1 Análise Matemática II

Apontamentos
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2021/22 - ISEC - LEI

→ Equação Diferencial Ordinária

(1) $\underbrace{F(x,y,y')}_{\text{f. implícita}} = 0 \Leftrightarrow$ (2) $\underbrace{y' = f(x,y)}_{\text{f. explícita}}$

Solução Geral

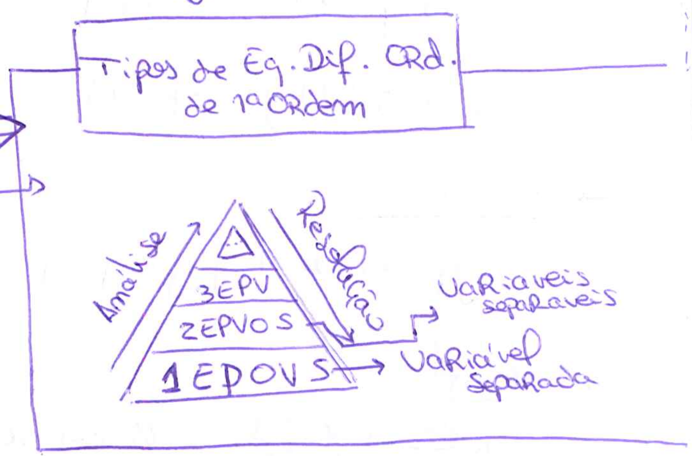
(3) $G = (x, y; C) = 0, \; C \in \mathbb{R} \Leftrightarrow y = g(x; C)$

. objetivo Principal $y(x) = ?$ → função Incógnita

$F(x,y,y') = \emptyset \Leftrightarrow y' = (x,y)$

$y(x) = ?$

$\boxed{y = g(x; C), \; C \in \mathbb{R}}$



→ Equações variáveis Separadas

$y' = f(x)$

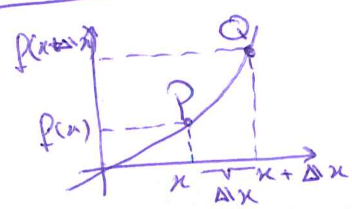
Resolução

$$\boxed{\int y' dx = \int f(x)\,dx \Leftrightarrow y = f(x)\,dx}$$

Exemplo

$y' = 1 \Leftrightarrow y = \int 1\,dx \Leftrightarrow y = x + C, \; C \in \mathbb{R}$

→ Acréscimo e Diferencial



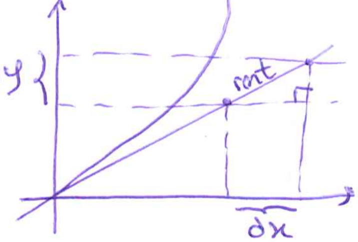
Δx = acréscimo de V. independente x

Δy = acréscimo de V dependente y

(1) $\Delta y = f(x + \Delta x) - f(x) \Leftrightarrow$

$\Leftrightarrow f(x + \Delta x) = f(x) + \Delta y$

$f(x) \simeq P_1(x)$



$\partial y = ?$

$mt = \frac{\partial y}{\partial x}$ | Eq. final

$mt = f(x)$ | $\partial y = f'(x)\,\partial x$

$$f(x)\,\partial x + g(y)\,\partial y = \emptyset \Leftrightarrow$$

$$\Leftrightarrow f(x) + g(y)\,\partial y/\partial x = \emptyset \text{ (dividir tudo por } \partial x) \Leftrightarrow$$

$$\Leftrightarrow f(x) + g(y)\,y' = \emptyset \Leftrightarrow \qquad \left(\frac{\partial y}{\partial x} = y'\right)$$

$$\Leftrightarrow g(y)\,y' = -f(x)$$

→ Resolução

$$\int f(x)\,\partial x + \int g(y)\,\partial y = \int \emptyset\,\partial x \Leftrightarrow \int f(x)\,\partial x + \int g(y)\,\partial y = C,\ C \in \mathbb{R}$$

→ Exemplo

$$x\frac{\partial x}{\partial x} + y\frac{\partial y}{\partial x} = \emptyset \Leftrightarrow x + y' = \emptyset \Leftrightarrow y' = -x \Leftrightarrow y = \int x \Leftrightarrow y = 1 + C,\ C \in \mathbb{R}$$

→ EDO de 1ª ordem e variáveis separadas

- Resolução

$$y' = 2x \Leftrightarrow y = \int 2x\,dx \Leftrightarrow y = x^2 + C,\ C \in \mathbb{R}$$

- $P_m(x) = a_0 + a_1 m + a_2 m^2 + (\ldots)\, a_m m^m$
- $P_m(x) = \sum_{i=\emptyset}^{m} a_i x^i,\ a \in \mathbb{R}$
- $a = \boxed{a_0 \mid a_1 \mid \ldots \mid a_m}$
- Operações associadas: $+, *$
- Caso $m = 2$

→ $P_2(m) = a_0 + a_1 x + a_2 x^2$

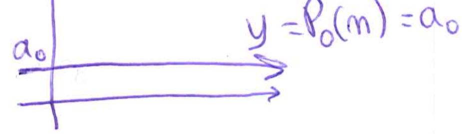
$P_2(m) = a_0 + a_1 * x + a_2 * x * x$

$P_2(m) = a_1 * x + x + a_2 x * x + a_0$

$P_2(m) = (a_2 * x + x) * x \begin{cases} \# + 1 \\ \# * = 2 \end{cases}$

- Caso $m = \emptyset$

$P_0(m) = a_0 \begin{cases} \# + = \emptyset \\ \# * = \emptyset \end{cases}$



$$P_3(y) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$\Big\downarrow$ transformar $\begin{cases} \# + = 3 \\ \# * = 6 \end{cases}$

$$P_3(y) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

$\Big\downarrow$ transformar

$$P_3(y) = ((a_3 x + a_2 x) x + a_1) x + a_0 \quad \begin{cases} \# + = 3 \\ \# * = 3 \end{cases}$$

(Método dos parênteses encaixados)

Algoritmo

Input = a, x
Output = P
m ← length(a);
P ← a(m);
Para i = m-1 até 1
    P ← P*m+a(i);
Fim

$$P_m(x) = a_0 + a_1 + a_2^{n^2} + \dots + a_m^{n^m}$$

$\Big\downarrow$ transformar $\begin{cases} \# + = m \\ \# * = ? \end{cases}$

$$P_m(x) = a_m x^m + a_{m-1} x^{m-1} + \dots + a_1 x + a_0$$

$\Big\downarrow$ "trabalhar"

$$P_m(x) = (((a_m^x + a_{m-1}) x + a_{m-2}) x + \dots + a_1) x + a_0$$

$$\begin{cases} \# + = m \\ \# * = m \end{cases}$$

Exercício 1 - Exame

a)

$$f(x) \begin{cases} \text{se } \emptyset < x \le 2 \\ \quad \text{então } y = 3\sqrt{1 - x^2/4} \\ \text{senão se } -x \le x \le 0 \\ \quad \text{então } y = 3\cos(1/2\, x) \end{cases}$$

$f(x)$ encontra-se na parte positiva

$f(0) = 3\cos(1/2 \times \emptyset) = 3\cos(0) = 3 \times 1 = 3$

$f(-\pi/2) = 3\cos(1/2 \times -\pi/2) = 3\cos(-\pi/4) = 3\cos(\pi/4) = 3\sqrt{2}/2$

x_i	$f(x_i)$
$-\pi$	0
$-\pi/2$	$\frac{3\sqrt{2}}{2}$
0	3

Declive:

$$\frac{\frac{3\sqrt{2}}{2} - 0}{-\pi/2 - (-\pi)} = \frac{\frac{3\sqrt{2}}{2}}{\frac{\pi}{2}} = \frac{3\sqrt{2}}{\pi}$$

$$\frac{3 - \frac{3\sqrt{2}}{2}}{0 - (-\pi/2)} = \frac{\frac{6 - 3\sqrt{2}}{2}}{\frac{\pi}{2}} = \frac{6 - 3\sqrt{2}}{\pi}$$

Calculo final:

$$\frac{\frac{3\sqrt{2}}{\pi} - \frac{6 - 3\sqrt{2}}{\pi}}{0 - (-\pi)} = \frac{\frac{6 - 6\sqrt{2}}{\pi}}{\pi} = \frac{6 - 6\sqrt{2}}{\pi^2}$$

x_i	$f(x_i)$		
$-\pi$	0 a_0		
		$\frac{3\sqrt{2}}{\pi}$ a_1	
$-\pi/2$	$\frac{3\sqrt{2}}{2}$		$\frac{6-6\sqrt{2}}{\pi^2}$ a_2
		$\frac{6-\sqrt{2}}{2}$	
0	3		

O polinómio interpolador de grau 2 para f em $[-\pi, 0]$ é

$$P_2(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1)$$
$$= 0 + \frac{3\sqrt{2}}{\pi}(x-(-\pi)) + \frac{6-6\sqrt{2}}{\pi^2}(x-(-\pi))\cancel{(x-(-\pi))}(x-(-\pi/2))$$
$$= \frac{3\sqrt{2}}{\pi}(x+\pi) + \frac{6-6\sqrt{2}}{\pi^2}(x+\pi)(x+\pi/2)$$

- fazer o mesmo para o $\underbrace{\text{segmento de Reta}}_{\text{grau 1 = 2 pontos}}$ negativo

- Pontos: $(-f, 1)$
  $(-6, 0.5)$

Tabela

x_i	y_i	
$-f$	1 a_0	
		$\frac{0.5-1}{-6-(-f)} = \frac{-0{,}5}{1} = 0{,}5$ a_1
-6	0.5	

- logo, o segmento de Reta é:

$$P_1(x) = a_0 + a_1(x-x_0)$$
$$= 1 + 0.5(x-(-f)) = (1-0.5)(x+f)$$

//

→ EDO de 1ª ordem e de variáveis separáveis

① $y' = f(x) * g(y) \Leftrightarrow$ ② $\frac{1}{g(y)} y' = f(x)$

③ $f_1(x) * g(x)\partial x + f_2(x) * g_2(y)\partial y = 0$

→ Fato Integrante

1° Passo: $\frac{1}{g_1(y) * f_2(x)}$

2° Passo: $\frac{1}{g_1(y) * f_2(y)} (f_1(x) * g_1(y)) + \frac{1}{g_1(y) * f(x)} f_2(x) * g_2(y) dy =$

$= \frac{1}{g_1(y) * f_2(x)} = 0 \iff \frac{f_1(x)}{f_2(x)} dx + \frac{g_2(y)}{g_1(y)} dy = 0$

3° Passo: $\int \frac{f_1(x)}{f_2(x)} dx + \int \frac{g_2(y)}{g_1(y)} dy = C$

... $y(x) = ? \iff y = g(x, C)$, $C \in \mathbb{R}$

→ Exercício exemplo

$\boxed{y' = -2xy}$

a) Caracterize

b) Determine a solução geral

a) EDO de 1ª ordem → A eq.ão só tem uma variável derivada "y" que é de 1° grau

EDO de variáveis separáveis

$y' = -2xy \Rightarrow \frac{dx}{dy} = -2xy \iff$

$\iff dy = -2xy\,dx \iff 2xy\,dx + dy = 0 \iff$

$\iff f_1(x) g_1(y) dx + f_2(x) g_2(y) dy = 0$

com $f_1(y) = 2x$, $g_1(y) = y$

$f_2(y) = g_2(y) = 1$

b) $y(x) = ?$

1° Passo: Fixar fator integrante

$\mu(x, y) = 1/y$

2° Passo: $\frac{2x\cancel{y}\,dx}{\cancel{y}} + \frac{dy}{y} = \frac{0}{y} \iff$

$\iff 2x\,dx + \frac{dy}{y} = 0$

3° Passo: Integrar

$\int 2x\,dx + \int \frac{dy}{y} = \int 0\,dx \iff$

$\iff x^2 + \ln(y) = C$

$\boxed{\text{obter função } y}$

$\ln|y| = C - x^2 \iff$

$\iff |y| = e^{C - x^2} \iff$

$\iff y = C e^{-x^2}$

$$y' = P(x)y = Q(x)$$

1º Passo: Homogénea

2º Passo: Método do fator Integrante

$$y = e^{-\int P(x)dx} * \left[\int e^{\int P(x)dx} * Q(x)dx + C\right]$$

→ Exemplo

$y + y' = x \Leftrightarrow y' + P(x)y = Q(x)$ com $P(y) = 1$ e $Q(x) = x$

$$y = e^{-\int 1 dx} * \left[\int e^{\int 1 dx} * x\, dx + C\right], C \in \mathbb{R} \Leftrightarrow$$

$$\Leftrightarrow y = e^{-x}\left[\int e^{x} * x\, dx + C\right], C \in \mathbb{R} \Leftrightarrow$$

$$\Leftrightarrow y = e^{-x}(e^{x}x - e^{x} + C) \Leftrightarrow$$

$$\Leftrightarrow y = x - 1 + Ce^{-x}$$

(2) $\left(i(0) = \frac{609}{101} + \frac{3}{101} = \frac{619}{101} \neq 6\right)$

e → força eletromotriz

i → intensidade

R → Resistência, R = 10 Ω

L → indutância, de = 0,5

EDO

$$e = Ri + L\frac{di}{dt}, i(t) = ?$$

Pressuposto

$e = e(t) = 3\,\text{sen}(2t)$ → logo, depende de t

condição inicial $i(0) = 6$

. Validar se:

$$i(t) = \frac{609}{101}e^{-20t} - \frac{30}{101}\text{sen}(2t) + \frac{3}{101}\cos(2t)$$

$$\boxed{3\,\text{sen}(2t) = 10i + 0{,}5\frac{di}{dt}, i(0) = 6}$$

Usando $i(t)$ dado, calcular

① $\frac{di}{dt} = ?$

② $10i + 0,5\frac{di}{dt} = ?$

$$i(t) = \left(\frac{609}{101}e^{-20t} - \frac{30}{101}\operatorname{sen}(2t) + \frac{3}{101}\cos(2t)\right)$$

$$= \left(\frac{609}{101}(-20)e^{-20t} - \frac{60}{101}\cos(2t) - \frac{6}{101}\operatorname{sen}(2t)\right)$$

$$= -\frac{12180}{101}e^{-20t} - \frac{60}{101}\cos(2t) - \frac{6}{101}\operatorname{sen}(2t)$$

Aplicar fórmula

$$0.5\frac{di}{dt} = \frac{-6090}{101}e^{-20t} - \frac{30}{101}\operatorname{sen}(2t) + \frac{3}{101}\operatorname{sen}(2t)$$

$$\rightarrow 10i + 0,5\frac{di}{dt} = 10 \times \left(\frac{609}{101}e^{-20t} - \frac{30}{101}\operatorname{sen}(2t) - \frac{3}{101}\cos(2t)\right)$$

$$-\frac{6090}{101}e^{-20t} - \frac{30}{101}\cos(2t) - \frac{3}{101}\operatorname{sen}(2t)$$

$$= \cancel{\frac{6090}{101}e^{-20t}} - \frac{300}{101}\operatorname{sen}(2t) - \cancel{\frac{6090}{101}e^{-20t}} + \frac{30}{101}\cos(2t) - \frac{3}{101}\operatorname{sen}(2t)$$

$$= -\frac{303}{101}\operatorname{sen}(2t) = -3\operatorname{sen}(2t)$$

Proposição Falsa $\Rightarrow$ $i(t)$ não é particular da EDO

$$3\operatorname{sen}(2t) \neq 10i + 0,5\frac{di}{dt}$$

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n

$$F(x, y, y') = \phi \iff y' = f(x,y)$$

$$y' = f(x,y) \qquad y = G(x; c)$$

→ PVI

$y' = f(x,y)$ ED

$x \in [a,b]$ VI

$y_a = y_0$ CI (Condição Inicial)

$y_0 = y_0 \iff x = a \wedge y = y_0$

1º Passo: Determinar solução geral de $y' = f(x,y)$, $y = f(x;c)$

2º Passo Determinar c de $y = f(x;c)$

$y(a) = y_0 \rightarrow C = ?$

3º Passo

$y(a) = y_0 \rightarrow C = ?$ $\quad y = (x;c)$

$y = f(x)$

Exemplo

$$\begin{cases} y' = -2x & ED \\ I \in [0; 1,5] & VI \\ y(0) = 1 & CI \end{cases}$$

1º Passo

EDV separaveis $y = Ce^{-x^2}, C \in \mathbb{R}$

2º Passo $C = ?$

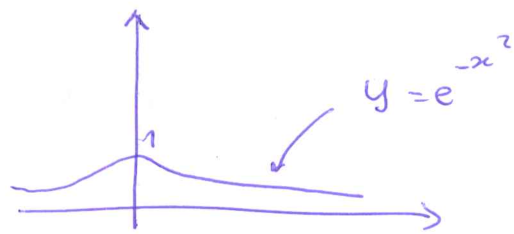
$y(0) = 1 \Leftrightarrow x = 0 \wedge y = 1$

Substituir

$1 = Ce^{-0^2} \Leftrightarrow 1 = Ce^0 \Leftrightarrow 1 = C$

3º Passo

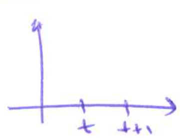
$y = 1e^{-x^2} \Leftrightarrow y = e^{-x^2} \longrightarrow$ solução particular de P



→ Métodos Numéricos para Eq. Dif. Ord. / PVI

$$(P) \begin{cases} y' = f(t,y) & ED \\ t \in [a,b] & VI \\ y(a) = y(0) & CI \end{cases}$$

1º Passo Descritização Regular de $t \in [a,b]$



(Partição Regular) → $t_0 = a$, $t_1 = t_0 + h$; $t_x = x_i + h$

$t(t_i) = t_i + h$; $t_m = t_{m-1}$; $t_h = b$

$h = \frac{b-a}{m}$, amplitude

$h = t_{i+1} - t_i \Leftrightarrow t_{i+1} = t_i + h$

2º Passo $y(i+1) = ? \Leftrightarrow t = t_{i+1} \wedge y = ?$

Aplicar fórmula de um dos métodos

$\boxed{\text{Tabelas}}$
- Euler
- Euler Melhorado/Modificado
- Runge Kutta 2/4 (RK2/RK4)
- ODE45
- ...

$$y_{i+1} = y_i + h \times f(t_i, y_i)$$

$$i = 0, 1, 2, n$$

Deduzir fórmula de Euler

$$t = t_{i+1}$$

$$y = y_i + (t_{i+1} - t_1)\ f(t_i, y_i) \Leftrightarrow$$

$$\Leftrightarrow y = y_i + h \times f(t_i, y_i) \Leftrightarrow$$

$$\Leftrightarrow y_{i+1} = y_i + h \times f(t_i, y_i)$$

1º Algoritmo Euler

- Input: f, a, h, n, y_0
- Output: y $\quad \boxed{y = y_0 \mid y_1 \mid \ldots \mid y_n}$

- $h \leftarrow (b-a)/n$

$t(1) \leftarrow a$

$y(1) \leftarrow y_0$

Para $i = 1$ até n

$y(i+1) = y(i) + h \times f(t(i), y(i))$

$t(i+1) = t(i) + h(i)$

Fim

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a) $\frac{dy}{dx} - yx^2 = -y \Leftrightarrow dy - yx^2\, dx = -y\, dx \Leftrightarrow$

$dy - yx^2\, dx + y\, dx = 0 \Leftrightarrow dy + y(-x^2+1)\, dx = 0 \Leftrightarrow$

$\Leftrightarrow dy - yx^2\, dx + (-x^2+1)\, dx = 0 \Leftrightarrow$

$\Leftrightarrow \frac{dy}{y} + (-x^2+1)\, dx = 0 \Leftrightarrow$

$\Leftrightarrow \int \frac{dy}{y} + \int -x^2+1 = \int 0 \Leftrightarrow \ln(y) + \frac{-x^3}{3} + 1 = 0 \Leftrightarrow$

$\Leftrightarrow y = e^{\frac{-x^3}{3} + 1}$

$(=) \frac{dy}{y} + (t^2-1)\,dx = 0 (=)$

$(=) \int \frac{dy}{y} + \int t^2-1 = \int 0 (=) \ln(y) + \frac{-x^3}{3} - x = 0 (=)$

$(=) \ln(y) = \frac{-x^3}{3} - x (=) y = e^{\frac{-x}{3} - x}$

No exercício

$6 = e^{-\frac{0^3}{3}} \times C (=) 6 = C$

4) $2y\,dx + (xy + 5x)\,dy = 0$

1º Passo : $\frac{1}{g_1(y) \times f_2(x)} (=) \frac{1}{2yx} (=)$

$(=) \frac{\cancel{2y}\,dx}{\cancel{2y}\,x} + \frac{(\cancel{x}(y+5))\,dy}{2y\cancel{x}} = 0 (=)$

$(=) \frac{dx}{x} \frac{(y+5)dy}{2y} = \frac{1}{x} dx + \frac{(y+5)\,dy}{2y} (=)$

$(=) \int \frac{1}{x} dx + \int \frac{(y+5)}{2y} dy = 0 (=)$

$(=) \ln(x) + 1/2 \int \frac{y}{y} + \frac{5}{y} dy = \int 0 (=)$

$(=) \ln(x) + 1/2 \int 1 dy + 1/2 \int \frac{5}{y} dy = 0 (=)$

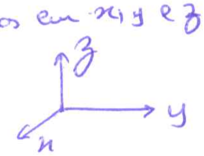
$(=) \ln(x) + 1/2 \int 1 dy + 5/2 \int \frac{1}{y} dy = \int 0 (=)$

$(=) \ln(x) + y/2 + \frac{5\ln(y)}{2} = 0 (=)$

$(=) 2\ln(x) + y + 5\ln(y)$

$f : D \in \mathbb{R}^2 \longrightarrow \mathbb{R}$
$(x,y) \rightsquigarrow z = f(x,y)$

Consideramos coordenadas em x, y e z

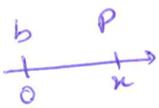


$\mathbb{R}^n = \mathbb{R} \times \mathbb{R} \times \dots \times \mathbb{R}$ $\quad x = (x_1, x_2, \dots, x_n)$

$\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$

1º objetivo : Ponto e Distância

Caso $\mathbb{R}$



$d(b,p) = d(p,b) = d(\overline{bp}) = |p-b| = |x-0| = |x| = \sqrt{x^2}$

Definição

$d(P,Q) = \sqrt{(x_1-y_1)^2 + (x_2-y_2)^2 + \dots + (x_n - y_n)^2}$

A Origem : $d = \sqrt{x^2+y^2}$

No Plano 2D : $d(P, Q) = ?$

$$d(P,Q) = \|\overline{PQ}\|$$
$$= \|(x_2, y_2) - (x_1, y_1)\|$$
$$= \|(x_2 - x_1, y_2 - y_1)\|$$
$$= d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$= d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

No Plano 3D : $d(O,P) = d(P,O)$

$$= \|\overline{OP}\|$$
$$= \|P - O\|$$
$$= \|(x, y, z) - (0, 0, 0)\|$$
$$= \|(x-0, y-0, z-0)\|$$
$$= d^2 = x^2 + y^2 + z^2$$
$$= d = \sqrt{x^2 + y^2 + z^2} \quad \text{(ORigem)}$$

Fórmula Geral

$$d(P,Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Análise Matemática II

Resumo
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2021/22 - ISEC - LEI

→ Resumo Interpolação Polinomial

Definição de Polinómio Interpolador

- $f(x) \approx P_m(x)$, $f \in C([a,b])$; $x_i \in [a,b]$ $(i = 0, 1, \dots, m) \rightarrow P_m(x_i) = f(x_i)$

- $P_m(x)$ aproxima $f(x)$, que é uma função desconhecida, apenas temos acesso a alguns pontos de $f(x)$.

Métodos de I.P.

- Interpolação Quadrática ($f(x)$ e $P_m(x)$) de ordem 2
- Interpoladore de Newton das diferenças divididas
- Séries de Taylor
- Interpolação de Lagrange

Interpolação Quadrática

→ Dados

Passo

Intervalo a a b $[a,b] = [-1;1]$

x_i	-1	0	1
$f(x)$	6	3	2

função desconhecida

$\Rightarrow \begin{cases} 3 \text{ pontos} \\ 3 \text{ interseções} \\ 3 \text{ equações} \end{cases} \rightarrow$ 1 sist. equações

$i = 0 = P_2(x_0) = f(x_0) \Leftrightarrow a_0 + a_1(-1) + a_2(-1)^2 = 6$

$\Leftrightarrow a_0 - a_1 + a_2 = 6$

$i = 1 = P_2(x_1) = f(x_1) \Leftrightarrow a_0 + a_1(0) + a_2(0)^2 = 3$

$\Leftrightarrow a_0 = 3$

$i = 2 \quad P_2(x_2) = f(x_2) \Leftrightarrow a_0 + a_1(1) + a_2(1)^2 = 2$

$\Leftrightarrow a_0 + a_1 + a_2 = 2$

$$\begin{cases} a_0 - a_1 + a_2 = 6 \\ a_0 = 3 \\ a_0 + a_1 + a_2 = 2 \end{cases}$$

Resolvendo o sistema...

$$\begin{cases} a_0 = 3 \\ a_1 = -2 \\ a_2 = 1 \end{cases} \Rightarrow \boxed{P_2(x) = x^2 - 2x + 3} \rightarrow \text{Polinómio que aproxima } f(x)$$

$$P_n(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \dots + a_n(x-x_0)(x-x_1)\cdots(x-x_{n-1})$$

1ª iteração

- $x = x_0$
- $P_n(x_0) = a_0 + \cancel{a_1(x_0-x_0)}^{0} + \cancel{a_2(x_0-x_0)(x_0-x_1)}^{0} + \dots + \cancel{a_n(x_0-x_0)\cdots(x_0-x_{n-1})}^{0}$
- $f(x_0) = a_0$

2ª iteração

$x = x_1$

$$P_n(x_1) = a_0 + a_1(x_1-x_0) + a_2(x_1-x_0)\cancel{(x_1-x_1)}^{0} + \dots + a_n(x_1-x_0)\cancel{(x_1-x_1)}^{0}$$

$$f(x_1) = a_0 + a_1(x_1-x_0)$$

e quantas vezes necessárias

$$f(x_1) = a_0 + a_1(x_1-x_0) \Leftrightarrow a_1 = \frac{f(x_1)-a_0}{x_1-x_0}$$

$$\Leftrightarrow a_1 = \frac{f(x_1)-f(x_0)}{x_1-x_0} \rightarrow \text{diferença dividida} \rightarrow a_1 = f[x_0, x_1]$$

$\boxed{\text{Logo}}$ $$f[x_i, x_{i+1}] = \frac{f(x_{i+1})-f(x_i)}{x_{i+1}-x_i}$$

Tabela das diferenças divididas

$$f[0,1] = \frac{f[1]-f[0]}{1-0}$$

x_i	$f(x_i)$	D_1	D_2	D_3
(x_0) 0	$f[0]$			
		$f[0,1]$		
(x_1) 1	$f[1]$		$f[0,2]$	
		$f[1,2]$		$f[0,3]$
(x_2) 2	$f[2]$		$f[1,3]$	
		$f[2,3]$		
(x_3) 3	$f[3]$			

$$\frac{f[1,3]-f[0,2]}{3-0}$$

$$\frac{f[2,3]-f[1,2]}{3-1}$$

$$P_2 = f[0] + f[0,1](x-x_0) + f[0,2](x-x_0)(x-x_1) + f[0,3](x-x_0)(x-x_1)(x-x_2)$$

1 ▶ Análise Matemática II - Aulas a limpo

Apontamentos
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◦ Teórico-Prática - 19/04/2022

→ EDO lineares ordem 2 com coeficientes constantes

$$\boxed{a_2 y'' + a_1 y' + a_0 y = b(x)} \longrightarrow \text{Equação Característica}$$

- $b(x) = 0 \longrightarrow$ homogénea
- $b(x) \neq 0 \longrightarrow$ completa

→ Exemplo/Exercício

$$y'' - 2y' + y = 1$$

① Resolver equação homogénea

$$\boxed{y'' - 2y + y = 0}$$

Fórmula Resolvente 2º grau

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

binómio discriminante

$$(\lambda - 1)^2 = 0$$

1a) Eq. caract.

$a_2\lambda^2 + a_1\lambda + a_0 = 0$: $\lambda^2 - 2\lambda + 1 = 0$

$$\lambda^2 - 2\lambda + 1 = 0 \Leftrightarrow \lambda = \frac{-(-2) \pm \sqrt{(-2)^2 - 4\times1\times1}}{2\times1}$$

$$\Leftrightarrow \lambda = 1$$

com multiplicidade 2 //

$m = 2$

1b)

Real λ	$m = 1$	$e^{\lambda x}$
	$m > 1$	$x^j e^{\lambda x}$, $0 \leq j \leq m-1$
complexa $\lambda_1 + \lambda_2 i$	$m = 1$	$e^{\lambda_1} \operatorname{sen}(\lambda_2 x)$, $e^{\lambda_1} \cos(\lambda_2 x)$
	$m > 1$	$x^j e^{\lambda x} \operatorname{sen}(\lambda_2 x)$, $x^j e^{\lambda x} \cos(\lambda_2 x)$, $0 \leq j \leq m-1$

$\lambda = 1$ com $m = 2$

$\Rightarrow$ S.F.S. $= \{x^0 e^x, x^1 e^x\}$

$= \{e^x, x e^x\}$

$\quad y_1(x) \quad y_2(x)$

Sistema fundamental de equações

1 opç. verificar independência linear do SFS

wronskiano ← $W(y_1, y_2) = w(e^x, xe^x) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^x & xe^x \\ e^x & e^x + xe^x \end{vmatrix}$

$= e^x(e + xe^x) - e^x \times xe^x = e^{2x} + \cancel{xe^{2x}} - \cancel{xe^{2x}} =$

$= e^{2x} \neq 0 \Rightarrow y_1$ e y_2 são linearmente indep.

$$y_H = C_1 y_1(x) + C_2 y_2(x$$
$$= C_1 e^x + C_2 x e^x$$

② Determinar solução particular

$$y_p = C_1(x) y_1(x) + C_2(x) y_2(x)$$

2a) Sistema de Lagrange

$$\begin{cases} C_1'(x) y_1(x) + C_2'(x) y_2(x) = 0 \\ C_1'(x) y_1'(x) + C_2'(x) y_2'(x) = b(x) \end{cases}$$

$$\begin{cases} C_1'(x) e^x + C_2'(x) x e^x = 0 \\ C_1'(x) e^x + C_2'(x)(e^x + x e^x) = 1/1 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{array}{cc} C_1'(x) & C_2'(x) \end{array}$$
$$\begin{bmatrix} e^x & x e^x \\ e^x & e^x + x e^x \end{bmatrix} \begin{bmatrix} C_1'(x) \\ C_2'(x) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Regra de Cramer

$$C_1'(x) = \frac{\begin{vmatrix} 0 & x e^x \\ 1 & e^x + x e^x \end{vmatrix}}{W(y_1, y_2)}, \quad C_2'(x) = \frac{\begin{vmatrix} e^x & 0 \\ e^x & 1 \end{vmatrix}}{W(y_1, y_2)}$$

$$= \frac{-x e^x}{e^{2x}}, \quad = \frac{e^x}{e^{2x}}$$

$$= -\frac{x}{e^x}, \quad = \frac{1}{e^x}$$

2b) $\cdot C_1(x) = \int C_1'(x)\,dx =$

$$= \int -x/e^x\,dx = \int x(-e^{-x})\,dx =$$

$$= \int -e^{-x}\,dx \times x - \int \int -e^{-x}\,dx \times x'\,dx =$$

$$= e^{-x} \times x - \int e^{-x}\,dx = x e^{-x} + \int -e^{-x}\,dx =$$

$$= x e^{-x} + e^{-x} + \emptyset \quad (C = \emptyset)$$

$$\cdot C_2(x) = \int C_2'(x)\,dx = \int \frac{1}{e^x}\,dx = \int e^{-x}\,dx = -\int -e^{-x}\,dx =$$

$$= -e^{-x} + \emptyset \quad (C = \emptyset)$$

2.c) Solução Particular

$$y_p = (x e^{-x} + e^{-x}) e^x + (-e^{-x})(x e^x) =$$

$$= x e^{-x} e^x + e^{-x} e^x - x e^{-x} e^x = x + 1 - x = 1$$

③ Solução da eq. completa

$$y = y_H + y_p = \boxed{C_1 e^x + C_2 x e^x + 1}$$

• Teórico-Prática (26/04/2022)

→ PVI

$$\begin{cases} y' = f(t,y) \\ t \in [a,b] \\ y(\theta) = y_0 \end{cases}$$

→ Método de Euler

$$\begin{cases} h = \frac{b-a}{n} \\ \begin{cases} t_0 = a \\ t_{i+1} = t_i + h \end{cases}, \quad i = 0, \dots, m-1 \end{cases}$$

$y_{i+1} = y_i + h f(t_i, h_i)$

$i = 0, \dots, m-1$

→ PVI Sist. eq. dif. (SED)

$$\begin{cases} u' = f(t,u,v) \\ v' = g(t,u,v) \\ t \in [a,b] \\ u(a) = u_0 \\ v(a) = v_0 \end{cases}$$

$\boxed{\text{Método Euler SED}}$

$u_{i+1} = u_i + h f(t_i, u_i, v_i)$, $i = 0, \dots, m-1$

$v_{i+1} = v_i + h g(t_i, u_i, v_i)$

$\boxed{\text{Algoritmo Euler SED}}$

In: f, g, a, b, m, u_0, v_0

Out: u_i, v_i

$t \leftarrow a : (b-a)/m : b$

$u \leftarrow \text{zeros}(1, m+1)$

$v \leftarrow u$

$u_1 \leftarrow u_0$

$v_1 \leftarrow v_0$

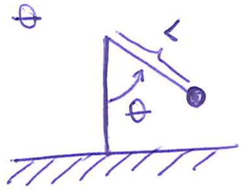
Para $i \leftarrow 1 : m$

$u(i+1) \leftarrow u(i) + h * f(t(i), u(i), v(i))$

$v(i+1) \leftarrow v(i) + h * f(t(i), u(i), v(i))$

→ Problema do Pêndulo

• objetivo: Determinar o deslocamento angular θ



m = massa

L = comp. linha

c = coef. amortec.

g = const. grav.

$$\theta'' + \frac{c}{mL}\theta' + \frac{g}{L}\operatorname{sen}\theta = 0$$

→ ordem 2

(zero) → não linear

$$\boxed{a_2 y'' + a_1 y' + a_0 y = b(t) \rightsquigarrow \text{SED}}$$

① Isolar y''

$a_2 y'' + a_1 y' + a_0 y = b(t)$

$\Leftrightarrow y'' = \frac{1}{a_2} b(t) - \frac{a_1}{a_2} y' - \frac{a_0}{a_2} y$

$\Leftrightarrow y'' = B(t) + a_4 y' + a_3 y$

② Mudança variáveis

$$\begin{cases} u = y \\ v = y' \end{cases}$$

③ Derivar

$$\begin{cases} u' = y' \\ v' = y'' \end{cases}$$

$u' = v$

PVI

$y(a) = \beta$

$y'(a) = \alpha$

④ Definir f e g

$f(t,u,v) = v$

$g(t,u,v) = B(t) + a_4 v + a_3 u$

$\Downarrow$

$u' = f(t,u,v)$

$v' = g(t,u,v)$

$u(a) = y(a) = \beta$

$v(a) = y'(a) = \alpha$

PVI SED

$$\begin{cases} u' = f(t,u,v) \\ v' = g(t,u,v) \end{cases} \quad \begin{cases} u(a) = \beta \\ v(a) = \alpha \end{cases} \Rightarrow \text{Métodos Numéricos}$$

→ **Resolução do Problema do Pêndulo**

$$\theta'' = \boxed{\frac{c}{mL}}\,\theta' + \boxed{\frac{g}{L}}\,\text{sen}\,\theta = \emptyset$$

$\frac{c}{mL} \to 0,3$ $\qquad$ $\frac{g}{L} \to 1$

Suposições

① $t \in [\emptyset, 15]$

② $\theta = y$ $\quad \begin{cases} y(0) = \theta(0) = \pi/2 \\ y'(0) = \theta'(0) = \emptyset \end{cases}$

③ PVI

$$1 \times y'' + 0,3\,y' + 1 \times y = 0$$

$\boxed{\text{Isolar } y''}$ ①

$$y'' = -0,3\,y' - y$$

$\boxed{\text{Mudar variáveis}}$ ② $\to \begin{cases} u = y \\ v = y' \end{cases}$

$\boxed{\text{Derivar}}$ ③ $\quad \begin{cases} u' = y' \\ v' = y'' \end{cases} \Leftrightarrow \begin{cases} u' = v \\ v' = -0,3v - u \end{cases}$

$\boxed{\text{Definir } f \text{ e } g}$ $\quad \begin{cases} f(t,u,v) = v \\ g(t,u,v) = -0,3v - u \end{cases}$

$\boxed{\text{Valores Iniciais}}$

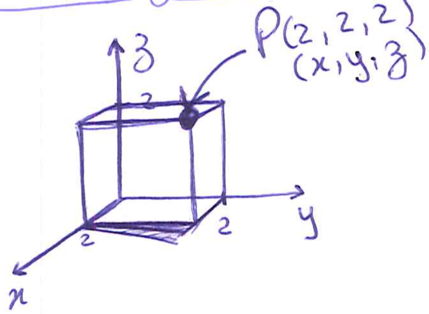
$$\begin{cases} u(0) = \pi/2 \\ v(0) = 0 \end{cases}$$

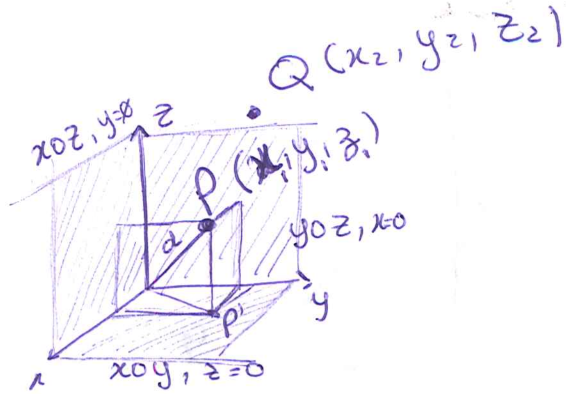
$\boxed{\text{PVI SED}}$

$$\begin{cases} u' = v \\ v = -0,3v - u \end{cases} \quad \begin{cases} u(0) = \pi/2 \\ v(0) = 0 \end{cases}$$

Teórica - 27/04/2022

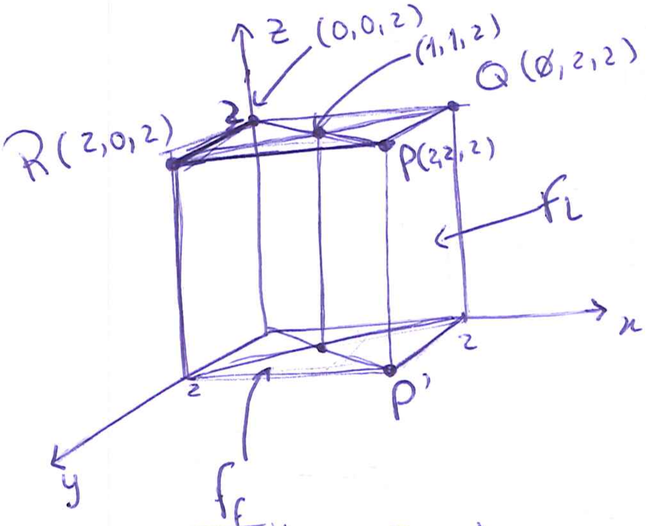
→ Cubo Mágico





$$\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$$

- $d(O,P) = \sqrt{(x_1-0)^2 + (y_1-0)^2 + (z_1-0)^2}$
  $= \sqrt{x_1^2 + y_1^2 + z_1^2}$
- $d^2 = x_1^2 + y_1^2 + z_1^2$
- $d = \sqrt{x_1^2 + y_1^2 + z_1^2}$
- $d(P,Q) = \|\overline{PQ}\| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

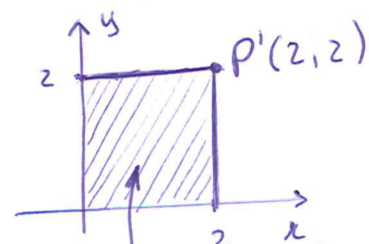


$$\boxed{P(x,y,z)}$$

- $f_s = \{(x,y,z) \in \mathbb{R}^3 : (x,y) \in D \wedge z = 2\}$

face superior do cubo

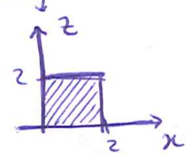
$\Leftrightarrow f_s = \{(x,y,z) \in \mathbb{R}^3 : 0 \leq x \leq 2 \wedge 0 \leq y \leq 2 \wedge z = 2\}$



$D = \{(x,y) \in \mathbb{R}^2 : 0 \leq x \leq 2 \wedge 0 \leq y \leq 2\}$

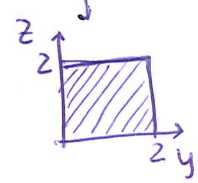
- $F_L = \{(x,y,z) \in \mathbb{R}^3 : 0 \leq x \leq 2 \wedge y = 2 \wedge 0 \leq z \leq 2\}$

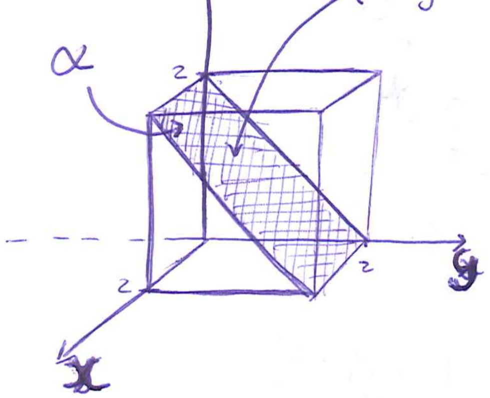
face lateral do cubo



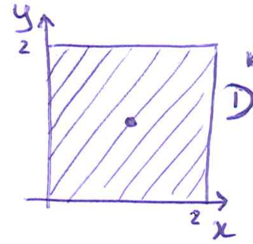
- $F_F = \{(x,y,z) \in \mathbb{R}^2 : x = 2 \wedge 0 \leq z \leq 2 \wedge 0 \leq y \leq 2\}$

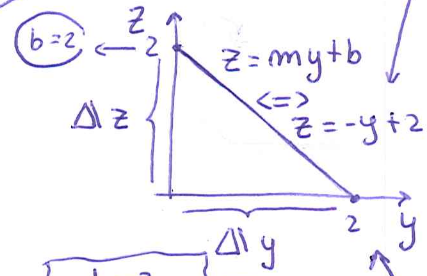
face frontal do cubo





$$\alpha = \{(x,y,z) \in \mathbb{R}^2 : (x,y) \in D \wedge z = -y+2\}$$



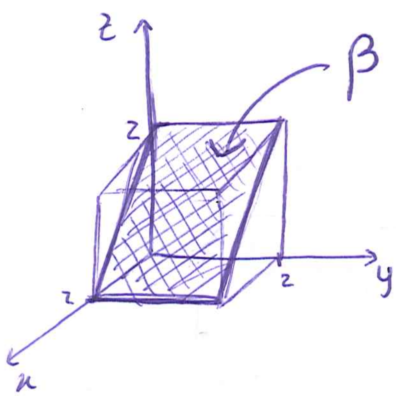


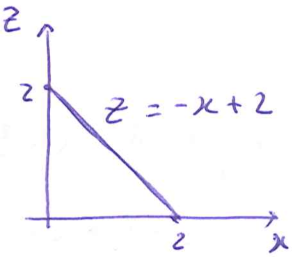
- $b = 2$
- $m = -\frac{\Delta z}{\Delta y} = -2/2$
- $z = -y+2$

Projeção no plano yOz

ou seja

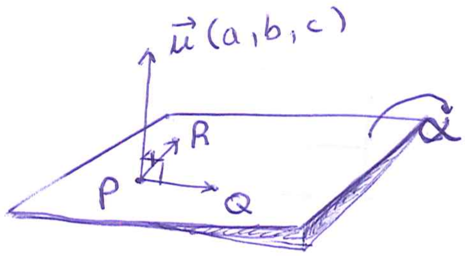
$$\alpha = \{(x,y,z) \in \mathbb{R}^3 : 0 \le x \le 2 \wedge 0 \le y \le 2 \wedge z = -y+2\}$$





ou seja

$$\beta = \{(x,y,z) \in \mathbb{R}^2 : 0 \le x \le 2 \wedge 0 \le y \le 2 \wedge z = -x+2\}$$



$$ax + by + cz + d = 0$$

$$\begin{cases} \vec{u} \cdot \overrightarrow{PQ} = 0 \\ \vec{u} \cdot \overrightarrow{PR} = 0 \end{cases} \Leftrightarrow \{ (\dots)\ \vec{u}(a,b,c)$$

Teórico - Prática - 10/05/2022

→ Derivação Numérica

$u \in C^m([a,b])$, m suficientemente grande, i.e., $m = 2, 3, 4$

x_K,
$[x_{K+1}]$ partição de $[a,b]$

- $\Delta x = \dfrac{b-a}{N}$, $N \geq 2$
- $x_i = a + i\Delta x$, $i = 0, \dots, N$

① Considerar o desenvolvimento em série de Taylor de u em torno de x_K

$$u(x) = \underbrace{u(x_K) + u'(x_K)(x - x_K)}_{P_1(x)} + \underbrace{u''(\xi_K)\frac{(x - x_K)^2}{2}}_{P_1(x)}$$

ξ_K entre x e x_K, $K = 0, \dots, N-1$

a) $x = x_{K+1}$

$m = 2$

$$u(x_{K+1}) = u(x_K) + u'(x_K)\overbrace{(x_{K+1} - x_K)}^{\Delta x} + u^{(11)}(\xi_K)\frac{(x_{K+1} x_K)^2}{2}$$

$$= u(x_K) + u'(x_K)\Delta x + u''(\xi_K)\frac{\Delta x^2}{2}$$

$$\Rightarrow u(x_{K+1}) - u(x_K) = u'(x_K)\Delta x + u''(\xi_K)\frac{\Delta x^2}{2}$$

$$\Rightarrow \boxed{\frac{u(x_{K+1}) - u(x_K)}{\Delta x} = u'(x_K)} + \underbrace{u''(\xi_K)\frac{\Delta x}{2}}_{\text{ERRO}}; \ \xi_K \in]x_K, x_{K+1}[, \ K = 0, \dots, N-1$$

$\cdot \frac{}{\Delta x}$

Nota
Não calculam $D_x u(b)$

Diferenças Progressivas

$$D_x(u_K) = \frac{u(x_{K+1}) - u(x_K)}{\Delta x}, \ K = 0, \dots, N-1$$

b) $m = 2$, $x = x_{K-1}$

$$u(x_{K-1}) = u(x_K) + u'(x_K)\overbrace{(x_{K-1} - x_K)}^{\Delta x} + u''(\xi_K)\left(\frac{x_{K-1} - x_K}{2}\right)^2$$

$$= u(x_K) - u'(x_K)\Delta x + u''(\xi_K)\frac{\Delta x^2}{2}$$

$$\Rightarrow \boxed{\frac{u(x_K) - u(x_{K-1})}{\Delta x} = u'(x_K)} - u''\frac{\Delta x}{2}, \ \xi_K \in (x_{K-1}, x_K), \ K = 1, \dots, N$$

Diferenças Regressivas

$D_{-x} u(x_K)$ ...

Nota
Não calculam [redacted] $D_{-x} u(a)$

m = 3

$$x = x_{k+1} \to u(x_{k+1}) = u(x_k) + u'(x_k)\Delta x + u''(x_k)\frac{\Delta x^2}{2} + u'''(\xi_k)\frac{\Delta x^3}{6}, \quad \xi \in]x_k, x_{k+1}[, \quad k=0,\dots,N-1$$

$$x = x_{k-1} \to \ominus\, u(x_{k-1}) = u(x_k) - u'(x_k)\Delta x + u''(x_k)\frac{\Delta x^2}{2} - u'''(\eta_k)\frac{\Delta x^3}{6}, \quad \eta \in]x_{k-1}, x_k[, \quad k=1,\dots N$$

$$u(x_{k+1}) \ominus u(x_{k-1}) = 2u'\boxed{(x_k)\Delta x} + \frac{\Delta x^3}{6}\left(u'''(\xi_k) - u'''(\eta_k)\right)$$

$$\Rightarrow \boxed{\frac{u(x_{k+1}) - u(x_{k-1})}{2\Delta x} = u'(x)} + \frac{\Delta x^2}{12}\left(u'''(\xi_k) + u'''(\eta_k)\right), \quad \xi_k, \eta_k \in]x_{k-1}, x_{k+1}[, \quad k=1,\dots N-1$$

$\downarrow$

Diferenças centradas

Nota

Não calcula $D_c u(a)$ nem $D_c u(b)$

3 m = 4

$$u(x) = u(x_k) + u'(x_k)(x-x_k) + u''(x_k)\frac{(x-x_k)^2}{2} + u'''(x_k)\frac{(x-x_k)^3}{3!} + u^{(4)}(\xi_k)\frac{(x-x_k)^4}{4!}$$

$$x = x_{k+1} \to u(x_{k+1}) = u(x_k) + u'(x_k)\Delta x + u''(x_k)\frac{\Delta x^2}{2} + u'''(x_k)\frac{\Delta x^3}{6} + u^{(4)}(\xi_k)\frac{\Delta x^4}{24}$$

$$x = x_{k-1} \to \oplus\, u(x_{k+1}) = u(x_k) - u'(x_k)\Delta x + u''(x_k)\frac{\Delta x^2}{2} - u'''(x_k)\frac{\Delta x^3}{6} + u^{(4)}(\eta_k)\frac{\Delta x^4}{24}$$

$$u(x_{k+1}) \oplus u(x_{k-1}) = 2u(x_k) + u''(x_k)\Delta x^2 + \frac{\Delta x^4}{24}\left(u^{(4)}(\xi_k) + u^{(4)}(\eta_k)\right)$$

$$\boxed{\frac{u(x_{k+1}) - 2u(x_k) + u(x_{k-1})}{\Delta x^2} = u''(x_k)} + \frac{\Delta x^2}{24}\left(u^{(4)}(\xi_k) + u^{(4)}(\eta_k)\right)$$

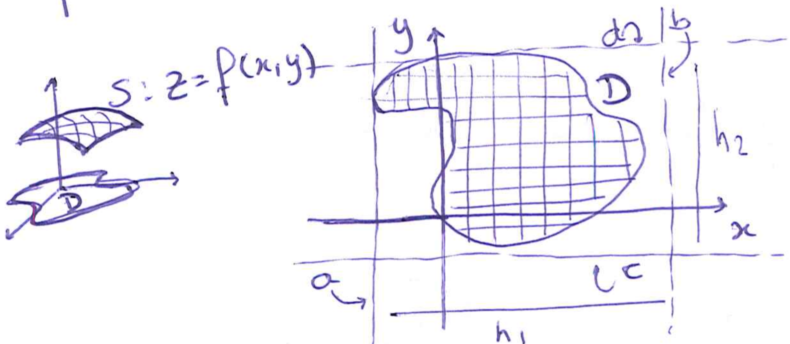
$\downarrow$

$D_2 u(x_k)$

Teórica - 11/05/2022

1▶ Função Real de 2 variáveis Reais

$$f : D \subset \mathbb{R}^2_{(x,y)} \longrightarrow \mathbb{R}$$
$$\longrightarrow z = f(x,y)$$



$$\boxed{f(x,y) = \sqrt{32-x^2-y^2}} \quad (1)$$

· Passo Ø1 : Caracterização de (1)

→ f é uma função irracional cujo operador principal é a Raiz quadrada

· Passo Ø2

· Processo 1 : caminho + curto

Δ!

$$f(x,y) = \sqrt{32-x^2-y^2} \Leftrightarrow \boxed{z = \sqrt{32-x^2-y^2} \ (2)} \Leftrightarrow z^2 = \left(\sqrt{32-x^2-y^2}\right)^2$$

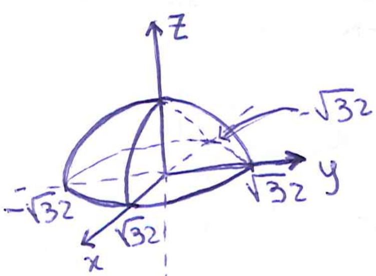
$$\Leftrightarrow z^2 \overset{!!}{=} 32-x^2-y^2 \Leftrightarrow \boxed{x^2+y^2+z^2 = 32 \ (3)} \Leftrightarrow x^2+y^2+z^2 = \left(\sqrt{32}\right)^2$$

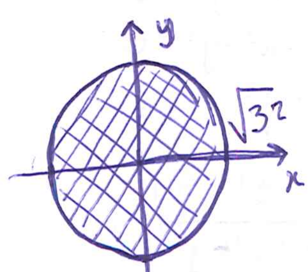
$$\Leftrightarrow \boxed{x^2+y^2+z^2 = R^2 \text{ com } R = \sqrt{32} \ (4)}$$

→ Equação da superfície esférica centrada na origem e Raio $\sqrt{32}$

→ Domínio de f

1. Graficamente





$$D = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 32\}$$

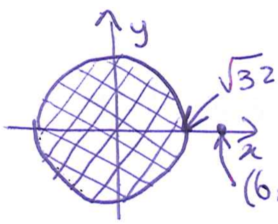
- $f(x,y) = \sqrt{32-x^2-y^2}$
- $Df = \{(x,y) \in \mathbb{R}^2 : 32-x^2-y^2 \geq 0\}$

$$32-x^2-y^2 \geq 0 \Leftrightarrow -x^2-y^2 \geq -32 \Leftrightarrow x^2+y^2 \leq 32$$

$\rightarrow Df = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 32\}$

$\underbrace{f(6,0)}_{\text{"Pertence ao Domínio?"}} = 6^2 + 0^2 = \underbrace{36 > 32}_{\text{Não verifica condição } x^2+y^2 \leq 32}$

$\rightarrow \sqrt{32-6^2-0^2} = \sqrt{\underbrace{32-36}_{<\emptyset}}$ X Não é possível



- Passo Ø2
  - Processo Ø2: Processo "mais Longo"

- $f(x,y) = \sqrt{32-x^2-y^2}$
- Algoritmo de Esboço 3D

→ Passo Ø1: Domínio de f

$Df = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 32\}$

→ Passo Ø2: Curvas de nível

$C_\emptyset = \{(x,y) \in Df : f(x,y) = \emptyset\}$

$f(x,y) = 0 \Leftrightarrow \sqrt{32-x^2-y^2} = 0$
$\Leftrightarrow 32-x^2-y^2 = 0$
$\Leftrightarrow \underline{x^2+y^2 = 32}$

$C_4 = \{(x,y) \in Df : f(x,y) = 4\}$

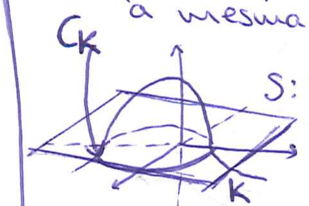
$f(x,y) = 4 \Leftrightarrow \sqrt{32-x^2-y^2} = 4$
$\Leftrightarrow 32-x^2-y^2 = 16$
$\Leftrightarrow \underline{x^2+y^2 = 16}$

$C_{\sqrt{32}} = \{(x,y) \in Df : f(x,y) = \sqrt{32}\}$

$f(x,y) = \sqrt{32} \Leftrightarrow \sqrt{32-x^2-y^2} = \sqrt{32} \Leftrightarrow x^2+y^2 = \emptyset$ $\quad (x,y) = (0,0)$

Nota

— Curvas de nível —
Linhas que unem pontos à mesma cota

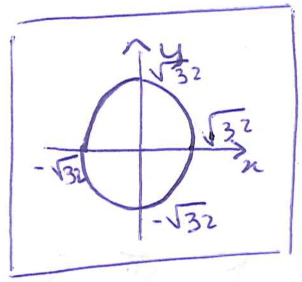


- $C_K = \begin{cases} z = f(x,y) \\ z = k \end{cases} \Leftrightarrow f(x,y) = k$
- $C_K = \{(x,y) \in Df : f(x,y) = k\}$

Passo 03 : Interseções com planos cartesianos

i) XOY, $z = 0$

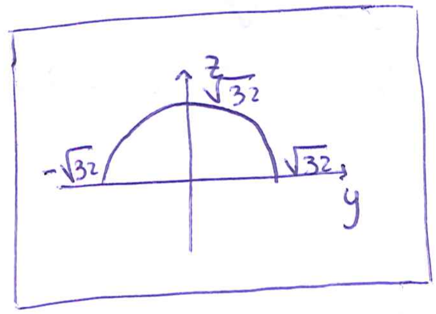
$$\begin{cases} z = f(x,y) \\ z = 0 \end{cases} \equiv C_0 : x^2+y^2=32$$



ii) YOZ, $x = 0$

$$\begin{cases} z = f(x,y) \\ x = 0 \end{cases} \Leftrightarrow \begin{cases} z = \sqrt{32-x^2-y^2} \\ x = 0 \end{cases}$$

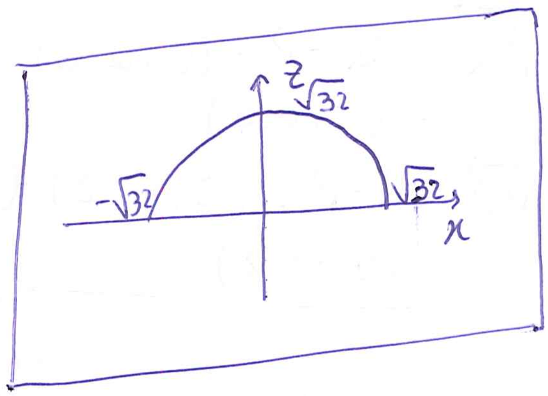
$$\Leftrightarrow \begin{cases} z = \sqrt{32-y^2} \\ x = 0 \end{cases}$$



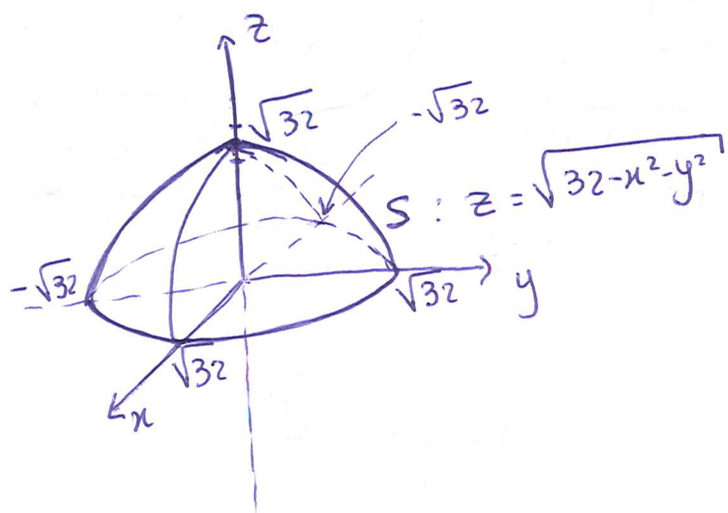
iii) XOZ, $y = 0$

$$\begin{cases} z = f(x,y) \\ y = 0 \end{cases} \Leftrightarrow \begin{cases} z = \sqrt{32-x^2-y^2} \\ y = 0 \end{cases}$$

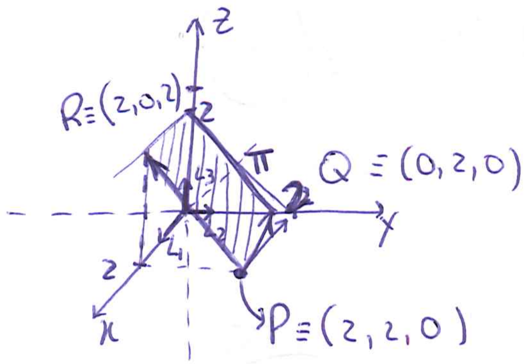
$$\Leftrightarrow \begin{cases} z = \sqrt{32-x^2} \\ y = 0 \end{cases}$$



Passo 04



▶ $\mathbb{R}^3$: Espaço



① Equação vetorial

Passam por um ponto

$$(x,y,z)\in\pi,\ (x,y,z) = \begin{vmatrix}P\\R\\Q\end{vmatrix} + \alpha\overrightarrow{PQ} + \beta\overrightarrow{PR},\quad \alpha,\beta\in\mathbb{R}$$

$$\boxed{ax+by+cz+d=0}$$ → Equação Cartesiana (1)

② Vetor Diretor de um Plano: ($\vec{u}\neq 0$)

- $\overrightarrow{PQ} = Q - P = (0,2,0)-(2,2,0) = (-2,0,0)$
- $\overrightarrow{PR} = R - P = (2,0,2)-(2,2,0) = (0,-2,2)$

Nota

– vetor Diretor de um Plano – ($\vec{u}\neq 0$)

$\vec{u}\neq 0$: $\vec{v}\in\mathbb{R}$, $\vec{u}\perp\vec{v}$

(tal que) (perpendicular)

$\vec{u}\perp\vec{v} \iff \vec{u}\cdot\vec{v} = \emptyset$

Def. $\vec{u}\cdot\vec{v} = \|\vec{u}\|\cdot\|\vec{v}\|\cdot\cos\theta$

Teo. $\vec{u}\cdot\vec{v} = u_1v_1 + u_2v_2 + u_3v_3$

onde $\vec{u} = (u_1,u_2,u_3)$ e $\vec{v} = (v_1,v_2,v_3)$

$$\begin{cases}\vec{u}\perp\overrightarrow{PQ}\\ \vec{u}\perp\overrightarrow{PR}\end{cases} \Rightarrow \begin{cases}\vec{u}(-2,0,0)=0\\ \vec{u}(0,-2,2)=0\end{cases}$$

$$\Rightarrow \begin{cases}-2u_1 = 0\\ -2u_2+2u_3 = 0\end{cases} \Rightarrow \begin{cases}u_1 = 0\\ u_2 = u_3\end{cases}$$

$$\Rightarrow \vec{u} = (0,\alpha,\alpha),\ \alpha\in\mathbb{R}$$

$\alpha = 1 \Rightarrow \vec{u}_\alpha = \vec{u}_1 = (0,1,1)$

$\alpha = 4 \Rightarrow \vec{u}_\alpha = u_4 = (0,4,4)$

$\boxed{\vec{u}_1 = \theta\vec{u}_4 \text{ c/ } \theta\in\mathbb{R}}$

$\vec{u} = (a,b,c)$ (v. (1))

↓

$\emptyset x + \alpha y + \alpha z + d = 0$

$\Rightarrow$ ($\alpha = 1$) $y + z + d = 0$

tem que se verificar para $(x,y,z) = P = (2,2,0)$ (p. exemplo)

$\Rightarrow 2 + 0 + d = 0 \Rightarrow d = -2$

$$\boxed{y + z - 2 = \emptyset}$$

3) Produto Externo

|Pseudo-determinante|

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{L_1} & \vec{L_2} & \vec{L_3} \\ -2 & 0 & 0 \\ 0 & -2 & -2 \end{vmatrix} = (0\times 2 - 0\times(-2))\vec{L_1} - (-2\times 2 - 0\times 0)\vec{L_2} + (-2\times(-2) - 0\times 0)\vec{L_3} =$$

linha para calculo do determinante

$$= 0\vec{L_1} + 4\vec{L_2} + 4\vec{L_3} = (0,4,4) = \vec{u}$$

4) $f(x,y) = \ln(4-y)$

$$g(x,y) = \begin{cases} e^{f(x,y)}, & 0 \leq y \leq -x^2+4 \\ 4, & x^2+y^2 \leq 4 \wedge y<0 \end{cases}$$

Nota

$\boxed{ax+by+cz+d=0} \Leftrightarrow$

Eq. cartesiana (1)

$\Leftrightarrow z = -\dfrac{ax+by+d}{c} \Leftrightarrow$

$\Leftrightarrow z = f(x,y) \rightsquigarrow f: \mathbb{R}^2 \to \mathbb{R}$

Conjunto Partida

Conjunto chegada

Função Escalar

D C

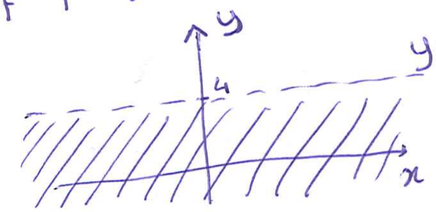
Domínio

CD_f

Contradomínio

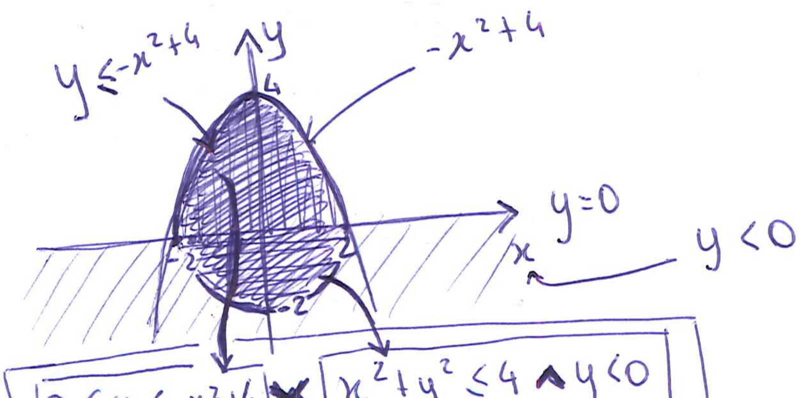
a) Domínios

$\boxed{f}$ $D_f = \{(x,y) \in \mathbb{R}^2 : 4-y>0\}$



$\boxed{g}$ $= \begin{cases} 4-y, & 0 \leq y \leq -x^2+4 \\ 4, & x^2+y^2 \leq 4 \wedge y<0 \end{cases}$

$$D_g = \{(x,y) \in \mathbb{R}^2 : (0 \leq y \leq -x^2+4) \vee (x^2+y^2 \leq 4 \wedge y<0)\}$$



$\boxed{\boxed{0 \leq y \leq -x^2+4} \vee \boxed{x^2+y^2 \leq 4 \wedge y<0}}$

$f : \mathbb{R}^n \longrightarrow \mathbb{R}$

$(x_1, \dots, x_n) \longmapsto y = f(x_1, x_2, \dots x_n)$

$$\frac{f(x_1, \dots, x_i + h, \dots x_n) - f(x_1, \dots, x_i, \dots, x_n)}{h}, \quad i = 1, \dots, n$$

$$\boxed{\frac{df}{dx}} = \lim_{h \to 0}$$

Notação de Leibniz → Derivação de f em ordem à var. x_i

(Outras: f_{x_i}, $D_x f$)

$f : \mathbb{R} \longrightarrow \mathbb{R}$ ⟿ Derivada

$x \longrightarrow y = f(x)$ $\quad f' : \frac{df}{dx}$

Quantas derivadas parciais de ordem K existem?

↓

n^K

$f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ $\quad n = 2$

$(x, y) \longmapsto z = f(x, y)$

$$\frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$$

$$\frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h}$$

ordem ∅: $\boxed{f(x, y)}$ $\quad 2^0 = 1$

ordem 1: $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ $\quad 2^1 = 2$

ordem 2:

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \partial y}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2}$$

$2^2 = 4$ $\quad (\dots)$

Muitas vezes esse problema não se verifica, porque Teorema de Schwarz

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y}$$

Se verificar a notação

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$$

Mas também pode significar

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$$

Equação de LaPlace

$$(1) \quad \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0 : \text{EDP}$$

Def. → Uma função diz-se Harmónica se verificar a equação de LaPlace

- Teórica - 18/05/2022

Derivadas Parciais

- Seja $z = f(x,y)$ (1)

$$z \begin{cases} x \to \dfrac{\partial z}{\partial x} \\ y \to \dfrac{\partial z}{\partial y} \end{cases} \quad (2)$$

- $$f_x(x,y) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x} \quad (3)$$

- $$f_y(x,y) = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y} \quad (4)$$

Nota (6)

$y = f(x)$ (5)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad (7)$$

$$y' = f'(x) = \frac{dy}{dx} \quad (8)$$

- $$f_x(x,y) = \frac{\partial f}{\partial x}(x,y) = \frac{\partial z}{\partial x} \quad (9)$$

- $$f_y(x,y) = \frac{\partial f}{\partial y}(x,y) = \frac{\partial z}{\partial y} \quad (10)$$

$$\boxed{\begin{array}{l} f(x) = x \\ f'(x) = 1 \Leftrightarrow x' = 1 \end{array}} \quad \boxed{f(x+h) = x+h}$$

PROVAR: $f'(x) = \lim\limits_{h \to 0} \dfrac{x + h - x}{h} = \lim\limits_{h \to 0} \dfrac{h}{h} = \lim\limits_{h \to 0} 1$ ($\frac{0}{0}$)

$\Leftrightarrow f'(x) = 1$ c.f.m.

$$\boxed{\begin{array}{l} f(x,y) = 4 - x^2 - y^2 \\ \Leftrightarrow z = 4 - x^2 - y^2 \end{array}} \quad \begin{cases} \dfrac{\partial z}{\partial x} = ? \\ \dfrac{\partial z}{\partial y} = \end{cases}$$

Nota

– Regras de derivação –

1. $(f \pm g)' = f' \pm g'$
2. $c' = 0$
3. $(f^p)' = p f^{p-1} f'$
4. $x' = 1$

Mantém-se constante pois estamos $\frac{\partial z}{\partial x}$ (3)

- $$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(4 - x^2 - y^2) = \underbrace{\frac{\partial}{\partial x}(4)}_{0} - \underbrace{\frac{\partial}{\partial x}(x^2)}_{} - \underbrace{\frac{\partial}{\partial x}(y^2)}_{0} =$$

$$= 0 - 2x^{2-1} \underbrace{\frac{\partial}{\partial x}(x)}_{1} - 0 = -2x \times 1 = \underline{-2x}$$

- $$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}(4 - x^2 - y^2) = \underbrace{\frac{\partial}{\partial y}(4)}_{0} - \underbrace{\frac{\partial}{\partial y}(x^2)}_{0} - \frac{\partial}{\partial y}(y^2)$$

$$= 0 - 0 - 2y \underbrace{\frac{\partial}{\partial y}(y)}_{} = -2y \times 1 = \underline{-2y}$$

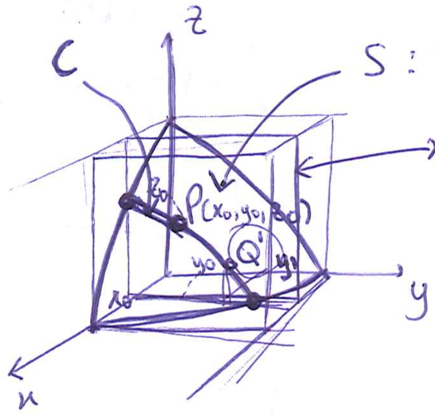
$$\partial x^2 \qquad \qquad \partial x^2$$

$$\Downarrow$$

$$\boxed{\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)} \longrightarrow \text{Derivar o que Derivamos Antes}$$

$$\boxed{\frac{\partial}{\partial x}\underbrace{(-2x)}_{\left(\frac{\partial z}{\partial x}\right)} = -2}$$

$$\boxed{\cdot\ f_{xy}(x,y) = \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial y}(-2x) = \emptyset}$$



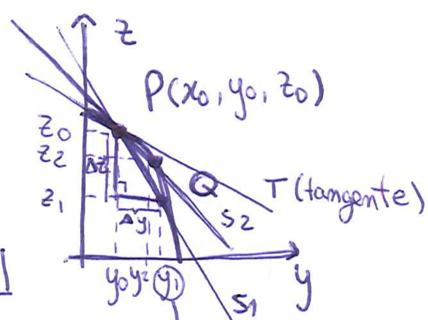
$$C = \begin{cases} z = f(x,y) \\ x = x_0 \end{cases} \Leftrightarrow \begin{cases} z = f(x_0, y) \\ x = x_0 \end{cases}$$

$$C : [\underbrace{x_0}_{\text{Constante}}, y, f(x_0, y)]$$

$$Q'(x_0, y_1, \partial) = Q'(x_0, y_1, z_1)$$

$$\hookrightarrow z_1 = f(x_0, y_1)$$

$$\Leftrightarrow z_1 = f(x_0, y_0 + \Delta y)$$



$\boxed{x = x_0}$

$\boxed{y_1 = y_0 + \Delta y}$

- $m_{S_1} = \dfrac{\Delta z}{\Delta y} = \dfrac{z_1 - z_0}{\Delta y}$

$$\boxed{m_{S_1} = \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y} \quad (11)}$$

- $z = m_{S_1} y + b \wedge x = x_0$
- $z_1 - z_0 = m_{S_1}(y - y_0) \wedge x = x_0$
- $m_{S_2} = \dfrac{\Delta z}{\Delta y} = \dfrac{z_2 - z_0}{\Delta y}$

$$m_{S_2} = \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y}$$

$$\{S_1, S_2, \dots\} \xrightarrow{\Delta y \to 0} \{T\}$$

$$\boxed{m_T = \lim_{\Delta y \to 0} m_S \quad (12)} \Leftrightarrow$$

$$\Leftrightarrow \boxed{m_T = \lim_{\Delta y \to 0} \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y} \quad (13)}$$

$\therefore$ (Conclusão) (13) e (4) $\boxed{m_T = f(x_0, y_0)}$

- Exemplo

$$f(x,y) = \sqrt{32-x^2-y^2}$$

→ Determine a equação da Reta tangente à curva C de interseção de $z = f(x,y)$ com o plano $x=2$, no ponto $P(2,2,\sqrt{24}) \equiv P(x_0,y_0,z_0)$.

- $m_T = f_y(x_0,y_0)$
- $z - z_0 = m_T(y-y_0) \wedge x = x_0$

Processo "de olhos fechados"

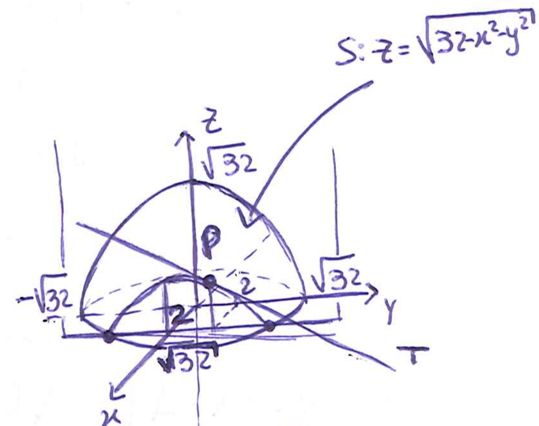
$$\cdot f_y(x,y) = \frac{\partial f}{\partial y}(x,y)$$

$$= \frac{\partial}{\partial y}\left(\sqrt{32-x^2-y^2}\right) = \frac{\partial}{\partial y}\left((32-x^2-y^2)^{1/2}\right) =$$

$$= 1/2\,(32-x^2-y^2)^{-1/2} \times \frac{\partial}{\partial y}(32-x^2-y^2) =$$

$$= \frac{-\cancel{2}y}{\cancel{2}\sqrt{32-x^2-y^2}}$$

$$\boxed{f_y(x,y) = -\frac{y}{\sqrt{32-x^2-y^2}}}$$



Equação na Reta tangente

$$\cdot m_T = f_y(2,2) = -\frac{2}{\sqrt{24}}$$

$$z - \sqrt{24} = -\frac{2}{\sqrt{24}}(y-2) \wedge x = 2$$

$$\Leftrightarrow z = -\frac{2}{\sqrt{24}}y + \frac{4}{\sqrt{24}} + \sqrt{24} \wedge x = 2$$

$$\Leftrightarrow z = -\frac{2}{\sqrt{24}}y + \frac{28}{24} \wedge x = 2$$

$$\Leftrightarrow z = my + b \wedge x = 2$$

Equação no ponto P

$$\cdot C = \begin{cases} z = \sqrt{32-x^2-y^2} \\ x = 2 \end{cases}$$

$$\cdot C = [2, y, \sqrt{28-y^2}]$$

$$\cdot C : \begin{cases} z = \sqrt{28-y^2} \\ x = 2 \end{cases}$$

1 ▶ Limites

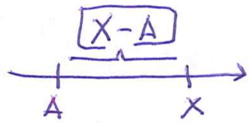
$f: \mathbb{R}^n \longrightarrow \mathbb{R}$

$X \longmapsto y = f(X)$

$X = (x_1, x_2, \dots, x_n)$

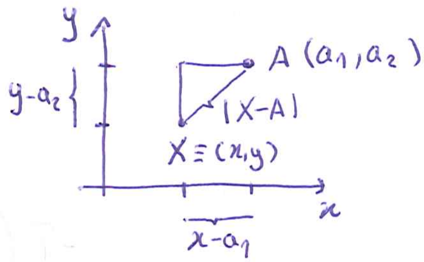
$\lim_{X \to A} f(X) = L \iff \forall_{\varepsilon} > 0 \ \exists \ \delta > 0 : \boxed{0 < |X - A| < \delta} \Rightarrow |f(X) - L| < \varepsilon$

- $n = 1$: $|X - A|$



$\Rightarrow \sqrt{(x-a)^2}$

- $n = 2$: $|X - A| = |(x, y) - (a_1, a_2)|$



$\Rightarrow \sqrt{(x-a_1)^2 + (y-a_2)^2}$

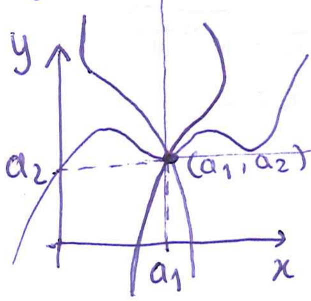
$\downarrow$

Teorema de Pitágoras

- $n = 3$: $|X - A| = \sqrt{(x-a_1)^2 + (y-a_2)^2 + (z-a_3)^2}$

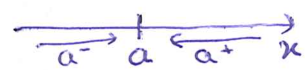
- $n = 2$ $\quad f: \mathbb{R}^2 \longrightarrow \mathbb{R}$

$(x, y) \longmapsto z = f(x, y)$



$\lim_{(x,y) \to (a_1, a_2)} f(x, y)$

- Em $\mathbb{R}$ ($n = 1$)



$\lim_{x \to a} f(x) = \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

$\boxed{\text{Teo. o limite é único}}$

→ limites iterados

- $n = 2$ → $\lim_{x \to a_1} \lim_{y \to a_2} f(x, y) = \lim_{y \to a_2} \lim_{x \to a_1} f(y, x)$

$2!$

- $n = 3$ → (x, y, z) $\quad (y, z, x)$

$3!$ $\quad (x, z, y)$ $\quad (z, y, x)$

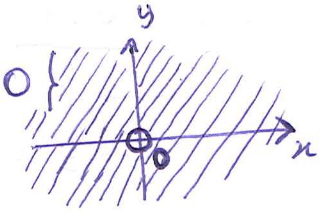
(y, x, z) $\quad (z, x, y)$

- $n = 4$ → $(\dots)$

$4!$

→ Exercícios

$\cdot f(x,y) = \dfrac{x^2+2y^2}{x^2+y^2}$ | $\cdot D_f = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \neq 0\}$
$= \mathbb{R}^2 \setminus \{0,0\}$



· Calcular, se existir, o $\lim\limits_{(x,y)\to(0,0)} f(x,y)$

$\rightarrow \lim\limits_{(x,y)\to(0,0)} f(x,y) = \lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+2y^2}{x^2+y^2} = \dfrac{0}{0}$

$\rightarrow \lim\limits_{x\to 0} \lim\limits_{y\to 0} f(x,y) = \lim\limits_{x\to 0} \lim\limits_{y\to 0} \dfrac{x^2+2y^2}{x^2+y^2} = \lim\limits_{x\to 0} \dfrac{\cancel{x^2}}{\cancel{x^2}} = \lim\limits_{x\to 0} 1 = \underline{\underline{1}}$ (1)

$\rightarrow \lim\limits_{y\to 0} \lim\limits_{x\to 0} f(x,y) = \lim\limits_{y\to 0} \lim\limits_{x\to 0} \dfrac{x^2+2y^2}{x^2+y^2} = \lim\limits_{y\to 0} \dfrac{2y^2}{y^2} = \lim\limits_{y\to 0} 2 = \underline{\underline{2}}$ (2)

· De (1) e (2) temos $\lim\limits_{x\to 0} \lim\limits_{y\to 0} f(x,y) = 1 \neq 2 = \lim\limits_{y\to 0} \lim\limits_{x\to 0} f(x,y)$

Assim, não existe $\lim\limits_{(x,y)\to(0,0)} f(x,y)$.

▶ Continuidade : f é contínua em A se $\boxed{\lim\limits_{X\to A} f(x) = f(a)}$

· $A \in D_f$

$\cdot f(x,y) = \begin{cases} \dfrac{x^2+2y^2}{x^2+y^2} &, (x,y) \neq (0,0) \\ 0 &, (x,y) = (0,0) \end{cases}$

– Para $\boxed{(x,y) \neq (0,0)}$, $f(x,y) = \dfrac{x^2+2y^2}{x^2+y^2}$ é uma função racional cujo denominador nunca se anula, logo é contínuo

· Problema: $(x,y) = (0,0)$ — $f(0,0) = 0$; $\lim\limits_{(x,y)\to(0,0)} f(x,y) \longrightarrow$ Não existe

→ A função é contínua em $\mathbb{R} \setminus \{0,0\}$

$$f:\mathbb{R}^2 \longrightarrow \mathbb{R}$$
$$(x,y) \longmapsto z=f(x,y)$$

- $$\frac{\partial f}{\partial \boxed{x}} = \lim_{h\to 0} \frac{f(x+h,y)-f(x,y)}{h}$$

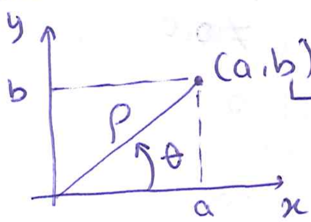
- $$\frac{\partial f}{\partial \boxed{y}} = \lim_{h\to 0} \frac{f(x,y+h)-f(x,y)}{h}$$

- $f(x,y) = x^3 + x^2y + xy^2 + y^3$

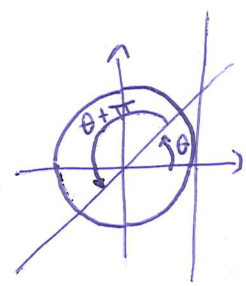
$$\rightarrow \frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^3+x^2y+xy^2+y^3)$$
$$= 3x^2 + 2x \times y + y^2 + 0 = \underline{\underline{3x^2+2xy+y^2}}$$

$$\rightarrow \frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^3+x^2y+xy^2+y^3)$$
$$= 0 + x^2 + x\,2y + 3y^2 = \underline{\underline{x^2+2xy+3y^2}}$$

Coordenadas Polares



$(a,b) \simeq a+bi$
↳ coordenadas cartesianas
$(\rho,\theta) \longrightarrow$ coordenadas Polares



→ Polares → Cartesianas

$(\rho,\theta) \rightsquigarrow (x,y)$

$$\begin{cases} x=\rho\cos\theta \\ y=\rho\,\mathrm{sen}\,\theta \end{cases}$$

→ Cartesianas → Polares

$(x,y) \rightsquigarrow (\rho,\theta)$

- $\rho = \sqrt{x^2+y^2}$ ⟶ Aplicação direta do T. Pitágoras

- $\mathrm{tg}\,\theta = \frac{\mathrm{sen}\,\theta}{\cos\theta} = \frac{\rho\,\mathrm{sen}\,\theta}{\rho\cos\theta} = \frac{y}{x}$

$\Rightarrow \mathrm{tg}\,\theta = \frac{y}{x} \Rightarrow \theta = \mathrm{arctg}\,\frac{y}{x}$
$(x,y) \in 1^{\circ}, 4^{\circ}$ Quad.
↳ $(x,y) \in 2^{\circ}, 3^{\circ}$ Quad.
$\theta + \pi$

• E se $x = \emptyset$?

Ponto sobre o eixo das coordenadas

• $x^2 + y^2 = R^2$ em coord. polares

$$\begin{aligned}\rightarrow x &= \rho\cos\theta \\ \rightarrow y &= \rho \operatorname{sen}\theta\end{aligned} \Bigg\} \quad \begin{aligned}&(\rho\cos\theta)^2 + (\rho\operatorname{sen}\theta)^2 = R^2 \\ &\Leftrightarrow \rho^2(\cos^2\theta + \operatorname{sen}^2\theta) = R^2 \\ &\Leftrightarrow \rho^2 = R^2 \Leftrightarrow \boxed{\rho = R}\end{aligned}$$

$$P_2C \text{ (polar 2 cartes.)} \Big| \begin{aligned} \mathbb{R}^+ \times \mathbb{R} &\longrightarrow \mathbb{R}^2 \\ (\rho, \theta) &\longmapsto (x, y) \end{aligned}$$

Gradiante

(não tenho certeza)

$$\cdot\ x(\rho,\theta) = \rho\cos\theta \Longrightarrow \nabla x = \begin{bmatrix} \frac{\partial x}{\partial \rho} \\ \frac{\partial x}{\partial \theta} \end{bmatrix} = \begin{bmatrix} \frac{\partial \rho}{\partial \rho} \times \cos\theta + \left(\frac{\rho}{\partial\rho}\right) \times \frac{\partial}{\partial \rho}(\cos\theta) \\ \rule{5cm}{0.4pt} \end{bmatrix} =$$

$$= \begin{bmatrix} \cos\theta \\ -\rho\operatorname{sen}\theta \end{bmatrix}$$

$$\cdot\ y(\rho,\theta) = \rho\operatorname{sen}\theta \Rightarrow \nabla y = \begin{bmatrix} \frac{\partial y}{\partial \rho} \\ \frac{\partial y}{\partial \theta} \end{bmatrix} = \begin{bmatrix} \operatorname{sen}\theta \\ \rho\cos\theta \end{bmatrix}$$

$$J_{P_2C} = \begin{bmatrix} \nabla x^T \\ \nabla y^T \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial \rho} & \frac{\partial y}{\partial \theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\rho\operatorname{sen}\theta \\ \operatorname{sen}\theta & \rho\cos\theta \end{bmatrix} : \underline{\text{Matriz Jacobiana}}$$

$$\underline{\text{Jacobiano}} : |J_{P_2C}| = \rho\cos^2\theta + \rho\operatorname{sen}^2\theta = \underline{\underline{\rho}}$$

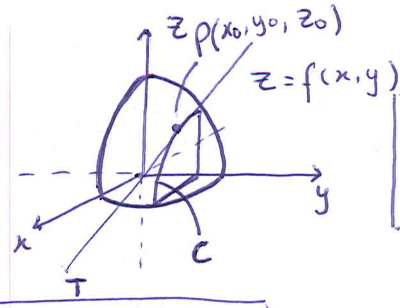
Derivadas Parciais

[redacted]

$$\boxed{z = f(x,y) \quad (1)} \quad \longrightarrow \frac{\partial f}{\partial x}$$

$$\boxed{z \begin{matrix} x \\ y \end{matrix}} \longrightarrow \frac{\partial z}{\partial y}$$

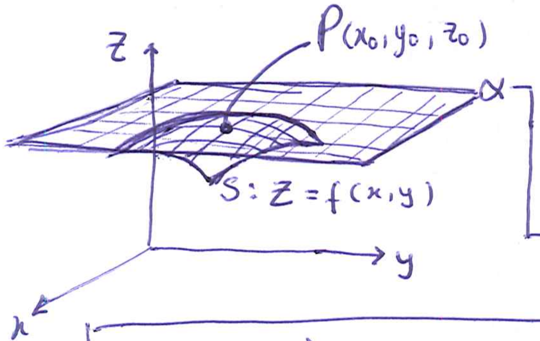
→ Interpretação Geométrica



declive de uma Reta tangente a uma curva

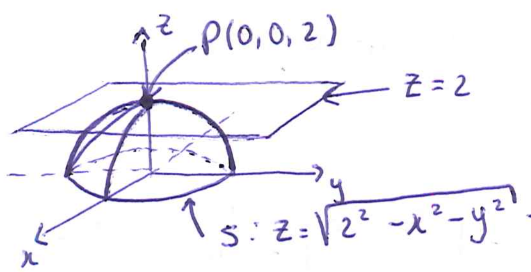
$$\boxed{m_T = \frac{\partial f}{\partial x}(x_0, y_0)}$$

$$\boxed{z - z_0 = m_T (x - x_0) \text{ n } y = y_0}$$



Plano tangente à superfície de: $z = f(x,y)$ no $P(x_0, y_0, z_0)$

$$\boxed{z - z_0 = \frac{\partial f}{\partial x}(x_0, y_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0, y_0)(y - y_0)}$$



$\to z = f(x,y)$

$\Leftrightarrow f(x,y) = \sqrt{4 - x^2 - y^2}$

→ Cálculos Auxiliares

i) $\frac{\partial f}{\partial x}(x,y) = \frac{\partial}{\partial x}\left(\sqrt{4 - x^2 - y^2}\right) =$

$= \frac{\partial}{\partial x}\left(\left(4 - x^2 - y^2\right)^{1/2}\right) =$

$\boxed{(f^p)' = p f^{p-1} f'}$

$= \frac{1}{2}\left(4 - x^2 - y^2\right)^{1/2 - 1} \frac{\partial}{\partial x}\left(4 - x^2 - y^2\right) =$

$= \frac{1}{\cancel{2}} \frac{1}{(4 - x^2 - y^2)^{1/2}} (0 - \cancel{2}x - 0) =$

$$\frac{\partial f}{\partial x}(x,y) = -\frac{x}{\sqrt{4 - x^2 - y^2}}$$

ii) $\frac{\partial f}{\partial y}(x,y) = (\dots) = -\frac{y}{\sqrt{4 - x^2 - y^2}}$

$$\to z - \underbrace{2}_{z_0} = \frac{\partial f}{\partial x}\underbrace{(0,0)}_{(x_0, y_0)} \times (x - \underbrace{0}_{x_0}) + \frac{\partial f}{\partial y}(x_0, y_0)(y - \underbrace{0}_{y_0})$$

$$\Leftrightarrow z - 2 = 0x + 0y \Leftrightarrow z - 2 = 0 \Leftrightarrow \boxed{z = 2}$$

Derivada Direcional

→ Definição

$$\boxed{f_{\vec{u}}(x,y)=\lim_{h\to 0}\frac{f(x+h\cos\theta,\ y+h\,\text{sen}\,\theta)-f(x,y)}{h},\ \text{se o limite existir}}$$

→ Se $\|\vec{u}\|=1$

Então $f_{\vec{u}}(x,y)=\nabla f(x,y)\cdot\vec{u}$

onde $\nabla f(x,y)=\dfrac{\partial f}{\partial x}(x,y)\,\vec{i}+\dfrac{\partial f}{\partial y}(x,y)\,\vec{j}$

$$\boxed{\nabla f(x,y)\equiv \text{Vetor Gradiante}}$$

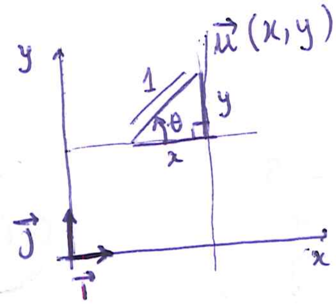
e $\boxed{\vec{u}=\cos\theta\,\vec{i}+\text{sen}\,\theta\,\vec{j}}$

↑

$\boxed{\vec{u}=x\,\vec{i}+y\,\vec{j}}$

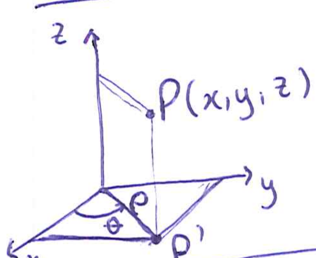
↑

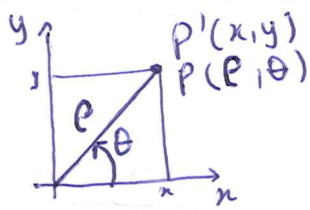
$$\boxed{\begin{aligned}\cos\theta&=\frac{x}{1}\Leftrightarrow x=\cos\theta\\ \text{sen}\,\theta&=\frac{y}{1}\Leftrightarrow y=\text{sen}\,\theta\end{aligned}}$$



Teórico-Prática - 07/06/2022

Coordenadas cilindricas → (ρ,θ,z)





$$\boxed{\begin{aligned}&\rho\geq 0\\ &0\leq\theta\leq 2\pi\\ &z\in\mathbb{R}\qquad (2)\end{aligned}}$$

$$\boxed{T_{\text{polares}}=\begin{cases}x=\rho\cos\theta\\ y=\rho\,\text{sen}\,\theta\end{cases}\Leftrightarrow\begin{cases}\cos\theta=\dfrac{x}{\rho}\\ \text{sen}\,\theta=\dfrac{y}{\rho}\end{cases}\quad(1)}$$

$$\boxed{T_{\text{cilindricas}}=\begin{cases}x=\rho\cos\theta\\ y=\rho\,\text{sen}\,\theta\\ z=z\end{cases}\quad(3)}$$

$$J=\begin{vmatrix}\frac{\partial x}{\partial\rho}&\frac{\partial x}{\partial\theta}&\frac{\partial x}{\partial z}\\ \frac{\partial y}{\partial\rho}&\frac{\partial y}{\partial\theta}&\frac{\partial y}{\partial z}\\ \frac{\partial z}{\partial\rho}&\frac{\partial z}{\partial\theta}&\frac{\partial z}{\partial z}\end{vmatrix}=\begin{array}{ccc}\text{i)}&\text{ii)}&\text{iii)}\end{array}\begin{vmatrix}\cos\theta&-\rho\,\text{sen}\,\theta&0\\ \text{sen}\,\theta&\rho\cos\theta&0\\ 0&0&1\end{vmatrix}=$$

$$=(\dots)\Leftrightarrow\boxed{J=\rho\ (4)}$$

i) $\dfrac{\partial x}{\partial\rho}=\dfrac{\partial}{\partial\rho}(\rho\cos\theta)$

$=\cos\theta\,\dfrac{\partial}{\partial\rho}(\rho)$

$=\cos\theta\,(1)$

$=\underline{\cos\theta}$

ii) $\dfrac{\partial x}{\partial\theta}=\dfrac{\partial}{\partial\theta}(\rho\cos\theta)$

$=\rho\dfrac{\partial}{\partial\theta}(\cos\theta)$

$=\rho\,(-\text{sen}\,\theta)$

$=\underline{-\rho\,\text{sen}\,\theta}$

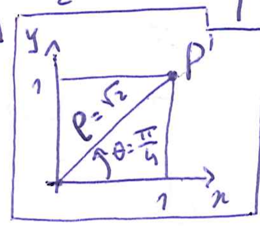
iii) $\dfrac{\partial x}{\partial z}=\dfrac{\partial}{\partial z}(\rho\cos\theta)=\underline{0}$

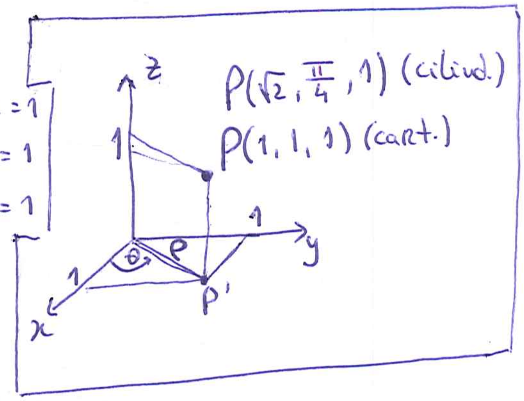
a) $P(\sqrt{2}, \frac{\pi}{4}, 1) = P(\rho, \theta, z)$

$P(x, y, z) = ?$ (coordenadas cartesianas)

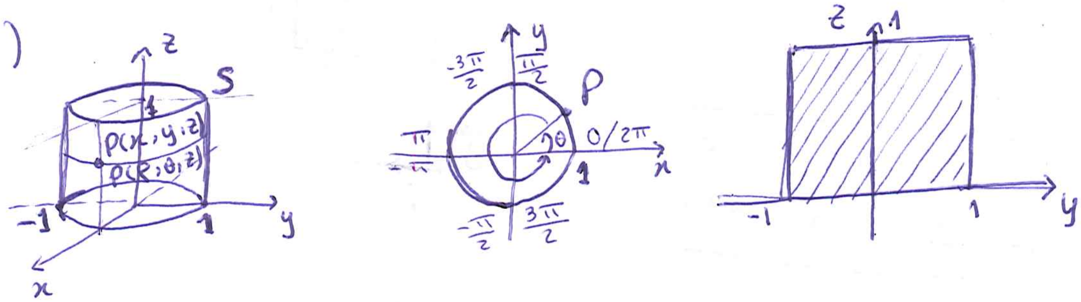
$$\begin{cases} x = \sqrt{2}\cos\frac{\pi}{4} \\ y = \sqrt{2}\,\text{sen}\,\frac{\pi}{4} \\ z = 1 \end{cases} \Leftrightarrow \begin{cases} x = \sqrt{2}\,\frac{\sqrt{2}}{2} \\ y = \sqrt{2}\,\frac{\sqrt{2}}{2} \\ z = 1 \end{cases} \Leftrightarrow \begin{cases} x = 1 \\ y = 1 \\ z = 1 \end{cases}$$

$P(1, 1, 1) = P(x, y, z)$





b)



→ Coordenadas cartesianas:

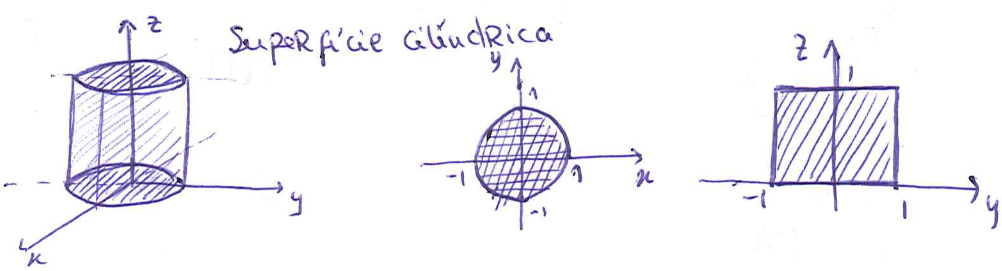
$S = \{(x, y, z) \in \mathbb{R}^2 : x^2 + y^2 = 1 \wedge 0 \leq z \leq 1\}$

→ Coordenadas cilíndricas:

$S = \{(\rho, \theta, z) : \rho = 1 \wedge 0 \leq \theta \leq 2\pi \wedge 0 \leq z \leq 1\}$

c)

Superfície cilíndrica



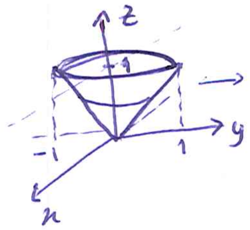
→ Coordenadas cartesianas

$S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \leq 1 \wedge 0 \leq z \leq 1\}$

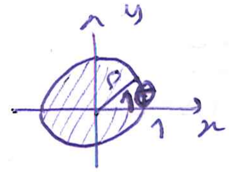
→ Coordenadas cilíndricas

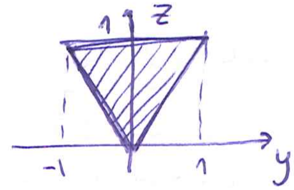
$S = \{(\rho, \theta, z) : 0 \leq \rho \leq 1 \wedge 0 \leq \theta \leq 2\pi \wedge 0 \leq z \leq 1\}$

d)



$\rightarrow z = f(x,y)$
$\Leftrightarrow z = \sqrt{x^2+y^2}$

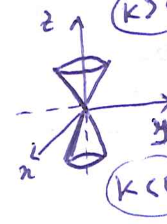




→ Coordenadas cartesianas

$$S = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2 \leq 1 \wedge z = \sqrt{x^2+y^2}\}$$

Nota



$(k>0)$

$\boxed{z = k\sqrt{x^2+y^2}}$

$(k<0)$

→ Coordenadas cilíndricas

$$\boxed{z = \sqrt{x^2+y^2}} \leftarrow \text{Aplicar (3)}$$

$$z = \sqrt{\rho^2\cos^2\theta + \rho^2 \operatorname{sen}^2\theta}$$

$$\Leftrightarrow z = \sqrt{\rho^2(\underbrace{\cos^2\theta + \operatorname{sen}^2\theta}_{1})}$$

$$\Leftrightarrow \boxed{z = \rho} \quad S = \{(\rho,\theta,z) : 0 \leq \rho \leq 1 \wedge 0 \leq \theta \leq 2\pi \wedge z = \rho\}$$

e) Cone "com matéria dentro"

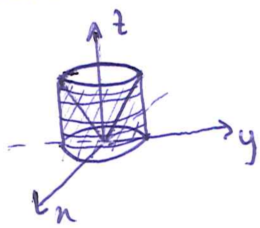
→ coordenadas cartesianas

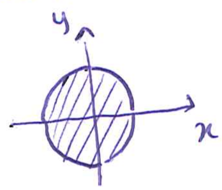
$$S = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2 \leq 1 \wedge \sqrt{x^2+y^2} \leq z \leq 1\}$$

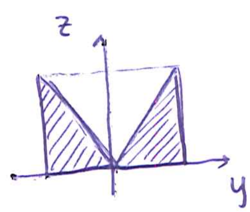
→ coordenadas cilíndricas

$S = \{(\rho,\theta,z) : 0 \leq \rho \leq 1 \wedge$ [redacted]
$0 \leq \theta \leq 2\pi \wedge$
$\rho \leq z \leq 1\}$

f) Cilindro com corte de cone dentro







→ coordenadas cartesianas

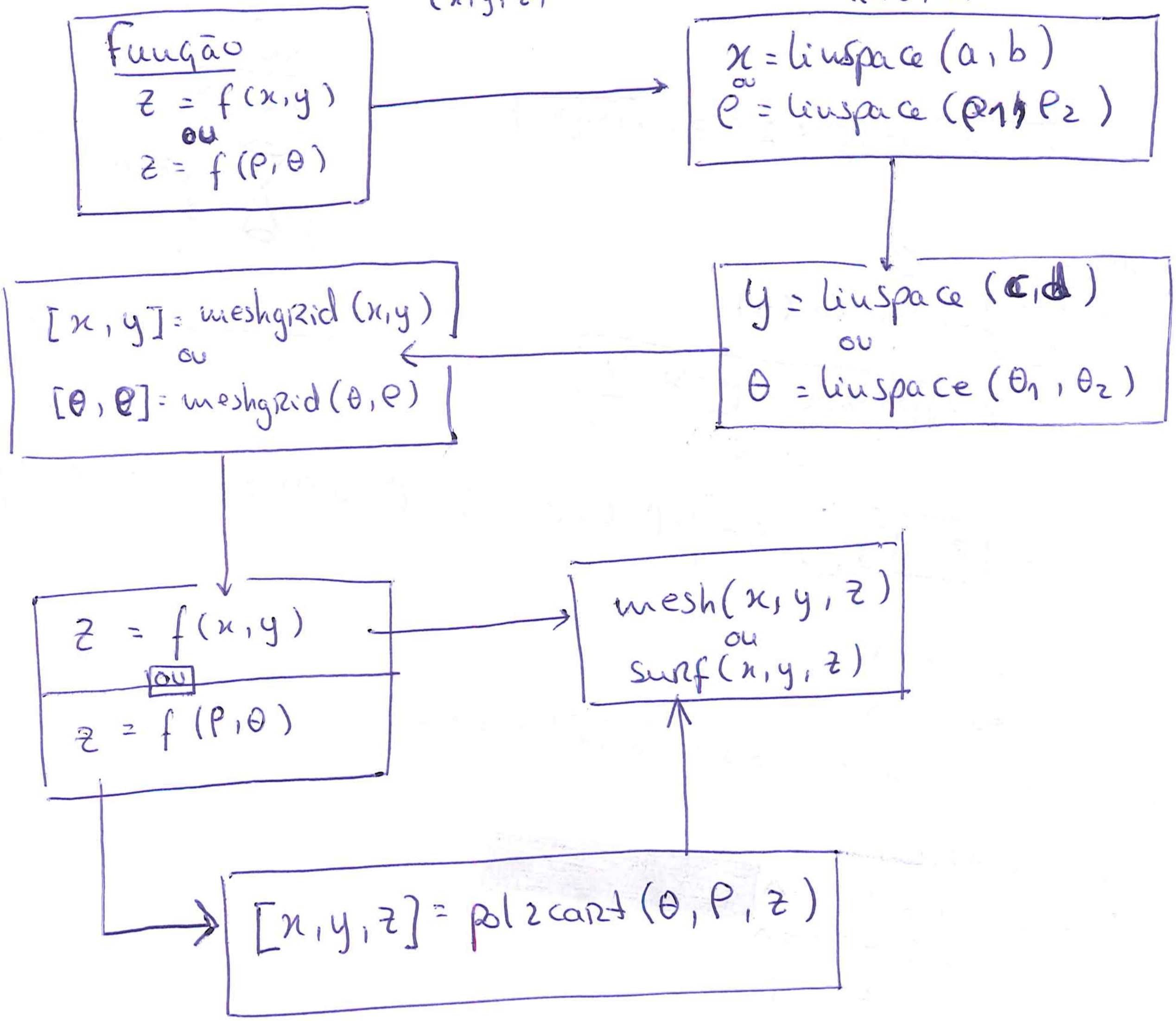
$$S = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2 \leq 1 \wedge 0 \leq z \leq \sqrt{x^2+y^2}\}$$

→ coordenadas cilíndricas

$$S = \{(\rho,\theta,z) : 0 \leq \rho \leq 1 \wedge 0 \leq \theta \leq 2\pi \wedge 0 \leq z \leq \rho\}$$

- Representação 3 D em MATLAB

• Coordenadas cartesianas / Coordenadas cilíndricas
(x, y, z) (ρ, θ, z)



Coordenadas Cilindricas (aula prática 07/06/2022)

[help pol2cart]

```
help pol2cart
```

```
pol2cart Transform polar to Cartesian coordinates.
   [X,Y] = pol2cart(TH,R) transforms corresponding elements of data
   stored in polar coordinates (angle TH, radius R) to Cartesian
   coordinates X,Y.  The arrays TH and R must the same size (or
   either can be scalar).  TH must be in radians.

   [X,Y,Z] = pol2cart(TH,R,Z) transforms corresponding elements of
   data stored in cylindrical coordinates (angle TH, radius R, height
   Z) to Cartesian coordinates X,Y,Z. The arrays TH, R, and Z must be
   the same size (or any of them can be scalar).  TH must be in radians.

   Class support for inputs TH,R,Z:
      float: double, single

   See also cart2sph, cart2pol, sph2cart.

   Documentation for pol2cart
   Other functions named pol2cart
```

a) Coordenadas cilindricas em coordenadas cartesianas

$$P\left(\frac{\pi}{4}, \sqrt{2}, 1\right) = P(\rho, \theta, z)$$

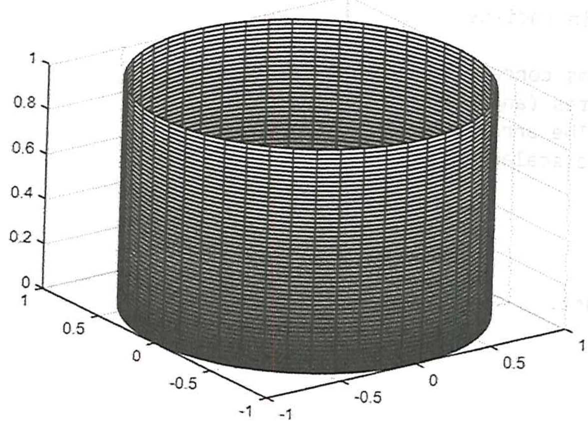
```
[x,y,z]=pol2cart(pi/4,sqrt(2),1)
```

```
x = 1.0000
y = 1
z = 1
```

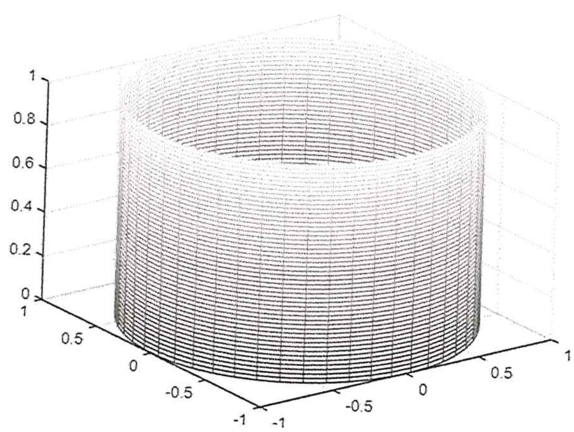
b) Superfície cilindrica de raio 1 e altura 1

$$S = \{(\rho, \theta, z) : \rho = 1 \wedge 0 \leq \theta \leq 2\pi \wedge 0 \leq z \leq 1\}$$

```
theta = linspace(0,2*pi,50); %matriz 100 elem de 0 a 2*pi
z=linspace(0,1,50); %matriz de 100 elem de 0 a 1
[theta,z] = meshgrid(theta,z);%array bidim. de theta e z
r = 1; %r = raio (p)
[x,y,z] = pol2cart(theta,r,z);
surf(x,y,z)
```



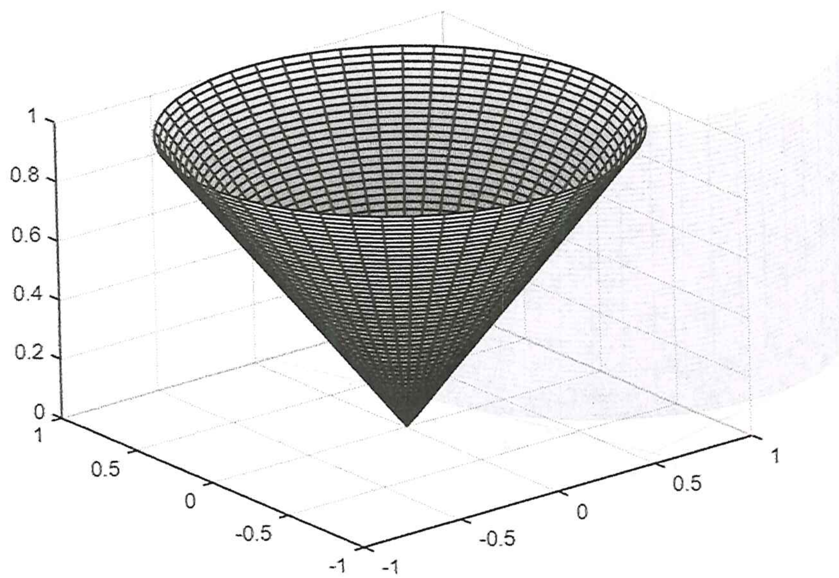

```
mesh(x,y,z)
```



c) Superficie cónica de raio 1 e altura 1

$$S = \{(\rho, \theta, z) : 0 \leq \rho \leq 1 \wedge 0 \leq \theta \leq 2\pi \wedge z = \rho\}$$

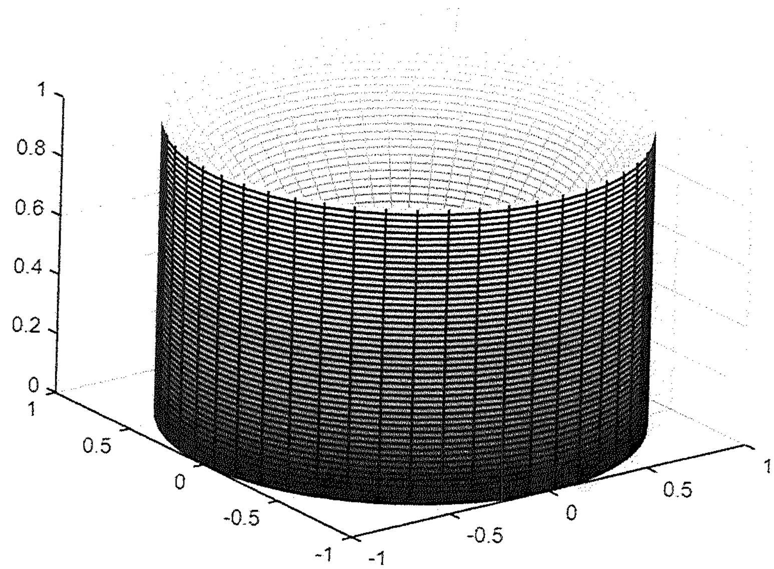
```
theta = linspace(0,2*pi, 50); %matriz 100 elem de 0 a 2*pi
r=linspace(0,1,50); %matriz de 100 elem de 0 a 1 // r = roh (p)
[theta,r] = meshgrid(theta,r);%array bidim. de theta e r
z=r;
[xx,yy,zz] = pol2cart(theta,r,z);
surf(xx,yy,zz)
```



f) Cilindro com um buraco cónico

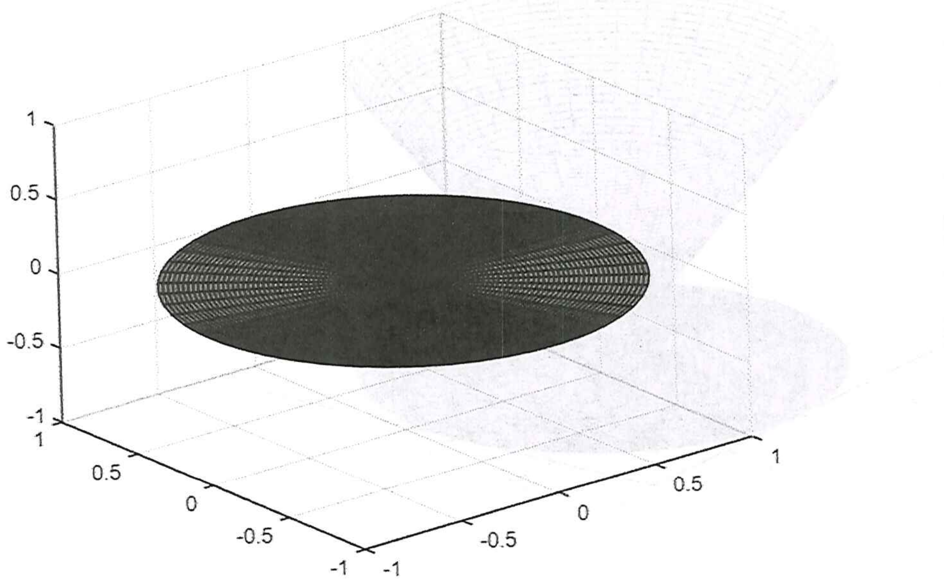
$$S = \{(\rho, \theta, z) : 0 \le \rho \le 1 \wedge 0 \le \theta \le 2\pi \wedge 0 \le z \le \rho\}$$

```
surf(x,y,z)
hold on
mesh(xx,yy,zz)
hold off
```



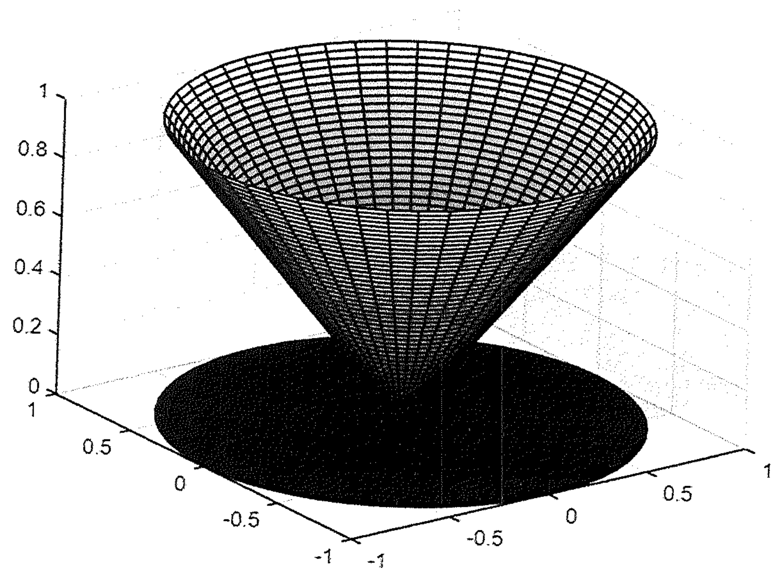
g) Chapa / Disco circular de raio 1

```
theta = linspace(0,2*pi, 50); %matriz 100 elem de 0 a 2*pi
r=linspace(0,1,50); %matriz de 100 elem de 0 a 1 // r = rdh (p)
[theta,r] = meshgrid(theta,r);%array bidim. de theta e r
z=zeros(50);
[xxx,yyy,zzz] = pol2cart(theta,r,z);
surf(xxx,yyy,zzz)
```



h) Cilindro com um buraco cónico e chapa circular de raio 1

```
surf(x,y,z)
hold on
surf(xx,yy,zz)
surf(xxx,yyy,zzz)
hold off
```



• Teorico - Prática - 07/06/2022 (continuação)

→ Teste B (2018/19)

$f(x,y) = \sqrt{x^2+y^2}$ (1)

1 f) $j(x,y) = sen(f^2(x,y) - x^2 - x) - (y^2+y))$ (2)

$j(x,y) = sen(\sqrt{x^2+y^2})^2 - x^2 + x - y^2 - y$

$\Leftrightarrow j(x,y) = sen(\cancel{x^2} + \cancel{y^2} - \cancel{x^2} + x - \cancel{y^2} - y)$

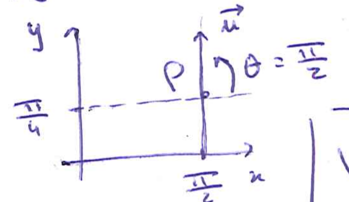
$\Leftrightarrow j(x,y) = sen(x-y)$ (3)

i) $V = j(x,y) \Leftrightarrow V = sen(x-y)$

Taxa de variação do Potencial

• $P(\frac{\pi}{2}, \frac{\pi}{4})$

• $\theta = 90°$



$\boxed{V_y(\frac{\pi}{2}, \frac{\pi}{4}) = ?}$

$\boxed{1^o \text{ Passo}}$

• $V_y(x,y) = \frac{\partial V}{\partial y} = \frac{\partial}{\partial y}(V) = \frac{\partial}{\partial y}(sen(x-y)) = \frac{\partial}{\partial y}(x-y)\cos(x-y)$

$= (0-1)\cos(x-y) = \boxed{-\cos(x-y)}$

$\boxed{\text{Nota}}$

$\boxed{(sen(f))' = f'\cos(f)}$

$\boxed{2^o \text{ Passo}}$

• $V_y(\frac{\pi}{2}, \frac{\pi}{4}) = -\cos(\frac{\pi}{2} + \frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$

→ O potencial diminui à taxa de $-\frac{\sqrt{2}}{2}$

iv) Plano tangente

$\boxed{\begin{array}{l} z = f(x,y) \\ P(0, -\frac{\pi}{2}, 1) \\ P(x_0, y_0, z_0) \end{array}}$

$\boxed{z - z_0 = \frac{\partial j}{\partial x}(x_0, y_0) \times (x - x_0) + \frac{\partial j}{\partial y}(x_0, y_0) \times (y - y_0)}$

i) $\frac{\partial j}{\partial x}(x,y) = \cos(x-y)$

$\frac{\partial j}{\partial x}(0, -\frac{\pi}{2}) = 0$

ii) $\frac{\partial j}{\partial y}(x,y) = -\cos(x-y)$

$\frac{\partial j}{\partial y}(0, -\frac{\pi}{2}) = 0$

$z - 1 = \overset{i)}{0}(x-0) + \overset{ii)}{0}(y + \frac{\pi}{2}) \Leftrightarrow z - 1 = 0$

$\Leftrightarrow \boxed{z = 1}$

iii) $z = arcsen(j(x-1)^2, (y-1)^2)$ (1)

$\boxed{x = 1 + \cos\theta}$ (2)

$\boxed{y = 1 + sen\,\theta}$ (3)

$\boxed{j(x,y) = sen(x-y)}$ (4)

então

$\boxed{1/2 \frac{\partial z}{\partial \theta} = -sen(2\theta)}$ (5)

$$\boxed{j((x-1)^2,(y-1)^2) = \text{sen}((x-1)^2-(y-1)^2) \quad (6)}$$

(6) ↪ (1)

- $Z = \text{arcsen}(\text{sen}((x-1)^2-(y-1)^2))$

$$\Leftrightarrow \boxed{Z = (x-1)^2-(y-1)^2 \quad (7)}$$

→ Passo 02

(2), (3) ↪ (7)

$$z = (1+\cos\theta-1)^2 - (1+\text{sen}\,\theta-1)^2$$

$$\Leftrightarrow \boxed{z = \cos^2\theta - \text{sen}^2\theta \quad (8)}$$

Nota

$$\cos^2\theta + \text{sen}^2\theta = 1$$
$$\cos^2\theta = 1-\text{sen}^2\theta$$

$$z = \cos^2\theta - \text{sen}^2\theta$$
$$\Leftrightarrow z = 1-\text{sen}^2\theta - \text{sen}^2\theta$$

→ Passo 03

- $\frac{\partial z}{\partial\theta} = \frac{\partial}{\partial\theta}(\cos^2\theta - \text{sen}^2\theta)$

$$= \frac{\partial}{\partial\theta}(1-2\,\text{sen}^2\theta) = -2\frac{\partial}{\partial\theta}(\text{sen}^2\theta) = -2\times 2\,\text{sen}\,\theta\frac{\partial}{\partial\theta}(\text{sen}\,\theta) =$$

$$= -2\times 2\,\text{sen}\,\theta\cos\theta$$

$$\boxed{\frac{\partial z}{\partial\theta} = -2\,\text{sen}\,2\theta \quad (9)}$$

→ Passo 04

(9) ↪ (5)

$$\frac{1}{\cancel{2}}(-\cancel{2}\,\text{sen}(2\theta)) = -\text{sen}(2\theta)$$

$$\Leftrightarrow -\cancel{\text{sen}(2\theta)} = -\cancel{\text{sen}(2\theta)}$$

$$\Leftrightarrow \boxed{0 = 0}$$

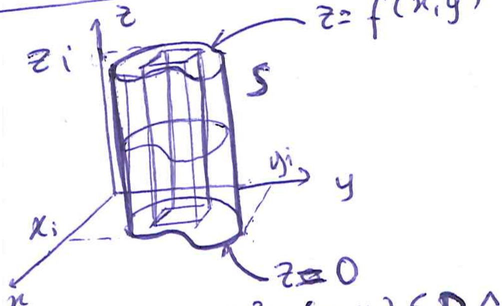
P.V. (preposição verdadeira)

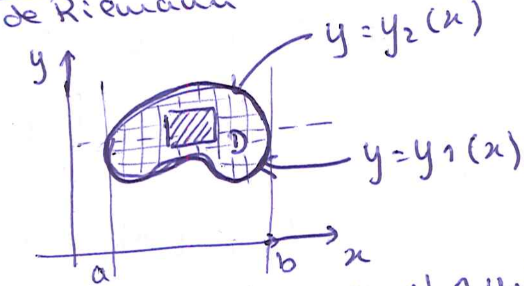
• Teórica - 08/06/2022

1) Integral Duplo

• $$\iint_D f(x,y)\,dy\,dx = \lim_{n\to+\infty}\lim_{m\to+\infty}\sum_{i=1}^{n}\sum_{j=1}^{m} f(x,y)\,\Delta x_i\,\Delta y_j \quad (1)$$

Somas de Riemann





$$D=\{(x,y)\in\mathbb{R}^2 : a\le x\le b \wedge y_1(x)\le y\le y_2(x)\}$$

$$S=\{(x,y,z)\in\mathbb{R}^3 : (x,y)\in D \wedge 0\le z\le f(x,y)\}$$

$$V(\square) = A(\text{base}) \times \text{altura} = \Delta x_i \times \Delta y_i \times f(x_i,y_i)$$

$$V(S) \approx \underbrace{\sum_{i=1}^{m}\sum_{j=1}^{m} f(x_i,y_i) \times \Delta x_i \times \Delta y_j}_{\text{Soma de Riemann (SR)}} \quad (2)$$

$$V(S) = \lim_{n\to+\infty}\lim_{m\to+\infty} (SR) \quad (3)$$

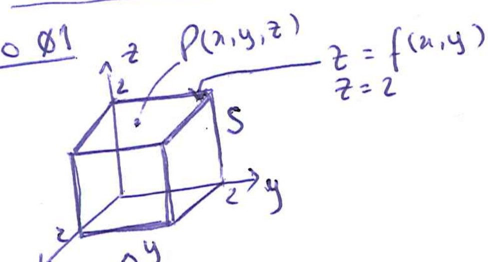
De (3) e (1)

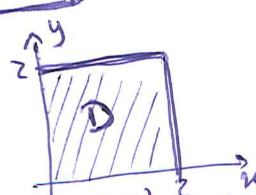
$$\boxed{V(S) = \iint_D f(x,y)\,dy\,dx} = \int_a^b\left[\int_{y_1(x)}^{y_2(x)} f(x,y)\,dy\right]dx$$

→ Exemplos

a) Cubo de Aresta 2

• Passo 01





• $D=\{(x,y)\in\mathbb{R}^2 : 0\le x\le 2 \wedge 0\le y\le 2\}$

• $S=\{(x,y,z)\in\mathbb{R}^3 : (x,y)\in D \wedge 0\le z\le 2\}$

• Passo 02

$$V(S) = \iint_D f(x,y)\,dy\,dx$$

$$= \int_0^2\left[\int_0^2 2\,dy\right]dx$$

$z=f(x,y)$
$z=2$

• Passo 03

$$V(S) = \int_0^2\left[\int_0^2 2\,dy\right]dx = 2\int_0^2\int_0^2 1\,dy\,dx =$$

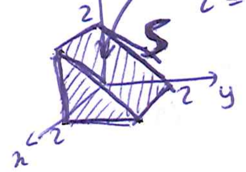
$$= 2\int_0^2 [y]_0^2\,dx = 2\int_0^2 (2-0)\,dx =$$

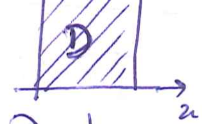
$$= 4\int_0^2 1\,dx = 4\,[x]_0^2 = 4\times 2 = 8$$

(2-0)

$$\boxed{V(S) = 8}$$

C.f.m.





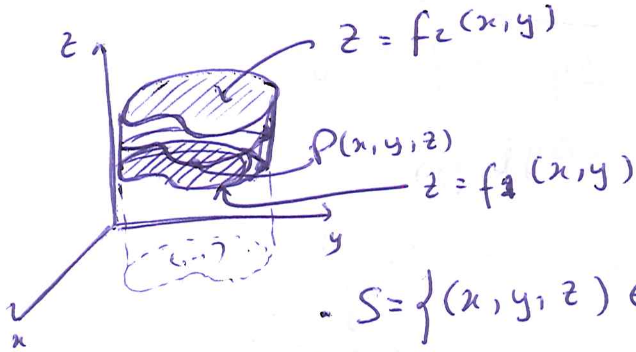
$D = \{(x,y) \in \mathbb{R}^2 : 0 \le x \le 2 \wedge 0 \le y \le 2\}$

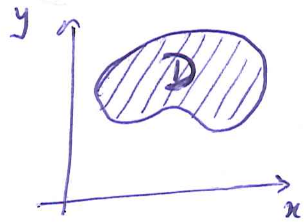
$S = \{(x,y,z) \in \mathbb{R}^3 : (x,y) \in D \wedge 0 \le z \le -y+2\}$

$$V(S) = \int_0^2 \int_0^2 -y+2 \, dy\, dx = \int_0^2 \left[-\frac{y^2}{2} + 2y \right]_0^2 dx = \;\Big|\; \boxed{V(S)=4}$$

$$= \int_0^2 2 \, dx = 2 [x]_0^2 = 2 \times 2 = \underline{\underline{4}}$$

Cfm



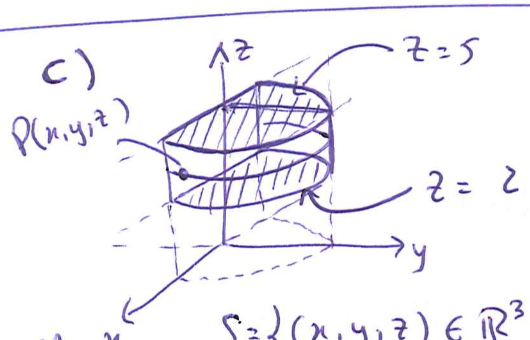


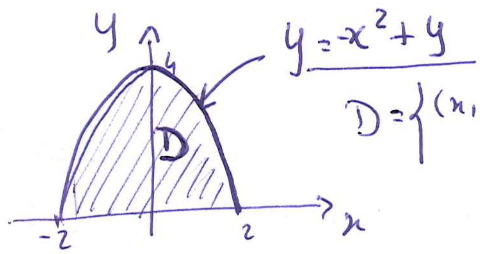
- $S = \{(x,y,z) \in \mathbb{R}^3 : (x,y) \in D \wedge f_1(x,y) \le z \le f_2(x,y)\}$
- $V(S) = \iint_D (f_2(x,y) - f_1(x,y))\, dy\, dx$
- $V(S) = V(S_2) - V(S_1) = \iint_D f_2(x,y)\, dy\, dx - \iint_D f_1(x,y)\, dy\, dx$

$$= \iint_D f_2(x,y) - f_1(x,y)\, dy\, dx$$

---

c)





$D = \{(x,y) \in \mathbb{R}^2 : -2 \le x \le 2 \wedge 0 \le y \le 4\}$

. <u>Passo 01</u> $S = \{(x,y,z) \in \mathbb{R}^3 : (x,y) \in D \wedge 2 \le z \le 5\}$

. <u>Passo 02</u>

$$V(S) = \iint_D (f_2(x,y) - f_1(x,y))\, dy\, dx = \iint_D 5-2\, dy\, dx = 3 \int_{-2}^{2} \int_0^{-x^2+4} 1 \, dy\, dx =$$

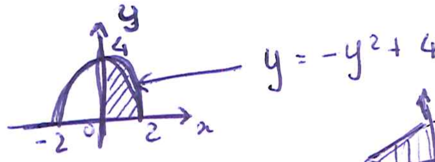
[redacted] $= 3 \int_{-2}^{2} [y]_0^{-x^2+4} dx = 3 \int_{-2}^{2} -x^2+4 \, dx$

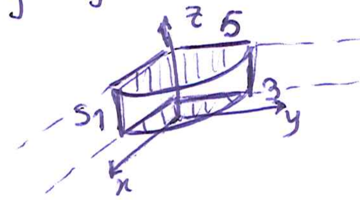
$$= 3 \left[-\frac{x^3}{3} + 4x \right]_{-2}^{2} = 3 \left(\left(-\frac{2^3}{3} + 8 \right) - \left(-\frac{(-2)^3}{3} - 8 \right) \right) = 3\left(-\frac{8}{3}+8\right) - \left(\frac{8}{3} - 8\right) =$$

$$= 3 \left(-\frac{8}{3} + \frac{24}{3} - \frac{8}{3} + \frac{24}{3} \right) = 3 \left(\frac{16}{3} + \frac{16}{3} \right) = 3 \left(\frac{32}{3} \right) = \underline{\underline{32}}$$

c) Simplificação

→ Processo Ø1: Simetria em S?





Sim → $V(S) = 2\,V(S_1)$

$$= 2\int_0^2 \int_0^{-x^2+4} (5-2)\,dy\,dx =$$

$$= 6\int_0^2 [y]_0^{-x^2+4}\,dx = 6\int_0^2 -x^2+4\,dx = 6\left[-\frac{x^3}{3} + 4x\right]_0^2 =$$

$$= 6 \times \left(-\frac{2^3}{3} + 4 \times 2\right) = 6 \times \left(-\frac{8}{3} + 8\right) = 6 \times \frac{16}{3} = \frac{96}{3}$$

$V(S) = 32$

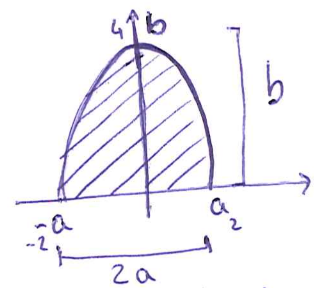
→ Processo Ø2

. $V(S) = A(\text{base}) \times \text{altura}$

$= \frac{4}{\cancel{3}} \times 2 \times 4 \times \cancel{3} =$

$= 4 \times 2 \times 4 = 32$

$V(S) = 32$



$A(\blacktriangle) = \frac{4}{3}ab$