(3) $\dot{x}_e = A x_e + B u_2 - B u_1$

$\hookrightarrow$ external disturbance

$$x_e = \begin{bmatrix} e \\ \dot{e} \end{bmatrix}$$

$u_2 = -K x_e$

$u_1(t)$ is bounded

$V(x_e) = x_e^T P x_e > 0$ good

$P > 0 \Rightarrow (A - BK)^T P + P(A - BK) = -Q_1 > 0$

$$\dot{V}(x_e) = \dot{x}_e^T P x_e + x_e^T P \dot{x}_e$$

$$= 2 x_e^T P \dot{x}_e$$

$$= 2 x_e^T P (A x_e + B u_2 - B u_1)$$

$$= 2 x_e^T P ((A - BK) x_e - B u_1)$$

$$= 2 x_e^T P (A - BK) x_e - 2 x_e^T P B u_1$$

$$= x_e^T \underbrace{\left[(A-BK)^T P + P(A-BK) \right]}_{-Q_1} x_e - 2 x_e^T P B u_1 < 0$$

$$= \underbrace{- x_e^T Q_1 x_e}_{\text{always } < 0} - 2 x_e^T P B u_1 < 0$$

u_1 is bounded

$\|u_1\| \le u_{1\,max}$

$$\boxed{- x_e^T Q_1 x_e < 2 x_e^T P B u_1}$$

I have this condition

$u_2 = -K x_e + \hat{u}_1$ → estimate of the disturbance

$\dot{x}_e = A x_e + B u_2 - B u_1$

$\dot{\hat{u}}_1 = f(x_e)$

$x_e(t) \to 0$

$\tilde{u}_1(t) \to 0$

$\tilde{u}_1(t) = u_1(t) - \hat{u}(t)$

u_1 is const.

$V(x_e, \tilde{u}_1) = x_e^T P x_e + \frac{1}{2} \tilde{u}_1^2$

$\dot{V}(x_e, \tilde{u}_1) = 2 x_e^T P \dot{x}_e + \tilde{u}_1 \cdot \dot{\tilde{u}}_1$

$\dot{\tilde{u}}_1 = \underbrace{\cancel{\dot{u}_1}}_{=0} - \dot{\hat{u}}_1$

$= 2 x_e^T P \dot{x}_e - \tilde{u}_1 \dot{\hat{u}}_1$

$\dot{x}_e = A x_e + B(-K x_e + \hat{u}_1) - B u_1$

$= (A - BK) x_e + B \hat{u}_1 - B u_1$

$= (A - BK) x_e - B \tilde{u}$

$= 2 x_e^T P \left((A - BK) x_e - B \tilde{u} \right. - \tilde{u}_1 \cdot \dot{\hat{u}}_1$

$= 2 x_e^T P (A - BK) x_e - 2 x_e^T B \tilde{u} - \tilde{u}_1 \cdot \dot{\hat{u}}_1$

$= - x_e^T Q x_e - 2 x_e^T P B - \tilde{u}_1 \dot{\hat{u}} < 0$

0

$\Rightarrow \quad - 2 x_e^T P B \cancel{\tilde{u}_1} \quad \cancel{\tilde{u}_1} \dot{\hat{u}}$

$\Rightarrow \quad \dot{\hat{u}} = -2 x_e^T P B$