

Latihan 1

1. Tentukan panjang dan besar vektor berikut:

$$\begin{aligned} \text{a. } \vec{a} = \begin{bmatrix} -12 \\ 5 \end{bmatrix} &= \sqrt{a_1^2 + a_2^2} \\ &= \sqrt{(-12)^2 + (5)^2} \\ &= \sqrt{144 + 25} \\ &= \underline{\underline{13}} \end{aligned}$$

$$\begin{aligned} \text{b. } \vec{b} = \begin{bmatrix} 7 \\ 24 \end{bmatrix} &= \sqrt{a_1^2 + a_2^2} \\ &= \sqrt{(7)^2 + (24)^2} \\ &= \sqrt{49 + 576} \\ &= \sqrt{625} \\ &= \underline{\underline{25}} \end{aligned}$$

$$\begin{aligned} \text{c. } \vec{c} = \begin{bmatrix} -10 \\ -24 \end{bmatrix} &= \sqrt{a_1^2 + a_2^2} \\ &= \sqrt{(-10)^2 + (-24)^2} \\ &= \sqrt{100 + 576} \\ &= \sqrt{676} \\ &= \underline{\underline{26}} \end{aligned}$$

2. Diketahui $\vec{a} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} -8 \\ 2 \end{bmatrix}$, $\vec{c} = \begin{bmatrix} 10 \\ -9 \end{bmatrix}$, $\vec{d} = \begin{bmatrix} -16 \\ -3 \end{bmatrix}$. tentukan

vektor berikut:

$$\begin{aligned} \text{a. } 5\vec{a} + 2\vec{b} &= 5 \begin{bmatrix} 6 \\ 4 \end{bmatrix} + 2 \begin{bmatrix} -8 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} 30 \\ 20 \end{bmatrix} + \begin{bmatrix} -16 \\ 4 \end{bmatrix} \\ &= \underline{\underline{\begin{bmatrix} 14 \\ 24 \end{bmatrix}}} \end{aligned}$$

$$\begin{aligned} b. -3\vec{c} + 4\vec{a} &= -3 \begin{bmatrix} 10 \\ -9 \end{bmatrix} + 4 \begin{bmatrix} 6 \\ 4 \end{bmatrix} \\ &= \begin{bmatrix} -30 \\ 27 \end{bmatrix} + \begin{bmatrix} 24 \\ 16 \end{bmatrix} \\ &= \begin{bmatrix} -6 \\ 43 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} c. 2\vec{a} - 4\vec{b} &= 2 \begin{bmatrix} -16 \\ -3 \end{bmatrix} - 4 \begin{bmatrix} -8 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} -32 - (-32) \\ -6 - 8 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ -14 \end{bmatrix} \end{aligned}$$

3 Tentukan cosinus sudut dari dua vektor berikut ini!

$$a. \vec{a} = \begin{bmatrix} -12 \\ 5 \end{bmatrix} \text{ dan } \vec{b} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$\begin{aligned} a. \vec{a} \cdot \vec{b} &= \begin{bmatrix} -12 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix} = (-12 \times 3) + (5 \times 4) \\ &= -36 + 20 \\ &= -16 \end{aligned}$$

$$\begin{aligned} b. |\vec{a}| &= \sqrt{(-12)^2 + 5^2} & c. |\vec{b}| &= \sqrt{3^2 + (-4)^2} \\ &= \sqrt{144 + 25} & &= \sqrt{9 + 16} \\ &= \sqrt{169} & &= \sqrt{25} \\ &= 13 & &= 5 \end{aligned}$$

$$\text{maka: } \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{-16}{13 \times 5} = \frac{-16}{65}$$

$$b. \vec{p} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \text{ dan } \vec{q} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$a. \vec{p} \cdot \vec{q} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 3 \end{bmatrix} = (1 \times 4) + (-1 \times 3) = 4 - 3 = 1$$

$$b. |\vec{p}| = \sqrt{1^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$$

$$c. |\vec{q}| = \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$$\text{maka } \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{1}{\sqrt{2} \cdot 5} = \frac{1}{5\sqrt{2}}$$

$$c. \vec{R} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{T} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$$

$$a. \vec{R} \cdot \vec{T} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ -2 \end{bmatrix} = (2 \times -3) + (3 \times -2) = -6 - 6 = -12$$

$$b. |\vec{R}| = \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$$

$$c. |\vec{T}| = \sqrt{(-3)^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$$

$$\text{maka } \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta = \frac{-12}{\sqrt{13} \times \sqrt{13}} = \frac{-12}{13}$$

4. Tentukan vektor satuan dari

$$a. \vec{p} = 6\vec{i} + 8\vec{j} = \frac{1}{10} \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

$$|\vec{p}| = \sqrt{6^2 + 8^2} = \sqrt{36+64} = \sqrt{100} = 10$$

$$= \begin{bmatrix} 6/10 \\ 8/10 \end{bmatrix}$$

No.

Date:

$$\begin{aligned}
 b. \vec{a} &= 12\vec{i} - 5\vec{j} &= \frac{1}{13} \begin{bmatrix} 12 \\ -5 \end{bmatrix} \\
 |\vec{a}| &= \sqrt{12^2 + (-5)^2} \\
 &= \sqrt{144 + 25} &= \begin{bmatrix} 12/13 \\ -5/13 \end{bmatrix} \\
 &= \sqrt{169} \\
 &= 13
 \end{aligned}$$

c. Jika vektor \vec{a} sama dengan vektor \vec{b} , tentukan x dan y berikut:

$$a. \vec{a} = (x+2y)\vec{i} + (-x-y)\vec{j} \text{ dan } \vec{b} = 4\vec{i} - \vec{j}$$

$$(1) x+2y=4 \Rightarrow x+2y=4$$

$$(2) -x-y=-1 \Rightarrow -x-y=-1$$

$$y=3$$

$$\text{maka } x+2(3)=4 \quad x=-2$$

$$x+6=4 \Rightarrow y=3$$

$$x=-2$$

$$b. \vec{a} = (2x-y)\vec{i} + \vec{j} \text{ dan } \vec{b} = 3\vec{i} - (-x-y)\vec{j}$$

$$(1) 2x-y=3 \Rightarrow 2x-y=3$$

$$(2) x+y=-7 \Rightarrow x+y=-7$$

$$3x=-4$$

$$x=-\frac{4}{3}$$

$$\text{maka } -\frac{4}{3}+y=-7$$

$$y=-7+\frac{4}{3} = \frac{-21+4}{3} = \frac{-17}{3} \Rightarrow x=-\frac{4}{3} \quad y=-\frac{17}{3}$$

6. Diketahui $\vec{a} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, dan $\vec{b} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$. Tentukan:

$$\begin{aligned}
 a. |\vec{a}| &= \sqrt{3^2 + 2^2} \\
 &= \sqrt{9+4} \\
 &= \sqrt{13}
 \end{aligned}$$

$$\begin{aligned}
 b. |\vec{b}| &= \sqrt{(-1)^2 + 4^2} \\
 &= \sqrt{1+16} \\
 &= \sqrt{17}
 \end{aligned}$$

$$c. |\vec{a} + \vec{b}| = \begin{bmatrix} 3 \\ -2 \end{bmatrix} + \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \sqrt{2^2 + 2^2}$$

$$= \sqrt{8} = 2\sqrt{2}$$

d. apakah $|\vec{a}| + |\vec{b}| = |\vec{a} + \vec{b}|$

$$|\vec{a}| + |\vec{b}| = \sqrt{13} + \sqrt{17} \neq \sqrt{30}$$

$$|\vec{a} + \vec{b}| = \begin{bmatrix} 3 \\ -2 \end{bmatrix} + \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \sqrt{2^2 + 2^2}$$

$$= \sqrt{4 + 4}$$

$$= \sqrt{8} = 2\sqrt{2}$$

Hasilnya Tidak sama

7. Diketahui $\vec{a} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$, tentukan:

$$a. |\vec{a}| = \sqrt{3^2 + (-2)^2} \quad b. |\vec{b}| = \sqrt{(-1)^2 + 4^2}$$

$$= \sqrt{9 + 4} \quad = \sqrt{1 + 16}$$

$$= \sqrt{13} \quad = \sqrt{17}$$

$$c. |\vec{a} - \vec{b}| = \begin{bmatrix} 3 \\ -2 \end{bmatrix} - \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ -6 \end{bmatrix} = \sqrt{4^2 + (-6)^2} = \sqrt{52}$$

$$= \sqrt{16 + 36}$$

d. Apakah $|\vec{a}| - |\vec{b}| = |\vec{a} - \vec{b}|$

Tidak

8 Diketahui $\vec{a} = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 12 \\ 3 \end{bmatrix}$. Tentukan nilai $\vec{a} \cdot \vec{b}$:

$$\begin{bmatrix} 4 \\ -10 \end{bmatrix} \cdot \begin{bmatrix} 12 \\ 3 \end{bmatrix} = (4 \times 12) + (-10 \times 3) \\ = (48 + (-30)) \\ = 18$$

9 Jika diketahui $|\vec{a}| = 4$ dan $|\vec{b}| = 8$ serta $\angle(\vec{a}, \vec{b}) = 45^\circ$. Tentukan nilai

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \angle(\vec{a}, \vec{b}) \\ = (4)(8) \cos 45^\circ \\ = 32 \cdot \frac{1}{\sqrt{2}} \\ = 16\sqrt{2}$$

10 Diketahui $|\vec{a}| = 4$, $|\vec{b}| = 5$ serta $|\vec{a} + \vec{b}| = 6$, tentukan nilai dari $|\vec{a} - \vec{b}|$

Diketahui:

$$\begin{aligned} |\vec{a}| &= 4 & \Rightarrow |\vec{a} + \vec{b}|^2 &= 6^2 \\ |\vec{b}| &= 5 & |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 &= 36 \\ |\vec{a} + \vec{b}| &= 6 & |4|^2 + 2\vec{a} \cdot \vec{b} + |5|^2 &= 36 \\ & & 16 + 2\vec{a} \cdot \vec{b} + 25 &= 36 \\ & & 2\vec{a} \cdot \vec{b} + 41 &= 36 \\ & & 2\vec{a} \cdot \vec{b} &= 36 - 41 \\ & & 2\vec{a} \cdot \vec{b} &= -5 \end{aligned}$$

$$\begin{aligned} |\vec{a} - \vec{b}|^2 &= |\vec{a}|^2 - 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 \\ &= |4|^2 - (-5) + |5|^2 \\ &= 16 + 5 + 25 \\ &= 46 \end{aligned}$$

$$|\vec{a} - \vec{b}| = \sqrt{46}$$