Summary of Project 9

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1 Introduction

Setup. Consider K targets and N sensors deployed in a given area. Let $t_k \in \mathbf{R}^2$ be the position of target k ($k = 1, \ldots, K$) and $s_n \in \mathbf{R}^2$ be the position of sensor n ($n = 1, \ldots, N$).

Each sensor reports its distance to the closest target plus some noise. More specifically, letting $d_n \in \mathbf{R}$ ($n = 1, \ldots, N$) be the measurement reported by sensor n ($n = 1, \ldots, N$), we have

$$d_n = \|s_n - x_n\| + \text{noise}, \tag{1}$$

where $x_n \in \mathbf{R}^2$ is the position of the target closest to sensor n (thus, x_n is an element of set of the targets' positions $\{t_1, \ldots, t_K\}$). Note that, for a vector $v = (v_1, \ldots, v_d) \in \mathbf{R}^d$, the symbol $\|v\|$ denotes its Euclidean norm,

$$\|v\| = \left(v_1^2 + \cdots + v_d^2\right)^{1/2}.$$

We will start with $T = 2$ targets and $N = 64$ sensors. The sensors are arranged in an 8×8 grid, with the grid points separated by one unit both in the horizontal and vertical directions. We say that sensor n is a neighbor of sensor m if they are neighbors in the grid, that is, if $\|s_n - s_m\| = 1$. We use the notation $n \sim m$ to indicate that sensors n and m are neighbors. Note that most sensors have four neighbors, whereas other sensors have only two or three neighbors.

Goal. We are given the sensors' positions s_n ($n = 1, \ldots, N$) and their respective measurements d_n ($n = 1, \ldots, N$). We aim to find the targets' positions t_k ($k = 1, \ldots, K$).

2 Step 1

As explained in the meeting, we will start by solving the convex optimization problem

$$\begin{array}{ll} \underset{x_1,\ldots,x_N,y_1,\ldots,y_N}{\text{minimize}} & \sum_{n=1}^N \left(\alpha_n^T \begin{bmatrix} x_n \\ y_n \end{bmatrix} - \beta_n\right)^2 + \rho \sum_{1\le n\le N} \sum_{n+1\le m\le N, m\sim n} \left\| \begin{bmatrix} x_n \\ y_n \end{bmatrix} - \begin{bmatrix} x_m \\ y_m \end{bmatrix} \right\| \\ \text{subject to} & \|x_n\|^2 \le y_n, \quad n = 1, \ldots, N, \end{array} \tag{2}$$

where

$$\alpha_n := \begin{bmatrix} -2s_n \\ 1 \end{bmatrix} \in \mathbf{R}^3, \quad (n = 1, \ldots, N),$$

$\beta_n := y_n^2 - \|s_n\|^2$ $(n = 1, \ldots, N)$, and $\rho > 0$ is a given positive constant—in this project, use $\rho = 10$. The optimization variables are $x_n \in \mathbf{R}^2$ and $y_n \in \mathbf{R}$ for $n = 1, \ldots, N$. Note that, in the second sum of the cost function in (2), we sum only over neighbor sensors.

Implementation. In the MATLAB file you received, the positions of the sensors are the columns of the matrix `sensors` $\in \mathbf{R}^{2\times N}$; thus, the position of the nth sensor (denoted a_n above) is the nth column of the matrix `sensors`. The measurements are given in the vector `d` $\in \mathbf{R}^{1\times N}$; thus, the measurement of sensor n (denoted y_n above) is the nth entry of the vector `d`.

After you solve problem (2), you should check if your solution matches the one given in the matrix `X` $\in \mathbf{R}^{2\times N}$ and vector `y` $\in \mathbf{R}^{1\times N}$: your optimal x_n should be the nth column of matrix `X`, and your optimal y_n should be the nth entry of vector `y`.