Lecture 15 - Spectral Clustering

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Spectral Clustering

Let G be a graph, $G(V, E)$, where $V = \{1, 2, 3, \cdots, n\}$

The Laplacian matrix:

$$L_G = \underbrace{D_G}_{\substack{\text{Diagonal matrix} \\ \text{with degrees of} \\ \text{each vertex}}} - \underbrace{A_G}_{\text{adjancency matrix}}$$

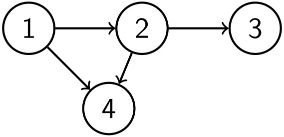
L_G is a PSD Matrix whose eigenvalues are

$$0 = \lambda_1 \leq \underbrace{\lambda_2}_{\text{Fiedler value}} \leq \lambda_3 \leq \cdots \leq \lambda_n$$

An eigenvector with eigenvalue λ_2 is called the <u>Fiedler vector</u> of $\vec{W}$.

Spectral Clustering

We will use $\vec{W}$, L_G to understand clusters in a graph.



$A = \{4, 5, 6, 7, 8\}, |A| = 5$
Notation
If X, Y are sets $X\ \ Y = \{x \in X | x \notin Y\}$
$|x| =$ size of x

Cut

Definition

A cut is G is a partition of V into two sets, A and V A, when AV. (The cut induced by A)

Notation

If X, YV, let E(X,Y) denote all edges with one certex in X and one vertex in Y.

$E(A, V\ A) = \{\{1, 6\}\}$

Definition

The density of the cut induced by A is:

Sparsest cut

Definition:

Let denote the smallest possible desnity of ca cut in G. Any cut with density is called a sparesest cut in G.

Example in Julia

Need to figure out how to make code look pretty in Latex

Exploring

$$Q\left(\vec{x}\right) = n \cdot \frac{\sum_{\{i,j\} \in E(G)} \left(x_i - x_j\right)^2}{\sum_{1 \le i < j \le n} \left(x_i - x_j\right)^2}$$

Let $A \subseteq V$,

$$A = \{4, 5, 6, 7, 8\}, \ \vec{C}_A = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Exploring

Need to add a visual representation demonstrating the Graph Laplacian and the example in code. About the information we can extract from the Graph Laplacian.

Conclusion

here we go: