MAD 6306 COMPLEX NETWORKS - SPRING 2025
HOMEWORK 1

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- DUE on 002/02/2025 11:59pm C.T.
- You can write on the separate work sheet or type your quiz. (Word or Latex or similar)
- If you use the handwriting, Solutions must be neat,clear and legible.
- If you need to scan you quiz, save it as a PDF file. Do not use jpeg, png, jpg etc. Do not submit more than one file.
- Please check your scanned file before submission. Make sure it is readable, correct order, properly oriented. Make sure it does include all pages.
- Please name your file as follows: $LastnameInitials - MAD6306hw1.pdf$.If your name is Alan David Roberts, file name is $RobertsAD - MAD6306hw1.pdf$.
- Try to keep the file size less than 4MB.
- You can resubmit the quiz if you want. Please specify which one is the one to be graded. Otherwise I will grade the most recent version.
- DO NOT EMAIL me the quiz. All quizzes are submitted via Canvas.

Date: 02/02/2025.

(1) Consider the following adjacency matrix of a network

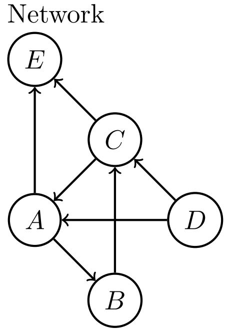
$$A = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

(a) Is the network directed or undirected? (Explain why).
(b) Draw the network.

Solution

(a) The nework is directed because the adjacency matrix is asymmetric. $A \neq A^T$

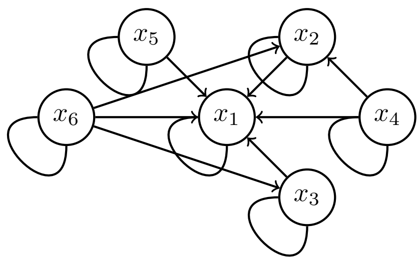
(b) Network



(2) Given the set of node V with $|V| = 6$ in which each node i is labelled by a natural number between 1 and 6,$i = 1, 2, 3, 4, 5, 6$, consider the directed network$G = (V, E)$ where each link from node j to node i indicates that j is a multiple of i.
  (a) Draw the network.
  (b) Write down the adjacency matrix of the network.

Solution

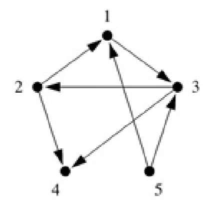
(a) network

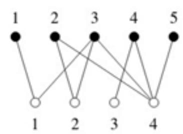


(b) adjacency matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(3) Consider the following two networks: Notwork(a) is directed and Network (b) is undirected but bipartite. Find the following:





(a) Find the adjacency matrix of network (a)
(b) Find the incidence matrix of network (b)

Solution

(a) Find the adjacency matrix of network (a)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) Find the incidence matrix of network (b)

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(4) Which word or words from the following list describe each of the five networks below: *directed, undirected, cyclic, acyclic, approximately acyclic, planar, approximately planar, tree, approximate tree.*

(a) The internet, at the level of autonomous systems
(b) A food web
(c) The stem and branches of a plant
(d) A spider web
(e) A complete clique of four nodes

Solution

(a) The internet, at the level of autonomous systems
Undirected, Acyclic, Approximately planar

(b) A food web
Directed, Cyclic

(c) The stem and branches of a plant
Tree, Acyclic

(d) A spider web
Undirected, Planar

(e) A complete clique of four nodes
Undirected, Cyclic

(5) A simple network consists of n nodes in a single component. What is the maximum possible number of edges it could have? What is the minimum possible number of edges it could have?

Solution

What is the maximum possible number of edges it could have?
By the Handshaking Theorem:

$deg(v_1) + deg(v_2) + deg(v_3) + ... + deg(v_n) = 2|E|$

G is a simple network of n nodes in a single component. The maximum possible number of edges is:

$(n-1) + (n-1) + (n-1) + ... + (n-1) = 2|E|$

$n\,(n-1) = 2|E|$

$\frac{n(n-1)}{2} = |E|$

What is the minimum possible number of edges it could have?

$n\,(n-1)$

Such graph will be a tree.