ADM 2304 - Assignment 3

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1. Midterm Scores

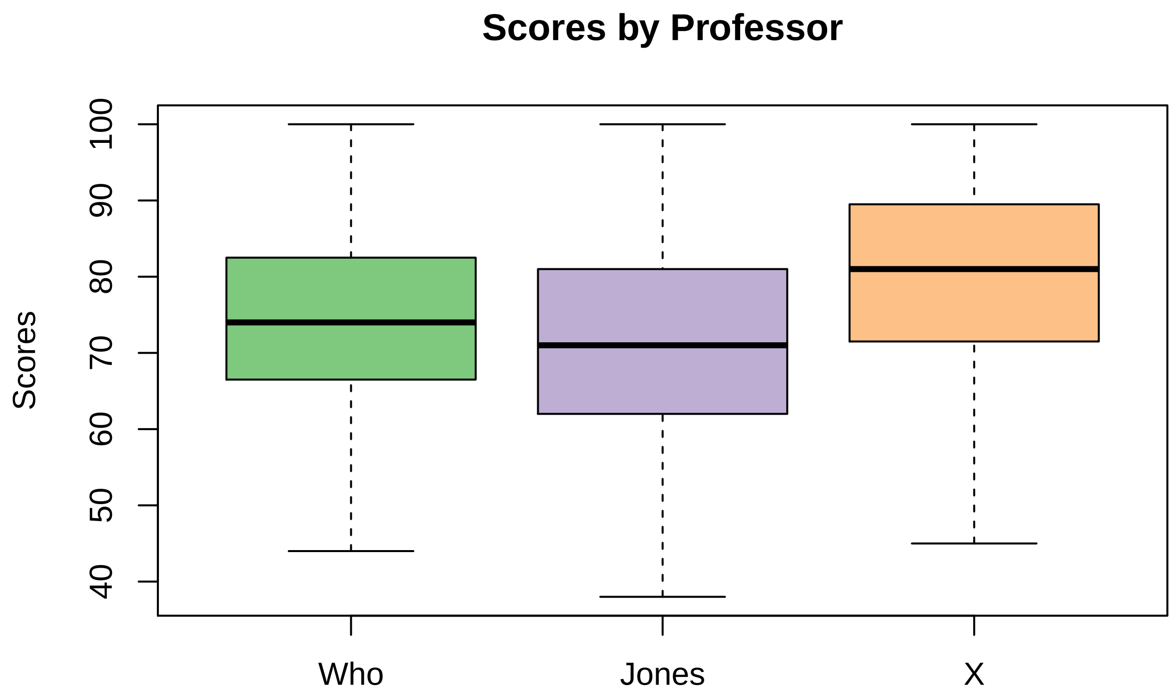
A Business School offers multiple sections of a second-year introduction to statistics course, which is taught by three instructors. Their names are Dr. Who, Dr. Jones (Jr.), and Prof. X. The dataset **Midterm Scores** contains the midterm scores of a random sample of students who took the course with one of these instructors over the last several years.The Business School would like to determine whether the average midterm score differs among these three instructors.

1.a

Create a side-by-side boxplot of the data and explain whether the similar variance and the nearly normality conditions for conducting an ANOVA seem to be satisfied.

Boxplots

```
##SBS Boxplots
boxplot(scores,
        main = "Scores by Professor",
        ylab = "Scores",
        col = brewer.pal(3,"Accent"))
```



The near normality condition for conducting an ANOVA seems to be satisfied. By looking at the side-by-side boxplot, we can see that the distributions for Dr. Who and Dr. Jones are fairly normal because of the equal length wicks on both sides of the boxes, although the Dr. Who boxplot has some very slight skewness (upper wick slightly shorter than lower wick) but it's not very severe, so it does not affect the near normality assumption. As for the Professor X boxplot, it does have more skewness (upper wick significantly shorter than lower wick) than the other two boxplots but it's only slight skewness, so it once again, does not affect the near normality condition.

The equal variance condition also seems to be satisfied. The Jones and X boxplots' box sizes seem to be roughly the same size, while the Who boxplot's box seems slightly smaller than the other two, however the difference is fairly small/negligible so the equal variance condition should still be satisfied.

1.b

> In addition to a side-by-side boxplot, what other graphs can you use to check whether the conditions for using an ANOVA are satisfied? Note: You don't need to produce the graphs; only explain how you would produce them.

To verify the equal variance condition, we can also graph a plot of residuals vs fitted values, which would first require us to first calculate the residual values ($\varepsilon_{ij} = X_{ij} - \mu_i$) and then plot them against the fitted values X_{ij} to look for patterns of the data points being unevenly spread. Furthermore, we can also use a boxplot of residuals to look for roughly same-sized box lengths to confirm the equal variance condition. We once again would need the residual values ($\varepsilon_{ij} = X_{ij} - \mu_i$) and then just plot them as a boxplot to do the comparison.

To verify the near-normality assumption, we can plot a normal probability of residuals plot to see whether they approximately align with the diagonal line of the plot. First, calculate the residual values (ε_{ij}) again using the same formulas as above and then just plot them as a normal probability plot. We can also use a histogram of residuals to see whether the histogram bars are relatively symmetric to confirm the near-normality assumption.

We, again, first have to calculate the residual values (ε_{ij}) and then plot them as a histogram to check for relative symmetry.

> **Answer the following questions, with exception of part g), assuming that the conditions for conducting an ANOVA are satisfied.**

1.c

> Use software to calculate the sample variance for each instructor and then use it to calculate the pooled variance manually. Verify that your pooled variance value is the same as the MSE value displayed in the partial ANOVA table in part d) below.

Sample Variances

```
##Sample variances
scores_who_var <-
  round(var(scores$Who), 3)
scores_who_var # Prof Who
```

```
## [1] 175.975
```

```
scores_jones_var <-
  round(var(scores$Jones), 3)
scores_jones_var # Prof Jones
```

```
## [1] 197.567
```

```
scores_x_var <-
  round(var(scores$X), 3)
scores_x_var # Prof X
```

```
## [1] 172.136
```

Pooled Variance

```
##Pooled variance
scores_pvar <- round(mean(c(
  scores_who_var,
  scores_jones_var,
  scores_x_var)), 2)
scores_pvar
```

```
## [1] 181.89
```

1.d

> Fill in manually the missing values (1) to (6) in the ANOVA table below. Show your computations (maximum of 2 decimal places). Check your results using software.

```
##Converting to long format
scores_long <- gather(
  scores, Professors, Scores)
head(as.data.frame(scores_long))
```

```
##   Professors Scores
## 1        Who     67
## 2        Who     59
```

```
## 3        Who    100
## 4        Who     81
## 5        Who     80
## 6        Who     63
```

```r
##1 way ANOVA
scores_anova <- aov(
  scores_long$Scores~scores_long$Professors,
  scores)
summary.aov(scores_anova)
```

```
##                          Df Sum Sq Mean Sq F value Pr(>F)
## scores_long$Professors   2   1449   724.7   3.984 0.0206 *
## Residuals              150  27284   181.9
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

1.e

> Use the one-way ANOVA table you produced in part d) to test whether there is a significant difference in the true mean midterm score among the three instructors. Use the critical value approach and a 5% confidence level.

Defining Hypotheses

$$\begin{cases} H_0 : \mu_{Who} = \mu_{Jones} = \mu_X \\ H_A : \text{At least one of the means is different} \end{cases}$$

The null hypothesis H_0 states that the means of midterm scores in all three of the professors' sections are the same. There is no variance in the means.

The alternate hypothesis H_A states that at least one of the means of the midterm scores among the three professors' sections are different. There is variance in the means.

Validating Test

```r
##Critical value
scores_cv <- round(qf(
  0.05,
  2,
  150,
  lower.tail = FALSE), 3)
scores_cv
```

```
## [1] 3.056
```

```r
##Validating test
summary.aov(scores_anova)[[1]][1,4] > scores_cv
```

```
## [1] TRUE
```

Since $F_{stat} > F_{\alpha,I-1,N-I} \rightarrow 3.98 > 3.06$, we reject the null hypothesis in favour of the alternative. There's sufficient evidence that the means of the midterm scores for the students taught by different professors are not all the same; there is at least one mean that is different from the others.

1.f

> Use the Bonferroni method for multiple comparisons to determine which population means differ (if any) at $\alpha = 0.05$. Show your computations and clearly state your conclusion for each pairwise comparison.

```r
##Bonferroni pairwise CIs
scores_bonf_ci <- PostHocTest(
  scores_anova,
  method = "bonferroni",
  conf.level = 0.95)
scores_bonf_ci
```

```
##
##   Posthoc multiple comparisons of means : Bonferroni
##     95% family-wise confidence level
##
## $`scores_long$Professors`
##                diff    lwr.ci    upr.ci   pval
## Who-Jones 4.098039 -2.368250 10.564328 0.3811
## X-Jones   7.529412  1.063123 13.995701 0.0164 *
## X-Who     3.431373 -3.034916  9.897661 0.6025
##
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

In order to establish that the means are significantly different from one another, 0 must be excluded from the interval such as:

$$0 \notin (\text{Lower bound, Upper bound})$$

Two of the pairwise comparisons have relatively similar means. The Jones-Who and X-Who pairs have similar means, while the X-Jones pair have significantly different means.

1.g

> Perform a Kruskal-Wallis non-parametric test to determine whether there is a difference in the midterm scores across the three instructors. Use a 5% significance level and the critical value approach. Is your
> conclusion consistent with your results in part e) above?

Defining Hypotheses

$$\begin{cases} H_0 : \theta_{Who} = \theta_{Jones} = \theta_X \\ H_A : \text{At least one of the medians is different} \end{cases}$$

The null hypothesis H_0 states that the medians of midterm scores in all three of the professors' sections are the same. There is no variance in the medians.

The alternate hypothesis H_A states that at least one of the medians of the midterm scores among the three professors' sections are different. There is variance in the medians.

Hypothesis Test

```r
##Kruskal-Wallis test
scores_kw_test <- kruskal.test(
  scores_long$Scores~
    scores_long$Professors,
  scores)
scores_kw_test
```

```
##
##  Kruskal-Wallis rank sum test
##
## data:  scores_long$Scores by scores_long$Professors
## Kruskal-Wallis chi-squared = 7.8399, df = 2, p-value = 0.01984
```

```r
##Critical value
scores_kw_cv <- round(qchisq(
  0.05,
  scores_kw_test$parameter,
  lower.tail = FALSE), 3)
scores_kw_cv
```

```
## [1] 5.991
```

Validating Test

```r
##Validating the test
scores_kw_test$statistic > scores_kw_cv
```

```
## Kruskal-Wallis chi-squared
##                       TRUE
```

Since the test statistic H_{stat} is greater than the critical value $\chi^2_{\alpha,I-1}$, ($H_{stat} > \chi^2_{0.05,2} \to 7.84 > 5.991$), we reject the null hypothesis in favour of the alternative. There is sufficient evidence to indicate that at least one of the medians for the midterm scores for the professors' students is different.

This result is consistence with what we obtained from the ANOVA test in part (e) because we also rejected the null hypothesis there (at least one of the means was different).

2. Utility Bills

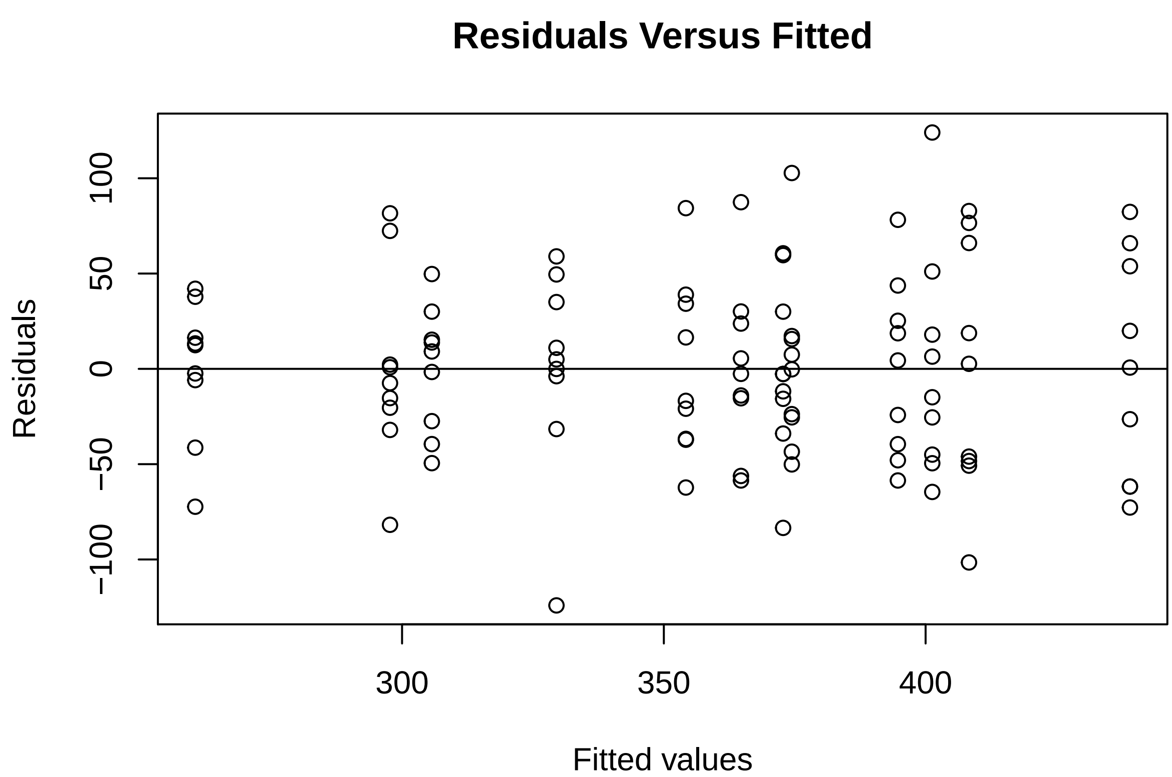
Supposed that, when comparing utility bills, a researcher was interested in determining whether residential utility bills differed among different cities in Canada, and also whether they differed depending on the number of bedrooms in a house. As part of the study, a random sample of households in different Canadian cities was selected, and their monthly utility bills and number of bedrooms were recorded. The data is provided in the dataset **Utility Bills**.

2.a

Plot the residuals against the fitted values corresponding to the two-way ANOVA model for this analysis. What two key model assumptions can be examined with this plot and do they appear to be warranted?

```r
##2 way ANOVA
util_bill_anova <- aov(
  util_bill$`Utility Bill ($)`~
    util_bill$Bedroom*util_bill$City,
  data = util_bill)
```

```r
##Calculating residuals
util_bill_res <- resid(util_bill_anova)
##Plotting residuals
plot(fitted(
  util_bill_anova),util_bill_res,
  main = "Residuals Versus Fitted",
  xlab = "Fitted values",
  ylab = "Residuals")
##Adding mean line
abline(0,0)
```



The two key model assumptions we can make using the residuals versus fitted values: **equal variance** and **near normality**.

The equal variance assumption in this dataset seems to be satisfied because the data looks reasonably scattered across the x-axis. There difference in the spreads depending on the fitted values is not that significant, thus this assumption seems to be satisfied.

The data points in the residuals versus fitted values plot seem to reasonably symmetric around 0 (the mean line) (above and below the line) with no significant outliers thus we can conclude that the near-normality assumption is satisfied.

2.b

> Test the following hypothesis at the 1% significance level. Use software to generate the corresponding two-way ANOVA table, but show any other computations.

- Significant interaction effect between number of bedrooms and city;
- Significant main effect of the number of bedrooms factor (if warranted);
- Significant main effect of the city factor (if warranted).

Interaction Between Factors A and B

$$\begin{cases} H_0 : \text{No interaction between factors A and B} \\ H_A : \text{Interaction between factors A and B} \end{cases}$$

The null hypothesis H_0 states that there is no interaction effect between the number of bedrooms and city.

The alternate hypothesis H_A states that there is an interaction effect between the number of bedrooms and the city.

```
##2 way ANOVA
summary.aov(util_bill_anova)
```

```
##                                    Df Sum Sq Mean Sq F value   Pr(>F)    
## util_bill$Bedroom                   2 187032   93516  37.710 8.21e-13 ***
## util_bill$City                      3  57339   19113   7.707 0.000115 ***
## util_bill$Bedroom:util_bill$City  6  21616    3603   1.453 0.202760    
## Residuals                          96 238068    2480                     
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Because the p-value is not smaller than the significance level ($p < \alpha \to 0.203 \not< 0.01$), we fail to reject the null hypothesis. There is insufficient to evidence to conclude that there is an interaction between the number of bedrooms and city.

Main Effect - Factor A

$$\begin{cases} H_0 : \alpha_i = 0 \quad \forall i \\ H_A : \alpha_i \neq 0 \quad \text{for some } i \end{cases}$$

Because the p-value of the Bedroom factor is smaller than the significance level (($\approx 0.000 < 0.01$)), we reject the null hypothesis in favour of the alternative. There is sufficient evidence to conclude that there is an effect due to the number of bedrooms.

Mean Effect - Factor B

$$\begin{cases} H_0 : \beta_j = 0 \quad \forall i \\ H_A : \beta_j \neq 0 \quad \text{for some } i \end{cases}$$

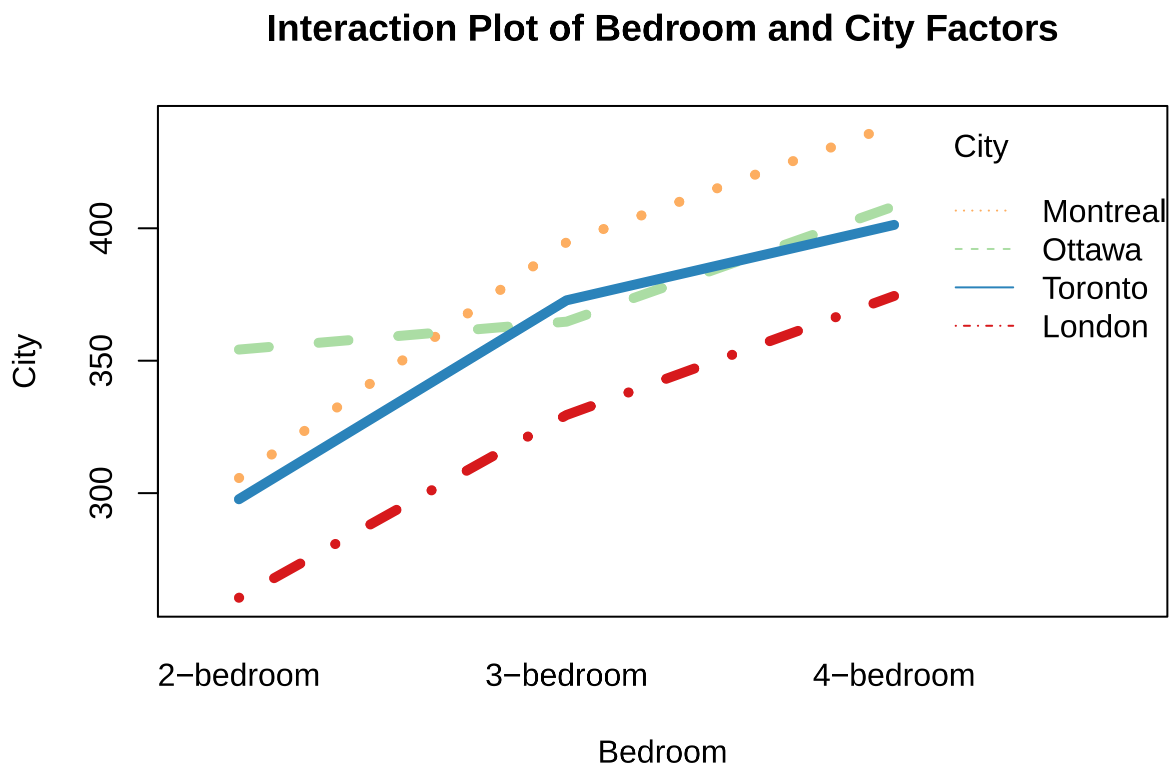
Because the p-value of the City factor is smaller than the significance level (($\approx 0.000 < 0.01$)), we reject the null hypothesis in favour of the alternative. There is sufficient evidence to conclude that there is an effect due to the city.

2.c

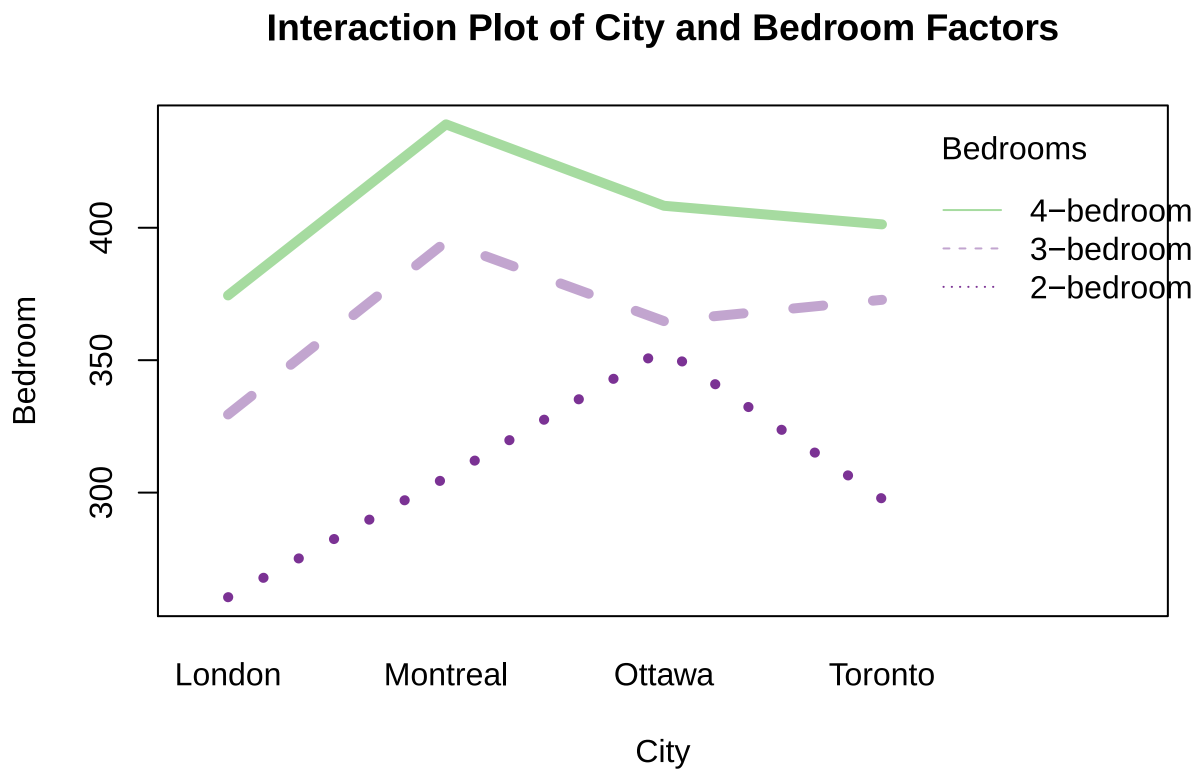
> Create the corresponding interaction plot and explain if it shows interaction between number of bedrooms and city. Is this consistent with your results for part b) above?

```
##Bedroom (x), City (y)
interaction.plot(
  util_bill$Bedroom,
 util_bill$City,
 util_bill$`Utility Bill ($)`,
```

```
main = "Interaction Plot of Bedroom and City Factors",
xlab = "Bedroom",
ylab = "City",
lwd = 5,
col = brewer.pal(
  4,
  "Spectral"),
fun = "mean",
trace.label = "City")
```




```
##City (x), Bedroom (y)
interaction.plot(
  util_bill$City,
  util_bill$Bedroom,
  util_bill$`Utility Bill ($)`,
  main = "Interaction Plot of City and Bedroom Factors",
  xlab = "City",
  ylab = "Bedroom",
  lwd = 5,
  col = brewer.pal(4,"PRGn"),
  fun = "mean",
  trace.label = "Bedrooms")
```



Although the bedroom effect plot seems to have some lines crossing each other, those interactions are not necessarily significant; thus, we can say that the interaction plot is consistent with the results obtained from the hypothesis test in part (b). There is no significant interaction between the number of bedrooms and the city factors.

2.d

> Calculate the Bonferroni margin of error for the confidence intervals based on all pairwise differences between the treatment means. Show your manual calculations and use an overall 95% confidence level.

```
##Number of pairwise comparisons
adj_alpha <- round(
  0.05 / (2*choose(12,2)),6)

##T-value for Bonferroni CI
bonf_ci_t_val <- qt(
  adj_alpha,
  util_bill_anova[["df.residual"]])

##Bonferroni CI ME
util_bonf_me <- round(
  -bonf_ci_t_val *
    sqrt(2480*(1/9+1/9)),2)
util_bonf_me
```

```
## [1] 81.68
```

2.e

Using the calculated margin of error from part d) and the pairwise confidence interval approach, determine whether there is sufficient evidence (at the 5% significance level) of a difference in mean monthly utility bills between 2-bedroom houses in Ottawa and 2-bedroom houses in London

```r
##Pairwise Bonf CI
util_bonf_pairs <- as.list(PostHocTest(
  util_bill_anova,
  method = "bonferroni",
  conf.level = 0.95,
  digits = 4,
  ordered = TRUE))
head(util_bonf_pairs[3], 5)
```

```
## $`util_bill$Bedroom:util_bill$City`
##                                             diff      lwr.ci     upr.ci
## 3-bedroom:London-2-bedroom:London       68.998889  -12.678690 150.676468
## 4-bedroom:London-2-bedroom:London      113.948889   32.271310 195.626468
## 2-bedroom:Montreal-2-bedroom:London     45.188889  -36.488690 126.866468
## 3-bedroom:Montreal-2-bedroom:London    134.211111   52.533532 215.888690
## 4-bedroom:Montreal-2-bedroom:London    178.542222   96.864644 260.219801
## 2-bedroom:Ottawa-2-bedroom:London       93.713333   12.035755 175.390912
## 3-bedroom:Ottawa-2-bedroom:London      104.241111   22.563532 185.918690
## 4-bedroom:Ottawa-2-bedroom:London      147.794444   66.116866 229.472023
## 2-bedroom:Toronto-2-bedroom:London      37.201111  -44.476468 118.878690
## 3-bedroom:Toronto-2-bedroom:London     112.292222   30.614644 193.969801
## 4-bedroom:Toronto-2-bedroom:London     140.776667   59.099088 222.454245
## 4-bedroom:London-3-bedroom:London       44.950000  -36.727579 126.627579
## 2-bedroom:Montreal-3-bedroom:London    -23.810000 -105.487579  57.867579
## 3-bedroom:Montreal-3-bedroom:London     65.212222  -16.465356 146.889801
## 4-bedroom:Montreal-3-bedroom:London    109.543333   27.865755 191.220912
## 2-bedroom:Ottawa-3-bedroom:London       24.714444  -56.963134 106.392023
## 3-bedroom:Ottawa-3-bedroom:London       35.242222  -46.435356 116.919801
## 4-bedroom:Ottawa-3-bedroom:London       78.795556   -2.882023 160.473134
## 2-bedroom:Toronto-3-bedroom:London     -31.797778 -113.475356  49.879801
## 3-bedroom:Toronto-3-bedroom:London      43.293333  -38.384245 124.970912
## 4-bedroom:Toronto-3-bedroom:London      71.777778   -9.899801 153.455356
## 2-bedroom:Montreal-4-bedroom:London    -68.760000 -150.437579  12.917579
## 3-bedroom:Montreal-4-bedroom:London     20.262222  -61.415356 101.939801
## 4-bedroom:Montreal-4-bedroom:London     64.593333  -17.084245 146.270912
## 2-bedroom:Ottawa-4-bedroom:London      -20.235556 -101.913134  61.442023
## 3-bedroom:Ottawa-4-bedroom:London       -9.707778  -91.385356  71.969801
## 4-bedroom:Ottawa-4-bedroom:London       33.845556  -47.832023 115.523134
## 2-bedroom:Toronto-4-bedroom:London     -76.747778 -158.425356   4.929801
## 3-bedroom:Toronto-4-bedroom:London      -1.656667  -83.334245  80.020912
## 4-bedroom:Toronto-4-bedroom:London      26.827778  -54.849801 108.505356
## 3-bedroom:Montreal-2-bedroom:Montreal   89.022222    7.344644 170.699801
## 4-bedroom:Montreal-2-bedroom:Montreal  133.353333   51.675755 215.030912
## 2-bedroom:Ottawa-2-bedroom:Montreal     48.524444  -33.153134 130.202023
## 3-bedroom:Ottawa-2-bedroom:Montreal     59.052222  -22.625356 140.729801
## 4-bedroom:Ottawa-2-bedroom:Montreal    102.605556   20.927977 184.283134
## 2-bedroom:Toronto-2-bedroom:Montreal    -7.987778  -89.665356  73.689801
## 3-bedroom:Toronto-2-bedroom:Montreal    67.103333  -14.574245 148.780912
## 4-bedroom:Toronto-2-bedroom:Montreal    95.587778   13.910199 177.265356
```

```
## 4-bedroom:Montreal-3-bedroom:Montreal   44.331111  -37.346468 126.008690
## 2-bedroom:Ottawa-3-bedroom:Montreal    -40.497778 -122.175356  41.179801
## 3-bedroom:Ottawa-3-bedroom:Montreal    -29.970000 -111.647579  51.707579
## 4-bedroom:Ottawa-3-bedroom:Montreal     13.583333  -68.094245  95.260912
## 2-bedroom:Toronto-3-bedroom:Montreal   -97.010000 -178.687579 -15.332421
## 3-bedroom:Toronto-3-bedroom:Montreal   -21.918889 -103.596468  59.758690
## 4-bedroom:Toronto-3-bedroom:Montreal     6.565556  -75.112023  88.243134
## 2-bedroom:Ottawa-4-bedroom:Montreal    -84.828889 -166.506468  -3.151310
## 3-bedroom:Ottawa-4-bedroom:Montreal    -74.301111 -155.978690   7.376468
## 4-bedroom:Ottawa-4-bedroom:Montreal    -30.747778 -112.425356  50.929801
## 2-bedroom:Toronto-4-bedroom:Montreal  -141.341111 -223.018690 -59.663532
## 3-bedroom:Toronto-4-bedroom:Montreal   -66.250000 -147.927579  15.427579
## 4-bedroom:Toronto-4-bedroom:Montreal   -37.765556 -119.443134  43.912023
## 3-bedroom:Ottawa-2-bedroom:Ottawa       10.527778  -71.149801  92.205356
## 4-bedroom:Ottawa-2-bedroom:Ottawa       54.081111  -27.596468 135.758690
## 2-bedroom:Toronto-2-bedroom:Ottawa     -56.512222 -138.189801  25.165356
## 3-bedroom:Toronto-2-bedroom:Ottawa      18.578889  -63.098690 100.256468
## 4-bedroom:Toronto-2-bedroom:Ottawa      47.063333  -34.614245 128.740912
## 4-bedroom:Ottawa-3-bedroom:Ottawa       43.553333  -38.124245 125.230912
## 2-bedroom:Toronto-3-bedroom:Ottawa     -67.040000 -148.717579  14.637579
## 3-bedroom:Toronto-3-bedroom:Ottawa       8.051111  -73.626468  89.728690
## 4-bedroom:Toronto-3-bedroom:Ottawa      36.535556  -45.142023 118.213134
## 2-bedroom:Toronto-4-bedroom:Ottawa    -110.593333 -192.270912 -28.915755
## 3-bedroom:Toronto-4-bedroom:Ottawa     -35.502222 -117.179801  46.175356
## 4-bedroom:Toronto-4-bedroom:Ottawa      -7.017778  -88.695356  74.659801
## 3-bedroom:Toronto-2-bedroom:Toronto     75.091111   -6.586468 156.768690
## 4-bedroom:Toronto-2-bedroom:Toronto    103.575556   21.897977 185.253134
## 4-bedroom:Toronto-3-bedroom:Toronto     28.484444  -53.193134 110.162023
##                                               pval
## 3-bedroom:London-2-bedroom:London     2.720133e-01
## 4-bedroom:London-2-bedroom:London     3.090595e-04
## 2-bedroom:Montreal-2-bedroom:London   1.000000e+00
## 3-bedroom:Montreal-2-bedroom:London   8.057028e-06
## 4-bedroom:Montreal-2-bedroom:London   1.269410e-09
## 2-bedroom:Ottawa-2-bedroom:London     8.460759e-03
## 3-bedroom:Ottawa-2-bedroom:London     1.584368e-03
## 4-bedroom:Ottawa-2-bedroom:London     6.040953e-07
## 2-bedroom:Toronto-2-bedroom:London    1.000000e+00
## 3-bedroom:Toronto-2-bedroom:London    4.108044e-04
## 4-bedroom:Toronto-2-bedroom:London    2.332740e-06
## 4-bedroom:London-3-bedroom:London     1.000000e+00
## 2-bedroom:Montreal-3-bedroom:London   1.000000e+00
## 3-bedroom:Montreal-3-bedroom:London   4.343072e-01
## 4-bedroom:Montreal-3-bedroom:London   6.554274e-04
## 2-bedroom:Ottawa-3-bedroom:London     1.000000e+00
## 3-bedroom:Ottawa-3-bedroom:London     1.000000e+00
## 4-bedroom:Ottawa-3-bedroom:London     7.471687e-02
## 2-bedroom:Toronto-3-bedroom:London    1.000000e+00
## 3-bedroom:Toronto-3-bedroom:London    1.000000e+00
## 4-bedroom:Toronto-3-bedroom:London    1.907779e-01
## 2-bedroom:Montreal-4-bedroom:London   2.803113e-01
## 3-bedroom:Montreal-4-bedroom:London   1.000000e+00
## 4-bedroom:Montreal-4-bedroom:London   4.680155e-01
## 2-bedroom:Ottawa-4-bedroom:London     1.000000e+00
```

```
## 3-bedroom:Ottawa-4-bedroom:London     1.000000e+00
## 4-bedroom:Ottawa-4-bedroom:London     1.000000e+00
## 2-bedroom:Toronto-4-bedroom:London    9.881763e-02
## 3-bedroom:Toronto-4-bedroom:London    1.000000e+00
## 4-bedroom:Toronto-4-bedroom:London    1.000000e+00
## 3-bedroom:Montreal-2-bedroom:Montreal 1.722466e-02
## 4-bedroom:Montreal-2-bedroom:Montreal 9.455463e-06
## 2-bedroom:Ottawa-2-bedroom:Montreal   1.000000e+00
## 3-bedroom:Ottawa-2-bedroom:Montreal   8.940452e-01
## 4-bedroom:Ottawa-2-bedroom:Montreal   2.069615e-03
## 2-bedroom:Toronto-2-bedroom:Montreal  1.000000e+00
## 3-bedroom:Toronto-2-bedroom:Montreal  3.445818e-01
## 4-bedroom:Toronto-2-bedroom:Montreal  6.328540e-03
## 4-bedroom:Montreal-3-bedroom:Montreal 1.000000e+00
## 2-bedroom:Ottawa-3-bedroom:Montreal   1.000000e+00
## 3-bedroom:Ottawa-3-bedroom:Montreal   1.000000e+00
## 4-bedroom:Ottawa-3-bedroom:Montreal   1.000000e+00
## 2-bedroom:Toronto-3-bedroom:Montreal  5.065291e-03
## 3-bedroom:Toronto-3-bedroom:Montreal  1.000000e+00
## 4-bedroom:Toronto-3-bedroom:Montreal  1.000000e+00
## 2-bedroom:Ottawa-4-bedroom:Montreal   3.188215e-02
## 3-bedroom:Ottawa-4-bedroom:Montreal   1.371152e-01
## 4-bedroom:Ottawa-4-bedroom:Montreal   1.000000e+00
## 2-bedroom:Toronto-4-bedroom:Montreal  2.094548e-06
## 3-bedroom:Toronto-4-bedroom:Montreal  3.827250e-01
## 4-bedroom:Toronto-4-bedroom:Montreal  1.000000e+00
## 3-bedroom:Ottawa-2-bedroom:Ottawa     1.000000e+00
## 4-bedroom:Ottawa-2-bedroom:Ottawa     1.000000e+00
## 2-bedroom:Toronto-2-bedroom:Ottawa    1.000000e+00
## 3-bedroom:Toronto-2-bedroom:Ottawa    1.000000e+00
## 4-bedroom:Toronto-2-bedroom:Ottawa    1.000000e+00
## 4-bedroom:Ottawa-3-bedroom:Ottawa     1.000000e+00
## 2-bedroom:Toronto-3-bedroom:Ottawa    3.472882e-01
## 3-bedroom:Toronto-3-bedroom:Ottawa    1.000000e+00
## 4-bedroom:Toronto-3-bedroom:Ottawa    1.000000e+00
## 2-bedroom:Toronto-4-bedroom:Ottawa    5.487226e-04
## 3-bedroom:Toronto-4-bedroom:Ottawa    1.000000e+00
## 4-bedroom:Toronto-4-bedroom:Ottawa    1.000000e+00
## 3-bedroom:Toronto-2-bedroom:Toronto   1.234504e-01
## 4-bedroom:Toronto-2-bedroom:Toronto   1.766860e-03
## 4-bedroom:Toronto-3-bedroom:Toronto   1.000000e+00
```

```
##Ottawa 2bd - London 2bd pairwise CI
ott_2bd_lon_2bd <- as.vector(c(row.names(
  util_bonf_pairs$`util_bill$Bedroom:util_bill$City`)[6],
  util_bonf_pairs$`util_bill$Bedroom:util_bill$City`[6,]))
ott_2bd_lon_2bd
```

```
## [1] "2-bedroom:Ottawa-2-bedroom:London" "93.7133333333337"
## [3] "12.0357547211587"                  "175.390911945509"
## [5] "0.00846075869021918"
```

Since 0 is not included within the interval, $0 \notin (12.0, 175.4)$, there is sufficient evidence that there is a significant difference in the mean monthly utility bills paid between a 2-bedroom Ottawa household and a 2-bedroom London household.