

MEASURES OF SPATIAL SEGREGATION

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The measurement of residential segregation patterns and trends has been limited by a reliance on segregation measures that do not appropriately take into account the spatial patterning of population distributions. In this paper we define a general approach to measuring spatial segregation among multiple population groups. This general approach allows researchers to specify any theoretically based definition of spatial proximity desired in computing segregation measures. Based on this general approach, we develop a general spatial exposure/isolation index (\tilde{P}^), and a set of general multigroup spatial evenness/clustering indices: a spatial information theory index (\tilde{H}), a spatial relative diversity index (\tilde{R}), and a spatial dissimilarity index (\tilde{D}). We review these and previously proposed spatial segregation indices against a set of eight desirable properties of spatial segregation indices. We conclude that the spatial exposure/isolation index \tilde{P}^* —which can be interpreted as a*

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measure of the average composition of individuals' local spatial environments—and the spatial information theory index \tilde{H} —which can be interpreted as a measure of the variation in the diversity of the local spatial environments of each individual—are the most conceptually and mathematically satisfactory of the proposed spatial indices.

1. INTRODUCTION—SEGREGATION AND SPACE

Reliable and meaningful measurement of residential segregation is essential to the study of the causes, patterns, and consequences of racial and socioeconomic segregation. Nonetheless, prior work on residential segregation has been limited by a reliance on methodological tools that do not fully capture the spatial distributions of race and poverty. Scholars have repeatedly pointed out that the most commonly used measures of segregation—such as the dissimilarity index (D), the exposure index (P^*), the variance ratio index (V), and the entropy-based information theory index (H)—are “aspatial,” meaning that they do not adequately account for the spatial relationships among residential locations (Grannis 2002; Massey and Denton 1988; Morrill 1991; Reardon and Firebaugh 2002b; Wong 1993; Wong 2002).

In this paper, we take up the challenge of developing measures of spatial segregation that satisfactorily address the problems identified with existing measures of segregation. We begin by arguing for a set of criteria that would be met by a satisfactory spatial segregation measure. We then present a new and general approach to measuring spatial segregation that addresses the key limitations of prior spatial measures. This approach allows researchers to specify theoretically appropriate definitions of how spatial features constrain or enhance the possibility of social interaction. Finally, we review previously proposed measures of spatial segregation and evaluate both these and our new measures against our criteria.

1.1. *Methodological Issues in the Measurement of Spatial Segregation*

Segregation can be thought of as the extent to which individuals of different groups occupy or experience different social environments. A measure of segregation, then, requires that we (1) define the social

environment of each individual, and (2) quantify the extent to which these social environments differ across individuals. Traditional measures of segregation are aspatial, in that they differ from one another only on the second of these criteria, because they implicitly define the social environment as equivalent to some organizational or spatial unit (school, census tract), without regard for the patterning of these units in social space. Much prior discussion of segregation indices, then, has focused only on the matter of the most appropriate mathematical formulation for quantifying differences across social environments (James and Taeuber 1985; Reardon and Firebaugh 2002a; White 1986; Zoloth 1976).

Aspatial segregation measures have been repeatedly criticized in the residential segregation context for their failure to account for the spatial patterning of census tracts (Grannis 2002; Massey and Denton 1988; Morrill 1991; Wong 1993; Wong 2002). In particular, two flaws of aspatial measures are identified: the *checkerboard problem* (Morrill 1991; White 1983) and the *modifiable areal unit problem* (Openshaw and Taylor 1979; Wong 1997). Each of these can be seen as critiques of the definition of the social environment implicit in the traditional segregation measures.

The checkerboard problem stems from the fact that aspatial segregation measures ignore the spatial proximity of neighborhoods and focus instead only on the racial composition of neighborhoods. To visualize the problem, imagine a checkerboard where each square represents an exclusively black or exclusively white neighborhood. If all the black squares were moved to one side of the board, and all white squares to the other, we would expect a measure of segregation to register this change as an increase in segregation, since not only would each neighborhood be racially homogeneous, but most neighborhoods would now be surrounded by similarly homogeneous neighborhoods. Aspatial measures of segregation, however, do not distinguish between the first and second patterns, since in each case the racial compositions of individual neighborhoods are the same (White 1983).

The modifiable areal unit problem (MAUP) arises in residential segregation measurement because residential population data are typically collected, aggregated, and reported for spatial units (such as census tracts) that have no necessary correspondence with meaningful social/spatial divisions. This data collection scheme implicitly assumes

that individuals living near one another (perhaps even across the street from one another) but in separate spatial units are more distant from one another than are two individuals living relatively far from one another but within the same spatial unit. As a result—unless spatial subarea boundaries correspond to meaningful social boundaries—all measures of spatial and aspatial segregation that rely on population counts aggregated within subareas are sensitive to the definitions of the boundaries of these spatial subareas.¹

Essentially then, the definition of spatial segregation measures requires a redefinition of the social environment implicit in the traditional segregation measures. In fact, the checkerboard problem and the MAUP are both artifacts of a reliance on subarea (e.g., tract) boundaries in the computation of segregation measurement. In principle, a segregation measure that used information on the exact locations of individuals and their proximities to one another in residential space could eliminate the checkerboard problem and MAUP issues entirely.

1.2. *The Dimensions of Spatial Segregation*

Another confusion in the segregation literature also results from relying on census tract boundaries in computing segregation measures. In an often-cited article, Massey and Denton (1988) describe five conceptually distinct “dimensions” of residential segregation: (1) *evenness*, (2) *exposure*, (3) *clustering*, (4) *centralization*, and (5) *concentration*. In their formulation, evenness and exposure are aspatial dimensions

¹In fact, the MAUP is constituted by two interrelated effects: an *aggregation* (or scale) effect, and a *zoning* effect (Wong 1997). The aggregation effect leads to differences in statistical measures resulting purely from dealing with data that are “less detailed.” The difference between a statistic derived from tract data and the same statistic derived for block group data, for example, is an aggregation effect. For segregation measures, greater aggregation usually results in lower measured levels of segregation. The zoning effect refers to the fact that any measure derived from aggregated population data depends on the choice of aggregation zones (i.e., the “modifiable areas”), even if the scale and number of the zones remains fixed. With regard to the census tracts often used in studies of segregation, the effect is *initially* to exaggerate segregation (because tracts are designed to be relatively homogeneous internally). However, over time, if the same zones are retained, measured levels of segregation fall (Massey and Denton 1988).

(allowing that they are nonetheless implicitly spatial because they depend on census tract boundaries), while clustering, concentration, and centralization are explicitly spatial dimensions of segregation, and they require information on the location and size of census tracts to compute.

The distinction between aspatial evenness and spatial clustering, however, is an artifact of the reliance on spatial subareas (e.g., census tracts) at some chosen geographical scale of aggregation. Evenness, in Massey and Denton's formulation, refers to the degree to which members of different groups are over- and underrepresented in different subareas relative to their overall proportions in the population. Clustering refers to the proximity of subareas with similar group proportions to one another. However, evenness at one level of aggregation (say, census tracts), is clearly strongly related to clustering at a lower level of aggregation (say, block groups), since tracts where a minority group is overrepresented will tend to be clusters of block groups where the minority population is overrepresented. Unless subarea boundaries correspond to meaningful social boundaries, the distinction between evenness and clustering is thus arbitrary.

In principle, if we derived a segregation measure from information about the exact locations and spatial environments of individuals and their proximities to one another in residential space, there would be no conceptual distinction at all between evenness and clustering. Any movement of an individual that increased unevenness (by moving a person from a location where his or her group is underrepresented to one where it is overrepresented) would also increase clustering, because it would result in members of the same groups being nearer to one another.

As a result of this insight, we suggest an alternative to the Massey and Denton (1988) dimensions of residential segregation. We argue that there are two primary conceptual dimensions to spatial residential segregation: (1) spatial exposure (or spatial isolation) and (2) spatial evenness (or spatial clustering). *Spatial exposure* refers to the extent that members of one group encounter members of another group (or their own group, in the case of spatial isolation) in their local spatial environments. *Spatial evenness*, or clustering, refers to the extent to which groups are similarly distributed in residential space. Spatial exposure, like aspatial exposure, is a measure of the typical environment experienced by individuals; it depends in part on the

overall racial composition of the population in the region under investigation. Spatial evenness, in contrast, is independent of the population composition.

To see that spatial exposure and evenness are conceptually distinct, consider the four patterns of individual residential locations (not subarea proportions) shown in Figure 1. In the upper half of the diagram are two patterns where black and white households are evenly distributed throughout space. Both of these patterns have low levels of spatial clustering (or high levels of spatial evenness). In the pattern on the upper right, however, there are more black households in the local environment of each white household (and vice versa) than in the pattern on the upper left; this means that the white-black exposure is higher on the right, and the white isolation is higher on the left. In the bottom half of the figure, both patterns

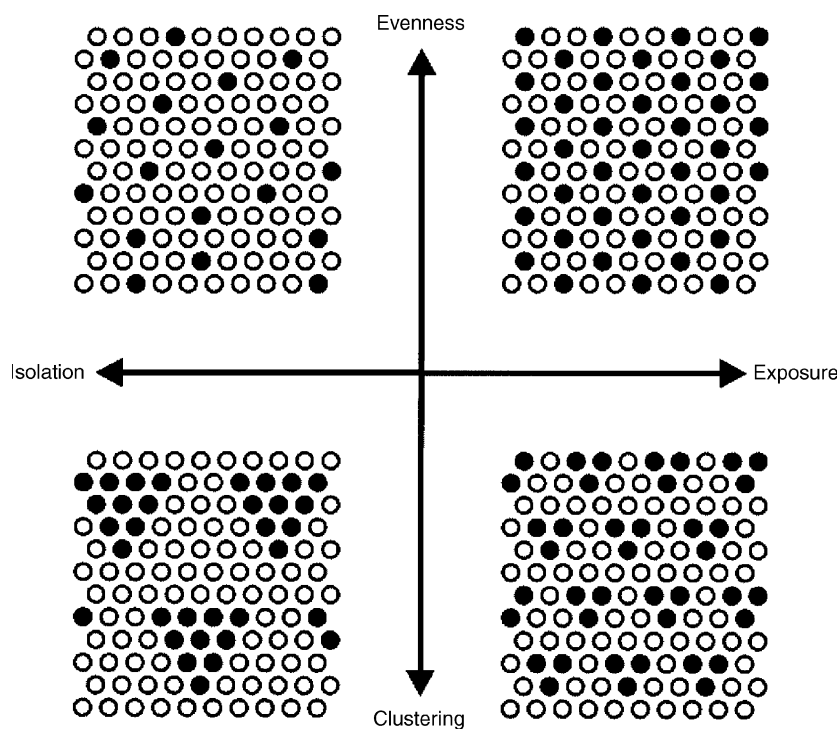


FIGURE 1. Dimensions of spatial segregation.

show greater clustering—but roughly the same levels of exposure—than the corresponding patterns above.

In this framework, Massey and Denton's evenness and clustering dimensions are collapsed into a single dimension. Their exposure dimension remains intact, but it is now conceptualized as explicitly spatial. Their centralization and concentration dimensions can be seen as specific subcategories of spatial unevenness. In some cases, centralization and concentration may be of sufficient theoretical interest to be considered distinct subdimensions; however, we do not consider them further in this paper.

1.3. *Existing Measures of Spatial Segregation*

Many spatial measures have been developed to address the methodological shortcomings identified above (for example, see Frank 2003; Grannis 2002; Jakubs 1981; Massey and Denton 1988; Morgan 1982, 1983a, 1983b; Morrill 1991; Waldorf 1993; White 1983, 1986; Wong 1993; Wong 1998, 1999, 2002), although it is not clear that any of the proposed measures fully solve the problem of measuring spatial segregation. Many of the measures have been developed in a relatively *ad hoc* manner, and none have been evaluated against a conceptually meaningful set of criteria, as has been done for the traditional aspatial measures (James and Taeuber 1985; Reardon and Firebaugh 2002a), so it is unclear whether they reliably produce results consistent with theoretically useful definitions of segregation.

At present, few of the proposed spatial segregation measures have been used in published empirical segregation research. These measures have been ignored in part because they typically are more difficult to compute than the aspatial measures. At present, there is also still little publicly available software to compute spatial segregation measures—Wong's extensions to the Arc/INFO (Wong and Chong 1998) and ArcView GIS software (Wong 2003), and Apparicio's extension to MapInfo GIS (Apparicio 2000) are the only examples that we are aware of. This limitation, however, is likely to become less relevant with the increased availability and ease of use of geographical information system (GIS) software (Longley et al. 2001). However, in the absence of a clear evaluation of the proposed

measures, the development of GIS software is likely to lead to a situation where researchers use a wide variety of different measures, resulting in findings that cannot be easily compared across studies.

2. MEASURES OF SPATIAL SEGREGATION

2.1. *Notation*

Throughout this paper, we use the following notation: consider a spatial region R populated by M mutually exclusive population subgroups (e.g., racial groups), indexed by m . Let p , q , and s index points within the region R ; and let r index subareas of the region R (e.g., census tracts). Let τ denote population density and π denote population proportion. In addition, we use a super-positioned tilde (\sim) to indicate that a parameter describes the spatial environment of a given point, rather than the point itself. Thus we have

τ_p = population density at point p ,

τ_{pm} = population density of group m at point p (note that $\sum_m \tau_{pm} = \tau_p$),

T = total population in R (note that $\int_p \tau_p dp = T$),

$\tilde{\tau}_{pm}$ = population density of group m in the local environment of point p ,

π_m = proportion in group m of total population (e.g., proportion black),

π_{pm} = proportion in group m at point p (defined as $\pi_{pm} = \tau_{pm}/\tau_p$),

$\tilde{\pi}_{pm}$ = proportion in group m in the local environment of point p .

Note that the population densities τ_p and τ_{pm} are defined by the population counts per unit area at location p . In practice, these must be estimated from census tract (or other subarea) population counts, most simply by dividing the population count of a tract by its area and assigning the population density this value at each point in the tract. Other density estimation procedures might be used as well, including pycnophylactic ("mass preserving") smoothing and dasy-metric mapping (for example, see Dent 1999; Mennis 2003; Tobler 1979). We leave discussion of these estimation methods and of the

sensitivity of segregation measurement to different choices of density estimators, however, for another paper.

2.2. Spatial Proximity and the Local Environment

The measurement of spatial segregation requires that we define the spatial proximity between all pairs of points in a region R . Let $\phi(p, q)$ be a non-negative function that defines the spatial proximity of locations q and p , such that $\phi(p, q) = \phi(q, p)$ and $\phi(q, q) = \phi(p, p)$ for all $p, q \in R$, and with larger values of $\phi(p, q)$ indicating greater proximity. Let $\Phi_p = \int_{q \in R} \phi(p, q) dq$, noting that we do not require $\Phi_p = \Phi_q$ for all $p, q \in R$. We define the population density of the local environment of a point p as the weighted average of the population densities of all other points in R , where points are weighted by their proximity to p :²

$$\tilde{\tau}_p = \frac{1}{\Phi_p} \int_{q \in R} \tau_q \phi(p, q) dq. \quad (1)$$

We define $\tilde{\tau}_{pm}$ similarly, substituting τ_{qm} for τ_q in equation (1). Now $\tilde{\tau}_p$ and $\tilde{\tau}_{pm}$ are, respectively, the spatially weighted average population density and the group m population density at point p . For each m , $\tilde{\tau}_{pm}$ describes a spatially smoothed population surface, where the value of $\tilde{\tau}_{pm}$ at location p indicates the group m population density at point p . We define

$$\tilde{\pi}_{pm} = \frac{\tilde{\tau}_{pm}}{\tilde{\tau}_p}. \quad (2)$$

It is trivial to show that, for each location p ,

$$\sum_{m=1}^M \tilde{\pi}_{pm} = 1. \quad (3)$$

We can think of the $\tilde{\pi}_{pm}$'s as indicating the population composition that a person living at point p would experience in his or her local

²Throughout this paper, we use a single integral to denote the summation over all points in a region.

environment, where the local environment is defined by the proximity function ϕ .³

The function $\phi(p, q)$ may take on a variety of possible forms, each implying a different definition of the local environment. For example, $\phi(p, q)$ might be a function that declines as the Euclidean distance from p to q increases, which means that the spatial environment of point p is influenced more by the population nearby than by those more distant. The spatial proximity function $\phi(p, q)$ might also incorporate information about physical barriers (such as rivers, railroads, or highways) and patterns of social interaction between locations p and q . Ideally, a spatial proximity function should capture theoretically meaningful patterns of social interaction.

One special case of the spatial proximity function is worth noting. Measures of aspatial segregation implicitly define the local environment of each individual as equivalent to the organizational unit (e.g., census tract, school) containing the individual. Reardon and Firebaugh (2002b) note that this can be seen as a special case of the above definition of the local environment, where spatial proximity is defined such that $\phi(p, q)$ equals some constant c if p and q are both in tract r and $\phi(p, q) = 0$ if p and q are in separate tracts. In this case, equations (1) and (2) yield $\tilde{\pi}_{pm} = \pi_{rm}$ for all m and all $p \in r$, indicating that the group composition of the local environment at each point in r is identical to the group proportions in tract r as a whole, regardless of how population groups are distributed within the tract, or how tracts are arranged in space (Reardon and Firebaugh 2002b). This insight—that the aspatial segregation indices can be seen as spatial indices that depend on a very specific notion of spatial proximity—will prove

³Note that we can rewrite equation (2) as

$$\tilde{\pi}_{pm} = \int_{q \in R} \frac{\tau_q \phi(p, q)}{\int_{s \in R} \tau_s \phi(p, s) ds} \pi_{qm} dq.$$

From this, we can see that $\tilde{\pi}_{pm}$ is a density- and proximity-weighted average of the π_{qm} 's for all q in R . In the aspatial case, population density and group proportions are assumed constant within tracts and the spatial proximity of each pair of distinct tracts is zero, so the above yields $\tilde{\pi}_{pm} = \pi_{rm}$, where p is in tract r (see Reardon and Firebaugh 2002b).

useful in our approach to developing spatial segregation measures in this paper.

2.3. *Criteria for Evaluating Spatial Segregation Measures*

Previous methodological work, drawing on the inequality measurement literature (for example, see Schwartz and Winship 1980), has defined a set of criteria for the evaluation of aspatial evenness measures of segregation (James and Taeuber 1985; Reardon and Firebaugh 2002a). Compliance with these criteria implies that a measure will register an appropriate change in segregation levels under specified conditions; conversely, noncompliance implies that it is possible for a measure to respond to changes in population distributions in ways that are inconsistent with conceptually appropriate definitions of segregation. Since the criteria were developed with aspatial measures in mind, Reardon and Firebaugh (2002b) suggest that they may need to be modified in order to apply them to spatial segregation measures. Here we describe a general set of criteria for segregation measures that apply to spatial evenness measures. A subset of these reduce to the Reardon and Firebaugh (2002a) criteria in the special case where a measure is aspatial.⁴ In addition to these criteria, we suggest several additional desirable properties that pertain specifically to spatial segregation indices. Five of the criteria—scale interpretability, arbitrary boundary independence, location equivalence, population density invariance, and additive spatial decomposability—apply to measures of spatial exposure as well.

1. *Scale interpretability*: A spatial segregation index should be equal to zero if the group proportions are the same in the local environment of each individual. A segregation index should reach its maximum value (typically normalized to equal 1) if the local environment of each individual is monoracial. An alternate way

⁴In addition to possessing the properties described here, a spatial segregation index should (1) be a continuous function of both the total and group population densities at each point and of the spatial proximity function between all points; (2) allow the computation of segregation among any number of population groups; and (3) correspond to a meaningful (aspatial) segregation measure in the aspatial special case.

of stating this is that a segregation index should reach its maximum value only if the proximity of any two members of different groups is zero. A segregation index should take on a negative value if the population is “hyper-integrated”—if individuals, on average, experience greater diversity in their local environments than the diversity of the population as a whole.⁵

2. *Arbitrary boundary independence*: A spatial segregation measure should be independent of the definitions of census tract (or other subarea) boundaries. In principle, a spatial segregation measure should be computed from information about the exact locations and spatial proximities of individuals in residential space (although in practice, it may be necessary to use tract or other subarea data to estimate local population densities). This will ensure that a measure will not be susceptible to MAUP issues.
3. *Location equivalence*: If the local environments of two points p and q have the same population composition (i.e., if $\tilde{\pi}_{pm} = \tilde{\pi}_{qm}$ for all m) and the same proximity to all other points (i.e., if $\phi(p, s) = \phi(q, s)$ for all $s \neq p, q$),⁶ then segregation is unchanged if we treat the two points as one point with a population density equal to the sum of the two original points. While this criterion may seem to have little concrete application, it is a spatial generalization of the aspatial organizational equivalence criterion, which states that if two organizational units (schools, tracts) have the

⁵In the spatial case, unlike the aspatial case, it may be possible—and meaningful—for the average individual to experience greater diversity in his or her local environment than the diversity of the population as a whole. Consider the residential pattern shown in the upper-right corner of Figure 1. If we define the local environment of each household as consisting of itself plus the six households immediately adjacent to it, then each white household will be in a local environment that is 3/7 black, despite the fact that the overall population is only 1/3 black. Likewise, each black household inhabits a local environment that is 6/7 white, despite the fact that the total population is only 2/3 white. In such a case, the population may be said to be hyper-integrated. A segregation index should be negative in this case, indicating that the population is more integrated than expected given the population composition.

⁶In general, this can occur only if the two points have the same population composition ($\pi_{pm} = \pi_{qm}$ for all m) and either (1) the points have the same population density ($\tau_p = \tau_q$), or (2) the points have the same population composition as their local environments ($\tilde{\pi}_{pm} = \tilde{\pi}_{qm} = \pi_{pm} = \pi_{qm}$ for all m).

same composition and are combined into a single unit, segregation is unchanged (James and Taeuber 1985).⁷

4. *Population density invariance*: If the population density τ_{pm} of each group m at each point p is multiplied by a constant factor c , segregation is unchanged. This is a spatial generalization of the aspatial size invariance criterion (James and Taeuber 1985).
5. *Composition invariance*: In general, a measure of spatial evenness should be independent of the population composition and should depend only on the distribution of groups in space. More formally, the composition invariance criterion states that if the proportions of groups change in the population while the relative distribution of groups in space remains the same, then segregation is unchanged. The key to operationalizing this seemingly intuitive concept is to define what it means for the relative distribution of groups to remain the same. As Coleman, Hoffer, and Kilgore (1982) point out, determining whether an index is composition invariant always “depends on a specific definition of what it means to say that the distribution [of individuals among organizational units] is ‘kept the same’ while the [group] proportion changes” (p. 177).⁸

The literature on segregation measurement provides several different definitions of composition invariance. James and Taeuber (1985) say that the distribution of individuals in space is the same if the population density of group m at each point is multiplied by a constant c and the population density of all other groups at each point is unchanged.⁹ Coleman, Hoffer, and Kilgore (1982), however,

⁷Note that this criterion implies that if $\phi(p, q)$ is defined so that $\phi(p, q) = c$ for all points p and q in tract r and $\phi(p, s) = \phi(q, s)$ for all points p and q in tract r and all points s not in tract r , then a segregation measure that satisfies the locational equivalence criterion will be unchanged if we consider the entire population of the tract to be located at a single point within the tract (e.g., the centroid).

⁸We would like to thank a thoughtful anonymous reviewer who suggested the importance of clarifying the meaning of the composition invariance criterion.

⁹Another proposed definition of composition invariance is given by Gorard and Taylor (2002), who argue that the distribution of individuals in space is the same if the *proportion* of group m in each location is multiplied by a constant (while the total enrollment in each school remains the same). This definition, however, is not internally consistent, since it is not symmetric—multiplying group m ’s proportion by c implies multiplying each other group’s composition in each location p by $d_p = (1 - c\pi_{pm})/(1 - \pi_{pm})$. Unless π_{pm} is constant for all p , then d_p varies across p , so composition invariance under this definition is dependent on which group is considered.

argue that this is an arbitrary definition of composition invariance; they imply that any segregation measure can be considered composition invariant under an appropriate definition of segregation. For example, if we define segregation as the ratio of actual to potential pair relations between members of different groups (as Coleman and colleagues do), then any change in the population composition that leaves this ratio unchanged should leave a composition invariant segregation index unchanged. The variance ratio index (V) can be defined as simply (one minus) the ratio of actual to potential pair relations between members of different groups, which means that V is composition invariant under this definition of segregation (Coleman, Hoffer, and Kilgore 1982).

This definition of composition invariance is somewhat tautological, of course, since it suggests that if we believe that an index appropriately measures what we take to be segregation (specifically, spatial evenness/clustering) in some meaningful sense, then that measure will be composition invariant by an appropriate definition of the criterion—a change in the population composition that leaves what is measured by a particular index unchanged will necessarily leave segregation unchanged, as measured by that index. As a result, we take the position that the traditional composition invariance criterion espoused by James and Taeuber (1985) is less important than is ensuring that a measure of segregation has a sound conceptual basis. If a segregation index measures exactly that quantity that we believe defines spatial segregation, then the index will be composition invariant by definition. That said, we nonetheless evaluate the measures discussed in the paper against the traditional (James and Taeuber) composition invariance criterion, in order to preserve continuity with prior research.

6. *Transfers and exchanges*: A key criterion for a segregation measure is a definition of how segregation should change in response to the movement of individuals in social space. Transfers and exchanges, as we define them here, are specific types of such movement. We suggest here spatial extensions of the Reardon and Firebaugh (2002a) multigroup transfer and exchange criteria; in addition, we suggest an additional exchange criterion.
 - *Transfers*: If an individual of group m is transferred from point p to q , and if the proportion of group m in the local

environments of all points closer to p than q is greater than the proportion of group m in the local environments of all points closer to q than p , segregation is reduced. In the aspatial case, this reduces to the usual transfer criterion (James and Taeuber 1985; Reardon and Firebaugh 2002a).

- *Exchanges (Type 1)*: If an individual of group m from point p is exchanged with an individual of group n from point q , and if the proportion of group m in the local environments of all points closer to p than q is greater than the proportion of group m in the local environments of all points closer to q than p , and if the proportion of group n in the local environments of all points closer to q than p is greater than the proportion of group n in the local environments of all points closer to p than q , segregation is reduced. In simpler terms, if an exchange moves two individuals of different groups to locations where they are less likely to encounter members of their own group (and hence, more likely to encounter members of other groups), then segregation should be reduced. In the aspatial case, this reduces to the usual exchange criterion (James and Taeuber 1985; Reardon and Firebaugh 2002a).
- *Exchanges (Type 2)*: If an individual of group m from point p is exchanged with an individual of group n from point q , and if the proportion of group m is greater than the proportion of group n in the local environments of all points closer to p than q , and if the proportion of group n is greater than the proportion of group m in the local environments of all points closer to q than p , segregation is reduced. Although this formulation of the exchange criterion does not reduce to the familiar exchange criterion, it has a compelling logic: if two individuals of groups m and n change places in a way that makes the proportions of groups m and n more similar in the local environments of at least some places (and leaves them unchanged in all others), while leaving the proportions of all other groups unchanged everywhere, then segregation should be reduced.¹⁰

¹⁰Note that in the two-group case, the type 2 exchange criterion is a special case of the type 1 criterion; in the multigroup case, however, they are distinct criteria—the conditions of a type 2 exchange can be met without meeting those of a type 1 exchange, and vice versa.

7. *Additive spatial decomposability*: If X spatial subareas are aggregated into Y larger spatial areas, then a segregation measure should be decomposable into a sum of within- and between-area components.
8. *Additive grouping decomposability*: If M groups are clustered in N supergroups, then a segregation measure should be decomposable into a sum of independent within- and between-supergroup components.

3. A GENERAL APPROACH TO MEASURING SPATIAL SEGREGATION

We now turn to developing and evaluating new and proposed measures of spatial segregation. We begin by describing a new approach to measuring spatial segregation and use this approach to develop several measures of spatial exposure and spatial evenness. Conceptually, we measure spatial exposure and spatial evenness as follows. We begin by computing the spatially weighted group composition of the local environment of each location (or person) in the region of interest. Typically, we will weight this measure so that locations near another location contribute more to its local spatial environment than do more distant locations (a "distance-decay" effect).

To measure spatial exposure, we compute the average composition of the local environments of members of each group. To measure spatial evenness, we examine how much variation there is among the racial compositions of the local environments of everyone living in the region of interest. If each person's spatial environment is relatively similar in racial composition, there is little spatial unevenness; conversely, if there is considerable variation across persons in the racial composition of their spatial environments, there is high spatial segregation (unevenness).

Our approach in this paper provides a general framework for measuring spatial segregation among multiple population groups. This approach encompasses, as special cases, traditional aspatial measures, both two-group and multigroup. Our approach here assumes complete data about the residential locations of individuals (though these data may be estimated from tract or other aggregated data, of course). Our approach does not, however, assume any specific functional form defining spatial proximities between locations. In fact, we deliberately do not specify a functional form for the spatial proximity function, as we wish to call attention to the fact that many meaningful definitions are possible. The flexibility of our approach allows researchers to

specify a definition of local social environments derived from theoretical considerations of patterns of social interaction.

3.1. *A General Spatial Exposure Segregation Index*

Equation (2) above defines a surface $\tilde{\pi}_{pm}$, which gives, at each point p in R , the proportion of the population in the local neighborhood who are members of group m . This can be interpreted as the exposure to group m for a person residing at location p . These $\tilde{\pi}_{pm}$ surfaces are the basis of the class of spatial segregation measures we develop here.

We define the spatial exposure of group m to group n as the average percentage of group n in the local environments of each member of group m .

$${}_m\tilde{P}_n^* = \int_{q \in R} \frac{\tau_{qm}}{T_m} \tilde{\pi}_{qn} dq. \quad (4)$$

We likewise define the spatial isolation of group m as simply the spatial exposure of group m to itself:

$${}_m\tilde{P}_m^* = \int_{q \in R} \frac{\tau_{qm}}{T_m} \tilde{\pi}_{qm} dq. \quad (5)$$

In the aspatial case, equations (4) and (5) are equivalent to the usual exposure and isolation indices (Bell 1954; Lieberson and Carter 1982a, 1982b). Although formulated slightly differently, Morgan's (1983b) distance-decay interaction index, ${}_mPC_n$, can be seen as a special case of equation (4), where the spatial proximity function used to compute $\tilde{\pi}_{qm}$ is defined based on estimated contact rates between each tract and its surrounding areas.¹¹

¹¹Schnell and Yoav (2001) develop sociospatial isolation measures using a related approach. Their approach differs from ours, however, in that they construct $\tilde{\pi}_{pm}$ as a sociospatially weighted average of the π_{qm} 's (see footnote 3) without weighting for population density. In addition, they average population compositions in the logistic scale, a technique that makes their measure difficult to interpret. Finally, they construct sociospatial isolation measures for individuals, rather than populations, though it would be a simple matter to average their individual isolation measures over all individuals to construct population-average exposure measures as we do in equations (4) and (5).

3.2. *A General Approach to Measuring Spatial Evenness*

Now recall that we define the evenness dimension of spatial segregation as the extent to which individuals of different groups occupy or experience different social environments. Given the population density distribution and the $\tilde{\pi}_{pm}$ exposure surfaces, we know the population density at each location and the group proportions in the local environment of each location; these are all we will need to construct a set of spatial segregation measures.

Knowing the population density (τ_p) at each location and the group proportions (the $\tilde{\pi}_{pm}$'s) in the local environment of each location, we can construct a variety of potentially useful multigroup spatial segregation measures. By substituting the $\tilde{\pi}_{pm}$'s for the π_{pm} 's in Reardon and Firebaugh (2002a, table 2), we can derive spatial generalizations of all their aspatial multigroup segregation measures (D , G , H , C , P , R). Because the aspatial measures are special cases of the spatial measures, and because the aspatial criteria described by Reardon and Firebaugh (2002a) are special cases of the spatial criteria described above, spatial measures derived this way cannot, in general, meet any of the spatial criteria that are not met by their aspatial analogs. We focus here, therefore, on deriving and describing a spatial version of the entropy-based information theory segregation index (H), since the aspatial multigroup H has been shown to be preferable to other aspatial measures on the basis of these criteria (Reardon and Firebaugh 2002a).

In addition, we describe and evaluate two additional measures—spatial versions of the dissimilarity index (D) and the relative diversity index (R). We evaluate the spatial dissimilarity index because the aspatial dissimilarity index has been used so commonly in segregation research. We evaluate the spatial relative diversity index because the aspatial R meets most criteria for an aspatial index (Reardon and Firebaugh 2002a), suggesting that it may make a useful spatial index as well.

3.3. *The Spatial Information Theory Segregation Index*

Following Theil (1972), we compute the spatially weighted entropy—a measure of population diversity (see Pielou 1977; White 1986)—at each point p as

$$\tilde{E}_p = - \sum_{m=1}^M (\tilde{\pi}_{pm}) \log_M (\tilde{\pi}_{pm}). \quad (6)$$

This is the entropy of the local environment of p . It is analogous to the entropy of an individual tract, E_r , that is used in the computation of the aspatial segregation index H (and in fact, if we define the local environment of p to be tract r , then $\tilde{E}_p = E_r$), except that \tilde{E}_p incorporates spatially weighted information on the racial composition at all points in R , not only the racial composition of the tract where p is located.

Now we define the spatial information theory index, denoted \tilde{H} :

$$\tilde{H} = 1 - \frac{1}{TE} \int_{p \in R} \tau_p \tilde{E}_p dp, \quad (7)$$

where E is the overall regional entropy of the total population given by

$$E = - \sum_{m=1}^M (\pi_m) \log_M (\pi_m). \quad (8)$$

The spatial information theory index, \tilde{H} , is a measure of how much less diverse individuals' local environments are, on average, than is the total population of region R . It will be equal to 1—indicating maximum segregation—only when each individual's local environment is monoracial. If each individual's local environment has the same racial composition as the total population, then $\tilde{E}_p = E$ for all p , and \tilde{H} will be zero—indicating complete integration.

3.4. *Additional Spatial Segregation Indices*

We define a spatial relative diversity index \tilde{R} as

$$\tilde{R} = 1 - \int_{p \in R} \frac{\tau_p \tilde{I}_p}{TI} dp, \quad (9)$$

where I is the interaction index, a measure of population diversity (Liebersohn 1969; White 1986):

$$I = \sum_{m=1}^M (\pi_m)(1 - \pi_m), \quad (10)$$

and where \tilde{I}_p is the spatially-weighted interaction index at point p :

$$\tilde{I}_p = \sum_{m=1}^M (\tilde{\pi}_{pm})(1 - \tilde{\pi}_{pm}). \quad (11)$$

Like \tilde{H} , \tilde{R} is a measure of how much less diverse individuals' local environments are, on average, than is the total population of region R .¹²

Finally, we define a spatial dissimilarity index as

$$\tilde{D} = \sum_{m=1}^M \int_{p \in R} \frac{\tau_p}{2TI} |\tilde{\pi}_{pm} - \pi_m| dp. \quad (12)$$

Unlike its aspatial analog, the spatial dissimilarity index cannot be interpreted as the proportion of the population who would have to

¹²Unlike in the aspatial case, \tilde{R} is not easily related to the \tilde{P}^* spatial exposure indices. In the aspatial case, in a two-group population, we have (Reardon and Firebaugh 2002a):

$$1 - R = \frac{mP_n^*}{\pi_n} = \frac{nP_m^*}{\pi_m}.$$

In general, the spatial version of these equalities does not hold, since the spatial versions of the quantities above are given by

$$\begin{aligned} 1 - \tilde{R} &= \int_{p \in R} \frac{\tau_p \tilde{\pi}_{pm} \tilde{\pi}_{pn}}{T \pi_m \pi_n} dp \\ \frac{m\tilde{P}_n^*}{\pi_n} &= \int_{p \in R} \frac{\tau_p \pi_{pm} \tilde{\pi}_{pn}}{T \pi_m \pi_n} dp \\ \frac{n\tilde{P}_m^*}{\pi_m} &= \int_{p \in R} \frac{\tau_p \tilde{\pi}_{pm} \pi_{pn}}{T \pi_m \pi_n} dp. \end{aligned}$$

These are equal only if $\tilde{\pi}_{pm} = \pi_{pm}$ holds for all p and m (see footnote 3).

move to achieve complete integration. However, it can be interpreted as a measure of how different the composition of individuals' local environments are, on average, from the composition of the population as a whole.

3.5. *Prior Proposed Measures of Spatial Segregation*

As we noted above, we are not the first to propose measures of spatial segregation. Table 1 summarizes proposed spatial segregation measures. Those that rely explicitly on tract boundaries and contiguity patterns are noted in column 3; these measures will each be necessarily susceptible to MAUP issues. The other indices are computed, in principle, from more general functions of spatial or social distance, although tract boundaries and contiguity are generally used to approximate spatial distance.

Among the proposed measures of spatial evenness, most are modifications of the aspatial dissimilarity index D (Jakubs 1981; Morgan 1982, 1983a; Morrill 1991; O'Sullivan and Wong 2004; Waldorf 1993; Wong 1993; Wong 1998); these generally incorporate some spatial contiguity weight into the computation of D , or characterize the distance between tracts in terms of "relocation efforts." As each of these measures is a generalization of D , they will necessarily fail to meet any of the criteria that the aspatial D fails to meet (Reardon and Firebaugh 2002a). In particular, they fail to meet the exchange criterion and the decomposition criteria. Moreover, most of these are based explicitly on tract boundaries, and so are susceptible to MAUP issues. Because of these shortcomings, we do not consider these measures further here.

Morgan (1983b:215) defines a symmetric spatial segregation index IC_2 that is a spatial analog to the variance ratio index or the standardized exposure index. However, IC_2 is well-defined only for spatial proximity functions where $\tilde{\pi}_{pm} = \pi_{pm}$ holds for all p and m , since otherwise the standardized versions of the exposure indices are not, in general, equal (see footnote 9). When IC_2 is well-defined, it can be seen as a special case of our relative diversity index \tilde{R} ; thus we do not consider IC_2 further here.

Among the other proposed measures of spatial evenness, the remainder (save our new measures) do not correspond to any known

TABLE 1
Proposed Spatial Segregation Measures

Measure	Aspatial Analog	Tract- based?	Original Citation	Brief Description
<i>Spatial Evenness Measures</i>				
$D(adj)$	D	Y	(Morrill 1991)	D adjusted for tract contiguity
$D(w)$	D	Y	(Wong 1993)	$D(adj)$ adjusted for contiguous tract boundary lengths
$D(s)$	D	Y	(Wong 1993)	$D(w)$ adjusted for tract perimeter/area ratio
$SD(m)$	D	Y	(Wong 1998)	Multigroup D computed using composite population counts
DBI	D	Y	(Jakubs 1981; Morgan 1982)	D adjusted for relocation efforts needed to achieve integration
$MDBI$	D	Y	(Morgan 1983a)	DBI with alternate definition of complete segregation
RDI	D	N	(Waldorf 1993)	D adjusted for relocation efforts
S	D	N	(O'Sullivan and Wong 2004)	D based on population density surfaces
IC_2	V	N	(Morgan 1983b)	Standardized distance-decay exposure
S	—	N	(Wong 1999)	Intersection of deviational ellipses
SP	—	N	(White 1983)	Ratio of mean within-group proximity to mean total proximity
SP	—	N	(Grannis 2002)	Multigroup version of White's SP
I	—	Y	(Frank 2003)	Moran's I -spatial autocorrelation among adjacent tracts
\tilde{H}	H	N	This paper	H based on local environment group proportions
\tilde{R}	R	N	This paper	R based on local environment group proportions
\tilde{D}	D	N	This paper	D based on local environment group proportions
<i>Spatial Exposure Measures</i>				
PC^*	P^*	N	(Morgan 1983b)	P^* based on distance-decay exposure, special case of \tilde{P}^*
\tilde{P}^*	P^*	N	This paper	P^* based on local environment group proportions

aspatial measure. White (1983) proposed a spatial proximity index to measure spatial segregation; Grannis (2002) proposed a multigroup version of this index, which we will denote as SP . The index is a measure of the average spatial proximity between two members of the same group divided by the average proximity between two members of the population. In principle, this measure does not depend on tract boundaries (White uses tract boundaries in estimating proximities; the measure would, however, be independent of tract boundaries if we had information on individuals' exact locations). Moreover, it has an intuitive appeal as a measure of spatial segregation. We consider SP a potentially useful measure of spatial segregation, and evaluate it alongside our new measures later in this paper. In our notation, the White/Grannis spatial proximity index is defined as

$$SP = \sum_{m=1}^M \frac{\pi_m P_{mm}}{P_{tt}}, \quad (13)$$

where P_{mm} , the average proximity between two members of group m , is defined as

$$\begin{aligned} P_{mm} &= \frac{1}{T_m^2} \int_p \int_q \tau_{pm} \tau_{qm} \phi(p, q) dq dp \\ &= \frac{1}{T_m^2} \int_p \tau_{pm} \tilde{\tau}_{pm} \Phi_p dp. \end{aligned} \quad (14)$$

P_{tt} is defined similarly. White suggests using a decreasing function of the distance between p and q for the proximity function ϕ here, though in principle, any desired proximity function could be used in equation (14) (Grannis 2002; ; White 1983).

Scholars have suggested several additional spatial evenness measures. Wong's deviational ellipse (1999) introduces the novel idea of comparing the overall spatial distribution of different population subgroups. However the measure is problematic because the deviational ellipse provides only a very generalized approximation of subgroup spatial distributions. In more recent work O'Sullivan and Wong (2004) use a density estimation method to approximate and compare the spatial distributions of two population subgroups.

However, because the resulting measure is a spatial generalization of D , this approach will fail to meet a number of the criteria under consideration. Finally, Frank (2003) suggests a segregation measure based on the spatial autocorrelation of tract compositions. Like all other measures that depend explicitly on tract boundary definitions, however, it is susceptible to MAUP issues. Thus, of the proposed spatial evenness measures, the White/Grannis spatial proximity index SP appears the most promising candidate for satisfying the spatial segregation criteria above.

There are far fewer candidates for a spatial exposure index. As we noted above, Morgan's PC^* is a special case of our more general proposed spatial exposure index, so we will not evaluate it separately here (Morgan 1983b). We are not aware of any other proposed spatial exposure indices.

4. EVALUATION OF THE SPATIAL SEGREGATION INDICES

We now turn to evaluating the indices against the criteria articulated above. We evaluate here four measures of spatial evenness (\tilde{H} , \tilde{D} , \tilde{R} , and SP), and one measure of spatial exposure (\tilde{P}^*).

Scale Interpretability. Each of the three evenness measures we derive— \tilde{H} , \tilde{D} , and \tilde{R} —meets the scale interpretability criterion. Each has a maximum of 1, obtained under complete segregation, is equal to zero if each local environment has a composition equal to that of the whole population, and is negative in the case of hyperintegration. The spatial proximity index has no theoretical maximum and is equal to 1 under perfect evenness, with values less than one indicating hyperintegration (White 1983). The lack of a theoretical maximum makes comparative studies using SP potentially difficult. The spatial exposure index is, by definition, bounded between 0 and 1.

Arbitrary Boundary Independence. Each of the five indices is computed based on population density information at each point; as a result, each of the indices is free of MAUP issues in principle, though the estimation of population density information from aggregated (tract) data may still lead to some MAUP issues, but these are due

to data collection methods rather than segregation computation methods.

Population Density and Location Equivalence. Both of these criteria are easily assessed using simple algebra. Like their aspatial counterparts, all five indices satisfy the population density invariance criteria. Each of the measures except *SP* meets the location equivalence criterion.

Composition Invariance. As we noted above, the composition invariance criterion as stated by James and Taeuber (1985) may be inappropriate, since it rests on a particular definition of what it means for the distribution of individuals in space to remain the same as the population composition changes. Nonetheless, it is useful to evaluate the indices against the James and Taeuber formulation of composition invariance for the sake of continuity with prior work. When we do so, we find that, since *D*, *H*, and *R* do not satisfy the criterion in the aspatial multigroup case, their spatial analogs likewise do not meet it (though \tilde{D} is composition invariant in the two-group case). Likewise, simple algebra shows that the spatial proximity index is not composition invariant either. The criterion does not apply to the spatial exposure index \tilde{P}^* .

Although none of \tilde{H} , \tilde{D} , \tilde{R} , and *SP* are composition invariant (by the James and Taeuber definition) in the general spatial multigroup case, we have conducted a preliminary set of exploratory simulation analyses in order to determine whether one or more of the indices approximates composition invariance. In general, the behavior of each of the indices with respect to a change in the population composition of the type suggested by James and Taeuber is complex, particularly when one or more groups makes up a small portion of the population and/or when the group whose size is changing is highly segregated from some groups and not highly segregated from others. Nonetheless, a series of exploratory simulations analogous to those conducted by James and Taeuber (1985:16–18) show that \tilde{H} and \tilde{D} are less sensitive to changes in the population composition, in general, than are \tilde{R} and *SP*.

It is worth noting that a failure to meet the James and Taeuber composition invariance criterion does not imply that a measure may not be composition invariant by some other definition. For example,

if the population composition changes but if the ratio of the average diversity (as measured by the entropy function E) of individuals' local environments to the total diversity of the population remains constant, then \tilde{H} will be composition invariant under a corresponding definition of invariance. Finally, failure to meet a composition invariance criterion indicates that a segregation measure is affected by the population composition; this does not, however, imply that the measure is a measure of spatial exposure, rather than of spatial evenness. A measure of exposure should increase when the proportion of the group that others are exposed to grows in the population. But, as James and Taeuber (1985) show, the aspatial H and V (as well as the spatial \tilde{H} and \tilde{R}) both may increase or decline in response to increases in one group's share of the population, indicating that their responsiveness to population composition is not due to a confounding of exposure and evenness measurement.

*Transfers and Exchanges.*¹³

- *Transfers:* None of the spatial segregation measures we describe here meet the transfer criterion in the general spatial case.
- *Exchanges:* In the most general case, none of the evenness measures meet the type 1 exchange criterion, and only \tilde{H} meets the type 2 exchange criterion. Several of the measures, however, meet the exchange criteria if the region R is symmetric under ϕ . We say that points p and q are symmetric if there is a one-one mapping between the set of points closer to p than q and those closer to q than p , and if corresponding points and their local environments have similar population density ratios. This condition is unlikely to be met completely, but may be approximated in real residential patterns. (See Section A.1 in the Appendix for a more precise definition and related discussion.)

Both \tilde{H} and \tilde{R} meet the type 1 exchange criterion under conditions of spatial symmetry. Moreover, while only \tilde{H} meets the type 2 exchange criterion in the most general case, \tilde{R} also meets the criterion when the region is symmetric under ϕ .

Under conditions of spatial symmetry, the spatial dissimilarity index \tilde{D} , like its aspatial counterpart, satisfies only a weak version of

¹³See Sections A.2 and A.3 in the Appendix for all proofs.

each of the exchange criteria. An exchange that moves a group m member away from locations with higher proportions of group m and nearer to points with lower proportions of group m will never result in an increase in \tilde{D} . In most cases, however—as long as there is set of symmetric points (see Section A.1 in the Appendix) s and s' in R such that $\tilde{\pi}_{sm} > \pi_m > \tilde{\pi}_{s'm}$ or $\tilde{\pi}_{sn} < \pi_n < \tilde{\pi}_{s'n}$ —a type 1 exchange will register an appropriate decrease in \tilde{D} . Likewise, as long as there is some point s closer to p than q such that $\tilde{\pi}_{sm} > \tilde{\pi}_{sn}$ or some point s' closer to q than p such that $\tilde{\pi}_{s'n} > \tilde{\pi}_{s'm}$, then a type 2 exchange will register an appropriate decrease in \tilde{D} .

The spatial proximity index SP fails to meet either of the exchange criteria, even under conditions of spatial symmetry.

Additive Spatial Decomposability. Suppose a spatial region R is subdivided into K subregions. In the aspatial case, any rearrangement of individuals within subregion k will not affect segregation within any other subregion; nor will it affect the between-subregion segregation level. In this case, Reardon and Firebaugh (2002a) define a segregation measure as organizationally decomposable if it can be written as a sum of $K+1$ independent components—a between-subregion component and K within-subregion components, with each of the K within-subregion components indicating the amount by which total segregation would be reduced if segregation within subregion k were eliminated by rearranging individuals within k while leaving the location of all other individuals outside k unchanged.

An additive spatial decomposition is not so neatly defined, since rearranging individuals within subregion k may change the spatial environments of individuals in other subregions. As a result, the between- and within-subregion components of segregation are not necessarily independent. Nonetheless, we can define meaningful spatial decompositions of both \tilde{R} and \tilde{H} that incorporate a spatial interaction term that accounts for the spatial interaction between locations in different subregions.

To describe the spatial decomposition of \tilde{H} , we first require a refinement to our earlier notation. For some region S , define $\tilde{E}_{p|S}$ as the spatial entropy at point p as defined in equation (6), except that the $\tilde{\pi}_{pm}$'s are computed from equations (1) and (2) using only the points in region S .

In this notation, \tilde{E}_p as defined in equation (6) would be written $\tilde{E}_{p|R}$, since all points in region R might contribute to the spatial environment of point p . Now we can write \tilde{H} as the sum of three components:

$$\begin{aligned} \tilde{H} = & \sum_{k \in R} \frac{t_k}{TE} (E - E_k) + \sum_{k \in R} \left[\int_{p \in k} \frac{\tau_p}{TE} (\tilde{E}_{p|k} - \tilde{E}_{p|R}) dp \right] + \\ & \sum_{k \in R} \frac{t_k E_k}{TE} \int_{p \in k} \frac{\tau_p}{t_k E_k} (E_k - \tilde{E}_{p|k}) dp \end{aligned} \quad (15)$$

The first term on the right-hand side of equation (15) is simply the aspatial segregation between the K subregions (see Reardon and Firebaugh 2002a). The integral in the third term on the right-hand side is the spatial segregation within subregion k , ignoring points outside of k . We can rewrite equation (15) as

$$\tilde{H} = H_K + \sum_{k \in R} \left[\int_{p \in k} \frac{\tau_p}{TE} (\tilde{E}_{p|k} - \tilde{E}_{p|R}) dp \right] + \sum_{k \in R} \frac{t_k E_k}{TE} \tilde{H}_k. \quad (16)$$

The middle term is an interaction term that reflects the contribution to spatial segregation that results from the spatial proximity of points within different subareas. In general, for a given subregion k , the integral will be positive if, on average, subregions outside of k decrease the diversity of the local environments of individuals within subregion k , and negative if they increase it.

It is useful to consider a few special cases of equation (16). First, suppose that each population group is evenly distributed within each subregion k —this would be the case if, for example, the subregions were census tracts and we assumed that the population of each tract were evenly distributed throughout the tract. In this case, $\tilde{H}_k = 0$ and $\tilde{E}_{p|k} = E_k$ for each k . Then \tilde{H} is simply the sum of the aspatial segregation among the tracts and a between-tract spatial segregation term:

$$\tilde{H} = H_K + \sum_{k \in R} \left[\int_{p \in k} \frac{\tau_p}{TE} (E_k - \tilde{E}_{p|R}) dp \right]. \quad (17)$$

Second, consider a special case where the spatial proximity of points in separate subregions is zero. This is the case, of course, for aspatial measures, but it also might be the case if, for example, a city were divided into distinct subareas through natural or manmade barriers—rivers, major highways, and the like—across which there were no spatial interaction. In this case, the interaction term would be zero, and \tilde{H} can be written as the sum of a between-subarea aspatial segregation component and K within-subarea spatial segregation measures:

$$\tilde{H} = H_K + \sum_{k \in R} \frac{t_k E_k}{TE} \tilde{H}_k. \quad (18)$$

Finally, suppose that the K subregions are large compared to the size of the local spatial environments of individuals. Then, for most individuals—except for those located near the boundaries between subregions—we will have $\tilde{E}_{p|k} \approx \tilde{E}_{p|R}$. As a result, the spatial interaction term will be relatively small, and the decomposition in equation (18) will hold approximately.

The spatial relative diversity index \tilde{R} can be decomposed in the same way as \tilde{H} , by substituting \tilde{I} for \tilde{E} in equations (15) through (18). \tilde{D} , however, like its aspatial counterpart, cannot be meaningfully decomposed into between- and within-subregion components. Nor are we able to construct a decomposition of SP .

The spatial exposure index, however, does have a useful spatial decomposition. Using similar notation as in equation (16), we can write

$${}_m \tilde{P}_n^* = \sum_{k \in R} \frac{t_{km}}{T_m} [{}_m \tilde{P}_n^*]_k + \sum_{k \in R} \left[\int_{p \in k} \frac{\tau_{pm}}{T_m} (\tilde{\pi}_{pn|R} - \tilde{\pi}_{pn|k}) dp \right]. \quad (19)$$

The first term in the right-hand side of the equation is a weighted average of the spatial exposure within each subarea. The second term is an interaction term that reflects the contribution to spatial exposure that results from the spatial proximity of points within different subareas.

Additive Grouping Decomposability. Following Reardon and Firebaugh (2002a), a spatial segregation index \tilde{S} meets the grouping decomposability criterion if we can write

$$\tilde{S} = \tilde{S}_N + \sum_{n=1}^N g(\tilde{S}_n), \quad (20)$$

where \tilde{S}_N is the segregation calculated among the N supergroups, \tilde{S}_n is the segregation among the groups making up supergroup n , and g is a strictly increasing function on the interval $[0,1]$ with $g(0) = 0$. As in the aspatial case, only \tilde{H} yields a meaningful grouping decomposition. Because the between-supergroup decomposition of H depends only on the decomposition of E , and because the $\tilde{\pi}_{pm}$'s sum to 1 for all p ; the decomposition of \tilde{H} into between- and within-supergroup components has the same form as the aspatial H :

$$\tilde{H} = \tilde{H}_N + \sum_{n=1}^N \frac{t_n E_n}{TE} \tilde{H}_n. \quad (21)$$

Table 2 summarizes the compliance of the spatial segregation measures with the criteria we describe. Of the four spatial evenness measures, the spatial proximity index SP is clearly the least satisfactory, as it fails to meet almost every criterion. The spatial information theory index \tilde{H} appears the most satisfactory, as it satisfies the exchange criteria in the widest range of cases and is also the only index that has both a meaningful spatial and grouping decomposition. Of the remaining two, \tilde{D} is arguably less satisfactory than \tilde{R} , as \tilde{R} meets the exchange criteria in a wider range of cases and can also be spatially decomposed.

5. DISCUSSION AND CONCLUSION

Despite the existence of a number of proposed measures of spatial segregation, such measures have not been widely used in residential segregation research. In fact, a reading of the existing spatial segregation literature provides little guidance about which of the many proposed measures are most useful. In this paper, we have—at the risk of further cluttering an already cluttered field—developed several new measures of spatial segregation, based on a new spatial proximity approach. Several key features of our approach are notable. First, our approach avoids, in principle, MAUP issues by using point-to-point

TABLE 2
Properties of Spatial Segregation Indices

	Information (\bar{H})	Relative Diversity (\bar{R})	Dissimilarity (\bar{D})	Spatial Proximity (SP)	Spatial Exposure (P^*)
Scale interpretability	✓	✓	✓	X	✓
MAUP-free	✓	✓	✓	✓	✓
Location equivalence					
Aspatial	✓	✓	✓	X	✓
Spatial	✓	✓	✓	X	✓
Population density invariance					
Aspatial	✓	✓	✓	✓	✓
Spatial	✓	✓	✓	✓	✓
Compositional invariance					
Aspatial 2-Group	X	X	✓	X	—
Aspatial M -Group	X	X	X	X	—
Spatial 2-Group	X	X	✓	X	—
Spatial M -Group	X	X	X	X	—
Transfers					
Aspatial 2-Group	✓	✓	X ^a	X	—
Aspatial M -Group	✓	X	X	X	—
Spatial 2-Group	X	X	X	X	—
Spatial M -Group	X	X	X	X	—

continued

TABLE 2
Continued

	Information Theory (\tilde{H})	Relative Diversity (\tilde{R})	Dissimilarity (\tilde{D})	Spatial Proximity (SP)	Spatial Exposure (\tilde{P}^*)
Exchanges (type 1)					
Aspatial 2-Group	✓	✓	X^a	X	—
Aspatial M -Group	✓	✓	X^a	X	—
Spatial 2-Group	✓ ^b	✓ ^b	$X^{a,b}$	X	—
Spatial M -Group	✓ ^b	✓ ^b	$X^{a,b}$	X	—
Exchanges (type 2)					
Aspatial 2-Group	✓	✓	X^a	X	—
Aspatial M -Group	✓	✓	X^a	X	—
Spatial 2-Group	✓	✓ ^b	$X^{a,b}$	X	—
Spatial M -Group	✓	✓ ^b	$X^{a,b}$	X	—
Additive spatial decomposability					
Aspatial 2-Group	✓	✓	X	X	✓
Aspatial M -Group	✓	✓	X	X	—
Spatial 2-Group	✓	✓	X	X	✓
Spatial M -Group	✓	✓	X	X	—
Additive grouping decomposability					
Aspatial	✓	X	X	X	—
Spatial	✓	X	X	X	—

^aThe dissimilarity index satisfies only a weak form of the principles of transfers and exchanges in these cases; transfers and exchanges that move individuals away from local environments with higher group proportions and nearer to those with lower group proportions will never result in an increase in \tilde{D} , though they may result in no change in \tilde{D} .

^bThe indices do not meet the criterion in general, though they do meet it if the region R is symmetric under ϕ .

proximity functions rather than tract contiguity matrices. Second, our approach is nonspecific regarding the choice of a spatial proximity function. This enables (requires, actually) researchers to specify their underlying assumptions about socio-spatial proximity, and it facilitates research that compares segregation levels based on different theoretical bases for defining spatial proximity. Further, our approach yields, as special cases, traditional aspatial segregation measures (both two-group and multigroup) and makes clear the assumptions about spatial proximity inherent in such measures. Finally, our approach yields measures of both spatial exposure/isolation and spatial evenness/clustering.

In addition to developing a new set of spatial segregation measures, we review and evaluate all previously proposed measures of spatial evenness and exposure as well as our new measures. Here we conclude that the spatial information theory index \tilde{H} is the best of the spatial evenness measures, when judged against the criteria we have outlined. Likewise, we conclude that the spatial exposure/isolation index \tilde{P}^* —which is a spatial generalization of the familiar P^* exposure and isolation index—is a satisfactory measure of spatial exposure. We suggest that researchers rely on these measures in future research in order to ensure comparability across studies.

We do not, however, specify or recommend a particular proximity function for use in computing the measures. In fact, it seems likely that research that compares segregation levels of \tilde{H} based on different proximity functions could be useful in understanding the processes that organize residential space. For example, parallel studies of a number of cities might reveal that using a simple fast (short) distance decay formulation for the proximity function results in a different rank-ordering of cities by segregation levels than does using a proximity function with a slower distance decay characteristic. Interpretation of such results would indicate something about the geographical scale at which segregation occurs in the cities in question.

While \tilde{H} and \tilde{P}^* are, in principle, very satisfactory spatial segregation indices, several important issues remain in operationalizing and computing these measures. The first is that although our approach relies on complete data about individual residential locations, such data are rarely available, though they can be estimated from readily available tract data using a variety of methods. The simplest method would be to assume an even population density

within each tract, though this will result in sharp discontinuities in density at tract edges. Alternatively, spatial smoothing of population can be performed using various methods: kernel density estimation (Silverman 1986); group-specific pycnophylactic smoothing, which redistributes each population group within tracts such that tract totals are honored but population groups are moved toward neighboring tracts with large populations of the same group (Tobler 1979); or dasymetric mapping, which uses street networks or zoning patterns to estimate population densities (Mennis 2003). Although all of these methods are computationally intensive, they can be readily automated within typical GIS software packages. We are currently developing a set of tools that will allow researchers to estimate smooth population density surfaces using these methods and to use these smooth density surfaces in the computation of spatial segregation measures.

A second practical issue in our approach is that it requires the numerical evaluation of integrals over the study region R . In practice, this means dividing R into small cells for computational purposes, but it is unclear how sensitive the resulting measures of segregation will be to the choice of cell size. A third issue arises at edges of a study region R . Omitting data from outside the study region (say, a city, or metropolitan area), may be convenient (or necessary, if data are not available), but this may affect the estimation of the population density and racial composition for points near the edge of the study region, which will in turn affect the measured segregation.

The scale of the chosen proximity function relative to the scale of cells, tracts, and the study region is likely to be the critical factor determining how sensitive computed segregation measures are to variation, respectively, in the density estimation method, the choice of cell size, and the approach to treating edge conditions. Future work should determine how sensitive \tilde{H} and \tilde{P}^* are to choices regarding these issues.

An additional issue pertains to the potential need to use complex spatial proximity functions. It may be relatively simple to use a "bounded Gaussian" distance-decay spatial proximity function—a proximity function that is strictly a decreasing function of the Euclidean distance between two points and that goes to zero at some defined distance; such a function is computationally efficient, because it is defined identically at each point and because the cut-off distance removes the necessity to perform numerical integration over the entire study region R . Such a function, regardless of its precise mathematical

form (Gaussian, negative exponential, negative power law, etc.) has a certain intuitive appeal but nonetheless has only weak theoretical support and no supporting empirical evidence. It may be thought of as accounting for the varied behavior of individuals by aggregating many individual life-spaces into an overall average.

A more realistic spatial proximity function might take into account obstacles and discontinuities in the spatial fabric, such as highways or rivers. These will disrupt the neat mathematics of a bounded-Gaussian proximity function, however, and call for special treatment, with the proximity of locations on opposite sides of boundaries being set much lower than their simple Euclidean separation distance would dictate. Conversely “promoters” or channels for sociospatial interaction, such as street networks and public transportation services, would be treated in the opposite sense, increasing the proximity of locations connected by them (Grannis 2002).

While our approach to measuring spatial segregation enables us to account for complicated patterns of spatial proximity, such patterns do complicate the implementation of the measures, since special programming in GIS software is necessary to incorporate them. Moreover, any proximity function that is not the same at all locations requires the representation of the function by a (very large) interaction matrix that records the proximity between every pair of locations in the study region. Calculation and manipulation of such a matrix will impose significant computational burdens on any implementation of the proposed measure. Given the complexity of programming complex, user-defined spatial proximity functions for a number of locations, we expect that most users of these measures will initially prefer to use some simple distance-decay function until software tools are available to automate the use of more complex spatial proximity functions.

Finally, the measures we have developed here apply most obviously to the case of spatial residential racial segregation. In principle, however, we can extend this approach to measure segregation according to any population characteristic. For example, we could generate measures of spatial income segregation simply by computing some income variation statistic (such as the variance) within each local environment and then computing a measure of the variation in this statistic across all points in the region. In addition, we can extend this approach to measure other types of segregation, simply by defining an

appropriate proximity function. For example, we can measure the segregation of social networks by defining some social proximity function that indicates how near to one another any two individuals are within a social network (see Reardon and Firebaugh 2002b). Because of the generality of the measures with regard to the proximity function, our approach here may yield useful measures of social segregation in any domain, so long as an appropriate social proximity function is specified.

APPENDIX A: COMPLIANCE OF THE MEASURES WITH THE TRANSFER AND EXCHANGE CRITERIA

A.1. *The Spatial Symmetry Condition*

To evaluate the conditions under which a spatial segregation index meets the exchange criterion, we first provide a definition of spatial symmetry. Given a spatial proximity function $\phi(p, q)$ that is defined for all points $p, q \in R$, we say that R is symmetric under ϕ if for each pair of distinct points $p, q \in R$, we can divide R into three subregions, R_p , R_q , and R_0 , where $\phi(s, p) > \phi(s, q)$ for all $s \in R_p$; $\phi(s, q) > \phi(s, p)$ for all $s \in R_q$; and $\phi(s, p) = \phi(s, q)$ for all $s \in R_0$, and such that for each point $s \in R_p$ there exists a unique corresponding point $s' \in R_q$ such that $\phi(s, p) - \phi(s, q) = \phi(s', q) - \phi(s', p)$ and $\frac{\tau_s}{\tilde{\tau}_s} = \frac{\tau_{s'}}{\tilde{\tau}_{s'}}$. If we denote

$$\Delta_s(p, q) = \frac{\tau_s}{\tilde{\tau}_s} [\phi(s, p) - \phi(s, q)], \quad (\text{A} - 1)$$

then we have

$$\Delta_s(p, q) = -\Delta_{s'}(p, q) \quad (\text{A} - 2)$$

for all symmetric points s and s' in R .

Several examples of spatial symmetry are notable. First, in the usual aspatial case, the region R is divided into distinct subregions (tracts), and $\phi(p, q)$ is defined such that $\phi(p, q) = c$ if p and q are in the same subregion and $\phi(p, q) = 0$ otherwise. It is simple to show that R is symmetric under ϕ in this case. To see this, consider the case where p and q are in different tracts. Then R_p consists of the tract containing point p , R_q consists of the tract containing point q , and R_0 is the remainder of R . Now if we assume the populations in both R_p

and R_q are both located at a single point (this will not change segregation as measured by any index satisfying the locational equivalence criterion—see footnote 4), then the conditions for symmetry are met.

A second example of spatial symmetry results if region R extends infinitely in all directions, with constant population density (in which case $\tau_s = \tilde{\tau}_s$ for all s), and if $\phi(p, q)$ depends only on the Euclidean distance between points p and q . Finally, if R is large compared to the scale of local environments defined by ϕ , and if the population density changes relatively little over distances comparable to the scale of local environments, then $\tau_s \approx \tilde{\tau}_s$ for all s in R and R is approximately spatially symmetric under ϕ .

A.2. Evaluation of the Exchange Criteria

We can evaluate each index's compliance with the principles of transfers and exchanges by taking the derivative of the index with respect to a transfer or exchange x . Moreover, because an exchange consists of a pair of complementary transfers, failure to satisfy the type 1 exchange criterion implies that a measure does not satisfy the transfer criterion. Likewise, a measure that meets the transfer criterion will necessarily meet the type 1 exchange criterion.

We first evaluate the behavior of the indices with respect to an exchange. When x involves the exchange of a member of group m at point p with a member of group n at point q , then the derivative of \tilde{H} with respect to x is

$$\frac{d\tilde{H}}{dx} = \frac{1}{TE} \int_{s \in R} \Delta_s(p, q) \ln \frac{\tilde{\pi}_{sn}}{\tilde{\pi}_{sm}} ds. \quad (\text{A} - 3)$$

Now we divide the region R into three subregions, R_p , R_q , R_0 , such that $\phi(s, p) > \phi(s, q)$ for all $s \in R_p$; $\phi(s', q) > \phi(s', p)$ for all $s' \in R_q$; and $\phi(r, p) = \phi(r, q)$ for all $r \in R_0$. Now

$$\frac{d\tilde{H}}{dx} = \frac{1}{TE} \left[\int_{s \in R_p} \Delta_s(p, q) \ln \frac{\tilde{\pi}_{sn}}{\tilde{\pi}_{sm}} ds + \int_{s' \in R_q} \Delta_{s'}(p, q) \ln \frac{\tilde{\pi}_{s'n}}{\tilde{\pi}_{s'm}} ds' \right]. \quad (\text{A} - 4)$$

Now suppose that $\tilde{\pi}_{sm} > \tilde{\pi}_{sn}$ and $\tilde{\pi}_{s'm} < \tilde{\pi}_{s'n}$ for all $s \in R_p$ and all $s' \in R_q$. In this case, equation (A-4) yields $\frac{d\tilde{H}}{dx} < 0$, so \tilde{H} satisfies the type 2 exchange criterion.

In general, \tilde{H} does not satisfy the type 1 exchange criterion, since equation (A-4) can be negative under conditions of a type 1 exchange. If R is symmetric under ϕ , however, then we can exploit the one-to-one mapping of points in R_p and R_q to write equation (A-4) as

$$\frac{d\tilde{H}}{dx} = \frac{1}{TE} \int_{s \in R_p} \Delta_s(p, q) \left(\ln \frac{\tilde{\pi}_{sn} \tilde{\pi}_{s'm}}{\tilde{\pi}_{s'n} \tilde{\pi}_{sm}} \right) ds, \quad (\text{A} - 5)$$

where s' is the point in R_q corresponding to the point s in R_p . For every point $s \in R_p$, $\phi(s, p) - \phi(s, q) > 0$. When $\tilde{\pi}_{sm} > \tilde{\pi}_{s'm}$ and $\tilde{\pi}_{sn} < \tilde{\pi}_{s'n}$, then equation (A-5) yields $\frac{d\tilde{H}}{dx} < 0$, so \tilde{H} satisfies the type 1 exchange criterion if R is symmetric under ϕ .

The derivative of \tilde{R} with respect to an exchange x is

$$\frac{d\tilde{R}}{dx} = \frac{2}{TI} \int_{s \in R} \Delta_s(p, q) [(\pi_m - \tilde{\pi}_{sm}) - (\pi_n - \tilde{\pi}_{sn})] ds \quad (\text{A} - 6)$$

Note that, unlike \tilde{H} , the condition that $\tilde{\pi}_{sm} > \tilde{\pi}_{sn}$ and $\tilde{\pi}_{s'm} < \tilde{\pi}_{s'n}$ for all $s \in R_p$ and all $s' \in R_q$ is not sufficient to ensure that $\tilde{R} < 0$, so \tilde{R} does not, in general, satisfy the type 2 exchange criterion. However, under the condition of spatial symmetry, we can write equation (A-6) as

$$\frac{d\tilde{R}}{dx} = \frac{2}{TI} \int_{s \in R_p} \Delta_s(p, q) [(\tilde{\pi}_{s'm} - \tilde{\pi}_{sm}) + (\tilde{\pi}_{sn} - \tilde{\pi}_{s'n})] ds. \quad (\text{A} - 7)$$

When either $\tilde{\pi}_{sm} > \tilde{\pi}_{s'm}$ and $\tilde{\pi}_{sn} < \tilde{\pi}_{s'n}$ or $\tilde{\pi}_{sm} < \tilde{\pi}_{s'm}$ and $\tilde{\pi}_{sn} > \tilde{\pi}_{s'n}$, then equation (A-7) yields $\frac{d\tilde{R}}{dx} < 0$, so \tilde{R} satisfies both exchange criteria if R is symmetric under ϕ .

The derivative of \tilde{D} with respect to an exchange x is

$$\frac{d\tilde{D}}{dx} = \frac{1}{2TI} \int_{s \in R} \Delta_s(p, q) (z_{sn} - z_{sm}) ds, \quad (\text{A} - 8)$$

where

$$z_{sk} = \begin{cases} 1 & \text{if } \tilde{\pi}_{sk} > \pi_k \\ -1 & \text{if } \tilde{\pi}_{sk} < \pi_k \\ 0 & \text{if } \tilde{\pi}_{sk} = \pi_k. \end{cases}$$

In the aspatial case, D satisfies only a weak form of the type 1 exchange criterion; the specified exchange may not reduce segregation, but will never increase it (Reardon and Firebaugh 2002a). In the spatial case, however, \tilde{D} does not satisfy even this weak form of the type 1 exchange criterion, as the expression in equation (A-8) may be positive in some cases. Under the spatial symmetry condition, however, equation (A-8) can be written

$$\frac{d\tilde{D}}{dx} = \frac{1}{2TI} \int_{s \in R_p} \Delta_s(p, q) [(z_{sn} - z_{sm}) - (z_{s'n} - z_{s'm})] ds. \quad (\text{A-9})$$

When either (1) $\tilde{\pi}_{sm} > \tilde{\pi}_{s'm}$ and $\tilde{\pi}_{sn} < \tilde{\pi}_{s'n}$, or (2) $\tilde{\pi}_{sm} > \tilde{\pi}_{sn}$ and $\tilde{\pi}_{s'm} < \tilde{\pi}_{s'n}$ is true, then equation (A-9) yields $\frac{d\tilde{D}}{dx} \leq 0$, so \tilde{D} satisfies a weak form of both exchange criteria if R is symmetric under ϕ .

If we assume that $\Phi_p = \Phi_q = \Phi$ for all $p, q \in R$, then the derivative of SP with respect to an exchange x is

$$\frac{dSP}{dx} = \frac{2\Phi}{T_m T_n P_{it}} [\pi_n (\tilde{\tau}_q \tilde{\pi}_{qm} - \tilde{\tau}_p \tilde{\pi}_{pm}) + \pi_m (\tilde{\tau}_p \tilde{\pi}_{pn} - \tilde{\tau}_q \tilde{\pi}_{qn})] \quad (\text{A-10})$$

In general, this quantity can be positive under the conditions of either exchange criterion. This is true for both the spatial and aspatial cases and for the two-group and multigroup versions of the index, so SP does not satisfy either of the exchange criteria in any case.

A.3. Evaluation of the Transfer Criterion

We next examine the behavior of the indices with respect to a transfer x of a person of group m from point p to q . Because \tilde{H} and \tilde{R} meet the first exchange criterion only when R is symmetric under ϕ , we need only evaluate \tilde{H} and \tilde{R} with respect to the transfer criterion in the case when R is symmetric under ϕ . In this case, we have

$$\frac{d\tilde{H}}{dx} = \frac{1}{TE} \int_{s \in R_p} \Delta_s(p, q) \left[(\tilde{E}_{s'} - \tilde{E}_s) + \ln \frac{\tilde{\pi}_{s'm}}{\tilde{\pi}_{sm}} \right] ds + \frac{1}{TE} (\tilde{E}_p - \tilde{E}_q) \quad (\text{A} - 11)$$

$$\frac{d\tilde{R}}{dx} = \frac{2}{TI} \int_{s \in R_p} \Delta_s(p, q) [(\tilde{I}_{s'} - \tilde{I}_s) + (\tilde{\pi}_{s'm} - \tilde{\pi}_{sm})] ds + \frac{1}{TI} (\tilde{I}_p - \tilde{I}_q) \quad (\text{A} - 12)$$

Both of these quantities may be positive when $\tilde{\pi}_{sm} > \tilde{\pi}_{s'm}$ for all $s \in R_p$ and all $s' \in R_q$, so neither \tilde{H} nor \tilde{R} meets the transfer criterion.

Because the aspatial D satisfies the transfer criterion only in the two-group case (Reardon and Firebaugh 2002a) and the spatial \tilde{D} satisfies only a weak form of the type 1 exchange criterion, and then only under conditions of spatial symmetry, we need only evaluate \tilde{D} with regard to the transfer criterion in the two-group case under conditions of spatial symmetry. The derivative of \tilde{D} with respect to a transfer x in this case is

$$\begin{aligned} \frac{d\tilde{D}}{dx} = \frac{-1}{TI} & \left[\int_{s \in R_p} \Delta_s(p, q) [(1 - \tilde{\pi}_{sm})z_{sm} - (1 - \tilde{\pi}_{s'm})z_{s'm}] ds \right. \\ & \left. + (\tilde{\pi}_{pm} - \pi_m)z_{pm} - (\tilde{\pi}_{qm} - \pi_m)z_{qm} \right], \end{aligned} \quad (\text{A} - 13)$$

where z_{pm} and z_{qm} are as in equation (A-8). This expression can be either positive or negative under the transfer criterion conditions, so \tilde{D} does not meet the transfer criterion.

Because the spatial proximity index SP does not meet the exchange criterion in any case, we know it will not meet the transfer criterion.

REFERENCES

- Apparicio, Phillipe. 2000. "Les indices de ségrégation résidentielle: un outil intégré dans un système d'information géographique." *Cybergéo, revue européenne de géographie* 134.

- Bell, W. 1954. "A Probability Model for the Measurement of Ecological Segregation." *Social Forces* 43:357–364.
- Coleman, James, Thomas Hoffer, and Sally Kilgore. 1982. "Achievement and segregation in secondary schools: A further look at public and private school differences." *Sociology of Education* 55:162–82.
- Dent, Borden D. 1999. *Cartography: Thematic Map Design*. Boston, MA: McGraw-Hill.
- Frank, Andrea I. 2003. "Using Measures of Spatial Autocorrelation to Describe Socio-economic and Racial Residential Patterns in US Urban Areas." Pp. 147–62 in *Socio-economic Applications of Geographic Information Science, Innovations in GIS*, edited by D. Kidner, G. Higgs, and S. White. London: Taylor and Francis.
- Gorard, Stephen, and Chris Taylor. 2002. "A Comparison of Measures in Terms of 'Strong' and 'Weak' Compositional Invariance." *Sociology* 36:875–95.
- Grannis, Rick. 2002. "Discussion: Segregation Indices and Their Functional Inputs." Pp. 69–84 in *Sociological Methodology*, Vol. 32, edited by Ross M. Stolzenberg. Boston, MA: Blackwell Publishing.
- Jakubs, John F. 1981. "A Distance-Based Segregation Index." *Journal of Socio-economic Planning Sciences* 15:129–36.
- James, David R. and Karl E. Taeuber. 1985. "Measures of Segregation." Pp. 1–32 in *Sociological Methodology*, Vol. 14, edited by Nancy Brandon Tuma. San Francisco, CA: Jossey-Bass.
- Liebertson, Stanley. 1969. "Measuring Population Diversity." *American Sociological Review* 34:850–62.
- Liebertson, Stanley, and Donna K. Carter. 1982a. "A Model for Inferring the Voluntary and Involuntary Causes of Residential Segregation." *Demography* 19:511–26.
- . 1982b. "Temporal Changes and Urban Differences in Residential Segregation: A Reconsideration." *American Journal of Sociology* 88:296–310.
- Longley, P., M. F. Goodchild, D. J. Maguire, and D. W. Rhind. 2001. *Geographic Information Systems and Science*. New York: Wiley.
- Massey, Douglas S., and Nancy A. Denton. 1988. "The Dimensions of Residential Segregation." *Social Forces* 67:281–315.
- Mennis, J. 2003. "Generating Surface Models of Population Using Dasymetric Mapping." *Professional Geographer* 55:31–42.
- Morgan, Barrie S. 1982. "An Assessment of Some Technical Problems in the Comparative Study of Residential Segregation." *Transactions-Institute of British Geographers* 7:227–32.
- . 1983a. "An Alternate Approach to the Development of a Distance-Based Measure of Racial Segregation." *American Journal of Sociology* 88:1237–49.
- . 1983b. "A Distance-Decay Interaction Index to Measure Residential Segregation." *Area* 15:211–16.
- Morrill, R. L. 1991. "On the Measure of Spatial Segregation." *Geography Research Forum* 11:25–36.
- Openshaw, S., and P. Taylor. 1979. "A Million or So Correlation Coefficients: Three Experiments on the Modifiable Area Unit Problem." Pp. 127–44 in *Statistical Applications in the Spatial Sciences*, edited by N. Wrigley. London: Pion.

- O'Sullivan, David, and David W. S. Wong. 2004. "A Density Surface-Based Approach to Measuring Spatial Segregation." Presented at the Annual Meeting of the Association of American Geographers, March 14–19, Philadelphia, PA.
- Pielou, E. C. 1977. *Mathematical Ecology*. New York: Wiley.
- Reardon, Sean F., and Glenn Firebaugh. 2002a. "Measures of Multigroup Segregation." Pp. 33–67 in *Sociological Methodology*, Vol. 32, edited by Ross M. Stolzenberg. Boston, MA: Blackwell Publishing.
- . 2002b. "Response: Segregation and Social Distance—A Generalized Approach to Segregation Measurement." Pp. 85–101 in *Sociological Methodology*, Vol. 32, edited by Ross M. Stolzenberg. Boston, MA: Blackwell Publishing.
- Schnell, Izhak, and Benjamini Yoav. 2001. "The Sociospatial Isolation of Agents in Everyday Life Spaces as an Aspect of Segregation." *Annals of the Association of American Geographers* 91:622–36.
- Schwartz, Joseph, and Christopher Winship. 1980. "The Welfare Approach to Measuring Inequality." Pp. 1–36 in *Sociological Methodology*, Vol. 9, edited by Karl F. Schuessler. San Francisco, CA: Jossey-Bass.
- Silverman, B. W. 1986. *Density Estimation for Statistics and Data Analysis*. London: Chapman and Hall.
- Theil, Henri. 1972. *Statistical Decomposition Analysis*, Vol. 14, edited by H. Theil. Amsterdam: North-Holland Publishing Company.
- Tobler, Waldo R. 1979. "Smooth Pycnophylactic Interpolation for Geographical Regions." *Journal of the American Statistical Association* 74:519–29.
- Waldorf, Brigitte S. 1993. "Segregation in Urban Space: A New Measurement Approach." *Urban Studies* 30:1151–64.
- White, Michael J. 1983. "The Measurement of Spatial Segregation." *American Journal of Sociology* 88:1008–18.
- . 1986. "Segregation and Diversity Measures in Population Distribution." *Population Index* 52:198–221.
- Wong, David S. 1993. "Spatial Indices of Segregation." *Urban Studies* 30:559–72.
- Wong, David W. S. 1997. "Spatial Dependency of Segregation Indices." *Canadian Geographer* 41:128–36.
- . 1998. "Measuring Multiethnic Spatial Segregation." *Urban Geography* 19:77–87.
- . 1999. "Geostatistics as Measures of Spatial Segregation." *Urban Geography* 20:635–47.
- . 2002. "Spatial Measures of Segregation and GIS." *Urban Geography* 23:85–92.
- . 2003. "Implementing Measures of Spatial Segregation in GIS." *Computers, Environment and Urban Systems* 27:53–70.
- Wong, David W. S., and Wing K. Chong. 1998. "Using Spatial Segregation Measures in GIS and Statistical Modeling Packages." *Urban Geography* 19:477–85.
- Zoloth, Barbara S. 1976. "Alternative Measures of School Segregation." *Land Economics* 52:278–98.