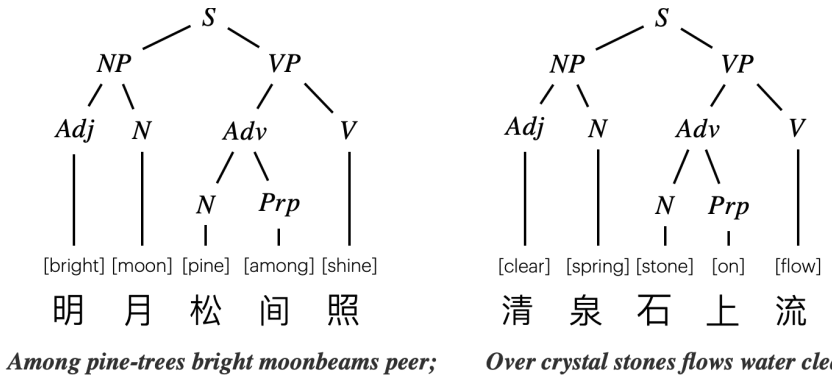

Figure 1

1 General repetition

Patterns are ubiquitous across domains, arising from the repetition of invariance—whether explicit or abstract—among their instances. From recognizing recurring architectural structures in cityscapes to identifying parallel syntactic forms in poetry, humans effortlessly detect and utilize patterns despite variations in their individual elements. This capacity extends to diverse domains: we discern shared narrative structures across novels, films, and music, and we uncover universal laws governing phenomena as distinct as ocean currents and atmospheric dynamics. Moreover, humans can abstract patterns of patterns, as seen in the application of category theory to mathematics and programming. These examples illustrate our remarkable ability to not only detect abstract repetitions but also to productively employ them in navigating both the physical and mental worlds. Such capacity is believed to be a fundamental aspect of human cognition (Pomiechowska et al., 2024).

The examples of poetry (Fig.1), music (Fig.2), and hierarchical planning (Fig.3), progressively illustrate the complexities of structural repetition and the computational mechanisms required to process them. In poetry, parallel syntactic structures demand a hierarchical interpretation of sequences and the ability to identify the repeated substructures, such as derivation trees. In the musical example, repeated computations can involve incomplete structures or "holes," where repetitions are not exact subtrees but partial ones. A top-down derivation reveals that notes $\hat{5} - \hat{4} - \hat{3}$ are first elaborated in the same way using ascending thirds (second row in Fig.2). This parallel construction then changes when the first two bars are further elaborated using neighboring motion whereas the third bar uses a passing motion (third row). Fig.3 presents a hierarchical planning of actions involved in the task of making a cup of coffee from its ingredients. Notice that the actions involving preparing coffee ground and preparing water are "repeated" not in the

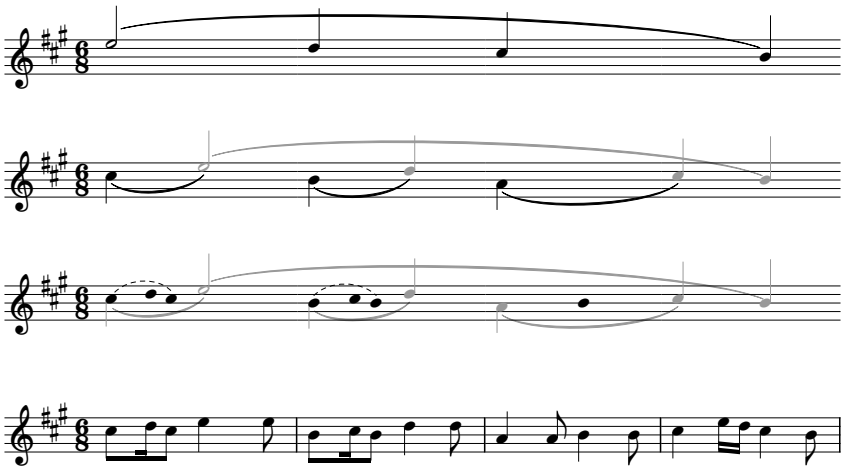

Figure 2

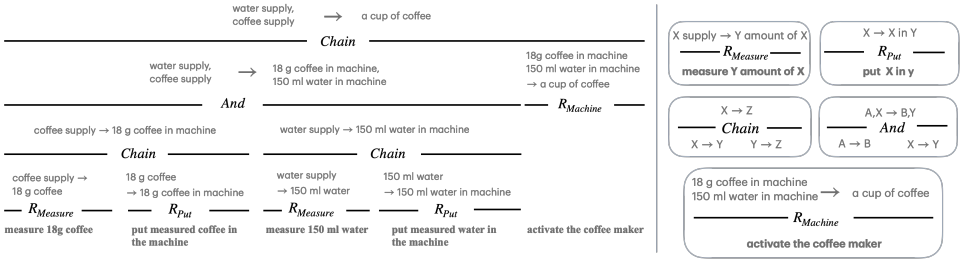

Figure 3

literal sense but in terms of the relational structure of the underlying tasks. This example highlights the role of relations as the basic repeating units, emphasizing the repetition of computational processes (e.g., polymorphic production rules) rather than the input/output states of the computation (e.g., (non)terminals symbols).

To summarize, we characterize structural repetition of hierarchical relations by two key properties. First, a single relation can manifest in multiple forms, similar to logical clauses involving meta-variables. Second, these relations can be inductively constructed via composition and duplications. Such construction process further entails the ability to express (a) incomplete/suspended computations and (b) various ways to repeat (the bindings of function variables in a composition expression).