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EE 342: Classical Control Systems Laboratory
Winter 2024
Instructor: Siavash Farzan
Lab Assignment 5: Lead Compensation for the Pendubot's Inverted Pendulum System

5.1. Introduction

Building on our understanding of Root Locus analysis from Lab 4, this lab assignment focuses on applying Lead Compensation techniques to improve the performance of control systems. Lead Compensation is a crucial control strategy for enhancing system response characteristics such as transient response, stability, and bandwidth. In this lab, students will explore the design of a lead compensator for Pendubot, exploring its impact on system performance and stability.

The process involves designing and implementing a lead compensator for the Pendubot's *inverted* pendulum control system. Lead Compensation modifies the system's pole-zero configuration in the s-domain to improve response times, increase stability margins, and enhance transient performance without significantly increasing system order or complexity. A Lead Compensator introduces a pole and a zero to the system, with the zero placed closer to the origin in the s-plane, leading to faster and more stable system responses.

This lab introduces a forward loop compensator – a controller that adjusts the control error e (the difference between the desired output and the actual output) to generate a control input for the inverted pendulum plant. Positioned just ahead of the plant, as illustrated in Figure 1, it is sometimes referred to as a cascade compensator.

With the system's output θ_{pend} directly compared with the input $\theta_{\text{reference}}$, the architecture embodies a unity-gain feedback system, where $H(s) = 1$. Unlike the proportional-derivative controller from Lab Assignment 2, which was adjusted by varying K_p and K_d gains, the compensator adds an additional zero and pole to the open-loop transfer function. The zero is positioned at a lower frequency than the pole, allowing for finer adjustments of their locations. This capability enables targeting specific stability properties, overshoot ratios, and response times, providing a borader range of control over the system's transient behavior.

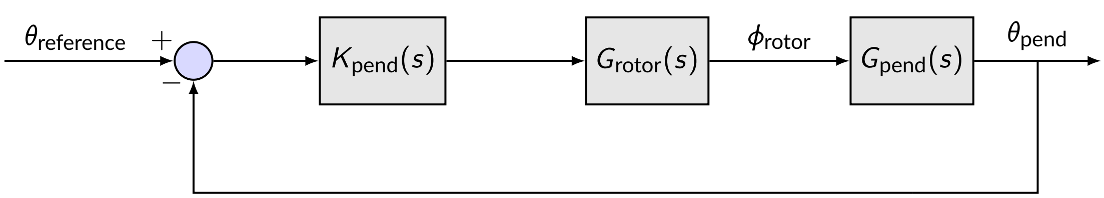


Figure 1: Pendulum angle control system.

The loop transfer function is derived by multiplying the compensator by the plant and sensor transfer functions. The root locus plot in Figure 2 shows how the locus for the dominant pole pair moves further into the left half-plane, indicating an increase in the system's natural frequency and a decrease in transient response time constants while maintaining the desired overshoot ratio.

The lab provides hands-on experience in designing a lead compensator using MATLAB's Control

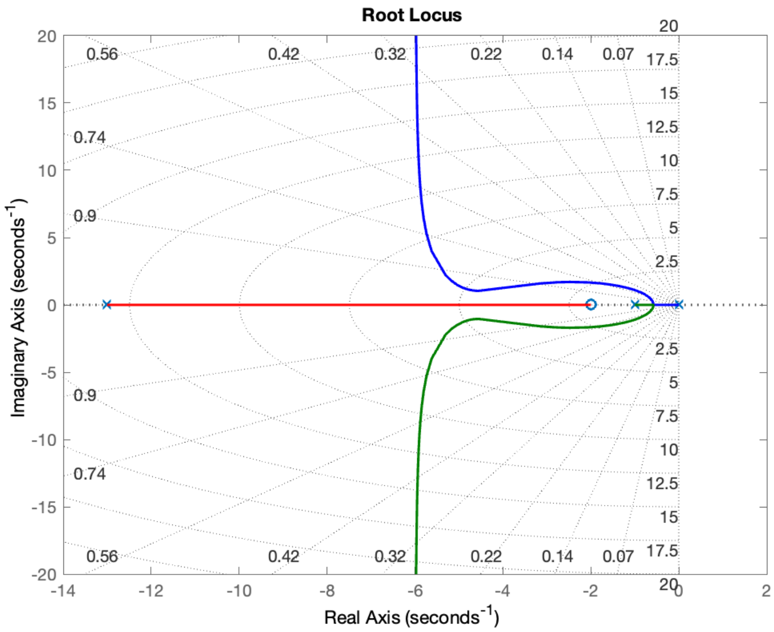


Figure 2: A compensated root locus plot via lead compensation..

System Designer tool. Students will see the effects of their designs on the system's stability, transient, and steady-state responses through simulations and analysis.

5.2. Lead Compensator Design for the Inverted Pendulum System

Part 1. Uncompensated System Analysis. Initially, we evaluate the stability and performance of the *uncompensated* inverted pendulum system. The system's block diagram is shown in Figure 3.

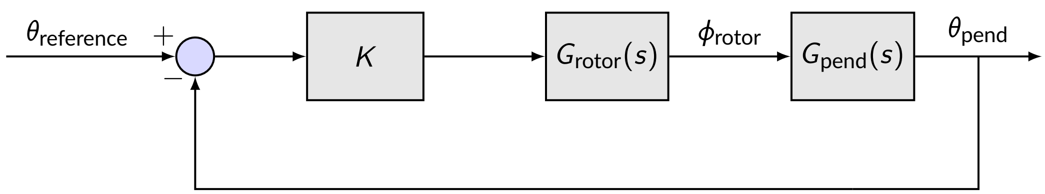


Figure 3: Pendulum angle control as an uncompensated system.

Use MATLAB to derive the uncompensated system's loop transfer function $KG(s)H(s)$, representing proportional control. Utilize the numerical plant models for the rotor $G_{\text{rotor}}(s)$ and the inverted pendulum $G_{\text{pend}}(s)$ blocks specified in Lab Assignment 2, Section 2.2.

Then, employ MATLAB's `controlSystemDesigner()` (as seen in Lab Assignment 4) to examine the system's root locus, stability, damping, and transient response. The goal is to configure the uncompensated system to be underdamped with a percent overshoot of $p.o. = 15\%$, corresponding to a specific damping ratio ζ.

Determine the required gain K to achieve the targeted percent overshoot/damping ratio. Additionally, identify the system's natural frequency ω_n and settling time t_s. Evaluate the feasibility of designing a gain for the uncompensated system that allows a shorter settling time without altering the damping ratio/percent overshoot.

In your report, summarize your findings, including the required gain, damping ratio, percent overshoot, natural frequency, settling time, peak time, and rise time. Include Root Locus and Step Response plots to support your analysis. Discuss the potential to achieve a shorter settling time with the specified percent overshoot, referencing your MATLAB findings.

Part 2. Compensated System Analysis The focus now shifts to analyzing and enhancing the inverted pendulum system's stability and performance with a lead compensator, defined as:

$$C(s) = \frac{K(s+a)}{s+b}$$

Refer to Figure 4 for the system's diagram with lead compensation.

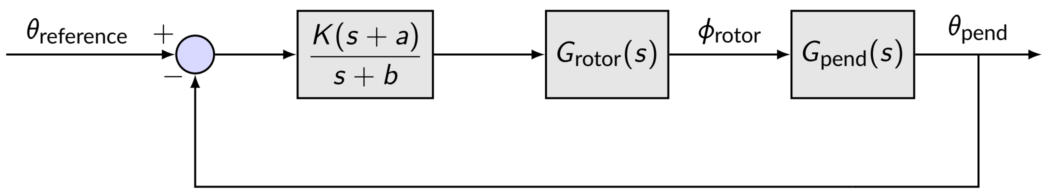


Figure 4: Pendulum angle control system with Lead Compensation.

Derive the loop transfer function $C(s)G(s)H(s)$ for the compensated inverted pendulum system. The design goal is to adjust K, a, and b to meet specific closed-loop system specifications:

i) The magnitude of the compensator pole $s = -b$ must be at least three times of the uncompensated plant pole on LHP (for $K = 0$).

ii) The magnitude of the compensator zero $s = -a$ should be at least 1.25 times that of the uncompensated plant pole on LHP (for $K = 0$, i.e. avoid canceling the plant pole with the compensator zero).

iii) The percent overshoot of $10\% < p.o. < 15\%$.

iv) Settling time of $0.75 < t_s < 1$ seconds.

v) Ensure the natural frequency ω_n is at least twice the uncompensated system's value (refer to your root locus plot for Part 1).

Open the Control System Designer with the uncompensated system in Part 1. Then, use the Control System Designer tool to add the compensator's zero and pole to meet the specifications above.

Hint: Click on the "Root Locus Editor" and then use the icons "Add real zero" and "Add real pole" in the "MODIFY POLES & ZEROS" menu bar (see the figure below) to shape the locus appropriately.

On the Control System Designer's root locus plot, adjust the positions of the new pole and zero with your mouse, aiming to shape the desired root locus. Your goal is to deflect the locus towards the left, which will increase the natural frequency ω_n for a given damping ratio ζ.

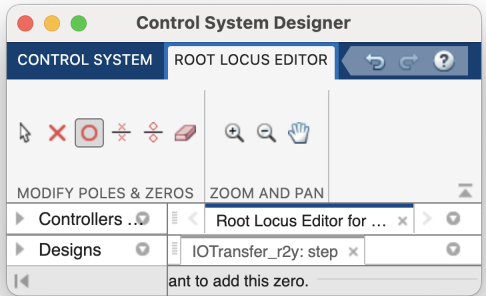


Figure 5: Adding a pole or zero to a root locus using MATLAB's Control System Designer tool.

Once the locus aligns with the above specifications, drag the closed-loop poles along this locus to achieve the specified overshoot. After positioning the closed-loop poles to match the desired overshoot, calculate the forward loop gain K corresponding to these locations.

As discussed in lectures, if the dominant pole approximation holds, the intersection point of a line at the required pole angle (θ) from the negative real axis to the locus indicates the necessary value of K to achieve the target damping ratio ζ.

With the compensator, the system shifts from a second-order all-pole to a third-order with an additional zero. If the compensator pole is far enough into the left half-plane, its transient response impact is minimal. However, the zero significantly influences the step response due to its proximity to the dominant poles.

Despite the system's increased order, the second-order system formulas relating rise time (t_r), peak time (t_p), percent overshoot $p.o.$, and settling time (t_s) to ζ and ω_n (as seen in Lab Assignment 2) can still offer a reasonable step response approximation. Use these formulas with your ω_n and ζ values from the Control System Designer to estimate your closed-loop system's performance metrics.

> Document your compensator design process, including calculations and MATLAB configurations in your report. Include the final root locus plot with measurements of ζ and ω_n. Provide estimated overshoot ratio, rise time, peak time, and settling time based on your design.

After designing the lead compensator and adjusting the system's parameters to meet the desired specifications, analyze the step response of θ_{pend} using the Control System Designer tool. This tool simulates a *unit* step input at $\theta_{\text{reference}}$, but for the purposes of this lab, you may rescale the plot for a 20° step input. Ensure the time duration is long enough for the system to reach a steady state. Pay attention to the steady-state error (e_{ss}) in the step response, as it will be the focus of our next lab assignment.

> Include the step response plot in your report. Clearly mark and measure t_r, t_p, $p.o.$, t_s, and e_{ss} on the plot.

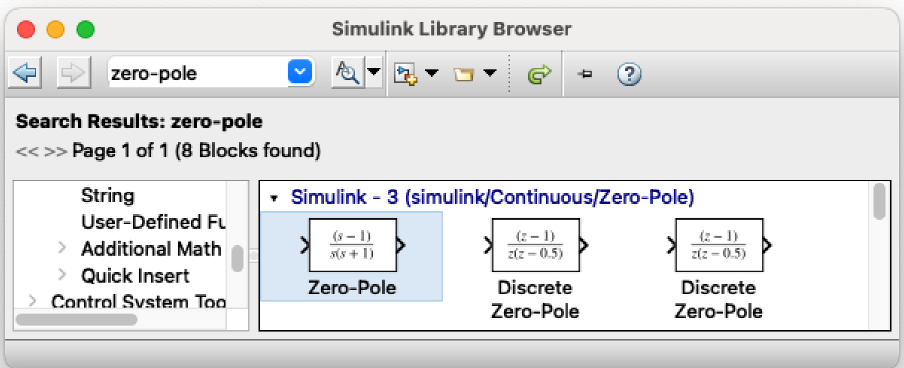
5.3. Simulation of the Inverted Pendulum Transient Behavior Under the Lead Compensator

i. Download the Simulink model "Pendubot_Inverted_Controller_Simulink.slx" from Canvas and open it in MATLAB.

ii. Ensure that the G_{rotor} and G_{pend} blocks reflect the correct numerical values as determined in Section 2.2 of Lab Assignment 2.

iii. Set the PD gains in the K_{rotor} block to zero to deactivate the rotor plant control.

With the loop gain, compensator zero, and pole locations determined to meet the design specifications, you will now implement the compensator in Simulink. Use the "Zero-Pole" block available in Simulink to set up your compensator with the specified values. You need to replace the existing PD block of K_{pend} with the new Compensator block.



Utilize the Simulink model and the compensator design to generate Step Response plots (with a step magnitude of $20°$), clearly indicating the Peak Time (t_p), Percent Overshoot ($p.o$) and Settling Time (t_s).

Hint: To apply a step input, activate the *"Pendulum Tracking Angle Step"* input in Simulink, ensuring all other input switches are disabled.

> Include the Step Response plot generated from your Simulink simulation in your report. Clearly indicate and measure (t_p), Percent Overshoot ($p.o$) and Settling Time (t_s) on these plots.

Finally, adjust the zero and pole locations (and possibly the gain value) by trial and error until the system meets the overshoot and settling time specifications. Try to get the step response as close as possible to the outcomes predicted during the design phase.

> Include the Step Response plot generated from your Simulink simulation for the tuned system in your report. Clearly indicate and measure (t_p), Percent Overshoot ($p.o$) and Settling Time (t_s) on these plots.

Summarize your results from your Control System Designer and Simulink simulation in the table below, showing $p.o.$, t_s, t_r and t_p found for each case.

	t_r (sec)	t_s (sec)	t_p (sec)	$p.o.\%$
Dominant pole pair approximation				
Control System Designer				
Simulink Simulation (Initial)				
Simulink Simulation (Tuned)				

5.4. Submission

- Lab Report: group submission via Canvas. Submit your report as a single .pdf file.
- Report Contents: your report must include all the plots, data, and results asked throughout the assignment, along with their description and detailed analysis. Here are the essential contents included in each section of the lab report:
  - **Introduction:** Briefly describe the main objectives and problems investigated in the lab and provide relevant context, key terms, and concepts so your reader can understand the report.
  - **Development and Implementation:** Provide details on the theory, derivation, design and implementation of your algorithms and programs, include any diagrams, schematics, flowcharts, or code blocks that help to illustrate your derivation and design, and discuss the experimental procedures. Clearly describe the steps you took, what decisions you made and why, and your system architecture.
  - **Results:** Present the data, plots, and measurements that you obtained during your experiments, include any relevant graphs, tables, or images to help display your results, and explain any trends or patterns that you observed in your data along with appropriate engineering analysis. Feel free to further elaborate or add additional plots and results if it helps with your explanation.
  - **Discussion:** Interpret the results of your experiments, and explain how they relate to your objectives and goals, discuss any limitations or challenges that you encountered during your lab work, suggest any future work or improvements that could be made to your design and implementation, and summarize your main findings and their implications.
  - **Conclusion:** Follows naturally from the discussion, includes suggestions and comments on the lab.
- Due: as specified on Canvas.