Université de Bourgogne
UFR Sciences et Techniques
L2 - Elec4A

Licence Sciences Technologies Santé
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# **TD 1 & 2** – *Frequency Signal Analysis and Filtering*

- **Question 1** – *Frequency analysis of 1D signals*

**1.1** Calculate the frequency representation (spectrum) of the rectangular signal $f_1(t)$, with $a = 1$ :

$$f_1(t) = \begin{cases} \frac{1}{a}, & \text{if } t \in \left[-\frac{a}{2}, \frac{a}{2}\right] \\ 0, & \text{otherwise} \end{cases}$$

**1.2** Calculate the frequency representation (spectrum) of the signal $f_2(t)$, with $a = 1$ :

$$f_2(t) = \begin{cases} \frac{1}{a}, & \text{if } t \in \left[-\frac{a}{2}, 0\right] \\ -\frac{1}{a}, & \text{if } t \in \left[0, \frac{a}{2}\right] \\ 0, & \text{otherwise} \end{cases}$$

**1.3** Plot the two spectra of the two windows. Interpret these spectra by indicating which window corresponds to a "high-pass" and which to a "low-pass".

- **Question 2** – *Frequency analysis of images*

**2.1** Calculate the frequency representation (spectrum) of the 2D box filter rectangular signal with sides $X$ and $Y$:

$$f(x,y) = \begin{cases} \frac{1}{XY}, & \text{if both } x \in \left[-\frac{X}{2}, \frac{X}{2}\right] \text{ and } y \in \left[-\frac{Y}{2}, \frac{Y}{2}\right] \\ 0, & \text{otherwise} \end{cases}$$

$$g(x,y) = \begin{cases} 4\frac{1-|xy|}{XY}, & \text{if both } x \in \left[-\frac{X}{2}, \frac{X}{2}\right] \text{ and } y \in \left[-\frac{Y}{2}, \frac{Y}{2}\right] \\ 0, & \text{otherwise} \end{cases}$$

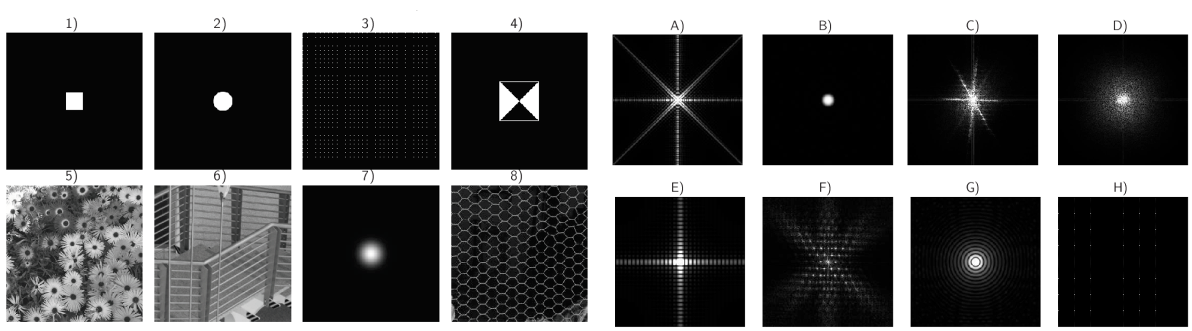
**2.2** Calculate the frequency representation (spectrum) of the 2D Gaussian centered at the origin and with isotropic standard deviation $\sigma$. What can you conclude about the relation between the two in the spatial and frequency domains? What happens when we increase $\sigma$? Please provide a discussion in terms of low and high pass filters.

**2.3** Find the frequency representation (spectrum) of the 2D signal $f(x,y) = \delta(x,y)$.

**2.4** Find the frequency representation (spectrum) and display a visualization of spatially and on frequency (magnitude) of the 2D signal

$$f(x,y) = \frac{1}{2}\left(\delta(x, y-a) + \delta(x, y+a)\right)$$

- **Question 3** – *Please match the following images with their respective Fourier spectrum.*



- **Question 4** – *Convolution in frequency*

Please consider the following functions:

- $f(x) = e^{-x^2}$,(Gaussian)
- $g(x)$ (Box Filter):

$$g(x) = \begin{cases} 1 & \text{for } |x| \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

- $h(x) = \delta(x)$ (Dirac delta function).

**4.1** Compute the Fourier transforms of the Gaussian function $\mathcal{F}\{f(x)\}$, the box filter $\mathcal{F}\{g(x)\}$, and of the Dirac delta $\mathcal{F}\{h(x)\}$.

**4.2** Compute the Fourier transform of the convolution of the Gaussian and the box filter $(f * g)(x)$, using the convolution theorem.

- **Question 5** – *Sampling and Discrete Fourier Transform*

**5.1** What sampling frequency should be chosen for the signal for not loosing information when performing reconstruction $x(t) = 2\sin(2\pi f_1 t)\sin(2\pi 7 f_1 t)$?

**5.2** Let $s[k]$ be a discrete signal of length $N$ and sampling frequency $f_e$. Establish the frequency dictionary that will allow the representation of the frequency content of this signal using the DFT.

**5.3** Illustrate the form of the result (the spectrum) of the Discrete Fourier Transform (DFT) in magnitude for the signal $x[k] = \sin(2\pi 20 k f_e)$.

**5.4** Given the discrete signal $x[n] = \{1, 1, 1, 1\}$ (i.e., a constant sequence of length 4):
(a) Compute the DFT of $x[n]$.
(b) Verify the result by applying the inverse DFT.

**5.5** Let $y[n] = \{1, -1, 1, -1\}$. Compute the DFT of $y[n]$ and explain the obtained result.