Spatial Indices of Segregation

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Summary. Segregation is commonly measured by means of an index of dissimilarity. A recent 'boundary modified' version of the index was formulated. It was based upon the concept that segregation is a separation created by spatial structure imposed upon the social space and thus interaction between racial groups is limited. The index takes into account one of the spatial elements—contiguity—but ignores the others. This paper argues that the length of the common boundary between two areal units and the shape of the areal units are important spatial components in determining segregation. Thus, a family of segregation indices is derived by incorporating these spatial components and can be applied to various spatial configurations. One of the indices possesses a distinctive property which is useful for comparing segregation levels in regions of various scales.

Introduction

Segregation is the spatial separation of population groups and is usually manifested by the spatial distribution of minority population. It also implies a limitation on the interaction between population groups. Thus, the degree of segregation can be indicated by the chance of interaction available between groups. Instead of investigating processes that generate segregation, this paper proposes modifications to an existing measure of segregation.

The most popular measure of segregation among geographers and sociologists is the index of dissimilarity, D, or the segregation index. Various versions of this index were derived (for example, Duncan and Duncan, 1955; Jakubs, 1979) partly due to the diverse conceptualisations of segregation and the different emphases in segregation study.

The role of geographers in augmenting the utility of the segregation index is unique. In contrast to many sociologists,

geographers attempt to conceive and describe the segregation issue from a spatial perspective. Thus, modifications of the segregation index introduced by geographers are also focused on capturing the spatial components involved in racial segregation. A recent attempt by Morrill (1991) deserved much attention because his modification of the index D is based upon the concept of spatial separation, which implies that the degree of segregation is a function of the intensity of interaction between population groups. However, Morrill's modified index of dissimilarity suffers from several limitations and ignores several very important spatial components which affect the opportunity for spatial interaction. This paper discusses these components and proposes further extensions of the index to incorporate these spatial components.

In the following section, some of the important contributions by geographers,

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including the recent modification proposed by Morrill, in improving the index D are highlighted. In section 2, it is demonstrated how the index can be modified if the common boundary of any two areal units is weighted by its length. In the third section, after the discussion of segregation from a spatial perspective and the relationship between segregation and the shape geometry of an area, a segregation index incorporating the weighted boundary factor and a measure of compactness for the areal units being studied is proposed. The last section presents results for several hypothetical spatial configurations, and these results can demonstrate the superiority of this modified index of D over the others.

1. Geography and Spatial Indices of Segregation

As was pointed out by Morrill (1991), an exhaustive review of research in segregation index is not necessary at this moment because Massey and Denton (1988) have provided a very in-depth and extensive review. However, there are several points in their review worth mentioning. Massey and Denton (1988) reiterated the meaning of segregation and concluded that segregation has five dimensions: evenness, exposure, concentration, centralisation and clustering. After they reviewed and empirically tested various measures of these dimensions of segregation, they concluded that D, the index of dissimilarity, is still the best to measure evenness, or the opportunity of intra-zonal interaction between racial groups. The index of dissimilarity can be expressed as

$$D = \frac{1}{2} \sum_{i} \left| \frac{b_i}{B} - \frac{w_i}{W} \right| \tag{1}$$

where b_i and w_i are the black and white population in areal unit i; and B and W are the total black and white population in the whole area.

Massey and Denton (1988) also recog-

nised the contributions of geographers in creating and modifying measures of segregation. For example, they discussed the works by White (1983), Jakubs (1981) and Morgan (1983), which incorporated the distance between the centroids of two areal units as an element in formulating their indices of segregation. These authors believed that distance between areal units was a spatial factor in affecting interaction among racial groups. Thus, the distances between centroids were combined with other elements to reflect the degree of segregation.

Based upon some early works in spatial statistics (Geary, 1968; Dacey, 1968), Massey and Denton (1988) derived an index to measure the clustering dimension of segregation. This index employs a connectivity matrix (or contiguity matrix) C, which is a binary matrix with 0s and 1s. The presence of a 0 value in the ith row and the ith column of the matrix indicates that areal units i and j are non-contiguous, while 1 represents adjacency. Massey and Denton criticised this approach on the grounds that it is not practical to use the connectivity matrix to study large areas because its creation has to involve intensive visual inspection of maps to detect contiguity information and laborious work to input the large n by n matrices. They argued that "contiguity is simply a dichotomous measure of distance" (Massey and Denton, 1988, p. 294) and the values in the matrix can be replaced by inter-zonal centroid distances. However, this implies that the probability of interaction is solely reflected by the single measure of distance, which fails to capture other aspects of accessibility such as the number of inter-zonal linkages and the points of contact. As indicated by research in spatial analysis (for example, Gould, 1969; Morrill and Pitts, 1967), the intensity of interaction diminishes drastically as distance increases. Therefore, inter-zonal distance may not be very useful and replacing distance by a dichotomous measure of contiguity may not lose much information.

Morrill (1991) also reviewed some of the segregation indices discussed by Massey and Denton (1988), but focusing on their spatial attributes. Based upon his conceptualisation of segregation, which "refers to the spatial separation of groups, resulting from certain physical or social processes" (Morrill, 1991, p. 25), Morrill regarded the shared boundary between areal units as an important component of spatial interaction. He proposed a "boundary modified" index of dissimilarity which explicitly regards the probability of contact between groups as a function of spatial structure. This index is defined as

$$D(adj) = D - \frac{\sum\limits_{i}\sum\limits_{j}|c_{ij}(z_i - z_j)|}{\sum\limits_{i}\sum\limits_{j}c_{ij}}$$
(2)

where c_{ij} is the value of the cell in row *i* and column *j* of the connectivity matrix, and z_i is the proportion of black in areal unit *i*.

Thus the index includes boundary information which is regarded as an important spatial component in affecting interaction. The index is evolved from the concept of spatial autocorrelation and takes into account "the pattern of differences in proportions minority across all adjacent boundaries" (Morrill, 1991, p. 34). This boundary-modified D index includes the original D, which represents the total deviation of the proportion of black population from the average or expected proportion, or the aggregate of the inter-racial interaction opportunity within each areal unit. The additional term in the boundarymodified D is used to adjust for the opportunity of interaction between areal units.

As was demonstrated by Morrill (1991), the original D is totally insensitive to the type of clustering and even to the number and size of the clustering minorities (Figures 1 and 2). In addition, a high value D does not necessarily mean that the minority groups are spatially segregated and not interacting with the majority population. If the spatial configuration provides plenty of

opportunities for the minorities to interact with others in other areal units and can induce desegregation, then the index of segregation should be adjusted for this spatial characteristic. In order to include this relationship of segregation and interzonal interaction opportunity, the second component in the boundary-modified index is introduced to indicate the opportunities of moving across boundaries if there are differences in minority proportion between any two adjacent areas. What Morrill has exploited is the topographical relation among areal units. This spatial information is combined with the interzonal variation of minority ratio to form an index of segregation.

2. Modifying D by Weight Matrix W

Morrill's modification of the D index has taken only the first step to incorporate concepts developed in spatial analysis and spatial statistics into the study of segregation. This paper attempts to propose two subsequent modifications on D due to the limitations of the boundary-modified D.

The second term in the boundary-modified D (equation 2) implies that if there is a difference in minority proportion between two adjacent areal units, interaction between the people in the two units is permitted. Thus the boundary information is captured by the binary connectivity matrix. Undoubtedly, this is the most convenient method to handle spatial information. However, adopting a binary connectivity matrix may simplify the segregation phenomenon to the extent that interactions between two areas will take place as long as they share a common boundary regardless of the ease of crossing the border. In many situations, the ease of crossing the border is proportional or inversely proportional to the length of the common boundary. However, according to Morrill's formulation, if the common boundary is reduced to a point, the interaction intensity between the area units is the same as having a very long common

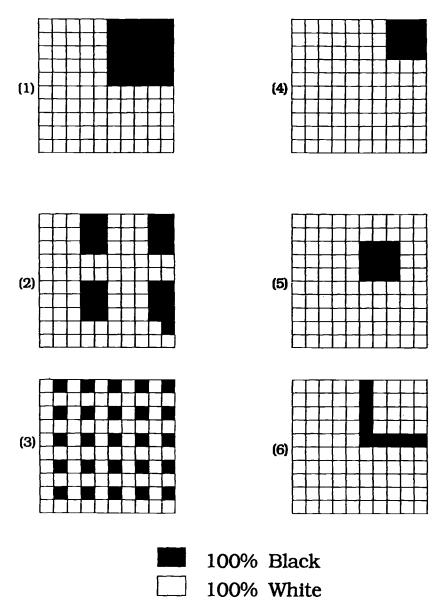


Figure 1. Six hypothetical spatial configurations: edge number = 180, MAX(P/A) = 4.

boundary. In other words, using the binary matrix fails to capture the variation of interaction intensity across boundaries due to the variation of the ease of crossing boundary indicated by the boundary length. This criticism has been raised by Kennedy and Tobler (1983) in the context of spatial interpolation. To facilitate further discussion, it is assumed that interaction between two areas is proportional to

the length of the shared boundary. Consider the three areal units A, B, C in Figure 3. The shared boundary between B and C is longer than the shared boundary between A and C, and we expect that the intensity of interaction between B and C should be higher than that between A and C. In the context of calculating indices of spatial autocorrelation, Cliff and Ord (1981) had discussed the use of a general-

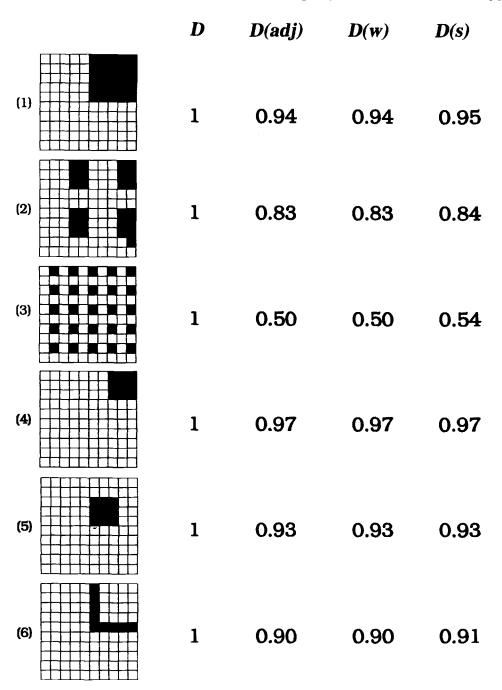


Figure 2. The family of Ds with 100 areal units in each configuration.

ised weighing matrix W, in which the element w_{ij} , can be the scaled or normalised length of the common boundary between areal units i and j. This method is

supported by Kennedy and Tobler (1983). In fact, W, which is a stochastic matrix, is usually discussed in conjunction with the binary connectivity matrix in spatial statis-

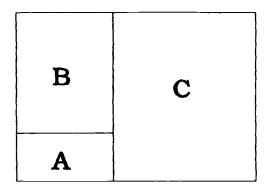


Figure 3. A variable-length boundary configuration.

tics (Odland, 1988). Then D can be further modified by these weights, w_{ij} s, and can be rewritten as

$$D(w) = D - \frac{1}{2} \sum_{i} \sum_{j} w_{ij} |z_i - z_j|$$
 (3)

and

$$w_{ij} = \frac{d_{ij}}{\sum_{i} d_{ij}} \tag{4}$$

In equation (4), d_{ij} is the length of the common boundary of areal units i and j. We may call this index the "weight-modified D" in contrast to Morrill's "boundary-modified D". Thus inter-zonal interaction, which can lower the degree of segregation, is a function of both the inter-zonal difference in racial mix and the length of the common boundary.

Adopting the stochastic W matrix not only can incorporate the length of common boundary as an important spatial factor in influencing interaction, it can also deal with boundaries of irregular shape and can also model the pattern of interaction more realistically and accurately. Massey and Denton (1988) criticised the geographers' method of using the binary connectivity matrix to capture contiguity information as too tedious, laborious and also incomprehensible. Using the weight-modified D

may seem to be even more difficult because besides the contiguity information, the lengths of common boundary are also required. However, with the advances in geographical information systems (GIS) technology, these two sets of topological information are readily available because they are stored as part of the spatial database in most geographical information systems.

In situations where a factor other than length is the major determinant in affecting the ease of crossing a common boundary, the weight matrix W can be modified to incorporate the factor to determine the potential of inter-zonal interaction.

3. Shape Geometry and Segregation

It is quite obvious that the second component in the two modified D indices is always related to the notion of spatial interaction. There is no need to discuss the long tradition of spatial interaction modelling research at this point, but the research in investigating the impacts of shapes and geometry of areal units on spatial interaction could offer some clues in developing a spatial index of segregation.

Geographers have long realised the impacts of spatial structure on spatial interaction patterns and the calibration of models (Fotheringham, 1981; Fotheringham and Webber, 1980). Griffith (1982) also demonstrated how the shape geometry of areal units could affect spatial interaction. In the context of segregation, we may consider cases in Figure 4(a), (b) and (c). X, Y, Z and P are individuals resident in the minority area, and the minority area in Figure 4(a) is larger than in 4(b). If both Y and Z are at the centre of the two minority regions, and since Z is closer to the area boundary than Y, then Z needs to spend less effort to cross the boundary to interact with the majority class in the other region. If the effort of movement is translated into probability, then the probability of interaction with the majority is

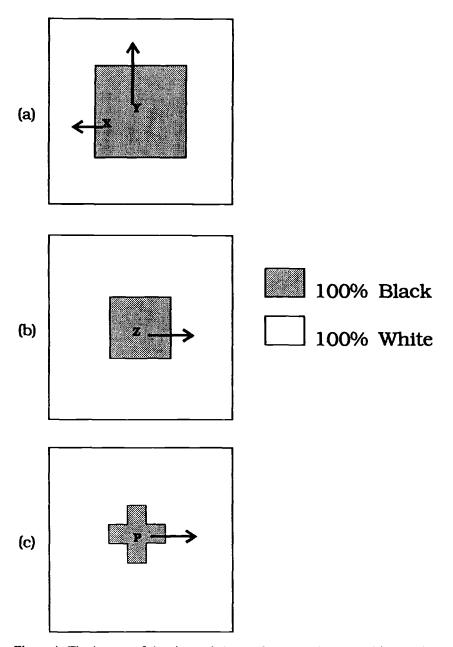


Figure 4. The impact of the size and shape of areas on inter-zonal interaction.

higher for Z than for Y. Using the same principle, the chance to interact with another group is higher for X than for Y. This example illustrates the fact that the size of a region or the proximity to the area boundary can effect the opportunity for interaction. Assuming that the areas of the minority regions in Figure 4(b) and 4(c) are

the same, we expect the chance to encounter people of another group is higher for P than for Z. Using shape indices can relate the structural or geometrical characteristics of the area to the probability of interaction.

Even though both Boyce and Clark (1964) and Taylor (1971) had analysed the

characteristics and evaluated the utilities of different shape indices, they could not agree upon an ideal shape index. In fact, each index possesses some unique characteristics and is suitable for particular types of application. In the context of deriving an index for segregation, a simple shape index of compactness employed by Pounds (1963) is useful. It is simply the ratio of the perimeter to the area of a region. In measuring the degree of segregation, this simple index has extreme powerful utility and can provide very meaningful interpretation. This will be elaborated in detail.

In the cases described by Figures 4(a), (b) and (c), we can assume that identical individuals in each groups are evenly distributed in the regions and the area of each region is a proxy of the population size. If one wants to interact with another group, a major determinant will be one's access to the boundary. We expect P is highly accessible to the boundary as compared to Z because the boundary of the minority region in Figure 4(c) is longer than in 4(b) though the two minority regions have the same size.

However, the degree of inter-racial interaction is not only a function of the perimeter of the region. Figures 5(a) and 5(b) demonstrate the importance of another attribute—area. The length of the shared boundary of the two areal units in Figures 5(a) and 5(b) are the same, but the sizes of the areal units are smaller in Figure 5(b) than in Figure 5(a). Given the small sizes of the areal units in Figure 5(b), people will find it quite easy to cross the boundary while on average people in Figure 5(a) have to make more effort to do so. We can conceptualise inter-zonal interaction as a process of individuals competing with each other for the access to the boundary. Then, the ratio of perimeter to area, which is inversely related to the degree of compactness, can indicate the proportion of boundary shared by each person in the population. If the population size of each region is known, it will be unnecessary to use the area of the region as a proxy of the population, and the index of compactness—in the context of segregation—will be the ratio of perimeter to population. However, for the rest of the discussion, the perimeter—area ratio will still be used.

Given the above argument and the modifications on D suggested earlier, D can be modified once again by incorporating the shape index and can be expressed as

$$D(s) = D - \frac{1}{2} \sum_{i} \sum_{j} w_{ij} |z_{i} - z_{j}|$$

$$* \frac{\frac{1}{2} [(P_{i}/A_{i}) + (P_{j}/A_{j})]}{MAX(P/A)}$$
(5)

where P_i and A_i are the perimeter and area of unit i, respectively.

Since it is assumed that residents cannot interact with someone outside the study region, and only interactions within the study region are counted, therefore the edges of an areal unit overlapped with the boundary of the study region are not counted towards the calculation of P_i .

In equation (5), MAX (P/A), is the maximum perimeter-area ratio or the minimum compactness of an areal unit found in the study region. This ratio is used to standardise compactness of different units. So, the ratio of the average compactness of any two adjacent units i and j to MAX (P/A) can be interpreted as the extent to which the two adjacent units have approached the ideal (or minimum) compactness facilitating inter-zonal interaction.

It is expected that the more compact the areal units are (i.e. low perimeter-area ratio), the lower the chance for the members to interact with members of other units. However, the degree of interaction also depends upon the opportunity of contact. Thus, interaction intensity is weighted by w_{ij} , the length of the common boundary of the two adjacent units i and j.

The properties of the terms subtracted from D in equation (5) have meaningful interpretations. As Morrill (1991) sug-

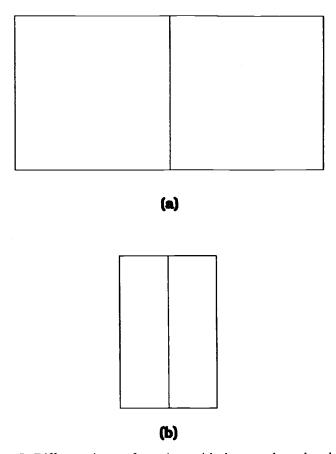


Figure 5. Different size configurations with the same boundary length.

gested, the second term in the boundarymodified D is to adjust for interaction opportunity. Similarly, in equation (5), $w_{ii}|z_i-z_i|$ reflects the contact opportunity created by the shared boundaries. Assuming that the original D is 1, then D(s)approaches 1 if all terms of the double sum approach zero. A term approaches 1 if all terms of the double sum approach zero. A term approaches zero if at least one of its factors approaches zero, or equivalently if: (1) i and j do not share a common boundary; or (2) two neighbouring areas iand j have the same minority proportion. Then, D(s) indicates purely the evenness of the minority population over space or the chance of intra-zonal interaction between the two groups.

On the other hand, the original D measure is adjusted substantially depending on

the two spatial factors captured by equation (5). If there are plenty of inter-zonal interaction opportunities created by the long common boundaries, then $w_{ii}|z_i-z_i|$ will be large. But D will not be discounted fully by the $w_{ij}|z_i-z_j|$ unless the ratio of the average compactness of two adjacent units to MAX (P/A) is close to 1. If this ratio is close to 1, it means that all areal units are very similar to the unit having the highest perimeter-area ratio or minimum compactness in the study region (for example, a spatial configuration of regular lattice with 'dense' grids). As a result, D(s)will approach 0. So theoretically, D(s) can range from 0 to 1.

The original D is criticised as not being very useful in comparing the segregation level in different areas (Jakubs, 1981). However, the MAX (P/A) in equation (5)

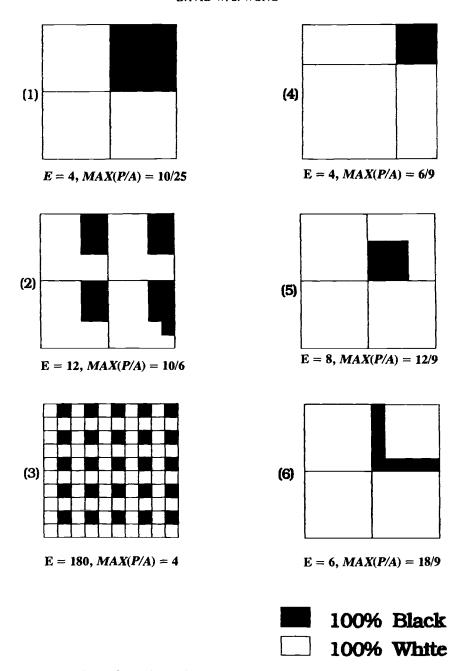


Figure 6. Six configurations with aggregated sub-areas; (E = number of edges).

can serve as an absolute scale for comparing different study areas if the MAX(P/A) is the largest among all the areal units in all study regions. In other words, the compactness of all the areal units is compared with the empirically ideal geometry facili-

tating inter-zonal interaction in all study areas.

4. Simulation Results

In order to demonstrate the drawbacks and

merits of the family of modified D indices, a set of simulated spatial configurations is created after Morrill (Figure 1). The values for the family of modified Ds are reported in Figure 2. It is assumed that each small square block is of dimension 1 by 1. Thus, its perimeter is 4 and area is 1.

As was pointed out by Morrill (1991) D is invariant to the spatial dispersion of minority population. As long as the difference of minority proportion is 100 per cent, D is 1 regardless of the number of minority cluster. It seems that the three modified Ds perform similarly. For most spatial configurations, D(s) is slightly higher than the others.

To a certain extent, all three modified Ds perform according to our expectation. We expect that the degree of segregation decreases from configuration (1) to (3) and all modified Ds show this trend. We also expect that configuration (5) should be less segregated than configuration (4), and configuration (6) should be the least segregated among the latter three patterns. These indices agree with our expectations. Even though D(s)s for configurations (1), (2), (3) and (6) are different from other modified Ds, the differences do not seem to be significant enough to claim that D(s) is better.

However, there is a 'flaw' in the calculations of these indices. All calculations are based upon the artifact that all areal units are of size 1 by 1. Even in configuration (1), the calculation processes count 25 areal units instead of 1 big unit with 100 per cent black. It is very unusual that areal units having the same attribute value are not grouped together when performing spatial analysis. If the boundaries are redrawn by grouping areal units having the same minority proportion together, then we obtain the configurations described in Figure 6 and the corresponding segregation measures are reported in Figure 7.

The first four columns in Figure 7 are the same indices described in Figure 2, but some of the index values are different. It is quite obvious that Morrill's D(adj) is sub-

stantially affected by the number of edges. while it fails to account for the variation in the size of the minority group. Among configurations (1), (4) and (5), we expect that they should have different degrees of segregation because they are either different in the size of the minority regions or in the lengths of the common boundaries between groups. But the D(adi) gives the same value to all three configurations. This implies that as long as the topological structures of different configurations are the same (configurations 1, 4, 5), D(adi) will not change and is insensitive to size or scale differences among configurations. D(adj) is the same for configurations (2) and (6). However, because of the smaller size of the minority area and the relatively long common boundaries between groups in configuration (6), it is reasonable to expect that (6) should have a higher rate of contact between groups as compared to (2). Once again, this demonstrates that D(adi)is not an effective measure.

D(w) seems to be more effective than D(adj) in terms of differentiating the size of minority groups and the opportunity of contact. Especially, D(w) takes into account the length of the common boundary or the opportunity for interaction. This characteristic of D(w) becomes evident when comparing configurations (4) and (5). According to D(w), (5) is less segregated than (4) due to the longer shared edges surrounding the minority unit. However, D(w) fails to take into account the size of the area and it is evidently shown by the same D(w) values for configurations (1) and (3). Intuitively, (3) should be less segregated than (1) because everyone in configuration (3) has abundant opportunity to interact with the other group.

D(w) is higher for configuration (4) than for (1) basically because the ratio of the length of the shared boundary and the total length of edges for (4) is smaller. However, these values may not represent the actual degree of segregation because the opportunity of contact is a function of both the length of common edges and the access to

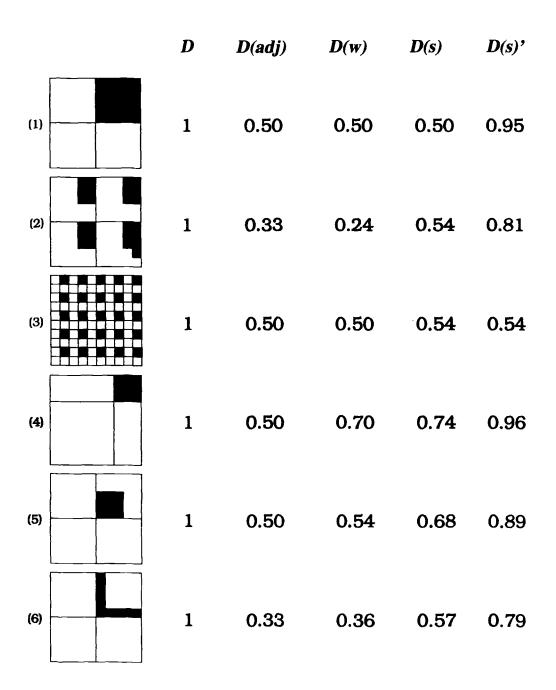


Figure 7. The family of Ds with aggregated sub-areas.

the common boundary. The latter factor sented by the index of compactness. can be interpreted as the competition for the access to the boundary among the minority population. This can be repre-

D(s) considers both the size and the length of common boundary of the configuration. However, column four in Figure 7 should be interpreted with extreme caution. D(s) is partly a relative measure of segregation. The last term in equation (5) involves the value MAX (P/A), the maximum perimeter-area ratio possibly found in each configuration. Since MAX (P/A) are different for each configuration in Figure 7 except for configurations (4) and (5), it will be misleading to compare D(s) for different spatial patterns. However, since the MAX (P/A) is the same for (4) and (5), we can conclude that (5) is less segregated.

D(s)' in column 5 demonstrates the usefulness and superiority of the D(s) by setting all MAX (P/A)s to 4, the highest MAX(P/A) found in all six configurations. The value 4 is also the index of compactness for a 1 by 1 square block. By doing so, we are comparing the compactness of each unit in all six configurations with the ideal shape geometry for spatial interaction and the D(s)' can be used to compare the degree of segregation among different regions. The results in column 5 generally resemble our intuition. Configurations (1) and (4) have the highest degree of segregation, while on the other end of the spectrum are configurations (3) and (6).

5. Conclusion

Morrill's boundary-modified D is limited by the fact that it regards the topological relation among areal units as the only spatial attribute in studying segregation. As a result, the scale or size of minority regions is not taken into consideration. This paper provides another approach to modify the popular index of dissimilarity to measure segregation. However, the new index is not just another version of the original index, but is also able to capture the information about the shape or geometry of areas, which has significant impact on segregating races territorially and limiting the chance of interaction across regional boundaries.

One of the long-standing problems of using D to measure segregation is its

aspatial nature and its insensitivity to changes in the size of areal unit (Jakubs, 1981). Thus using D to compare the segregation levels of any two study regions having different sets of spatial resolutions is not meaningful. Employing the MAX(P/A) measures enables a correct comparison in the degree of segregation in two areas, even if they are of different size.

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