Computation and Neural Manifolds

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Neural Manifolds

Neural manifolds are being uncovered in many tasks aacrros the brain that help explain cognition

- What are neural manifolds?
  1. Neural manifolds are abstractions of neural trajectories, neurodynamical structure, often low-dimensional, in high-dimensional neural activity.
- What do neural manifolds do ?
  1. Neural manifolds provide evidence for what is being computed by the brain.
  2. Neural manifolds provide evidence for how the brain computes.

Neural Activity

Neural activity can be described as a high-dimensional neural spcace. We start at some prat of the brain, the dimensionality is the number of neurons on that part, and each dimension is the firing rate of those neurons. The trajectory is the evolution of time
Manifolds are the abstraction fo the Neural N-Space to represent the activity over time

Manifolds

Manifolds can be described as subspaces of vector spaces.
each vector is called the Basis vector
Basis Vectors: every point on manifold is a linear combination of these vectors.

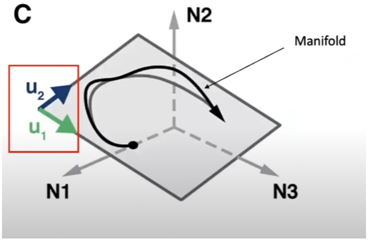

Figure: manifold

Trajectory of time-varying population activity in the neural space of the three recorded neurons (black). The trajectory is mostly confined to the neural manifold, a plane shown in gray and spanned by the neural modes u_1 and u_2.

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Temporal interval estimation

In the temporal interval estimation task, monkeys estiamte the duration of a temporal interval.

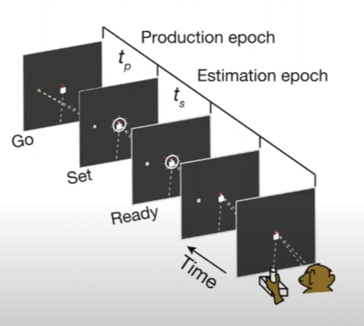

Figure: temporal interval estimation

Schematic of a single trial of the ReadySet-Go task. The animal has to estimate a sample interval, ts, between Ready and Set (estimation epoch), and produce a matching interval, tp, after Set with a delayed response (Go) via a saccade or a movement of the joystick (production epoch).

what is being computed?

What counts as evidence for what is being computed ?

Generally, physical systems provide evidence for what is computed if:

1. the proposed computation maps on to the behavior
2. the behavior maps on to the properties of the part
3. the part maps on to the proposed computation

But first, what was the computation ?

Bayesian Computation

A Bayesian computation was hypothesized for estimating intervals.
The temporal interval estimation task is being analyzed to discover how the brain is computing.

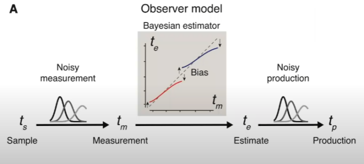

Figure: temporal interval estimation

Behavior on Temporal Estimation Task

Cool graphs go here

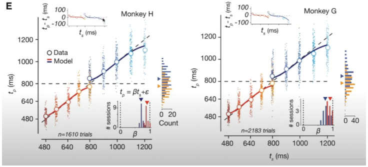

Figure: temporal interval estimation

what is being computed?

What counts as evidence for what is being computed ?

Generally, physical systems provide evidence for what is computed if:

1. the proposed computation maps on to the behavior
   The Bayesian computation maps the bias in estimates of the interval
2. the behavior maps on to the properties of the part
   How about the behaviour part mapping ?
3. the part maps on to the proposed computation

But first, what was the computation ?

Neural Manifolds on Temporal Estimation Task

Using Principal component Analysis, we can reduce and project this neural activity in a 3D space formed from the 3 principal components.
So know we can see the behavior maps on the properties of the part, for longer intervals there is a longer manifolds for shorter intervals there is shorter manifolds.

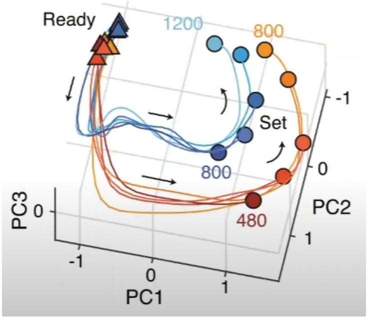

Figure: temporal interval estimation

what is being computed?

What counts as evidence for what is being computed ?
Generally, physical systems provide evidence for what is computed if:

1. the proposed computation maps on to the behavior
   The Bayesian computation maps the bias in estimates of the interval
2. the behavior maps on to the properties of the part
   the position along the curved manigolds maps the estimate of the interval
3. the part maps on to the proposed computation
   Does the manifolds maps on to the computation?

manifolds maps on to the computation?

Different inputs determine the identity of the manifolds and the position in manifold space.
Different manifolds for different priors
Different states for different intervals within a distribution provide the input to the projection.

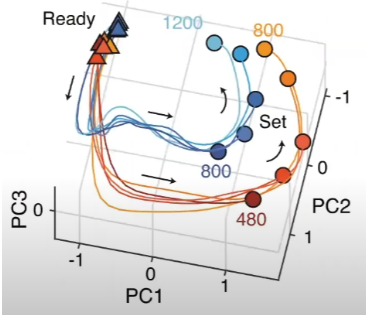

Figure: temporal interval estimation

what is being computed?

What counts as evidence for what is being computed ?
Generally, physical systems provide evidence for what is computed if:

1. the proposed computation maps on to the behavior
   The Bayesian computation maps the bias in estimates of the interval
2. the behavior maps on to the properties of the part
   the position along the curved manigolds maps the estimate of the interval
3. the part maps on to the proposed computation
   the position along the curved manifold maps the Bayesian computation

Evidence for what is being computed

A three-way mapping between computation, part, and behavior is evidence for what is being computed.
The Bayesian computation maps the bias in estimates of the interval, the position along the curved manifolds maps the estimate of the interval, and the position along the curved manigolds maps the Bayesian computation.
Now we know how the Brain computes, can we use the Manifolds as evidence for What the Brain Computes ?

How it is being computed?

Manifolds are also evidence for how something is being computed.
How the Bayesian estimates are being computed by neural activity.
The hypothesized computation was a projection from the manifold on to an encoding dimension.

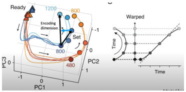

Figure: temporal interval estimation

the second computation was a projection from the manifold onto what they called an encoding dimension.

This encoding Dimension was simply the vector described by the line that connected two points onto the manifold:

- The point corresponding to the shortest interval to be estimated in a distribution
- The point corresponding to the longest interval in the distribution to be estimated

The geometry of projecting a curve onto a line is the compression of the

How it is being computed?

Intermediate intervals are mapped onto intermediate points on the manifold.
So the differences in the input to the vector projection give rise to differences in speed.
x-axis is the projection onto the encoding dimension u and the y-axis is the speed through that manifold space along the curved 2D manifold and there is a law like correlation where the greate the projection onto U slower the speed .
We can observe a preservation in the projections of the relations between the speeds between the output of the computations and the properties of the manifolds and the proposed computation of vector projection

How it is being computed?

Manifolds provide evidence for how somehting is being computed.
Points on manifold map onto inputs and points on manifold or in manifold space map onto outputs of computation, and the relations between inputs and outputs of the computation have corresponding relations between points on manifold or in manifold space.
There is a Homomorphism, which is the preservation of relations between the outputs of the proposed computation and there is a corresponding relation in manifold space that preserves those relations.

Objections

Arent all these computations the result of a single neuron activity ?

Example of computation by neurons: perceptual decision making

Example of computation by neurons: perceptual decision making
Stage 1: MT - Sensory Evidence
Stage 2: LIP - Accumulated Sensory Evidence
Stage 3: Choice - Categorization of evidence

Objections Reply

Arent all these computations the result of a single neuron activity ?
Reply:
Single Neurons cannot span a ¿1D space
But manifolds are often 1D
But often they are ¿1D

Objections

1. Arent all these computations the result of a single neuron activity ?
2. Arent all these computations the result of population activity ?

Objections Reply

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Reply

- Different population patterns can result in the same manifold.
- Population equivalence classes are not the right type of thing to transform inputs into outputs or to structure dynamics.
- What all the elements of the population equivalence class have in common are the dynamics -¿ collapses into manifold view

In other words, different patterns of spiking activity across the population results in the same low dimensional dynamical structure.
In other works, the neural trajectories whose abstractions are the neural manifolds.
This implies that there are not dedicated neurons that encode the manifold dimensions because there is a variety of diversity of population activity patterns that can give rise to it.

Objections

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Objections Reply

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Reply

- The weight matrix is not sufficient: difference numbers of neurons and different sets of weights are sufficient for the same manifold.
- What difference weight matrices have in common are just those dynamics -¿ collapses into manifold view.
- Similar point applies for other features of population (e.g., noise correlations)

Neuraocomputational Manifold Hypothesis

NCM - Neural Trajectory n on a manifold m in manifold space S is a computation of a function f over inputs i and outputs o only if there is:

- input mapping: neural trajectory n starts on m that maps onto i;
- output mapping: neural trajectory n ends at a point in S that maps onto o; and
- relation mapping: for every relation R between pairs of i and o of f, there is a relation R' between pairs of points from n on m and in S.

Objections

1. Arent all these computations the result of a single neuron activity ?
2. Arent all these computations the result of population activity ?
3. Arent all these computations the result of other features of the population ?
4. Manifolds arent real.

Inpute-State-Output Analysis of Computation

A physical system P implements a CSA M if there is a vectorization of internal states of P into components $\left[s^1, s^2, \cdots\right]$, and a mapping f from the substates s^j into corresponding substates S^j of M, along with similar vectorizations and mapping for inputs and outputs, such that for every state-transition rule
$\left(\left[I^1, \cdots, I^k\right], \left[S^1, S^2, \cdots\right]\right) -> \left(\left[S'^1, S'^2, \cdots\right], \left[O^1, \cdots, O^i\right]\right)$ of M: if P is in internal state $\left[s^1, s^2, \cdots\right]$ and receiving input $\left[i^i, \cdots, i^n\right]$ which map to formal state and input $\left[S^1, S^2, \cdots\right]$ and $\left[I^1, \cdots, I^k\right]$ respectively, this reliably causes it to enter an internal state and produce an output that map to $\left[S'^1, S'^2, \cdots\right]$ and $\left[O^1, \cdots, O^i\right]$ respectively.
Chalmers 2011, p.329

Inpute-State-Output Analysis of Computation Reply

This is different than the computations outlined in the temporal interval estimation that rely on manifolds.
The computation or those trajectories does not rely on these sort of state to state transformations.
Because across different manifolds such as the transformation from the estimation epic one-dimensional manifolds to the production epic two-dimensional may involve state-to-state changes, but it also may involve changes in manifold space altogether.
Also part of the Manifold hypothesis is the preservation of the relations, the homomorphism and that requirement is absent from the Inpute-State-Output Analysis of Computation. Thus this analysis is insufficient.

Input-Output Analysis of Computation

Cummins says that "to compute a function g is to execute a program that gives o as its output on input i just in case $g(i) = o$. Computing reduces to program execution, so our problem reduces to explaining what it is to execute a program ... program exectuion involves steps, and to treat each elementary step as a function that the executing system simply satisfies. To execute a program is to satisfy steps" - Cummins 1989, p. 91-92

Temporal Interval estimation cannot be described as program execution

Modeling Analysis of Computation

A physical system P is a computing system just in case:

1. **Input-Output Mirroring**. The input-output function, g, of a given process in **P** preserves a certain relation, $\underline{R}$, in a target domain **T**: there is a mapping from **P** to **T** that maps g to $\underline{R}$, x to $\underline{x}$, y to $\underline{y}$, . . . , such that $g(x) = y$ iff $\left\langle \underline{x}, \underline{y} \right\rangle \ \epsilon \ \underline{R}$. This means that g and $\underline{R}$ share some formal relation **f**.
2. **Implementing**. This process of **P**, whose input-output function is g, implements some formalism **S** whose input-output (abstract) function is **f**.
3. **Representing**. The input variables x of **P** represent the entities $\underline{x}$ of **T**, and the output variables y of **P** represent the entities $\underline{y}$ of **T**.

Shagrir 2022, p.240

Modeling Analysis of Computation Reply

The same function describes a relation between states of the physical system and states of the computation, but this seems false because some functional relations between physical system States simply do not stand in a counterpart relation in the sates of the computation for example different lenghts of temporal intervals stand in longer than and shorter than temporal relations, but the manifold states which have the best claim to satisfy the modeling account do not stand in temporal relations rather they stand in manifold spatial relations.
Shagrir uses a lot the occular motion integrator as example. The occular motor integrator maps certain neural inputs to neural outputs and this mapping input output process mirrors the relation between movements and positions of the eye.

Conclusion

Manifolds provide evidence for what is being computed and how it is being computed in the brain

- Manifolds are abstractions from sets of neural trajectory and are structure in neural dynamics
- Manifolds are evidence for what and how the brain computes
- Manifolds can constrain accounts of neural computation.

References

The Slides 1 through 30 are based on the presentation:

Computation and Neural Manifolds

https://www.youtube.com/watch?v=zgP8SipIuHA

Galego et al. 2017

Neural Manifolds for the Control of Movement

https://pubmed.ncbi.nlm.nih.gov/28595054/

Sohn et al 2019

Bayesian Computation through Cortical Latent Dynamics

https://www.sciencedirect.com/science/article/pii/S0896627319305628

Crazy Ideas

Take the idea of the Feynman's path integral as inspiration.
The brain computes by creating and manipulating representations of reality, effectively building models that allow for prediction and manipulation of the world. Computing is not just about processing data (the change of states), but about creating and using representations that have meaning.
These representations can be expressed as manifolds.
As it tranverses the manifold we observe a change of phase which can be descirbed in information geometry and the former statistical mechanics as related to likelihood. (attention to the Hodge operator). If we consider that there are many paths to start at a simplicial representation and to achive a simplicial go and the path is the computation. We can say that all computations that are within the path of least ACTION constructively interferes and all other paths destructively interferes.
Need to extend the Approximate Bayesian Computation (ABC)

Path Integral

Take the idea of the Feynman's path integral as inspiration.
$A = e^{i\phi}$
$\Delta\phi = \frac{2\pi}{\lambda}\Delta x - 2\pi f\Delta t$
$A = e^{i\sum\Delta\phi} = e^{i\sum\frac{2\pi}{\lambda}\Delta x - 2\pi f\Delta t}$
de Broile: $\lambda = \frac{h}{mv}$, Einstein-Planck: $E = hf -> f = \frac{E}{h}$, Simplified Planck Constant $\hbar = \frac{h}{2\pi}$
$A = e^{i\sum\frac{2\pi mv}{h}\Delta x - 2\pi\frac{E}{h}\Delta t}$
$A = e^{i\frac{\sum mv\Delta x - E\Delta t}{\hbar}}$
$A = e^{i\frac{\sum\left(mv\frac{\Delta x}{\Delta t} - E\right)\Delta t}{\hbar}}$
For a very small Δt:
$A = e^{i\frac{\int\left(mv\frac{dx}{dt} - E\right)dt}{\hbar}}$
$mv\frac{dx}{dt} = mv^2$ and $E = T + V = \frac{1}{2}mv^2 + V$

Path Integral

$A = e^{i \frac{\int \left(mv^2 - \frac{1}{2} mv^2 - V \right) dt}{\hbar}}$
$A = e^{i \frac{\int \left(\frac{1}{2} mv^2 - V \right) dt}{\hbar}}$
Classical Action: $S = \int \left(\frac{1}{2} mv^2 - V \right) dt$ therefore $A = e^{i \frac{S}{\hbar}}$
$A = e^{i \frac{S}{\hbar}}$ given $\hbar \approx 10^{-34} J \cdot s$
$dS = S[q] - S[p] = 0$
So the path chosen is the path of least action which is the path where all phases interfere cosntructively.
If you observe closely this resembles the Lagrangian.
$S = \int_{t_1}^{t_2} (T - V)\, dt = \int_{t_1}^{t_2} L dt$
T being kinect energy and V being potential energy.

Lagrangian Examples
Classical mechanics: $L_{CM} = T - V$
Special relativity $L_{SR} = -mc^2\sqrt{1 - \frac{v^2}{c^2}}$
Electrodynamics $L_{EM} = -1\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{c}j_\mu A_\mu$
General Relativity $L_{GR} = -mc^2\sqrt{-c^2 g_{\mu\nu}}$