COMPLEX NETWORKS - SPRING 2024
HOMEWORK 3

INSTRUCTOR: JIA LIU
SOLUTION BY: RENAN MONTEIRO BARBOSA

- DUE on 03/30/2025 11:59pm C.T.
- You can write on the separate work sheet or type your quiz. (Word or Latex or similar)
- If you use the handwriting, Solutions must be neat,clear and legible.
- If you need to scan you quiz, save it as a PDF file. Do not use jpeg, png, jpg etc. Do not submit more than one file.
- Please check your scanned file before submission. Make sure it is readable, correct order, properly oriented. Make sure it does include all pages.
- Please name your file as follows: $LastnameInitials-MAP5990quiz1.pdf$.If your name is Alan David Roberts, file name is $RobertsAD-MAP5990quiz1.pdf$.
- Try to keep the file size less than 4MB.
- You can resubmit the quiz if you want. Please specify which one is the one to be graded. Otherwise I will grade the most recent version.
- DO NOT EMAIL me the quiz. All quizzes are submitted via Canvas.

Date: 03/31/2025.

(1) Consider the adjacency matrix A of a directed network of size $N = 4$ given by

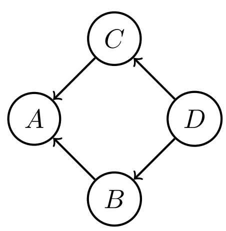
$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

In the following we will indicate with $\mathbf{1}$ the column vector with elements $i_i = 1$ for $i = 1, 2, \cdots, N$ and we will indicate with $\mathbf{I}$the identity matrix.

(a) Draw the network
(b) Calculate the eigenvector centrality using its definition.
(c) Calculate the Katz centrality.
(d) Calculate the PageRank centrality.

Answers:

(a) Draw the network.



(b) Calculate the eigenvector centrality using its definition.

If all eigenvalues of a graph's adjacency matrix are zero, and the graph is not strongly connected, then the eigenvector centrality scores for all nodes will be zero, as the eigenvector associated with the largest eigenvalue will be a zero vector.

By the iterative method:

$$X^{(0)} = \begin{pmatrix} \frac{1}{N} \\ \frac{1}{N} \\ \frac{1}{N} \\ \frac{1}{N} \end{pmatrix} = \begin{pmatrix} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{pmatrix}$$

$$X^{(n)} = A^{(n)} X^{(0)}$$

$$X^{(1)} = A^{(n)} X^{(0)}$$

$$X^{(1)} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{pmatrix} = \begin{pmatrix} \frac{2}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ 0 \end{pmatrix}$$

$$X^{(2)} = A^{(n)} X^{(1)}$$

$$X^{(2)} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \frac{2}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{2}{4} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$X^{(3)} = A^{(n)} X^{(2)}$

$$X^{(3)} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \frac{2}{4} \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

(c) Calculate the Katz centrality.

Considering alpha $\alpha = 0.1 and \beta = 1$

$X = \beta \left(I - \alpha A\right)^{-1}$

$$X = \left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \alpha \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \right)^{-1}$$

$$X = \left(\begin{bmatrix} 1 & -0.1 & -0.1 & 0 \\ 0 & 1 & 0 & -0.1 \\ 0 & 0 & 1 & -0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right)^{-1} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$X = \left(\begin{bmatrix} 1 & -0.1 & -0.1 & 0 \\ 0 & 1 & 0 & -0.1 \\ 0 & 0 & 1 & -0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right)^{-1} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$X = \begin{pmatrix} 1.22 \\ 1.1 \\ 1.1 \\ 1 \end{pmatrix}$$

Notes:
Using Python code obtained a different result:

Katz centrality of 0(A): 0.5

Katz centrality of 1(B): 0.5

Katz centrality of 2(C): 0.5

Katz centrality of 3(D): 0.5

There was a typo, I haven't heard about a Kate centrality.

(d) Calculate the PageRank centrality.

$X = \beta \left(I - \alpha A D^{-1}\right)^{-1} = \beta \sum_{n=0}^{\infty} \alpha^n \left(A D^{-1}\right)^n I$

Solution with python code:

With damping of $\alpha = 0.1$, $\beta = 1$ and 10000 iterations

PageRank: [0.27225 0.23625 0.23625 0.225]

With damping of 0.85 and 10000 iterations

PageRank: [0.12834375 0.0534375 0.0534375 0.0375]

(2) Consider the adjacency matrix A of a directed network of size $N = 4$ given by

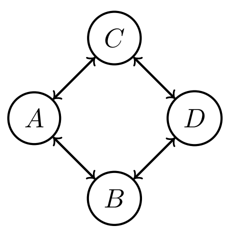
$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

In the following we will indicate with $\mathbf{1}$ the column vector with elements $i_i = 1$ for $i = 1, 2, \cdots, N$ and we will indicate with $\mathbf{I}$the identity matrix.

(a) Draw the network
(b) Calculate the degree centrality.

Answers:

(a) Draw the network



(b) Calculate the degree centrality.

Degree Centrality, where n=4

$C'_D(i) = \frac{d_0(i)}{n-1}$

We can observe that each node has the same degree centrality:

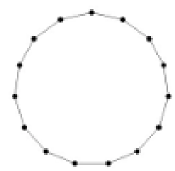
$C'_D(i) = \frac{2}{3}$

$$X = \begin{pmatrix} \frac{2}{3} \\ \frac{2}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{pmatrix}$$

The python code output:

0: 0.6666666666666666, 1: 0.6666666666666666, 2: 0.6666666666666666, 3: 0.6666666666666666

(3) A network consists of n nodes in a ring, where n is odd. All the nodes have the same closeness centrality. What is it, as a function of n?



Answers:

Since n is odd, the shortest path between any two nodes is $\frac{n-1}{2}$. Therefore, the closeness centrality is $\frac{4n}{n^2-1}$.

Calculation:

- $C(x) = \frac{n}{\sum_y d(x,y)}$, where $d(x,y)$ is the distance between node i and j (or x and y).
-
- $C(x) = \frac{n}{\frac{n-1}{2}\frac{n+1}{2}} = \frac{4n}{n^2-1}$

(4) Study the real-world complex networks on Neuman's website $http://www-personal.umich.edu/\ mejn/netdata/$, choose five real-world networks listed in the table and fill the table:

Network	directed or not	node#	edge#	community#
Karate				
Dolphin				
Les Miserable				
American College Football				
Power Grid				

Answers:

Network	directed or not	node#	edge#	community#
Karate	directed	34	78	4
Dolphin	un-directed	62	159	5
Les Miserable	directed	77	254	6
American College Football	un-directed	115	613	9
Power Grid	un-directed	4941	6594	37

(5) Choose one network from the previous question:

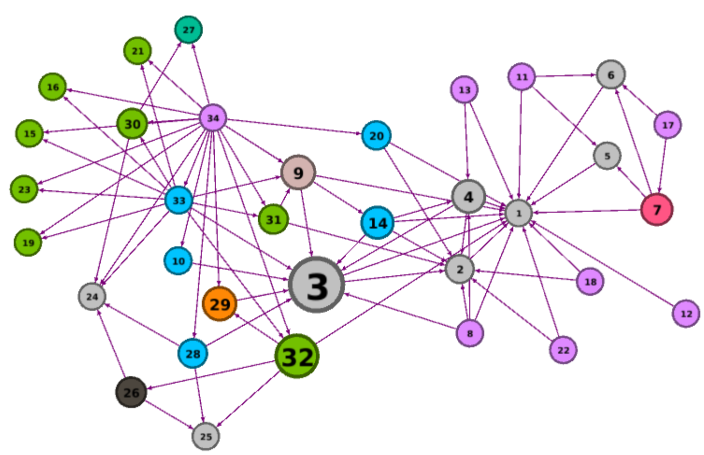
(a) Use Gephi to plot the network. Make sure to use centrality and communities so that you can show the properties of the network.

(b) Use Gephi to find the largest two nodes with the betweenness centrality, degree centrality, and pagerank centrality. Use the table to report your data.

Answers:

I choose the Karate Network. It is the smallest and is easier to work with Gephi.

(a) Use Gephi to plot the network.



(b) Use Gephi to find the largest two nodes.

Betweenness Centrality

Label	Betweenness Centrality
3	8.833333
32	5.083333

Degree Centrality

Label	Degree Centrality
34	17
1	16

Pagerank Centrality

Label	Pagerank Centrality
1	0.246433
2	0.087712