Philosophical Logic Paper 210611

1ai $\{(P\to Q), (R\to S)\} \vDash_{LP} ((P\to S)\vee(R\to Q))\}$

Consider arbitrary trivalent interpretation I.
Suppose for conditional proof that
(1) $KV_I(P\to Q)=1$ or #
(2) $KV_I(R\to S)=1$ or #
Suppose for reductio that
(3) $KV_I((P\to S)\vee(R\to Q))=0$
(3), $\vee$ $\Rightarrow$
(4) $KV_I(P\to S)=0$
(5) $KV_I(R\to Q)=0$
(4), $\to$ $\Rightarrow$
(6) $KV_I(P)=1$
(7) $KV_I(S)=0$
(5), $\to$ $\Rightarrow$
(8) $KV_I(R)=1$
(9) $KV_I(Q)=0$
(6), (9), $\to$ $\Rightarrow$
(10) $KV_I(P\to Q)=0$
(1), (10), reductio $\Rightarrow$
(11) $KV_I((P\to S)\vee(R\to Q))=1$ or #
(11), conditional proof, generalisation, definition of $\vDash_{LP}$ $\Rightarrow$
(12) $\{(P\to Q), (R\to S)\} \vDash_{LP} ((P\to Q)\vee(R\to S))$

$\{(P\to Q), (R\to S)\} \vDash_{LP*} ((P\to S)\vee(R\to Q))$

Consider arbitrary trivalent interpretation I.
Suppose for conditional proof that
(1) $V^*_I(P\to Q)=1$ or #
(2) $V^*_I(R\to S)=1$ or #
(1), $\to$ $\Rightarrow$
(3) ~~E+~~ Either (4), (5), ~~(6)~~ or (6)
(4) $V^*_I(P=0)$
(5) $V^*_I(Q=1)$
(6) $V^*_I(P)=V^*_I(Q)=\#$
(4), $\to$ $\Rightarrow$
(7) ~~KV_I~~ $V^*_I(P\to S)=1$
(7), $\vee$ $\Rightarrow$
(8) $V^*_I((P\to S)\vee(R\to Q))=1$
(5), $\to$, $\vee$ $\Rightarrow$
(9) $V^*_I((P\to S)\vee(R\to Q))=1$
(6), (2) $\Rightarrow$
(10) ~~E+~~ Either (11), (12), or (13)
(11) $V^*_I(P)=V^*_I(Q)=\#$ and $V^*_I(R)=0$
(12) $V^*_I(P)=V^*_I(Q)=\#$ and $V^*_I(S)=1$
(13) $V^*_I(P)=V^*_I(Q)=$ ~~...~~ $V^*_I(R)=V^*_I(S)=\#$
(11), $\to$ $\Rightarrow$
(14) $V^*_I(R\to Q)=1$
(14), $\vee$ $\Rightarrow$
(15) $V^*_I((P\to S)\vee(R\to Q))=1$
(12), $\to$, $\vee$ $\Rightarrow$
(16) $V^*_I((P\to S)\vee(R\to Q))=1$
~~(17)~~
(13), $\to$, $\wedge$ $\Rightarrow$
(17) $V^*_I((P\to S), (R\to Q))=1$
(8), (9), (15), (16), (17), cases, conditional proof, definition of $\vDash_{LP*}$ $\Rightarrow$
(8) $\{(P\to Q), (R\to S)\} \vDash_{LP*} ((P\to S)\vee(R\to Q))$.

ii $\{(P\to Q), P\} \nvDash_{LP} Q$

Consider the following counterexample.
$I(P)=\#$, $I(Q)=0$, $I(\alpha)=0$ for all other sentence letters α.

$KV_I(P\to Q)=\#$, $KV_I(P)=\#$, $KV_I(Q)=0$, $\Rightarrow$
$\{(P\to Q), P\} \nvDash_{LP} Q$

$\{(P\to Q), P\} \vDash_{LP*} Q$

Consider arbitrary trivalent interpretation I.
Suppose for conditional proof that
(1) ~~≠~~ $V^*_I(P\to Q)=1$ or #
(2) $V^*_I(P)=1$ or #
Suppose for reductio that
(3) $V^*_I(Q)=0$
(1), (3), $\to$ $\Rightarrow$
(4) $V^*_I(P)=0$
(2), (4), reductio $\Rightarrow$
(5) $V^*_I(Q)=1$
(5), conditional proof, generalisation, definition of $\vDash_{LP*}$ $\Rightarrow$
(6) $\{(P\to Q), P\} \vDash_{LP*} Q$

iii $\{(P\vee Q), \sim P\} \nvDash_{LP} Q$
$\{(P\vee Q), \sim P\} \nvDash_{LP*} Q$

The following is a counterexample for both cases.

$I(P)=\#$, $I(Q)=0$, $I(\alpha)=0$ for all other sentence letters α. KV_I and V^*_I agree on the valuations of wffs not containing $\to$.

$KV_I(P\vee Q)=$ ~~≠~~ $V^*_I(P\vee Q)=\#$, $KV_I(\sim P)=V^*_I(\sim P)=\#$, $KV_I(Q)=V^*_I(Q)=0$ $\Rightarrow$
$\{(P\vee Q), \sim P\} \nvDash_x Q$ for $x = LP, LP*$.

bi ~~Suppose that ⊨~~ Consider arbitrary PL-wff ϕ. Suppose that $\nvDash_{PL} \phi$. Then there exists bivalent interpretation I such that $V_I(\phi)=0$. ~~V_I and V_I and V^*_I agree~~ PL-valuation functions and LP* valuation function have identical rules for bivalent interpretations, so agree for bivalent interpretations. So $V^*_I(\phi)=V_I(\phi)=0$. Then $\nvDash_{LP*} \phi$. So if $\vDash_{LP*} \phi$ then $\vDash_{PL} \phi$.

ii Consider the counterexample $\phi = \cancel{\neg(P \wedge \neg P)}$ $P \to (Q \to P)$. It is trivial that $\models_{PC} \phi$. Consider trivalent interpretation I such that $I(P) = \#$, $I(Q) = 1$, $I(\alpha) = 0$ for all other sentence letters α. $V^*_I(Q \to P) = 0$, $V^*_I(P \to (Q \to P)) = 0$, so $\not\models_{CP*} \phi$.

Interpretation of # as "both true and false" is in general motivated by such cases as the liar sentence "this sentence is true". If the liar sentence is true, then it is also false, and if it is false then it is also true, so it seems in either case, it must be both true and false. Then both the liar sentence and its negation are true, so a paraconsistent logic is necessary. Such a logic rejects ~~ex~~ ex falso quodlibet as a logical consequence. This is necessary because we do not want just any sentence to be a logical consequence of ~~to~~ the liar sentence (or any such paradox) and its negation.

Both CP and CP* are paraconsistent, so neither has an advantage here.

It is not clear that we would want a paraconsistent logic to shrink the set of logical truths as CP* does ~~eviden~~ ~~proven~~ (from (b)). CP does not shrink the set of logical truths. We think that if some sentence is ~~true~~ logically true then it is at least logically true, ~~so it is not clear why~~ (and could also be both true and false), so it is not clear why some PC-logical truths as basic as those formalised by $P \to (Q \to P)$ should cease to be so in ~~a~~ a paraconsistent logic. This is reason to favour ~~CP*~~ CP over CP*.

The Kleene truth table ~~also~~ for $\to$ also seems more accurate to this interpretation than the V^*_I truth table. For example, if the antecedent is both true and false and the consequent is false, we think that the conditional is both true and false, rather than merely false.

CP* obeys modus ponens but CP does not. This is reason to favour CP* because it is not clear why we should think a paraconsistent logic would reject MP. One reason to think a paraconsistent logic should reject MP is the following ~~argume~~ argument. This sentence is false. If the liar sentence is true, then pigs fly. So pigs fly. We do not want the conclusion to be ~~a~~ true. CP* would respond that the second premise is false, and is some attempt to get ex falso quodlibet in through a "backdoor", so ~~the~~ it is not a problem that CP* obeys MP.

2ai $\Diamond(P \wedge \Box P) \wedge \Diamond(\sim P \wedge \Box \sim P)$

ii By flipping a railway switch from one state to another, ~~it~~ accessing one of the ~~world~~ relevant worlds, a trolley can be directed onto one ~~set of~~ track, the ~~#~~ ϕ world ~~track~~, from which only further bits of that track, further ϕ worlds, are immediately ~~access~~ accessible. By flipping the railway switch to the other state, accessing the relevant $\sim\phi$ world, the metaphorical trolley is directed onto the $\sim\phi$ track from which only further $\sim\phi$ worlds are immediately accessible.

iii At some time just before the results of this exam are determined, the sentence "I passed the exam (on the first attempt)" is a railway switch. Supposing that ~~timeli~~ timelines "branch" and there are possible alternative timelines, there is a time accessible from this such that in all further accessible times, the given sentence is true and another time accessible from this such that in all further accessible timelines, the given sentence is false.

Another example is the sentence "I die tomorrow", supposing that there is some accessible timeline where I die tomorrow and some accessible timeline where I do not, and "tomorrow" is interpreted as 5th June 2023 at any time.

bi The required S4 model ~~is~~ and sentence are as follows.

$M = \langle W, R, I \rangle$
$W = \{0, 1, 2\}$
$R = \{\langle 0,1\rangle, \langle 0,2\rangle, \dots\}$
(the remaining ordered pairs are "filled in" by reflexivity, transitivity)
~~I(P) =~~ $I(P,1) = 1$, $I(\alpha, \omega) = 0$ for all other sentence~~s~~ letters and worlds α, ω.

P is a railway switch at world 0 which accesses 1, where P is true and all ~~furth~~ P is true in all further accessible worlds (namely 1) and accesses 2 where P is false and false in all further ~~acce~~ accessible worlds (namely 2).

ii No.

Consider arbitrary S4 model $M = \langle W, R, I\rangle$ and arbitrary ~~sente~~ MPL-wff ϕ such that ~~ϕ~~ and arbitrary world $w \in W$ such that ϕ is a railway switch at w in M.
~~Suppose that for reductio th~~
~~(1)~~
(1) $V_M(\Diamond(\phi\wedge\Box\phi)\wedge\Diamond(\sim\phi\wedge\Box\sim\phi), w) = 1$
Suppose for reductio that
(2) $\forall$ MPL-wff ψ, $u \in W$: $V_M(\Diamond\Box\psi \to \Box\Diamond\psi, u) = 1$
(1), $\wedge \Rightarrow$
(3) $V_M(\Diamond(\phi\wedge\Box\phi), w) = 1$
(4) $V_M(\Diamond(\sim\phi\wedge\Box\sim\phi), w) = 1$
(3), $\Diamond \Rightarrow$
(5) $\exists v \in W, Rwv: V_M(\phi\wedge\Box\phi, v) = 1$
(5), $\wedge\Rightarrow$
(6) $\exists v\in W, Rwv: V_M(\phi, v) = 1$
(7) $\exists v \in W, Rwv: V_M(\Box\phi, v) = 1$
(7), $\Diamond \Rightarrow$
(8) $V_M(\Diamond\Box\phi, w) = 1$
(2), ~~(8)~~ $\Rightarrow$
(9) $V_M(\Diamond\Box\phi\to\Box\Diamond\phi, w) = 1$
(8), (9), $\to$, $\Rightarrow$
(10) $V_M(\Box\Diamond\phi, w) = 1$
(10), $\Box \Rightarrow$
(11) $\forall v' \in W$ ~~$V_M(\Diamond\phi, v') = 1$~~, Rwv': $V_M(\Diamond\phi, v') = 1$
(4), $\Diamond\Rightarrow$
(12) $\exists v \in W$, Rwv: $V_M(\sim\phi\wedge\Box\sim\phi, v) = 1$
(12), $\wedge \Rightarrow$
(13) $\exists v\in W, Rwv$: ~~$(\sim\phi, v) = 1$~~ $V_M(\sim\phi, v) = 1$
(14) $\exists v \in W, Rwv$: $V_M(\Box\sim\phi, v) = 1$
(11), (14), $\Rightarrow$
(15) $\exists v\in W, Rwv$: $V_M(\Box\sim\phi, v) = 1$ and $V_M(\Diamond\phi, v) = 1$
(15), $\Box$, $\Diamond \Rightarrow$
(16) $\exists v \in W, Rwv$: $(\forall v' \in W, Rvv': V_M(\sim\phi, v') = 1)$ and $(\exists v' \in W, Rvv': V_M(\phi, v) = 1)$
(16), $\sim \Rightarrow$
(17) $\exists v\in W, Rwv$: $(\forall v' \in W, Rvv': V_M(\phi, v') = 0)$ and $(\exists v' \in W, Rvv': V_M(\phi, v') = 1)$
(17) $\Rightarrow$
(18) $\exists v \in W, Rwv$: $\exists v' \in W, Rvv'$: $V_M(\phi, v') = 0$ and $V_M(\phi, v') = 1$
(18), reductio, generalisation.
(19) For all such M, w, ϕ, it is not the case that all instances of $\Diamond\Box\psi\to\Box\Diamond\psi$ are valid. So there is no such model where all instances of $\Diamond\Box\psi\to\Box\Diamond\psi$ are valid.

iii If ϕ is a railway switch at w in S4-model M, then there exists $w' \in W$ such that Rww', $V_M(\phi, w') = 1$, and for all $w'' \in W$ such that $Rw'w''$, $V_M(\phi, w'') = 1$, so there is no world accessible from w' ~~such that~~ where ϕ is false, then ϕ is not a railway switch at w', so ϕ being a railway switch is contingent in the sense that ϕ is not a railway switch in all worlds even if it is a railway switch in some world. If ϕ is a railway switch at w in S4 $\mathfrak{M}$, then w accesses at least one world at which ϕ is a railway switch, namely w. So w accesses some ϕ-is-a-railway-switch world and some ϕ-is-not-a-railway-switch world, so ϕ's being a railway switch is contingent at w, and in general, every railway switch is contingently so in every world where it is a railway switch. This is necessary in the metalogical sense that it follows from the definition of a railway switch. This is not necessarily so within the S4 model because in ~~the~~ w', it is necessary that ϕ is not a railway switch, because, by transitivity, all worlds w'' accessible from w' access only other worlds accessible from w', which are such that ϕ is true. So no world accessible from w' accesses any world where ϕ is false, so ϕ is not a railway switch in any such world, so ϕ is necessarily not a railway switch at w', it is not contingent whether ϕ is a railway switch at w' so at w, it is not necessarily contingent that ϕ is a railway switch.

ii Every S5-model is a S4-model. Every instance of $\Diamond\Box\psi \to \Box\Diamond\psi$ is S5-valid, so by the result in (bii), there is no S5-model where some sentence ϕ is a railway switch.

S5: $\Diamond\Box\phi \to \Box\phi$
S4: $\Box\phi \to \Box\Box\phi$
D: $\Box\phi \to \Diamond\phi$

(1) $\Diamond\Box\psi \to \Box\psi$ (S5)
(2) $\Box\psi \to \Box\Box\psi$ (S4)
(3) $\Box\psi \to \Diamond\psi$ (D)
(4) $\Box\Box\psi \to \Box\Diamond\psi$ (3, NEC, K, MP)
(5) $\Diamond\Box\psi \to \Box\Diamond\psi$ (1, 2, 4, PC syllogism)

$\vdash_{S5} \Box\phi \to \Box\Box\phi$
(1) $\Box\phi \to \Diamond\Box\phi$ (T)
(2) $\Diamond\Box\phi \to \Box\Diamond\Box\phi$ (S5$\Diamond$)
(3) $\Diamond\Box\phi \to \Box\phi$ (S5)
(4) $\Box\Diamond\Box\phi \to \Box\Box\phi$ (3, NEC, K, MP)
(5) $\Box\phi \to \Box\Box\phi$ (1, 2, 4, PC syllogism)

$\vdash_{S5} \Diamond\phi \to \Box\Diamond\phi$
(1) $\Diamond\Box{\sim}\phi \to \Box{\sim}\phi$ (S5)
(2) ${\sim}\Box{\sim}\phi \to {\sim}\Diamond\Box{\sim}\phi$ (1, PC contraposition)
(3) ${\sim}\Diamond\Box{\sim}\phi \to \Box\Diamond\phi$ (PC DNE)
(4) $\Diamond\phi \to \Box\Diamond\phi$ (2, 3, PC syllogism)

$\vdash_{S5} \Box\phi \to \Diamond\phi$
(1) $\Box\phi \to \phi$ (T)
(2) $\Box{\sim}\phi \to {\sim}\phi$ (T)
(3) ${\sim}{\sim}\phi \to {\sim}\Box{\sim}\phi$ (2, PC contraposition)
(4) $\phi \to {\sim}{\sim}\phi$ (PC DNI)
(5) $\Box\phi \to \Diamond\phi$ (1, 4, 3, PC syllogism).

c There are bifurcating choices of possibility in the sense that, having chosen to assign this ~~ex~~ paper a passing grade, it is not possible thereafter (presumably) to rescind the grade. But the sense of possible does not seem to be metaphysical but rather "possible subject to the constraints imposed by university rules", so this is no reason to reject S5 as a language of metaphysical necessity.

There are bifurcating choices of possibility in another sense. It is possible that I die on 5th June 2023. But once ~~it~~ it is 6th ~~Ju~~ June, this is either necessarily true or necessarily false. But again the sort of modality here is temporal rather than metaphysical.

The sort of bifurcation that would count against S5 is some ~~metaphysically~~ ~~contingent~~ sentence's being contingent but possibly necessary and possibly necessarily false in a metaphysical sense.

We can construct Salmon-like cases for this. Suppose Woody actually originated from matter $m@$. It is possible that Woody originated from ~~a~~ wood older than m_0 and necessarily so originated. Similarly, it is possible did not so originate and necessarily did not so originate. "Possible" in the above has metaphysical modality. Then "Woody originated from wood older than m_0" is a railway switch in the actual world. Even if there is no reason to think the above setup is ~~pos metaphys~~ metaphysically possible, it certainly is logically possible. So we would not want the non-existence of ~~to~~ railway switches to be a logical truth.

Logic should remain in some sense topic-neutral here.