ⇒

(5) ∀ C of I: $Vm_I(\Delta\phi, C)=1$

(5), SV^*_I ⇒

(6) $SV^*_I(\Delta\phi)=1$

(6), conditional proof, generalisation, definition of $\models_{S^*}$ ⇒

(7) $\phi \models_{S^*} \Delta\phi$

ii Not: if $\phi \models_{S^*} \psi$ then $\sim\psi \models_{S^*} \sim\phi$

Consider the following counterexample.

~~$\phi = \Delta P$ $\psi = P$~~ $\phi = P$, $\psi = \Delta P$. ~~$I(P) = \#$~~

$I(P)=\#$

$I(\alpha)=0$ for all other sentence letters α.

$SV^*_I(\sim\psi)=1$, $SV^*_I(\sim\phi)=\#$ ⇒ $\sim\psi \not\models_{S^*} \sim\phi$

From (ai), $\phi \models_{S^*} \psi$.

bi Suppose ϕ is not a PC-semantic consequence of Γ. Then there exists bivalent interpretation I such that for all $\gamma \in \Gamma$, $V_I(\gamma)=1$, and $V_I(\phi)=0$. Then there exists trivalent interpretation, namely I, such that for all precisifications, namely I, for all $\gamma\in\Gamma$, $V_I(\gamma)=1$ and $V_I(\phi)=0$, then for all $\gamma \in \Gamma$, $SV_I(\gamma)=1$ and $SV_I(\phi)=0$, then $\Gamma \not\models_S \phi$.

Suppose $\Gamma \not\models_S \phi$. Then there exists trivalent interpretation I such that for all $\gamma\in\Gamma$, $SV_I(\gamma)=1$, and $SV_I(\phi)\neq 1$. Then, there exists some ~~bivalent interpretation~~ precisification I^+ of I such that for all $\gamma\in\Gamma$, $V_{I^+}(\gamma)=1$, and $V_{I^+}(\phi)=0$. ~~Then~~ I^+ is a bivalent interpretation, so $\Gamma \not\models_{PC} \phi$.

By biconditional proof, $\Gamma \not\models_{PC}\phi$ iff $\Gamma\not\models_S\phi$, then $\Gamma\models_{PC}\phi$ iff $\Gamma\models_S\phi$.

ii Lemma: for all trivalent interpretations I, for all precisifications C of I, for all PC-wff (containing no Δ), $Vm_I(\phi, C) = V_C(\phi)$.

Base case.

Consider arbitrary ~~trivalent inter~~ PC-wff ϕ such that complexity $c(\phi)=0$. Then ϕ is some sentence letter α. $Vm_I(\phi, C) = K(\phi, C) = C(\alpha) = V_C(\alpha)$. ~~For all such ϕ,~~ By generalisation, for all such α, $Vm_I(\phi, C)=V_C(\phi)$.

Induction Hypothesis.

Given n, for all m<n, for all PC-wff ϕ such that $c(\phi)=m$, $Vm_I(\phi,C)=V_C(\phi)$.

Induction step

Consider arbitrary PC-wff ϕ such that $c(\phi)=n$. Then $\phi = \sim\psi$ or $\psi\to\chi$. Suppose $\phi=\sim\psi$. Then $Vm_I(\phi,C)=1$ iff $Vm_I(\psi,C)=0$ iff by IH ($c(\psi)=n-1$) $V_C(\psi)=0$ iff $V_C(\phi)=1$. Suppose $\phi=\psi\to\chi$. Then $Vm_I(\phi,C)=1$ iff $Vm_I(\psi,C)=0$ or $Vm_I(\chi,C)=1$ iff by IH ($c(\psi), c(\chi) < n$) $V_C(\psi)=0$ or $V_C(\chi)=1$ iff $V_C(\phi)=1$. By cases, generalisation, for all such ϕ, $Vm_I(\phi,C)=V_C(\phi)$.

By induction, for all PC-wff ϕ, $Vm_I(\phi,C)=V_C(\phi)$.

$SV^*_I(\phi)=1$ iff for all C of I, $Vm_I(\phi,C)=1$ iff for all C of I $V_C(I)=1$ iff $SV_I(\phi)=1$. $SV^*_I(\phi)=0$ iff for all C of I, $Vm_I(\phi,C)=0$ iff for all C of I, $V_C(I)=0$ iff $SV_I(\phi)=0$. $SV^*_I(\phi)=\#$ iff $SV^*_I \neq 1,0$ iff $SV_I(\phi)\neq 1,0$ iff $SV_I(\phi)=\#$. ~~So for all trivalent interpretat~~ By generalisation, for all trivalent interpretations I, for all PC-wff ϕ, $SV^*_I(\phi)=SV_I(\phi)$.

iii Lemma : $\Gamma \models_{S^*} \phi$ iff $\Gamma\models_S\phi$.

Suppose $\Gamma\not\models_{S^*}\phi$, then there exists trivalent I such that for all $\gamma\in\Gamma$, $SV^*_I(\gamma)=1$ and $SV^*_I(\phi)=0$, then there exists trivalent interpretation namely I such that for all $\gamma\in\Gamma$, $SV_I(\gamma)=1$ and $SV_I(\phi)=0$, then $\Gamma\not\models_S\phi$.

Suppose $\Gamma\not\models_S\phi$ then there exists trivalent I such that for all $\gamma\in\Gamma$, $SV_I(\gamma)=1$, and $SV_I(\phi)=0$, then there exists trivalent interpretation, namely I, such that for all $\gamma\in\Gamma$, $SV^*_I(\gamma)=1$, and $SV^*_I(\phi)=0$. then $\Gamma \not\models_{S^*}\phi$

By biconditional proof ~~So~~ $\Gamma\not\models_S\phi$ iff $\Gamma\not\models_{S^*}\phi$, so $\Gamma\models_S\phi$ iff $\Gamma\models_{S^*}\phi$.

c Supervaluationist semantics seem to fail when a definitely operator like Δ is ~~included~~ included. (But not ~~when~~ when such an operator is not included. From the result of (b), for such sentences, supervaluationist semantic consequence and validity exactly tracks the semantic consequence and validity, which is largely unproblematic). The two objectionable results ~~with~~ of supervaluationist semantic consequence with the determinately operator are (1) that ~~defini~~ $\phi \models \Delta\phi$ and (2) that not: if $\phi \models \psi$ then $\sim\psi \models \sim\phi$.

(1) is problematic because we do not think, for example that "Middling Mary is definitely rich" is a semantic consequence of "Middling Mary is rich" where Middling Mary is a vague, borderline, indefinite case.

(2) is problematic because we think ~~t~~ semantic consequence should obey contraposition, ~~i.e. that~~ ~~if~~ for example, if "all men are mortal and Socrates is a man" logically implies "Socrates is mortal, then "Socrates is not mortal", we intuitively think, logically implies "not-all men are mortal and Socrates is a man".

The ~~pr~~ superficial problem with (1) rests on a misunderstanding of supervaluationism. That $M \models \Delta M$ means not that if Middling Mary is rich then, logically, she is definitely rich, but that if Middling Mary is rich on all sharpenings, i.e. it is supertrue that Middling Mary is rich then it is supertrue that Middling Mary is definitely rich. This is not entirely intuitive because our intuitions will remain mostly silent on such "second-order" notions as supertruth, but is not straightforwardly problematic.

The deeper problem with (1) is that $M \models \Delta M$ but not $\models M \to \Delta M$ because ~~on some sharpenings~~ where M is a borderline case, on some ~~e~~ sharpenings M is true but ΔM is false. But we think that if ~~M logically~~ "Middling Mary is rich" logically implies "Middling Mary is definitely rich", then ~~"Middling M~~ the former materially (which is weaker than logically) implies the latter.

With a determinately operator, we must take seriously the fact that supervaluationism deals in supertruth and not simple truth. Then, it does not really "say" anything about ordinary logical inferences.