Applied Welfare Rough Notes

Questions

- How does the Vickrey-Clarke-Groves mechanism work when some public good is not costless? Do the given definitions of $p_i = \sum_{j \neq i} r_j$ and $t_i = \max\{\sum_{j \neq i} r_j, 0\}$ apply? (These were the values given in the lecture.) Does it make more sense to rely on these definitions or to use the understanding of a VCG mechanism as a tax equal to the externality each consumer imposes on others.
  - Coerce non-zero-cost cases to zero-cost cases by imposing a fixed cost sharing rule, then consumers report their net valuations.
- Definition (Indirect Utility)
  - The indirect utility function v is a function from price vector $\overrightarrow{\mathbf{p}}$ and income m (or endowment $\overrightarrow{\mathbf{w}}$) to utility. Formally, $v(\overrightarrow{\mathbf{p}}, m) = \max_{\overrightarrow{\mathbf{x}}} u(\overrightarrow{\mathbf{x}}) \text{ s.t. } \overrightarrow{\mathbf{p}} \cdot \overrightarrow{\mathbf{x}} \leq m$. Equivalently, let $\overrightarrow{\mathbf{x^*}}(\overrightarrow{\mathbf{p}}, m)$ denote the optimal (solving the above maximisation problem) consumption bundle given $\overrightarrow{\mathbf{p}}$ and m, then $v(\overrightarrow{\mathbf{p}}, m) = u(\overrightarrow{\mathbf{x^*}}(\overrightarrow{\mathbf{p}}, m))$.
- Definition (Hicksian Demand)
  - The Hicksian demand function h is a function from price vector $\overrightarrow{\mathbf{p}}$ and utility level $\bar{u}$ to an optimal consumption bundle $\overrightarrow{\mathbf{x^*}}$ that minimises expenditure $\overrightarrow{\mathbf{p}} \cdot \overrightarrow{\mathbf{x}}$ subject to $u(\overrightarrow{\mathbf{x^*}}) = \bar{u}$, where u is the (direct) utility function. Formally, $\overrightarrow{\mathbf{x^*}} = h(\overrightarrow{\mathbf{p}}, \bar{u}) = \min_{\overrightarrow{\mathbf{x}}} \overrightarrow{\mathbf{p}} \cdot \overrightarrow{\mathbf{x}} \text{ s.t. } u(\overrightarrow{\mathbf{x^*}}) = \bar{u}$.
- Definition (Marshallian Demand)
  - The Marshallian demand function is the familiar function from price vector $\overrightarrow{\mathbf{p}}$ and income m (or endowment $\overrightarrow{\mathbf{w}}$) to an optimal consumption bundle $\overrightarrow{\mathbf{x^*}}$ that maximises utility $u(\overrightarrow{\mathbf{x}})$ subject to budget constraint $\overrightarrow{\mathbf{p}} \cdot \overrightarrow{\mathbf{x}} \leq m$.
- Definition (Compensating Variation)
  - The compensating variation for change in price from $\overrightarrow{\mathbf{p^0}}$ to $\overrightarrow{\mathbf{p^1}}$ given income m is the difference between (1) the minimum expenditure required to achieve the ex-post utility u^1 at ex-post prices $\overrightarrow{\mathbf{p^1}}$ and (2) the minimum expenditure required to achieve the ex-ante utility u^0 at ex-post prices $\overrightarrow{\mathbf{p^1}}$. Formally, let $e(\overrightarrow{\mathbf{p}}, u) = \min_{\overrightarrow{\mathbf{x}}} \overrightarrow{\mathbf{p}} \cdot \overrightarrow{\mathbf{x}} \text{ s.t. } u(\overrightarrow{\mathbf{x}}) = u$, then the compensating variation for change in price from $\overrightarrow{\mathbf{p^0}}$ to $\overrightarrow{\mathbf{p^1}}$ given income m is $CV(\overrightarrow{\mathbf{p^0}}, \overrightarrow{\mathbf{p^1}}, m) = e(\overrightarrow{\mathbf{p^1}}, u^1) - e(\overrightarrow{\mathbf{p^1}}, u^0) = m - e(\overrightarrow{\mathbf{p^1}}, u^0)$. Informally, this is the change in income that, together with the change in price, leaves the consumer no worse off.
- Definition (Equivalent Variation)
  - The equivalent variation for change in price from $\overrightarrow{\mathbf{p^0}}$ to $\overrightarrow{\mathbf{p^1}}$ given income m is the difference between (1) the minimum expenditure required to achieve the ex-post utility u^1 at ex-ante prices $\overrightarrow{\mathbf{p^0}}$ and (2) the minimum expenditure required to achieve the ex-ante utility u^0 at ex-ante prices $\overrightarrow{\mathbf{p^0}}$. Formally, let $e(\overrightarrow{\mathbf{p}}, u) = \min_{\overrightarrow{\mathbf{x}}} \overrightarrow{\mathbf{p}} \cdot \overrightarrow{\mathbf{x}} \text{ s.t. } u(\overrightarrow{\mathbf{x}}) = u$, then the equivalent variation for change in price from $\overrightarrow{\mathbf{p^0}}$ to $\overrightarrow{\mathbf{p^1}}$ given income m is $EV(\overrightarrow{\mathbf{p^0}}, \overrightarrow{\mathbf{p^1}}, m) = e(\overrightarrow{\mathbf{p^0}}, u^1) - e(\overrightarrow{\mathbf{p^0}}, u^0) = e(\overrightarrow{\mathbf{p^0}}, u^1) - m$. Informally, this is the change in income that would leave the consumer no worse off if it occurred instead of the price change.
- Result
  - Where consumers have quasilinear preferences, a given price change has no income effect, then compensating variation and equivalent variation are equal, and both are equal to the change in consumer surplus.

Public Goods

- Parameters
  - Suppose that there is a set of agents $N = \{1, \ldots, n\}$, there are two goods, the public good x, and the private good y, each agent i has quasilinear (in y_i) utility $u(x, y_i) = v_i(x) + y_i$, and budget constraint $x + y_i \leq m_i$ (which implies that the price of y is 1 and the unit cost of x is 1, which is without loss of generality). This budget constraint does not apply in the usual sense because a consumer can free-ride on other consumers' provision of the public good.
- Theorem (Samuelson Rule)

- The socially optimal quantity x^* of a public good x is such that the marginal benefit $\sum_i v_i'(x)$ is equal to the marginal cost of the public good.
- Given that utilities are quasilinear in the private good y, supposing that this good is transferrable, the unique Pareto-optimal allocation is the allocation that maximises total utility. This maximisation problem is $\max_x \sum_{i \in N} u_i(x, y_i)$ subject to the social budget constraint $x + \sum_{i \in N} y_i \leq \sum_{i \in N} m_i$. At the optimum, the social budget constraint binds. Any candidate optimum such that the social budget constraint does not bind fails to deviation by increasing some y_i by some sufficiently small amount ϵ such that the constraint remains satisfied. The maximisation problem can be rewritten as $\max_x(\sum_{i \in N} v_i(x) + \sum_{i \in N} y_i)$, which reduces to $\max_x(\sum_{i \in N} v_i(x) + \sum_{i \in N} m_i - x)$. The first-order condition is $\sum_{i \in N} v_i'(x) = 1$. This is the Samuelson rule, and can be read as marginal social benefit equal to marginal social cost.
- For public goods, or goods with negative externalities, where utilities are not quasilinear in money, the Samuelson rule is that the sum of marginal rates of substitution is equal to the marginal rate of transformation. The relevant marginal rate of substitution is found by dividing the marginal utility of the public good by the marginal utility of money. This marginal rate of substitution can be interpreted as the marginal private benefit for the computation of the Lindahl price.
  - Given quasilinear utilities, the Pareto-optimum is found by equating MB and MC, not by maximising total utility.

- Result (Underprovision)
  - At the Nash equilibrium, only the consumer with the highest marginal valuation pays for the public good, every other consumer free rides, and the public good is underprovided.
- Definition (Lindahl Pricing)
  - Under Lindahl pricing, each consumer i (reports demand function $x_i(t_i)$ or equivalently valuation function $v_i'(x)$, and) pays t_i per unit of the public good (which has price 1), where $\sum_{i \in N} t_i = 1$, and each consumer's demand $x_i(t_i)$ for the good given tax t_i (which solves the utility-maximisation problem given the "budget constraint" $t_i x_i + y_i \leq m_i$) is equal to the socially optimal quantity x^*, or equivalently $t_i = v_i'(x^*)$. Supposing that each consumer i truthfully reports $x_i(t_i)$ or equivalently v_i', the equilibrium is Pareto-optimal.
- Result (Lindahl Pricing Not Strategy-Proof)
  - Each consumer with marginal utility from consumption of the public good less than the maximum (among consumers) has strict incentive to report zero demand for the public good.
  - The lecture illustrates graphically that there is incentive to under-report valuations and demands.

Vickrey-Clarke-Groves Mechanism

- Definition (Groves Payment)
  - In the case of a discrete public good that is either provided or not provided, the Groves payment p_i to consumer i is defined by $p_i = \sum_{j \neq i} r_j$ where r_j is the valuation reported by consumer j, if the public good is provided, and 0 otherwise.
- Result
  - Truthful reporting is a dominant strategy.
- Definition (Clarke Tax)
  - In the same context, the Clarke tax t_i on player i is defined by $t_i = \max\{\sum_{j \neq i} r_j, 0\}$.
- Result
  - By inspection, the Clarke tax is independent of each player i's action r_i. Then, truthful reporting remains a dominant strategy. Further, for all i, $t_i - p_i \geq 0$, then no expenditure is incurred in inducing truthful reporting.

Negative Externalities

- Where there is uncertainty over marginal social benefits, whether it is preferable to use a tax or a quota to achieve efficiency depends on the relative elasticity of marginal social costs. If marginal social costs are elastic, then there is large variance in the socially optimal quantity (which is uncertain), so it is preferable to set a tax and allow quantities to vary. If marginal social costs are inelastic, then there is small variance in the socially optimal quantity, so it is preferable to set a quota. Where marginal social benefits are certain but marginal social costs are uncertain, tax and quota are equivalent because any tax yields some equilibrium quantity with certainty, so any outcome achievable by a quota is achievable by some tax (that can be computed with certainty).