

$$2. \quad u) \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} \quad A = U \Sigma U^T$$

$$\lambda_1 = 3 \quad \lambda_2 = -2 \quad \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$f) \begin{bmatrix} 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} =$$

$$\lambda_1 = 4 = \lambda_2 = \lambda_3 = 0 \quad \Sigma = \begin{bmatrix} 4 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$u_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad u_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad u_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = A A^T =$$

$$A^T A = \begin{pmatrix} 0 & 0 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$(1-5) \times 120$$

$$x^2 - 2x + 0$$

$$\begin{matrix} x^2 & x & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 0 \end{matrix} \Rightarrow \begin{pmatrix} 2 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$U_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$U_2^T = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

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