Oligopoly Notes

Bertrand Model

Static Bertrand Model

- Parameters
  - Consider the static Bertrand duopoly. Each of two firms indexed by $i \in \{1, 2\}$ produces a homogenous good at common, constant marginal cost c, and chooses price p_i to maximise $\pi_i = (p_i - c)q_i$, where q_i is the quantity of firm i. Demand is downward-sloping and consumers buy only from the firm(s) with the lowest price.
- Analysis
  - The unique Nash equilibrium is the strategy profile under which each firm i chooses $p_i = c$ and enjoys $\pi_i = 0$. If, instead, $p_i > c$ and $p_j = c$ for $i \neq j$, then $q_i = 0$ hence $\pi_i = 0$. If, instead, $p_i < c$ and $p_j = c$ for $i \neq j$, then $(p_i - c) < 0$ and $q_i > 0$ hence $\pi_i < 0$. No firm i has incentive to deviate from this strategy profile. Under any other strategy profile, either $p_i < c$ for some i or $p_i > c$ for some i. In the former case, a lowest-price firm has negative margin and positive output, hence negative profit. A lowest-price firm can increase profit by choosing price equal to marginal cost. In the latter case, either a lowest-price firm can increase profit by increasing price to just undercut a highest-price firm, or a highest-price firm can increase profit by decreasing price to just undercut a lowest-price firm.
- Result (Efficiency)
  - Bertrand competition is allocatively efficient and productively efficient. Under the unique Nash equilibrium, all firms choose price equal to marginal cost, the valuation of the marginal consumer, given by the price, is equal to the marginal cost of production. Social welfare, given by the excess of consumers' valuations over producers' costs across all units of output, is maximised. Even under asymmetric Bertrand competition, at most one firm enjoys positive profit, which is modest if cost differences are small.
  - Under the unique Nash equilibrium, only firms with minimum marginal cost produce and sell any output.
  - Bertrand competition between two symmetric firms yields the perfectly competitive result.
- Result (Collusion Incentive)
  - Under Bertrand competition, firms have strong incentive to collude. Under the unique Nash equilibrium, both firms enjoy zero profit. Each firm would be better off if both instead choose a higher price. Bertrand competition has the structure of a prisoner's dilemma where the Nash equilibrium outcome is Pareto-dominated by another outcome which cannot be sustained in equilibrium. Each firm chooses price without accounting for the externalities imposed on competitors, hence aggregate profit (and individual profit given symmetry) is not maximised in equilibrium.
- Evaluation (Result)
  - The Bertrand model of oligopoly yields the implausible "knife-edge" result that competition between few firms yields a perfectly competitive outcome where firms are entirely unable to exercise market power to increase price above marginal cost and extract profit.
  - Even under asymmetric Bertrand competition, at most one firm enjoys positive profit, which is modest if cost differences are small.
- Evaluation (Assumptions)
  - The Bertrand model of oligopoly assumes that firms produce homogenous products, production is not subject to capacity constraints, and competition between firms is static rather than dynamic, none of these assumptions is particularly realistic.
- Case Study (Nynex)
  - In 1987, Nynex (monopolist) sold telephone directories on CD (relatively homogenous product) for USD 10,000 (monopoly price). Following the entry of competitor, Pro CD, price for telephone directories on CD fell to <USD 1,000 (much closer to marginal cost). (Cabral, 2017, pp. 256-257)
- Case Study (Nintentdo)
  - In 1999, Nintendo cut prices for its game console to match Sony within an hour of the Sony price cut. (Cabral, 2017, p. 273)

Sequential Bertrand Model

- Parameters

- Consider the sequential Bertrand duopoly. Each of two firms indexed by $i \in \{1, 2\}$ produces a homogenous good at common constant marginal cost c. Firm 1 chooses price p_1 then firm 2 chooses price p_2 with knowledge of p_1. Each firm i's payoff is given by $\pi_i = (p_i - c)q_i$ where q_i is the quantity of firm i. Demand is downward-sloping and consumers buy only from the firm(s) with the lowest price.

- Analysis
  - Any strategy profile such that firm 1 chooses some $p_1 \geq c$ and firm 2 chooses price p_2 equal to monopoly price p^M if $p_1 > p^M$, p_2 arbitrarily below p_1 if $p^M \geq p_1 > c$ and $p_2 = c$ otherwise is a Nash equilibrium. Under any such strategy profile, firm 1 enjoys zero profit while firm 2 enjoys weakly positive profit.
  - Suppose that firm 1 entertains some small but non-zero probability that firm 2 mistakenly sets $p_2 > p_1$. Then the unique Nash equilibrium is such that firm 1 chooses $p_1 = p^M$ to maximise expected profit (which is always non-positive if firm 2 does not make a mistake) and firm 2 chooses p_2 arbitrarily below $p_1 = p^M$.
- Evaluation (Assumptions)
  - The sequential Bertrand model appears unrealistic. In reality, prices are (relatively) easily adjusted, hence it is unlikely that a first-mover exists, which cannot change its price in response to its competitors.

Dynamic Bertrand Model

- Parameters
  - Consider the dynamic Bertrand oligopoly. Each of n firms indexed by $i \in \{1, 2, \ldots, n\}$, in each period $t \in \{1, 2, \ldots\}$ of an infinite number of periods produces a homogenous good at common constant marginal cost c and chooses price p_{it} to maximise discounted sum of profits $\Pi_i = \pi_{i1} + \delta\pi_{i2} + \ldots$ where δ is the discount rate and π_{it} is the profit of firm i in period t equal to $(p_{it} - c)q_{it}$ where q_{it} is the output of firm i in period t. In each period, firms compete in prices, demand is downward-sloping, and consumers buy only from the lowest-price firm(s).
- Analysis
  - The strategy profile under which each firm chooses price equal to marginal cost in each period is a subgame-perfect equilibrium. By the result of the static Bertrand game, no firm has incentive to deviate in any period.
  - The strategy profile under which each firm plays the grim trigger strategy whereby it chooses price equal to the monopoly price $p^M \equiv \arg\max_p (p - c)Q(p)$ where $Q(p)$ is the industry demand function, if no firm previously chose otherwise, and price equal to marginal cost otherwise, is a subgame-perfect equilibrium for sufficiently large discount factor δ.
  - Suppose that in period t no firm previously chose price not equal to monopoly price. Firm i optimally deviates from the given strategy profile by choosing price arbitrarily below monopoly price. Firm i captures all demand at this price, and enjoys profit arbitrarily below monopoly profit $\pi^M \equiv \max_p (p - c)Q(p)$. In all subsequent periods, all other firms choose price equal to marginal cost. By the result of the static Bertrand game, the deviating firm optimally responds by choosing price equal to marginal cost. The deviating firm enjoys zero profit in all subsequent periods. $\Pi_i = \pi^M + \delta(0) + \delta^2(0) + \ldots = \pi^M$ (re-labelling period t as period 1, since past-period profits are unchanged by present-period and future-period outcomes).
  - If firm i does not deviate from the given strategy profile, it captures its symmetrical share of demand, and enjoys its symmetrical share $\frac{\pi^M}{n}$ of monopoly profit in period t and all subsequent periods. $\Pi_i = \frac{\pi^M}{n} + \frac{\delta\pi^M}{n} + \frac{\delta^2\pi^M}{n} + \ldots = \frac{1}{1-\delta}\frac{\pi^M}{n}$.
  - No firm i has incentive to deviate from the given strategy profile iff $\frac{1}{1-\delta}\frac{\pi^M}{n} \geq \pi^M$, $\frac{1}{1-\delta} \geq n$, $1 - \delta \leq \frac{1}{n}$, $\delta \geq \frac{n-1}{n} \equiv \bar{\delta}$, i.e. the discount factor exceeds the critical discount factor $\bar{\delta}$.
- Result (Concentration and Collusion)
  - Collusion is easier to sustain in concentrated markets. By inspection, $\uparrow n \Rightarrow \uparrow \bar{\delta}$, i.e. as the number of firms increases, the critical discount factor increases, and the less likely it is that collusion is sustainable. Intuitively and anecdotally, we expect collusion to be easier to sustain in more concentrated markets because the more concentrated the market, the larger each firm, hence the smaller the externality each firm imposes on all other firms in choosing a low price and the less demanding the restriction faced by each firm when it is made to account for these externalities to maximise joint profit under collusion.
- Result (Symmetry and Collusion)
  - Collusion is easier to sustain among similar firms. Where firms are asymmetric, and one firm has lower marginal cost than each of the other firms, aggregate profit is maximised when the lowest-cost firm produces and sells at its monopoly price, and all other firms choose a higher price, sell no output, and enjoy zero profit. This is not sustainable in equilibrium because each of the higher-cost firms has incentive to undercut the lowest-cost firm. The lowest-cost firm cannot effectively retaliate since it cannot inflict negative profit on other firms.

- Under a collusive equilibrium, if the lowest-cost firm deviates, it enjoys high profit in the period that it deviates, and non-zero profit under the "punishment" phase. The lowest-cost firm has greatest incentive to deviate.
- If firms are asymmetric, collusion is conditional on all firms agreeing on profit distribution, but no "focal equilibrium" suggests itself since the symmetry rule is inappropriate.

- Result (Unilateral Price Wars)
  - If it is allowed that the discount rate varies between firms, collusion is sustainable only if no firm has incentive to deviate, i.e. if all firms have sufficiently high discount rates. Some firms may be more impatient than others, for example, because of financial distress hence a greater probability of bankruptcy and a greater need for immediate cash. If one firm is sufficiently impatient such that, for this firm, incentive to deviate exceeds incentive to collude, this firm deviates and unilaterally undermines the collusive arrangement.
- Evaluation (Over-prediction)
  - The dynamic Bertrand model appears to over-predict the occurrence of collusion. Given conservative (low) $\delta = 0.9$, tacit collusion is sustainable in a 10-firm oligopoly. Anecdotally, this result is unrealistic.
- Evaluation (Renegotiation)
  - The dynamic Bertrand model assumes that firms credibly commit to not renegotiate the collusive arrangement after some firm deviates, such that the "punishment" phase is indefinite. Given that each firm is better off under the collusive phase than under the "punishment" phase, each firm has incentive to renegotiate after one firm deviates. The possibility of renegotiation undermines the "punishment" for deviation, hence undermines the sustainability of collusion.

Cournot Model

Static Cournot Model

- Parameters
  - Consider the static Cournot oligopoly. Each of n firms indexed by $i \in \{1, 2, \ldots, n\}$ produces a homogenous good at constant marginal cost c_i and chooses output q_i to maximise profit $\pi_i = [P(q_i + Q_{-i}) - c_i]q_i$ where $Q_{-i} = \sum_{i \neq j} q_j$ is the total output of all other firms. Demand is downward-sloping. Suppose for simplicity that inverse demand is given by $P(Q) = a - bQ$.
- Analysis
  - Each firm i's best response, $q_i^\dagger$ is such that the first-order condition for q_i holds, i.e. $\frac{\partial \pi_i}{\partial q_i}|_{q_i = q_i^\dagger} = [P(q_i^\dagger + Q_{-i}) - c_i] + q_i^\dagger P'(q_i^\dagger + Q_{-i}) = 0$, $q_i^\dagger = -\frac{[P(q_i^\dagger + Q_{-i}) - c_i]}{P'(q_i^\dagger + Q_{-i})} = \frac{a - bq_i^\dagger - bQ_{-i} - c_i}{b}$.
  - At Nash equilibrium, each firm chooses its best response. Let q_i^* denote the Nash equilibrium output of firm i. $q_i^* = \max\{\frac{a - bQ^* - c}{b}, 0\}$ where $Q^* = \sum_{j=1}^n q_j^*$.
  - Suppose that firms have common constant marginal costant marginal cost c, i.e. $c_i = c$ for all i. Then by symmetry, $q_i^* = q^*$ for all i and Nash equilibrium aggregate output $Q^* = nq^*$. Suppose further that $a > c$ such that equilibrium aggregate (and individual) output is non-zero. Then $q^* = \frac{a - bnq^* - c}{b}$, $q^* = \frac{a - c}{(n+1)b}$.
  - $Q^* = nq^* = \frac{n}{n+1}\frac{a-c}{b}$, Nash equilibrium price $P^* = P(Q^*) = a - bQ^* = \frac{a}{n+1} + \frac{nc}{n+1}$, Nash equilibrium (individual) profit, $\pi^* = (P - c)q^* = \frac{a-c}{n+1}\frac{a-c}{(n+1)b} = \frac{(a-c)^2}{(n+1)^2 b}$, Nash equilibrium (aggregate) profit, $\Pi^* = n\pi^* = \frac{n(a-c)^2}{(n+1)^2 b}$.
  - Suppose that $n = 2$ and $c_1 < c_2$. Then, at Nash equilibrium, by substitution into the best response function above, $q_1^* = \frac{a - bq_1^* - bq_2^* - c_1}{b} = \frac{a - bq_2^* - c_1}{2b}$ and $q_2^* = \frac{a - bq_1^* - bq_2^* - c_2}{b} = \frac{a - bq_1^* - c_2}{2b}$. By substitution, $q_2^* = \frac{a - c_2}{2b} - \frac{a - bq_2^* + c_1}{4b} = \frac{a - 2c_2 + c_1}{4b} + \frac{q_2^*}{4}$, $q_2^* = \frac{a - 2c_2 + c_1}{3b}$.
- Result (Efficiency)
  - Cournot competition is allocatively inefficient and productively inefficient. Under the unique Nash equilibrium, if any firm produces non-zero output, price exceeds marginal cost (by inspection of firms' best response function). The valuation of the marginal consumer, given by the price, exceeds the marginal cost of production. The good is underproduced.
  - Under the unique Nash equilibrium, firms with marginal cost greater than the lowest marginal cost may produce non-zero output. By inspection of q_2^* in the case of asymmetric duopoly, $q_2^* = 0$ only if industry demand is small, i.e. a is small, or if there is a significant difference in cost between the two firms, i.e. c_1 is significantly lower than c_2.
- Result (Concentration and Efficiency)
  - In the limit as the number of firms becomes large, the Cournot Nash equilibrium outcome converges to the efficient perfectly competitive outcome. By inspection, in the symmetric Cournot model, as $n \to +\infty$, $Q^* \to \frac{a-c}{b}$, $P^* \to c$, $\pi^* \to 0$, and $\Pi^* \to 0$. At $P = c$, consumers' valuation of the marginal unit of output, which is equal to price, is equal to the marginal cost of production, hence social welfare is maximised under this outcome.

- Result (Collusion Incentive)
  - Firms in Cournot competition have incentive to collude by restricting output to increase price hence profits. Consider the monopolist facing the same demand and cost conditions that chooses q to maximise $[P(q)-c]q$. It can be shown that monopoly output $q^M = \frac{a-c}{2b}$ and monopoly profit $\pi^M = \frac{(a-c)^2}{4b}$. If each firm in the symmetric Cournot oligopoly produces its share of monopoly quantity, $\frac{q^M}{n}$, aggregate profit is equal to monopoly profit π^M, and by symmetry, individual firm profit $\frac{\pi^M}{n} = \frac{(a-c)^2}{4nb} > \pi^* = \frac{(a-c)^2}{(n+1)^2 b}$ for $n > 1$.
  - Cournot competition has the structure of a prisoner's dilemma where the Nash equilibrium outcome is Pareto-dominated by another outcome which cannot be sustained in equilibrium. Each firm chooses output without accounting for the externalities imposed on competitors, hence aggregate profit (and individual profit given symmetry) is not maximised in equilibrium.
- Result (Size and Cost)
  - Firm size distribution at the Cournot equilibrium is a reflection of underlying differences in (cost) efficiency of competing firms. By inspection, $\downarrow c_i \Rightarrow \uparrow q_i^*$.
  - The higher a firm's cost, the smaller its gross margin, hence the less incentive it has to increase output.
- Result (Margin and Volume)
  - Gross margin of each firm at the Cournot equilibrium is directly proportionate to its volume (revenue) share.
  - Let $L_i \equiv \frac{p-c_i}{p}$ denote firm i's Lerner index, which is equal to firm i's gross margin as a fraction of price, $s_i \equiv \frac{q_i}{Q}$ denote firm i's volume (equivalently revenue, under Cournot competition) share, and $\epsilon \equiv \frac{\partial Q}{\partial P}\frac{P}{Q}$ denote the price-elasticity of demand. At Cournot equilibrium, $q_i^* = -\frac{P(Q^*)-c_i}{P'(Q^*)}$. Multiplying throughout by $\frac{1}{Q^*}\frac{Q^*}{P}$, $\frac{q_i^*}{Q^*}\frac{Q^*}{P(Q^*)} = -\frac{P(Q^*)-c_i}{P(Q^*)P'(Q^*)}$. Rearranging, $\frac{q_i^*}{Q^*}|\frac{Q^*P'(Q^*)}{P(Q^*)}| = \frac{P(Q^*)-c_i}{P(Q^*)}$, $L_i = \frac{s_i}{|\epsilon|}$.
  - Under Cournot competition, all firms sell at the same price, hence differences in gross margin are determined by differences in cost. The higher a firm's cost, the smaller its gross margin, hence the less incentive it has to increase output.
- Result (Margin and Concentration)
  - Under Cournot competition, average (across firms) profitability is directly proportionate to the Herfindahl index of concentration.
  - Let $L \equiv \sum_{i=1}^n s_i L_i$ denote the aggregate Lerner index, which gives the volume-weighted average gross margin, $H = \sum_{i=1}^n s_i^2$ denote the Herfindahl index of concentration, and $\epsilon \equiv \frac{\partial Q}{\partial P}\frac{P}{Q}$ denote the price-elasticity of demand. At Cournot equilibrium, $L \equiv \sum_{i=1}^n s_i L_i = \sum_{i=1}^n \frac{s_i^2}{|\epsilon|} = \frac{H}{|\epsilon|}$.
- Evaluation (Result)
  - Cournot competition between few firms yields results intermediate between monopoly and perfectly competitive outcomes. Intuitively and anecdotally, such results are expected.
- Evaluation (Assumptions)
  - Cournot competition appears unrealistic. In reality, firms choose prices, and it is not necessarily the case that all firms choose the same market-clearing price, given total industry output.

Stackelberg Model (Sequential Cournot Model)

- Parameters
  - Consider the Stackelberg duopoly with linear demand and symmetric costs. Each of two firms indexed by $i \in \{1, 2\}$ produces a homogenous good at common, constant marginal cost c. Firm 1 chooses output q_1 then firm 2 chooses output q_2 with knowledge of q_1. Each firm i's payoff is given by $\pi_i = [P(q_i + q_{-i}) - c]q_i$ where $P(Q) = a - bQ$ is the inverse demand function and $Q = q_i + q_{-i}$ is the total output.
- Analysis
  - Firm 2's best response, $q_2^\dagger$ is such that the first-order condition for q_2 holds, i.e. $\frac{\partial \pi_2}{\partial q_2}|_{q_2=q_2^\dagger} = [P(q_2^\dagger + q_1) - c] + q_2^\dagger P'(q_2^\dagger + q_1) = 0$, $q_2^\dagger = -\frac{[P(q_2^\dagger+q_1)-c]}{P'(q_2^\dagger+q_1)} = \frac{a-bq_2^\dagger-bq_1-c}{b}$. $q_2^\dagger = \frac{a-bq_1-c}{2b}$.
  - Given common knowledge of rationality, firm 1 rationally expects that firm 2 chooses $q_2 = q_2^\dagger$. Therefore, firm 1's profit function, $\pi_q(q_1) = [P(q_1 + q_2^\dagger) - c]q_1 = [P(\frac{a+bq_1-c}{2b}) - c]q_1 = (\frac{a-bq_1-c}{2})q_1$. Firm 1 chooses q_1 equal to the Nash equilibrium output q_1^* such that the first-order condition for q_1 holds, i.e. $\frac{\partial \pi_1}{\partial q_1}|_{q_1=q_1^*} = \frac{a-bq_1^*-c}{2} - \frac{bq_1^*}{2} = \frac{a-c}{2} - bq_1^* = 0$, $q_1^* = \frac{a-c}{2b}$.
  - Firm 2's Nash equilibrium output $q_2^* = q_2^\dagger(q_1^*) = \frac{a-c}{2b} - \frac{q_1^*}{2} = \frac{a-c}{2b}$.
  - Nash equilibrium aggregate output $Q^* = q_1^* + q_2^* = \frac{3(a-c)}{4b}$, Nash equilibrium price $P^* = a - b(\frac{3(a-c)}{4b}) = \frac{a+3c}{4}$, Nash equilibrium profit for firm 1, $\pi_1^* = \frac{(a-c)^2}{8b}$, Nash equilibrium profit for firm 2, $\pi_2^* = \frac{(a-c)^2}{16b}$. Nash equiilibrium aggregate profit $\Pi^* = \pi_1^* + \pi_2^* = \frac{3(a-c)^2}{16b}$.

- Result (Profit vs Cournot)
  - The first-mover can increase profit by increasing output since, at the Cournot (simultaneous move) equilibrium, the direct effect of a marginal increase in output on the first-mover's profit is zero, since the first-order condition holds, but the indirect effect of such an increase on the first-mover's profit through the competitor's output is positive. In the Cournot and Stackelberg models, outputs are strategic substitutes, the second-mover optimally responds to increased aggression (increased output) from the first-mover by reducing output, hence increasing the first-mover's profit.
- Evaluation (Assumptions)
  - The Stackelberg model assumes a static one-shot game, which is unrealistic. Even if firms choose quantities non-simultaneously, it is more realistic that firms repeatedly take turns to choose quantities (i.e. firm 1 chooses, then firm 2, then firm 1 again, then firm 2 again, and so on). If firms choose quantities in this way, the Cournot outcome is restored in equilibrium.
  - The Stackelberg model assumes that there is some first-mover in output, but it is unclear how this is determined except in the case where an incumbent competes with a new entrant.
- Discussion (n-firm Stackelberg Model)
  - The n-firm Stackelberg model can be analysed by re-labelling of variables in the Stackelberg duopoly model. Suppose that the two firms considered in the duopoly model are the second-last mover and last mover, and that demand in the duopoly model is residual demand after all earlier movers have chosen output.
  - Suppose, for example, that there are 3 firms, and that firm 1 moves first, followed by firm 2, then by firm 3. Suppose further that the inverse demand function is $P = a - bQ$, then the inverse of residual demand facing firms 2 and 3 given firm 1's output q_1 is given by $P_{2,3}(q_1) = a - bq_1 - bQ_{2,3}$. By the result of the Stackelberg duopoly model, firm 2 chooses $q_2^\dagger(q_1) = \frac{a-bq_1-c}{2b}$ and firm 3 chooses $q_3^\dagger(q_1) = \frac{a-bq_1-c}{4b}$, hence $Q_{2,3}^\dagger(q_1) = \frac{3}{4}\frac{a-bq_1-c}{b}$.
  - By inspection, the "best response" of firms 2 and 3 (collectively) in the Stackelberg triopoly is greater than the best response of firm 2 in the Stackelberg duopoly. Firms 2 and 3 "collectively" compete "tougher" (i.e. respond more aggressively by choosing greater output, given any output by firm 1).

Dynamic Cournot Model

- Parameters
  - Consider the dyamic Cournot oligopoly with linear demand. Each of n firms indexed by $i \in \{1, 2, \ldots, n\}$, in each period $t \in \{1, 2, \ldots\}$ of an infinite number of periods produces a homogenous good at common, constant marginal cost c and chooses output q_{it} to maximise discounted sum of profits $\Pi_i = \pi_{i1} + \delta\pi_{i2} + \ldots$ where δ is the discount rate and π_{it} is the profit of firm i in period t equal to $[P(q_{it} + Q_{-it}) - c]q_{it}$ where $Q_{-it} = \sum_{i \neq j} q_{jt}$ is the total output of all other firms in period t. Demand is downward-sloping. Suppose for simplicity that inverse demand is given by $P(Q) = a - bQ$.
- Analysis
  - The strategy profile under which each firm plays a grim trigger strategy and chooses output equal to its share $\frac{q^M}{n}$ of monopoly output $q^M = \arg\max_q [P(q) - c]q$ in each period if no firm previously chose otherwise, and Cournot Nash output q^C otherwise, is a subgame-perfect equilibrium for sufficiently large discount factor.
  - Suppose that in period t no firm previously deviated from the given strategy profile. Firm i optimally deviates from the given strategy profile by choosing its best response given that all other firms choose output equal to their share of monopoly output. Firm i increases output, inflicting a negative externality on all other firms. Firm i enjoys greater profit $\pi^D > \frac{\pi^M}{n}$ in this period (than if it had not deviated). In all subsequent periods, all other firms choose their Cournot quantities, and firm i optimally responds by choosing its Cournot quantity. Firm i enjoys Cournot profit $\pi^C < \frac{\pi^M}{n} < \pi^D$, which is less than collusive profit, and still less than the profit from deviation in the period of deviation. $\Pi_i = \pi^D + \delta\pi^C + \delta^2\pi^C + \ldots = \pi^D + \frac{\delta\pi^C}{1-\delta}$ (re-labelling period t as period 1, since past-period profits are unchanged by present-period and future-period outcomes).
  - If firm i does not deviate from the given strategy profile, it enjoys its symmetrical share of monopoly profit $\frac{\pi^M}{n}$ in period t and all subsequent periods. $\pi_i = \frac{\pi^M}{n} + \delta\frac{\pi^M}{n} + \delta^2\frac{pi^M}{n} + \ldots = \frac{\pi^M}{n(1-\delta)}$.
  - No firm i has incentive to deviate from the given strategy profile iff $\frac{\pi^M}{n(1-\delta)} \geq \pi^D + \frac{\delta\pi^C}{1-\delta}$, iff δ is sufficiently high.
- Result (Collusion vs Bertrand)
  - It is ambiguous whether collusion is more sustainable under dynamic Cournot competition than under dynamic Bertrand competition. Under Cournot, the increase in deviation-period profit to the deviating firm is smaller than under Bertrand, but the decrease in punishment phase profit is smaller. Whether the incentive to collude is greater or smaller under Cournot than under Bertrand depends on the relative magnitude of these differences.

Kreps-Scheinkman (1983) Model

- Parameters
  - Consider the Kreps-Scheinkman (1983) model of price-setting oligopoly with capacity constraints. In the first stage of a two stage game, each of two firms indexed by $i \in \{1, 2\}$ chooses capacity $\bar{q}_i$ and incurs a fixed cost $c\bar{q}_i$ where c is the common unit cost of capacity. In the second stage, each of the two firms, given common knowledge of capacities, chooses price p_i. Each firm i's payoff is given by $\pi_i = p_i q_i - c\bar{q}_i$, where $q_i \leq \bar{q}_i$ is firm i's output. Demand is downward-sloping and consumers buy only from the firm(s) with the lowest price. Suppose for simplicity that inverse demand is given by $P(Q) = 1 - Q$. In the second stage, each firm supplies up to $\bar{q}_i$ units of output at zero marginal cost. Suppose that demand is efficiently rationed such that consumers with the highest valuations buy first.
- Analysis
  - In the second stage subgame, the strategy profile such that each firm i chooses $p_i = p^* = P(\bar{q}_1 + \bar{q}_2) = 1 - \bar{q}_1 - \bar{q}_2$ is a subgame Nash equilibrium.
  - Under the given strategy profile, market demand is equal to the sum of the two firms' capacities. Each firm supplies output equal to its capacity.
  - If firm i instead chooses $p_i < p^*$, since it is capacity constrained, its output remains unchanged. By inspection, its profit decreases.
  - If firm i instead chooses $p_i > p^*$, given sufficiently small capacities relative to market demand, its profit decreases.
  - $\pi_i = p_i q_i - c\bar{q}_i = (1 - q_i - \bar{q}_j)q_i - c\bar{q}_i$, $\frac{\partial \pi_i}{\partial q_i} = (1 - q_i - \bar{q}_j) - q_i = 1 - 2q_i - \bar{q}_j > 0$ if $\bar{q}_i, \bar{q}_j \in [0, \frac{1}{3})$ since $q_i \leq \bar{q}_i$, π_i decreases with decreasing q_i if $\bar{q}_i, \bar{q}_j \in [0, \frac{1}{3})$, hence π_i decreases with increasing p_i if $\bar{q}_i, \bar{q}_j \in [0, \frac{1}{3})$ since (residual) demand is downward-sloping.
  - Given sufficiently small capacities relative to market demand, no firm has incentive to deviate from the given strategy profile..
  - Under the unique subgame Nash equilibrium, profit to firm i, $\pi_i = p^*\bar{q}_i - c\bar{q}_i = [P(\bar{q}_i + \bar{q}_j) - c]\bar{q}_i$. The reduced-form profit function of the Kreps-Scheinkman game is identical to the Cournot profit function, in which quantities are to be interpreted as capacities. The unique subgame-perfect equilibrium of the Kreps-Scheinkman game thus coincides with the Cournot equilibrium, in which quantities are to be interpreted as capacities.
- Result (Profit vs Bertrand)
  - Firms under Kreps-Scheinkman competition enjoy non-zero profit because, given sufficiently small capacities relative to market demand, firms can (to some extent) raise prices since other firms are capacity-constrained and consumers (informally) "have no where else to shop from". Residual demand is (given sufficiently small capacities relative to market demand) downward-sloping (rather than perfectly elastic, as in the case of Bertrand competition).
- Evaluation (Assumptions)
  - The Kreps-Scheinkman result holds only if total capacity is small relative to market demand. This is more likely if the unit cost of investment is high, since each firm's capacity is constrained by the condition that net (of investment costs) profits must be positive.
  - The Kreps-Scheinkman result also requires that demand is efficiently rationed. This assumption is not necessarily realistic. For example, in certain markets, rationing is achieved by queueing. Efficient rationing realistically approximates rationing by queueing only if customers with higher valuations for the good are also more willing and able to queue.
- Discussion (Generalisation)
  - Capacity constraints are an extreme case of increasing marginal costs (or, equivalently, decreasing returns to scale), where marginal costs are zero up to the capacity and infinite thereafter. The Kreps-Scheinkman result that firms do not price efficiently (at marginal cost) where marginal costs are increasing, generalises effectively.

Rotemberg-Saloner (1986) Model

- Parameters
  - Consider the Rotemberg-Saloner (1986) model of dynamic Bertrand oligopoly with observable demand fluctuations. Each of n firms indexed by $i \in \{1, 2, \ldots, n\}$, in each period $t \in \{1, 2, \ldots\}$ of an infinite number of periods produces a homogenous good at common constant marginal cost c and chooses price p_{it} to maximise discounted sum of expected profits $\Pi_i = \pi_{i1} + \delta E(\pi_{i2}) + \ldots$ where δ is the discount rate and π_{it} is the profit of firm i in period t equal to $(p_{it} - c)q_{it}$ where q_{it} is the output of firm i in period t. In each period, firms compete in prices, demand is downward-sloping, and consumers buy only from the lowest-price firm(s).
  - Demand fluctuates stochastically and, in each period, demand is either high or low. Demand function is $Q = D_t(p) = \theta_t D(p)$, where θ_t is a discrete random variable that takes value θ^H with probability $\frac{1}{2}$ and value θ^L with

probability $\frac{1}{2}$, and $\theta^H > \theta^L$. In each period, each firm observes the state of demand before choosing price.

- Analysis
  - Since marginal costs are constant and the state of demand affects only the scale of demand, monopoly price p^M is constant. Let π_H^M and π_L^M denote the monopoly profit in the high-demand state and in the low-demand state respectively. $\pi_H^M = \max_p (p-c)\theta^H D(p) > \max_p (p-c)\theta^L D(p) = \pi_L^M$. Let π^M denote the average (over time) monopoly profit. $\pi^M = \frac{\pi_H^M + \pi_L^M}{2}$.
  - The strategy profile under which each firm plays a grim trigger strategy, and chooses price equal to the monopoly price if no firm previously chose otherwise, and price equal to marginal cost otherwise, is a subgame-perfect equilibrium for sufficiently large discount factor.
  - Suppose that demand is high in period t. If firm i optimally deviates in this period by choosing price arbitrarily below monopoly price, it captures all demand and enjoys profit arbitrarily below monopoly profit π_H^M. In all subsequent periods, all other firms choose price equal to marginal cost. By the result of the static Bertrand game, the deviating firm optimally responds by choosing likewise. The deviating firm enjoys zero profit in all subsequent periods. $\Pi_i = \pi_H^M + \delta(0) + \delta^2(0) + \ldots = \pi_H^M$ (re-labelling period t as period 1, since past-period profits are unchanged by present-period and future-period outcomes).
  - If firm i does not deviate from the given strategy profile, it captures its symmetrical share of demand and enjoys its symmetrical share $\frac{\pi_H^M}{n}$ of monopoly profit in period t and its symmetrical share of expected monopoly profit $\frac{\pi^M}{n}$ in each subsequent period. $\Pi_i = \frac{\pi_H^M}{n} + \frac{\delta\pi^M}{n} + \frac{\delta^2\pi^M}{n} + \ldots = \frac{\pi_H^M}{n} + \frac{\delta\pi^M}{(1-\delta)n}$.
  - No firm i has incentive to deviate in period t if $\frac{\pi_H^M}{n} + \frac{\delta\pi^M}{(1-\delta)n} \geq \pi_H^M$.
  - By similar arguments, supposing that demand is low, no firm i has incentive to deviate in period t if $\frac{\pi_L^M}{n} + \frac{\delta\pi^M}{(1-\delta)n} \geq \pi_L^M$. Since $\pi_H^M > \pi_L^M$, the latter condition holds if the former condition holds, no firm has incentive to deviate at all if no firm has incentive to deviate under high demand.
  - $\frac{\pi_H^M}{n} + \frac{\delta\pi^M}{(1-\delta)n} \geq \pi_H^M$ iff $\frac{\delta}{1-\delta}\pi^M \geq (n-1)\pi_H^M$ iff $(-1+\frac{1}{1-\delta})\pi^M \geq (n-1)\pi_H^M$ iff $1-\delta \leq \frac{\pi^M}{(n-1)\pi_H^M + \pi^M}$ iff $\delta \geq \hat{\delta} \equiv \frac{(n-1)\pi_H^M}{(n-1)\pi_H^M + \pi^M}$.
  - $\hat{\delta} > \bar{\delta}$, by inspection, since $\pi_H^M > \pi_L^M$ hence $\pi_H^M > \pi^M$.
  - If instead $\bar{\delta} < \delta < \hat{\delta}$, $\pi_H^M > \frac{\pi_H^M}{n} + \frac{\delta}{1-\delta}\frac{\pi^M}{n}$, collusion is sustainable at p^M in the low-demand state, but full collusion is not sustainable in the high-demand state since the payoff from deviation exceeds the payoff from collusion, when firms collude to set price equal to the monopoly price.
  - Partial collusion at the lower price p^P is sustainable in the high-demand state, where p^P is sufficiently low and the corresponding profit $\pi^P = (p^P - c)\theta_H D(p^P)$ is such that $\pi^P = \frac{\pi^P}{n} + \frac{\delta}{1-\delta}\frac{\pi^M}{n}$, where π^M is redefined as $\frac{\pi^P + \pi_L^M}{2}$.
- Result (Collusion vs Bertrand)
  - Full collusion at monopoly price is more difficult to sustain under the Rotemberg-Saloner model where there are observable demand fluctuations than under the dynamic Bertrand model where there are no such fluctuations. In the high demand state, the "reward" from deviation is greater, but forgone future profits are similar since demand is stochastic and future demand (hence collusion profit) is independent of present demand.
- Result (Price Wars)
  - Under the Rotemberg-Saloner model, price wars are observed in periods of high demand, since the collusive price is lower in such periods. Price wars are a necessary component of a sustainable collusive arrangement, and do not indicate a breakdown of the collusive arrangement. No firm deviates from the collusive strategy in equilibrium.
- Case Study (Armed Services Medical Procurement Agency)
  - In 1956, collusion in the market for the antibiotic tetracycline broke down when the Armed Services Medical Procurement Agency placed a large order. (Tirole, 1988, p. 250)

Green-Porter (1984) Model

- Parameters
  - Consider the Green-Porter (1984) model of dynamic Bertrand oligopoly with unobservable demand fluctuations. Each of n firms indexed by $i \in \{1, 2, \ldots, n\}$, in each period $t \in \{1, 2, \ldots\}$ of an infinite number of periods produces a homogenous good at common constant marginal cost c and chooses price p_{it} to maximise discounted sum of expected profits $\Pi_i = E(\pi_{i1}) + \delta E(\pi_{i2}) + \ldots$ where δ is the discount rate and π_{it} is the profit of firm i in period t equal to $(p_{it} - c)q_{it}$ where q_{it} is the output of firm i in period t. In each period, firms compete in prices, demand is downward-sloping, and consumers buy only from the lowest-price firm(s).
  - Demand fluctuates stochastically and, in each period, demand is zero with probability α. Demand function is $Q = D_t(p) = \theta_t D(p)$, where θ_t is a discrete random variable that takes value 0 with probability α and value 1 with probability $1-\alpha$. Firms do not observe the state of demand.
- Analysis

- Given discount factor δ sufficiently close to 1, the strategy profile under which each firm chooses the monopoly price until demand for its product is 0, at which point it chooses price equal to marginal cost for T periods before reverting to the monopoly price is a subgame-perfect equilibrium for some sufficiently large T.
  - If a firm deviates from the given strategy profile, it captures the entire market demand, and receives profit arbitrarily below monopoly profit in one period. In the subsequent T periods, it receives zero profit.
  - If a firm does not deviate from the given strategy profile, it receives its share of monopoly profit in one period and expects to receive its share of monopoly profits in each of the subsequent T periods with some probability (a function of α).
  - The given strategy profile is a subgame-perfect equilibrium if δ and T are both sufficiently large, such that no firm has incentive to deviate.
- Result (Price Wars)
  - Under the Green-Porter model, price wars are observed in periods of low demand, since, if firms do not cut prices after observing low demand for their own product, then each firm has incentive to deviate from the collusive arrangement because it will not be punished by rivals and will enjoy increased profit. Price wars are a necessary component of a sustainable collusive arrangement, and do not indicate a breakdown of the collusive arrangement. No firm deviates from the collusive strategy in equilibrium. Price wars are thus, in a sense, involuntary.
- Case Study (Joint Executive Committee)
  - Between 1880 and 1886, the Joint Executive Committee, a cartel controlling eastbound freight shipments from Chicago to the Atlantic seaboard, adopted a variant of the above trigger strategy, resulting in occasional price wars following unobservable demand slumps. (Cabral, 2017, pp. 307-308)
- Case Study (Danish Competition Council)
  - From 1993 to 1995, the Danish Competition Council gathered and regularly published actual transaction prices in three regional ready-mixed concrete markets for two grades of ready-mixed concrete. In this period, price dispersion between firms decreased dramatically, and the average price level increased significantly. (Cabral, 2017, pp. 318-319)

Facilitating Practices

- Discussion
  - Facilitating practices facilitate tacit collusion either by restructuring firms' incentives such that firms have greater incentive to cooperate and less incentive to compete or by facilitating information exchange that eliminates uncertainty about rivals' actions. Incentive management and information exchange devices embedded in contracts make the corresponding commitments credible since a pubic court will enforce the contract if necessary. If such devices are embedded in contracts with buyers, the opportunity for buyers to collect damages in the event that firms fail to abide by their commitments constitutes incentive for buyers to monitor and bear the cost of enforcing the commitments.

Retroactive Most-Favoured-Nation Clauses

- Discussion
  - Retroactive most-favoured-nation clauses penalise sellers for reducing price. When the seller reduces price, the seller is required to retrospectively reimburse recent customers the amount that they would have saved had they been offered the same discount.
  - The increase in deviation-period profit to the deivating firm decreases because the firm incurs the additional cost of reimbursement. Profit under collusion is unchanged since colluding firms do not cut prices. Hence firms have less incentive to deviate under a retroactive most-favoured nation clause.
- Case Study (GE)
  - In 1963, GE, a producer of large turbine generators, instituted a most-favoured-customer clause, under which it would be bound, in the event that it reduced prices, to offer all customers in the past six months an identical discount. Rival Westinghouse followed suit within a year. Except for a brief period of price cutting in 1964, prices of the two firms remained stable and identical until regulators intervened in 1975.

Contemporaneous Most-Favoured-Nation Clauses

- Discussion
  - Contemporaneous most-favoured-nation clauses prevent price discrimination. Under such a clause, when the seller offers a discounted price to another buyer, the seller must offer the same discount to the original buyer (with whom a

contract with a contemporaneous most-favoured-nation clause is binding). Sellers that universally adopt contemporaneous most-favoured-nation clauses commit to selling at a common price.
    - Suppose that each firm in a Bertrand duopoly chooses either a universal high price, a universal low price, or an "intermediate" option, where it sells at a high price but offers selective discounts to large buyers. The grim trigger strategy whereby each firm chooses the high monopoly price if no firm previously chose otherwise, and the low price equal to marginal cost otherwise may be sustainable only if the "intermediate" option were not available. This is because selective discounts are typically more difficult to detect than general discounts, hence selective discounts are detected with a greater lag, and the deviating firm enjoys deviation profit over a larger number of periods before it is punished, hence the incentive for deviation is greater (than if the deviating firm could only deviate by offering a general discount which is swiftly detected).
    - If both firms universally adopted contemporaneous most-favoured-nation clauses, neither firm can offer a selective discount, and each firm thus has less incentive to deviate from the collusive arrangement. Collusion is easier to sustain as a result.

Pricing Conventions

- Discussion
    - Pricing conventions create price transparency and enable buyers to negotiate for lower prices if and when a selective discount is offered. Buyers who discover that they have paid more than others have a powerful tool for negotiating a matching discount, which can make it costly or difficult for a seller to offer selective discounts.
    - The effect of such pricing conventions is analogous to that of contemporaneous most-favoured-nation clauses.
    - Price transparency is facilitated by the adoption of relative value scales, or product standardisation.
    - A relative value scale is a pricing system in which there is a fixed relationship among the prices of a number of products, which thereby restrict price movements to proportional changes in all prices. For example, a car repair shop might set an hourly rate and apply a standard job completion time table from a private or trade association publication.
    - Product standardisation eliminates some non-price competition: no seller can offer more or less of the standardised product attributes in an individual product. As a result, all competition must be in the price dimension. Firms cannot offer "selective discounts" by offering some customers products of greater quality than others.

Meet-or-Release Clauses

- Discussion
    - Meet-or-release clauses serve primarily as an information exchange device. If a buyer discovers a lower price elsewhere, he is not released from his obligation to buy from the original supplier unless he informs that supplier of the lower price and that supplier has the opportunity to meet the lower price. The buyer has incentive to inform the original supplier of the lower price since he thereby enjoys a lower price (either by buying from the competitor or by compelling the original supplier to match the lower price). Meet-or-release clauses reduce if not eliminate detection lags. The length of time over which a deviating firm enjoys greater profit before it is punished by competitors is reduced, hence the incentive to deviate decreases and collusion is easier to sustain.

Meeting Competition Clauses

- Discussion
    - Under no-release meeting competition clauses, if a buyer discovers a lower price elsewhere and informs his original supplier of this price, the original supplier must sell to the buyer at this lower price.
    - Suppose that each firm in a dynamic Bertrand duopoly universally adopts a no-release meeting competition clause. Then, if one firm cuts price, its competitor's (effective) price decreases accordingly. Retaliation is immediate and automatic. The deviating firm captures no additional consumers since no consumer has incentive to switch suppliers because each consumer can buy from his original supplier at the lower price. Deviation from the collusive arrangement yields no increase in profit, even in the short-term, hence firms have no incentive to deviate from the collusive arrangement and collusion is more sustainable.
    - A no-release meeting competition clause also facilitates the achievement of the collusive outcome. For example, a seller who provides a no-release meeting competition clause to current customers can raise price without losing any customers to a lower priced rival. Buyers are automatically given the rival's lower price until all firms raise their prices. This eliminates the transitional losses that might otherwise deter price increases. It also eliminates the rival's transitional gains and with it the incentive to delay a matching price increases.

Multimarket Contact

- Case Study (Airlines)
  - Evans and Kessides (1994) studied prices of and routes served by U.S. airlines Delta, American, and Northwest in 1988 and found that an index of "average contact" which measures the number of markets in which competing airlines face each other has a significant positive impact on airfares. This is consistent with the result that multimarket contact facilitates collusion since deviation from a collusive arrangement in one market triggers price wars in all markets.
- Case Study (Dog Food)
  - In 1986, Quaker Oats, a dominant player in the moist dog food segment acquired Anderson Clayton, strengthening Quaker's position in the moist dog food segment and acquiring a foothold for Quaker in the dry dog food segment. Ralston Purina, the dominant player in the dry segment responded by acquiring Benco Pet Food's Inc., Quaker's primary rival in the moist segment.
  - The behaviour of Quaker and Ralston Purina seem consistent with the intention of putting their respective rivals on notice. Were Quaker to initiate a price war in the dry segment, Ralston Purina would be capable of retaliating in the moist segment, and vice versa.

Symmetric Markets

Asymmetric Costs

Asymmetric Market Shares