Theoretical and practical study about fuzzy rule based systems and their activation methods

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Overview

Introduction

This work presents the Moser-Navara axioms, that is an attempt of axiomatization for the design of Fuzzy Inference Systems (FIS) that has a mathematical foundation instead of just basing in empirical results. Different FIS designs using conjunctive and implicative models and their link with Mathematical Morphology theory will be explored as well as their adherence to the aforementioned axioms. Finally, there are some experimental results of different FIS designs and a comparison of their performances.

Fuzzy Relations models

Conjunctive model

$$R_*^{\vee}(x, y) = \bigvee(A_i(x) * B_i(y))$$

where $*$ is a left-continuous t-norm.

Implicative model

$$R_{\rightarrow}^{\wedge}(x, y) = \bigwedge(A_i(x) \rightarrow B_i(y))$$

where $\rightarrow$ is a residual implication.

Fuzzy Inference compositions

Compositional Rule of Inference (sup-$*$ composition)

$$B'(y) = (A' \circ R)(y) = \bigvee(A'(x) * R(x, y))$$

Bandler-Kohout subproduct (inf-$\rightarrow$ composition)

$$B'(y) = (A' \triangleleft R)(y) = \bigwedge(A'(x) \rightarrow R(x, y))$$

Moser-Navara Axioms

1. For all $i \in 1, \ldots, n$
$$A_i \circ R_*^{\vee} = B_i$$

2. For each normal input $A' \in \mathcal{F}(X)$ there is an index i such that
$$A' \circ R_*^{\vee} \not\subseteq B_i$$

3. Output $A' \circ R_*^{\vee}$ belongs to the union of consequents B_i of "fired rules"
$$A' \circ R_*^{\vee} \subseteq \bigcup_{i \in F} B_i$$
$$F = \{i | Supp(A_i) \cap Supp(A') \neq \emptyset\}, (B_i \cup B_j)(y) = B_i(y) \vee B_j(y)$$

The similar results for implicative model $R_{\rightarrow}^{\wedge}$ follow in the same way.

Interpretation of Axioms

A literal interpretation of these axioms could be:

1. The fuzzy relation $R(x, y)$ should present recall (like an autoassociative memory)
2. For normal inputs, the system gives significant outputs with non-trivial information
3. If the input of the FIS is contained in the universe of antecedents, there should be a non-zero output for it

FIS Rule activation using Similarity measure

FIS Rule activation using Subsethood measure

Parking problem

References