

# Computer Control

## Project Presentation

### **Authors**

Renato Loureiro

Tiago Santos

Pedro Sarnadas

# Model Estimation

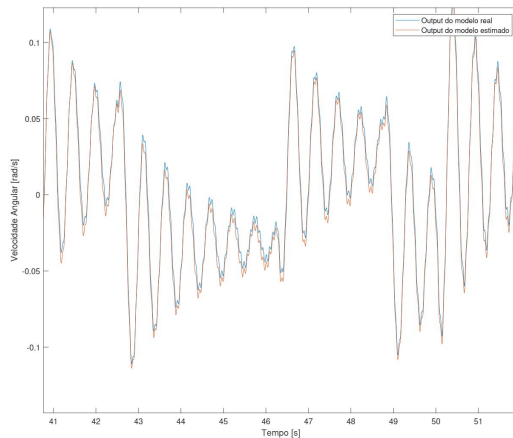
## ARMAX Parameters

$$n_a = 5 \quad n_b = 5$$

$$n_c = n_a \quad n_k = 1$$

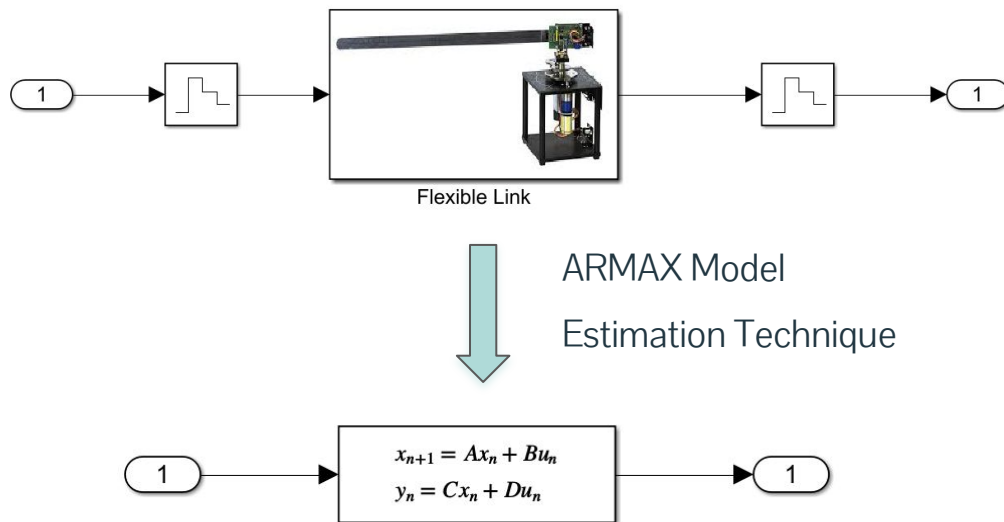
## Validation

PRBS B=0.008



## Simulation Parameters

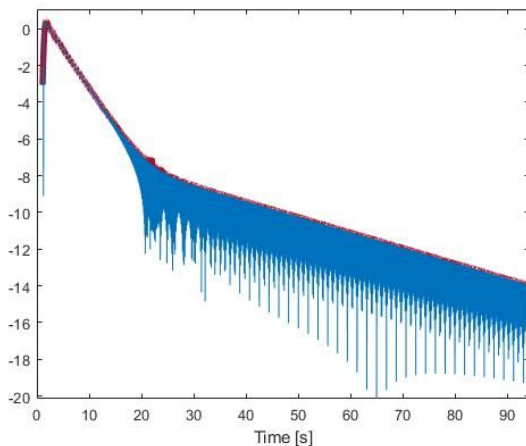
$$T_s = 0.01 \text{ s}$$



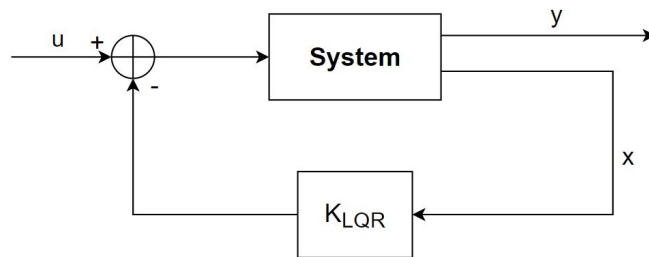
# LQR Implementation

Evaluate the envelope of the output

Eigenvalue confirmation - R = 50



$$\text{Slope} = -0.0842$$



$$u(k) = -K_{LQR}x(k)$$

$$J = \frac{1}{2} \sum_{k=0}^{\infty} [x^T(k)Qx(k) + Ru^2(x)]$$

Solve Riccati Equation

$$S = A^T \left[ S - SB^T BS \frac{1}{R} \right] A + C^T C$$

$$K_{LQR} = (R - B^T SB)^{-1} B^T SA$$

# LQR Implementation

Evaluate the envelope of the output

Eigenvalue confirmation - R = 50

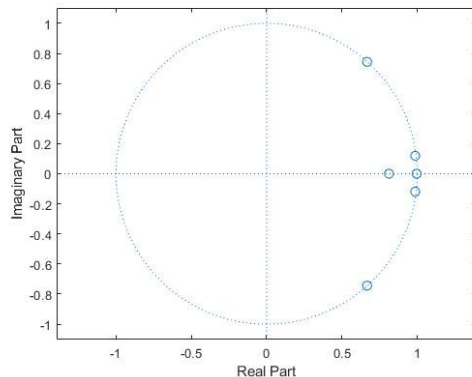
Theoretical Pole location

$$0.6669 \pm 0.7440i$$

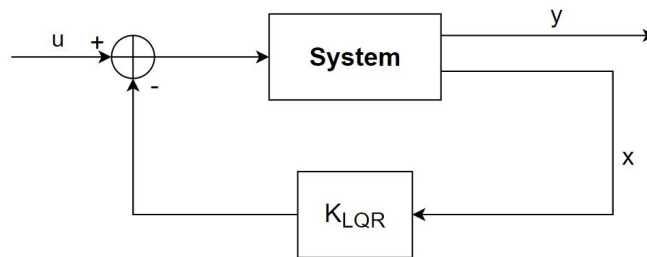
$$0.9958$$

$$0.9863 \pm 0.1192i$$

$$0.8122$$



$$\max(|\lambda_i|) = 0.9992$$

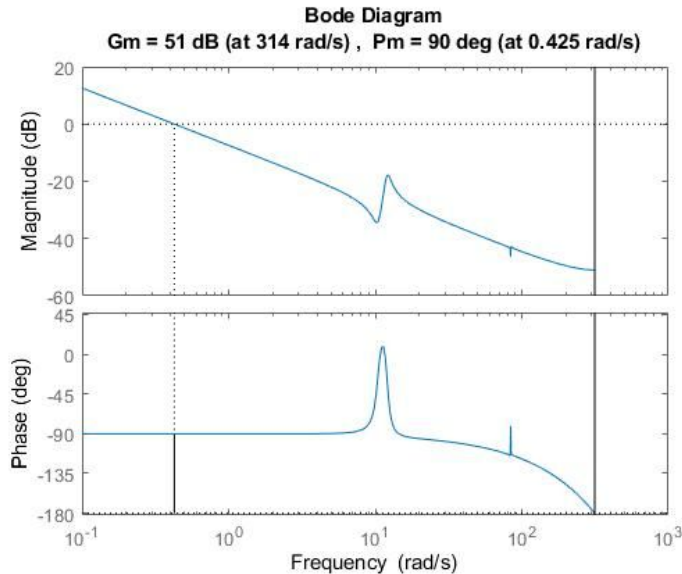


$$m \approx \frac{\log(\max\{|pole|\})}{T_s}$$

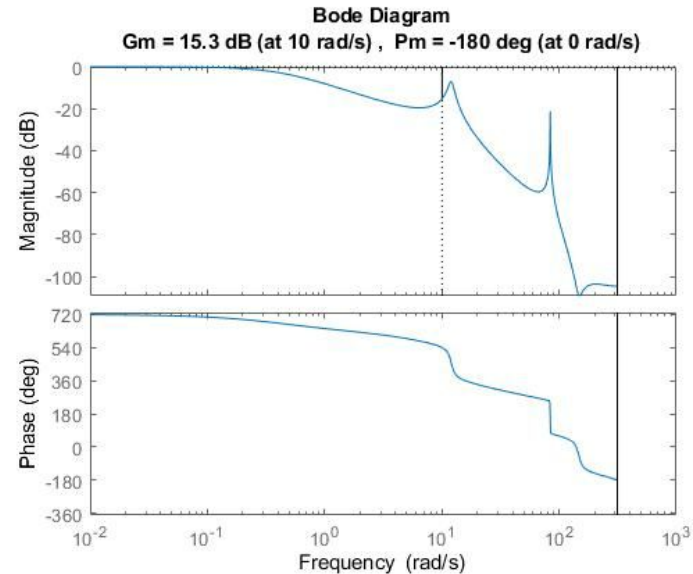
$$error = 4.8963 \cdot 10^{-7}\%$$

# LQR Implementation

## Open loop Bode Diagram for LQR Control



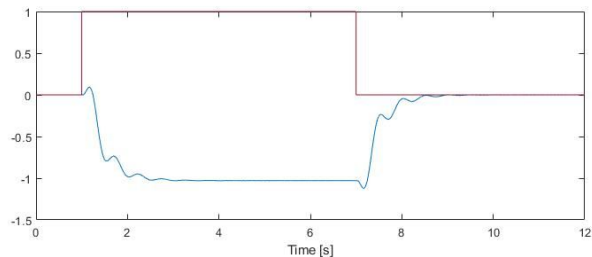
## Close Loop Bode Diagram for LQR Control



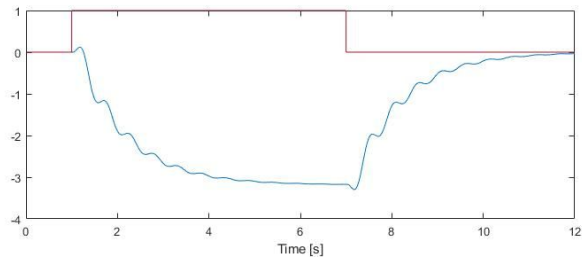
The close loop bode diagram has the reference tracking gain already

# LQR Implementation

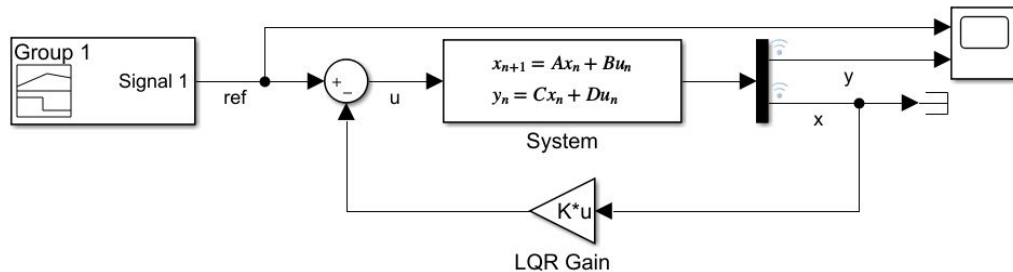
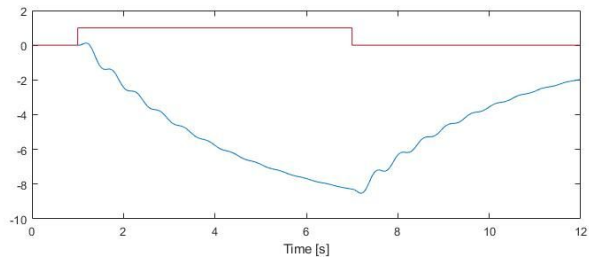
R=1



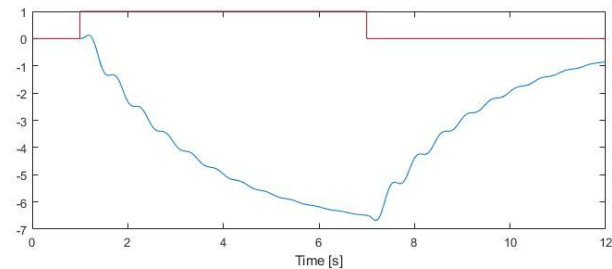
R=10



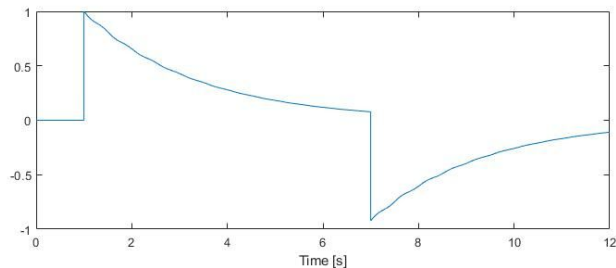
R=100



R=50



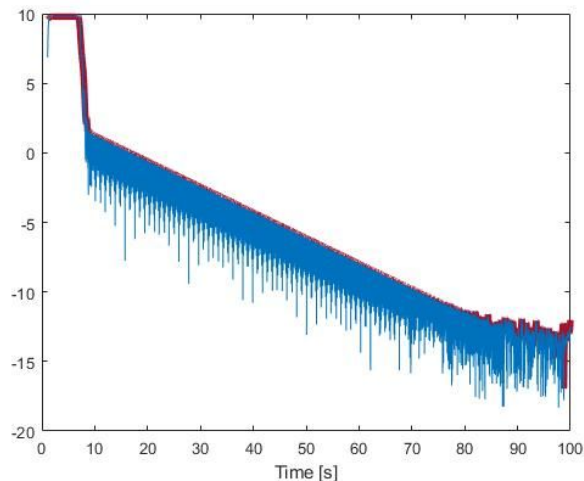
Tensão de  
entrada (V)



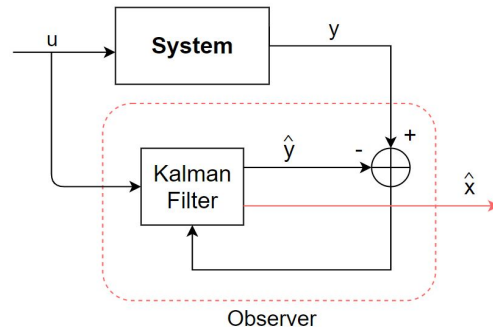
# LQE Implementation

Evaluate the envelope of the output

Eigenvalue confirmation - Re = 0.1



$$Slope = -0.1550$$



$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + B_1w(k) \\ y(k) &= Cx(k) + v(k) \end{aligned}$$

$$H(M) = E\{\|x(k) - \tilde{x}(k)\|^2\}$$

$$\begin{aligned} P &= APA^T + Q_e - \frac{APC^T CPA^T}{R_e + CPC^T} \\ M &= APC^T(R_e + CPC^T)^{-1} \end{aligned}$$

# LQE Implementation

Evaluate the envelope of the output

Eigenvalue confirmation - Re = 0.1

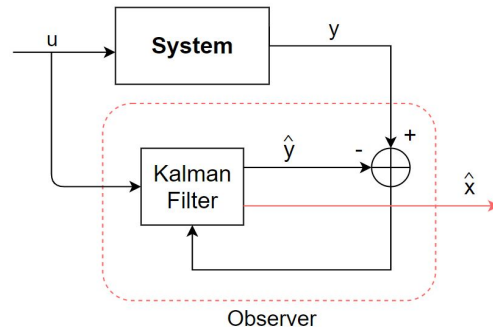
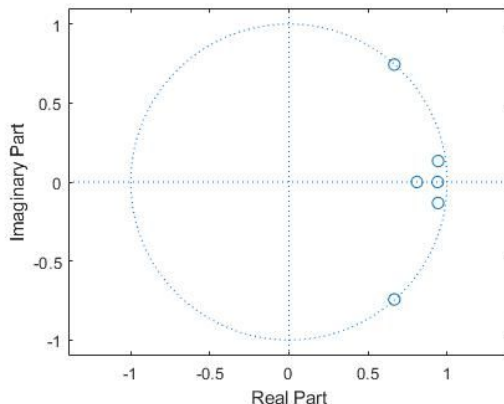
$$0.6662 \pm 0.7433i$$

$$0.9400$$

$$0.9430 \pm 0.1326i$$

$$0.8091$$

$$\max(|\lambda_i|) = 0.9982$$



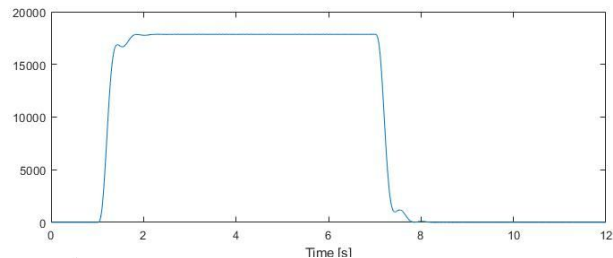
$$m \approx \frac{\log(\max\{|pole|\})}{T_s}$$

$$error = 0.1842 \%$$

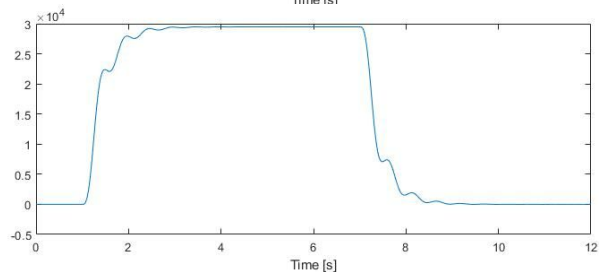


# LQE Implementation

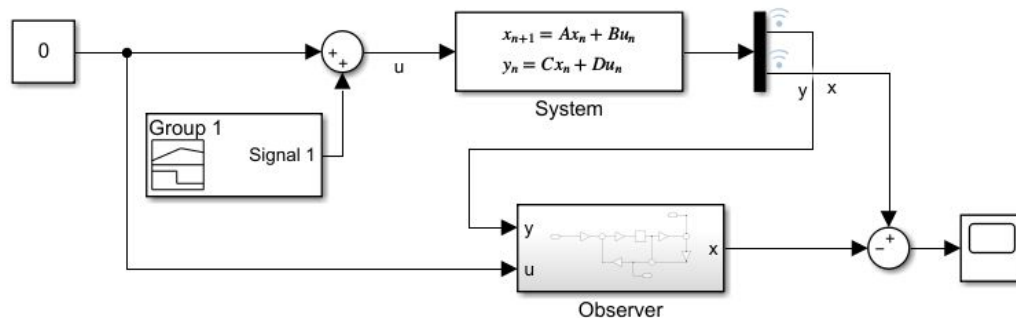
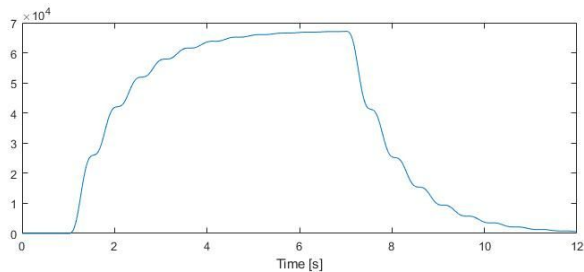
R=0.1



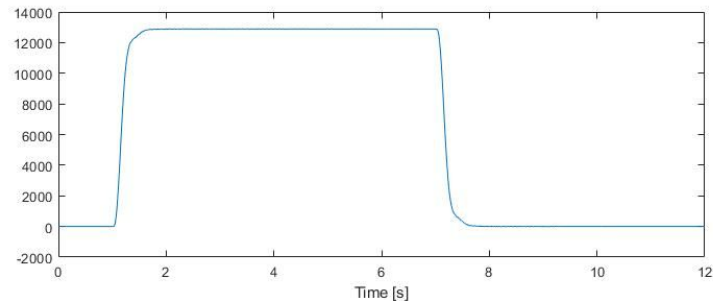
R=1



R=10



R=0.01



# LQG Implementation

Check poles of the closed loop

$$0.6662 \pm 0.7433i$$

$$0.6669 \pm 0.7440i$$

$$0.9430 \pm 0.1326i$$

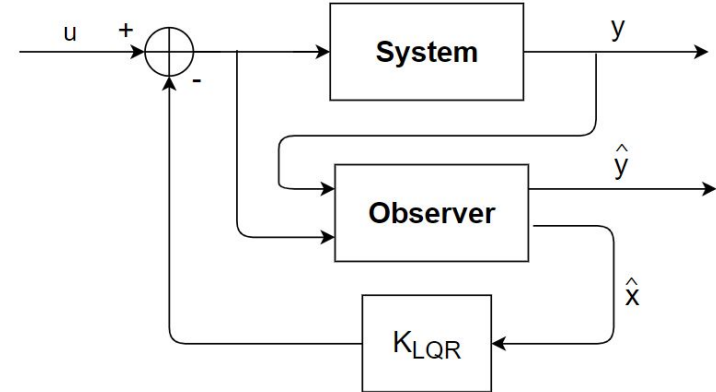
$$0.9863 \pm 0.1192i$$

$$0.9958$$

$$0.8122$$

$$0.9400$$

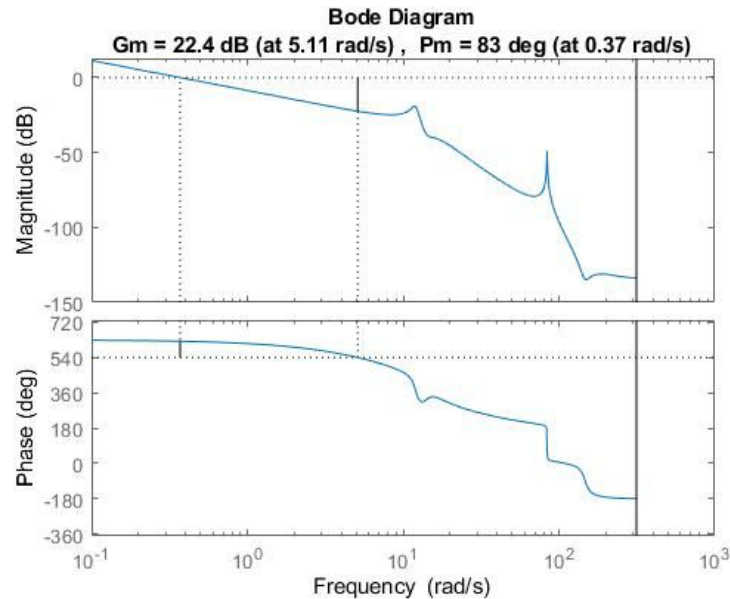
$$0.8091$$



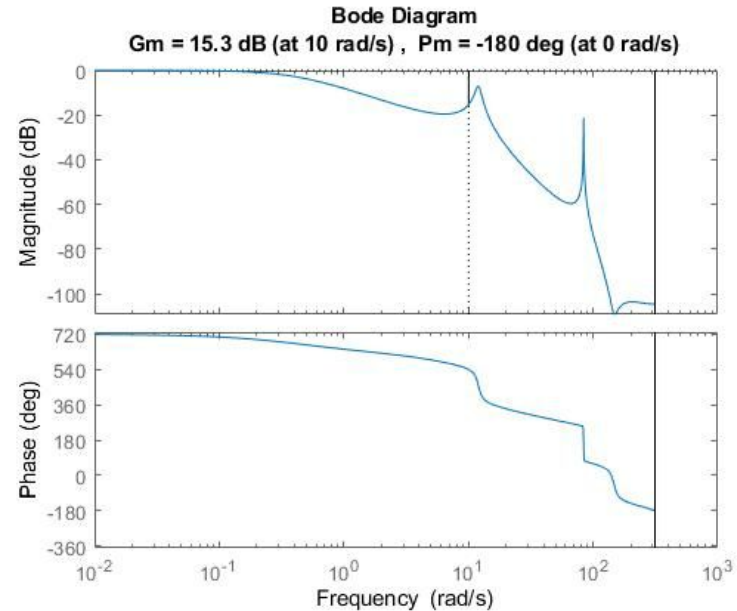
It is possible to notice the separation principle because the poles of the merge configuration (LQG) are the join poles of the LQR and LQE previously analyzed

# LQG Implementation

Open loop Bode Diagram for LQG Control

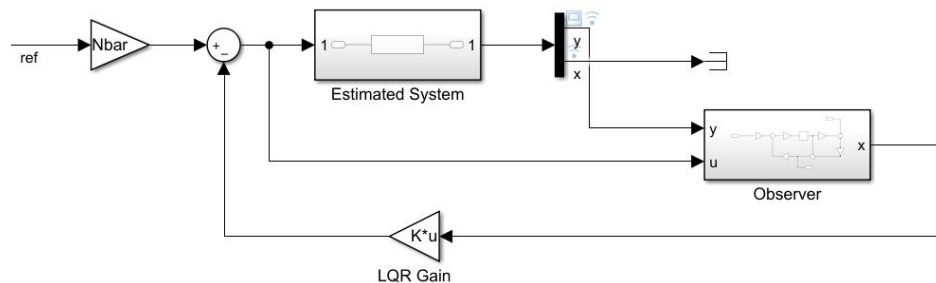
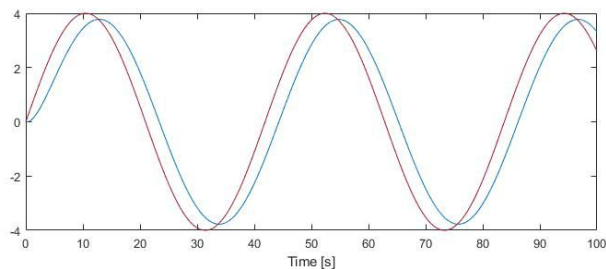
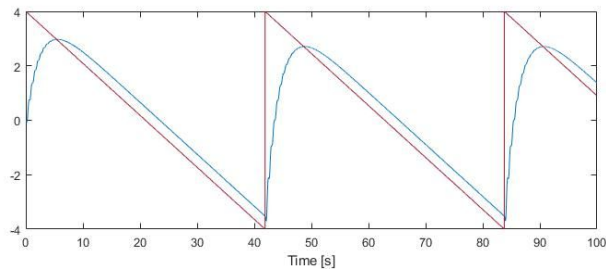
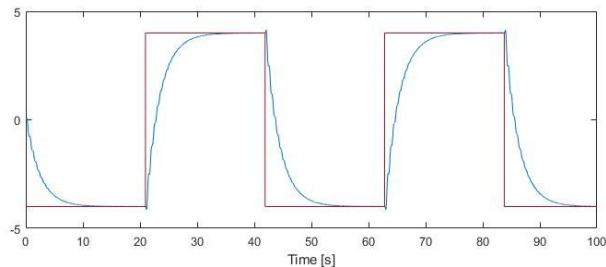


Close Loop Bode Diagram for LQG Control



The close loop bode diagram has the reference tracking gain already

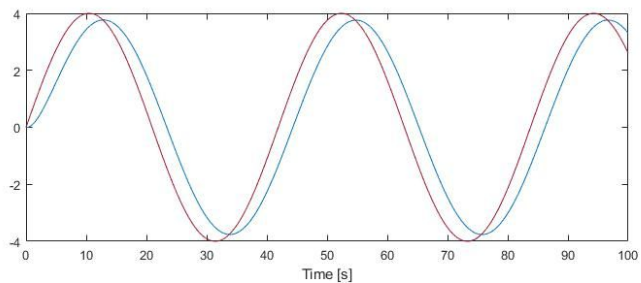
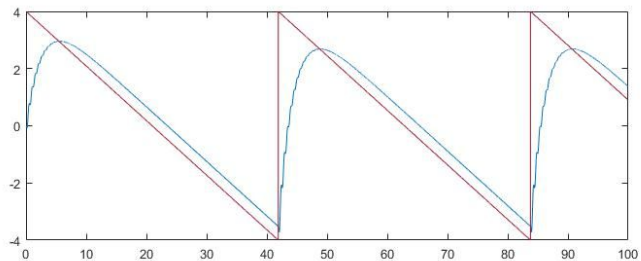
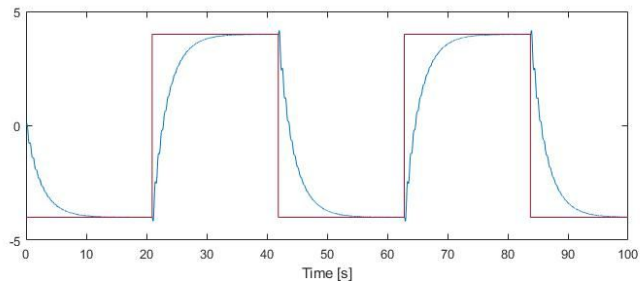
# Following Reference Approach



$$\begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} A - I & B \\ C & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix}$$

$$\bar{N} = N_u + K N_x$$

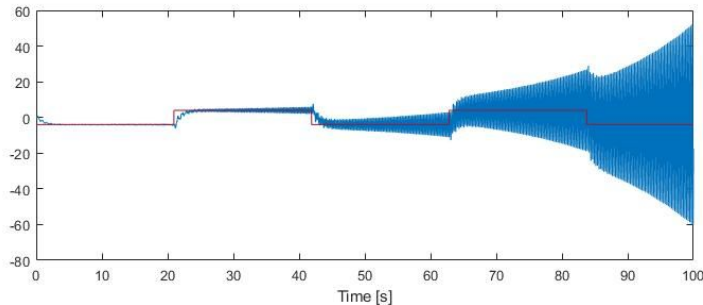
# Test on Real System



The controller works perfectly with the given real model but it may suffer a from the performance perspective if the real model has disturbances.

# Test on Perturbed System

Using the previous control and estimation in a perturbed system

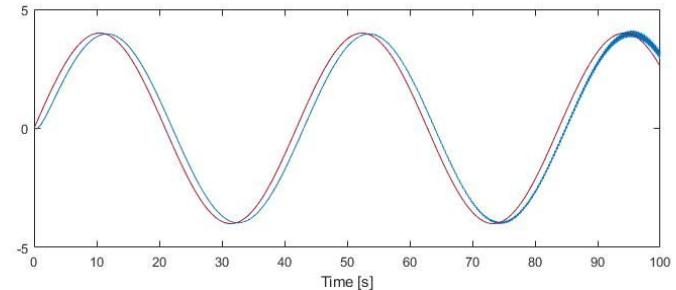


**Necessity to iterate - approach: be more conservative when choosing  $R$  and  $R_e$**

**First:** Change  $R_e$  and see if the problem is with the estimation optimization being too rigid

**Second:** Change  $R$  and see if the problem is with the control optimization being too optimistic

The system is unstable and it seems it is behaving with a high frequency - maybe smoothing the controller (make it a little bit more slower will solve the problem)

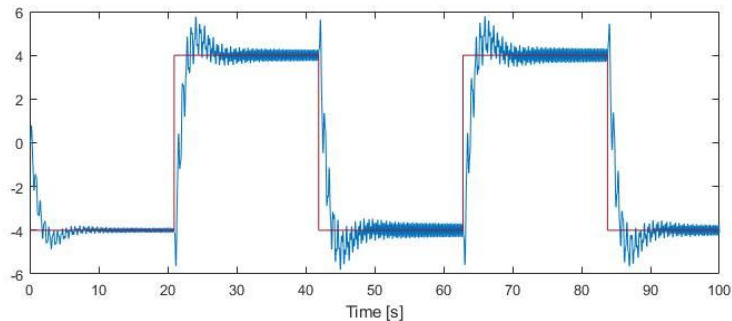


With the sine input the system has a better performance - it made us think the problem with our controller is with the high frequency response

# Test on Perturbed System

Re = 20

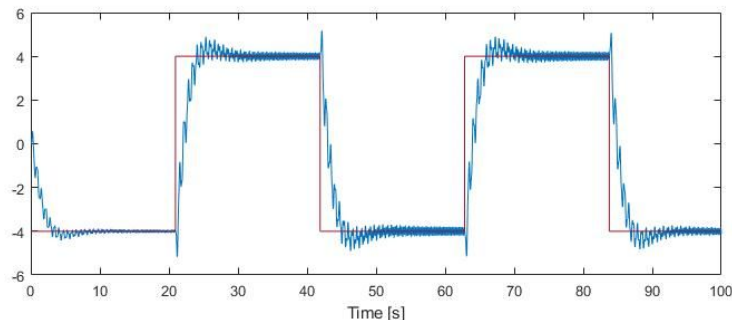
R = 50



It seems we were too optimistic with the value given to our Re related to the estimation process

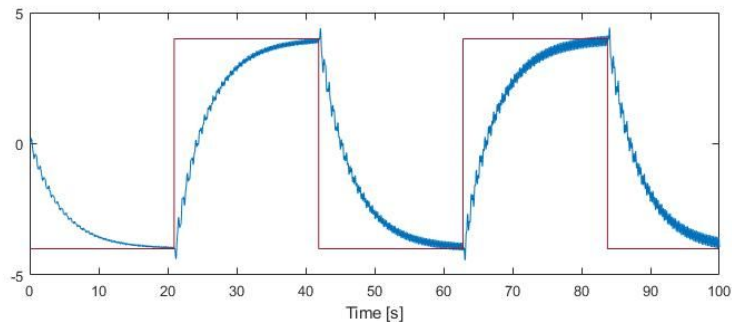
Re = 10

R = 100



Re = 0.1

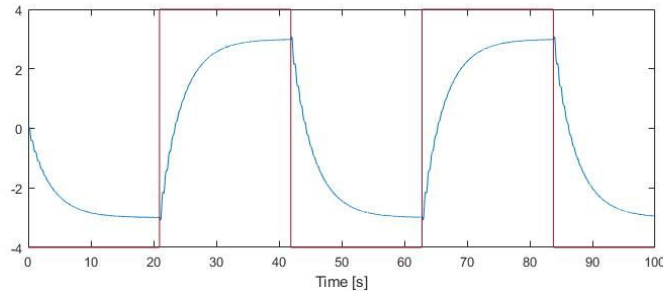
R = 800



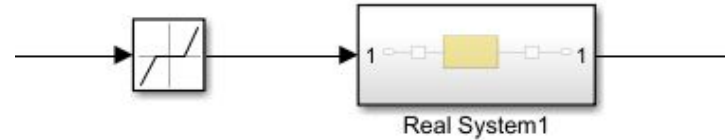
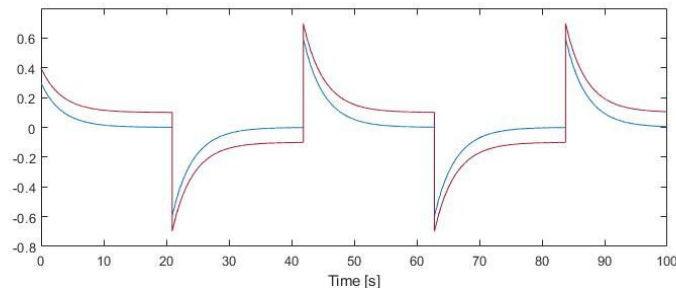
The previous Graphs correspond to the value of the parameter which we consider the plant to work correctly

# Evaluate Dead Zone Control Dynamic Problem

## Output Behaviour



## Control Input Behaviour



The current controller is unable to track the reference when the dead zone is present



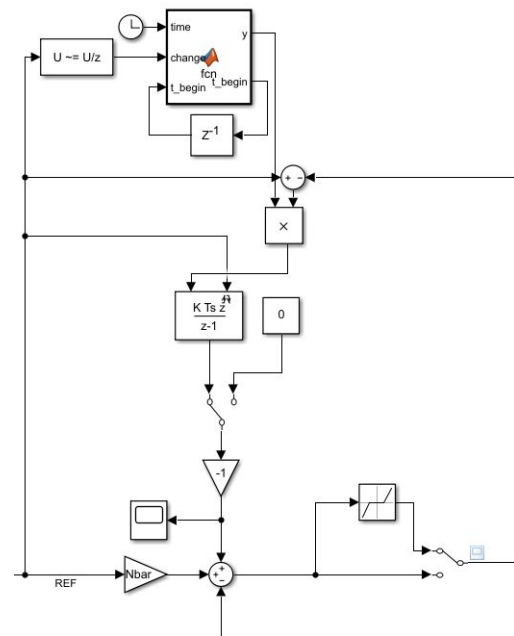
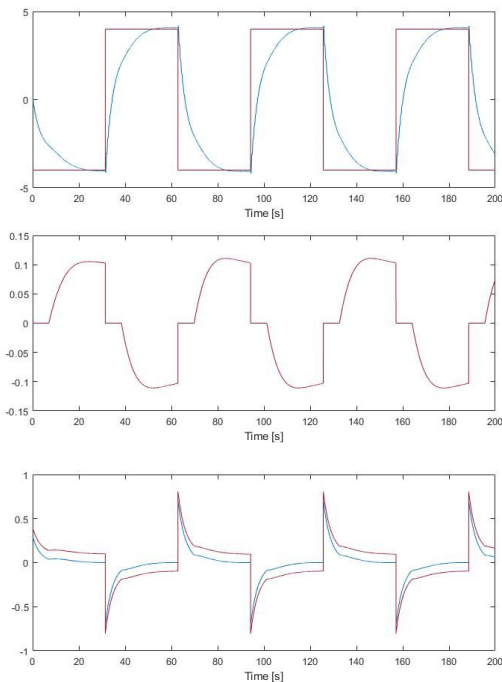
# Evaluate Dead Zone Control Dynamic Problem

## Solution 1

### Results from Iterations

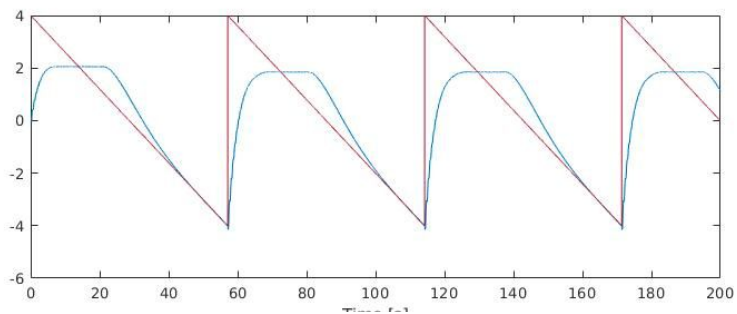
$$\Delta t = 7 \text{ s}$$

$$K_{integrator} = 0.012$$

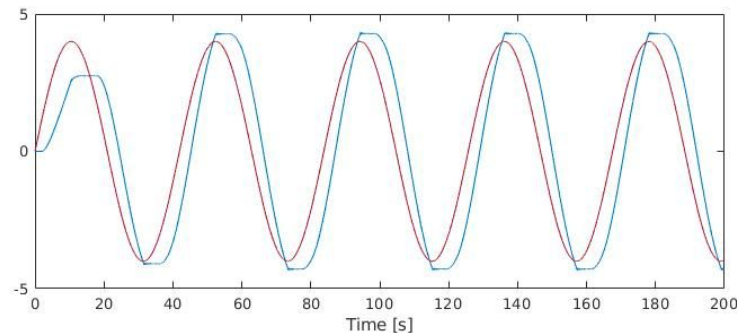


# Evaluate Dead Zone Control Dynamic Problem

## Performance



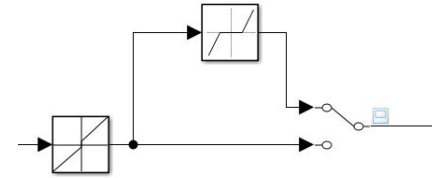
The previous control method only works good with square wave input



# Evaluate Dead Zone Control Dynamic Problem

## Solution 2

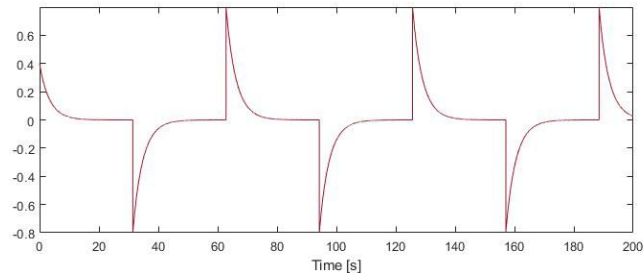
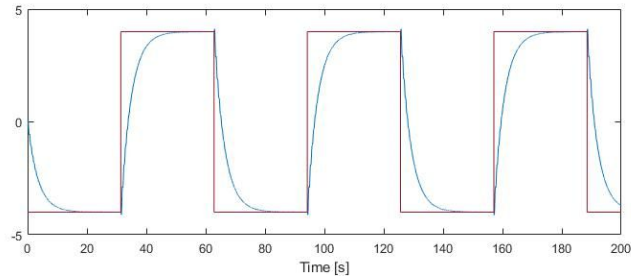
Invert non linear characteristic



Inversor Characteristic

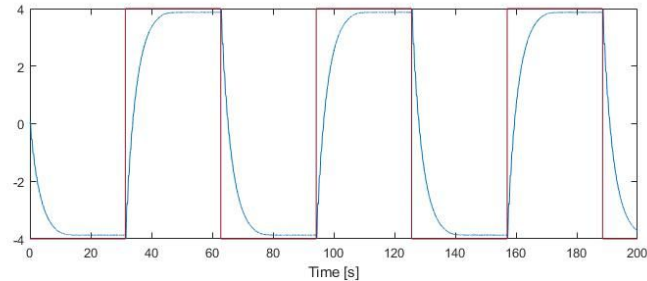
$$y = \begin{cases} u + 0.1, & \text{if } u > 0 \\ u - 0.1, & \text{if } u < 0 \end{cases}$$

This approach needs the exact knowledge of the dead zone and a possible problem is the implementation of the non linear block to invert the dead zone



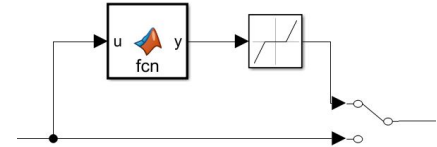
# Evaluate Dead Zone Control Dynamic Problem

For  $a = 0.01$

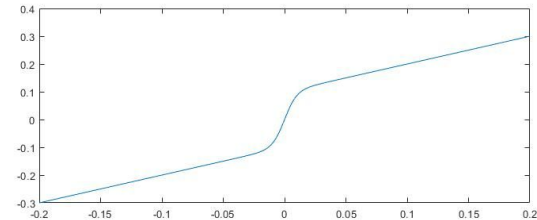


Solution 3

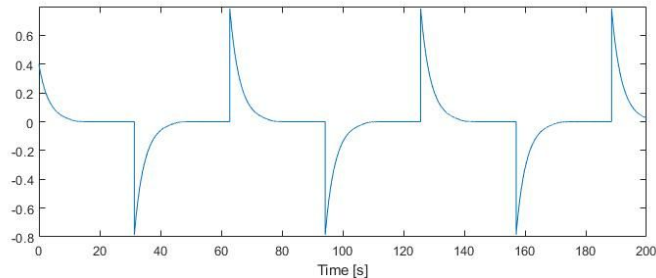
Invert non linear characteristic with  
differentiable function



Differentiable Pseudo Inverse Characteristic



$$y = \frac{e^{\frac{u}{a}}}{e^{\frac{u}{a}} + e^{-\frac{u}{a}}} \cdot (u + 0.1) + \frac{e^{-\frac{u}{a}}}{e^{\frac{u}{a}} + e^{-\frac{u}{a}}} \cdot (u - 0.1)$$



Q&A Session is Open