



THE DEVELOPER'S  
CONFERENCE

# Otimização de Decisões em Python

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Cansado disso?



E disso?



E disso?



# Seus problemas acabaram!



# 3 simples passos



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# 3 simples passos

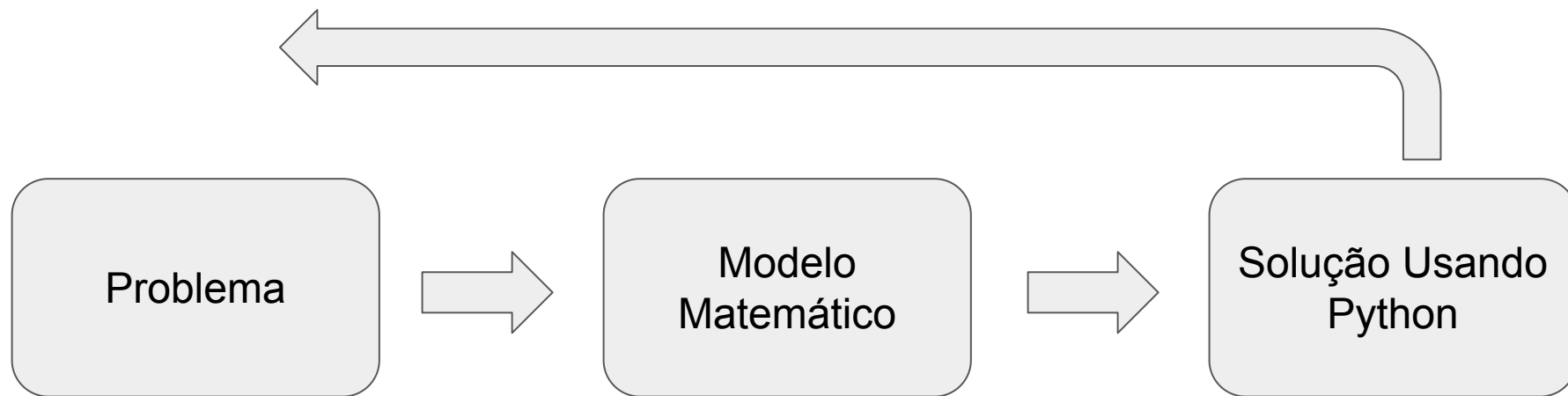




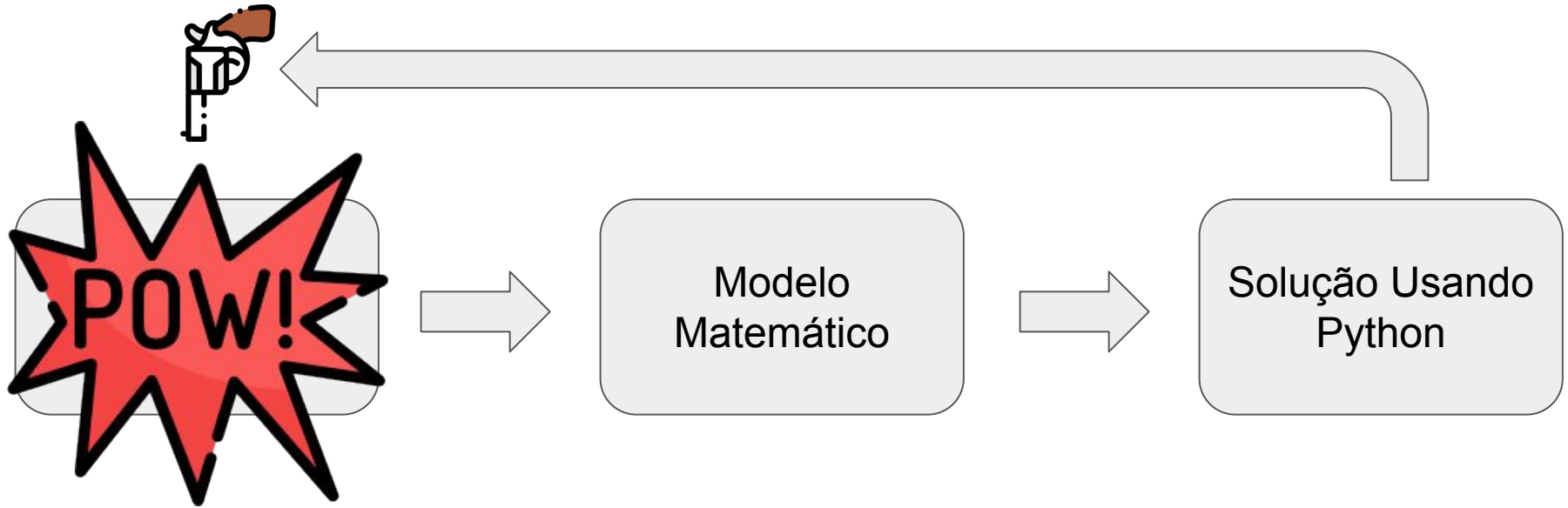
# 3 simples passos



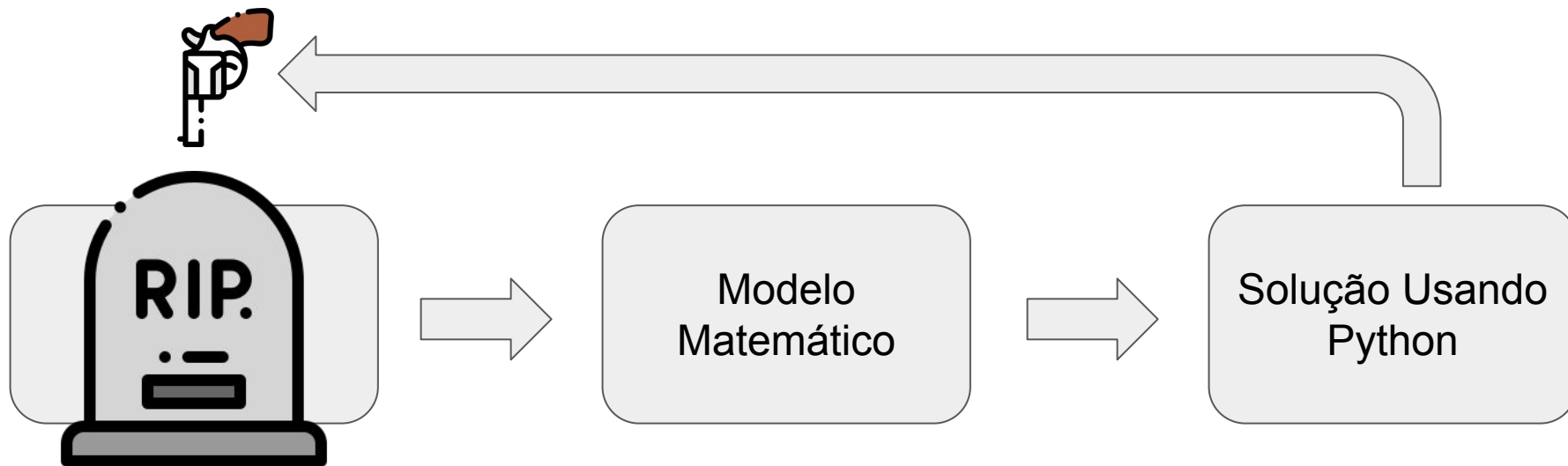
# 3 simples passos

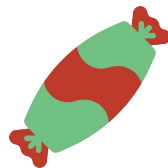
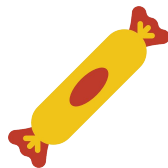


# 3 simples passos

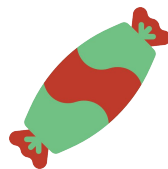
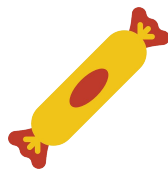
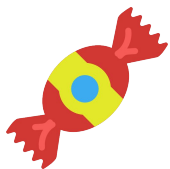


# 3 simples passos



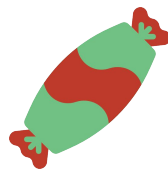
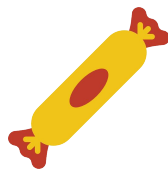
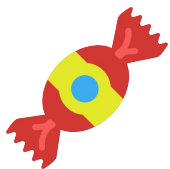


Você e seu irmão ganharam, de sua avó, uma caixa com 6 bombons diferentes.



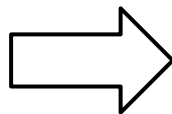
Você e seu irmão ganharam, de sua avó, uma caixa com 6 bombons diferentes.

Qual a melhor forma de dividi-los, fazendo com que vocês dois fiquem igualmente contentes com o resultado da divisão?

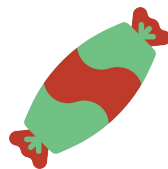
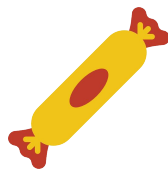
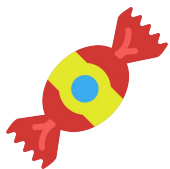


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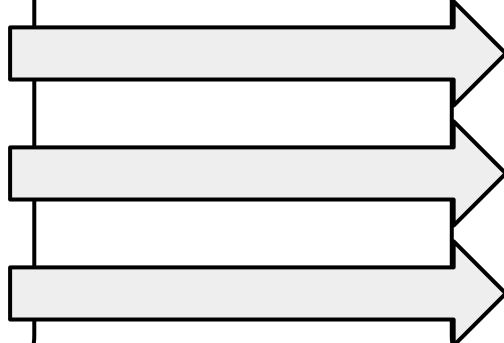


$$\begin{array}{ll} \min & f(x) \\ s. a. & g(x) \geq b \\ & x \geq 0 \end{array}$$



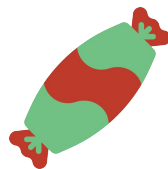
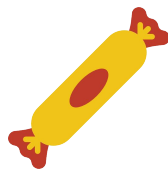
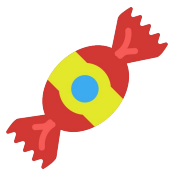
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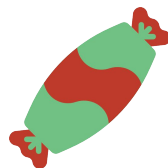
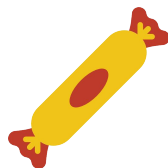
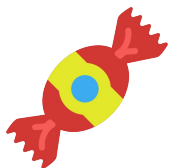


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**Função Objetivo**

$$\begin{aligned} \min \quad & f(x) \\ \text{s. a.} \quad & g(x) \geq b \\ & x \geq 0 \end{aligned}$$



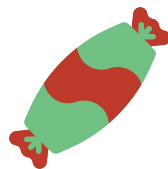
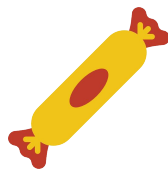
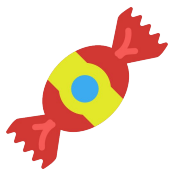
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**Função Objetivo**

**Restrições**

$$\begin{aligned} \min \quad & f(x) \\ \text{s. a.} \quad & g(x) \geq b \\ & x \geq 0 \end{aligned}$$



Você e seu irmão ganharam, de sua avó, uma caixa com 6 bombons diferentes.

Qual a melhor forma de dividi-los, fazendo com que vocês dois fiquem igualmente contentes com o resultado da divisão?

**Função Objetivo**

**Restrições**


**Variáveis de Decisão**

$$\min f(x)$$

$$s. a. \quad g(x) \geq b$$


$$x \geq 0$$

1




9

2




5

3




8

4




1

5



7

6



2

$B$

$g$



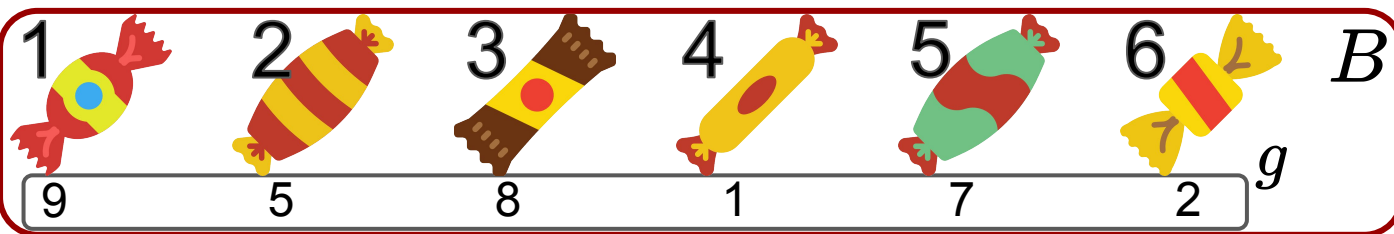
1



2

$C$

Variáveis de decisão



Variáveis de decisão



1	2	3	4	5	6	$B$
9	5	8	1	7	2	$g$

		$C$
1	2	

Variáveis de decisão

1 1

1	2	3	4	5	6	$B$
9	5	8	1	7	2	$g$

		$C$
1	2	

Variáveis de decisão

 1 1	
 1 2	

1

2







3

4

5

6

B

9

5

8

1

7

2

g




C

1

2

Variáveis de decisão

1 1

2 1

3 1

4 1

5 1

6 1















1 2

2 2

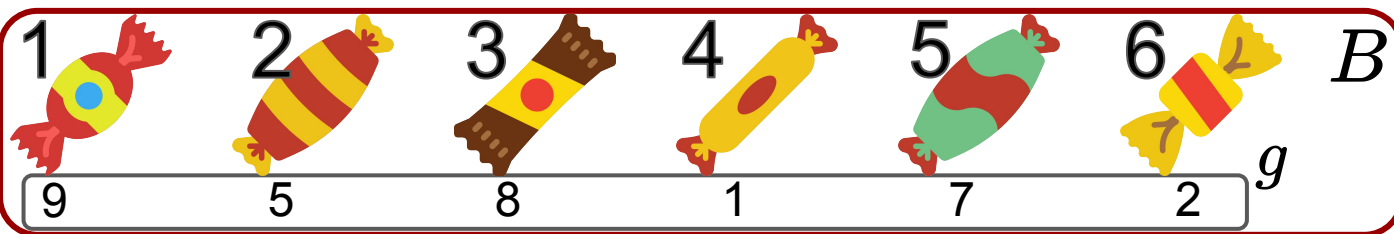
3 2

4 2

5 2

6 2

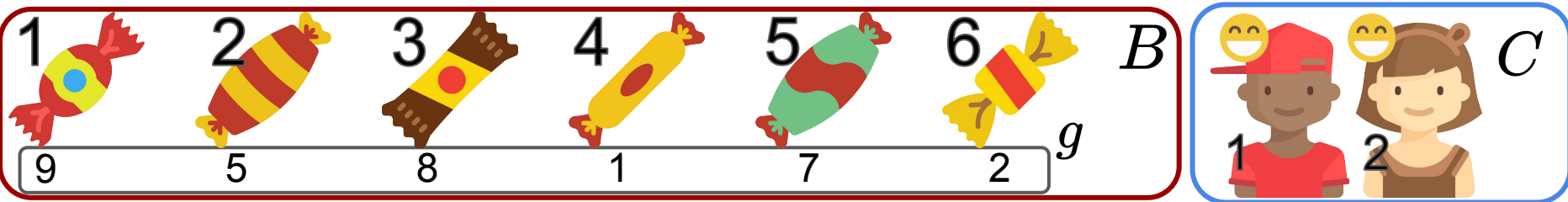




Variáveis de decisão

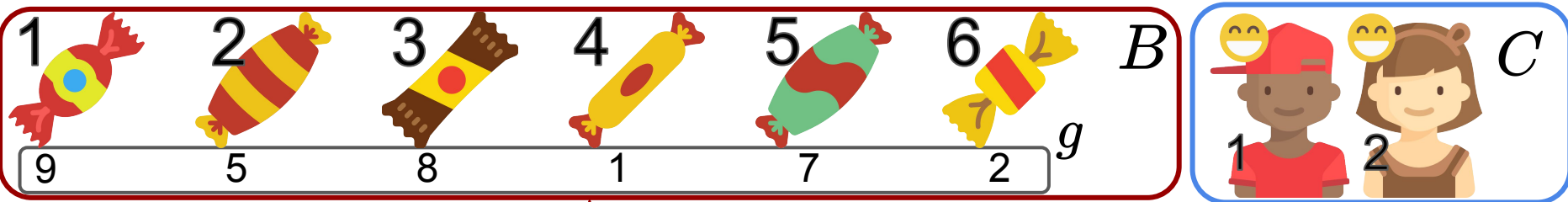
$$x_{11} \quad x_{21} \quad x_{31} \quad x_{41} \quad x_{51} \quad x_{61}$$

$$x_{12} \quad x_{22} \quad x_{32} \quad x_{42} \quad x_{52} \quad x_{62}$$



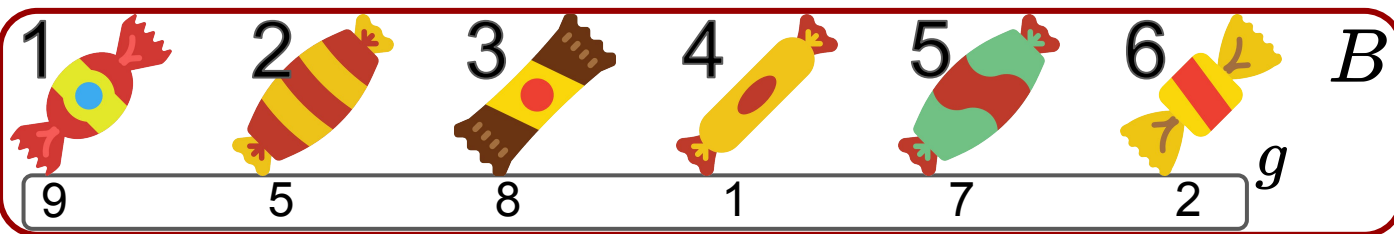
Variáveis de decisão

$$\begin{array}{cccccc}
 x_{11} & x_{21} & x_{31} & x_{41} & x_{51} & x_{61} \\
 x_{12} & x_{22} & x_{32} & x_{42} & x_{52} & x_{62}
 \end{array} = 0 \text{ ou } 1$$



Variáveis de decisão







$$x_{ij} = \{0, 1\}, \forall i \in B \wedge \forall j \in C$$







Modelo Matemático

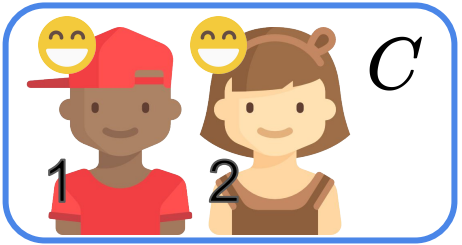
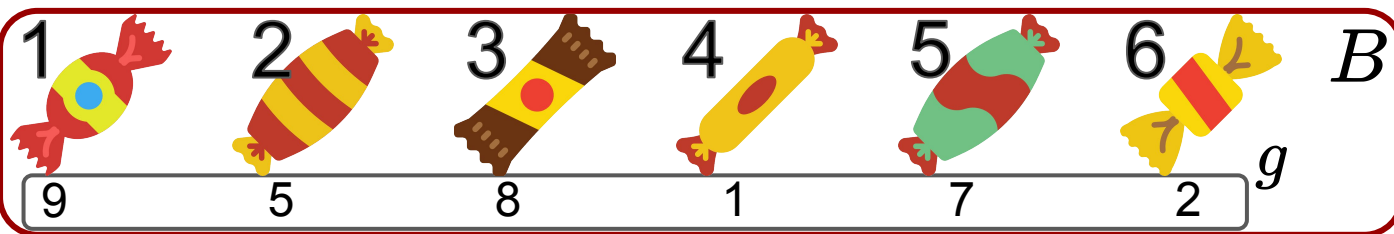
$$x_{ij} = \{0, 1\}, \forall i \in B \wedge \forall j \in C$$

- Função objetivo
- Restrições
- Variáveis de decisão

1	2	3	4	5	6	$B$
						
9	5	8	1	7	2	$g$







		$C$
		
1	2	





Função Objetivo



Função Objetivo







$$\min( \text{person 1} - \text{person 2} )$$

1	2	3	4	5	6	$B$
						
9	5	8	1	7	2	$g$

		$C$
		
1	2	

Função Objetivo

$$\min \left( \begin{array}{c} \text{Person 1} \\ 1 \end{array} \begin{array}{c} \text{Candy 1} \\ \text{Candy 2} \\ \text{Candy 3} \end{array} - \begin{array}{c} \text{Person 2} \\ 2 \end{array} \begin{array}{c} \text{Candy 4} \\ \text{Candy 5} \\ \text{Candy 6} \end{array} \right)$$

1	2	3	4	5	6	$B$
						
9	5	8	1	7	2	$g$

		$C$
		
1	2	

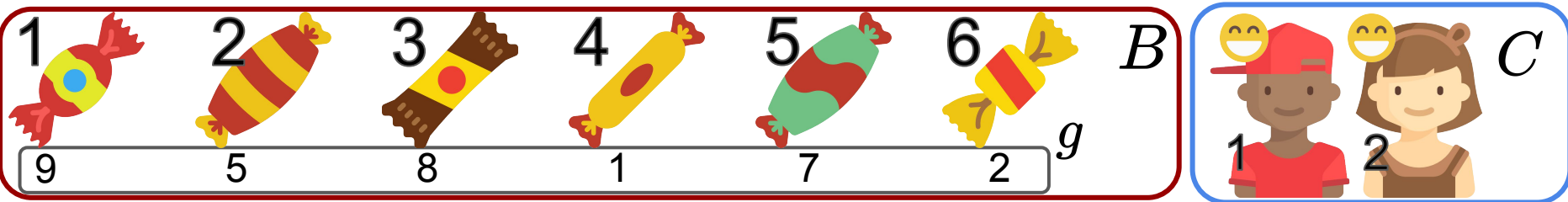
Função Objetivo

$$\min \left( \begin{array}{c} \text{Person 1} \\ 1 \end{array} \begin{array}{c} \text{Candy 2} \\ \text{Candy 4} \\ \text{Candy 6} \end{array} - \begin{array}{c} \text{Person 2} \\ 2 \end{array} \begin{array}{c} \text{Candy 3} \\ \text{Candy 1} \\ \text{Candy 5} \end{array} \right)$$

$5 + 1 + 2$

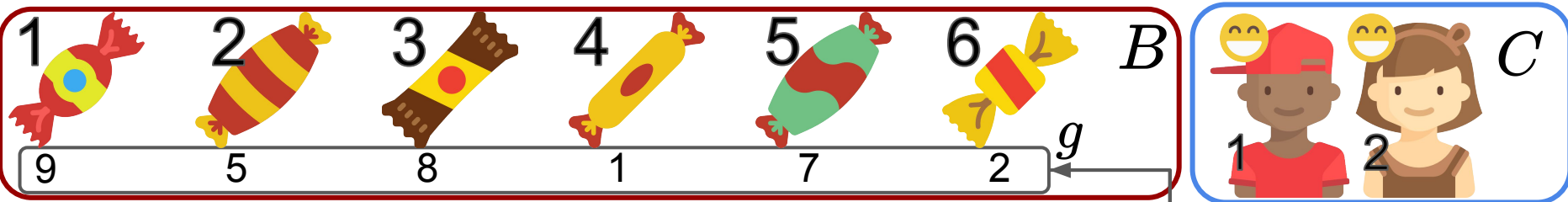
$8 + 9 + 7$





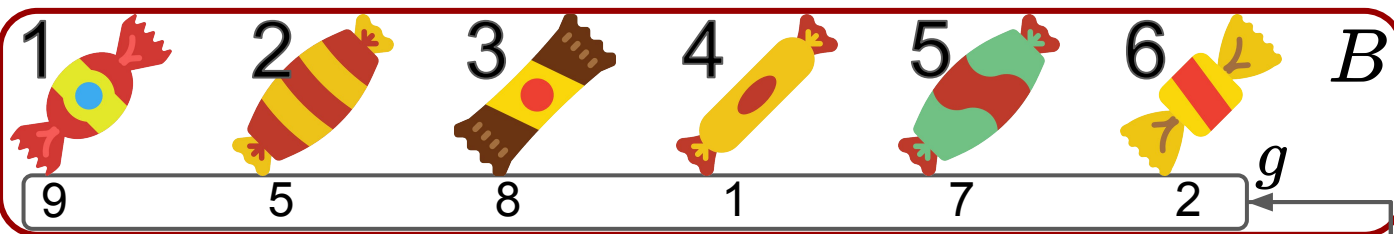
Função Objetivo

$$\min(\sum_B g_i x_{i1} - \sum_B g_i x_{i2})$$



Função Objetivo

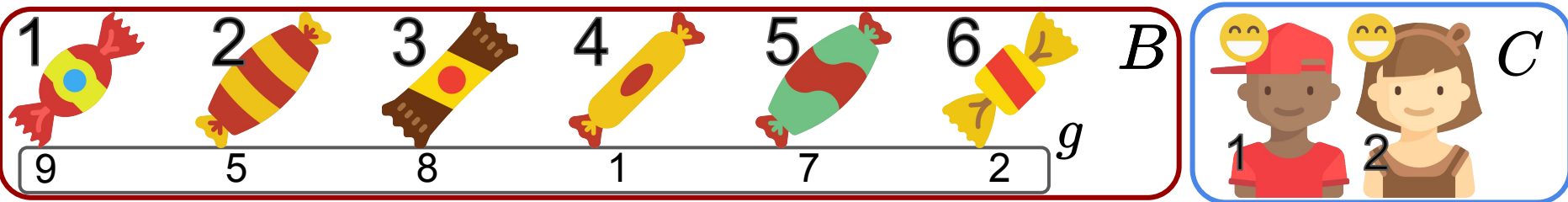
$$\min(\sum_B \boxed{g_i} x_{i1} - \sum_B \boxed{g_i} x_{i2})$$



Função Objetivo

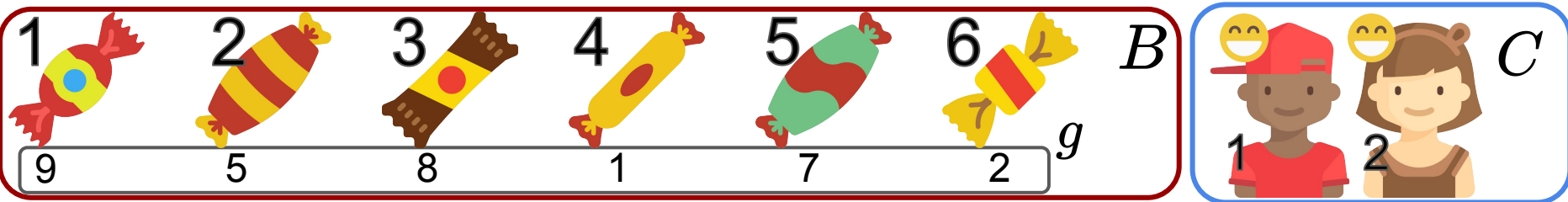
$$\min(\sum_B g_i x_{i1} - \sum_B g_i x_{i2})$$

Variáveis de decisão



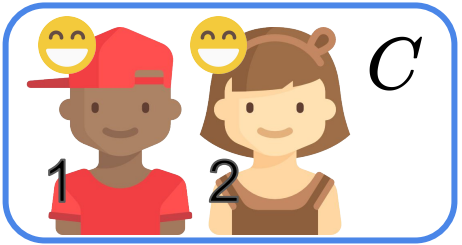
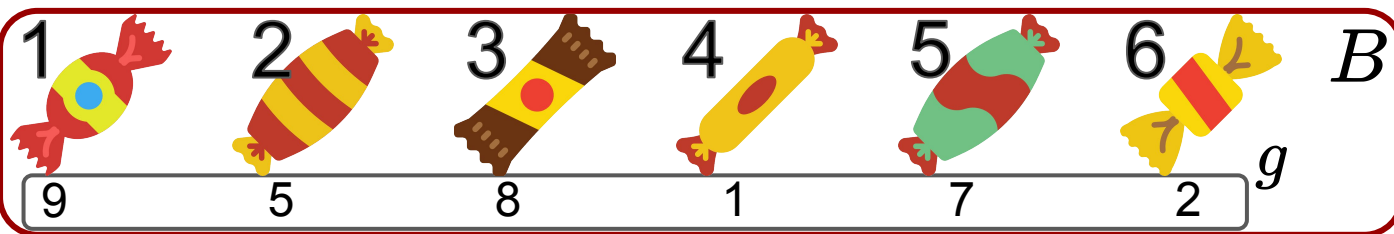
Função Objetivo

$$\min(\sum_B g_i x_{i1} - \sum_B g_i x_{i2})$$



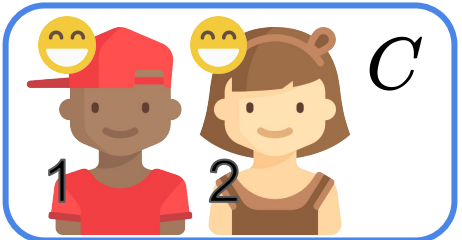
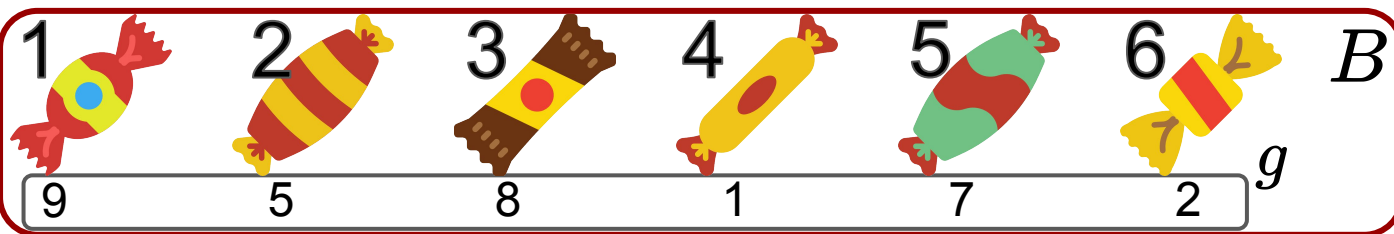
Função Objetivo

$$\min(\sum_B g_i x_{i1} - \sum_B g_i x_{i2})$$



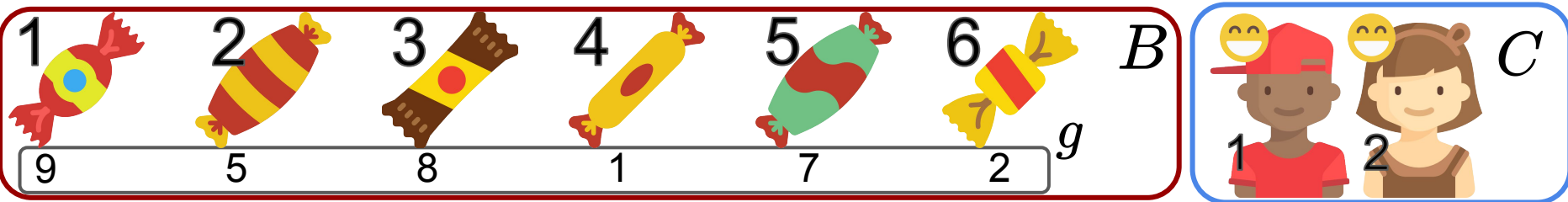
Função Objetivo

$$\min \left( \begin{matrix} \text{Person 1} \\ 1 \end{matrix} - \begin{matrix} \text{Person 2} \\ 2 \end{matrix} \right)$$



Função Objetivo

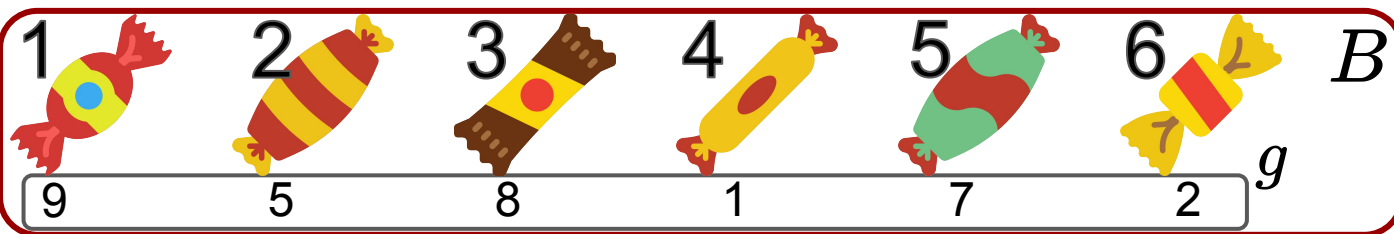
$$\min \left( \begin{matrix} \text{H} \\ 1 \end{matrix} - \begin{matrix} 2 \\ 2 \end{matrix} \right)$$



Função Objetivo

$$\min(\sum_B g_i x_{i1} - \sum_B g_i x_{i2})^2$$











# Modelo Matemático

$$\min(\sum_B g_i x_{i1} - \sum_B g_i x_{i2})^2$$

Função objetivo

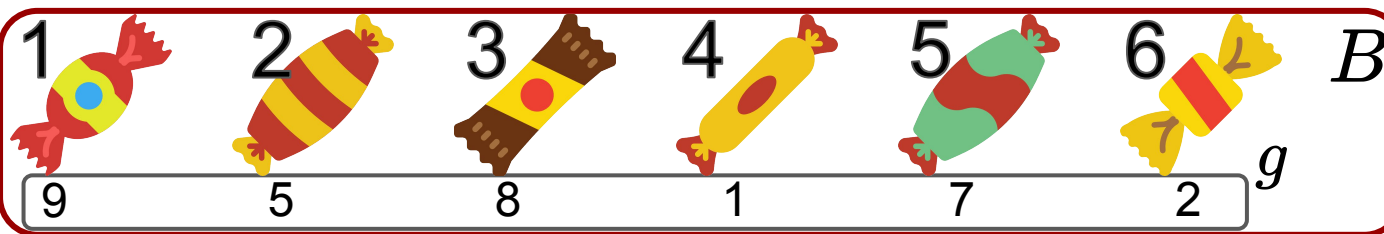
$$x_{ij} = \{0, 1\}, \forall i \in B \wedge \forall j \in C$$

Variáveis de decisão

1	2	3	4	5	6	$B$
						
9	5	8	1	7	2	$g$

		$C$
		
1	2	

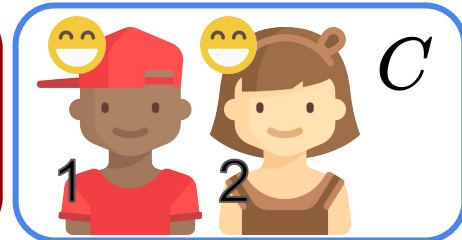
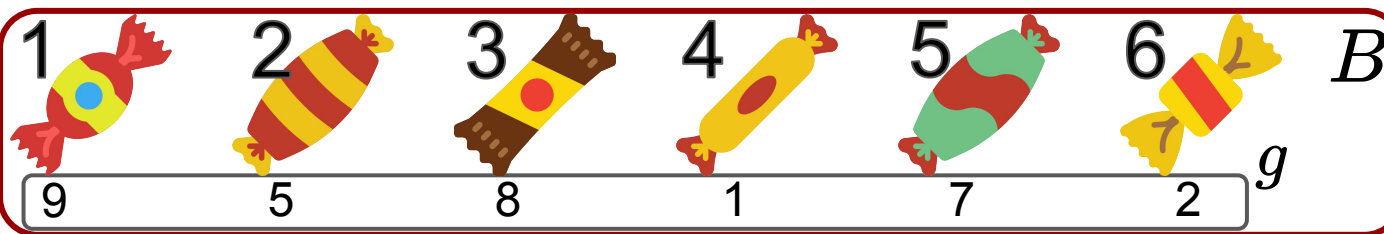
Restrições



Restrições

$$x_{ij} = \{0, 1\}, \forall i \in B \wedge \forall j \in C$$

$x_{11}$	$x_{21}$	$x_{31}$	$x_{41}$	$x_{51}$	$x_{61}$
$x_{12}$	$x_{22}$	$x_{32}$	$x_{42}$	$x_{52}$	$x_{62}$



Restrições

$$x_{ij} = \{0, 1\}, \forall i \in B \wedge \forall j \in C$$

$$\begin{aligned} x_{11} &= 0 & x_{21} &= 0 & x_{31} &= 0 & x_{41} &= 0 & x_{51} &= 0 & x_{61} &= 0 \\ x_{12} &= 0 & x_{22} &= 0 & x_{32} &= 0 & x_{42} &= 0 & x_{52} &= 0 & x_{62} &= 0 \end{aligned}$$

1

2

3

4

5

6

9
5
8
1
7
2

*B*
*g*

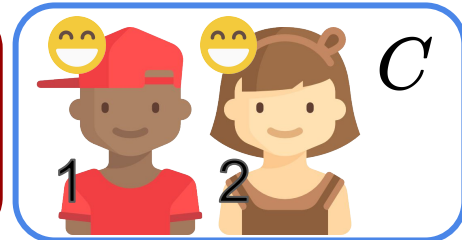
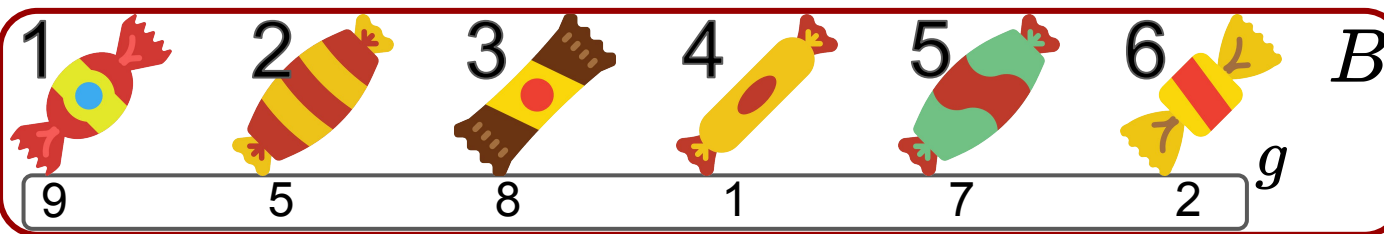
1
2

*C*

Restrições

$x_{ij} = \{0, 1\}, \forall i \in B \wedge \forall j \in C$

~~$x_{11} = 0 \ x_{21} = 0 \ x_{31} = 0 \ x_{41} = 0 \ x_{51} = 0 \ x_{61} = 0$~~   
 ~~$x_{12} = 0 \ x_{22} = 0 \ x_{32} = 0 \ x_{42} = 0 \ x_{52} = 0 \ x_{62} = 0$~~









Restrições

$$x_{ij} = \{0, 1\}, \forall i \in B \wedge \forall j \in C$$

$$\begin{aligned} x_{11} &= 1 & x_{21} &= 1 & x_{31} &= 1 & x_{41} &= 1 & x_{51} &= 1 & x_{61} &= 1 \\ x_{12} &= 1 & x_{22} &= 1 & x_{32} &= 1 & x_{42} &= 1 & x_{52} &= 1 & x_{62} &= 1 \end{aligned}$$

1
2
3
4
5
6

9
5
8
1
7
2

*B*
*g*






*C*


Restrições


$$x_{ij} = \{0, 1\}, \forall i \in B \wedge \forall j \in C$$


~~$$\begin{aligned}
&x_{11} = 1 \ x_{21} = 1 \ x_{31} = 1 \ x_{41} = 1 \ x_{51} = 1 \ x_{61} = 1 \\
&x_{12} = 1 \ x_{22} = 1 \ x_{32} = 1 \ x_{42} = 1 \ x_{52} = 1 \ x_{62} = 1
\end{aligned}$$~~


1  
  
9

2  
  
5

3  
  
8

4  
  
1

5  
  
7

6  
  
2

*B*  
*g*

  
1

  
2

*C*

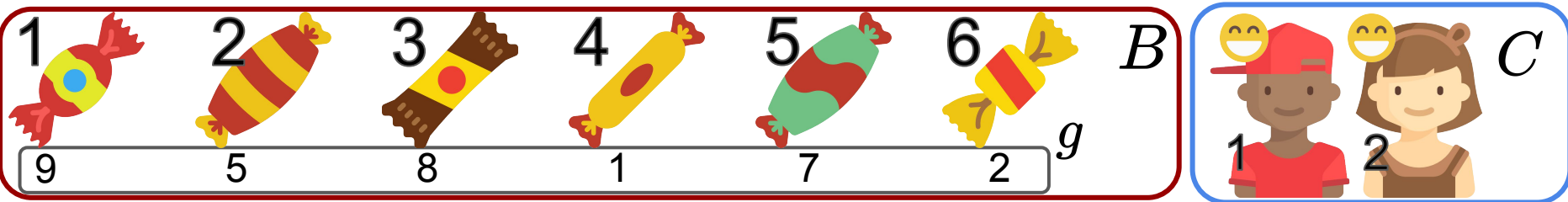
Restrições

$$x_{ij} = \{0, 1\}, \forall i \in B \wedge \forall j \in C$$



$$\begin{aligned}
 x_{11} &= 1 & x_{21} &= 0 & x_{31} &= 1 & x_{41} &= 0 & x_{51} &= 0 & x_{61} &= 1 \\
 x_{12} &= 0 & x_{22} &= 1 & x_{32} &= 0 & x_{42} &= 1 & x_{52} &= 1 & x_{62} &= 0
 \end{aligned}$$





# Restrições

$$x_{11} = 0 \quad x_{21} = 0 \quad x_{31} = 0 \quad x_{41} = 0 \quad x_{51} = 0 \quad x_{61} = 0$$

$$x_{12} = 0 \quad x_{22} = 0 \quad x_{32} = 0 \quad x_{42} = 0 \quad x_{52} = 0 \quad x_{62} = 0$$



$$x_{11} = 1 \quad x_{21} = 1 \quad x_{31} = 1 \quad x_{41} = 1 \quad x_{51} = 1 \quad x_{61} = 1$$

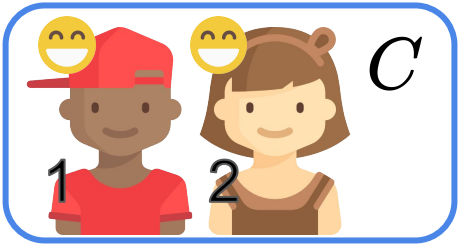
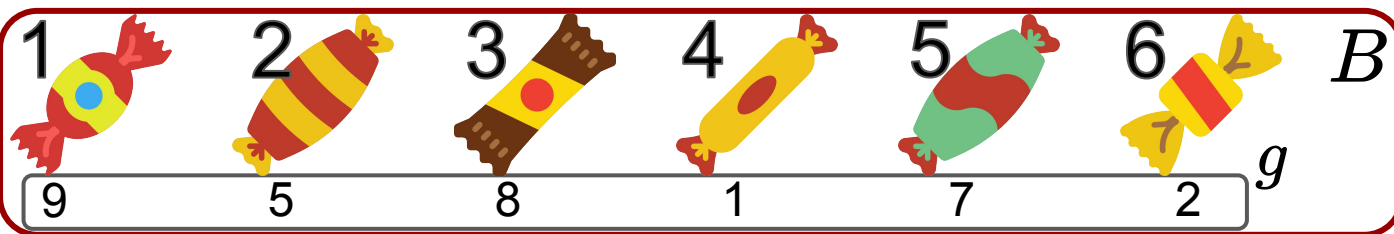
$$x_{12} = 1 \quad x_{22} = 1 \quad x_{32} = 1 \quad x_{42} = 1 \quad x_{52} = 1 \quad x_{62} = 1$$



$$x_{11} = 1 \quad x_{21} = 0 \quad x_{31} = 1 \quad x_{41} = 0 \quad x_{51} = 0 \quad x_{61} = 1$$

$$x_{12} = 0 \quad x_{22} = 1 \quad x_{32} = 0 \quad x_{42} = 1 \quad x_{52} = 1 \quad x_{62} = 0$$





$$x_{11} + x_{12} = 1$$

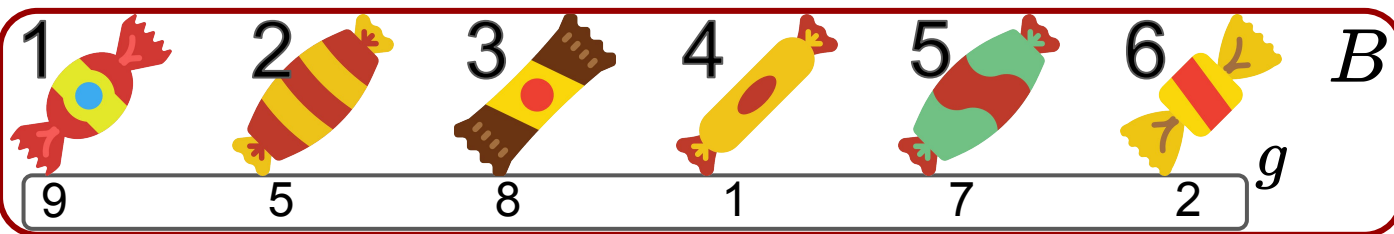
$$x_{21} + x_{22} = 1$$

$$x_{31} + x_{32} = 1$$

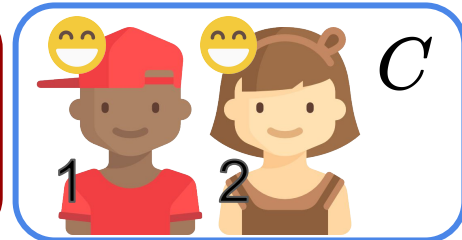
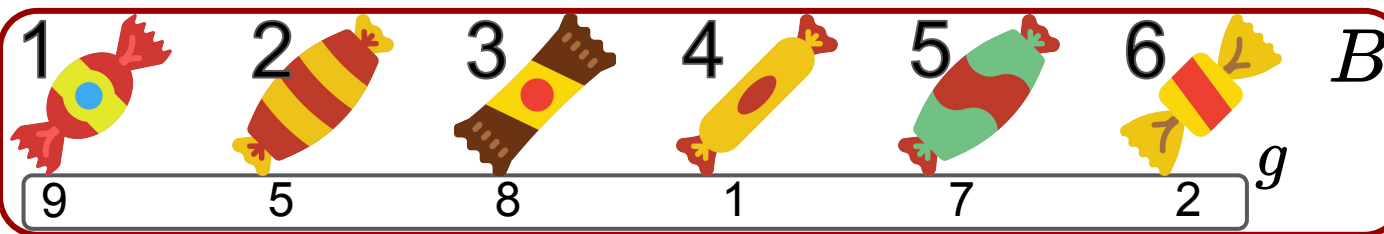
$$x_{41} + x_{42} = 1$$

$$x_{51} + x_{52} = 1$$

$$x_{61} + x_{62} = 1$$



$$x_{i1} + x_{i2} = 1, \forall i \in B$$



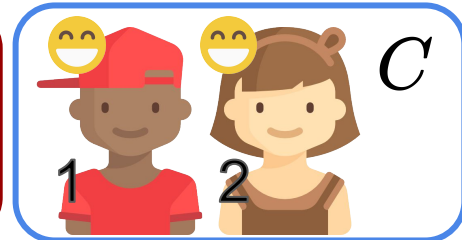
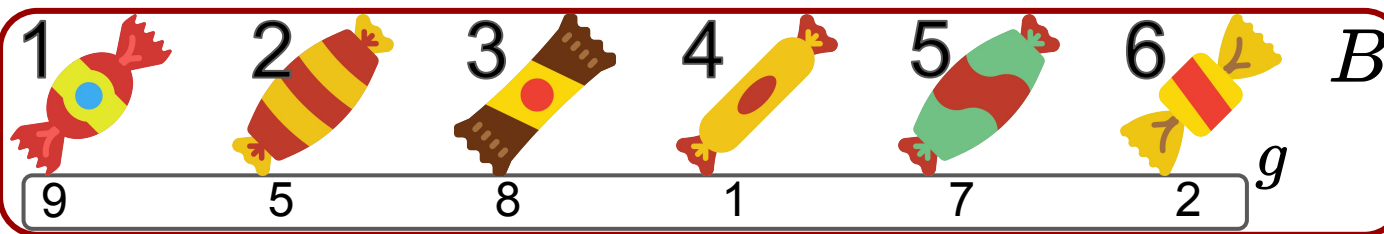
# Modelo Matemático

$$\min(\sum_B g_i x_{i1} - \sum_B g_i x_{i2})^2$$

$$x_{i1} + x_{i2} = 1, \forall i \in B$$

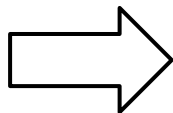
$$x_{ij} = \{0, 1\}, \forall i \in B \wedge \forall j \in C$$

- Função objetivo
- Restrições
- Variáveis de decisão

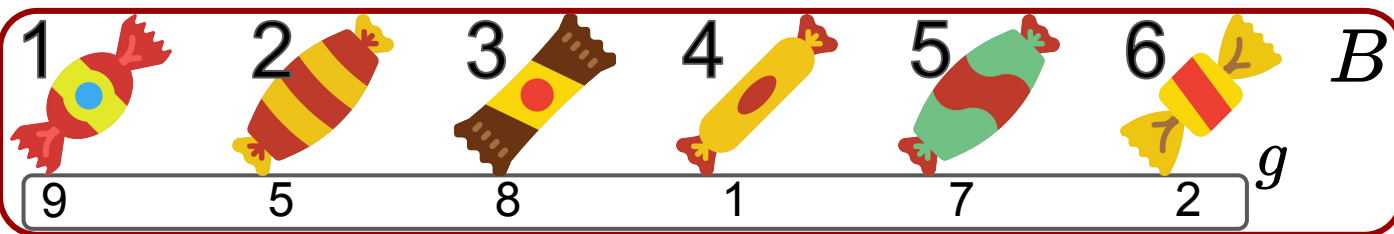


Você e seu irmão ganharam uma caixa com 6 bombons da sua avó.

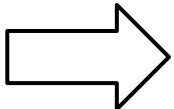
Qual a melhor forma de dividi-los, fazendo com que vocês dois fiquem igualmente contentes com o resultado da divisão?

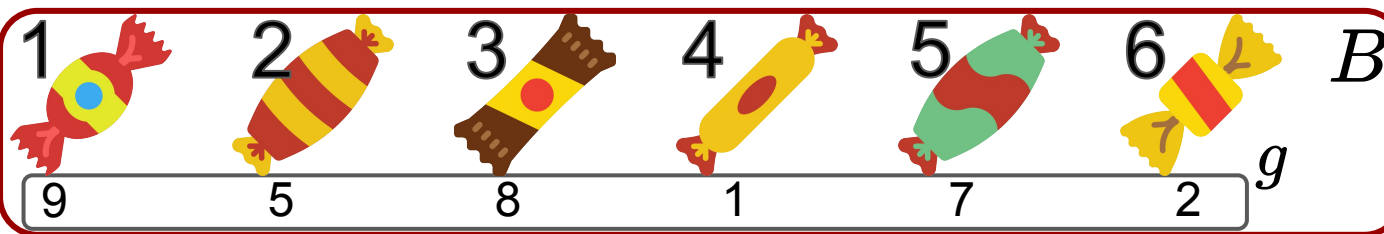


$$\begin{aligned} \min \quad & (\sum_B g_i x_{i1} - \sum_B g_i x_{i2})^2 \\ \text{s. t.} \quad & x_{i1} + x_{i2} = 1, \forall i \in B \\ & x_{ij} = \{0, 1\}, \forall i \in B \wedge \forall j \in C \end{aligned}$$

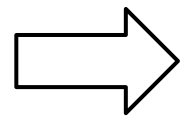


$$\begin{aligned}
 \min \quad & \left( \sum_B g_i x_{i1} - \sum_B g_i x_{i2} \right)^2 \\
 \text{s.t.} \quad & x_{i1} + x_{i2} = 1, \forall i \in B \\
 & x_{ij} = \{0, 1\}, \forall i \in B \wedge \forall j \in C
 \end{aligned}$$





$$\begin{aligned}
 \min \quad & (\sum_B g_i x_{i1} - \sum_B g_i x_{i2})^2 \\
 \text{s.t.} \quad & x_{i1} + x_{i2} = 1, \forall i \in B \\
 & x_{ij} = \{0, 1\}, \forall i \in B \wedge \forall j \in C
 \end{aligned}$$



## Modelo Matemático

$$\begin{aligned} \min \quad & (\sum_B g_i x_{i1} - \sum_B g_i x_{i2})^2 \\ \text{s. t.} \quad & x_{i1} + x_{i2} = 1, \forall i \in B \\ & x_{ij} = \{0, 1\}, \forall i \in B \wedge \forall j \in C \end{aligned}$$



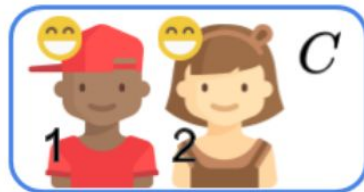


## Modelo Matemático

$$\begin{aligned} \min \quad & (\sum_B g_i x_{i1} - \sum_B g_i x_{i2})^2 \\ \text{s. t.} \quad & x_{i1} + x_{i2} = 1, \forall i \in B \\ & x_{ij} = \{0, 1\}, \forall i \in B \wedge \forall j \in C \end{aligned}$$

```
In [1]: import pyomo.environ as poe
        modelo = poe.AbstractModel()
```

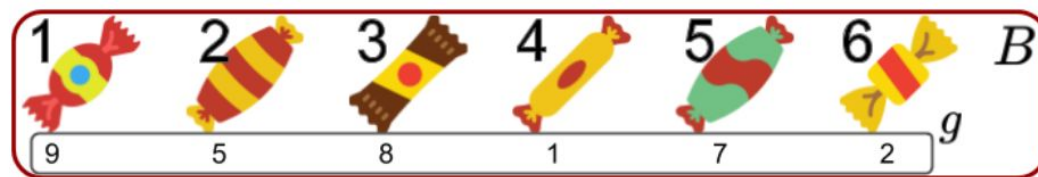




Conjunto representando as crianças:

```
In [2]: modelo.crianças = poe.RangeSet(1,2)
```

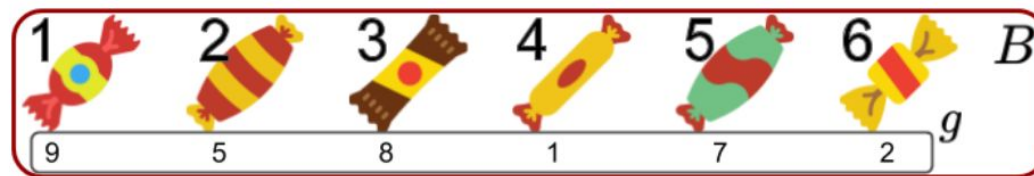




Conjunto representando os bombons:

```
In [3]: modelo.bombons = poe.Set()
```





Conjunto representando os bombons:

```
In [3]: modelo.bombons = poe.Set()
```

Nível de gostosura dos bombons:

```
In [4]: modelo.gostosura = poe.Param(modelo.bombons, within=poe.NonNegativeReals)
```



Variáveis de decisão:

$$x_{11} \ x_{21} \ x_{31} \ x_{41} \ x_{51} \ x_{61}$$
$$x_{12} \ x_{22} \ x_{32} \ x_{42} \ x_{52} \ x_{62}$$

- 1, se o bombom  $i$  for para a criança  $j$
- 0, caso contrário

```
In [5]: modelo.x = poe.Var(modelo.bombons, modelo.crianças, within=poe.Binary)
```



Função objetivo:

$$\min \quad (\sum_B g_i x_{i1} - \sum_B g_i x_{i2})^2$$

```
In [6]: def funcao_objetivo(modelo):  
        return((sum(modelo.gostosura[i] * modelo.x[i, 1] for i in modelo.bombons))  
               - sum(modelo.gostosura[i] * modelo.x[i, 2] for i in modelo.bombons))  
        **2  
  
        modelo.OBJ = poe.Objective(rule=funcao_objetivo)
```



Restrições:

$$s.t. \quad x_{i1} + x_{i2} = 1, \forall i \in B$$

```
In [7]: def funcao_restricao_de_cobertura(modelo, i):  
        return modelo.x[i, 1] + modelo.x[i, 2] == 1  
  
        modelo.restricao_de_cobertura = \  
            poe.Constraint(modelo.bombons, rule=funcao_restricao_de_cobertura)
```



```
In [8]: ! cat candy_box_problem_instance.dat
```

```
# AMPL format
```

```
set bombons := 1 2 3 4 5 6;
```

```
param gostosura :=
```

```
1 9
```

```
2 5
```

```
3 8
```

```
4 1
```

```
5 7
```

```
6 2
```

```
;
```





```
In [9]: !pyomo solve candy_box_problem.py candy_box_problem_instance.dat --solver=bonmin
```

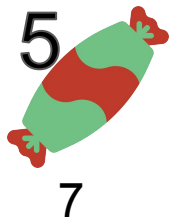
```
[ 0.00] Setting up Pyomo environment
[ 0.00] Applying Pyomo preprocessing actions
[ 0.00] Creating model
[ 0.01] Applying solver
[ 0.05] Processing results
      Number of solutions: 1
      Solution Information
          Gap: None
          Status: optimal
          Function Value: 0.0
      Solver results file: results.yml
[ 0.05] Applying Pyomo postprocessing actions
[ 0.05] Pyomo Finished
```



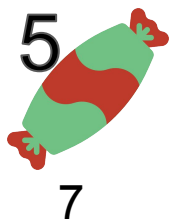
```
In [10]: !grep -B 1 -A 99 "# Solution Information" results.yml
```

```
# -----  
# Solution Information  
# -----  
Solution:  
- number of solutions: 1  
  number of solutions displayed: 1  
- Gap: None  
  Status: optimal  
  Message: bonmin\x3a Optimal  
  Objective:  
    OBJ:  
    Value: 0  
  Variable:  
    x[1,2]:  
    Value: 1  
    x[2,2]:  
    Value: 1  
    x[3,1]:  
    Value: 1  
    x[4,1]:  
    Value: 1  
    x[5,1]:  
    Value: 1  
    x[6,2]:  
    Value: 1  
  Constraint: No values
```





16



16





✓ Divisão de bombons

# Problema de particionamento de número



Divisão de bombons

# Problema de particionamento de número

- ✓ Divisão de bombons
- ✓ Divisão de tarefas domésticas

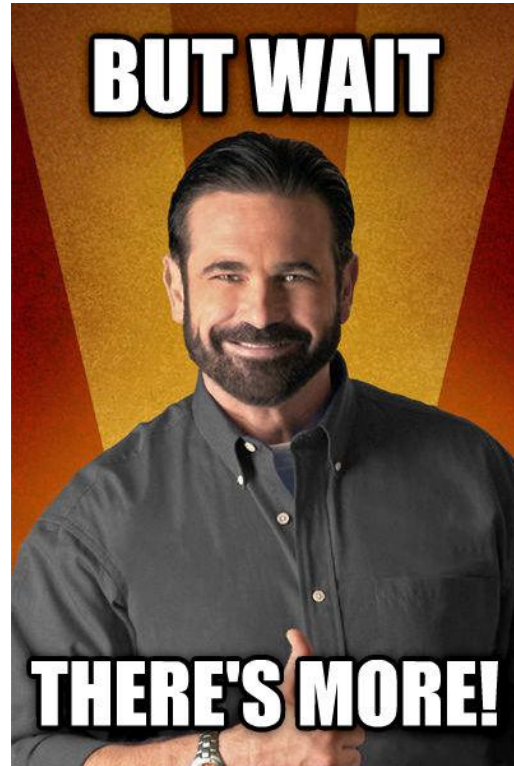


# Problema de particionamento de número

- ✓ Divisão de bombons
- ✓ Divisão de tarefas domésticas
- ✓ Divisão de tarefas entre processadores

# Problema de particionamento de número

- ✓ Divisão de bombons
- ✓ Divisão de tarefas domésticas
- ✓ Divisão de tarefas entre processadores
- ✓ Divisão de linhas entre empresas de ônibus



# 3 simples passos





- ✓ Alocação de viagens a ônibus
- ✓ Alocação de viagens a motoristas
- ✓ Escala semanal
- ✓ Planejamento de viagem

# Quer saber mais?

Hart, William E., Carl Laird, Jean-Paul Watson, David L. Woodruff, Gabriel A. Hackebeil, Bethany L. Nicholson, and John D. Sirola. **Pyomo – Optimization Modeling in Python**. Springer, 2017.

WPLEX. **Artigos – Pesquisa e Desenvolvimento para Mobilidade Urbana**. Disponível em: <<https://wplex.com.br/artigos.html>>

<https://github.com/renan-eccel/candy-box-problem>

# Muito obrigado pela oportunidade!

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