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Background Part I

1 The phenomenon of structural repetition

1.1 Defining repeat

We experience repetition in diverse domains: we discern shared narrative structures across novels, films, and music, and we uncover universal laws governing phenomena as distinct as ocean currents and atmospheric dynamics. Moreover, humans can abstract patterns of patterns, as seen in the application of category theory to mathematics and programming. These examples illustrate our remarkable ability to not only detect abstract repetitions but also to productively employ them in navigating both the physical and mental worlds.

Repeat, as we intuitively perceive it, is surprisingly nontrivial to formalize.

1.1.1 Repeated observations

Let us first approximate our experience of repeat with a simple characterization: **a specific observation occurs across time and space**. We can find examples across domains that fit this definition. For instance, an ongoing fire siren corresponds to a repeat of a specific sound across time. The decorative motifs found in mosaic tiling are repeats of a visual symbol across space. The periodic motion of a pendulum contains repeats of a specific movement. To put it formally:

$$[\![x_2 \textit{ repeats } x_1]\!]^1 = \{x_2 = x_1\} \qquad \text{(Approximation 1)}$$

Our first definition is simple and precise, and its mental processing of can be computationally modelled in a straight forward manner. However, it does not fully capture of our *intuitive experience* of repeat. Consider a postman, when frustrated by his mundane work, complains about "doing the same thing" over and over again. Clearly, by "doing the same thing," our imaginary postman does not mean it as repeating a list of concrete physical actions (e.g.,

[1]We adopt the notation $[\![\ldots]\!]$ from formal semantics as a function that maps a expression to its truth condition

turning right on the crossroad at 7:23 am, and stopping his car at the exact same spot). If the postman's job can be just explained as such, imagine the catastrophes caused by a mail robot programmed to do exactly that.

In order to make the postman's expression work, our model of repeat should allow some degrees of flexibility. For example, there are multiple ways to achieve the same task of "parking the car" depending on the situation at hand. What if the closest parking slot is occupied? What if there are kids walking by? What if there are construction works blocking the entire side of the street? "Just do the sensible things" would be a typical response from human. In those scenarios, the observed actions would be adapted to include extra stepping on gas and break, checking the mirrors/traffic lights/pedestrians, looking for available parking slots. Then to our robot friend, in what ground can it understand these "sensible adaptions" of actions as being the "same"?

One might try to incorporate this kind of non-exact repeat by proposing some kind of function that measure the similarity/dissimilarity among the instances. For example, one might say if two list of actions both ends with stopping the car and getting out the car, we define a high similarity among them. But there are cases where our experience of repeat is not about the similarity of the observations at all. Consider a cityscape of New York where we find repetitions of buildings. Each building can be very different from the image perspective.

The reason we perceive repeat is that we interpret these visuals as concrete instances of the same abstract concept: building, which we know which properties and functions they should have. Thus, the core problem of the first definition is that it describes the phenomenon of the perceived repeat purely as a property of the observations themselves, without involving a mental model that explains and thus predict the observations. This problem can not be fixed without incorporating the mental understanding of the observations in our formulation of repeat.

1.1.2 Repeated abstractions

If we think of the "mental understanding" as the ability to associate a specific token with its category/type, our next approximation of repeat would be "instantiations of the same category/type".

$$[\![x_2 \textit{ repeats } x_1]\!] = \{\mathrm{Type}(x_1) = \mathrm{Type}(x_2)\} \qquad \text{(equal abstraction)}$$

By shifting the focus from observations to their abstractions, we can explain the postman's notion of "doing the same thing" as multiple instantiations of abstract action "parking the car" or "inserting mail to mailboxes" on different situation-encoding parameters such as "the location of the next closest parking slot" and "whether there are objects blocking the path". In the case of New York's cityscape, each appearance of the buildings can be thought of as an instantiated token from the type "building" using different configurations such as the

[bright]	[moon]	[pine]	[among]	[shine]	[clear]	[spring]	[stone]	[on]	[flow]
明	月	松	间	照	清	泉	石	上	流

Among pine-trees bright moonbeams peer; ***Over crystal stones flows water clear.***

Figure 1.1: Exerpts from the poem () written by Wang Wei (699761 during Tang dynasty), translated by Xu Yuanchong

building's height, style, year, and the angle facing the viewer, etc.

Despite the generality of the approach, it may sometimes ignore too much information for our experience of repeat. Take the following exerpt from a Chinese poem for example, despite the different in the concrete words, we see a clear parallelism (as a form of repeat) in the two phrases. Is it the abstraction that is repeated? Technically yes, since each of the two phrases can be understood syntactically as a noun phrase. But something very important is missing if we simply say these two phrases contains repeat just because they are both noun phrases. If we substitute the second phrase with any other noun phrase, our previous experience of parallelism imediatelly disappear. One can notice that their are some correpondence between the structure of this two phrases. In term of grouping, "[bright] [moon]" correponds to "[clear] [spring]" and "[pine] [among] [shine]" correponds to "[stone] [on] [flow]". Not only each of the corresponding character are from same syntactic category, the syntactic structures of the two phrases are identitcal as demonstrated in Figure 1.2. Furthermore, the meaning of the correponding words all aims to convey a feeling of serenity.

1.1.3 Repeated interpretations

1.1.4 Repetition within a structure

So far we have only concerned repetition as a *perceived* property of observations.

$$[\![y_2 \textit{ repeats } y_1]\!] = \{\exists R\ x_1'\ x_2'\ x_1\ x_2, \quad x_1' R x_1 \wedge x_2' R x_2\} \qquad \text{(Approximation 3)}$$

1.2 Structural Repeat

Our experience of repeat can be evalated by structure. In an informal way, we mean "structure" as "how things are put together" or equivalently, "how things can be elaborated". In language, the syntactic structure describes how words can combine to form a correct sentence, the semantic structures describes how meanings can combine to form new meanings. In action planning, structure describes how actions and goals can be broken into smaller components. In music, tonal structure describes how voices and harmony can be prolonged (elaborated through time).

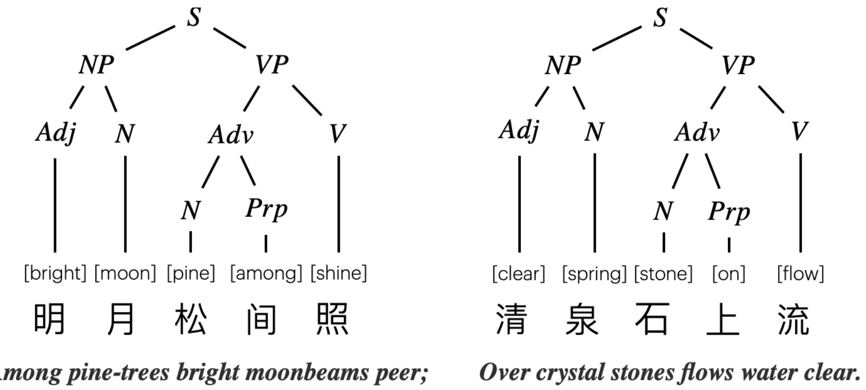

Figure 1.2: A poetry written by Wang Wei (699761 during Tang dynasty), translated by Xu Yuanchong, exhibiting parallel syntactic structure.

1.2.1 Example 1: Chinese Poetry

Parallel syntax is the phenomenon when two phrases have the identical syntactic structure. An example is shown in Figure 1.2. This kind of repeat is hardly a happy coincidence. It is often the case that the semantic meanings also forms either synonyms or contrast.

1.2.2 Example 2: Making coffee

Imagine John performed the following list of actions: he measures 18 grams of coffee ground, put these measured coffee ground into a coffee maker, he then measures 150 ml of water, and put the measured water into the coffee maker, then he activates the coffee machine.

The “structure” in this example is about the meaning of these actions: what does each action achieve and how does these list of actions accomplish a single goal? (How actions are put together). We analyze this scenario by interpreting an action as a particular way to achieve a goal. To make things simple, goals are interpreted as changes in the state of the world. For example, the overarching goal in this example is that John want to transform his state from "having coffee bean and water supply" into “having a cup of coffee”.

Goals can be combined in two general and natural ways, the first is sequential composition (chaining): if we know how to accomplish $x \to z$ and $z \to y$ we can accomplish the goal $x \to y$ by concatenating the two actions (also feeding the result of the action to the next action). The second is parallel composition, if we can accomplish $x_1 \to y_1$ and $x_2 \to y_2$, we can also accomplish $(x_1, x_2) \to (y_1, y_2)$. The tuple notation denotes the conjunction of states.

The action of measuring 18 grams of coffee bean is a solution of the goal “having a bag of coffee bean →having 18 grams of coffee bean”

The action of put

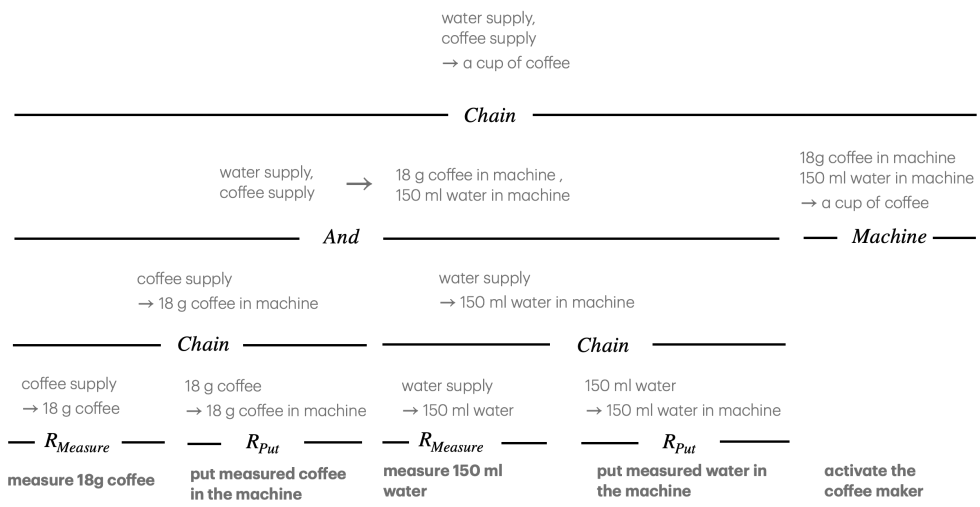

Figure 1.3

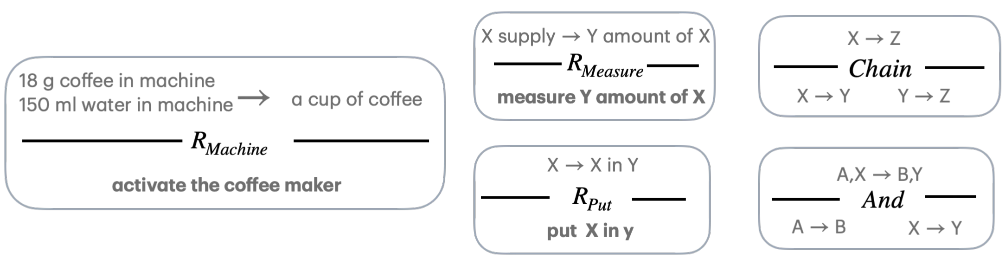

Figure 1.4

1.2.3 Making coffee

1.2.4 Comics

1.2.5 Classical Chinese poem

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