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3. $y'' + y' = 0$

Dimisalkan

$y = e^{rx}$

$y' = re^{rx}$

$y'' = r^2 e^{rx}$

$(r^2 e^{rx}) + (re^{rx}) = 0$

$e^{rx}(r^2 + r) = 0$

$r^2 + r = 0$

$r(r+1) = 0$

$r_1 = 0 \vee r_2 = -1$ → akar real

$y = c_1 e^{(0)x} + c_2 e^{(-1)x}$

$y = c_1 + c_2 e^{-x}$

6. $xy'' + 2y' + xy = 0,$

$y'' + \frac{2}{x} y' + y = 0$

$y_1 = \frac{\cos x}{x}$

diperoleh

$P(x) = \frac{2}{x}, \quad q(x) = 1$

$y - y_2 = x y_1 = x \frac{\cos x}{x}$

misal $x = \int x$

maka $x = \int \frac{1}{y_1^2} e^{-\int p\,dx}$

$= \int \frac{1}{(\frac{\cos x}{x})^2} e^{-\int \frac{2}{v} dx}$

$= \int \frac{x^2}{\cos^2 x} \cdot e^{-2\ln x} dx$

$= \int \frac{x^2}{\cos^2 x} x^{-2}$

$= \int \sec^2 x\,dx$

$= \tan x$

Substitusi nilai y_2

$y_2 = x y_1$

$= \tan x \frac{\cos x}{x}$

$= \frac{\sin x}{\cos x} \cdot \frac{\cos x}{x}$

$= \frac{\sin x}{x}$

sehingga diperoleh general solution

$y = c_1 \frac{\cos x}{v} + c_2 \frac{\sin x}{x}$

$= \frac{1}{x}(c_1 \cos x + c_2 \sin x)$

11. $y'' = 2y'$

$y'' - 2y' = 0$

misal

$y = e^{rx}$

$y' = re^{rx}$

$y'' = r^2 e^{rx}$

$r^2 e^{rx} - 2re^{rx} = 0$

$r^2 - 2r = 0$

$r(r-2) = 0$

$r_1 = 0 \vee r_2$ -> akar real

Sehingga

$y = C_1 e^{r_1 x} + C_2 e^{r_2 x}$

$= C_1 e^{0x} + C_2 e^{2x}$

$= C_1 + C_2 e^{2x}$

Gradien singgung di titik asal melalui gradien 1

$y' = 2C_2 e^{2x}$

$y'(0) = 2C_2 e^0$

$1 = 2C_2$

$C_2 = \frac{1}{2}$

kurva y yang melewati titik $(0, 0)$

$y(0) = C_1 + \frac{1}{2} e^{2(0)}$

$0 = C_1 + \frac{1}{2}$

$C_1 = -\frac{1}{2}$

Maka persamaan solusinya

$x = -\frac{1}{2} + \frac{1}{2} e^{2x}$

$= \frac{1}{2}(e^{2x} - 1)$

16. $y'' + 0,6y' + 0,09y = 0$

$y'(0) = 0,14$, $y(0) = 2,2$

$y_1 = e^{-0,3x}$, $xe^{-0,3x}$

a. $\frac{y_1}{y_2} = \frac{e^{-0,3x}}{xe^{-0,3x}}$

$= \frac{1}{x}$ -> tidak konstan

Sehingga $e^{-0,3x}$ dan $xe^{-0,3x}$ linear independen di semua interval

b. $y = C_1 e^{-0,3x} + C_2 x e^{-0,3x}$

$y' = -0,3C_1 e^{-0,3x} - 0,3C_2 e^{-0,3x} + C_2 e^{-0,3x}$

- ketika $y(0) = 2,2$

$2,2 = C_1 e^{-0,3(0)} + C_2(0) e^{-0,3(0)}$

$C_1 = 2,2$

- ketika $y'(0) = 0,14$

$0,14 = -0,3C_1 e^0 - 0,3(0)e^0 + C_2 e^0$

$0,14 = -0,66e^0 + C_2 e^0$

$C_2 = 0,8$

Sehingga persamaan kurva

$y = 2,2e^{-0,3x} + 0,8xe^{-0,3x}$

ketika, $y = 0$, maka $x = -2,75$

$x = 0$, maka $y = 2,2$