

COMPLEX NETWORKS - SPRING 2025
HOMEWORK 2

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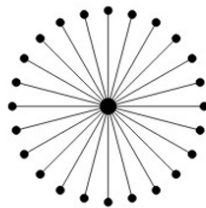
- DUE on 02/23/2025 11:59pm C.T.
- You can write on the separate work sheet or type your quiz. (Word or Latex or similar)
- If you use the handwriting, Solutions must be neat,clear and legible.
- If you need to scan you quiz, save it as a PDF file. Do not use jpeg, png, jpg etc. Do not submit more than one file.
- Please check your scanned file before submission. Make sure it is readable, correct order, properly oriented. Make sure it does include all pages.
- Please name your file as follows: *LastnameInitials–MAP5990quiz1.pdf*. If your name is Alan David Roberts, file name is *RobertsAD–MAP5990quiz1.pdf*.
- Try to keep the file size less than 4MB.
- You can resubmit the quiz if you want. Please specify which one is the one to be graded. Otherwise I will grade the most recent version.
- DO NOT EMAIL me the quiz. All quizzes are submitted via Canvas.

- (1) Demonstrate the following for undirected networks:
- (a) A 3 regular graph must have an even number of nodes.

- (b) The average degree of a tree is strictly less than 2.

- (2) A star graph consists of a single central node with $n - 1$ other nodes connected to it.

- (a) Find the adjacency matrix of the following star network with nodes N :



- (b) Find the largest eigenvalue of the adjacency matrix.

- (3) **Average degree of a growing network** Assume that you are observing a growing undirected network. The network evolves in time by this simple rules:

- At time $t = 1$ there is a single isolated node.
- At each time $t > 1$ a new node is added to the network and is connected to the existing network by a new link.

Consider the network at time $t = T$.

- (a) What is the total number of nodes N ?

- (b) What is the total number of links L ?

- (c) What is the average degree $\langle k \rangle$?

- (d) What is the average degree in the limit $T \rightarrow +\infty$?

- (4) One can calculate the diameter of certain types of network exactly. Assume that each of these network has network size N .

- (a) What is the diameter of a fully connected network?

- (b) What is the diameter of a star network?

- (c) What is the diameter of a linear chain of N nodes?

- (d) What is the small world diameter property?

- (e) Which of the above networks are small-world? In another word, which networks have SWDP?

- (5) Please submit the network for your final project, including the following. This is a group work. Please discuss with your team members but each one needs to submit the individual report.
- (a) What is your network?
 - (b) Where you get the data?
 - (c) Describe your network including the application area, size, nodes.
 - (d) Why you (or your team) want to study this network.