Philosophical Logic Paper 200526

1a Prove by induction that for every ~~#~~ PC-wff w no → ϕ, ~~I#~~ $V_{I\#}(\phi) = \#$ where $I\#(\alpha) = \#$ for all sentence letters α.

Base case
Consider arbitrary PC-wff ϕ such that complexity, i.e. number of connectives $C(\phi)=0$. ~~and ϕ~~ Suppose that ϕ contains no →. Then ϕ is some sentence letter α. $tV_{I\#}(\phi) = I\#(\alpha) = \#$ by definition of t-valuation, by construction of $I\#$. By conditional proof, generalisation, for all ϕ such that $C(\phi)=0$, if ϕ contains no →, $V_{I\#}(\phi) = \#$.

Induction Hypothesis
Given n, for all $m<n$, for all ϕ such that $C(\phi)=m$, if ϕ contains no →, then $V_{I\#}(\phi) = \#$.

Induction step
Consider arbitrary PC-wff ϕ such that $C(\phi)=n$. Suppose that ϕ contains no →. Then $\phi = \sim\psi$, $\psi \wedge \chi$, or $\psi \vee \chi$. Suppose $\phi = \sim\psi$, then $C(\psi) = n-1$, then by IH, $V_{I\#}(\psi) = \#$, then by $\sim$ clause, $V_{I\#}(\phi) = \#$. Suppose $\phi = \psi \circ \chi$, then $C(\psi) + C(\chi) = n-1$, so $C(\psi), C(\chi) < n$, then by IH, $V_{I\#}(\psi) = V_{I\#}(\chi) = \#$, then by $\wedge$ and $\vee$ clause, $V_{I\#}(\phi) = \#$ (for $\circ = \wedge, \vee$). By cases, conditional proof, generalisation, for all ϕ such that $C(\phi) = n$, if ϕ contains no → then $V_{I\#}(\phi) = \#$.

By induction, for all PC-wff ϕ, if ϕ contains no →, then $V_{I\#}(\phi) = \#$, then $\not\models_{t}\phi$, so if $\models_t \phi$, then ϕ contains at least one →.

bi Consider the following counterexample.

$I(P) = I(Q) = \#$, $I(R) = 0$, $I(\alpha) = 0$ for all other sentence letters α.
$tV_I(P \vee R) = \#$, $tV_I(Q \vee R) = \#$, $tV_I(\sim(P \check{\wedge} Q)) = \#$,
so $tV_I(\text{antecedent}) = \#$ hence $tV_I(\cdots \to R) = \#$
so $\not\models_t \cdots \to R$.

ii Consider arbitrary trivalent interpretation I.
Suppose for reductio that
(1) $tV_I((P\to Q)\vee(Q\to P)) = 0$ or #
 (1) ⟹
(2): (3) or (4)
(3) $tV_I((P\to Q)\vee(Q\to P)) = 0$
(4) $tV_I((P\to Q)\vee(Q\to P)) = \#$
 (3) ⟹, $\vee$ ⟹
(5) $tV_I(P\to Q) = 0$
(6) $tV_I(Q\to P) = 0$
 (5), → ⟹
(7) $tV_I(P) = 1$
 (6), → ⟹
(8) $tV_I(P) = 0$
 (7), (8), reductio ⟹
(9) $tV_I((P\to Q)\vee(Q\to P)) = 1$
 (4), $\vee$ ⟹
(10): (11), (12), or (13)
(11) $tV_I(P\to Q) = \#$, $tV_I(Q\to P) = 0$
(12) $tV_I(P\to Q) = 0$, $tV_I(Q\to P) = \#$
(13) $tV_I(P\to Q) = \#$, $tV_I(Q\to P) = \#$
 (11), → ⟹
(14) $tV_I(P) \neq 0$, $tV_I(P) = 0$
 (14), reductio ⟹
(9)
 (12), →, reductio ⟹
(9)
 (13), →, reductio ⟹
~~(15) $tV_I(P) =$~~
(9)
 by cases ⟹
(9)
 (9), generalisation, definition of $\models_t$ ⟹
(15) $\models_t (P\to Q)\vee(Q\to P)$

iii Consider the following counterexample.

$I(P) = I(Q) = \#$, $I(\alpha) = 0$ for all other sentence letters α.

$tV_I(P\to Q) = 1$, $tV_I(\sim(P\wedge\sim Q)) = \#$ ⟹
$P\to Q \not\models_t \sim(P\wedge\sim Q)$.

ci Denote the formula found in ci as ψ.

The required wff is $\sim\psi(\sim P/P, \sim Q/Q)$, i.e. the required wff is obtained by ~~substitu~~ uniformly substituting each instance of P in ψ with $\sim P$, likewise for Q, and then adding $\sim$ at the front.

d $\not\models_K \phi\to\phi$ but ~~$\not\models_K \phi$~~ $\models_t \phi\to\phi$, ~~so~~ and we think every English sentence of the form "if ϕ then ϕ" is true ~~and is~~, and is a logical truth, so t seems to do better here.

But we do not think if Henry is tall then the man with 100 hairs is bald is true when both are borderline or vague cases. The formalisation as ~~P→~~ $T\to B$ is true in KV but not in tV under $I(T) = I(B) = \#$, where # is

interpreted as denoting a sort of indeterminacy. So for such cases it seems tv does better than kv.
~~What goes wrong~~

Neither kv nor tv are entirely satisfying. Both seem to miss the underlying problem: penumbral connections, i.e. logical relationships between vague sentences. There is no such relationship between "Henry is tall" and "the man with 130 hairs is bald" but there is such a connection between "Henry is tall" and "Henry is not tall", even when all these sentences are indeterminate. So any truth tables would yield the same valuation of T→B and T→T or T→~T, because the truth values of each ~~component~~ immediate component is the same.

So a non-truth-functional ≠ valuation like supervaluation is required. This would represent an improvement over both kv and tv.

~~The problem here is "pervasive semantic~~

2a i $\Box[\Diamond(P\wedge Q)\wedge\Diamond(P\wedge\sim Q)\wedge\Diamond(\sim P\wedge Q)\wedge\Diamond(\sim P\wedge\sim Q)]$

ii The required model is as follows.

$M=\langle W,R,I\rangle$
$W=\{w, w_{11}, w_{10}, w_{01}, w_{00}\}$
$R=\{\langle u,v\rangle : u,v\in W\}$
$I(P,w_{11})=I(Q,w_{11})=I(P,w_{10})=I(Q,w_{01})=1,$
$I(\alpha,u)=0$ for all other sentence letters and worlds α, u.

b i $\Box\Diamond\Box P\wedge\Box\Diamond\Box Q\wedge\sim\Box P\wedge\sim\Box Q\wedge$
$\Diamond(\Box P\wedge\Box Q)\wedge\Diamond(\Box P\wedge\sim\Box Q)\wedge\Diamond(\sim\Box P\wedge\Box Q)$
$\cancel{\wedge\Diamond(\Box\sim P\wedge\Box\sim Q)}$

ii The required S4 model is as follows.

$M=\langle W,R,I\rangle$
$W=\{w, w_{11}, w_{10}, w_{01}\}$
$R=\{\langle w,w_{11}\rangle, \langle w,w_{10}\rangle, \langle w,w_{01}\rangle,$
$\langle w_{10},w_{11}\rangle, \langle w_{01},w_{11}\rangle, \dots\}$
where the remaining pairs in R are those implied by reflexivity and transitivity.
$I(P,w_{10})=I(Q,w_{01})=I(P,w_{11})=I(Q,w_{11})=1,$
$I(\alpha,u)=0$ for all other sentence letters and worlds α, u.

Neither button is pushed at w because w sees a $\sim P$ world, namely w, and a $\sim Q$ world, namely w. Both P and Q are buttons at w because each accessible world (all of them) sees a $\Box P$ world, namely w_{11} and a $\Box Q$ world, namely w_{11}. The buttons are independent because moving from w to w_{10} pushes P but not Q and ~~pe~~ moving from w to w_{01} pushes Q but not P.

c The notion of metaphysical necessity seems imprecise. For example, it is conventional to think that persons are (metaphysically) necessarily ~~it per humans~~, ~~and~~ rational agents, and not some other person, i.e. that such things as rational agency and personal identity are metaphysically essential to ~~humans~~ persons. But other sorts of metaphysical necessity are more controversial. For example, the existence of God is not uncontroversially metaphysically necessary. (Personal) identity is most ~~obvious~~ problematically imprecise. For example, ~~would some pers.~~ we think some person would still be the same person if he were marginally more vicious, but would doubt this identity if ~~the~~ he were entirely vicious. We would, naturally, say "that's a completely different person".

One reason for thinking that there is a correct logic of metaphysical necessity is ~~the~~ ~~metaphysical neces~~ the thought that metaphysical necessity is in some sense a limiting case. ~~(A a~~ This means that ~~a~~ something is metaphysically necessary iff it is true in all ~~meta~~ possible worlds, ~~(A~~ in contrast, something is physically necessary iff it is possible in all physically possible worlds, so metaphysical necessity is in some sense maximal and unqualified. Then S5 would seem to be correct logic for metaphysical ~~necess~~ modality because some sentence is S5-valid iff it is true in every possible world.

The above argument trades on an ambiguity in the phrase "possible world", and thereby buries the imprecision in metaphysical necessity. "Possible worlds" in the conventional, familiar logical sense refers to some qualifiedly maximal entity according to which such things as propositions or states of affairs obtain or do not obtain. ~~The sense of "~~ These are better ~~...~~ referred to as simply worlds. The sense of "possible worlds" that is necessary for the truth of "something is metaphysically necessary iff it is true in all possible worlds" is ~~a~~ as "metaphysically possible worlds". The two senses are not the same. This is evident from the metaphysical impossibility of a world in which Nathan Salmon is Henry Kissinger. ~~And~~ There certainly is ~~such~~ a world, i.e. a qualifiedly maximal ... according to which this is true. So we should not think that metaphysical necessity is truth ~~o~~ in all worlds. ~~Some s~~ We must consider what sorts of restrictions on metaphysical access are appropriate.

It seems we can know that some sorts of restrictions are plausible, i.e. ~~there ar~~. We are not completely clueless in constructing an appropriate logic for metaphysical necessity. For example, we think that each world is ~~not~~ a metaphysical possibility by its own lights, so it ~~is reasonable to impose~~ an ~~reflexivity~~ appropriate logic for metaphysical modality will have a reflexive accessibility relation.

Salmon argues that we can also know that some sorts of restrictions are inappropriate. ~~For example.~~ According to Salmon, transitivity is inappropriate because metaphysical necessity is not itself so necessary. Consider Woody, W, a table, that actually originates from woody ~~mat~~ matter m. Suppose that W ~~could instead~~, ~~if~~ given that it actually originates from m, could have originated from different woody matter m' and still have been W. Suppose further that had ~~m~~ W originated from m', it could have originated from m''. For m, m', m'' chosen with sufficient care such that m' is a limiting case, i.e. as different from m as can be such that W could still have originated from m', and m'' is a little more different from m than this, W could not actually have originated from m''. Then ~~it~~ it is metaphysically necessary that W did not originate from m'' but not ~~necess~~ necessarily so necessary. So a transitive ~~acc~~ accessibility relation that implies the logical truth of $\Box\phi \rightarrow \Box\Box\phi$ is inappropriate.

A friend of S4 could reject Salmon's argument by rejecting either (1) W could have originated from $m' \neq m$ or (2) W could not have originated from m''. (1) is a strict essentialism, (2) is a very loose essentialism.

~~We show~~ To think that there is no hope of finding a correct logic for metaphysical necessity is to think that there could be no resolution to the above arguments, i.e. that we can never know or even come close to knowing enough about the nature of ~~m~~ metaphysical necessity to resolve such disputes. This sort of fatalism seems unreasonable, and that we can make at least such claims as that the actual world is ~~by its own light~~ actually metaphysically possible should give us some hope.

3a i $\nvDash_{SOL} \forall X \forall Y \exists Z [\forall x(Zx \leftrightarrow (Xx \vee Yx)) \wedge \sim(\forall x(Zx \leftrightarrow Xx) \vee \forall x(Zx \leftrightarrow Yx))]$

This fails under any ~~variable assignment~~ model such that the extension of X ~~is~~ could be a subset of the extension of Y or vice versa.

Consider the concrete counterexample below.

$M = \langle D, I \rangle$
$D = \{0\}$
Any variant assignment $g^{X\,Y}_{U\,V}$ ~~for all~~ of any variable assignment g is such that $U \subseteq V$ or $V \subseteq U$, ~~then for $g^{X\,Y\,Z}_{U\,V\,W}$ where W~~ $= U \cup V = U$ or V, $V_{M,g^{XYZ}_{UVW}}(\forall x \ldots) = 0$ then $V_M, g^{XY}_{UV}(\exists Z \ldots) = 0$ because any Z with extension $W \subset U \cup V$ fails the first conjunct and any Z with extension $W \supseteq U \cup V \leftrightarrow W = U \cup V \leftrightarrow W = U$ or $W = V$ fails the second conjunct.

ii $\vDash_{SOL} \exists X \exists Y \sim \forall x(Xx \leftrightarrow Yx)$
~~(→) V_M,~~
Consider arbitrary SOL model $M = \langle D, I \rangle$ and ~~$\forall M, g^{XYx}_{D\emptyset u}$~~ arbitrary variable assignment g.
(1) $\forall u \in D: V_{M,g^{X\,Y\,x}_{D\,\emptyset\,u}}(Xx) = 1$
(because $|x|_{M,g^{XYx}_{D\emptyset u}} = u \in |X|_{M,g^{XYx}_{D\emptyset u}} = D$)
(2) $\forall u \in D: V_{M,g^{X\,Y\,x}_{D\,\emptyset\,u}}(Yx) = 0$
(because $|x|_{M,g^{XYx}_{D\emptyset u}} = u \notin |Y|_{M,g^{XYx}_{D\emptyset u}} = \emptyset$)
(1), (2), $\leftrightarrow \Rightarrow$
(3) $\forall u \in D: V_{M,g^{X\,Y\,x}_{D\,\emptyset\,u}}(Xx \leftrightarrow Yx) = 0$
(3), $\forall$, $\sim \Rightarrow$
(4) $V_{M,g^{X\,Y}_{D\,\emptyset}}(\sim\forall x(Xx \leftrightarrow Yx)) = 1$
(4), $\exists \Rightarrow$
(5) $V_{M,g}(\exists X \exists Y \sim\forall x(Xx \leftrightarrow Yx)) = 1$
(5), generalisation $\Rightarrow$
(6) $\forall$ SOL-models M ; $\forall$ variable assignments g: $V_{M,g}(\cdots) = 1$, i.e. $\vDash_{SOL} \cdots$

b i Consider the ~~SOL-wff~~ following SOL-wff.
$\exists R [\forall x \forall y (Rxy \rightarrow Fx \wedge Gy) \wedge$
$\forall x (Fx \rightarrow \exists y Rxy) \wedge$
$\sim\exists x \exists y_1 \exists y_2 (Rxy_1 \wedge Rxy_2 \wedge y_1 \neq y_2) \wedge$
$\sim\exists x_1 \exists x_2 \exists y (Rx_1y \wedge Rx_2y \wedge x_1 \neq x_2)]$
where ~~x≠y abb~~ $\alpha \neq \beta$ abbreviates ~~$\sim(\alpha=\beta)$~~ $\sim\alpha = \beta$.

This wff reads "exists a two-place predicate R such that R is a relation from F things to G things, every F thing bears R to something, R is functional and R is one-to-one". In other words, "there is a one-to-one mapping from F things to G things". This is true iff there are (weakly) more G things than F things. Let $F \preceq G$ abbreviate this wff.

Let $F \subseteq G$ abbreviate $\forall x(Fx \rightarrow Gx)$ which reads "every F thing is a G" and is true iff the set of F things is a (weak) subset of the set of G things. Let $F \subset G$ abbreviate $(F \subseteq G) \wedge \sim(G \subseteq F)$, which is true iff the set of F things is a strict subset of the set of G things.

$\exists F' [F' \subset F \wedge F' \preceq F \wedge F \preceq F]$ reads "there exists F', a subset of F, the same size of F" i.e. ~~F is De~~ there are (Dedekind) infinitely many F things". Abbreviate this as ∞F.

$\exists X \infty X$ is true ~~iff~~ only in models with infinite domains. $V_{M_g}(\exists X \infty X) = 1$ for all g for all M with infinite domains.

ii $\exists x Fx$ formalises "there exists at least one F thing". $\exists x_1 \exists x_2 (Fx_1 \wedge Fx_2 \wedge x_1 \neq x_2)$ formalises "there exist at least two F things". $\exists x_1 \exists x_2 \exists x_3 (Fx_1 \wedge Fx_2 \wedge Fx_3 \wedge x_1 \neq x_2 \wedge x_1 \neq x_3 \wedge x_2 \neq x_3)$ formalises "there exist at least three F things". The strategy generalises. Let $\exists nF$ denote the formalisation of "there exist at least n F things".

The set $\{\exists_n F : n \in \mathbb{N}\} \cup \{\neg\exists X \infty X\}$ is finitely satisfiable. Any finite subset of this contains a finite number of $\exists nF$-form wffs. Denote the maximum n among these as $\bar{n}$, then the finite subset is satisfiable by $M = \langle D, I \rangle$, $D = \{1, \ldots, \bar{n}\}$, $I(F) = D$. The set is not satisfiable because if the number of elements in the domain of any model is infinite, $\neg\exists X \infty X$ is false. Otherwise, some $\exists nF$-wff is false.

So SOL= is not compact because finite satisfiability fails to imply satisfiability.

c SOL= is incomplete. But PC= is complete (Godel) ~~and~~ as are PC ~~and~~ and PC. If we think one of the functions of logic is to identify logical truths, it is a virtue of PC, PC, PC= that they are complete, so an axiomatic proof system for each exists such that each logical truth of PC, PC, PC= can be axiomatically proven. ~~Completeness.~~ SOL= fails this desideratum for a logical theory. So completeness marks a notable distinction at which it is tempting to draw the line between logic and mathematics.

If we ~~are inclined~~ think completeness important for logic because an important function of logic is to identify logical truths, we should also think decidability ~~imp~~ important. A logic is decidable iff there exists some effective procedure for determining logical truth or falsehood that requires rigour but not ingenuity, and terminates in a finite number of steps. ~~PC~~ PC is not decidable, but we have strong intuitions that ~~PC~~ PC is logic. So it seems we cannot consistently reject ~~the~~ SOL= as logic while accepting PC as logic.

Further, if we think that one of the primary functions of logic is to identify logical truths (and ~~incompatib~~ inconsistencies), we have reason to think SOL= is logic. SOL= but not PC= can identify logical truths and ~~inconsist~~ inconsistencies to do with infinity and ancestral. For example, from (b), PC= cannot formalise, let alone find as inconsistent the set {"there is at least one F thing", "there are at least two F things", ..., "there are finitely many F things"}, whereas SOL= can, (at the cost of compactness).

One worry about considering SOL= to be logic rather than set theory is that this violates the topic-neutrality of logic. In other words, we think logic should not make set-theoretic claims in the same way that it should remain silent on the existence of dinosaurs. The SOL=-validity in (aii) seems to commit SOL= to the existence of two different sets.

This worry is unreasonable. The validity in (aii) should be interpreted in the logical sense of "given any non-empty set as a domain, there are ~~two subsets of~~ at least two subsets of this domain". This "set-theoretic" claim is not in fact a set-theoretic commitment of SOL= but a consequence of SOL= ~~being de~~ -truth being defined in set theoretic terms. ~~So~~ SOL is not committed to the existence of a two-membered set, a basic set-theoretic truth, so it can remain topic-neutral and can hardly be considered set theory or any other sort of mathematics.

4ai ~~No, it is, intuitively, not a logical consequence of the premises.~~

Yes, it is intuitively a logical consequence of the premises.

If I had rolled a ~~$~~ 5, then I would have rolled a 5 or a 6 then I would have won or at least deserved to win. So if I did not then win, I would still have at least deserved to win.

ii P1: $\sim(R_5 \vee R_6)$
P2: $(R_5 \vee R_6) \Box\!\!\rightarrow (P \vee Q)$
P3: $R_5 \Box\!\!\rightarrow \sim P$
C: $R_5 \Box\!\!\rightarrow Q$

R_5: I roll a 5.
R_6: I roll a 6.
P: I win
Q: I deserve to win

Consider the following countermodel.

$M = \langle W, \precsim, I \rangle$
$W = \{0, 1, 2\}$
$\precsim_0 = \{\langle 1,2 \rangle, \dots\}$
The remaining pairs are as implied by the base assumption that ~~$\forall u \in W$~~ $\forall w \in W : \forall u \in W$: $u \precsim_w u$, reflexivity, transitivity, antisymmetry, and connectedness.
$\precsim_1 = \{\langle 2,0 \rangle, \dots\}$
$\precsim_2 = \{\langle 1, 0 \rangle, \dots\}$
$I(R_6, 1) = 1$, $I(R_5, 2) = 1$, $I(P, 1) = 1$
$I(\alpha, w) = 0$ for all other sentence letters and worlds α, w.

~~It is~~ At world 0, it is trivial that P1 is true, P2 is satisfied because the 0-closest $R_5 \vee R_6$-world is 1 and at world 1, P hence $P \vee Q$ is true. P3 is true because the 0-closest R_5 world is 2 and there P is false. C is false because the 0-closest R_5 world is 2 and there Q is false.

iii P1: $\sim(R_5 \vee R_6)$
P2: $(R_5 \Box\!\!\rightarrow (P \vee Q)) \wedge (R_6 \Box\!\!\rightarrow (P \vee Q))$
P3: $R_5 \Box\!\!\rightarrow \sim P$
C: $R_5 \Box\!\!\rightarrow Q$

The sentence letters have the same interpretation as before.

Consider arbitrary SC-model $M = \langle W, \precsim, I \rangle$ and arbitrary world $w \in W$.
Suppose for ~~reductio that~~ conditional proof that

(1) ~~If~~ $V_M(\sim(R_5 \vee R_6), w) = 1$
(2) $V_M((R_5 \Box\!\!\rightarrow (P \vee Q)) \wedge (R_6 \Box\!\!\rightarrow (P \vee Q)), w) = 1$
(3) $V_M(R_5 \Box\!\!\rightarrow \sim P, w) = 1$
Suppose for reductio that
(4) $V_M(R_5 \Box\!\!\rightarrow Q, w) = 0$
(2), $\wedge$ $\Rightarrow$
(5) $V_M(R_5 \Box\!\!\rightarrow (P \vee Q), w) = 1$
(3) ~~+~~ $\Box\!\!\rightarrow$ $\Rightarrow$
(6) $V_M(Q, u) = 0$, where
u is the w-closest ~~for~~ R_5-world, i.e. $V_M(R_5, u) = 1$ and $\forall u' \in W$, $V_M(R_5, u') = 1 : u \precsim_w u'$.
(3), (6), $\Box\!\!\rightarrow$ $\Rightarrow$
(7) $V_M(\sim P, u) = 1$
~~(5)~~ (7), $\sim$ $\Rightarrow$
(8) $V_M(P, u) = 0$
(5), (6), $\Box\!\!\rightarrow$ $\Rightarrow$
(9) $V_M(P \vee Q, u) = 1$
(8), (9), $\vee$ $\Rightarrow$
(10) $V_M(Q, u) = 1$
(6), (10), reductio $\Rightarrow$
(11) $V_M(R_5 \Box\!\!\rightarrow Q, w) = 1$
(11), conditional proof, generalisation, definition of $\vDash_{SC}$ $\Rightarrow$
(12) P1, P2, P3 $\vDash_{SC}$ C.

iv For cc, the formalisation in ii is still such that the conclusion is not a semantic consequence of the premises. ~~The same countermodel applies.~~ The same countermodel applies.

~~Consider the countermodel $M = \langle W, \precsim, I \rangle$.~~

~~$W = \{0, 1, 2\}$~~
~~$\precsim_0 = \{\langle 1,2 \rangle,$~~

For cc, the formalisation in iii is such that the conclusion is ~~not~~ a semantic consequence of the premises.

~~Consider the following countermodel.~~

~~$M = \langle W, \precsim, I \rangle$~~
~~$W = \{0, 1, 2\}$~~
~~$\precsim_0 = \{\langle 1,2 \rangle, \langle 2,1 \rangle, \dots\}$~~
~~The remaining pairs are as implied by the modified base assumption, that reflexivity, transitivity, and connectivity.~~
~~$\precsim_1 = \{\langle 2,0 \rangle, \dots\}$~~
~~$\precsim_2 = \{\langle 1,0 \rangle, \dots\}$~~
~~$I(R_5, 1) = I(R_5, 2) = I(R_6, 1) = I(R_6, 2) = I(P, 1) = I(Q, 2) = 1$~~
~~$I(\alpha, w) = 0$ for all other sentence letters and worlds α, w.~~

~~At world 0, each premise is true but the~~

~~conclusion is false.~~

Consider arbitrary CC-model $M = \langle W, \preceq, I \rangle$, and arbitrary world $w \in W$.

Suppose for conditional proof that

(1) $V_M(\sim(R5 \vee R6), w) = 1$

(2) $V_M((R5 \Box\!\!\rightarrow (P \vee Q)) \wedge (R6 \Box\!\!\rightarrow (P \vee Q)), w) = 1$

(3) $V_M(R5 \Box\!\!\rightarrow \sim P, w) = 1$

Suppose for reductio that

(4) $V_M(R5 \Box\!\!\rightarrow Q, w) = 0$

  (2), $\wedge$ $\Rightarrow$

(5) $V_M(R5 \Box\!\!\rightarrow (P \vee Q), w) = 1$

  (4), $\Box\!\!\rightarrow$ $\Rightarrow$

(6) $\exists u \in W: V_M(R5, u) = 1$

  (5), (6), $\Box\!\!\rightarrow$ $\Rightarrow$

(7) $\exists v \in W: V_M(R5, v) = 1$ and

  $\forall v' \in W, v' \preceq_w v: V_M(R5 \rightarrow (P \vee Q), v') = 1$

  (3), (6), $\Box\!\!\rightarrow$ $\Rightarrow$

(8) $\exists v \in W: V_M(R5, v) = 1$ and

  $\forall v' \in W, v' \preceq_w v: V_M(R5 \rightarrow \sim P, v') = 1$

  (7), (8) $\Rightarrow$

(9) $\exists v \in W: V_M(R5, v) = 1$ and

  $\forall v' \in W, v' \preceq_w v:$ $V_M(R5 \rightarrow (P \vee Q), v') = 1$ and

  $V_M(R5 \rightarrow \sim P, v') = 1$

  (9), $\rightarrow$, $\vee$, $\sim$ $\Rightarrow$

(10) $\exists v \in W: V_M(R5, v) = 1$ and

  $\forall v' \in W, v' \preceq_w v: V_M(R5 \rightarrow Q, v') = 1$

  (10), $\Box\!\!\rightarrow$ $\Rightarrow$

(11) $V_M(R5 \Box\!\!\rightarrow Q, w) = 1$

  (4), (11), reductio $\Rightarrow$

(12) $V_M(R5 \Box\!\!\rightarrow Q, w) = 1$

  (12), conditional proof, generalisation, definition of $\models_{CC}$ $\Rightarrow$

(13) $P1, P2, P3 \models_{CC} C$.

b SC apparently cannot properly handle disjunctive consequents (while CC can) because SC obeys distribution, i.e. $\phi \Box\!\!\rightarrow (\psi \vee \chi) \models_{SC} (\phi \Box\!\!\rightarrow \psi) \vee (\phi \Box\!\!\rightarrow \chi)$ but not $\phi \Box\!\!\rightarrow (\psi \vee \chi) \not\models_{CC} (\phi \Box\!\!\rightarrow \psi) \vee (\phi \Box\!\!\rightarrow \chi)$. Stalnaker thinks this is a virtue of SC because in English, for example, when a person says "if I played baseball, I would have been either a third-basEman or a shortstop", it is natural to respond "so which of the two would you have been?". This seems to presuppose the truth of either "if I had played baseball, I would have been a third-baseman" or "if I had played baseball, I would have been a shortstop." so it seems to treat the disjunction of these as a logical consequence of the first statement.

But that simply is not true. It is natural to so respond even if the answer one expects is "it would certainly have been one of the two, but it really might have been either, so it is not that I would have been a ~~third-b~~ ~~that~~ third-baseman nor that I flatly would have ~~be~~ been a shortstop". This is a perfectly fair response, so the truth of the disjunction is not presupposed. Then, it is a fault of SC that it obeys distribution.

Neither ~~theory~~ SC nor CC seems adequate for disjunctive antecedents. Consider the simple English argument "if I had rolled a ~~five~~ 5 or 6, I would have won. So if I had rolled a 5, I would have won." This is intuitively valid. But it is not valid in either SC or CC. The premise is true and the conclusion false in some world of a simple model such that there is a near 6-world and a far 5-world, and I win in the 6-world but not the 5-world.

SC and CC seem to go ~~the~~ wrong here because what is meant by the premise seems to be "if I had rolled a 5, I would have won, and if I had rolled a 6, I would have won". If formalised in the way that reflects these ~~SC~~ semantics, the argument is valid in both SC and CC. But this is odd for two reasons. ~~First, a disjunction somehow turns into a conjunction.~~ ~~Second~~ a disjunctive consequent ~~somehow~~ The English premise is ~~disjunctive~~ has a disjunctive form, but we here allege that it has a conjunctive meaning. The disjunction in the English premise applies within the consequent, but here we allege the conjunction applies to the whole conditional.

The naive formalisation is not always correct. For example, it would be inappropriate "there aint no cake" as $\sim\sim C$. ~~Also~~ A more similar example is "if he is unmarried, he must be a bachelor" is uncontroversially formalised as $\Box(U \rightarrow B)$ rather than $U \rightarrow \Box B$.

We might reject the naive formalisation of the English premise (in a way that is not ad hoc) because when the English premise is spoken, ~~we to no~~ the speaker is not imagining some "roll a five or six" world in which he wins but imagining both a "roll a ~~five~~ 5" world in which he wins and a "roll a ~~5~~ 6" world in which he wins. We do not imagine the former because ~~that~~ that is too ~~in~~ indeterminate to imagine. Likewise in evaluating the truth of the premise, we separately imagine a roll a 5 world and a roll a 6 world, and check if ~~he~~ one wins in both.