Forecasting Using Exponential Smoothing

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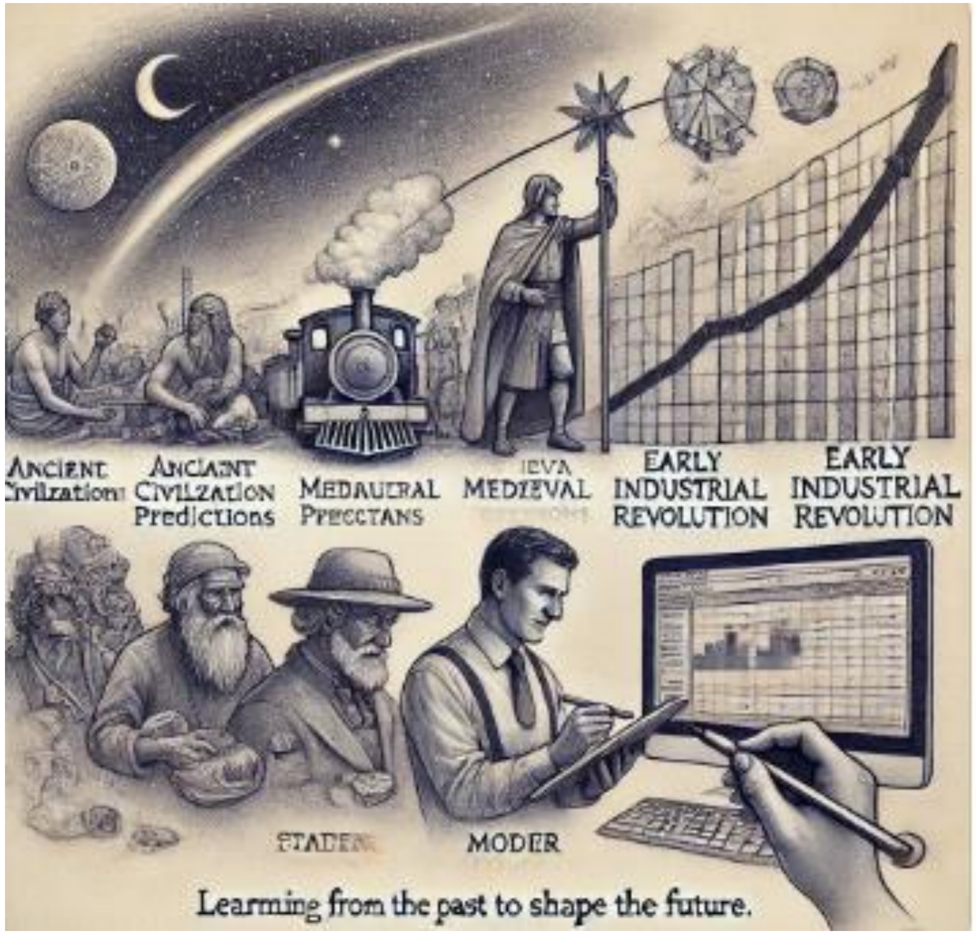
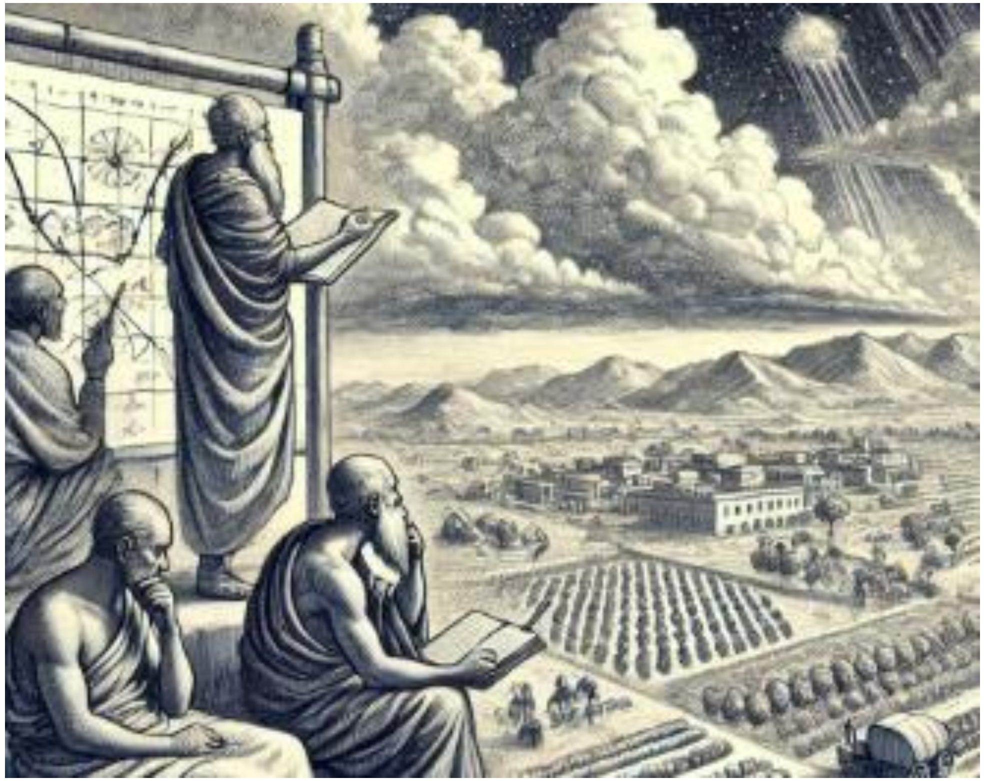


Motivation:

Forecasting is more than predicting numbers. Throughout history it has repeatedly determined the fate of civilizations.



Motivation:

Ancient societies relied on weather forecasting to ensure food security. They predicted rainfall or drought for their survival.

Motivation:

Today, most business rely on accurate forecasting for decision-making.

Motivation:

The COVID-19 pandemic illustrated the importance of forecasting that impact millions worldwide.

Motivation:

As we study time series forecasting, remember you’re acquiring skills that have the power to influence real-world outcomes significantly.

Forecasting Procedure

1. Select a model
2. Split data into train & test sets
3. Fit model on training set
4. Evaluate model on test set
5. Re-fit model on entire data set
6. Forecast for future data

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The Models

Exponential Smoothing

- A forecasting technique that applies decreasing weights exponentially to past observations

- Recent observations receive higher weights, making the method highly responsive to changes and trends in data

Single Exponential Smoothing (SES)

- Primarily used for short-term forecasting in stable environments without significant trends or seasonal patterns

- No explicit handling of trends or seasonal effects

Single Exponential Smoothing (SES)

Formula: $F_{t+1} = \alpha Y_t + (1 - \alpha)F_t$

$\boldsymbol{\alpha}$ (Level smoothing factor): Determines how quickly the forecast adapts to recent changes. Higher means faster adjustment

$\boldsymbol{Y_t}$: The actual observed value at the current time period.

$\boldsymbol{F_t}$: The previously forecasted value

Note: * *SES predicts future values; EWMA smooth data to reveal trends or anomalies (no prediction)* *

Single Exponential Smoothing (SES)

Formula: $F_{t+1} = \alpha Y_t + (1 - \alpha)F_t$

Example:

Suppose $\boldsymbol{\alpha}$ = 0.3, initial forecast F_1= 100, actual sales Y_1 = 120:

Forecast next period (F_2) = 0.3 * 120 + (0.7 * 100) = 106

Double Exponential Smoothing (Holt’s Method)

- Primarily used for data that demonstrates clear linear trend but no significant seasonality.

- Does not account for seasonality

Double Exponential Smoothing (Holt's Method)

Formula and Parameters:

The double exponential smoothing involves two main equations:

1. **Level (L):** This component captures the smoothed value of the series at time t.

2. **Trend (T):** This component estimates the trend (slope) of the series at time t.

Double Exponential Smoothing (Holt's Method)

Formula and Parameters:

The double exponential smoothing involves two main equations:

1. **Level (L):** $L_t = \alpha Y_t + (1 - \alpha)(L_{t-1} + T_{t-1})$
2. **Trend (T):** $T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1}$

Where:

Y_t is the actual value at time t.

α is the smoothing parameter for the level ($0 < \alpha < 1$)

β is the smoothing parameter for the trend ($0 < \alpha < 1$)

L_{t-1} and T_{t-1} are the level and trend estimates at time *t – 1*, respectively

Double Exponential Smoothing (Holt's Method)

Given the following data, assume Level smoothing factor (α) = 0.8, and Trend smoothing factor (β)= 0.2

Time (t)	**Observation (Y_t)**
1	50
2	53
3	56

Double Exponential Smoothing (Holt's Method)

Initialization:

Initial Level (L_t): Set the first observation: $\boldsymbol{L_1 = Y_1 = 50}$

Initial Trend (T_t): Estimated different between the two:
$T_1 = Y_2 - Y_1 = 53 - 50 = 3$

Double Exponential Smoothing (Holt's Method)

Time t = 2:

Level (L_2): $L_2 = \alpha Y_2 + (1 - \alpha)(L_1 + T_1)$

Substituting the values:

$$L_2 = 0.8 * 53 + (1 - 0.8)(50 + 3)$$
$$L_2 = 42.4 + (0.2)(53)$$
$$\boldsymbol{L_2 = 42.4 + 10.6 = 53}$$

Time t = 2:

Trend (T_2): $T_2 = \beta(L_2 - L_1) + (1 - \beta)T_1$

Substituting the values:

$$T_2 = 0.2(53 - 50) + (0.8)3$$
$$T_2 = 0.2(3) + (0.8)3$$
$$\boldsymbol{T_2 = 0.6 + 2.4 = 3}$$

Double Exponential Smoothing (Holt's Method)

Time t = 3:

Level (L_3): $L_3 = \alpha Y_3 + (1 - \alpha)(L_2 + T_2)$

Substituting the values:

$L_3 = 0.8 * 56 + (1 - 0.8)(53 + 3)$
$L_3 = 44.8 + (0.2)(56)$
$\boldsymbol{L_3 = 44.8 + 11.2 = 56}$

Time t = 3:

Trend (T_3): $T_3 = \beta(L_3 - L_2) + (1 - \beta)T_2$

Substituting the values:

$T_2 = 0.2(56 - 53) + (0.8)3$
$T_2 = 0.2(3) + (0.8)3$
$\boldsymbol{T_2 = 0.6 + 2.4 = 3}$

Double Exponential Smoothing (Holt's Method)

Forecasting (t = 4)

Forecast (F_4): $F_4 = L_3 + T_3$

Substituting the values:

$F_4 = L_3 + T_3$
$F_4 = 56 + 3$
$\boldsymbol{F_4 = 56}$

Triple Exponential Smoothing (Holt-Winter’s Method)

- Primarily used for data that shows clear trend and seasonal cycles

- Computationally intensive, requiring more historical data and careful parameter tuning

Triple Exponential Smoothing (Holt-Winter's Method)

Formula and Parameters:

The HW involves three main equations:

1. **Level** L_t: The smoothed value of the series at time t.
2. **Trend** T_t: Captures the direction and rate of change in the series
3. **Seasonality** S_t**:** Accounts for repeating patterns or cycles in the data

There are two variations of HW:

1. Additive Seasonality
2. Multiplicative Seasonality

Triple Exponential Smoothing (Holt-Winter's Method)

Additive Method:

$$L_t = \alpha(Y_t - S_{t-m}) + (1 - \alpha)(L_{t-1} + T_{t-1})$$
$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1}$$
$$S_t = \gamma(Y_t - L_t) + (1 - \gamma)S_{t-m}$$
$$F_{t+h} = L_t + hT_t + S_{t+h-m(k+1)}$$

Where:

Y_t is the actual value at time t.

α is the smoothing parameter for the level ($0 < \alpha < 1$)

β is the smoothing parameter for the trend ($0 < \alpha < 1$)

γ is the seasonal smoothing parameter ($0 < \gamma < 1$)

m is the number of periods in a full seasonal cycle

h is the number of periods ahead for forecasting

k is the integer part of $\frac{h-1}{m}$, ensuring the correct seasonal index is used for forecasting

Triple Exponential Smoothing (Holt-Winter's Method)

Multiplicative Method:

$$L_t = \alpha\left(\frac{Y_t}{S_{t-m}}\right) + (1 - \alpha)(L_{t-1} + T_{t-1})$$

$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1}$$

$$S_t = \gamma\left(\frac{Y_t}{L_t}\right) + (1 - \gamma)S_{t-m}$$

$$F_{t+h} = L_t + hT_t + S_{t+h-m(k+1)}$$

Where:

Y_t is the actual value at time t.

α is the smoothing parameter for the level (0 < α < 1)

β is the smoothing parameter for the trend (0 < α < 1)

γ is the seasonal smoothing parameter (0 < γ < 1)

m is the number of periods in a full seasonal cycle

h is the number of periods ahead for forecasting

k is the integer part of $\frac{h-1}{m}$, ensuring the correct seasonal index is used for forecasting

Triple Exponential Smoothing (Holt-Winter’s Method)

Given the following data, assume seasonal period (m) = 2, level smoothing (a) = 0.5, trend smoothing (B) = 0.3, and seasonal smoothing (y) = 02.

Time (t)	**Observation (Y_t)**
1	50
2	53
3	57

Triple Exponential Smoothing (Holt-Winter's Method)

Initial Level L0:

A common method is to take the average of the first season's observations. We use the first two data points (since m = 2):

$$L_0 = \frac{Y_1 + Y_2}{2} = \frac{50 + 53}{2} = 51.5$$

Triple Exponential Smoothing (Holt-Winter's Method)

Initial Trend T_0 :

A simple estimate is the difference between the first two observations:

$$T_0 = Y_2 - Y_2 = 53 - 50 = 3$$

Triple Exponential Smoothing (Holt-Winter's Method)

Initial Seasonal Indices S_1 and S_2 :

For an additive model, the seasonal index is estimated as the difference between observation and the initial level:

For t = 1 (Season 1)

$$S_1 = Y_1 - L_0 = 50 - 51.5 = -1.5$$

For t = 2 (Season 2)

$$S_2 = Y_2 - L_0 = 53 - 51.5 = 1.5$$

Triple Exponential Smoothing (Holt-Winter's Method)

L_2 and T_2 at t = 2

L_2 = Y_2 = 53
T_2 = T_0 = 3

This values serve as our 'previous' values when updating at *t = 3*

Triple Exponential Smoothing (Holt-Winter's Method)

Update Level L_3:

m = 2

Y_3 = 57

S_{3-2} = S_1 = -1.5

Previous Level L_2 = 53

Previous Trend T_2 = 3

Level:

$$L_3 = \alpha(Y_3 - S_{3-2}) + (1 - \alpha)(L_{3-1} + T_{3-1})$$
$$L_3 = \alpha(Y_3 - S_1) + (1 - \alpha)(L_2 + T_2)$$
$$L_3 = 0.5\big(57 - (-1.5)\big) + (1 - 0.5)(53 + 3)$$
$$L_3 = 0.5(58.5) + (0.5)(56)$$
$$L_3 = 29.25 + 28$$
$$L_3 = 57.25$$

Triple Exponential Smoothing (Holt-Winter's Method)

Update Trend T_3:

L_3= 57.25

L_3= 53

T_2= 3

Trend:

$$T_3 = \beta(L_3 - L_{3-1}) + (1 - \beta)T_{3-1}$$
$$T_3 = \beta(L_3 - L_2) + (1 - \beta)T_2$$
$$T_3 = 0.3(57.25 - 53) + (1 - 0.3)(3)$$
$$T_3 = 0.3(4.25) + (0.7)(3)$$
$$T_3 = 1.275 + 2.1$$
$$T_3 = 3.375$$

Triple Exponential Smoothing (Holt-Winter's Method)

Update Seasonal S_3:

Y_3= 57

L_3= 57.25

S_{3-2} = S_1 = -1.5

Seasonal:

$$S_3 = \gamma(Y_3 - L_3) + (1 - \gamma)S_{3-2}$$
$$S_3 = \gamma(Y_3 - L_3) + (1 - \gamma)S_1$$
$$S_3 = 0.2(57 - 57.25) + (1 - 0.2)(-1.5)$$
$$S_3 = 0.2(-0.25) + (0.8)(-1.5)$$
$$S_3 = -0.05 - 1.2$$
$$S_3 = -1.25$$

Triple Exponential Smoothing (Holt-Winter's Method)

Forecasting for t = 4

For an additive model, the forecast for h periods ahead is:

$$F_{t+h} = L_t + hT_t + S_{t+h-m(k+1)}$$

For a one-step-ahead forecast from *t = 3* (*h = 1*), we need a seasonal index corresponding to *t + h – m. Remember k =* $\frac{h-1}{m} = \frac{0-1}{2} = \frac{0}{2} = 0$

Thus, we use $S_{3+1-2(0+1)} = S_{4-2(1)} = S_2$

$$F_{t+h} = L_t + hT_t + S_{t+h-m(k+1)}$$

$$F_4 = L_3 + hT_3 + S_2$$

$$F_4 = 57.25 + 1(3.375) + 1.5 = 62.125$$

[Code Demo]

Thank you very much for listening.