4a Let $\vec{x} = (x_1, x_2, x_3)$ denote the observed student's choices where x_1 is her choice on Monday, x_2 on Tuesday, and x_3 on Wednesday. ~~Let $\succsim$ denote the observed student's preferences.~~ Let $\succ$ denote the observed student's rational strict preference relation.

Suppose $\vec{x} = (F, P, M)$, then ~~$F \succ P$~~ $F \succsim_c M$, $F \succsim_c P$, $P \succsim_c V$ and $M \succsim_c V$. We cannot infer whether $M \succ P$ or $P \succ M$, so the observed student did not choose (F, P, M)

Suppose $\vec{x} = (F, P, V)$, then $F \succsim_c M$, $F \succsim_c P$, $P \succsim_c V$ and $V \succsim_c M$, then $F \succsim_c P \succsim_c V \succsim_c M$, i.e. we can infer $\succ$, so the observed student could have chosen (F, P, V).

By symmetry, the observed student could have chosen (F, V, M).

Suppose $\vec{x} = (F, V, V)$, then we cannot infer whether $P \succ M$ or $M \succ P$, so the observed student did not choose (F, V, V)

Suppose $\vec{x} = (M, P, M)$, then we cannot infer whether $F \succ V$ or $V \succ F$, so $\vec{x} \neq (M, P, M)$. Suppose $\vec{x} = (M, P, V)$, then ~~we cannot infer whether~~ $M \succsim_c P$, $P \succsim_c V$, and $V \succsim_c M$, so $\succ$ is not transitive hence not rational. By reductio, $\vec{x} \neq (M, P, V)$. Suppose $\vec{x} = (M, V, M)$, then we cannot infer whether $F \succ P$ or $P \succ F$, so $\vec{x} \neq (M, V, M)$. Suppose $\vec{x} =$ ~~(M, V, P)~~ (M, V, V), then we cannot infer whether $F \succ P$ or $P \succ F$, so $\vec{x} \neq (M, V, V)$. So $x_1 \neq M$. By symmetry, $x_1 \neq P$.

$\vec{x} = (F, P, V) \overset{\text{or}}{=} (F, V, M)$

b Let $X_i' = \Delta_i / \{x_i\}$ for $i \in \{1, 2, 3\}$

$c(\Delta_i, \succ)$ directly reveals $x_i \succsim_c x_i'$ for all $x_i' \in X_i'$, i.e. each choice reveals that the chosen ~~item is~~ element is preferred by the observed student to each ~~eleme~~ other element in that day's menu.

All other inferred preferences are revealed indirectly by the transitivity of rational preferences, from the directly revealed preferences.

Suppose $\vec{x} = (F, P, V)$, then the following preferences are revealed directly: $F \succsim_c M$, $F \succsim_c P$, $P \succsim_c V$, $V \succsim_c M$, and the following preferences are revealed indirectly: $P \succsim_c M$, $F \succsim_c V$.

Suppose $\vec{x} = (F, V, M)$, then $F \succsim_c M$, $F \succsim_c P$, $V \succsim_c P$, and $M \succsim_c V$ are revealed directly and $F \succsim_c V$ and $M \succsim_c P$ are revealed indirectly.

c $\vec{x} = (P, V, V) \Rightarrow P \succsim_c F$, $P \succsim_c M$, $V \succsim_c P$, $V \succsim_c M$. (directly)

$\Rightarrow V \succsim_c F$ (indirectly)

So whether $M \succ F$ or $F \succ M$ cannot be inferred from ~~$c(\Delta_i, \succ)$~~ $\vec{x} = (P, V, V)$. So $\succ$ is fully mapped iff $\Delta_4 = \{F, M\}$.

d $\vec{x} = \overset{(P,V,M)}{\cancel{(P,M,V)}} \Rightarrow P \succ_c F, P \succ_c M, M \succ_c V, V \succ_c P$ (directly)
$\Rightarrow V \succ_c F, V \succ_c M, V \succ_c V$ (indirectly). We conclude that the observed student's preferences are irrational since it is irrational to strictly prefer some element to itself.

e Denote the Monday Meat dish MM. Then
$\Delta_1 = \{F, MM, P\}$, $\Delta_2 = \{P, V\}$, $\Delta_3 = \{M, V\}$, $\Delta_4 = \{M, F, V, P\}$
$c(\Delta_1, \succ) = P$, $c(\Delta_2, \succ) = V$, $c(\Delta_3, \succ) = M \Rightarrow$
$P \succ_c F$, $P \succ_c MM$, $V \succ_c P$, $M \succ_c V$ (directly) $\Rightarrow$
$V \succ_c F$, $V \succ_c MM$, $M \succ_c P$, $M \succ_c F$, $M \succ_c MM$ (indirectly) $\Rightarrow$
$c(\Delta_4, \succ) = M$