Combining Approximation Algorithms with Metaheuristics for the Facility Location problem

Renata Sarmet Smiderle Mendes
BEPE-FAPESP

The Facility Location problem

- It is sought to decide how many and which facilities to open in order to serve the connection demands of the clients.

Minimize the total cost

- Cost of opening each facility
- Cost of assigning each client to an open facility

Metric version of the problem

- The connection costs are symmetric
- It respects the triangular inequality

Motivated by practical applications, modelling problems such as

- Plant positioning
- Construction of computer networks
- Information clustering.

Approximation Limit

- Guha and Khuller [1] have shown that the MUFL is NP-hard and that it is not possible to get an approximation better than **1.463** for it, unless P equals NP.

What I have already done

- Studied some approximation algorithms for the MUFL
- Implemented (using C++) and tested (using Python) some of these with instances from the literature

Deterministic Rounding of Linear Programming

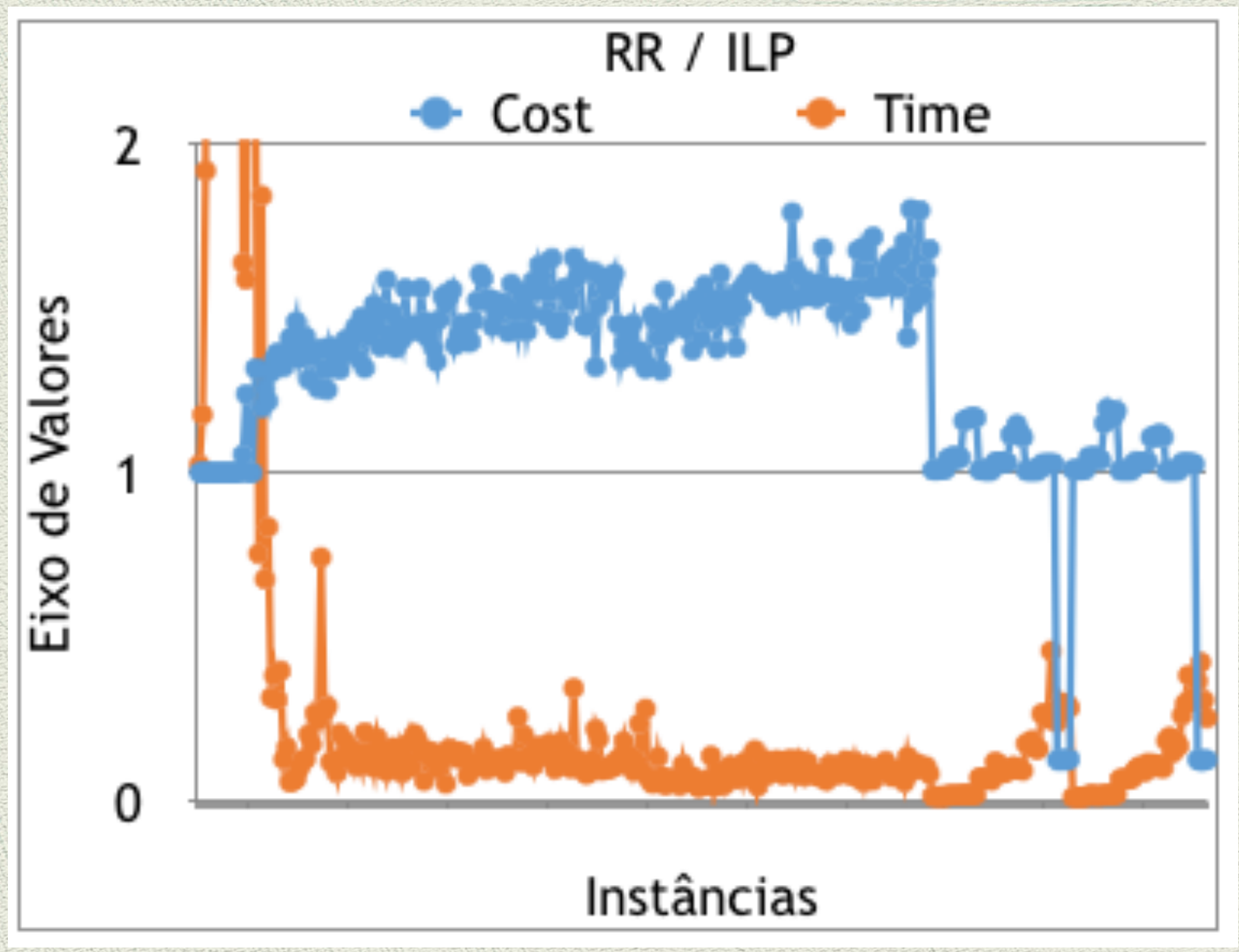
- Shmoys, Tardos and Aardal [2] obtained the first constant approximation algorithm for the MUFL, with a ratio of **3.16**, using the linear programming deterministic rounding technique.

- Studied analysis of **4-approximation.**

Randomized Rounding of Linear Programming

- This result was improved by Chudak and Shmoys [3], which proposed a randomized rounding algorithm with an approximation ratio of **1.736**.

Studied analysis of 3-approximation

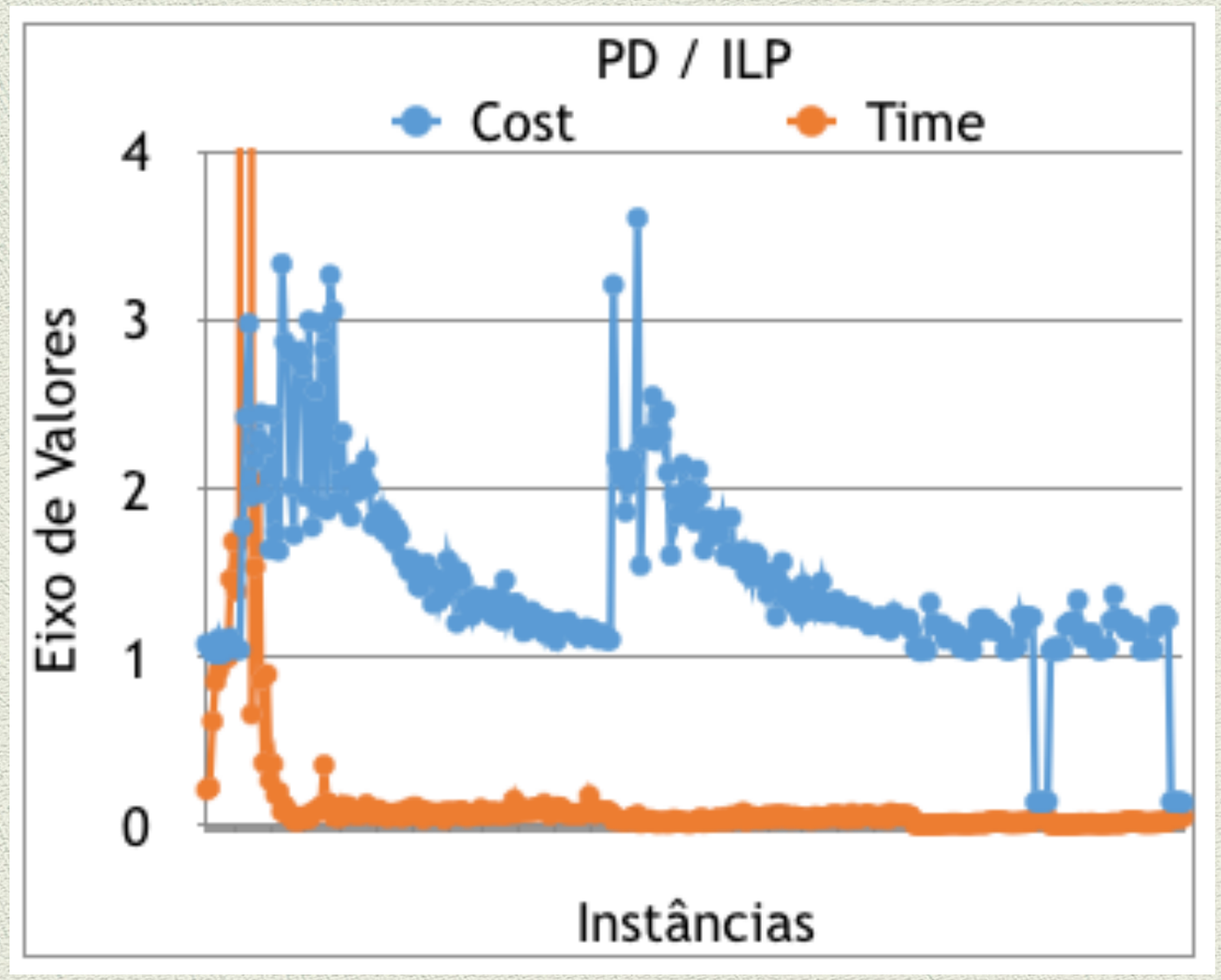


Primal-Dual Method

- The first primal-dual algorithm for the MUFL was a **3-approximation** obtained by Jain and Vazirani [4].

Studied analysis of 3-approximation

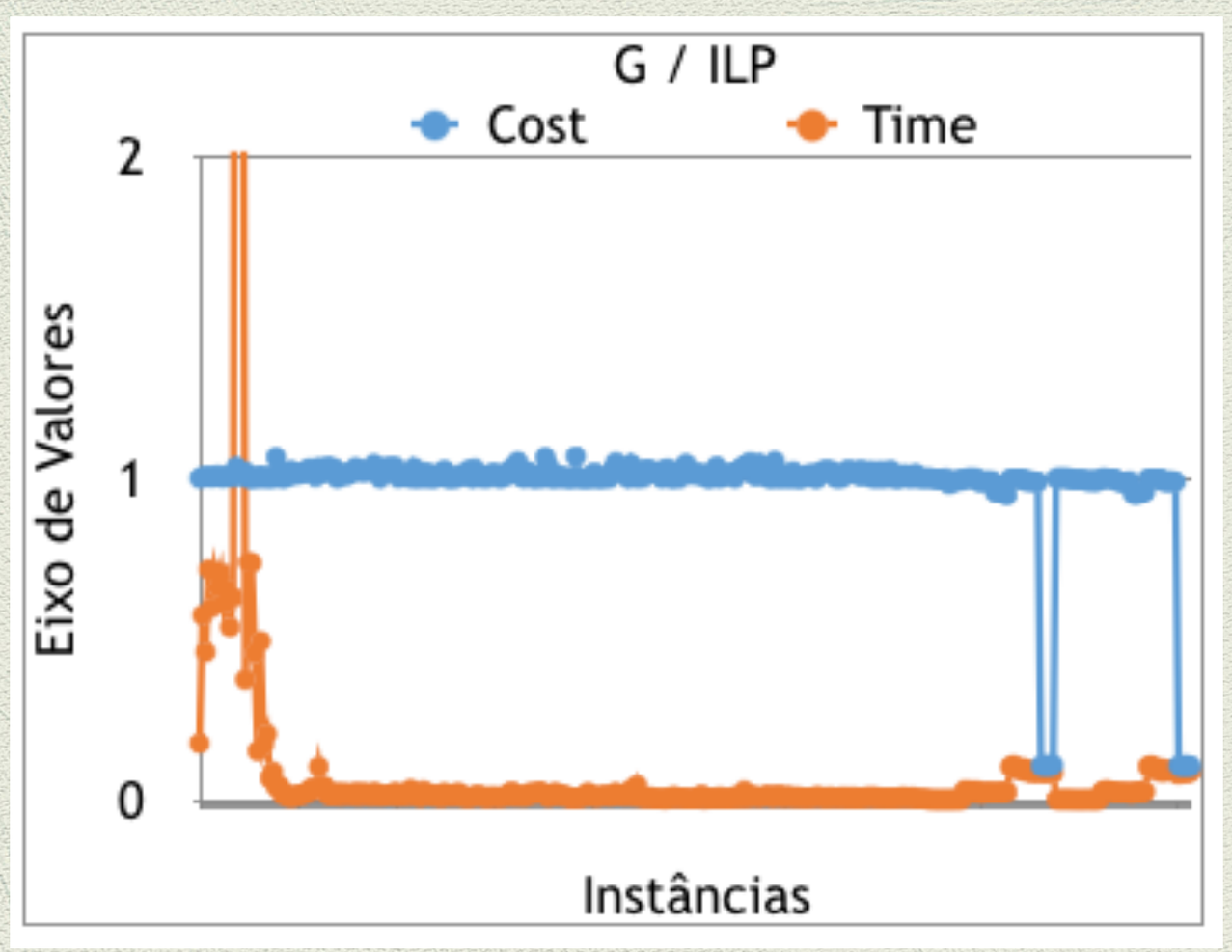
PROBLEM



Greedy Approach

- A greedy algorithm, whose analysis uses the dual-fitting technique, was obtained by Jain, Mahdian, Markakis, Saberi and Vazirani [7], reaching an approximation ratio of **1.61**.

Studied analysis of 2-approximation

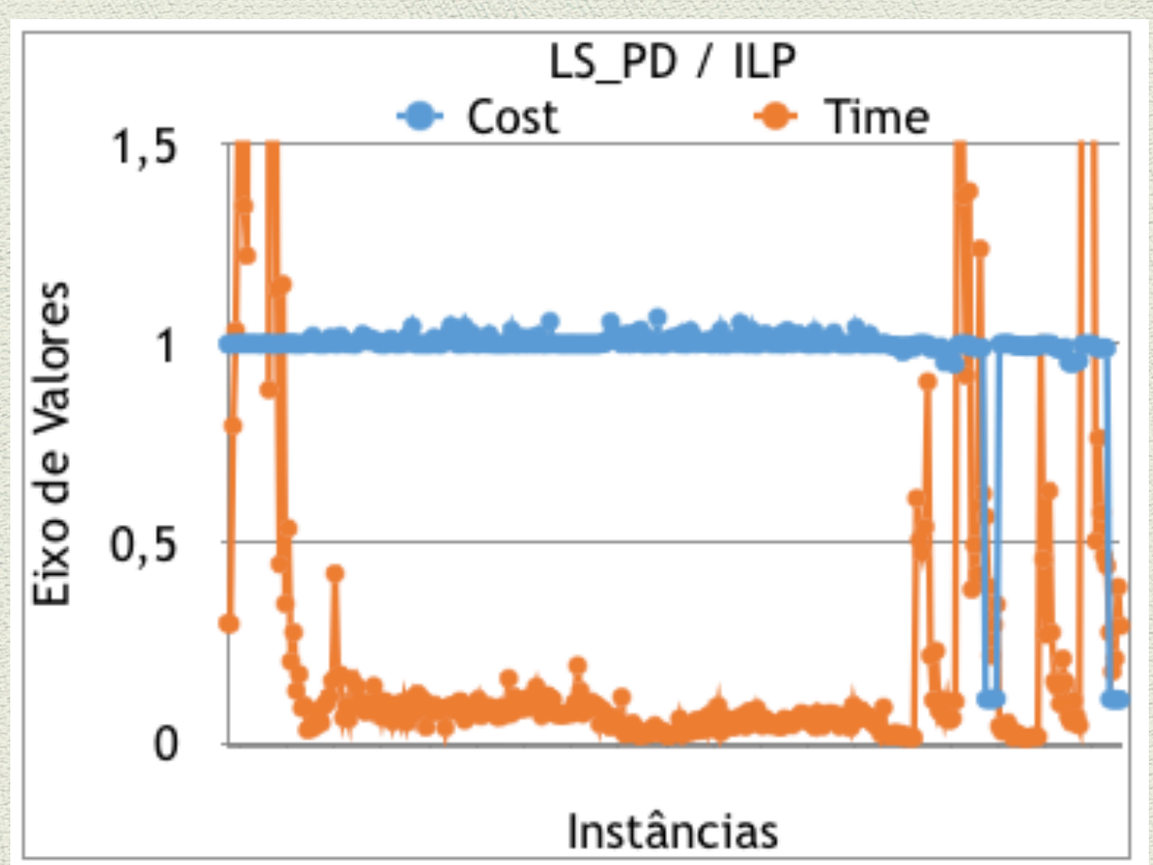


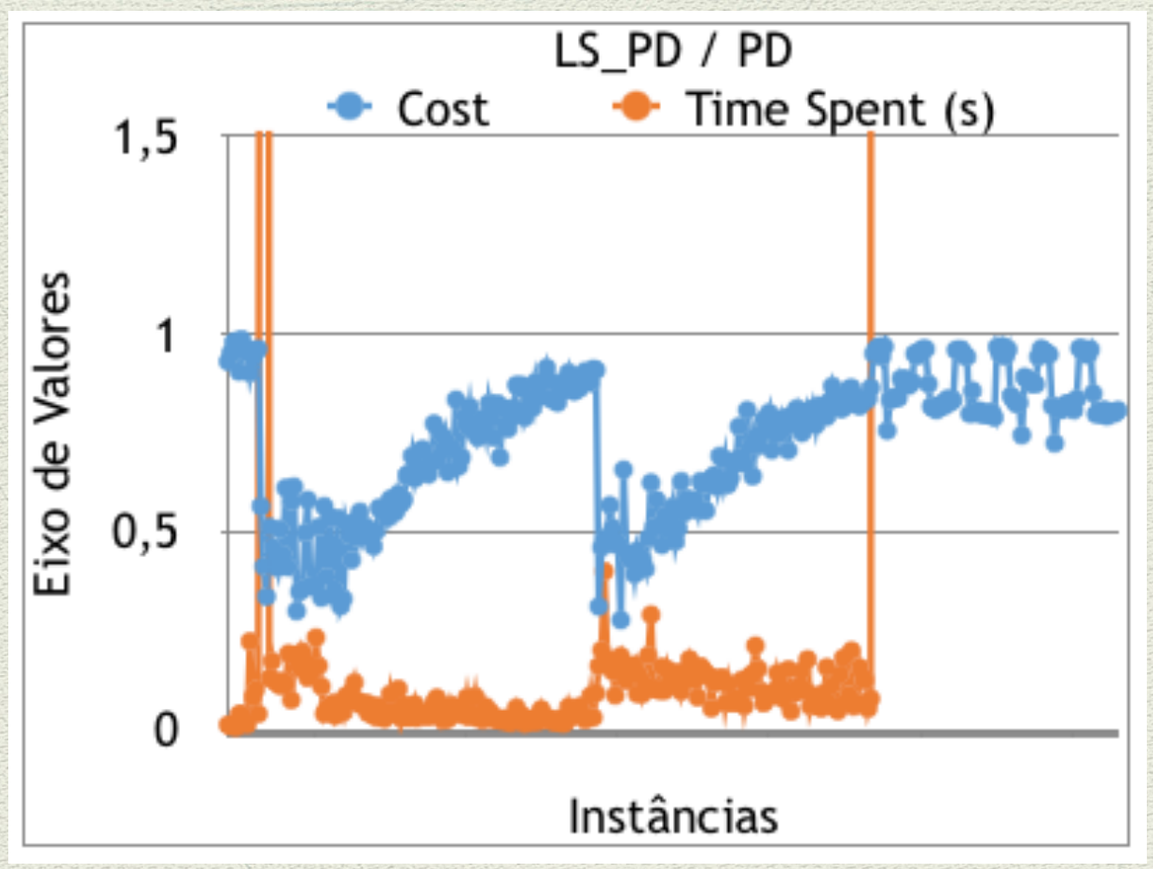
Local Search Approximation Algorithm

- Charikar and Guha [5] obtained a local search algorithm capable of improving the approximation ratio of both the best primal-dual algorithm and the best randomized rounding algorithm. The analysis of this algorithm was simplified by Gupta and Tangwongsan [6]. It has an approximation ratio of **2.414**.

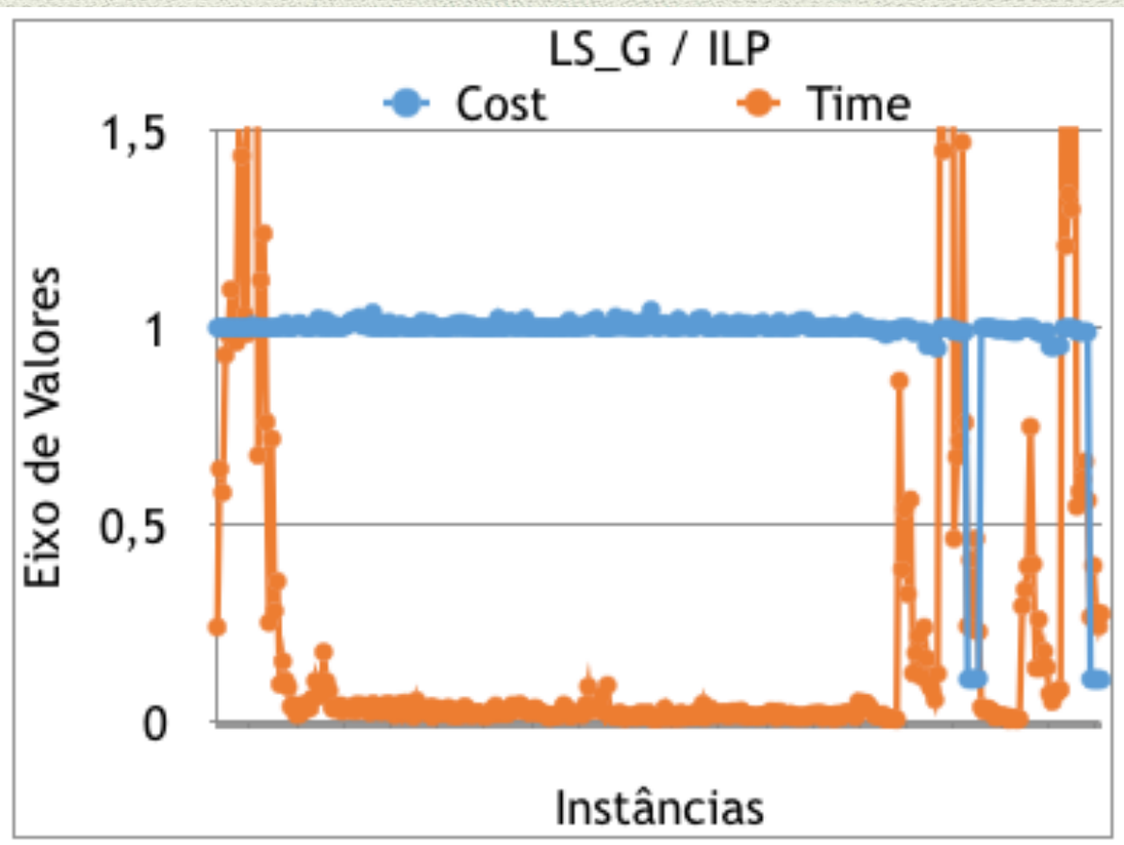
- Studied analysis of **3-approximation.**

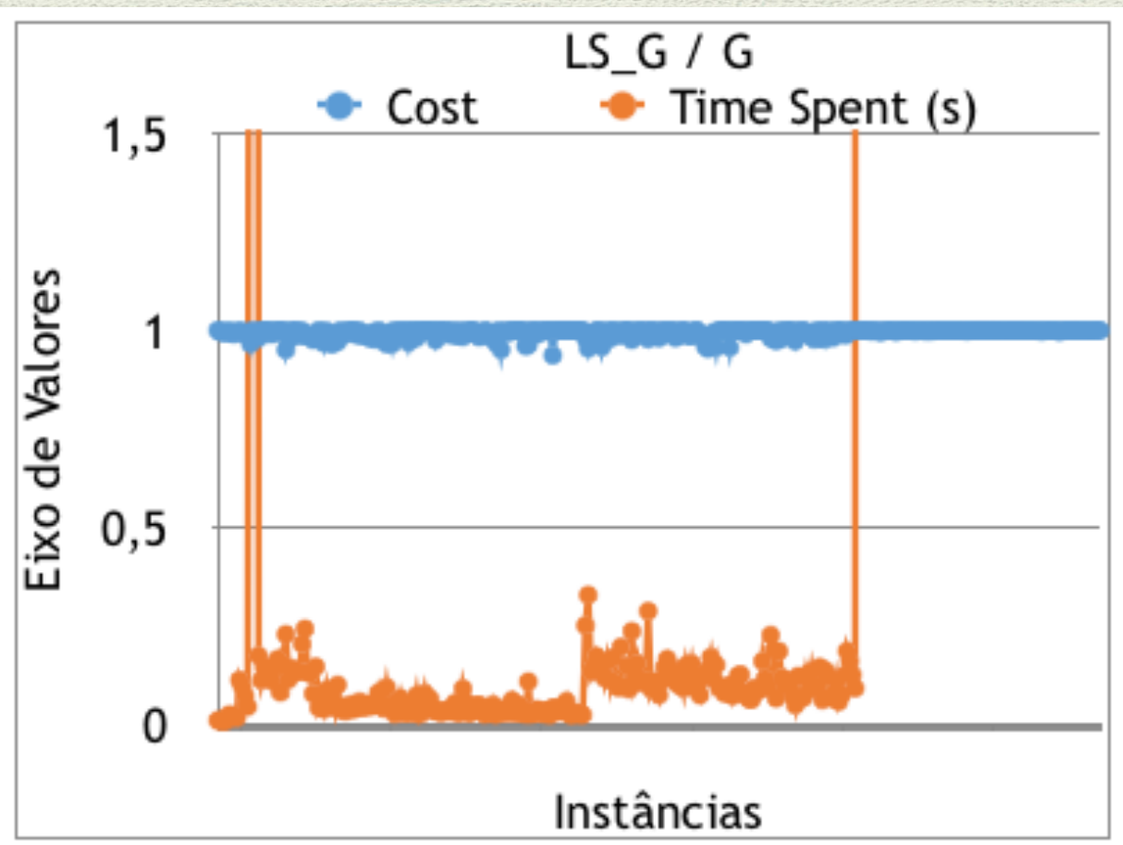
Local Search and Primal Dual



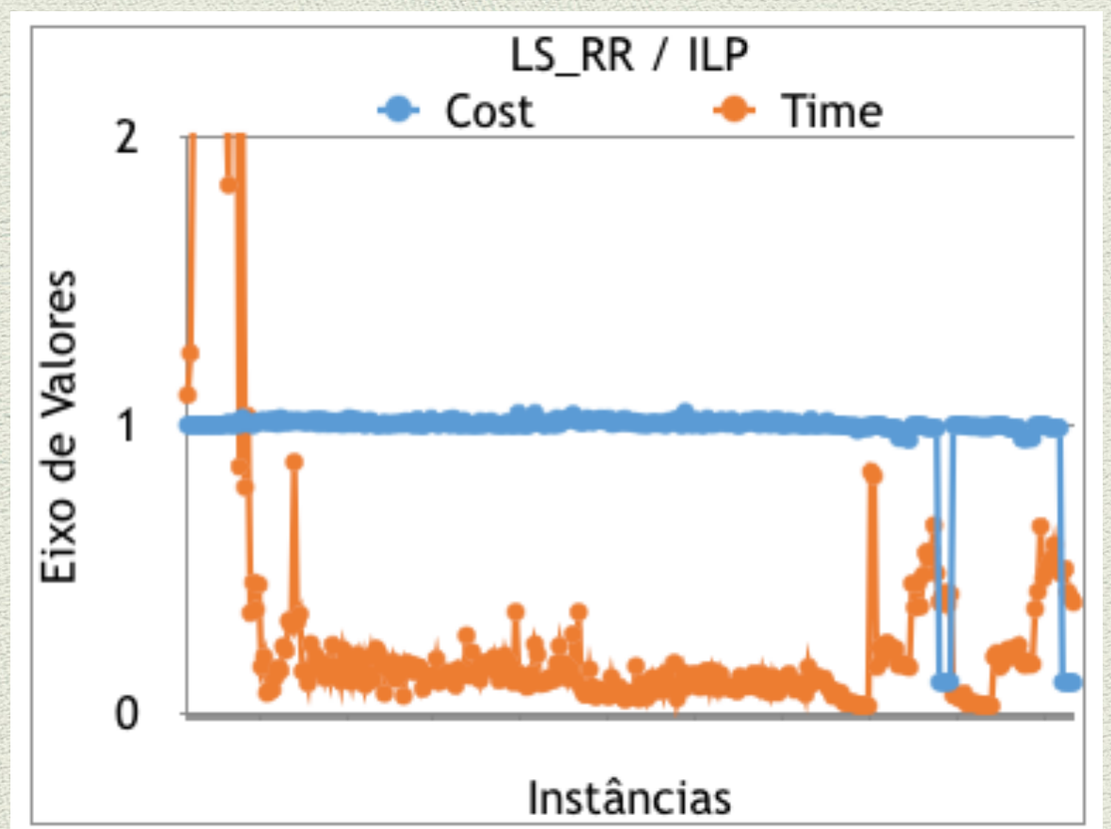


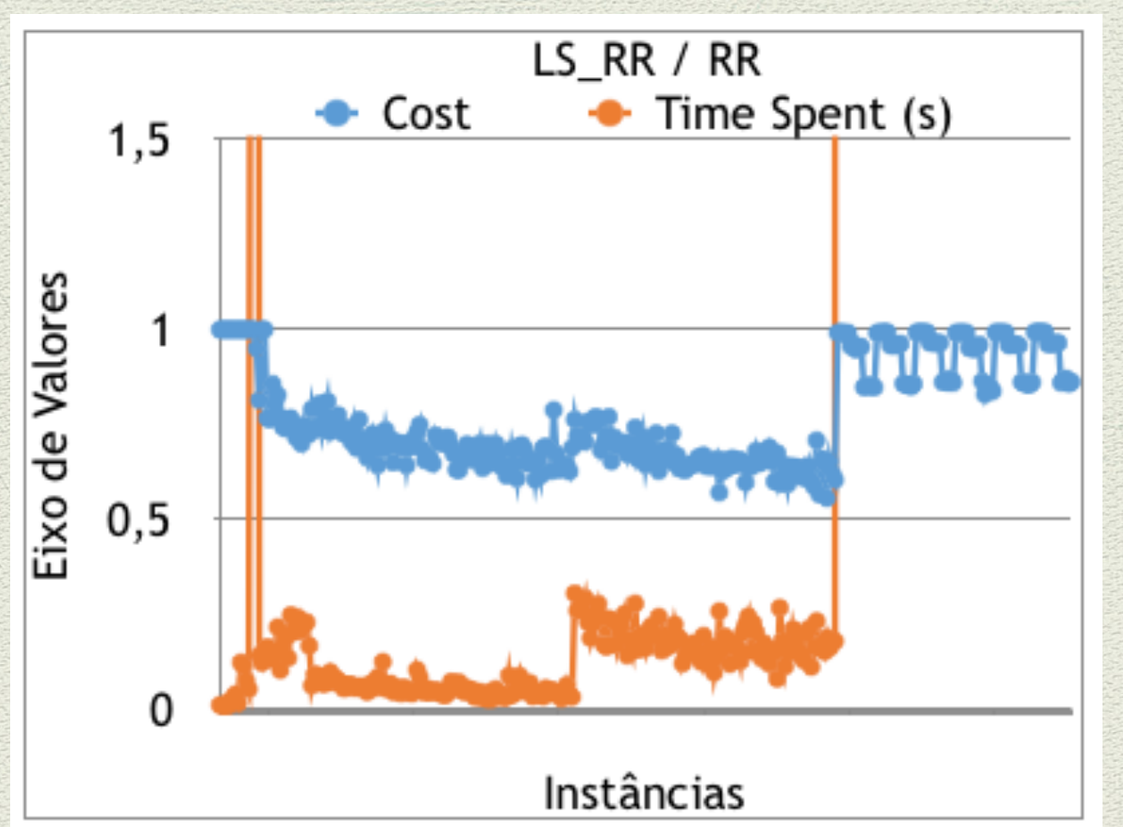
Local Search and Greedy



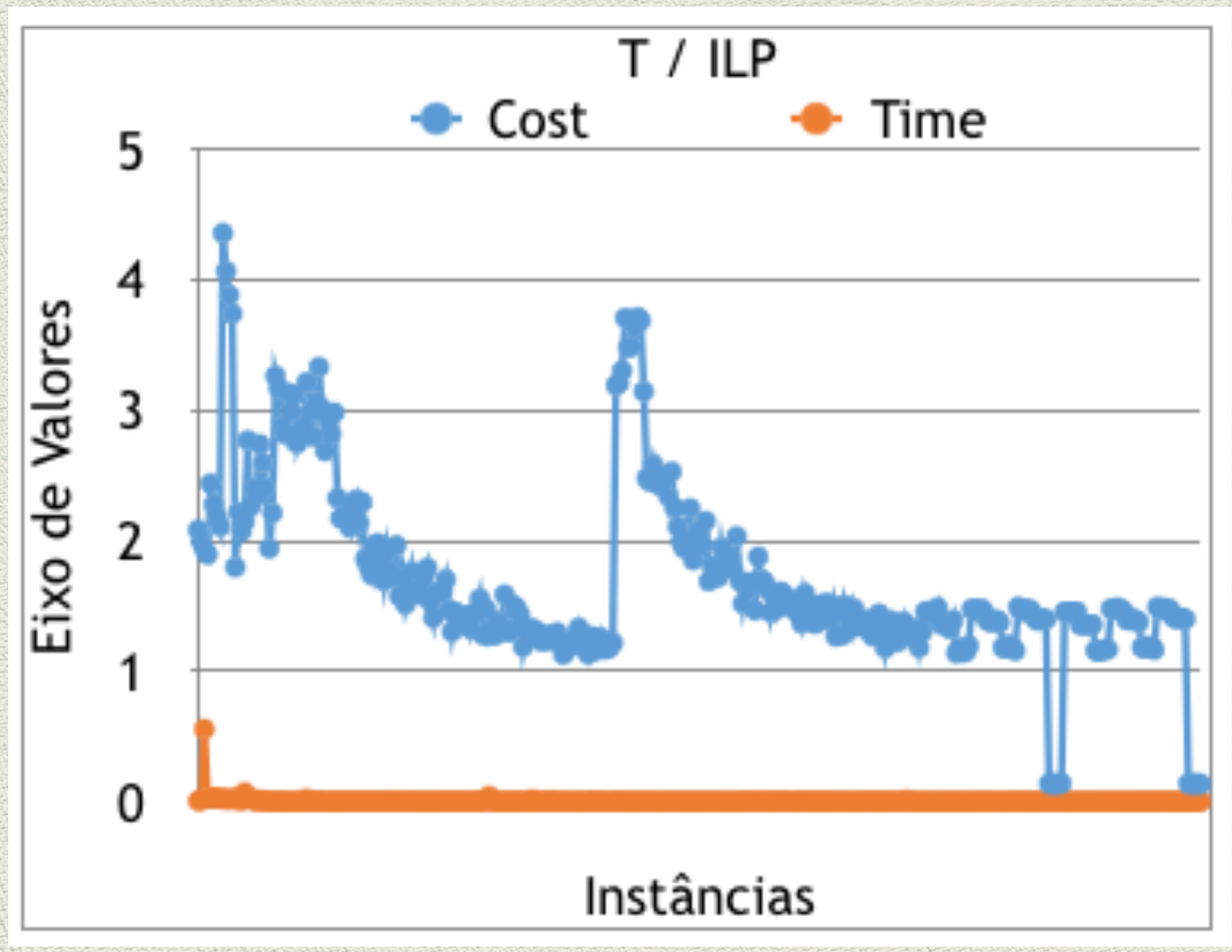


Local Search and Randomized Rounding

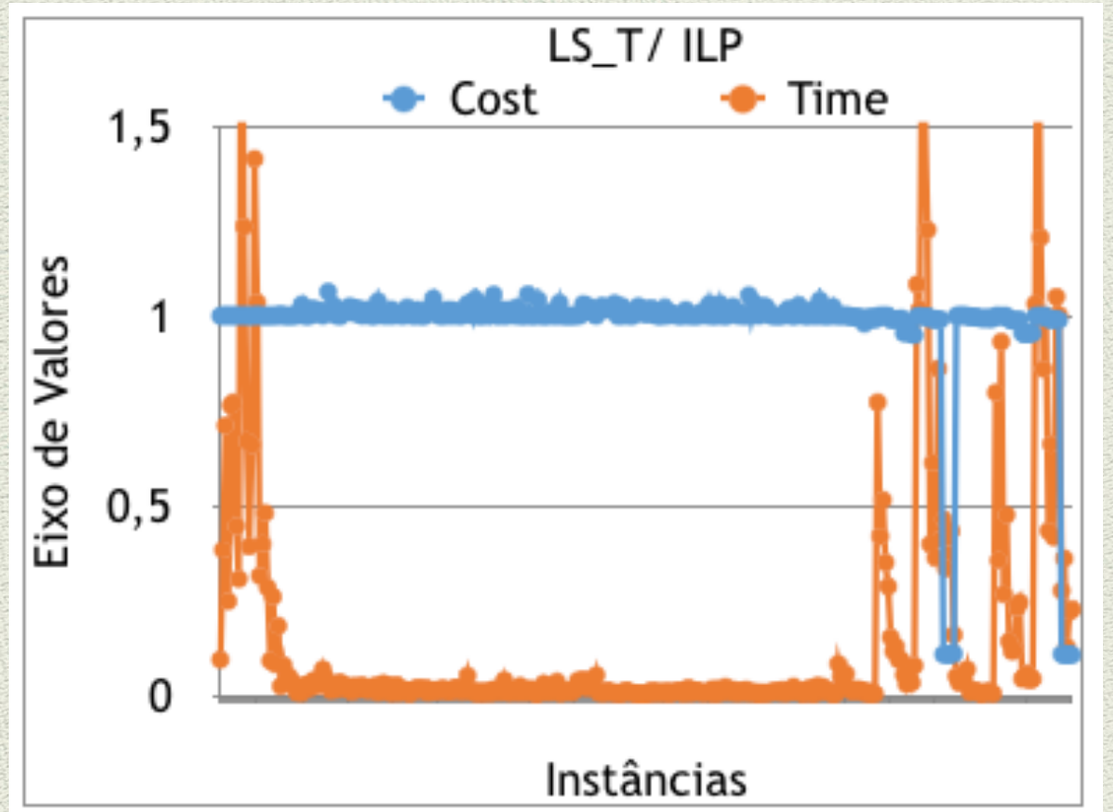


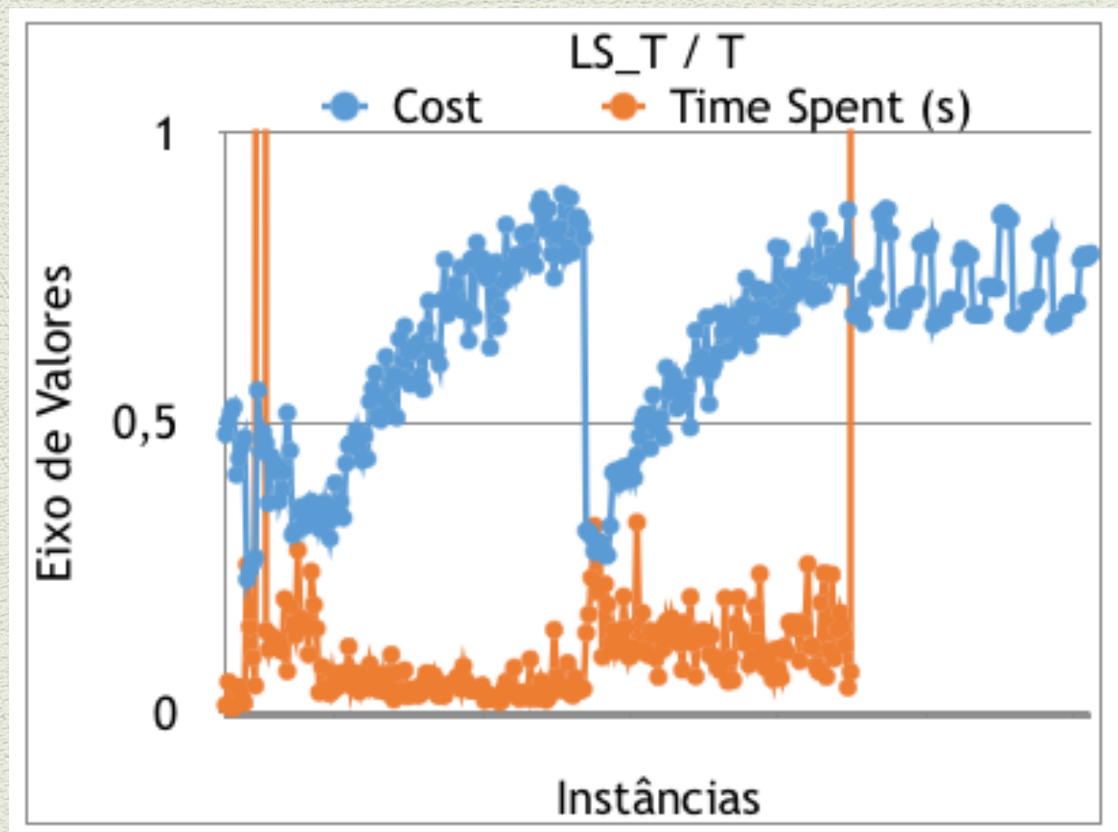


Trivial Algorithm

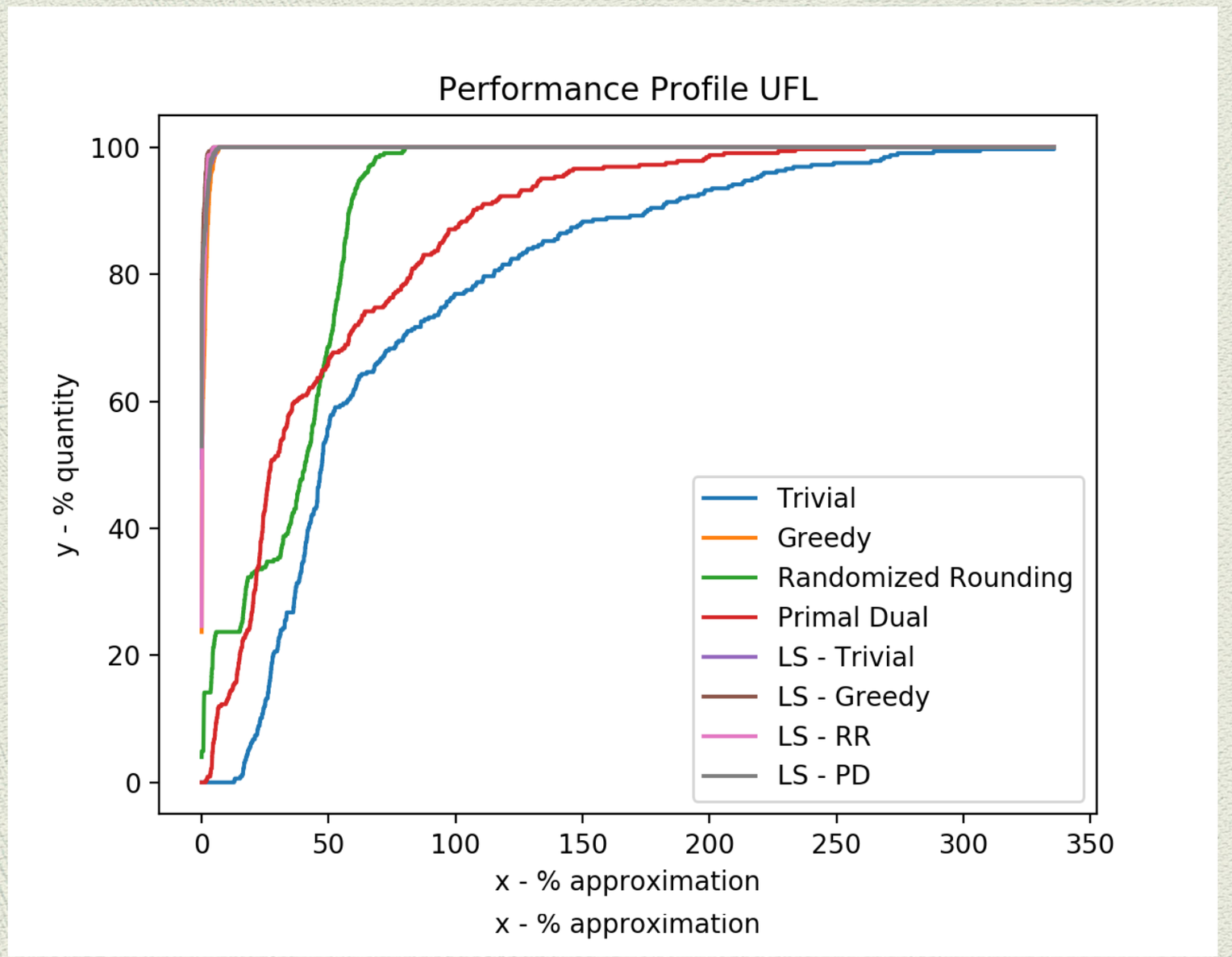


Local Search and a Trivial Algorithm

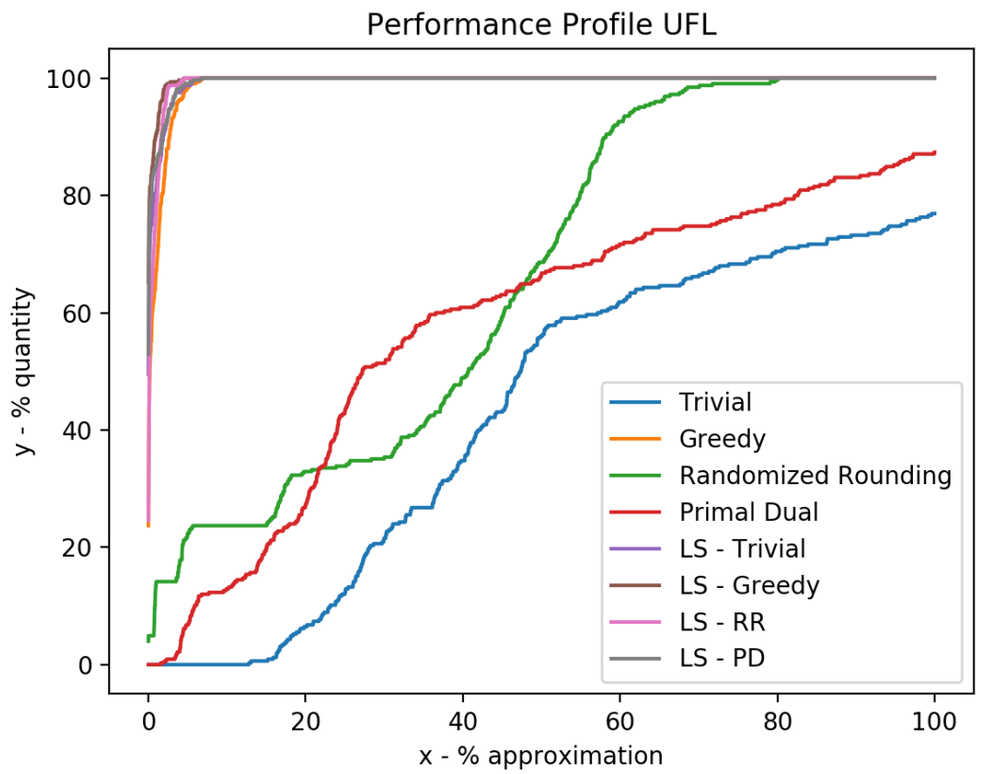


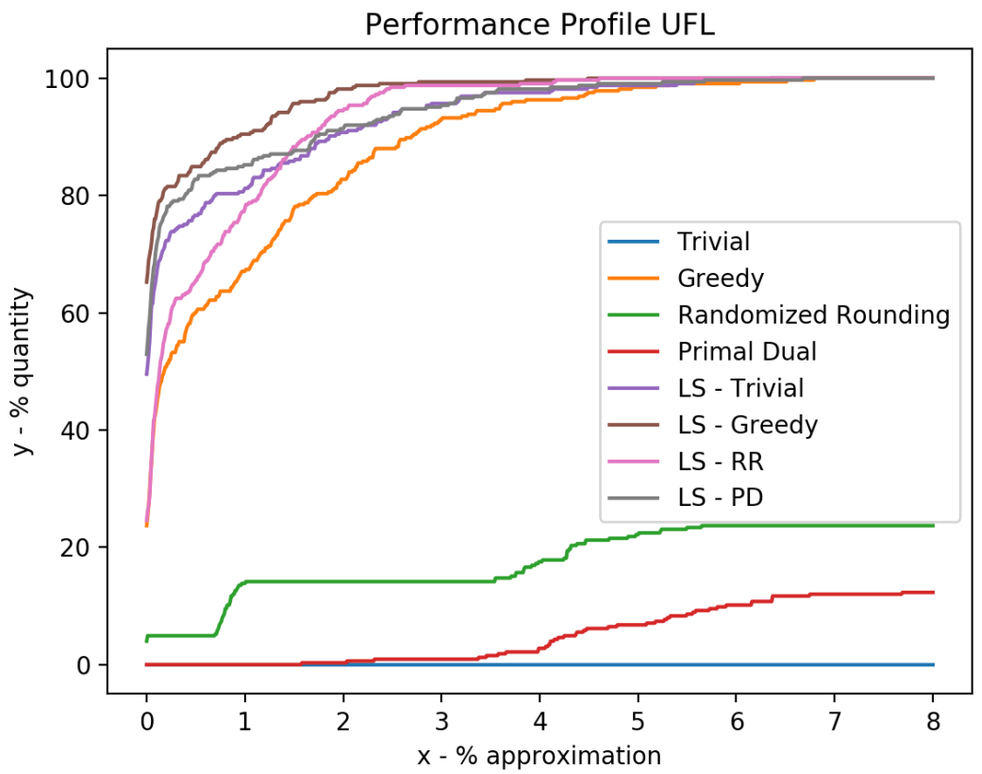


Performance Profile



Performance Profile - Zoom





Best Results - not studied yet

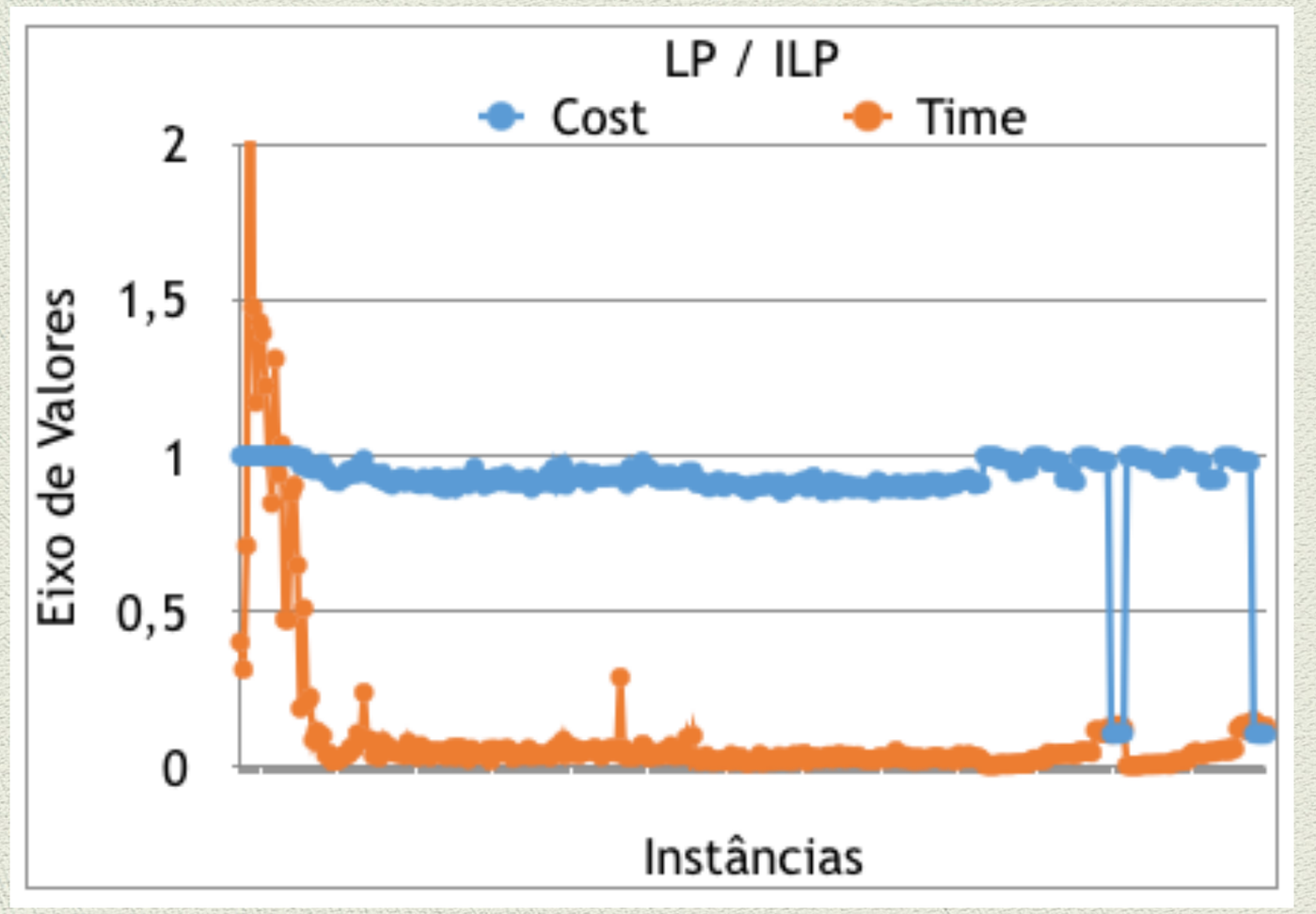
- By the combination of previous algorithms, Byrka and Aardal [8] obtained an algorithm with an approximation ratio of **1.5**.

- Finally, a **1.488-approximation** algorithm was obtained by Shi Li [9], by combining previous algorithms through the use of random choices and probabilistic analysis.

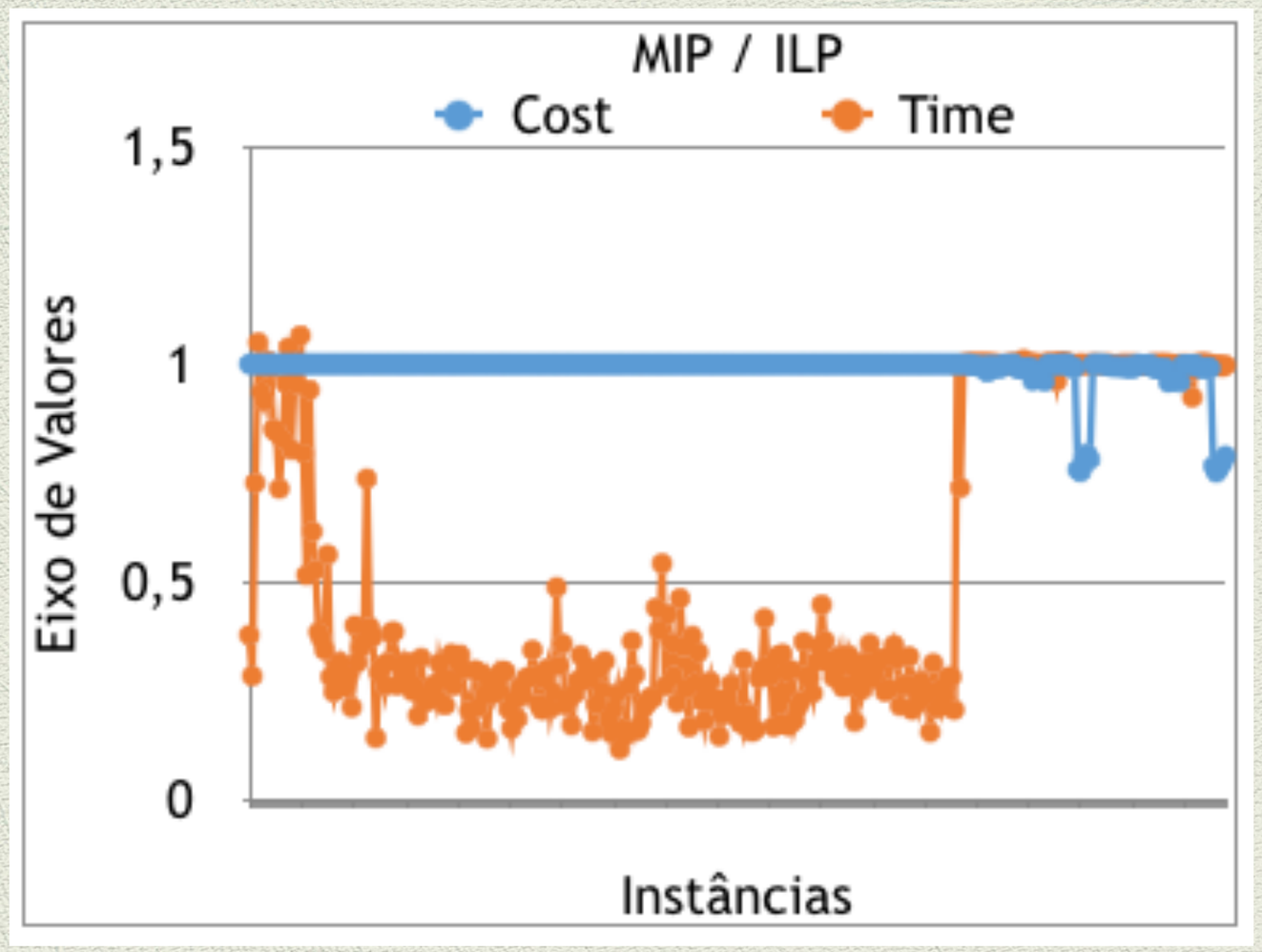
Gurobi

- Integer Linear Program
- Linear Program
- Mixed Integer Linear Program
- Dual Linear Program

Linear Program



Mixed Integer Linear Program



BEPE Project

- This project objective is to **study metaheuristics** for the Facility Location Problem and to learn how to **combine them with approximation algorithms** already studied, in order to achieve good solutions.

- The **implementation** of these algorithms followed by an **empirical analysis** of results is also an objective of this project.

Local and Tabu Search

- Modify the local search approximation algorithm for the MUFL into a tabu search (TS) and compare both the solutions quality and time efficiency of the two algorithms.

Randomized Rounding and Genetic Algorithm.

- Use the randomized rounding approximation algorithm for the MUFL to generate the initial population for a genetic algorithm (GA).

- The population generated this way may not be diverse enough, so we will test and adapt it if necessary, by introducing more random choices to the rounding algorithm, for example.

- After the initial population is built, crossover and mutation operators will be implemented to be applied to the members of the population.

Randomized Rounding, Genetic and Memetic Algorithms.

- Next, we will modify the GA to memetic algorithms (MA) by using either the LS or the TS discussed in the previous topic to improve the populations.
- We will decide if we apply these improvements to the whole population or just to part of it as well as if they should be applied in every generation or every multiple of generations based on experimentation and comparisons.
- We may also use other crossover and mutation strategies, as well as populations structures and restart strategies.

[If time permits]
Greedy Algorithm and GRASP.

- Design and implement a greedy randomized algorithm based on the greedy approximation algorithm for the MUFL.

- Then, combine this randomized algorithm with the LS algorithm discussed to build a Greedy Randomized Adaptive Search Procedure (GRASP).

- We may also use the TS discussed in a former topic to build a different GRASP.

[If time permits] Greedy Algorithm and GRASP.

- Moreover, the greedy randomized algorithm can be used both in the GA and the MA of the previous topic to give greater diversity to the initial population, particularly if the solutions of the randomized rounding algorithm are not diverse enough.

[If time permits]
Variants of the Facility Location Problem

- There are many variants of the Facility Location Problem, like the CFL. So, if time permits, it's possible to adapt our algorithms for this generalization.

Tests and Comparisons

- We will compare different algorithms and approaches, in order to attest their quality and then perform a statistical analysis on the results, mainly to properly evaluate our algorithms and also to fine tune their parameters.

References

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- [5] Moses Charikar and Sudipto Guha. Improved combinatorial algorithms for facility location prob- lems. SIAM Journal on Computing, 34(4):803–824, 2005.
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