

## COMPLEX NETWORKS - SPRING 2024

### HOMEWORK 3

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- DUE on 03/23/2025 11:59pm C.T.
- You can write on the separate work sheet or type your quiz. ( Word or Latex or similar)
- If you use the handwriting, Solutions must be neat,clear and legible.
- If you need to scan you quiz, save it as a PDF file. Do not use jpeg, png, jpg etc. Do not submit more than one file.
- Please check your scanned file before submission. Make sure it is readable, correct order, properly oriented. Make sure it does include all pages.
- Please name your file as follows: *LastnameInitials–MAP5990quiz1.pdf*. If your name is Alan David Roberts, file name is *RobertsAD–MAP5990quiz1.pdf*.
- Try to keep the file size less than 4MB.
- You can resubmit the quiz if you want. Please specify which one is the one to be graded. Otherwise I will grade the most recent version.
- DO NOT EMAIL me the quiz. All quizzes are submitted via Canvas.

- (1) Consider the adjacency matrix  $A$  of a directed network of size  $N = 4$  given by

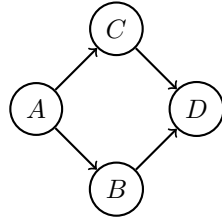
$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

In the following we will indicate with  $\mathbf{1}$  the column vector with elements  $i_i = 1$  for  $i = 1, 2, \dots, N$  and we will indicate with  $\mathbf{I}$  the identity matrix.

- Draw the network
- Calculate the eigenvector centrality using its definition.
- Calculate the Katz centrality.
- Calculate the PageRank centrality.

**Answers:**

- (a) Draw the network.



- Calculate the eigenvector centrality using its definition.
- Calculate the Katz centrality.

There was a typo, I haven't heard about a Kate centrality.

- Calculate the PageRank centrality.

- (2) Consider the adjacency matrix  $A$  of a directed network of size  $N = 4$  given by

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

In the following we will indicate with  $\mathbf{1}$  the column vector with elements  $i_i = 1$  for  $i = 1, 2, \dots, N$  and we will indicate with  $\mathbf{I}$  the identity matrix.

- (a) Draw the network
- (b) Calculate the degree centrality.

**Answers:**

- (a) Draw the network
- (b) Calculate the degree centrality.

- (3) A network consists of  $n$  nodes in a ring, where  $n$  is odd. All the nodes have the same closeness centrality. What is it, as a function of  $n$ ?



**Answers:**

- (4) Study the real-world complex networks on Neuman's website [http : //www-personal.umich.edu/ mejn/netdata/](http://www-personal.umich.edu/mejn/netdata/), choose five real-world networks listed in the table and fill the table:

Network	directed or not	node#	edge#	community#
Karate				
Dolphin				
Les Miserable				
American College Football				
Power Grid				

**Answers:**

- (5) Choose one network from the previous question:
- (a) Use Gephi to plot the network. Make sure to use centrality and communities so that you can show the properties of the network.
  - (b) Use Gephi to find the largest two nodes with the betweenness centrality, degree centrality, and pagerank centrality. Use the table to report your data.

**Answers:**

- (a) Use Gephi to plot the network.
- (b) Use Gephi to find the largest two nodes