MAD 6306 COMPLEX NETWORKS - SPRING 2024
HOMEWORK 1

INSTRUCTOR: JIA LIU

- DUE on 002/02/2025 11:59pm C.T.
- You can write on the separate work sheet or type your quiz. (Word or Latex or similar)
- If you use the handwriting, Solutions must be neat,clear and legible.
- If you need to scan you quiz, save it as a PDF file. Do not use jpeg, png, jpg etc. Do not submit more than one file.
- Please check your scanned file before submission. Make sure it is readable, correct order, properly oriented. Make sure it does include all pages.
- Please name your file as follows: $LastnameInitials - MAD6306hw1.pdf$.If your name is Alan David Roberts, file name is $RobertsAD - MAD6306hw1.pdf$.
- Try to keep the file size less than 4MB.
- You can resubmit the quiz if you want. Please specify which one is the one to be graded. Otherwise I will grade the most recent version.
- DO NOT EMAIL me the quiz. All quizzes are submitted via Canvas.
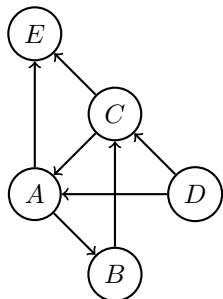
(1) Consider the following adjacency matrix of a network

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

(a) Is the network directed or undirected? (Explain why).
(b) Draw the network.

Solution

(a) The nework is directed because the adjacency matrix is asymmetric.
$A \neq A^T$

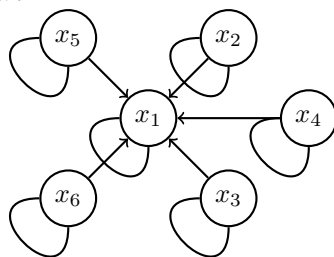(b) Network

(2) Given the set of node V with $|V| = 6$ in which each node i is labelled by a natural number between 1 and 6, $i = 1, 2, 3, 4, 5, 6$, consider the directed network $G = (V, E)$ where each link from node j to node i indicates that j is a multiple of i.
  (a) Draw the network.
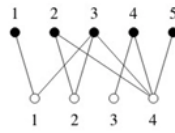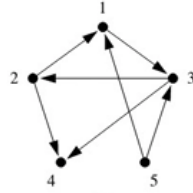 (b) Write down the adjacency matrix of the network.

Solution

(a) network

(b) adjacency matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(3) Consider the following two networks: Notwork(a) is directed and Network (b) is undirected but bipartite. Find the following:

(a) Find the adjacency matrix of network (a)
(b) Find the incidence matrix of network (b)

Solution

(a) Find the adjacency matrix of network (a)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) Find the incidence matrix of network (b)

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(4) Which word or words from the following list describe each of the five networks below: *directed, undirected, cyclic, acyclic, approximately acyclic, planar, approximately planar, tree, approximate tree.*
  (a) The internet, at the level of autonomous systems
  (b) A food web
  (c) The stem and branches of a plant
  (d) A spider web
  (e) A complete clique of four nodes

Solution

  (a) The internet, at the level of autonomous systems
    Undirected, Acyclic, Approximately planar

  (b) A food web
    Directed, Cyclic

  (c) The stem and branches of a plant
    Tree, Acyclic

  (d) A spider web
    Undirected, Planar

  (e) A complete clique of four nodes
    Undirected, Cyclic

(5) A simple network consists of n nodes in a single component. What is the maximum possible number of edges it could have? What is the minimum possible number of edges it could have?

Solution

By the Handshaking Theorem:

$deg(v_1) + deg(v_2) + deg(v_3) + ... + deg(v_n) = 2|E|$

G is a simple network of n nodes in a single component. The maximum possible number of edges is:

$(n-1) + (n-1) + (n-1) + ... + (n-1) = 2|E|$

$n(n-1) = 2|E|$

$\frac{n(n-1)}{2} = |E|$