

**MAD 6306 COMPLEX NETWORKS - SPRING 2024**  
**HOMEWORK 1**

INSTRUCTOR: JIA LIU

- DUE on 002/02/2025 11:59pm C.T.
- You can write on the separate work sheet or type your quiz. ( Word or Latex or similar)
- If you use the handwriting, Solutions must be neat,clear and legible.
- If you need to scan you quiz, save it as a PDF file. Do not use jpeg, png, jpg etc. Do not submit more than one file.
- Please check your scanned file before submission. Make sure it is readable, correct order, properly oriented. Make sure it does include all pages.
- Please name your file as follows: *LastnameInitials – MAD6306hw1.pdf*. If your name is Alan David Roberts, file name is *RobertsAD – MAD6306hw1.pdf*.
- Try to keep the file size less than 4MB.
- You can resubmit the quiz if you want. Please specify which one is the one to be graded. Otherwise I will grade the most recent version.
- DO NOT EMAIL me the quiz. All quizzes are submitted via Canvas.

(1) Consider the following adjacency matrix of a network

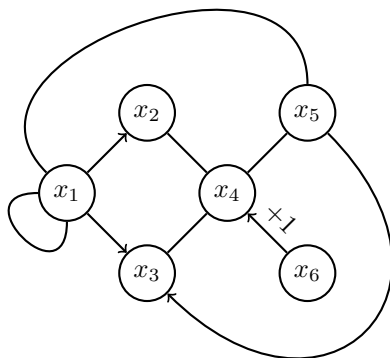
$$A = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

- (a) Is the network directed or undirected? (Explain why).  
 (b) Draw the network.

### Solution

(a) The network is directed because the adjacency matrix is asymmetric.

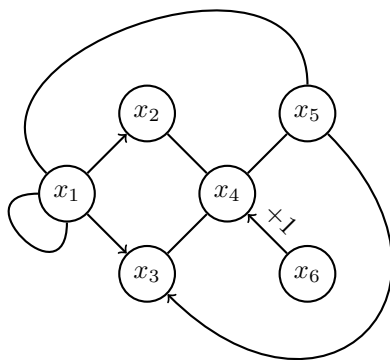
(b) Network



- (2) Given the set of node  $V$  with  $|V| = 6$  in which each node  $i$  is labelled by a natural number between 1 and 6,  $i = 1, 2, 3, 4, 5, 6$ , consider the directed network  $G = (V, E)$  where each link from node  $j$  to node  $i$  indicates that  $j$  is a multiple of  $i$ .
- (a) Draw the network.
- (b) Write down the adjacency matrix of the network.

## Solution

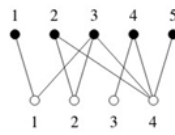
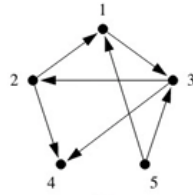
- (a) network



- (b) adjacency matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- (3) Consider the following two networks: Network(a) is directed and Network (b) is undirected but bipartite. Find the following:



- (a) Find the adjacency matrix of network (a)  
 (b) Find the incidence matrix of network (b)

- (4) Which word or words from the following list describe each of the five networks below: *directed*, *undirected*, *cyclic*, *acyclic*, *approximately acyclic*, *planar*, *approximately planar*, *tree*, *approximate tree*.
- (a) The internet, at the level of autonomous systems
  - (b) A food web
  - (c) The stem and branches of a plant
  - (d) A spider web
  - (e) A complete clique of four nodes

- (5) A simple network consists of  $n$  nodes in a single component. What is the maximum possible number of edges it could have? What is the minimum possible number of edges it could have?