Logistic Regression

Logistic Regression

- We've explored how to use Linear Regression and its many variations to predict a continuous label.
- But how can we predict a categorical label?

Logistic Regression

- We've explored how to use Linear Regression and its many variations to predict a continuous label.
- But how can we predict a categorical label?
    - Logistic Regression

Logistic Regression

- Logistic Regression
  - Don’t be confused by the use of the term “regression” in its name!
  - Logistic Regression is a **classification** algorithm designed to predict **categorical** target labels.

Logistic Regression

- Logistic Regression Section Overview
  - Transforming Linear Regression to Logistic Regression
  - Mathematical Theory behind Logistic Regression
  - Simple Implementation of Logistic Regression for Classification Problem

Logistic Regression

- Logistic Regression Section Overview
    - Interpreting Results
        - Odds Ratio and Coefficients
        - Classification Metrics
            - Accuracy
            - Precision
            - Recall
        - ROC Curves

Logistic Regression

- Logistic Regression Section Overview
    - Multiclass Classification with Logistic Regression
    - Logistic Regression Project
    - Logistic Regression Project Solutions

Logistic Regression

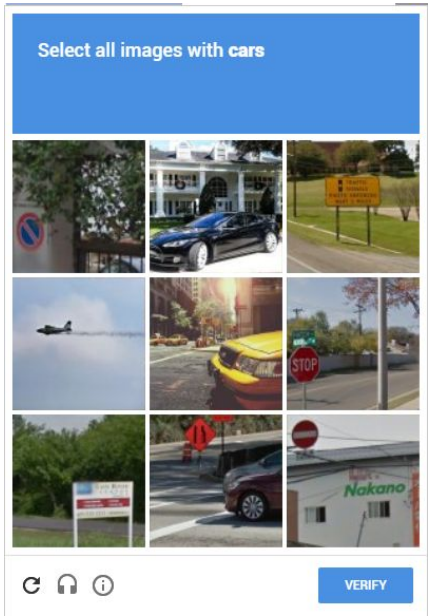
- Logistic Regression will allow us to predict a categorical label based on historical feature data.
- The categorical target column is two or more discrete class labels.

Logistic Regression

- Classification algorithms predict a class or category label:
    - Class 0: Car Image
    - Class 1: Street Image
    - Class 2: Bridge Image

Logistic Regression

- You may not have realized you are helping Google label class data!



Logistic Regression

- Keep in mind, any continuous target can be converted into categories through discretization.
    - Class 0: House Price $0-100k
    - Class 1: House Price $100k-200k
    - Class 2: House Price <$200k

Logistic Regression

- Classification algorithms also often produce a **probability** prediction of belonging to a class:
    - Class 0: 10% Probability
    - Class 1: 85% Probability
    - Class 2: 5% Probability

Logistic Regression

- Classification algorithms also often produce a **probability** prediction of belonging to a class:
    - Class 0: 10% Probability - Car Image
    - Class 1: 85% Probability - Street Image
    - Class 2: 5% Probability - Bridge Image
        - Model reports back prediction of Class 1, image is a street.

Logistic Regression

- Also note our prediction **ŷ** will be a category, meaning we won't be able to calculate a difference based on **y-ŷ.**
    - **Car Image - Street Image** does not make sense.
- We will need to discover a completely different set of error metrics and performance evaluation!

Let’s get started!

Logistic Regression Theory and Intuition

Part One: The Logistic Function

PIERIAN DATA

Logistic Regression

- Logistic Regression works by transforming a Linear Regression into a classification model through the use of the logistic function:

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

Logistic Regression

- Let’s begin by understanding the history and motivation behind the logistic function (a.k.a the sigmoid function):

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

Logistic Regression

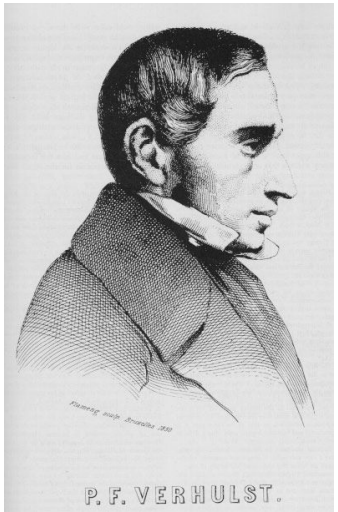
- Note:
  - For now, we're only referring to the logistic function itself, not the logistic regression model!

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

Logistic Regression

- 1830-1850: Under guidance of Adolphe Quetelet, Pierre François Verhulst developed the logistic function:

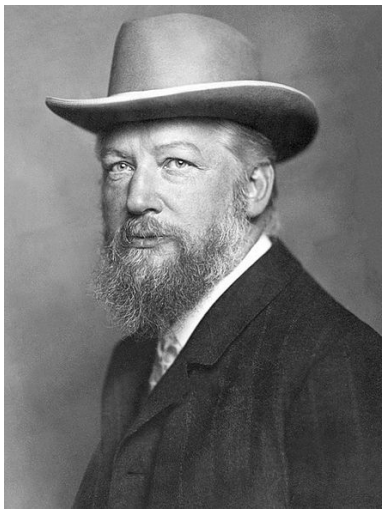
$$\sigma(x) = \frac{1}{1 + e^{-x}}$$



Logistic Regression

- 1883: Logistic function was independently developed in chemistry as a model of autocatalysis by Wilhelm Ostwald.

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$



Logistic Regression

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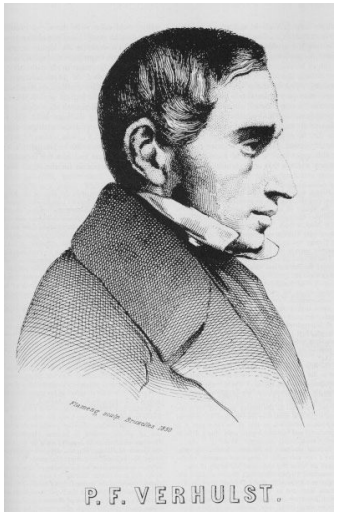
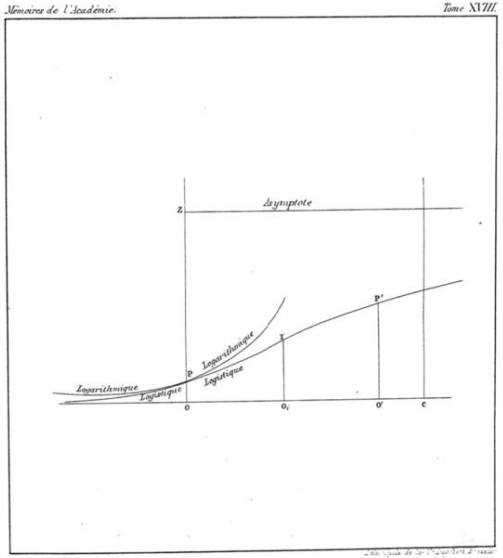


Logistic Regression

- 1830-1850: Developed for the purposes of modeling population growth.

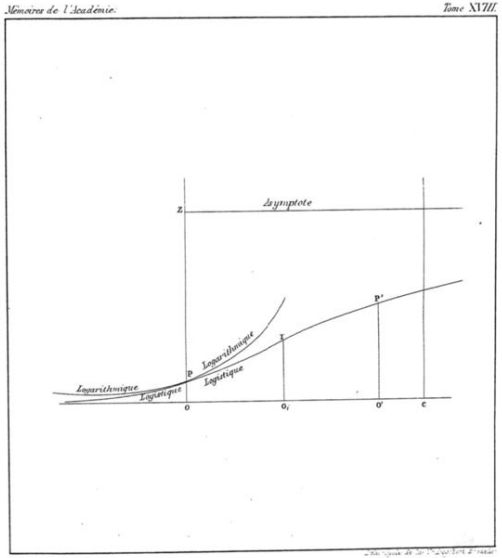


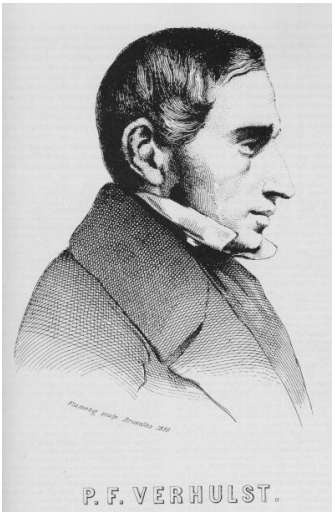
Mémoire sur la population par M. P. Verhulst.

PIERIAN DATA

Logistic Regression

- Why the need for a logistic function versus a logarithmic function?





PIERIAN DATA

Logistic Regression

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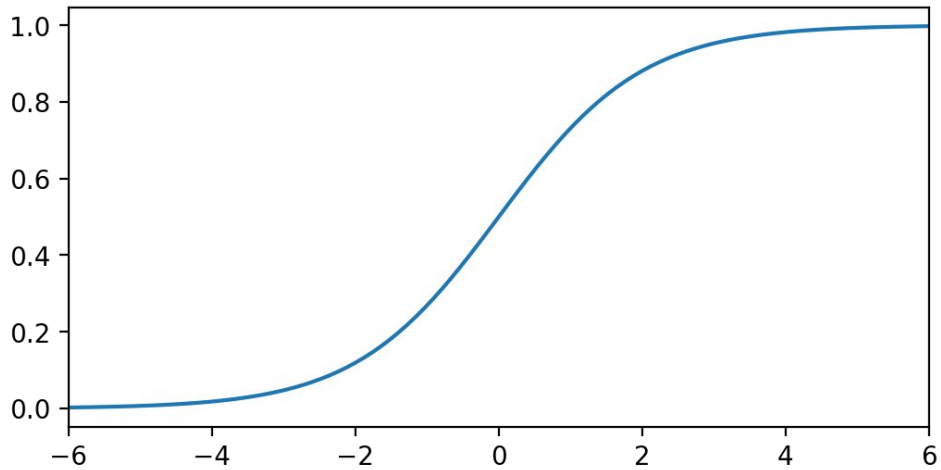
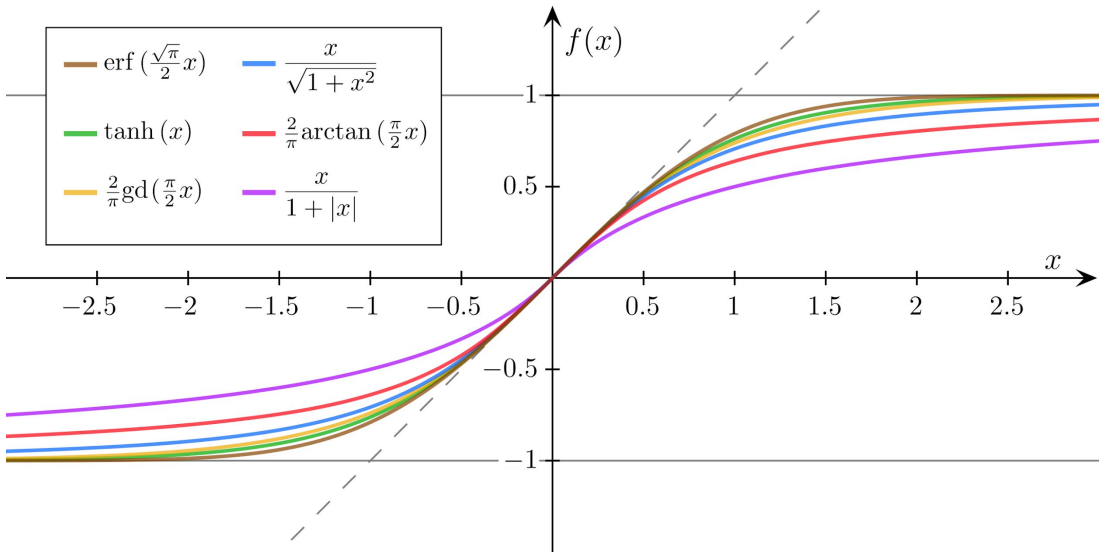


$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

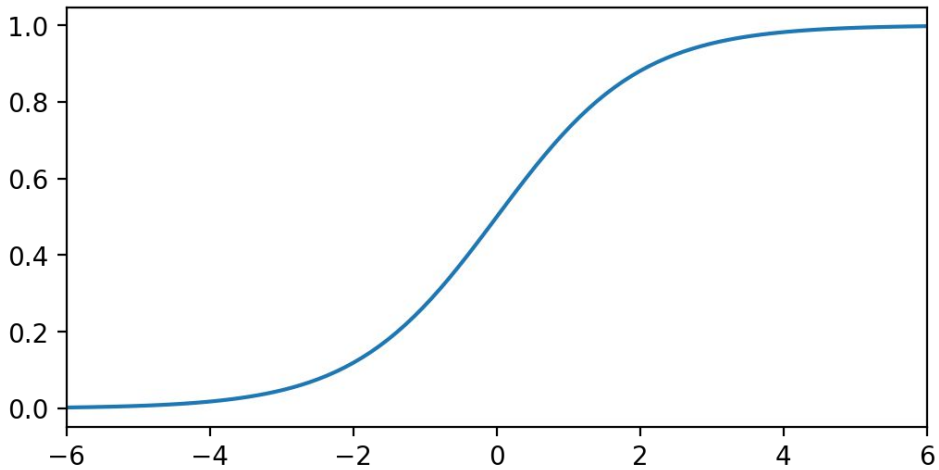
Logistic Regression

- Note: There is a “family” of logistic functions.



Logistic Regression

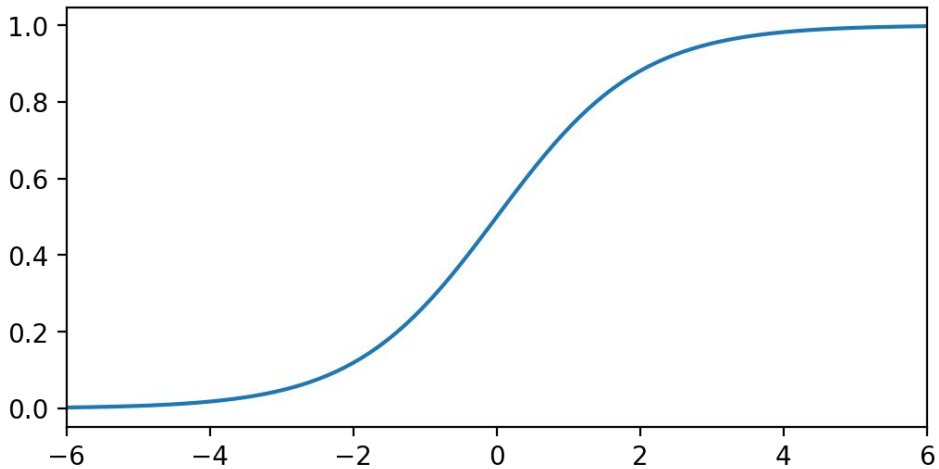
- Notice the “leveling off” behavior of the curve.



$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

Logistic Regression

- Also notice **any** value of **x** will have an output range between 0 and 1.



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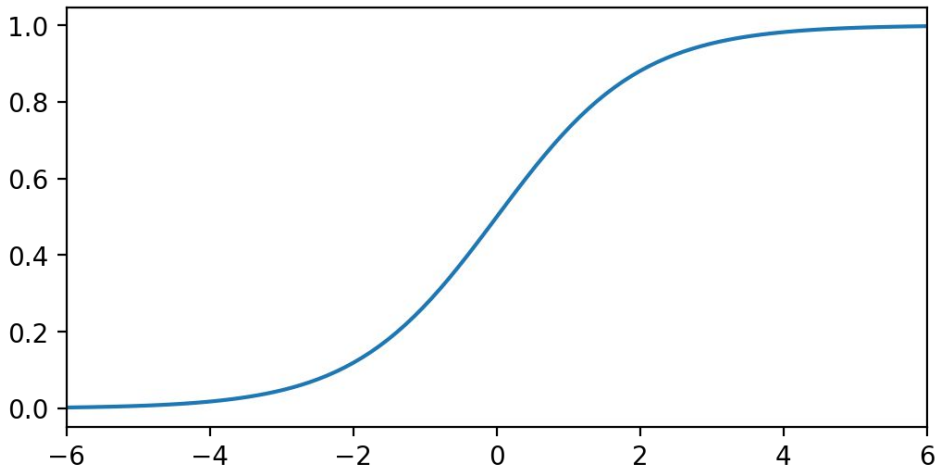
Logistic Regression

- Many natural real world systems have a “carrying capacity” or a natural limiting factor.



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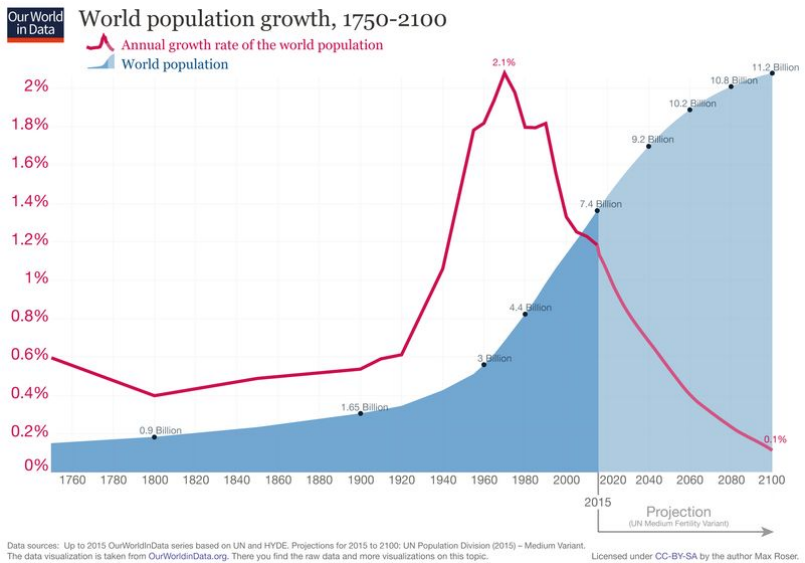

PIERIAN DATA

Logistic Regression

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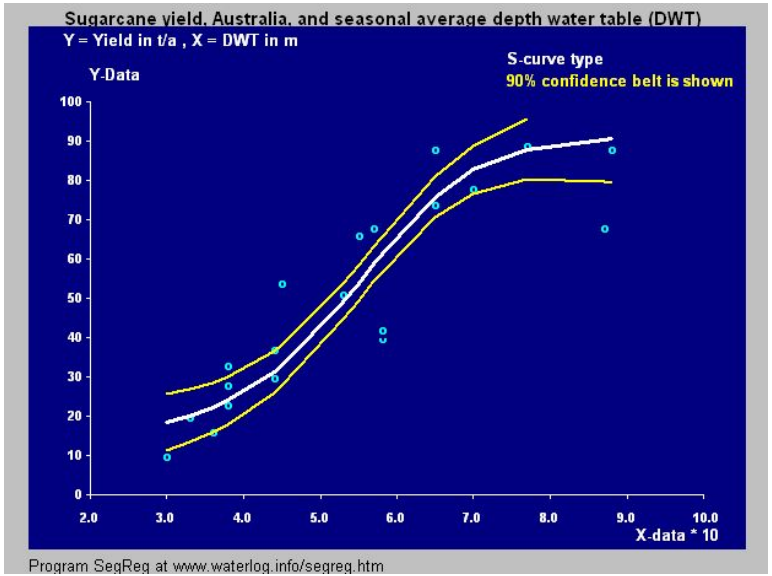
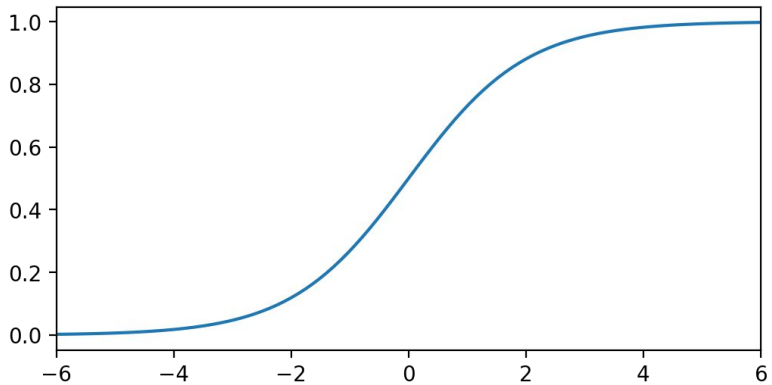


PIERIAN DATA

Logistic Regression

- 1940s: Using the logistic function for statistical modeling was developed by Joseph Berkson.



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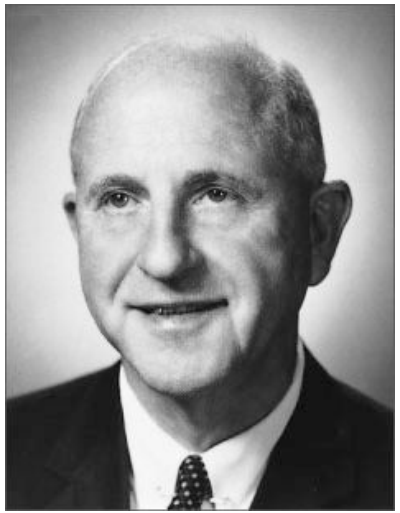
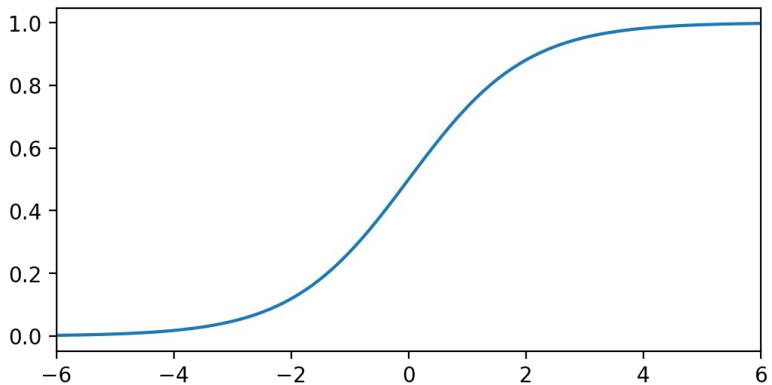


Logistic Regression

- 1944:*"Application of the logistic function to bio-assay"* in the Journal of the American Statistical Association



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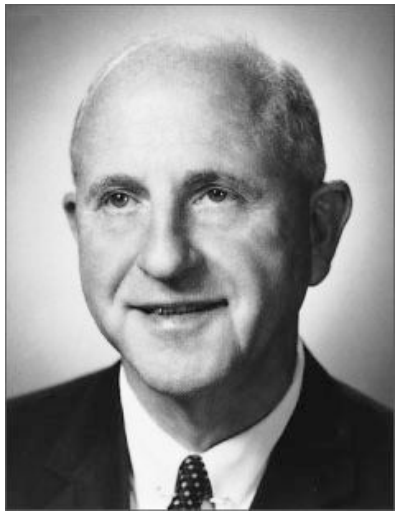


Logistic Regression

- Quirky fact: Berkson was a prominent opponent of the idea that cigarette smoking causes cancer.

PIERIAN DATA

Logistic Regression

- Life magazine, Berkson: "*...very doubtful that smoking causes cancer of the lung.*"

leaves another 'very doubtful'

and cell alteration than the lungs of nonsmokers. The particular kinds of damage also suggested how smoking might contribute to a variety of lung diseases—and how the lung damage might, in turn, lead to circulatory troubles and a harder-working heart—though this has never been directly demonstrated in any laboratory experiment.

It was these "many lines of converging evidence," in the committee's phrase, that led the committee to its most serious charges against cigarets:

► Lung Cancer: "Cigaret smoking is causally related to lung cancer in men; the magnitude of the effect of cigaret smoking far outweighs all other factors. The data for women, though less extensive, points in the same direction.

"The risk of developing lung cancer increases with duration of smoking and the number of cigarets smoked per day, and is diminished by discontinuing smoking."

► Other Lung Diseases: "Cigaret smoking is the most important of the causes of chronic bronchitis in the United States, and increases the risk of dying from chronic bronchitis," as well as from pulmonary emphysema, destruction of tiny air sacs that perform the lungs' basic breathing function.

► Heart Disease: "Male cigaret smokers have a higher death rate from coronary artery disease than nonsmoking males, but it is not clear that the association has causal significance."

► The report emphasized the much lesser risk of smoking cigars and pipes. But it did point out that pipe smoking seems to increase the risk of getting cancer of the lip.

► Most surprising was the verdict on nicotine. Long considered the most harmful ingredient in tobacco because it is believed to be the stimulant that gives the smoker his kicks, nicotine was found to be relatively harmless.

But the report and its conclusions are already being challenged by scientists as eminent as any on the surgeon general's committee. Though few in number, these critics have persistently refused to accept the evidence against cigarets as conclusive. An outspoken dissenter is Dr. Joseph Berkson of the Mayo Clinic, one of the nation's leading medical statisticians. Berkson has repeatedly challenged Hammond's studies, partly on the grounds that his population samples were not truly representative of the whole population. He and others have also pointed out that the incriminating statistics have never been backed up by conclusive laboratory proof —e.g., lung cancer has never been induced in any experimental animal by the inhalation of cigaret smoke alone. Berkson's objections were carefully considered by the committee and answered to its own satisfaction. Berkson, however, still believes it "very doubtful that smoking causes cancer of the lung."

Anti-report statistician is Dr. Joseph Berkson of the Mayo Clinic. He is one of a small but outspoken minority who insist that the case linking cigaret smoking with lung cancer remains essentially unproven.

Logistic Regression

- While we now know smoking is clearly bad for you, we still haven't learned how to convert a linear regression to a logistic regression!
- Let's continue on by seeing how a linear regression is unable to solve classification problems effectively and how the logistic function can fix this!

Logistic Regression Theory and Intuition

Part Two:
Linear to Logistic Intuition

PIERIAN DATA

Logistic Regression

- Let’s explore how to convert a Linear Regression model used for a **regression task** into a Logistic Regression model used for a **classification task.**
- Imagine a dataset with a single feature (previous year’s income) and a single target label (loan default)

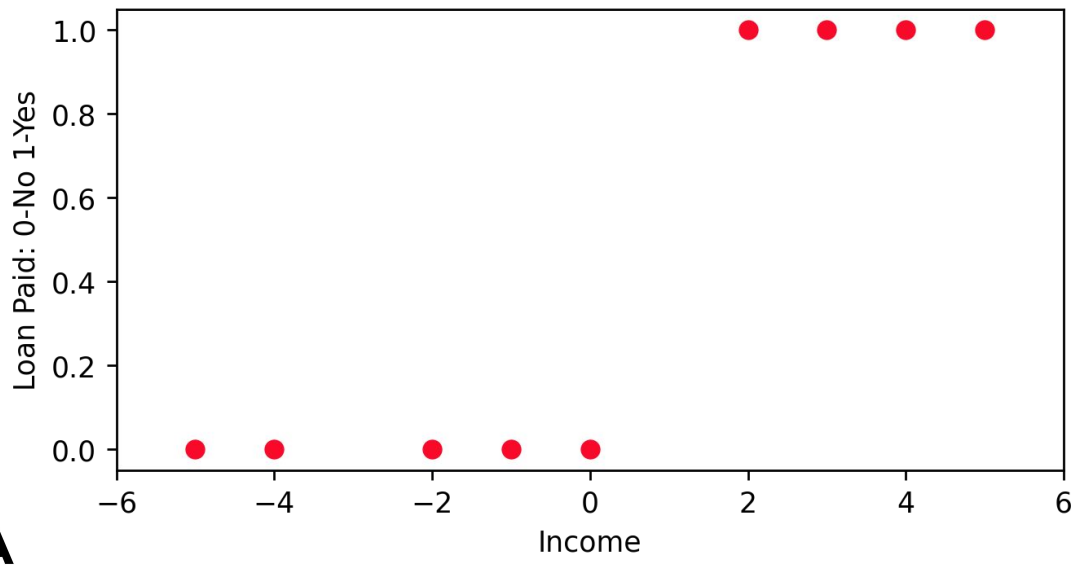
Logistic Regression

- Our data set:

Income	Loan Paid
-5	0
-4	0
-2	0
-1	0
0	0
2	1
3	1
4	1
5	1

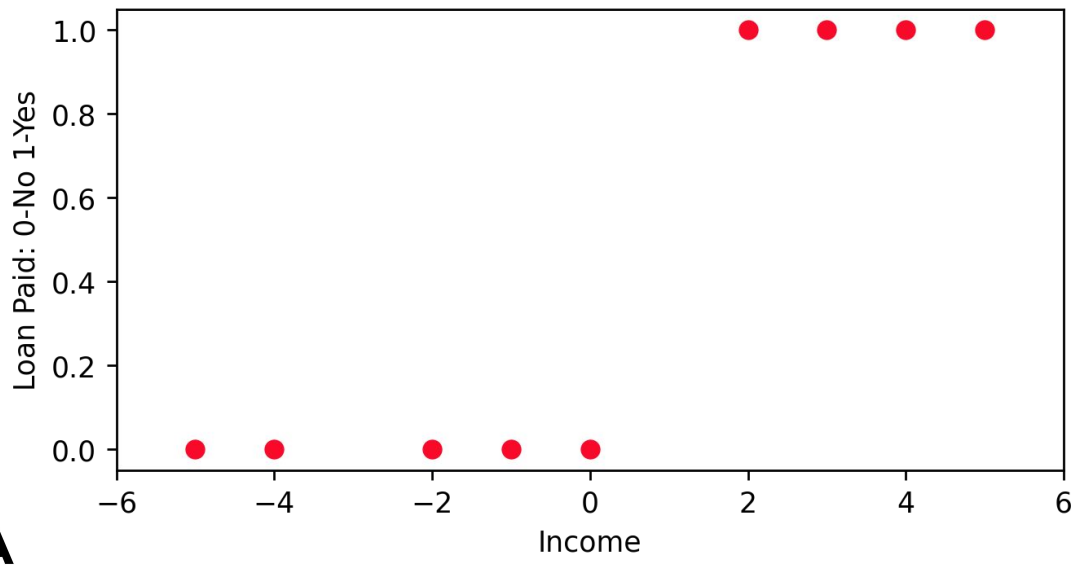
Logistic Regression

- Let's begin by plotting income versus default:



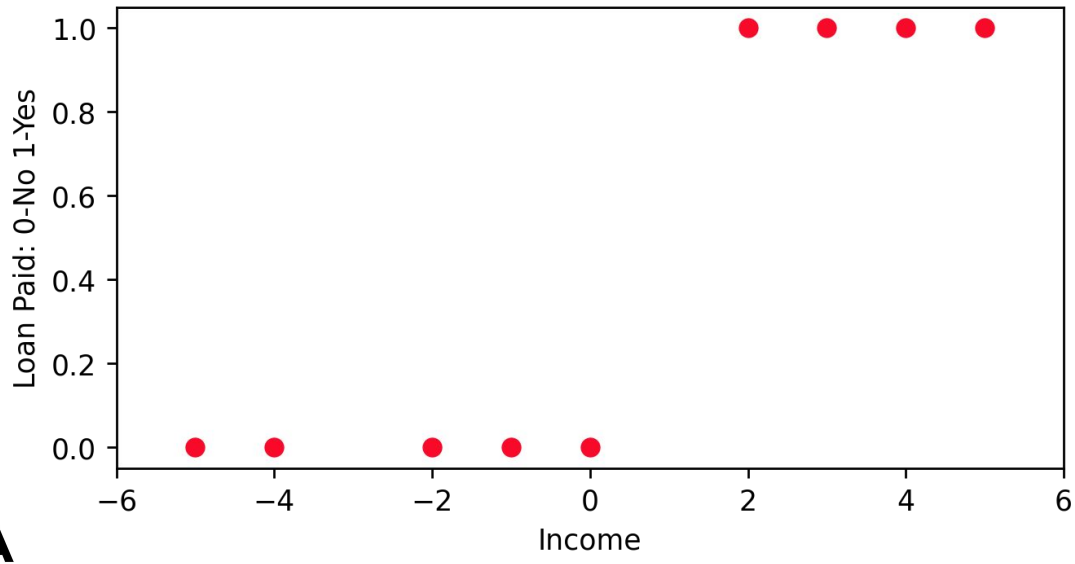
Logistic Regression

- Notice that people with negative income tend to default on their loans.



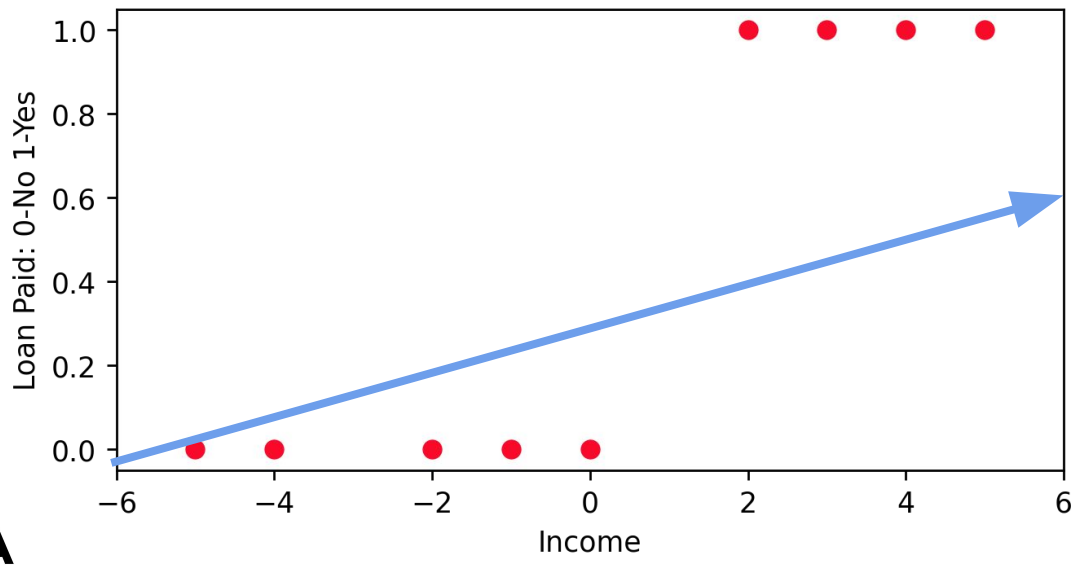
Logistic Regression

- What if we had to predict default status given someone's income?



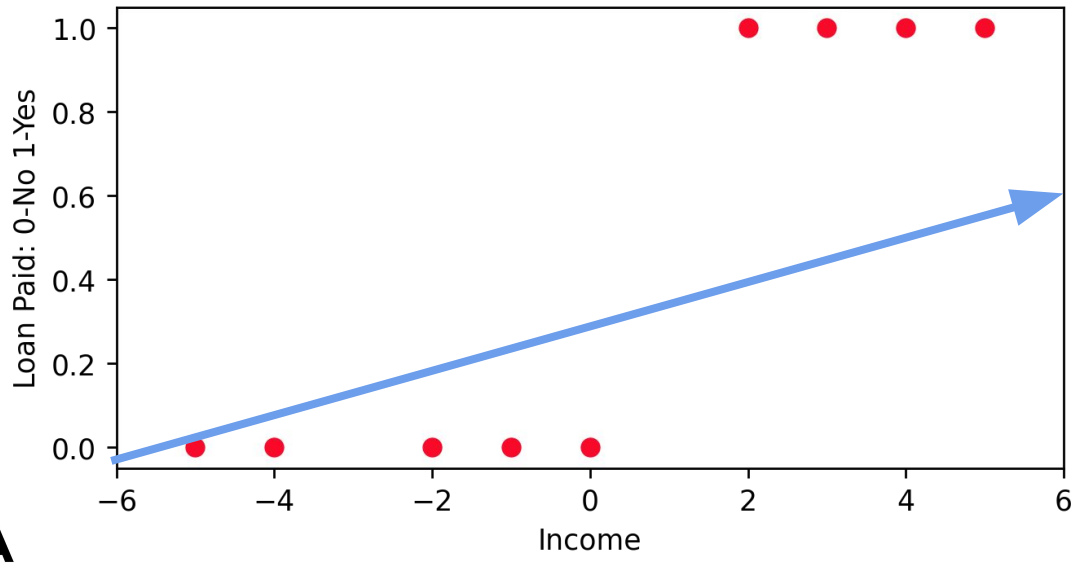
Logistic Regression

- Fitting a Linear Regression would not work (recall Anscombe's quartet):



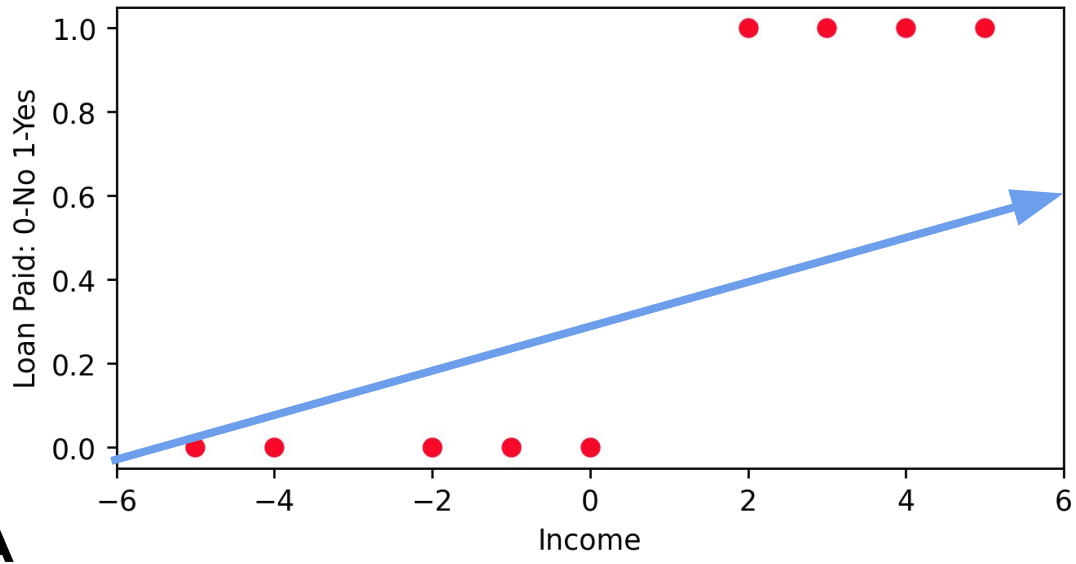
Logistic Regression

- Linear Regression easily distorted by only having 0 and 1 as possible y training values.



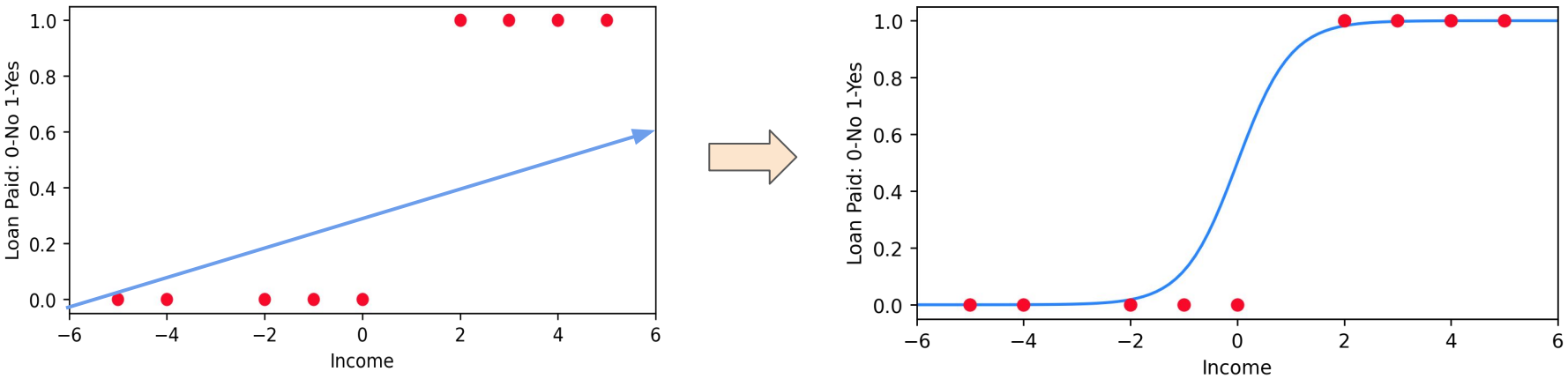
Logistic Regression

- Also would be unclear how to interpret predicted y values between 0 and 1.



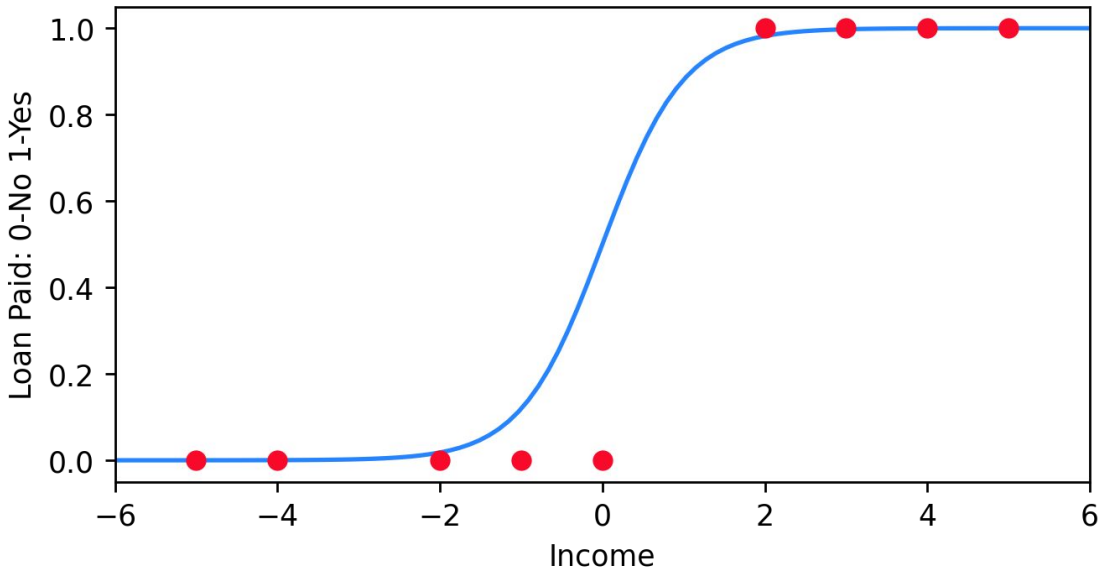
Logistic Regression

- We could make use of the Logistic Function for a conversion!



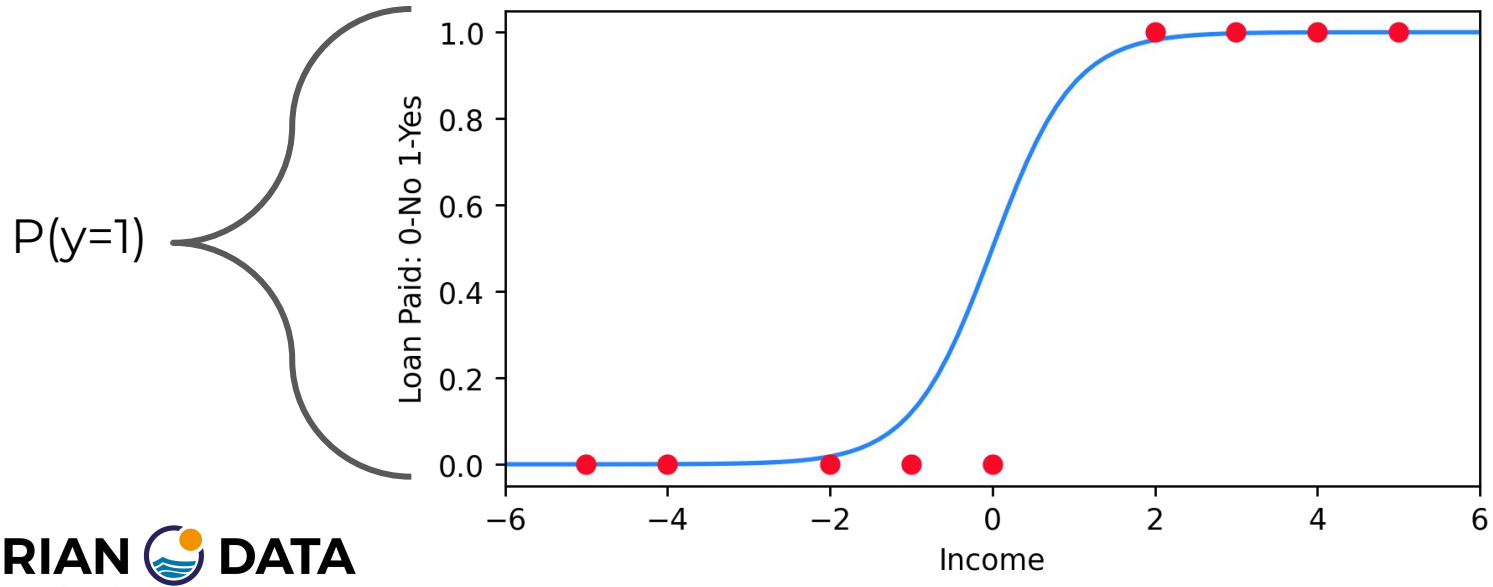
Logistic Regression

- Let's first focus on what this Logistic Regression would look like.



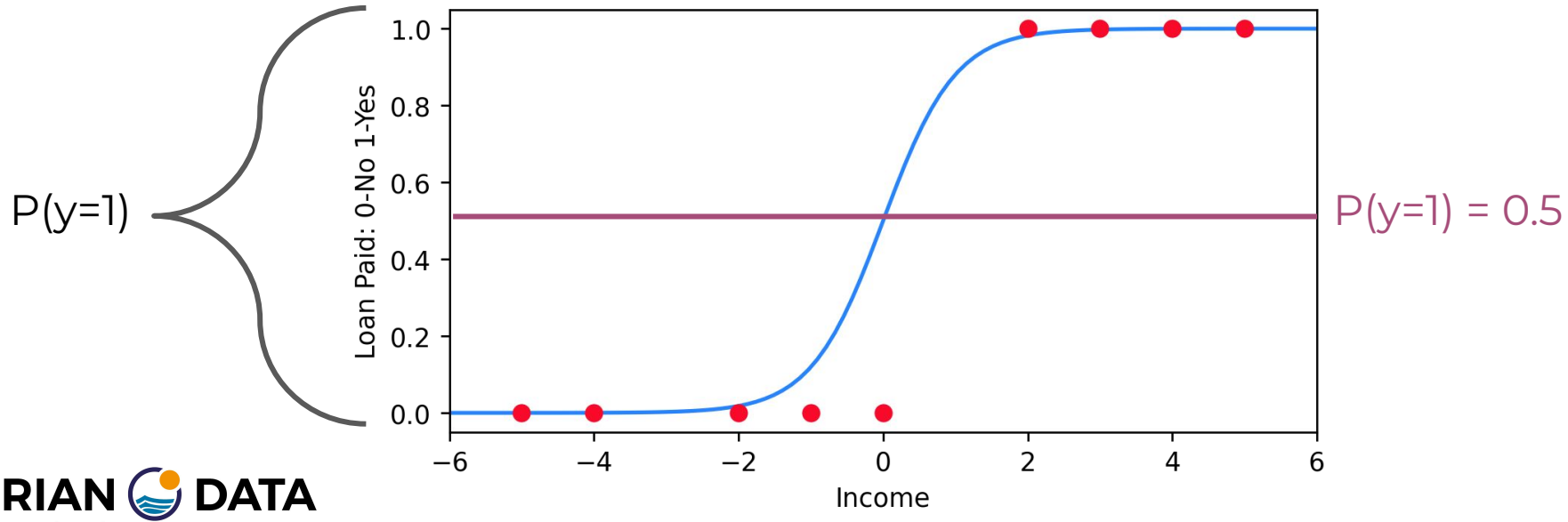
Logistic Regression

- Treat the y-axis as a probability of belonging to a class:



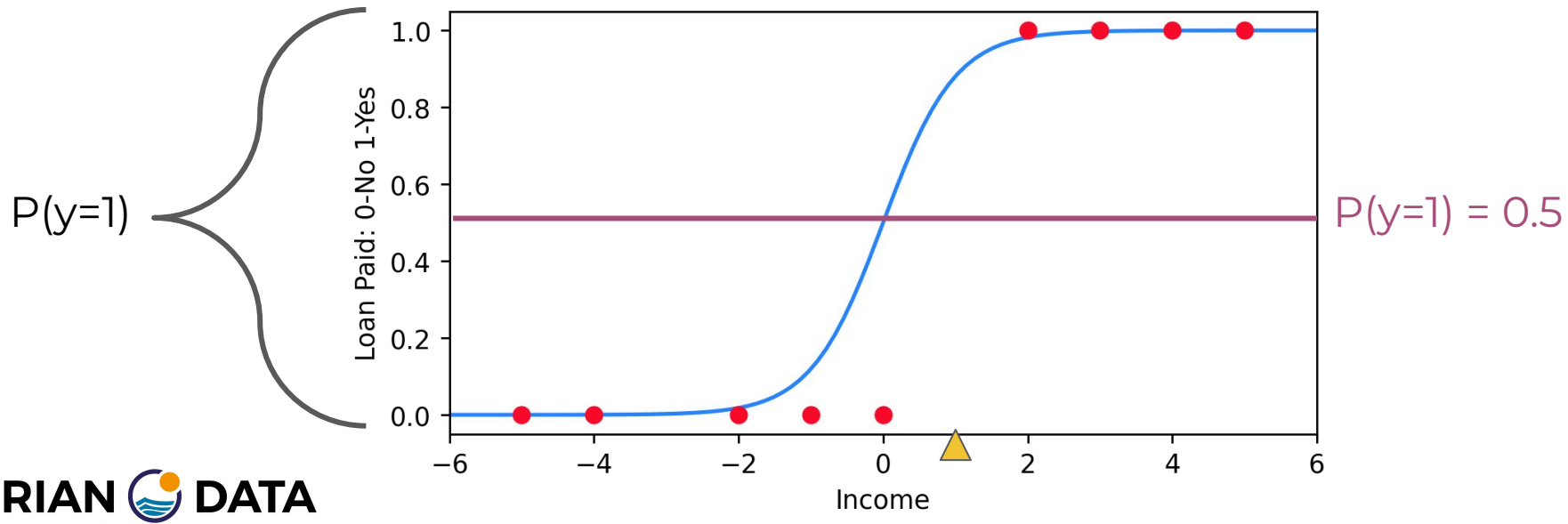
Logistic Regression

- Treating P(y=1) >= 0.5 as a cut-off for classification:



Logistic Regression

- For example, a new person with an income of 1:



Logistic Regression

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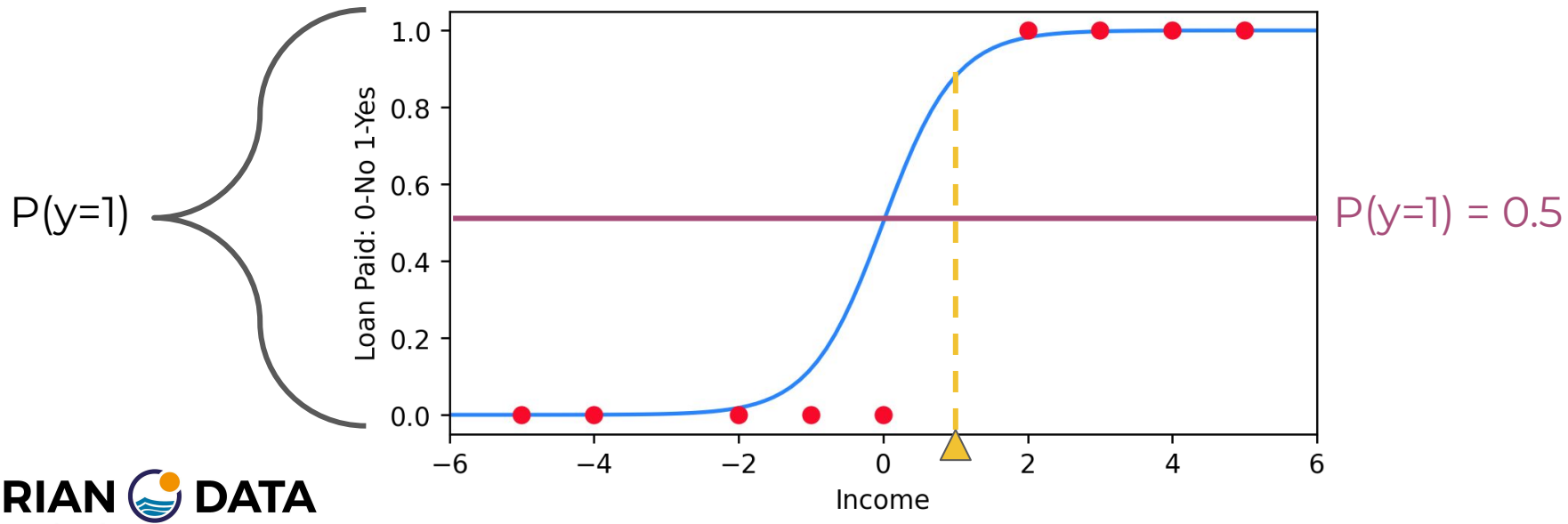
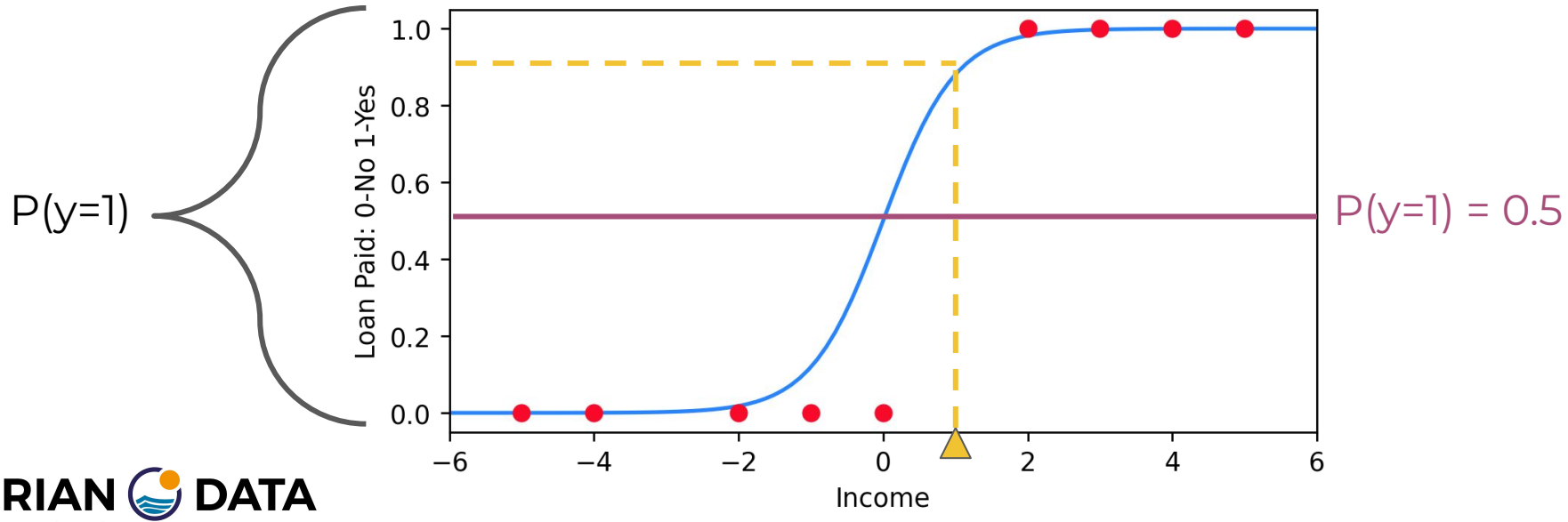


Logistic Regression

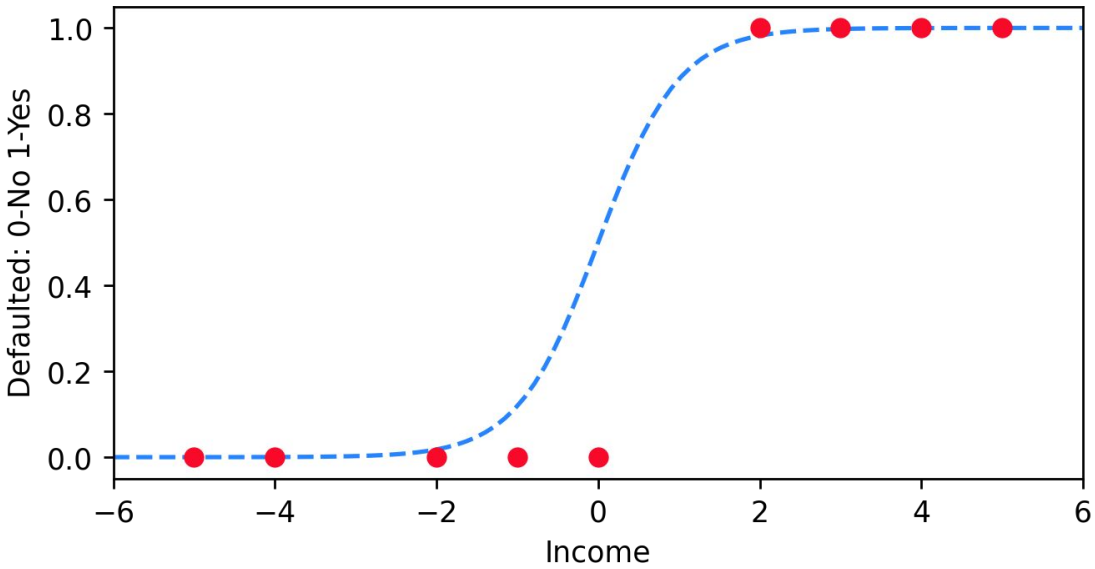
- Predict a 90% probability of paying off loan, return prediction of Loan Paid = 1.



PIERIAN DATA

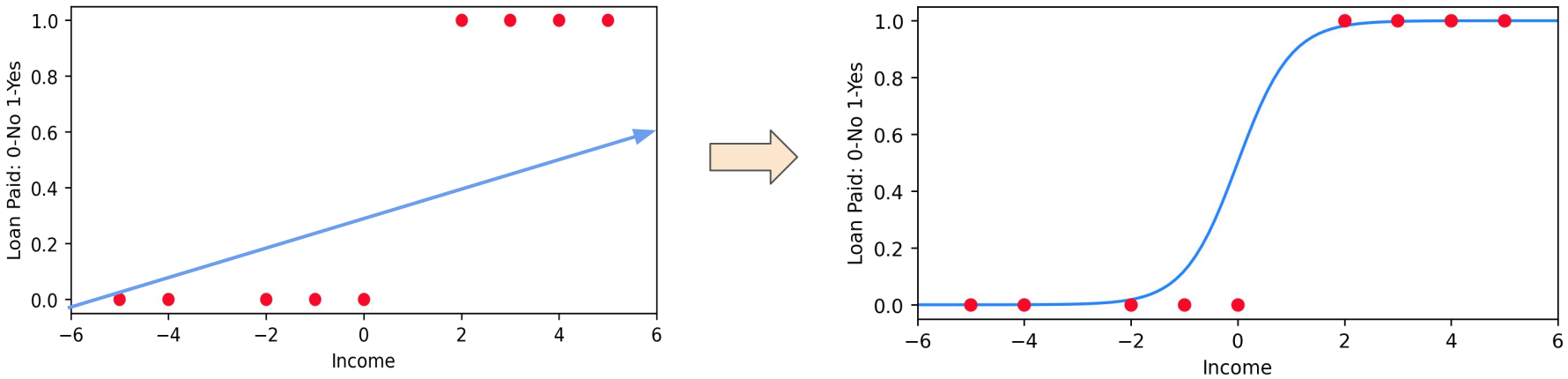
Logistic Regression

- But how do we actually create this line?



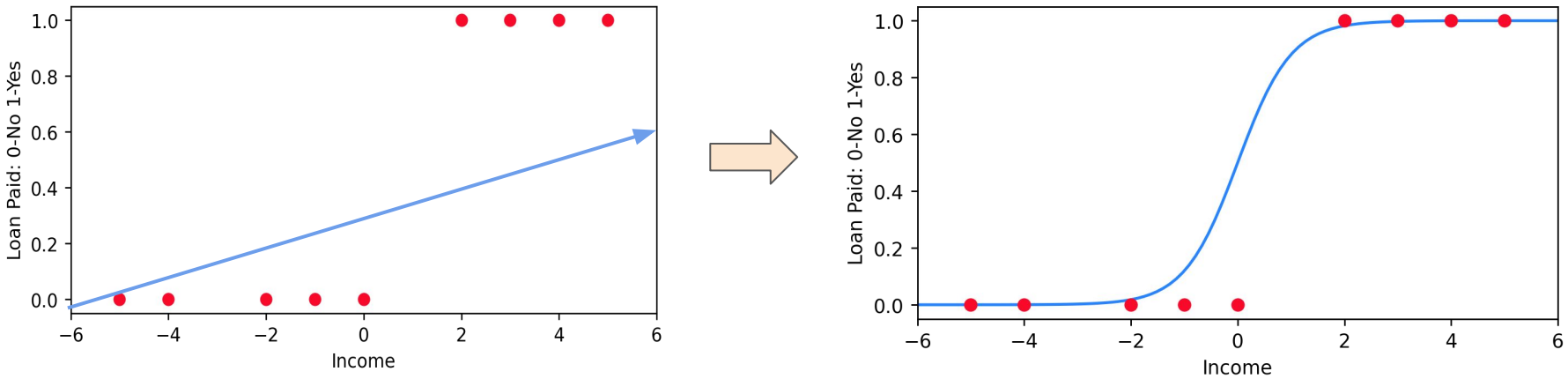
Logistic Regression

- Fortunately, the mathematics of the conversion are quite simple!



Logistic Regression

- In the next lecture we will go through the mathematical process of this conversion.



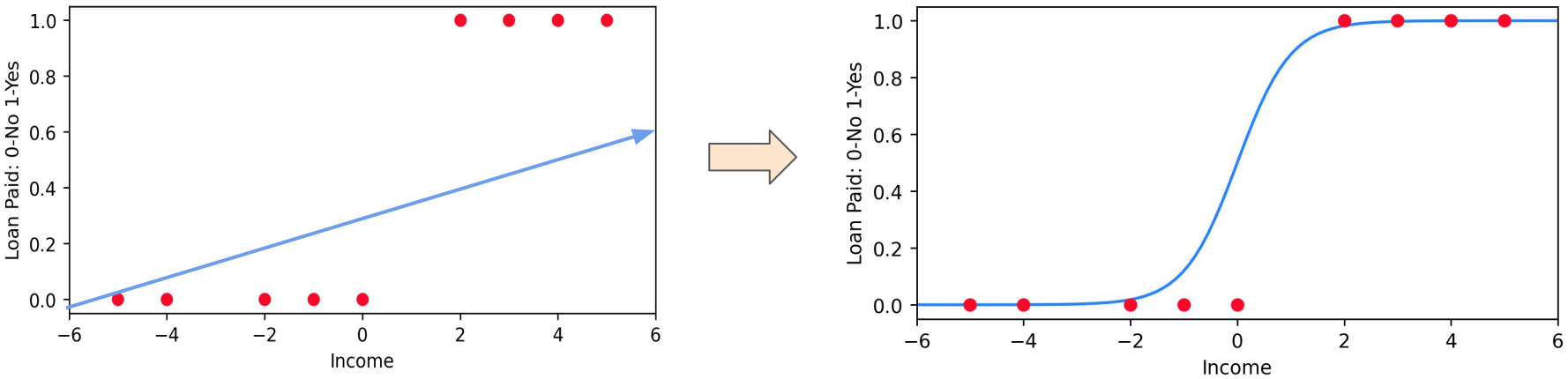
Logistic Regression Theory and Intuition

Part Two: Linear to Logistic Math

PIERIAN DATA

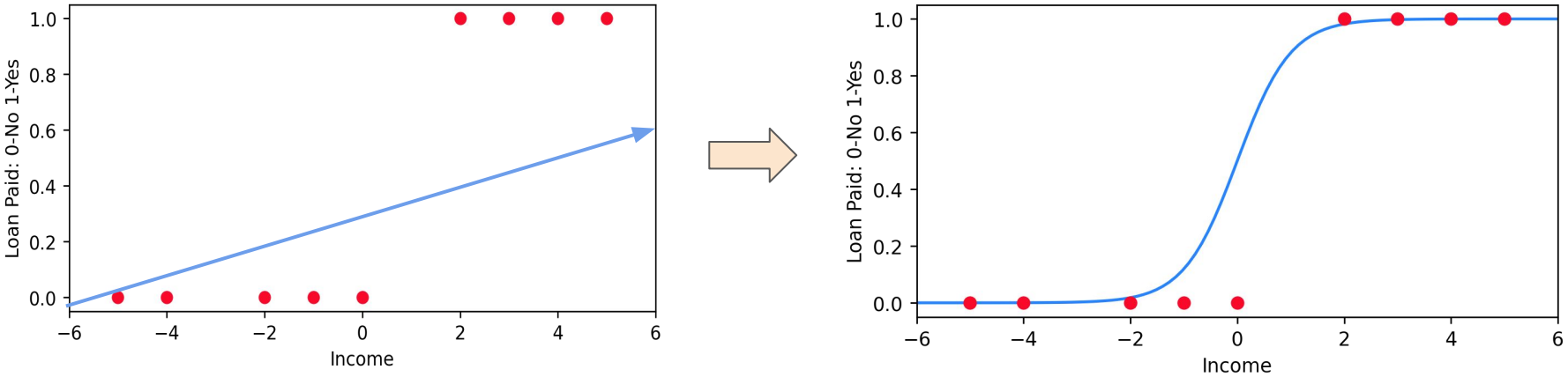
Logistic Regression

- Let's go through the math of converting Linear Regression to Logistic Regression.



Logistic Regression

- Relevant ISLR Reading:
  - Section 4.3 Logistic Regression



Logistic Regression

- We already know the Linear Regression equation:

$$\hat{y} = \beta_0 x_0 + \cdots + \beta_n x_n$$

$$\hat{y} = \sum_{i=0}^{n} \beta_i x_i$$

Logistic Regression

- We also know the Logistic function transforms any input to be between 0 and 1

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

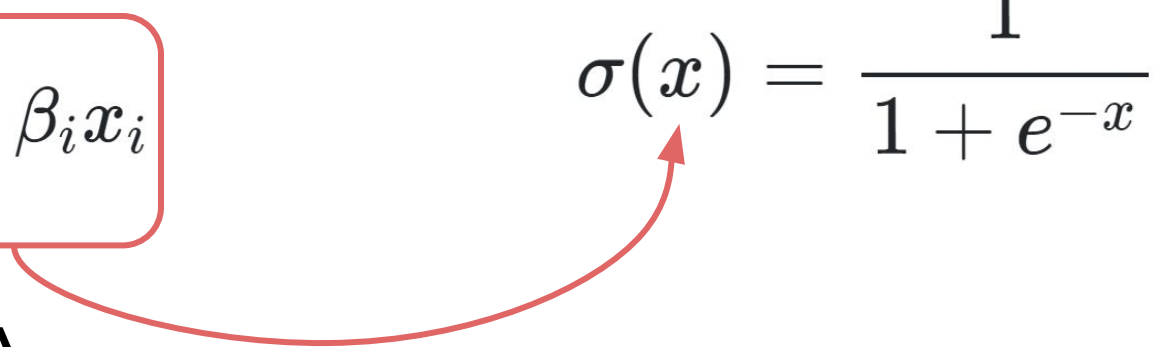
Logistic Regression

- All we need to do is plug the Linear Regression equation into the Logistic function to create a Logistic Regression!

$$\hat{y} = \beta_0 x_0 + \cdots + \beta_n x_n$$

$$\hat{y} = \sum_{i=0}^{n} \beta_i x_i$$

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$



Logistic Regression

- Simply put in terms of the logistic function:

$$\hat{y} = \sigma(\beta_0 x_0 + \cdots + \beta_n x_n)$$

$$\hat{y} = \sigma\left(\sum_{i=0}^{n} \beta_i x_i\right)$$

Logistic Regression

- Writing it out fully:

$$\hat{y} = \frac{1}{1 + e^{-\sum_{i=0}^{n} \beta_i x_i}}$$

Logistic Regression

- How do we interpret the coefficients and their relation to $\hat{\mathbf{y}}$?

$$\hat{y} = \frac{1}{1 + e^{-\sum_{i=0}^{n} \beta_i x_i}}$$

Logistic Regression

- First we need to understand the term **odds.**
- A term you may be familiar with from gambling **odds**.

Logistic Regression

- In gambling odds are often referred to in the sense of N to 1.
- But where does this actually come from?

PIERIAN DATA

Logistic Regression

- The odds of an event with probability **p** is defined as the chance of the event happening divided by the chance of the event not happening:

$$\frac{p}{1-p}$$

Logistic Regression

- Imagine an event with **50%** probability of occuring. This is **0.5/1-0.5** which is **0.5/0.5** , the same as **1/1** or **1 to 1 odds of occurring.**

$$\frac{p}{1-p}$$

Logistic Regression

- Taking the formula below, we can rearrange it to show that it is equivalent to modelling the log of the odds as a linear combination of the features.

$$\hat{y} = \frac{1}{1 + e^{-\sum_{i=0}^{n} \beta_i x_i}}$$

Logistic Regression

- This will allow us to solve for the coefficients and feature x in terms of **log odds.**

$$\hat{y} = \frac{1}{1 + e^{-\sum_{i=0}^{n} \beta_i x_i}}$$

Logistic Regression

- Solving for **log odds:**

$$\hat{y} = \frac{1}{1 + e^{-\sum_{i=0}^{n} \beta_i x_i}}$$

Logistic Regression

- Solving for **log odds:**

$$\hat{y} = \frac{1}{1 + e^{-\sum_{i=0}^{n} \beta_i x_i}}$$

$$\hat{y} + \hat{y} e^{-\sum_{i=0}^{n} \beta_i x_i} = 1$$

Logistic Regression

- Solving for **log odds:**

$$\hat{y} + \hat{y}e^{-\sum_{i=0}^{n} \beta_i x_i} = 1$$

Logistic Regression

- Solving for **log odds:**

$$\hat{y} + \hat{y}e^{-\sum_{i=0}^{n} \beta_i x_i} = 1$$

$$\hat{y}e^{-\sum_{i=0}^{n} \beta_i x_i} = 1 - \hat{y}$$

Logistic Regression

- Solving for **log odds:**

$$\hat{y} + \hat{y}e^{-\sum_{i=0}^{n} \beta_i x_i} = 1$$

$$\hat{y}e^{-\sum_{i=0}^{n} \beta_i x_i} = 1 - \hat{y}$$

$$\frac{\hat{y}}{1 - \hat{y}} = e^{\sum_{i=0}^{n} \beta_i x_i}$$

Logistic Regression

- Solving for **log odds:**

$$\frac{\hat{y}}{1-\hat{y}} = e^{\sum_{i=0}^{n} \beta_i x_i}$$

Logistic Regression

- Solving for **log odds:**

$$\frac{\hat{y}}{1-\hat{y}} = e^{\sum_{i=0}^{n} \beta_i x_i}$$

$$\ln\left(\frac{\hat{y}}{1-\hat{y}}\right) = \sum_{i=0}^{n} \beta_i x_i$$

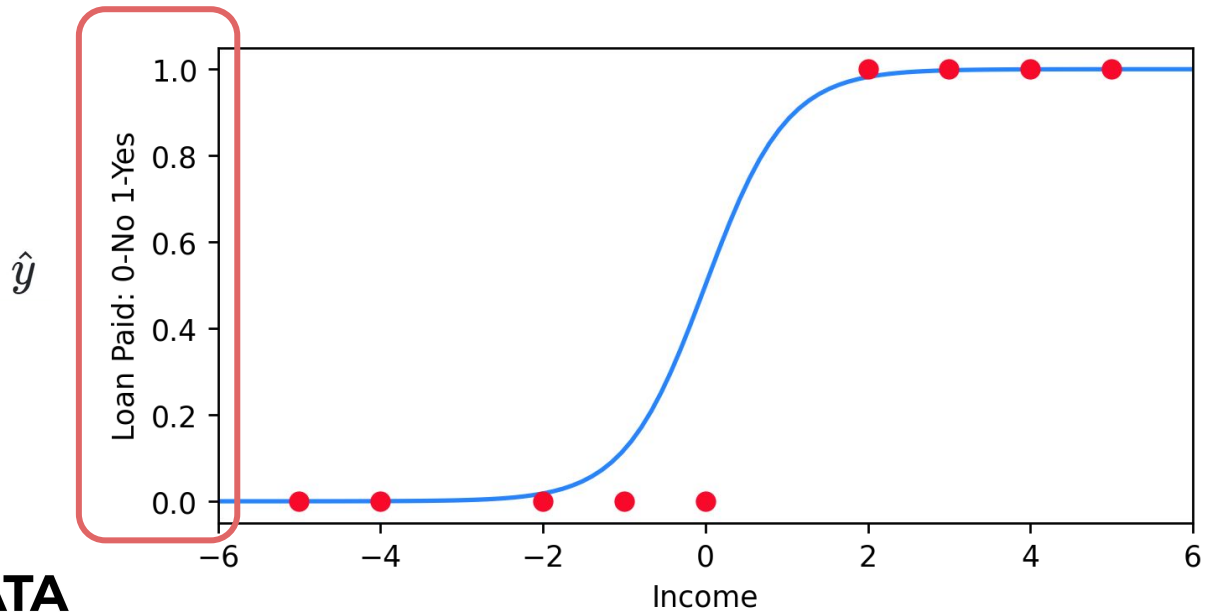
Logistic Regression

- What would the function curve look like in terms of log odds?

$$\ln\left(\frac{\hat{y}}{1-\hat{y}}\right) = \sum_{i=0}^{n} \beta_i x_i$$

Logistic Regression

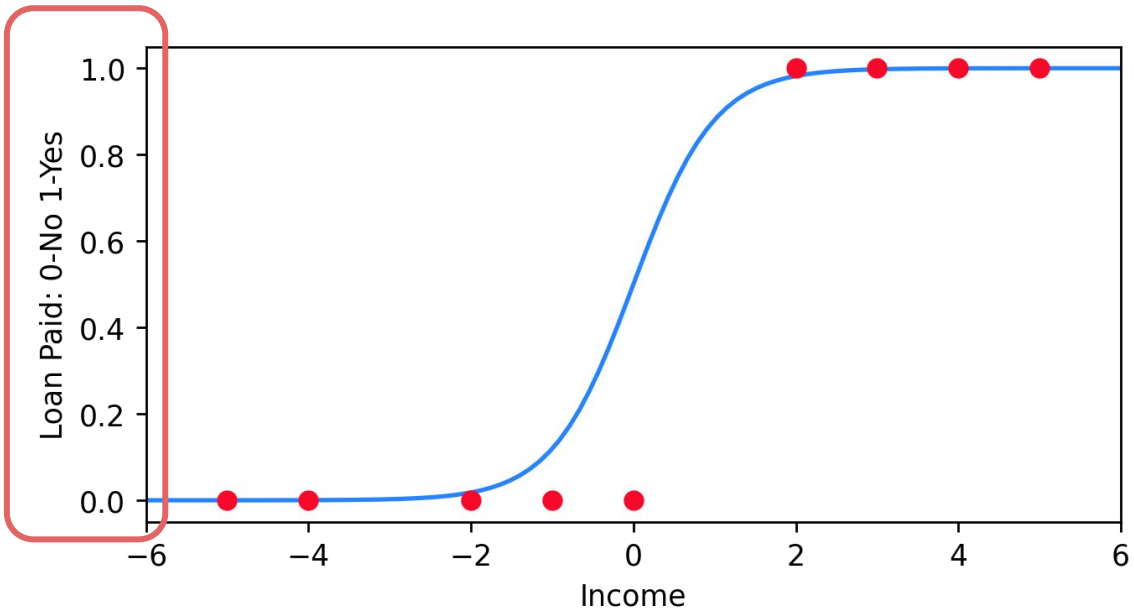
- What would the function curve look like in terms of log odds?



Logistic Regression

- What would the function curve look like in terms of log odds?

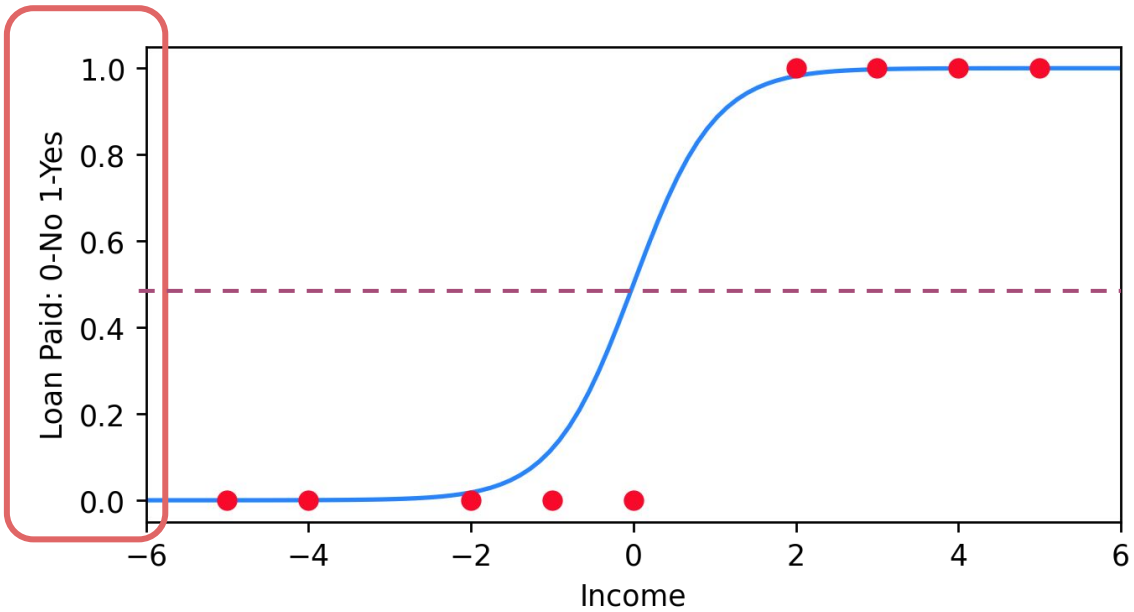
$$\ln\left(\frac{\hat{y}}{1-\hat{y}}\right) \Leftarrow \hat{y}$$



Logistic Regression

- Consider p=0.5

$$ln(\frac{0.5}{1-0.5}) = 0$$



Logistic Regression

- Consider p=0.5, halfway point now at 0.

$$ln(\frac{0.5}{1-0.5}) = 0$$

Logistic Regression

- As p goes to 1 then log odds becomes ∞

$$\lim_{p \to 1} ln(\frac{p}{1-p}) = \infty$$

$$ln(\frac{0.5}{1-0.5}) = 0$$

Logistic Regression

- As p goes to 0 then log odds becomes -∞

$$\lim_{p \to 1} ln(\frac{p}{1-p}) = \infty$$

$$ln(\frac{0.5}{1-0.5}) = 0$$

$$\lim_{p \to 0} ln(\frac{p}{1-p}) = -\infty$$

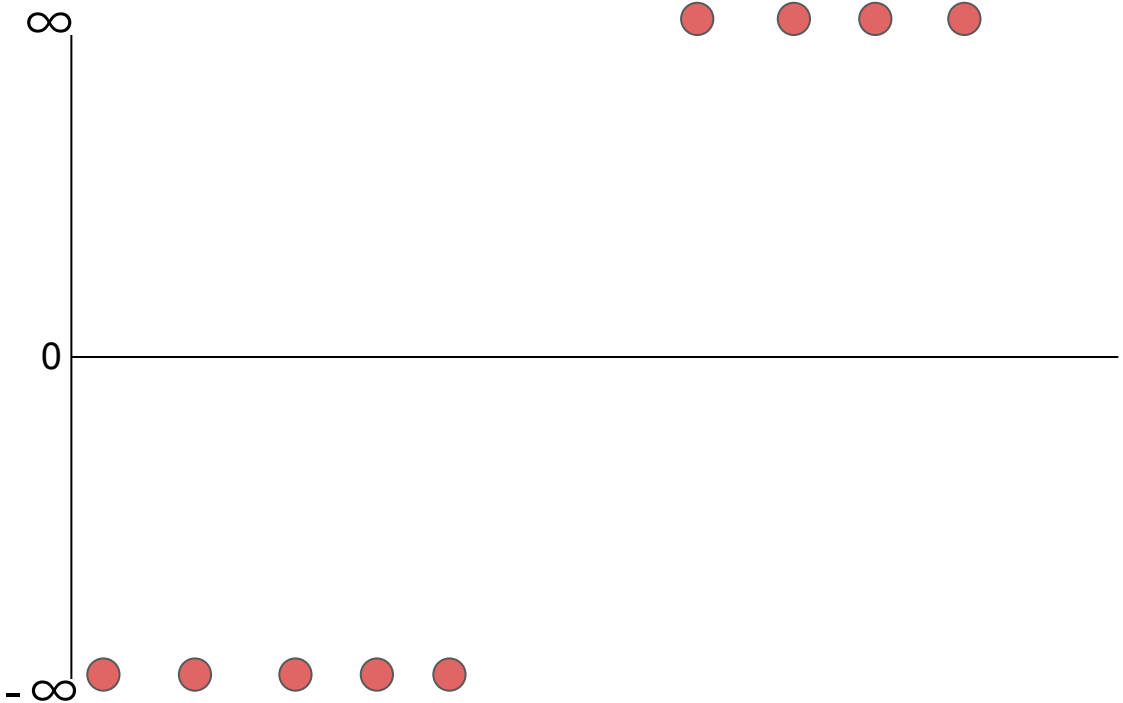
Logistic Regression

- Class points now at infinity

$$\lim_{p \to 1} ln(\frac{p}{1-p}) = \infty$$

$$ln(\frac{0.5}{1-0.5}) = 0$$

$$\lim_{p \to 0} ln(\frac{p}{1-p}) = -\infty$$



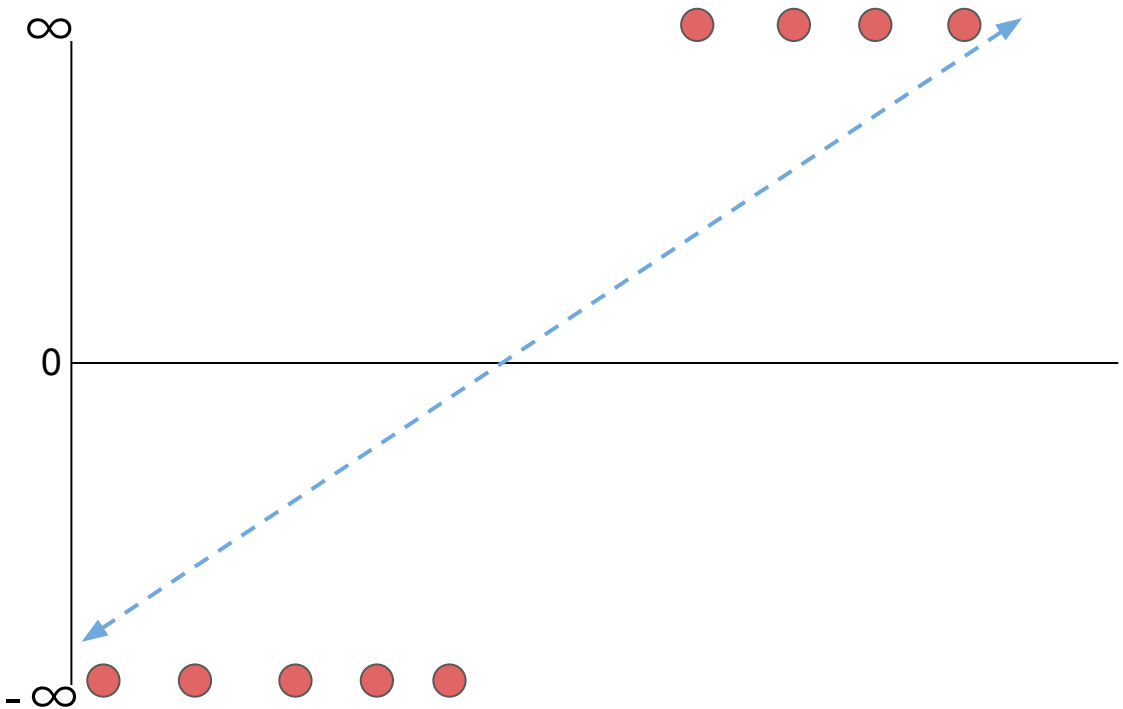
Logistic Regression

- On log scale logistic function is straight line

$$\lim_{p \to 1} ln(\frac{p}{1-p}) = \infty$$

$$ln(\frac{0.5}{1-0.5}) = 0$$

$$\lim_{p \to 0} ln(\frac{p}{1-p}) = -\infty$$



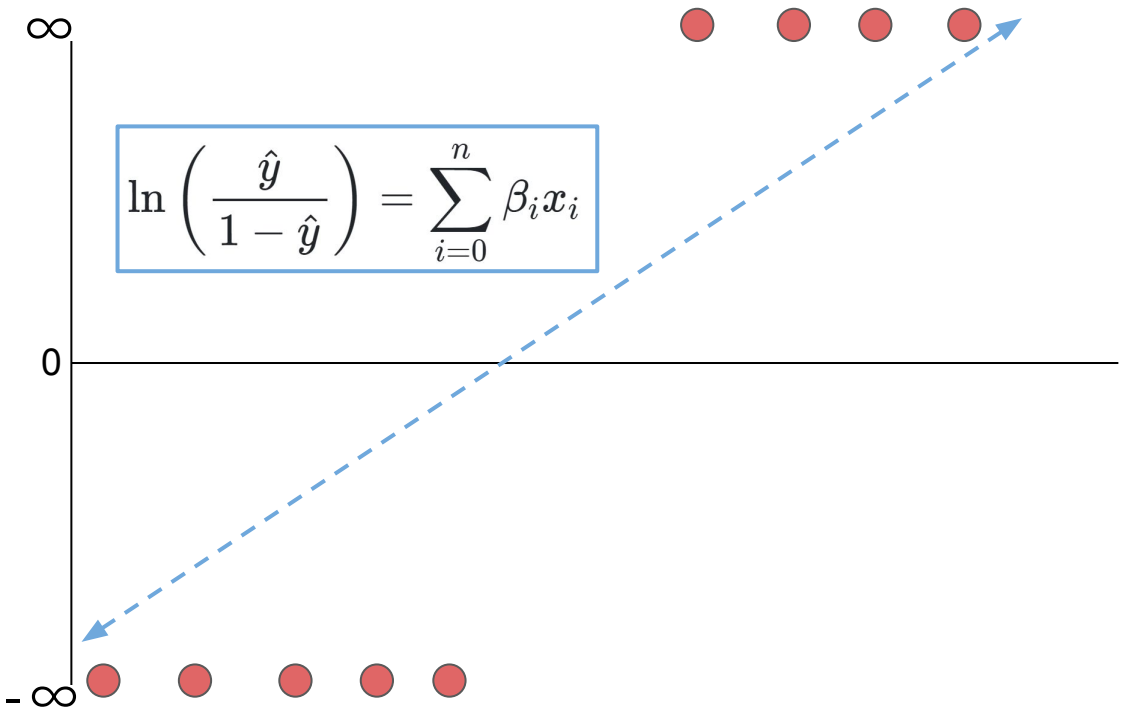
Logistic Regression

- Coefficients in terms of change in log odds.

$$\lim_{p \to 1} ln(\frac{p}{1-p}) = \infty$$

$$ln(\frac{0.5}{1-0.5}) = 0$$

$$\lim_{p \to 0} ln(\frac{p}{1-p}) = -\infty$$



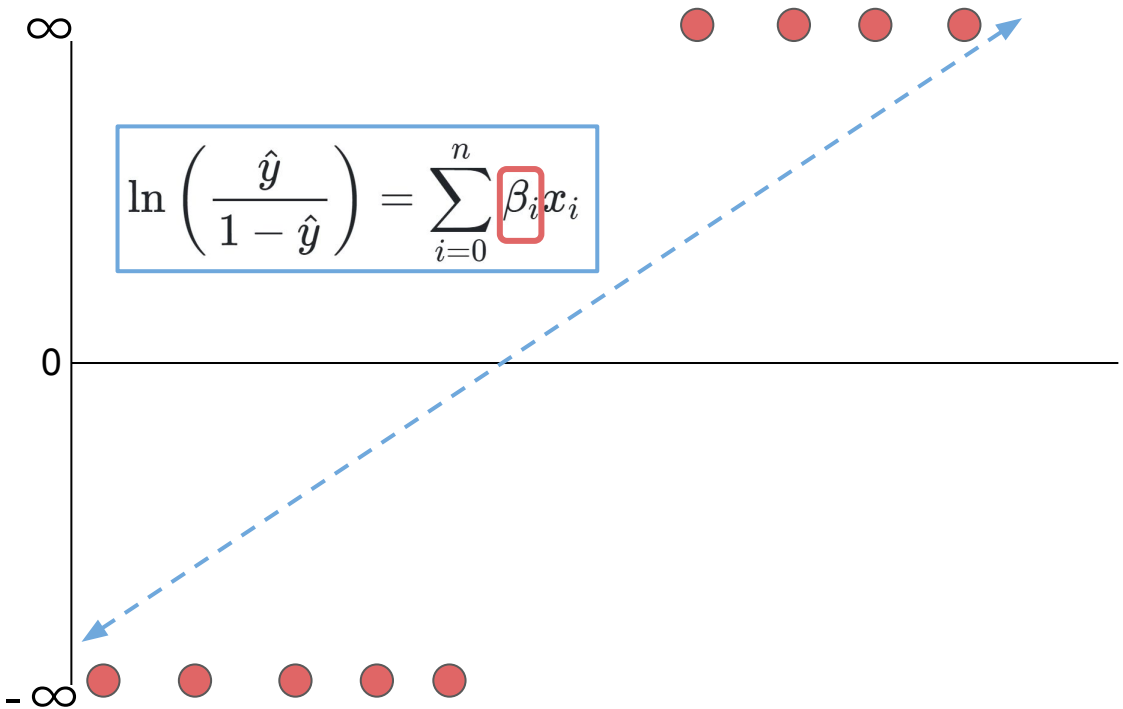
Logistic Regression

- Is **β** simple to interpret? Not really...

$$\lim_{p \to 1} ln(\frac{p}{1-p}) = \infty$$

$$ln(\frac{0.5}{1-0.5}) = 0$$

$$\lim_{p \to 0} ln(\frac{p}{1-p}) = -\infty$$



Logistic Regression

- Since the log odds scale is nonlinear, a **β** value can not be directly linked to “one unit increase” as it could in Linear Regression.

$$\ln\left(\frac{\hat{y}}{1-\hat{y}}\right) = \sum_{i=0}^{n} \beta_i x_i$$

Logistic Regression

- There are some straightforward insights we can gain however...

$$\ln\left(\frac{\hat{y}}{1-\hat{y}}\right) = \sum_{i=0}^{n} \beta_i x_i$$

Logistic Regression

- Sign of Coefficient
  - Positive **β** indicates an increase in likelihood of belonging to 1 class with increase in associated **x** feature.
  - Negative **β** indicates an decrease in likelihood of belonging to 1 class with increase in associated **x** feature.

Logistic Regression

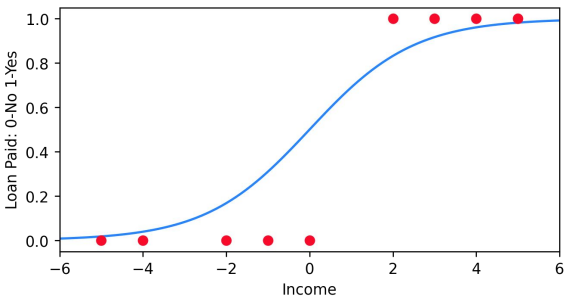
- Magnitude of Coefficient
  - Harder to directly interpret magnitude of **β** directly, especially when we could have discrete and continuous x feature values.
  - We can however begin to use **odds ratio**, essentially comparing magnitudes against each other.

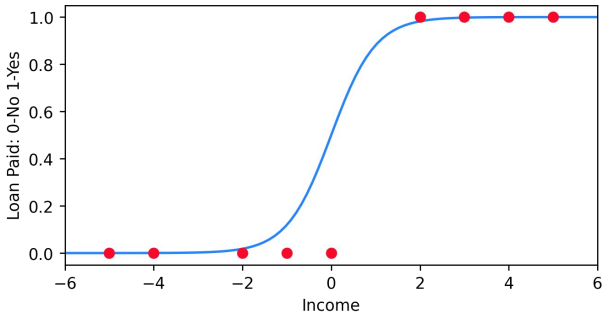
Logistic Regression

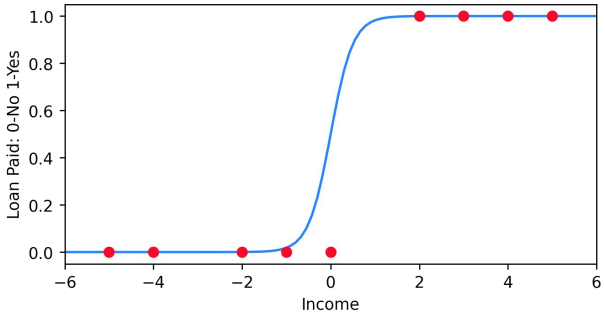
- Magnitude of Coefficient
  - Comparing magnitudes of coefficients against each other can lead to insight over which features have the strongest effect on prediction output.

Logistic Regression

- The last mathematical topic we need to discuss concerning Logistic Regression is how we actually fit this curve!

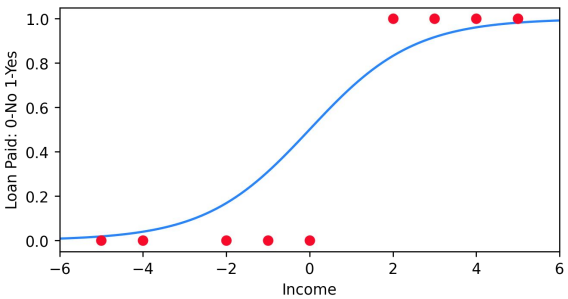


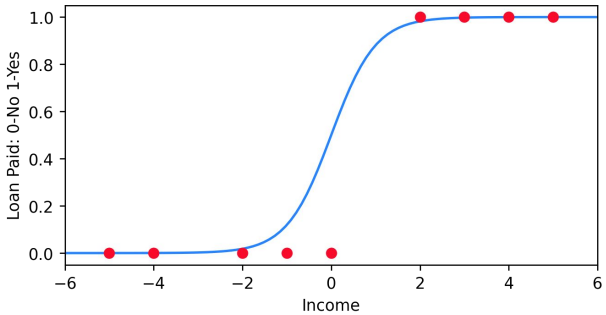


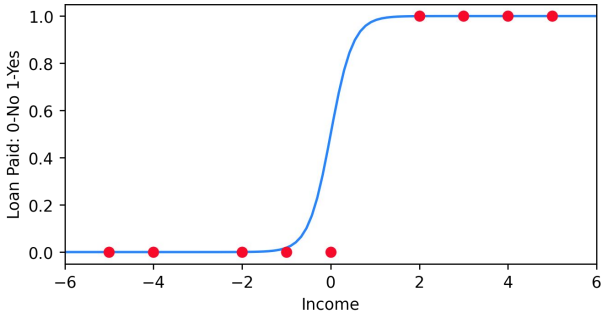


Logistic Regression

- We'll discuss the basics of fitting the best curve with maximum likelihood in the next lecture!







Logistic Regression Theory and Intuition

Part Three: Finding the Best Fit

PIERIAN DATA

Logistic Regression

- Logistic Regression uses Maximum Likelihood to find the best fitting model.
- This lecture will give you an intuition of how this method works.
- We'll also then display the cost function and gradient descent that is solved for by the computer.

Logistic Regression

- Quick Note: ISLR Section 4.3.2

default status. In other words, we try to find $\hat{\beta}_0$ and $\hat{\beta}_1$ such that plugging these estimates into the model for $p(X)$, given in (4.2), yields a number close to one for all individuals who defaulted, and a number close to zero for all individuals who did not. This intuition can be formalized using a mathematical equation called a *likelihood function*:

$$\ell(\beta_0, \beta_1) = \prod_{i:y_i=1} p(x_i) \prod_{i':y_{i'}=0} (1 - p(x_{i'})). \quad (4.5)$$

The estimates $\hat{\beta}_0$ and $\hat{\beta}_1$ are chosen to *maximize* this likelihood function.

Maximum likelihood is a very general approach that is used to fit many of the non-linear models that we examine throughout this book. In the linear regression setting, the least squares approach is in fact a special case of maximum likelihood. The mathematical details of maximum likelihood are beyond the scope of this book. However, in general, logistic regression

Logistic Regression

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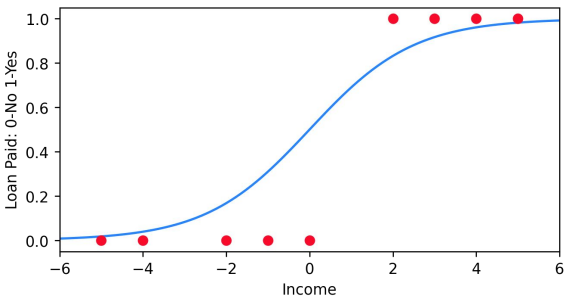
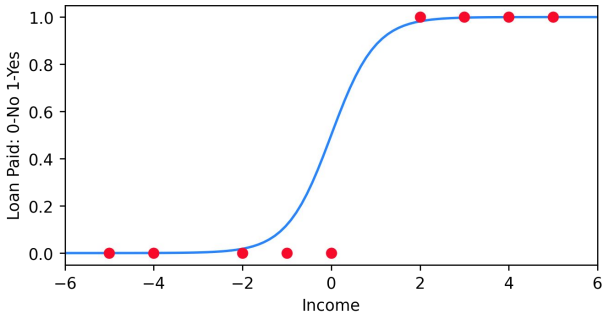
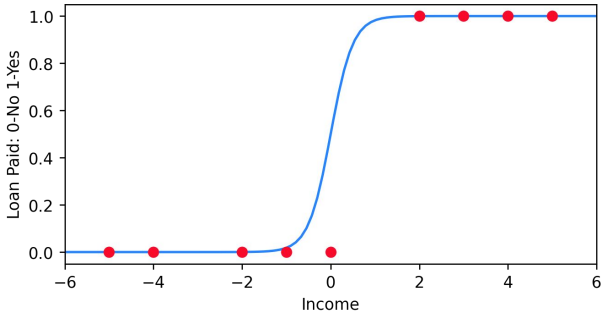
Logistic Regression

- Here we see three different Logistic Regression curves with different **β** values.
- How do we measure which is the best fit?

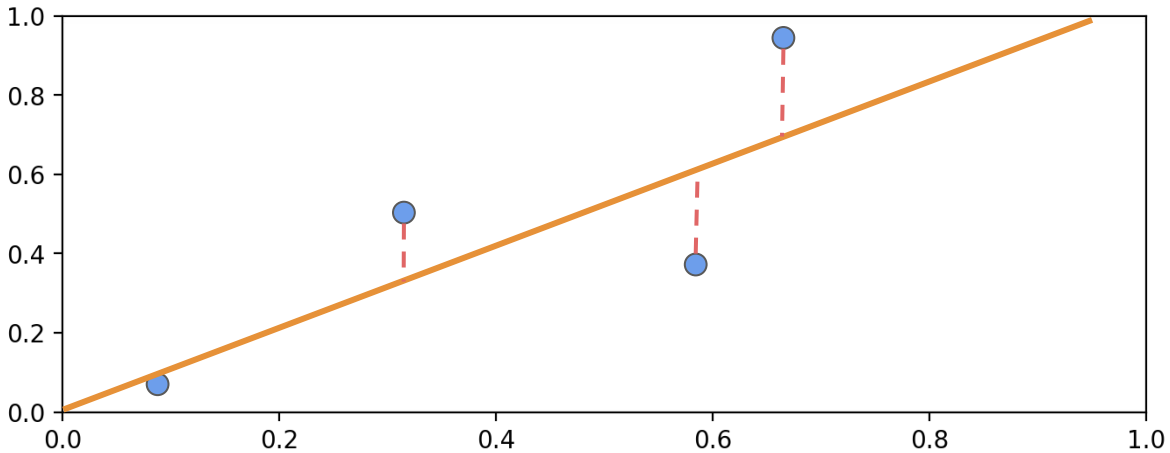






Logistic Regression

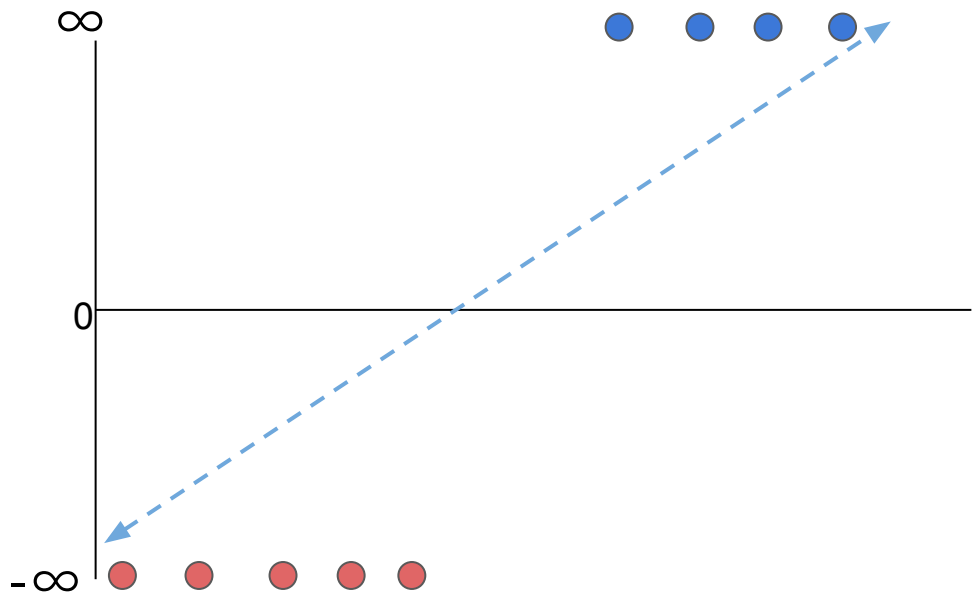
- Recall in Linear Regression we seek to minimize the Residual Sum of Squares (RSS).



Logistic Regression

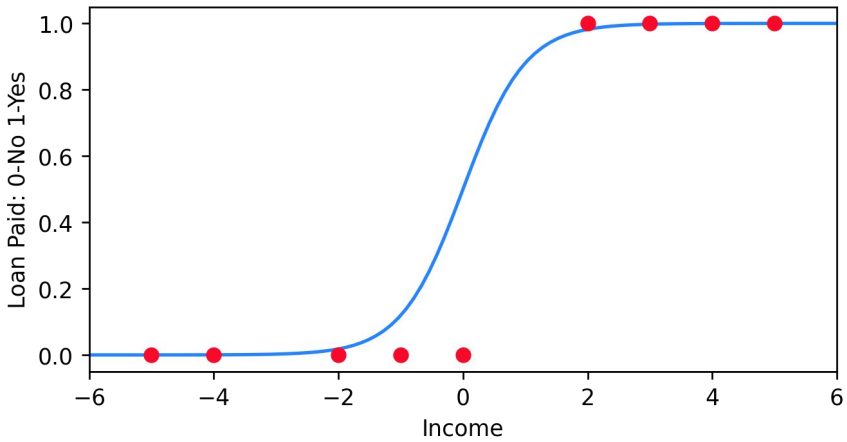
- Unfortunately, even in log odds targets are at infinity, making RSS unfeasible.

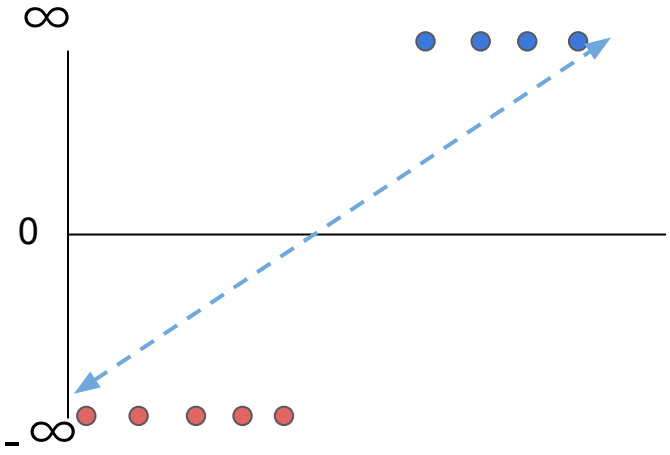
$$\ln\left(\frac{\hat{y}}{1-\hat{y}}\right) = \sum_{i=0}^{n} \beta_i x_i$$



Logistic Regression

- The first step for maximum likelihood is to go from log odds back to probability.





Logistic Regression

- The first step for maximum likelihood is to go from log odds back to probability.

$$\ln(\frac{p}{1-p}) = \ln(odds)$$

Logistic Regression

- The first step for maximum likelihood is to go from log odds back to probability.

$$\ln(\frac{p}{1-p}) = \ln(odds)$$

$$\frac{p}{1-p} = e^{\ln(odds)}$$

Logistic Regression

- The first step for maximum likelihood is to go from log odds back to probability.

$$\ln(\frac{p}{1-p}) = \ln(odds)$$

$$\frac{p}{1-p} = e^{\ln(odds)}$$

$$p = (1-p)e^{\ln(odds)}$$

Logistic Regression

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$$p = (1 - p)e^{\ln(odds)}$$

Logistic Regression

- The first step for maximum likelihood is to go from log odds back to probability.

$$p = (1 - p)e^{\ln(odds)}$$

$$p = e^{\ln(odds)} - pe^{\ln(odds)}$$

Logistic Regression

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$$p = (1 - p)e^{\ln(odds)}$$

$$p = e^{\ln(odds)} - pe^{\ln(odds)}$$

$$p + pe^{\ln(odds)} = e^{\ln(odds)}$$

Logistic Regression

- The first step for maximum likelihood is to go from log odds back to probability.

$$p = (1 - p)e^{\ln(odds)}$$

$$p = e^{\ln(odds)} - pe^{\ln(odds)}$$

$$p + pe^{\ln(odds)} = e^{\ln(odds)}$$

$$p(1 + e^{\ln(odds)}) = e^{\ln(odds)}$$

Logistic Regression

- The first step for maximum likelihood is to go from log odds back to probability.

$$p = e^{\ln(odds)} - pe^{\ln(odds)}$$

$$p + pe^{\ln(odds)} = e^{\ln(odds)}$$

$$p(1 + e^{\ln(odds)}) = e^{\ln(odds)}$$

$$p = \frac{e^{\ln(odds)}}{1 + e^{\ln(odds)}}$$

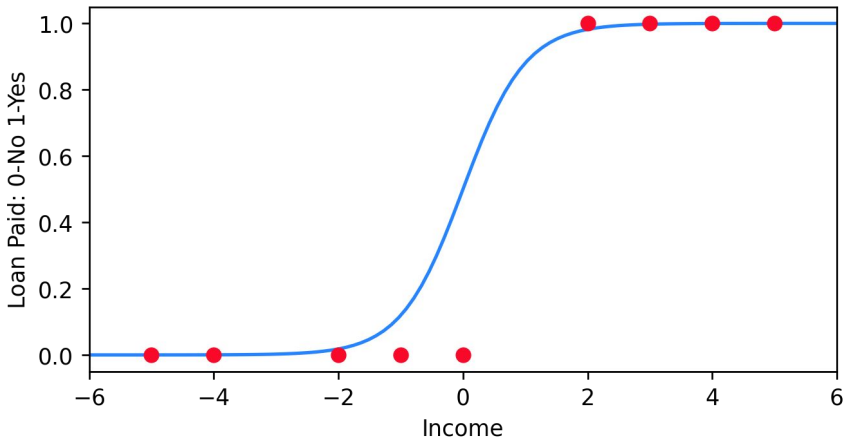
Logistic Regression

- The first step for maximum likelihood is to go from log odds back to probability.

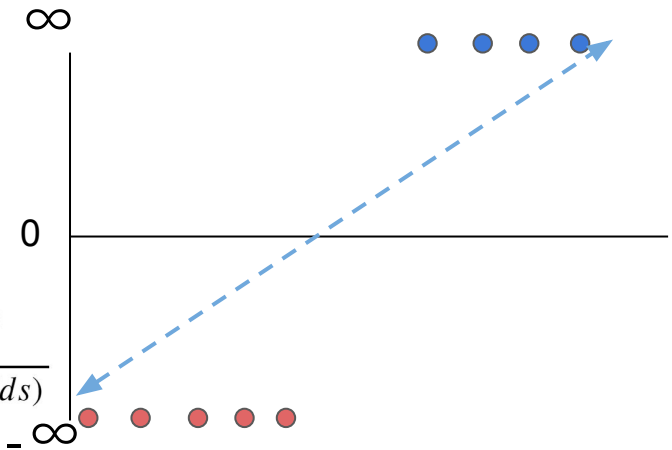
$$p = \frac{e^{\ln(odds)}}{1 + e^{\ln(odds)}}$$

Logistic Regression

- We are now able to convert ln(odds) into a probability.

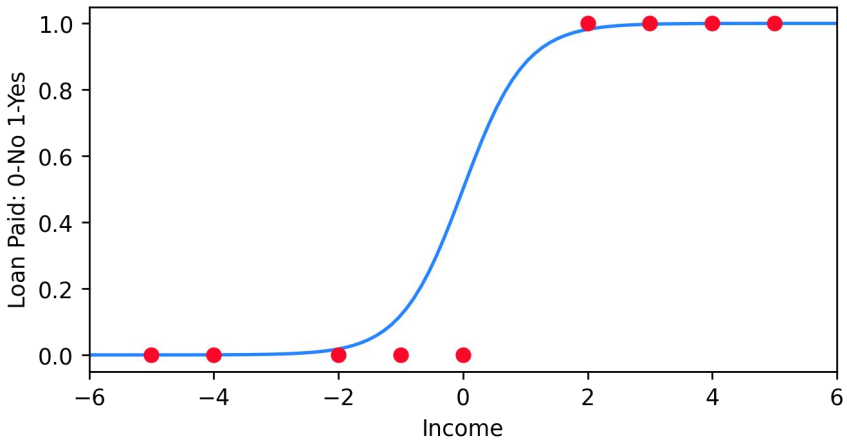


$$p = \frac{e^{\ln(odds)}}{1 + e^{\ln(odds)}}$$

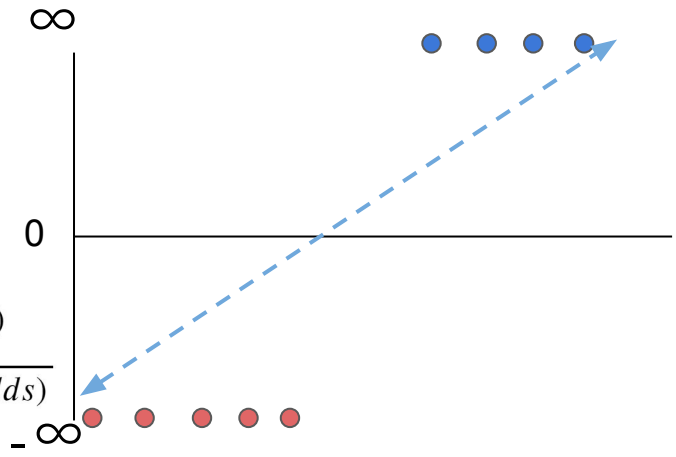


Logistic Regression

- Let's now explore the intuition behind **maximum likelihood.**

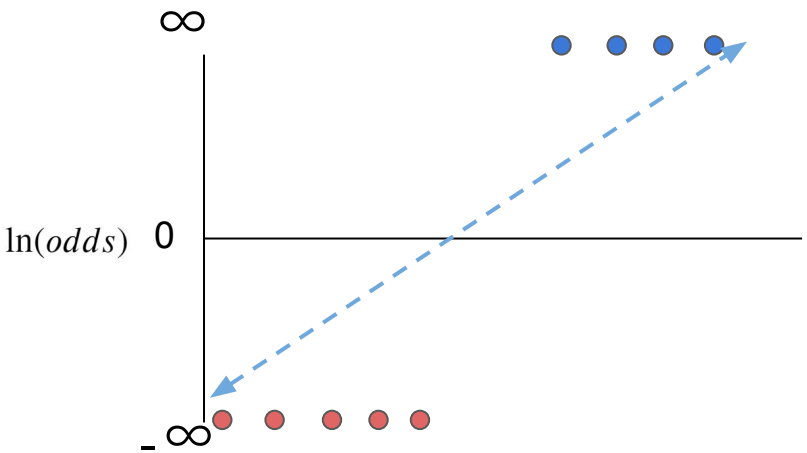


$$p = \frac{e^{\ln(odds)}}{1 + e^{\ln(odds)}}$$



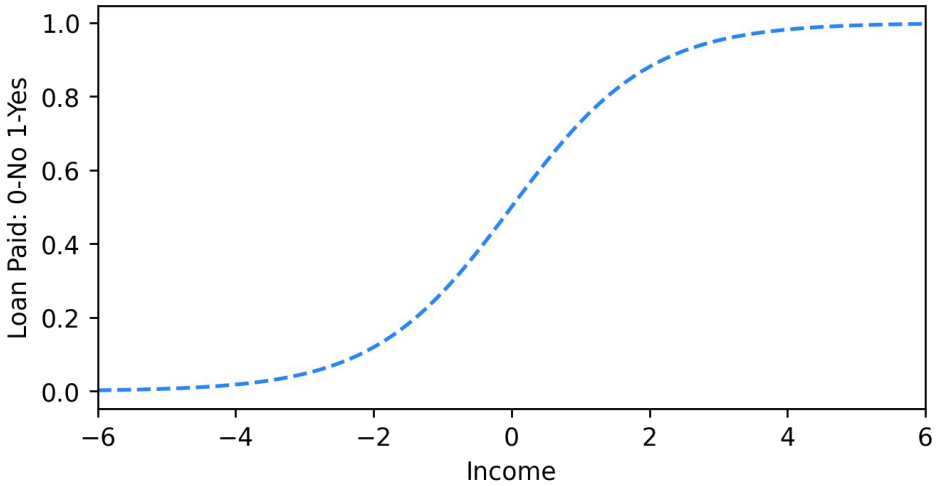
Logistic Regression

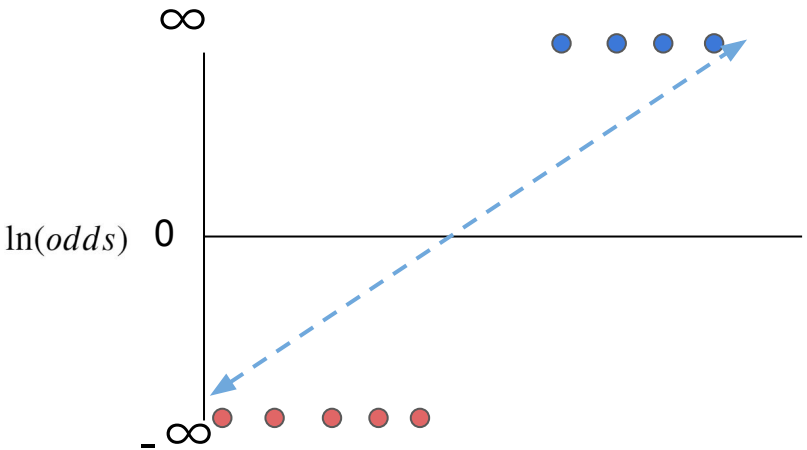
- We choose a line in the log(odds) axis and project the points on to the line:



Logistic Regression

- We also know this line has a form on the probability y-axis.





Logistic Regression

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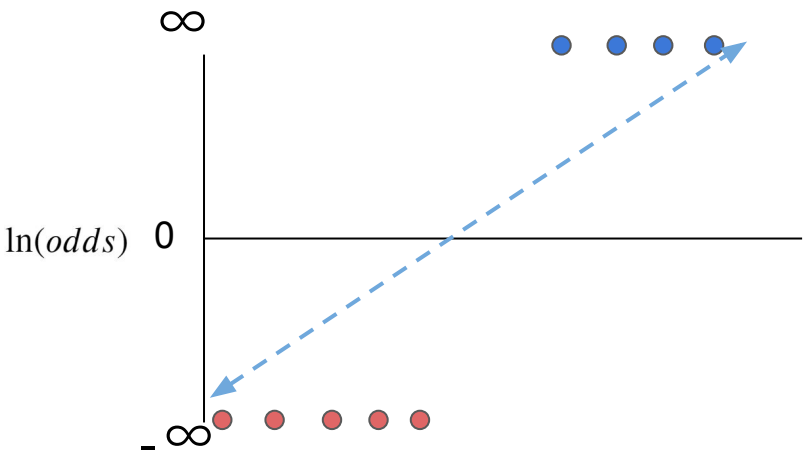

Logistic Regression

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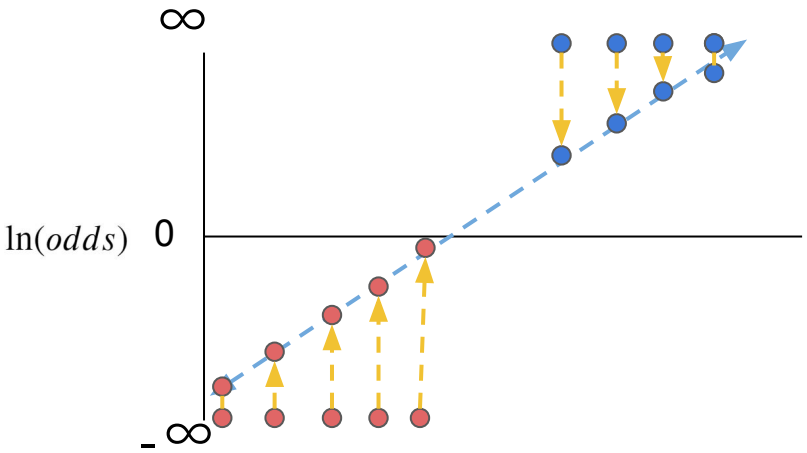
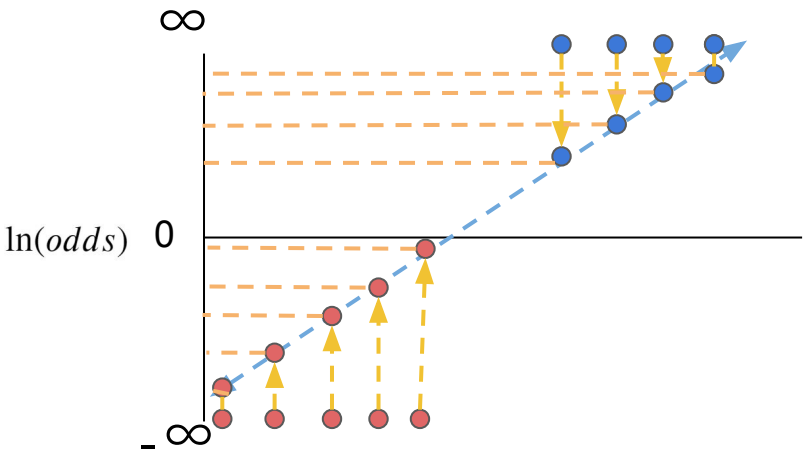


Logistic Regression

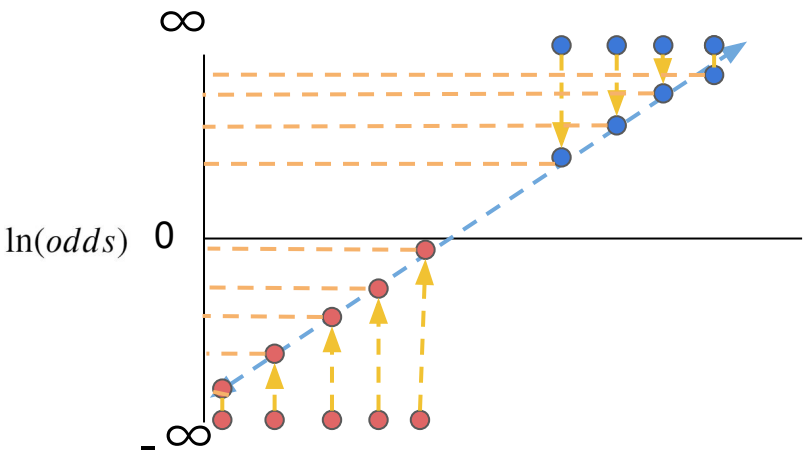
- Calculate the log odds for the projected points on this line.



Logistic Regression

- Plot these values as probabilities on the logistic regression model.

$$p = \frac{e^{\ln(odds)}}{1 + e^{\ln(odds)}}$$



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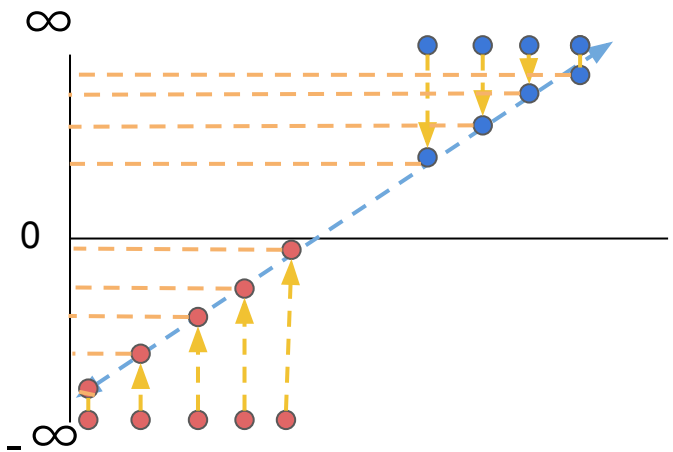

Logistic Regression

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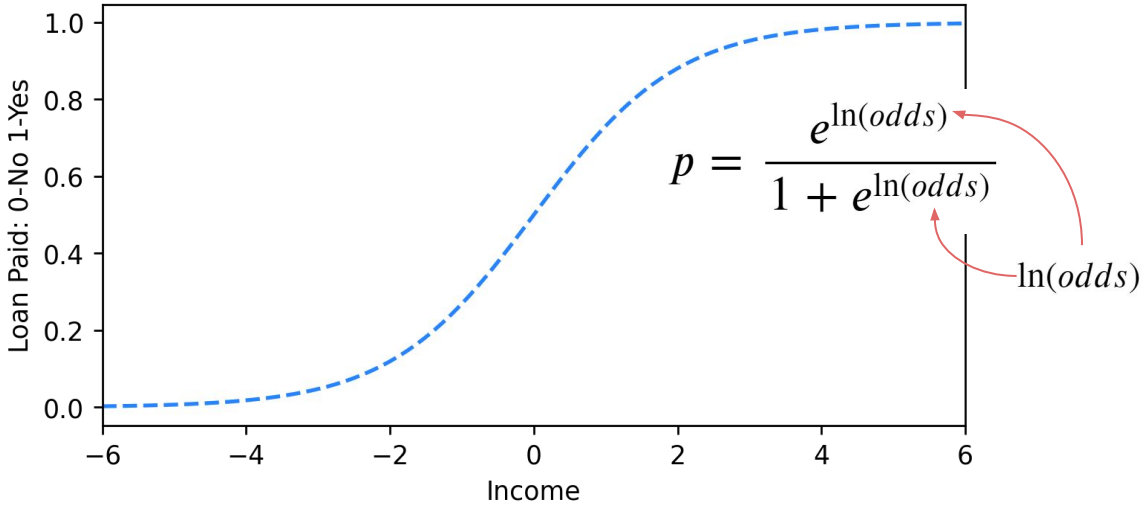

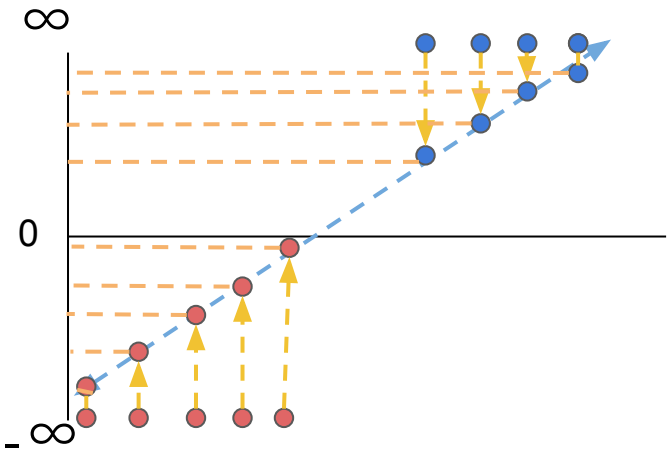

Logistic Regression

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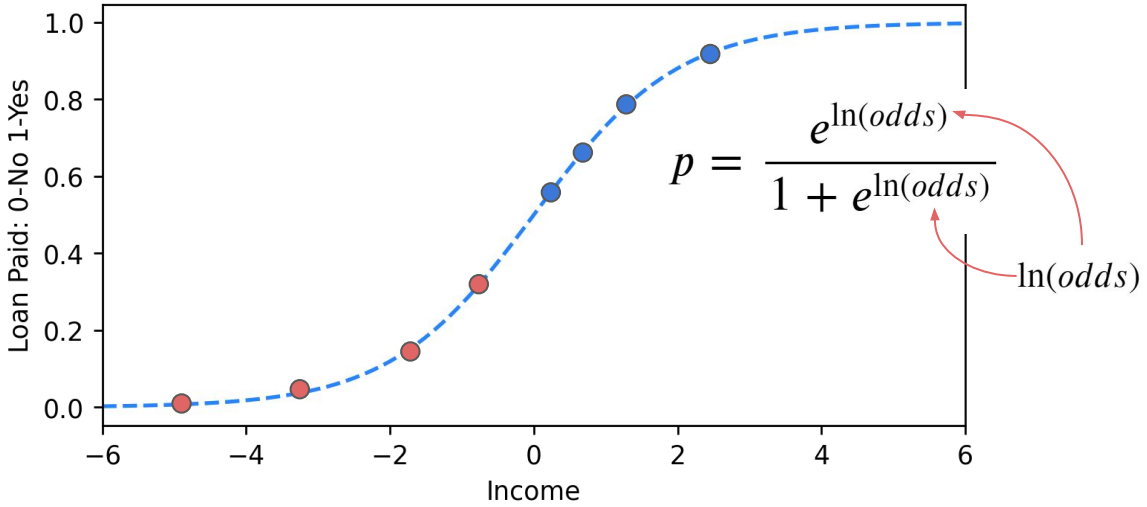
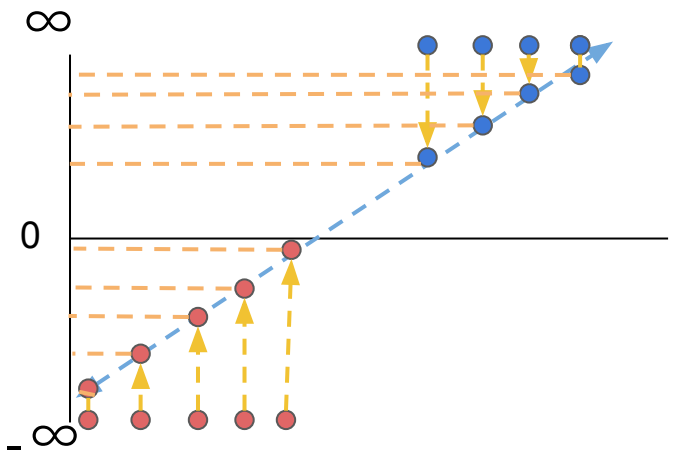
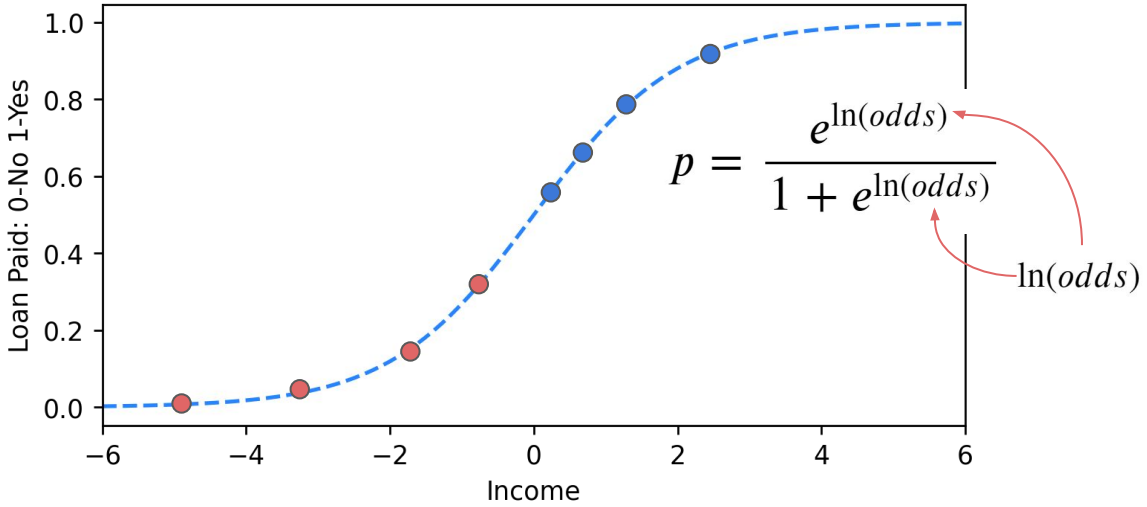
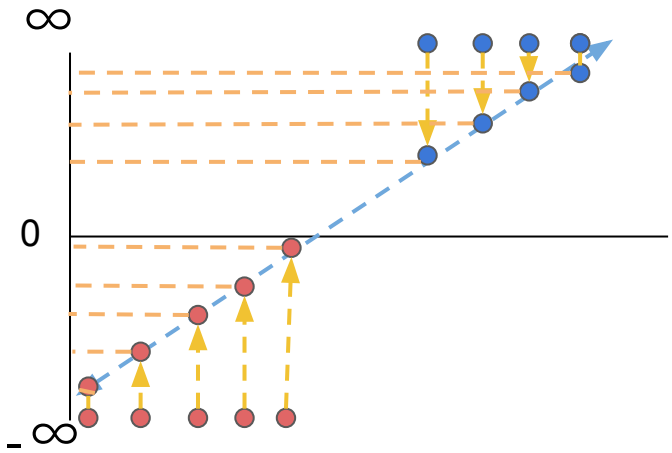




Logistic Regression

- We now measure the likelihood of these probabilities.





PIERIAN DATA

Logistic Regression

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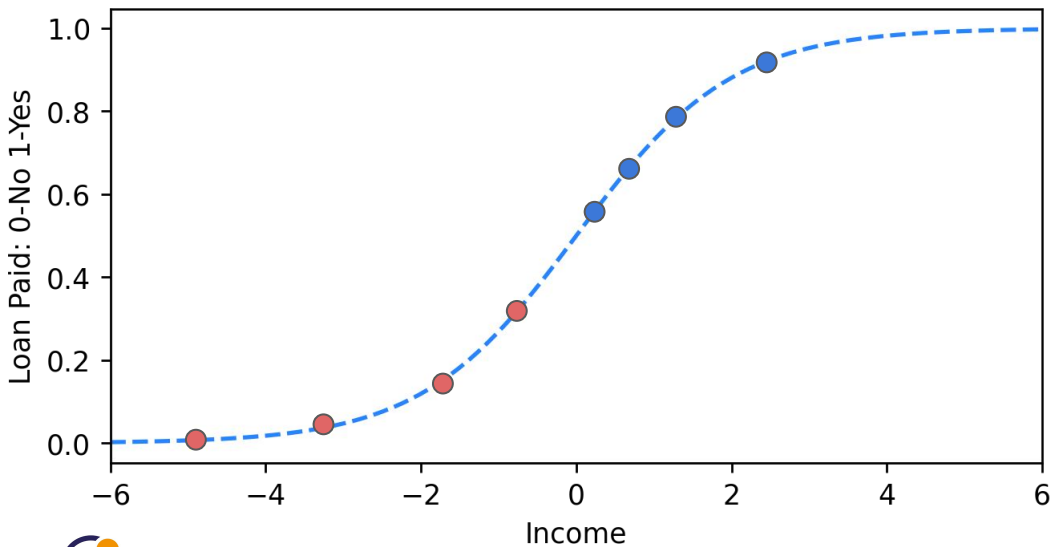
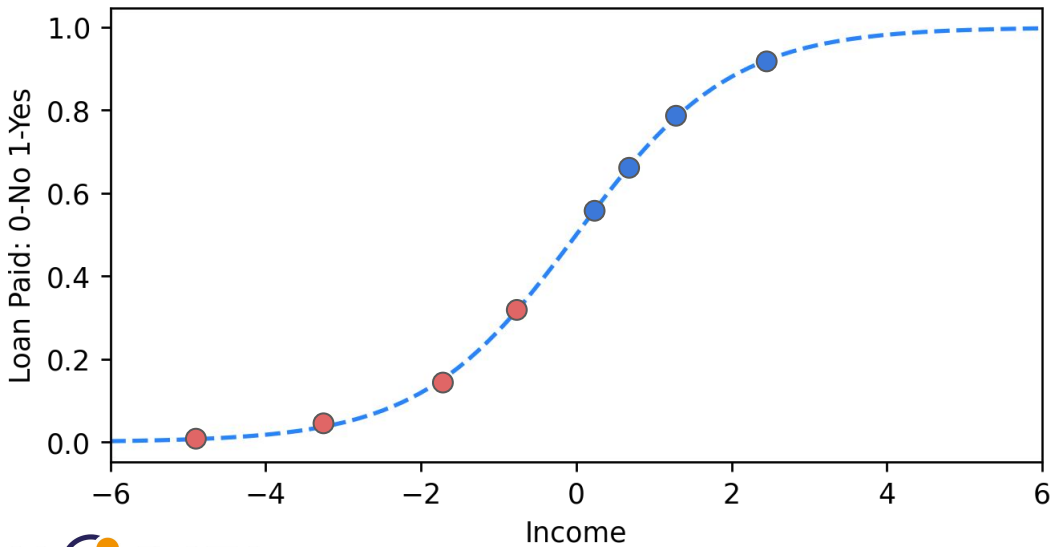


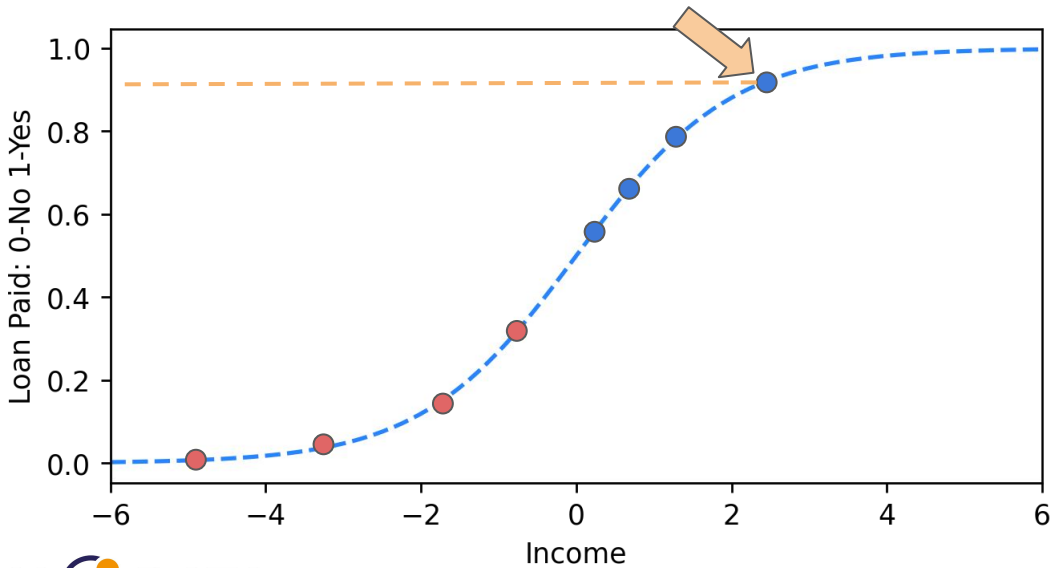
Logistic Regression

- Likelihood = Product of probabilities of belonging to class 1.



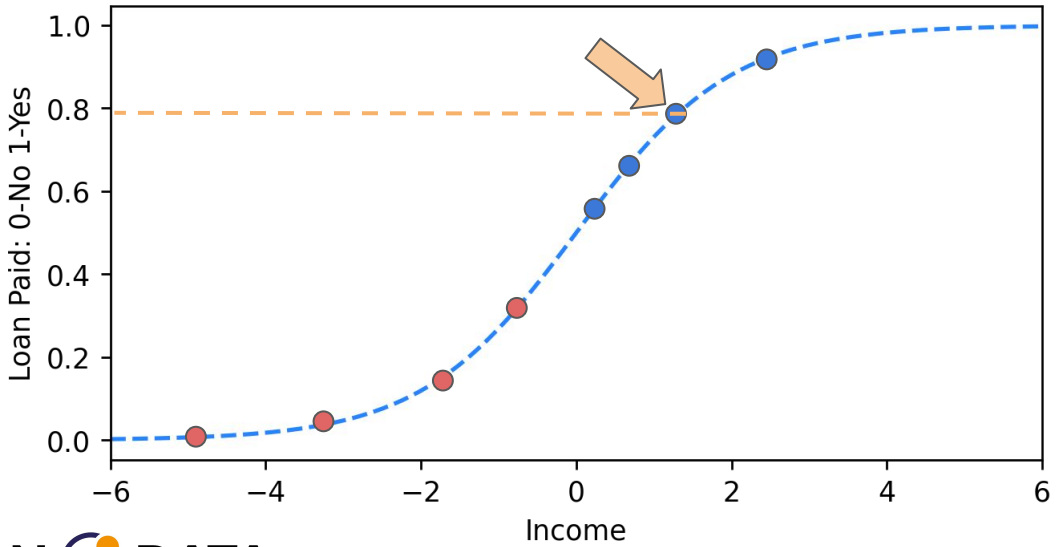
Logistic Regression

- Likelihood = 0.9 ...



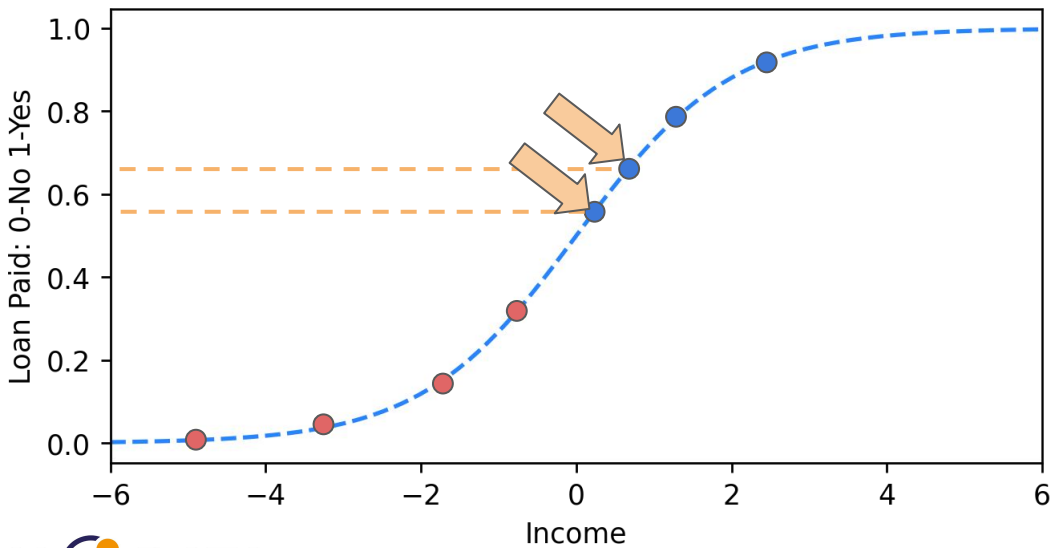
Logistic Regression

- Likelihood = 0.9 × 0.8 × ...



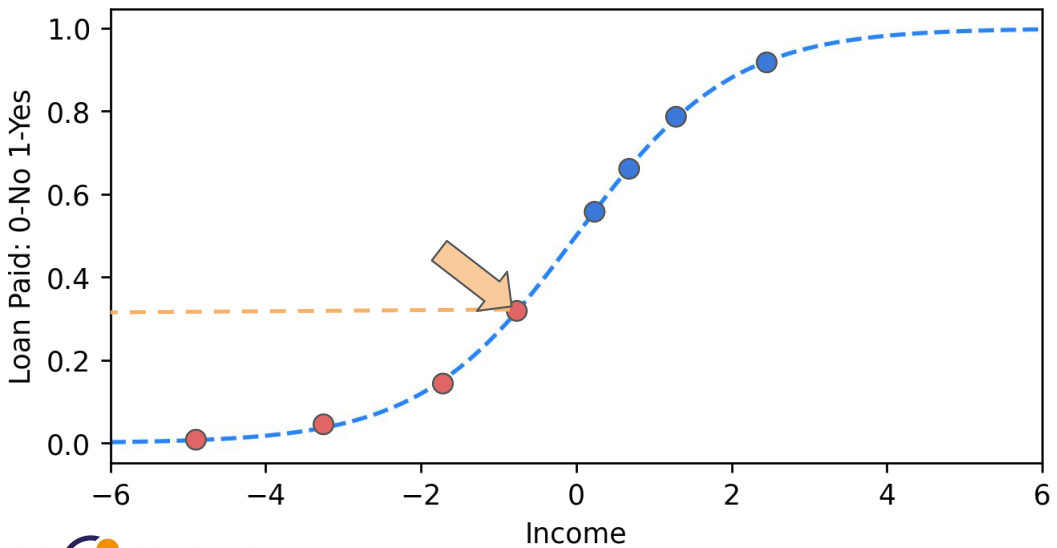
Logistic Regression

- Likelihood = 0.9 × 0.8 × 0.65 × 0.55 × ...



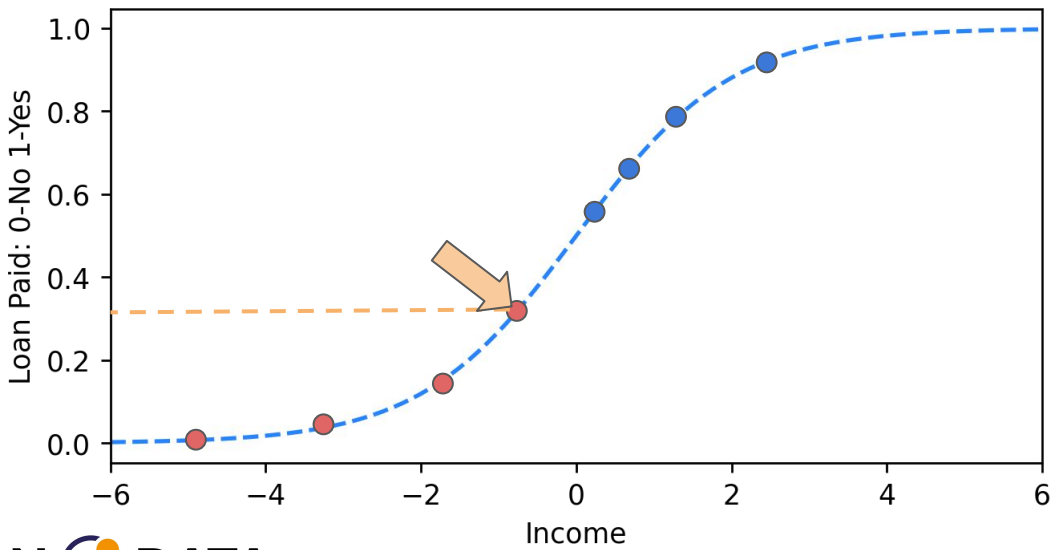
Logistic Regression

- Likelihood = 0.9 × 0.8 × 0.65 × 0.55 × (1-p) ×...



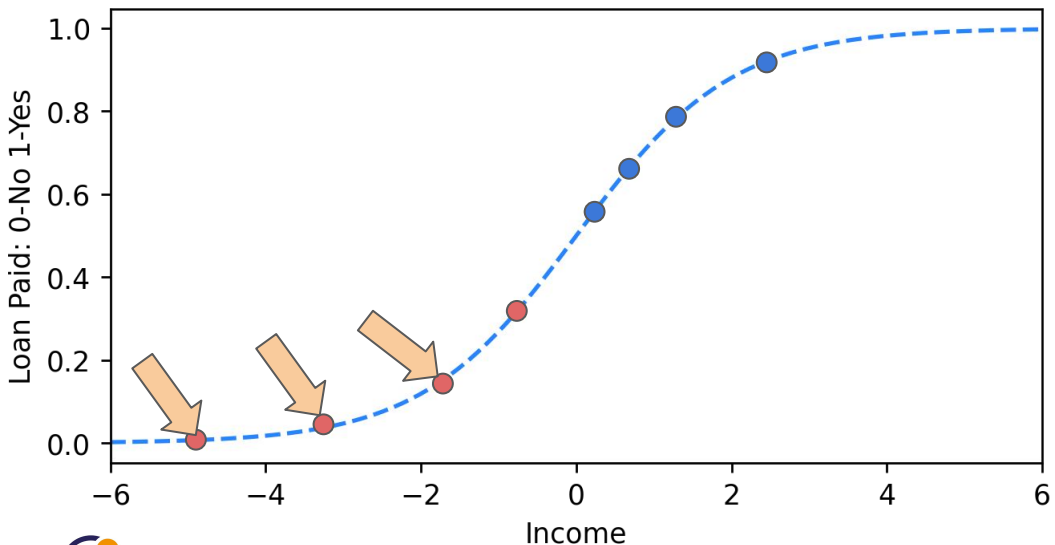
Logistic Regression

- Likelihood = 0.9 × 0.8 × 0.65 × 0.55 × (1-0.3) ×...



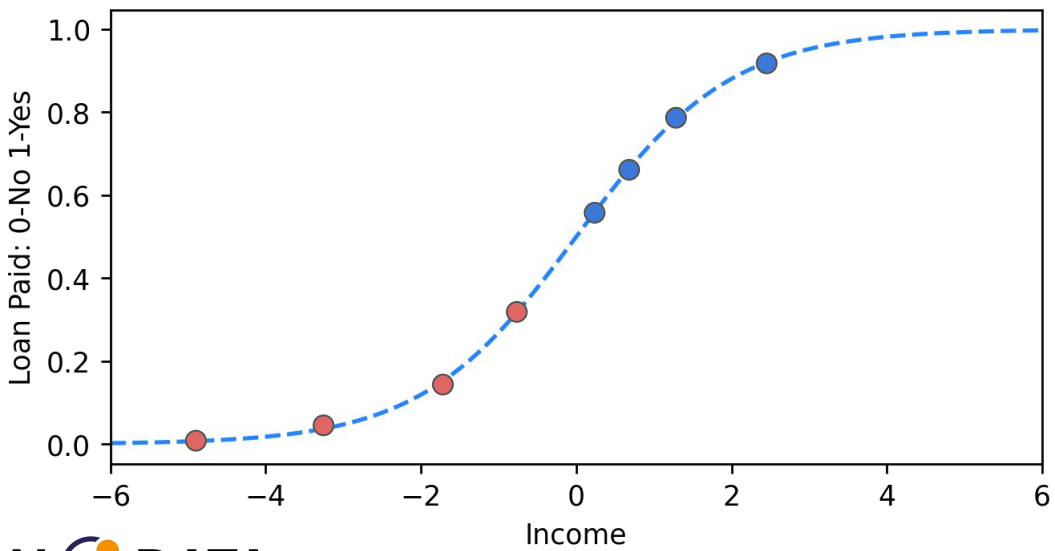
Logistic Regression

- Likelihood = 0.9 × 0.8 × 0.65 × 0.55 × (1-0.3) × (1-0.2) × (1-0.08) × (1-0.02)



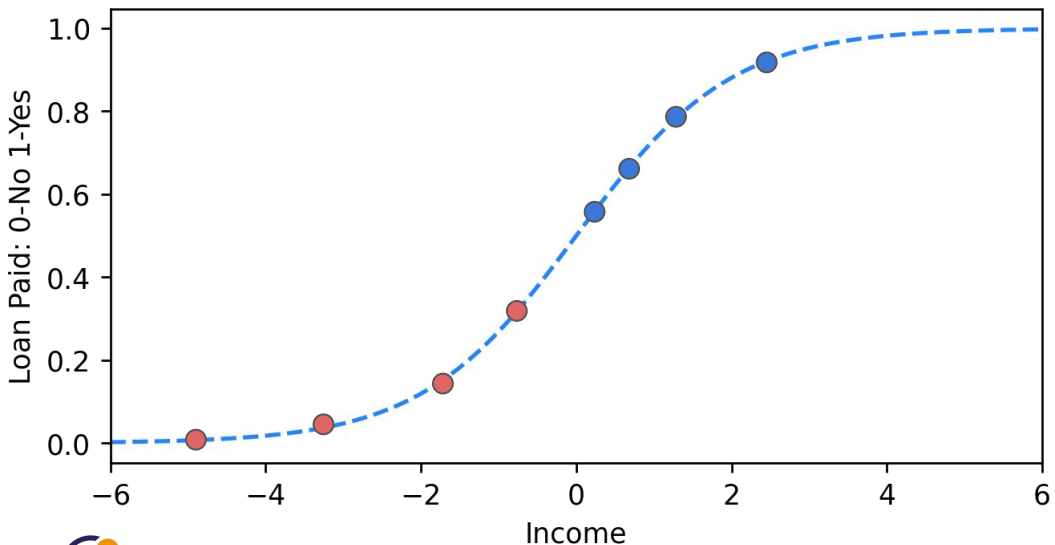
Logistic Regression

- Likelihood = 0.129



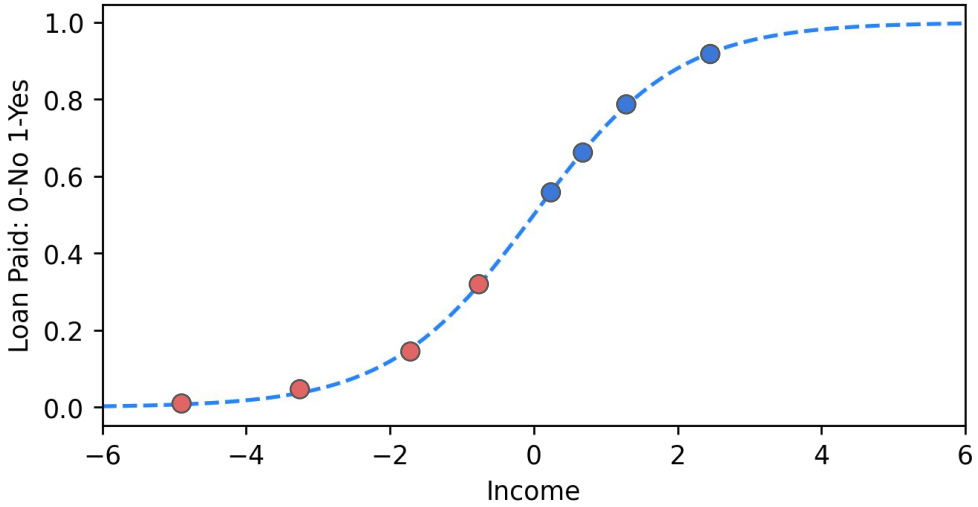
Logistic Regression

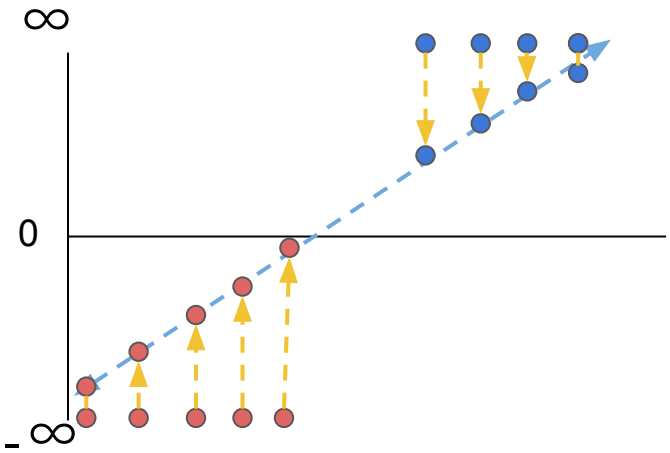
- Note in practice we actually maximize the **log** of the likelihoods. (e.g. ln(0.9)×ln(0.8)×...)



Logistic Regression

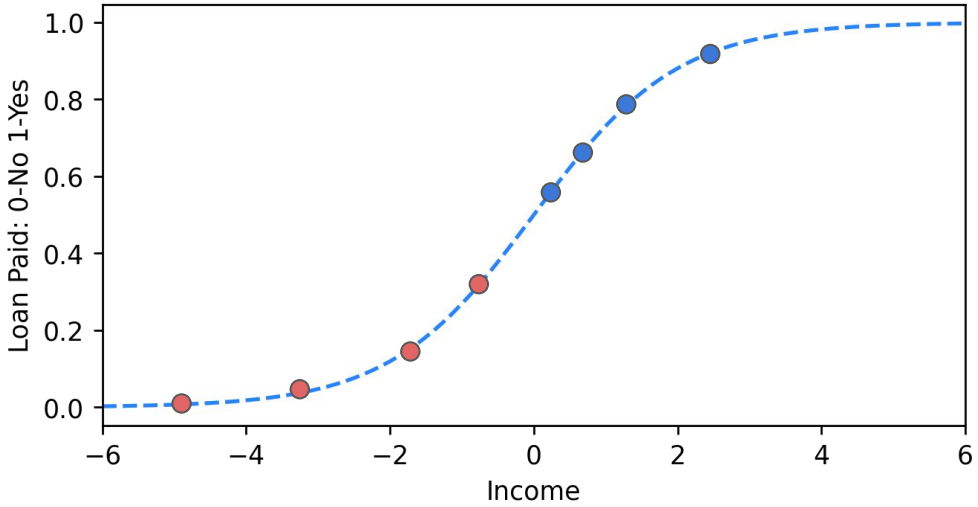
- There is some set of coefficients that will maximize these log likelihoods.

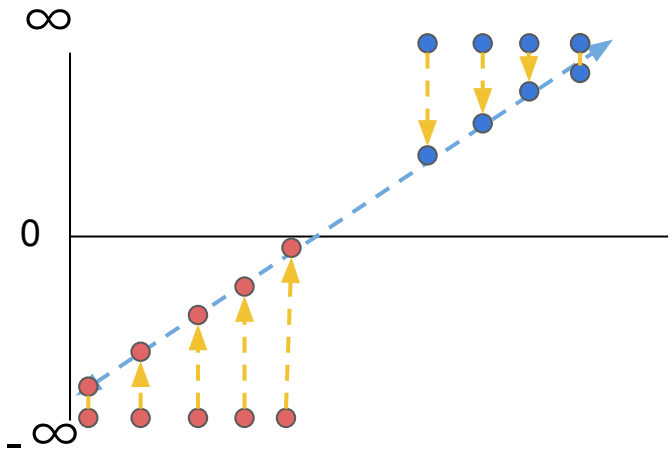




Logistic Regression

- Choose best coefficient values in log odds terms that creates maximum likelihood.





PIERIAN DATA

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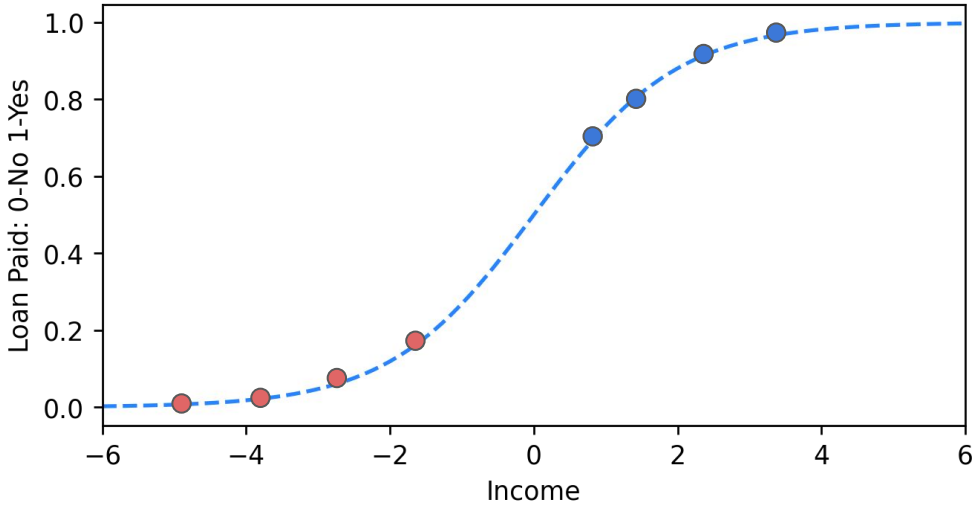

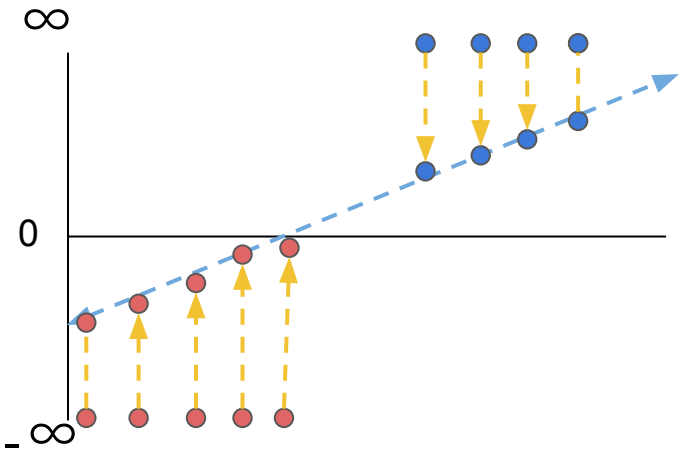

Logistic Regression

- While we are trying to **maximize** the likelihood, we still need something to **minimize**, since the computer's gradient descent methods can only search for minimums.

Logistic Regression

- In terms of a cost function, we seek to minimize the following (log loss):

$$J(\mathbf{x}) = -\frac{1}{m}\sum_{j=1}^{m} y^j \log\left(\hat{y}^j\right) + (1-y^j)\log\left(1-\hat{y}^j\right)$$

$$J(\mathbf{x}) = -\frac{1}{m}\sum_{j=1}^{m}\left(y^j \log\left(\frac{1}{1+e^{-\sum_{i=0}^{n}\beta_i x_i^j}}\right) + (1-y^j)\log\left(1-\frac{1}{1+e^{-\sum_{i=0}^{n}\beta_i x_i^j}}\right)\right)$$

Logistic Regression

- Just as with Linear Regression, gradient descent can solve this for us!

$$J(\mathbf{x}) = -\frac{1}{m}\sum_{j=1}^{m} y^j \log\left(\hat{y}^j\right) + (1-y^j)\log\left(1-\hat{y}^j\right)$$

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Logistic Regression

- Don’t worry about fully understanding this gradient descent.
- In practice we never have to implement it ourselves.
- Main takeaway should be the relationship between log odds and probability.

Logistic Regression

- Now that we have an intuition of what happens “behind the scenes”, let’s explore Logistic Regression with Python!

Logistic Regression with Scikit-Learn

Part One: Exploratory Data Analysis

PIERIAN DATA

Logistic Regression with Scikit-Learn

Part Two: Creating and Training a Model

PIERIAN DATA

Logistic Regression Understanding Coefficients

PIERIAN DATA

Classification Performance Metrics

Part One: Confusion Matrix Basics

PIERIAN DATA

Classification Metrics

- You’ve probably heard of terms such as “false positive” or “false negative”. As well as metrics like “accuracy”.
- But what do these terms actually mean mathematically?

Classification Metrics

- Imagine we've developed a test or model to detect presence of a virus infection in a person based on some biological feature.
- We could treat this as a Logistic Regression, predicting:
    - 0 - Not Infected (Tests Negative)
    - 1 - Infected (Tests Positive)

Classification Metrics

- It is unlikely our model will perform perfectly. This means there 4 possible outcomes:
    - Infected person tests positive.
    - Healthy person tests negative.

Classification Metrics

- It is unlikely our model will perform perfectly. This means there 4 possible outcomes:
    - Infected person tests positive.
    - Healthy person tests negative.
        - *Note, these are the outcomes we want! But it is unlikely our test is perfect...*

Classification Metrics

- It is unlikely our model will perform perfectly. This means there 4 possible outcomes:
    - Infected person tests positive.
    - Healthy person tests negative.
    - Infected person tests negative.
    - Healthy person tests positive.

Classification Metrics

- Based off these 4 possibilities, there are many error metrics we can calculate.
- First, let’s start by visualizing these four possibilities as a matrix.

Classification Metrics

- Confusion Matrix

		ACTUAL	
		INFECTED	HEALTHY

Classification Metrics

- Confusion Matrix

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED		
	HEALTHY		

Classification Metrics

- Confusion Matrix

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	TRUE POSITIVE	
	HEALTHY		

Classification Metrics

- Confusion Matrix

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	TRUE POSITIVE	
	HEALTHY		TRUE NEGATIVE

Classification Metrics

- Confusion Matrix

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	TRUE POSITIVE	FALSE POSITIVE
	HEALTHY		TRUE NEGATIVE

Classification Metrics

- Confusion Matrix

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	TRUE POSITIVE	FALSE POSITIVE
	HEALTHY	FALSE NEGATIVE	TRUE NEGATIVE

Classification Metrics

- Imagine a test group of 100 people:

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED		
	HEALTHY		

Classification Metrics

- 5 are infected. 95 are healthy.

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED		
	HEALTHY		

Classification Metrics

- We tested all of them with these results:

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	4	2
	HEALTHY	1	93

Classification Metrics

- What is accuracy?

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	4	2
	HEALTHY	1	93

Classification Metrics

- What is accuracy?

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	4	2
	HEALTHY	1	93

- Accuracy:
  - How often is the model correct?

Acc = (TP+TN)/Total

Classification Metrics

- Calculating accuracy:

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	4	2
	HEALTHY	1	93

(4+93)/100 = 97% Accuracy

- Accuracy:
  - How often is the model correct?

Acc = (TP+TN)/Total

Classification Metrics

- Is this a good value for accuracy?

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	4	2
	HEALTHY	1	93

(4+93)/100 = 97% Accuracy

- Accuracy:
  - How often is the model correct?

Acc = (TP+TN)/Total

Classification Metrics

- The accuracy paradox...

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	4	2
	HEALTHY	1	93

(4+93)/100 = 97% Accuracy

- Accuracy:
  - How often is the model correct?

Acc = (TP+TN)/Total

Classification Metrics

- Imagine we **always** report back “healthy”

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	4	2
	HEALTHY	1	93

Classification Metrics

- Imagine we **always** report back “healthy”

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	0	0
	HEALTHY	5	95

Classification Metrics

- Imagine we **always** report back “healthy”

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	0	0
	HEALTHY	5	95

(0+95)/100 = 95% Accuracy

- Accuracy:
  - How often is the model correct?

95% accuracy for a model that always returns “healthy”!

Classification Metrics

- You may be thinking, “*The numbers here are arbitrary, we just happen to get good accuracy in this made up case. Real world data would reflect poor accuracy if a model always returned the same result*”.

Classification Metrics

- This is the accuracy paradox!
  - Any classifier dealing with **imbalanced** classes has to confront the issue of the accuracy paradox.
  - **Imbalanced** classes will always result in a distorted accuracy reflecting better performance than what is truly warranted.

Classification Metrics

- **Imbalanced** classes are often found in real world data sets.
    - Medical conditions can affect small portions of the population.
    - Fraud is not common (e.g. Real vs. Fraud credit card usage).

Classification Metrics

- If a class is only a small percentage (**n%**), then a classifier that always predicts the majority class will always have an accuracy of (1-n).
- In our previous example we saw infected were only 5% of the data.
- Allowing the accuracy to be 95%.

Classification Metrics

- This means we shouldn’t solely rely on accuracy as a metric!
- This is where precision,recall, and f1-score will come in.
- Let’s explore these other metrics in the next lecture.

Classification Performance Metrics

Part Two: Precision and Recall

PIERIAN DATA

Classification Metrics

- We already know how to calculate accuracy and its associated paradox.
- Let's explore three more metrics that can help give a clearer picture of performance:
    - Recall (a.k.a. sensitivity)
    - Precision
    - F1-Score

Classification Metrics

- Let's begin with recall.

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	4	2
	HEALTHY	1	93

- Recall:
  - When it actually is a positive case, how often is it correct?

(TP)/Total Actual Positives

Classification Metrics

- Let's begin with recall.

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	4	2
	HEALTHY	1	93

Recall =
(TP)/Total Actual Positives

- Recall:
  - When it actually is a positive case, how often is it correct?

(TP)/Total Actual Positives

Classification Metrics

- Let's begin with recall.

		ACTUAL: INFECTED	ACTUAL: HEALTHY
PREDICTED	INFECTED	4	2
	HEALTHY	1	93

Recall = (TP)/5

- Recall:
  - When it actually is a positive case, how often is it correct?

(TP)/Total Actual Positives

Classification Metrics

- Let's begin with recall.

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	4	2
	HEALTHY	1	93

Recall = (4)/5

- Recall:
  - When it actually is a positive case, how often is it correct?

(TP)/Total Actual Positives

Classification Metrics

- Let's begin with recall.

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	4	2
	HEALTHY	1	93

Recall = 0.8

- Recall:
  - How many relevant cases are found?

(TP)/Total Actual Positives

Classification Metrics

- What's the recall if we always classify as "healthy"?

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	0	0
	HEALTHY	5	95

Recall =
(TP)/Total Actual Positives

- Recall:
  - How many relevant cases are found?

(TP)/Total Actual Positives

Classification Metrics

- What's the recall if we always classify as "healthy"?

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	0	0
	HEALTHY	5	95

Recall = (0)/5 !

- Recall:
  - How many relevant cases are found?

(TP)/Total Actual Positives

Classification Metrics

- A recall of 0 alerts you the model isn't catching cases!

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	0	0
	HEALTHY	5	95

Recall = (0)/5 !

- Recall:
  - How many relevant cases are found?

(TP)/Total Actual Positives

PIERIAN DATA

Classification Metrics

- Now let's explore **precision**.

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	4	2
	HEALTHY	1	93

Precision =
(TP)/Total Predicted Positives

- Precision:
  - When prediction is positive, how often is it correct?

(TP)/Total Predicted Positives

Classification Metrics

- Now let's explore **precision**.

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	4	2
	HEALTHY	1	93

Precision =
(TP)/Total Predicted Positives

- Precision:
  - When prediction is positive, how often is it correct?

(TP)/Total Predicted Positives

Classification Metrics

- Now let's explore **precision**.

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	4	2
	HEALTHY	1	93

Precision = (TP)/6

- Precision:
  - When prediction is positive, how often is it correct?

(TP)/Total Predicted Positives

Classification Metrics

- Now let's explore **precision**.

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	4	2
	HEALTHY	1	93

Precision = (TP)/6

- Precision:
  - When prediction is positive, how often is it correct?

(TP)/Total Predicted Positives

Classification Metrics

- Now let's explore **precision**.

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	4	2
	HEALTHY	1	93

Precision = (4)/6

- Precision:
  - When prediction is positive, how often is it correct?

(TP)/Total Predicted Positives

Classification Metrics

- Now let's explore **precision**.

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	4	2
	HEALTHY	1	93

Precision = 0.666

- Precision:
  - When prediction is positive, how often is it correct?

(TP)/Total Predicted Positives

Classification Metrics

- What's the **precision** if we always classify as "healthy"?

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	0	0
	HEALTHY	5	95

Precision =
(TP)/Total Predicted Positives

- Precision:
  - When prediction is positive, how often is it correct?

(TP)/Total Predicted Positives

PIERIAN DATA

Classification Metrics

- What's the **precision** if we always classify as "healthy"?

		ACTUAL	
		INFECTED	HEALTHY
PREDICTED	INFECTED	0	0
	HEALTHY	5	95

Precision = 0/0

- Precision:
  - When prediction is positive, how often is it correct?

(TP)/Total Predicted Positives

Classification Metrics

- Recall and Precision can help illuminate our performance specifically in regards to the relevant or positive case.
- Depending on the model, there is typically a trade-off between precision and recall, which we will explore later on with the ROC curve.

Classification Metrics

- Since precision and recall are related to each other through the numerator (TP), we often also report the F1-Score, which is the harmonic mean of precision and recall.

$$F = \frac{2 \times precision \times recall}{precision + recall}$$

Classification Metrics

- The harmonic mean (instead of the normal mean) allows the entire harmonic mean to go to zero if **either** precision or recall ends up being zero.

$$F = \frac{2 \times precision \times recall}{precision + recall}$$

Classification Metrics

- As a final note on the confusion matrix, there are **many** more metrics available:

		True condition				
	Total population	Condition positive	Condition negative	Prevalence = $\frac{\Sigma \text{ Condition positive}}{\Sigma \text{ Total population}}$	Accuracy (ACC) = $\frac{\Sigma \text{ True positive} + \Sigma \text{ True negative}}{\Sigma \text{ Total population}}$	
Predicted condition	Predicted condition positive	**True positive**	**False positive**, Type I error	Positive predictive value (PPV), Precision = $\frac{\Sigma \text{ True positive}}{\Sigma \text{ Predicted condition positive}}$	False discovery rate (FDR) = $\frac{\Sigma \text{ False positive}}{\Sigma \text{ Predicted condition positive}}$	
	Predicted condition negative	**False negative**, Type II error	**True negative**	False omission rate (FOR) = $\frac{\Sigma \text{ False negative}}{\Sigma \text{ Predicted condition negative}}$	Negative predictive value (NPV) = $\frac{\Sigma \text{ True negative}}{\Sigma \text{ Predicted condition negative}}$	
		True positive rate (TPR), Recall, Sensitivity, probability of detection, Power = $\frac{\Sigma \text{ True positive}}{\Sigma \text{ Condition positive}}$	False positive rate (FPR), Fall-out, probability of false alarm = $\frac{\Sigma \text{ False positive}}{\Sigma \text{ Condition negative}}$	Positive likelihood ratio (LR+) = $\frac{\text{TPR}}{\text{FPR}}$	Diagnostic odds ratio (DOR) = $\frac{\text{LR+}}{\text{LR−}}$	F_1 score = $2 \cdot \frac{\text{Precision} \cdot \text{Recall}}{\text{Precision} + \text{Recall}}$
		False negative rate (FNR), Miss rate = $\frac{\Sigma \text{ False negative}}{\Sigma \text{ Condition positive}}$	Specificity (SPC), Selectivity, True negative rate (TNR) = $\frac{\Sigma \text{ True negative}}{\Sigma \text{ Condition negative}}$	Negative likelihood ratio (LR−) = $\frac{\text{FNR}}{\text{TNR}}$		

Classification Metrics

- Finally, let’s explore a way to visualize the relationships between metrics such as precision and recall with curves.

Classification Performance Metrics

Part Three: ROC Curves

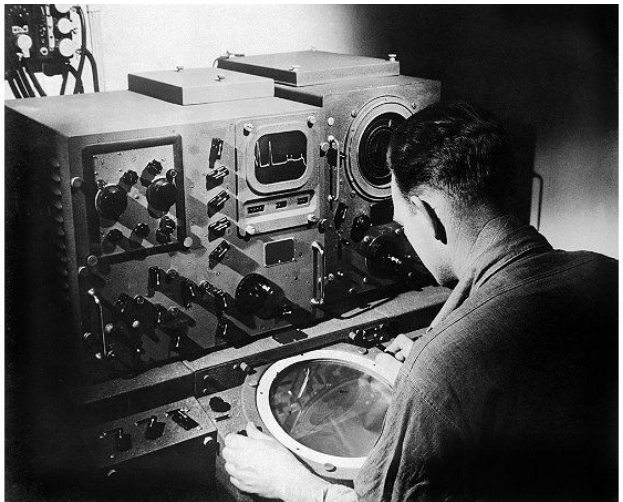
PIERIAN DATA

Classification Metrics

- During World War 2, Radar technology was developed to help detect incoming enemy aircraft.

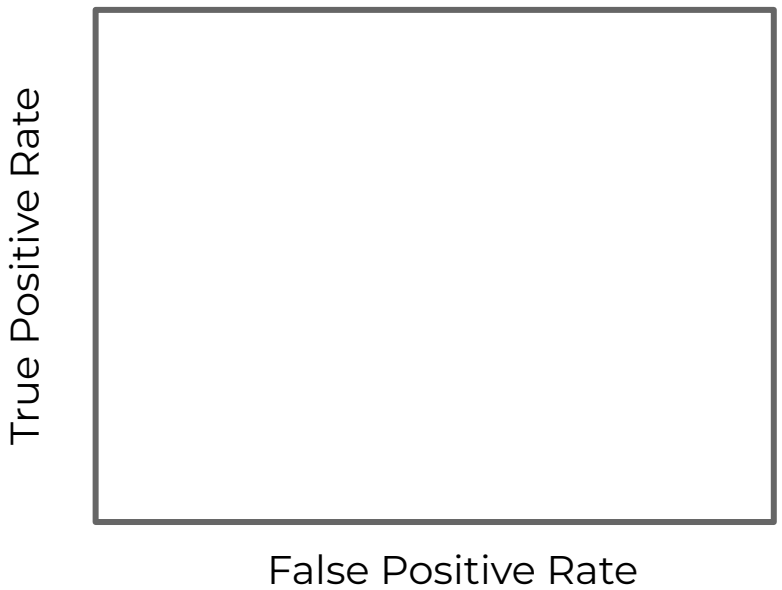
Classification Metrics

- The technology was so new, the US Army wanted to develop a methodology to evaluate radar operator performance.



Classification Metrics

- They developed the Receiver Operator Characteristic curve.



Classification Metrics

- They developed the Receiver Operator Characteristic curve.

Classification Metrics

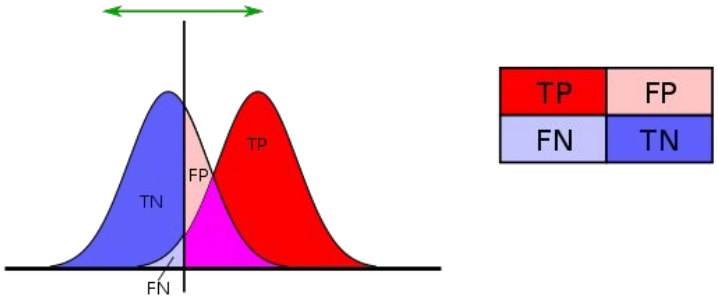
- They developed the Receiver Operator Characteristic curve.

Classification Metrics

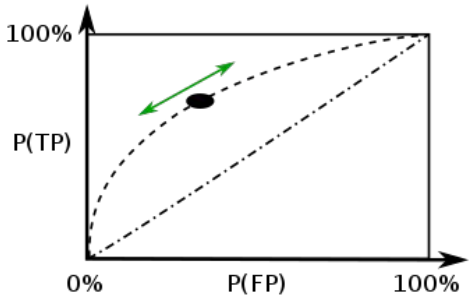
- There can be a trade-off between True Positives and False Positives.

Classification Metrics

- There can be a trade-off between True Positives and False Positives.

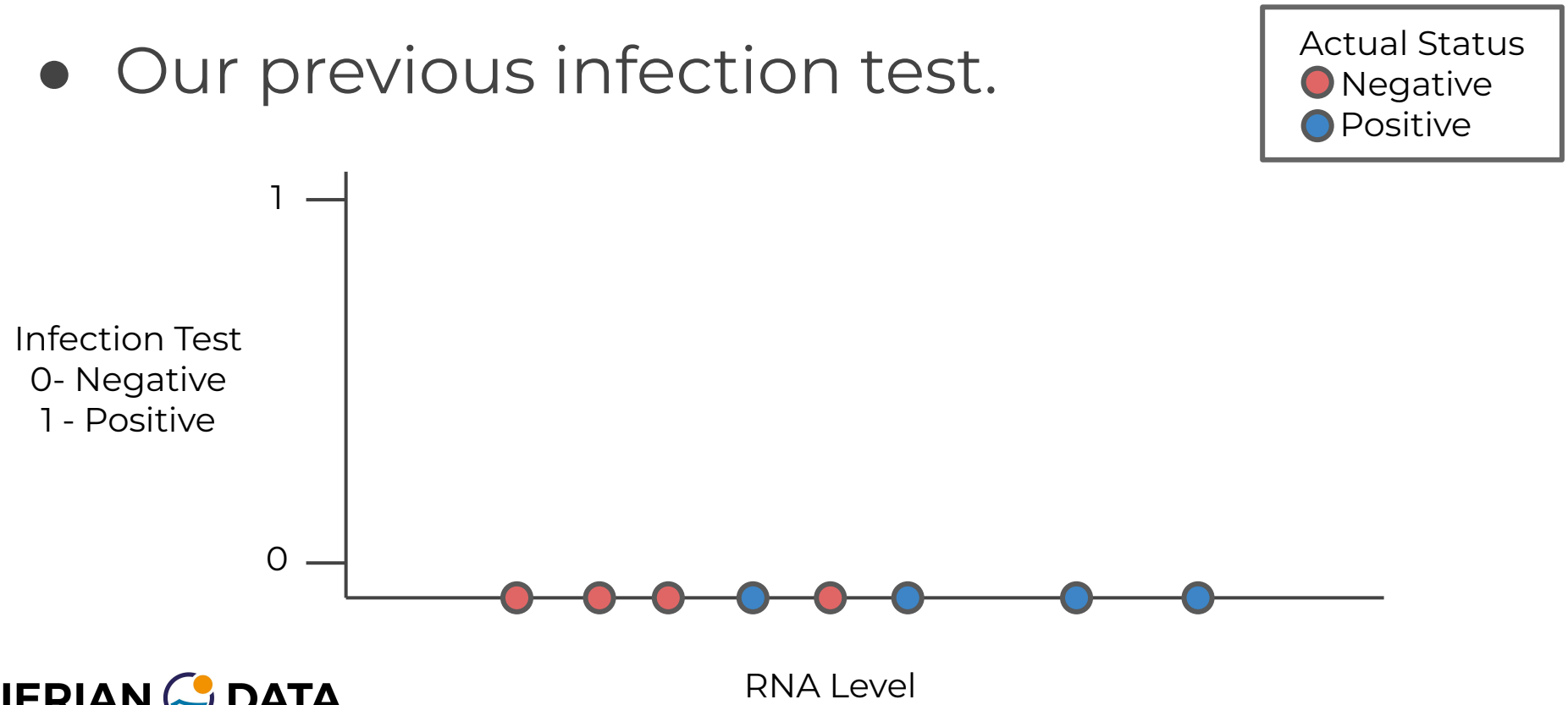


TP	FP
FN	TN



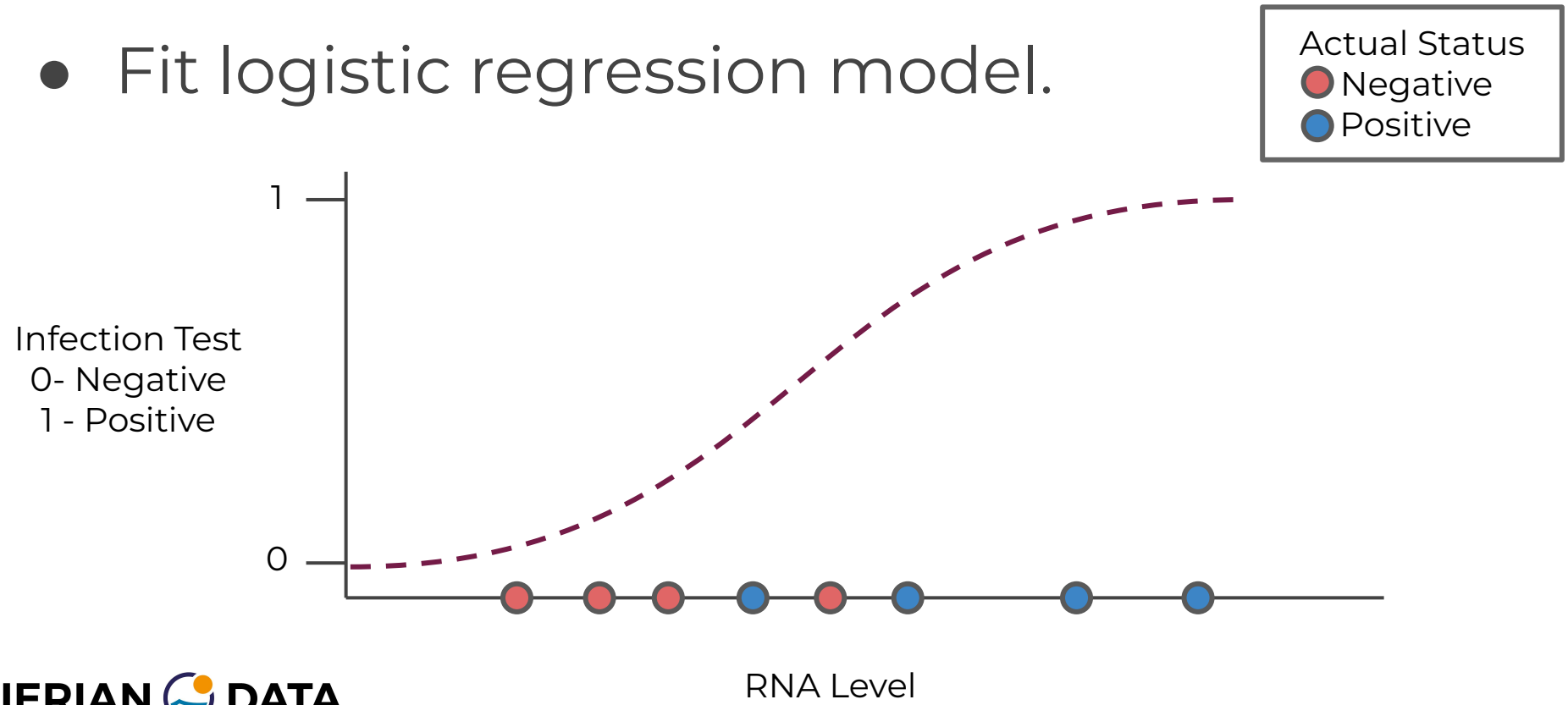
Classification Metrics

- Our previous infection test.



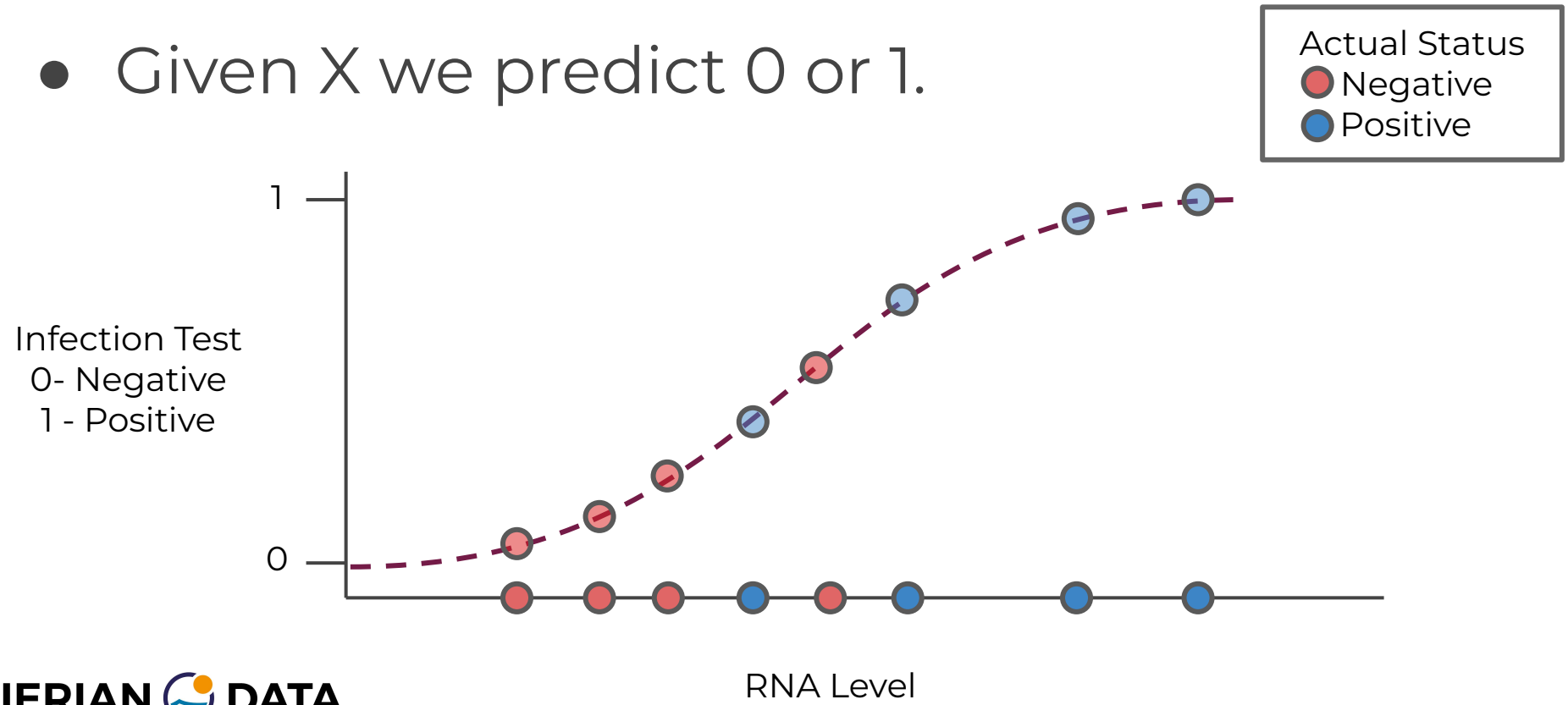
Classification Metrics

- Fit logistic regression model.



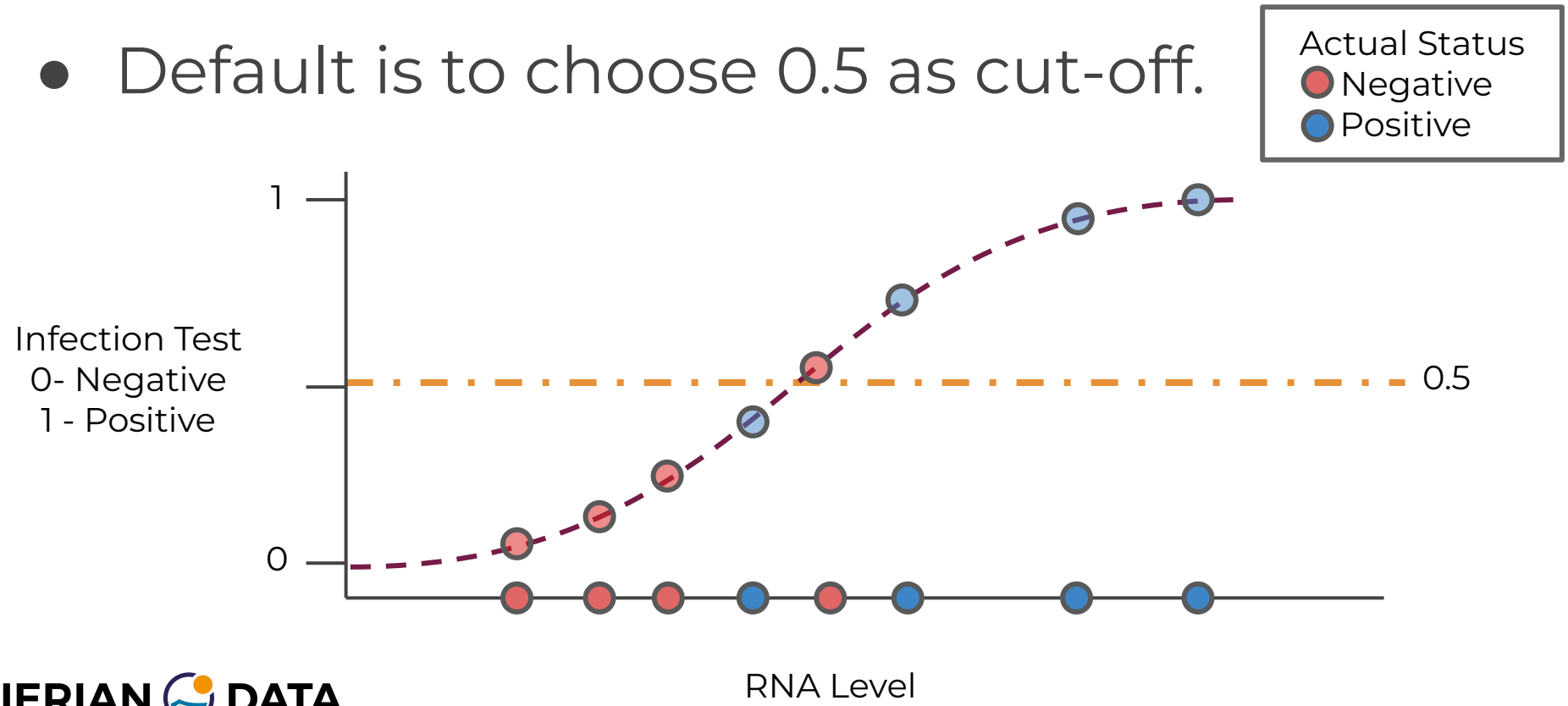
Classification Metrics

- Given X we predict 0 or 1.



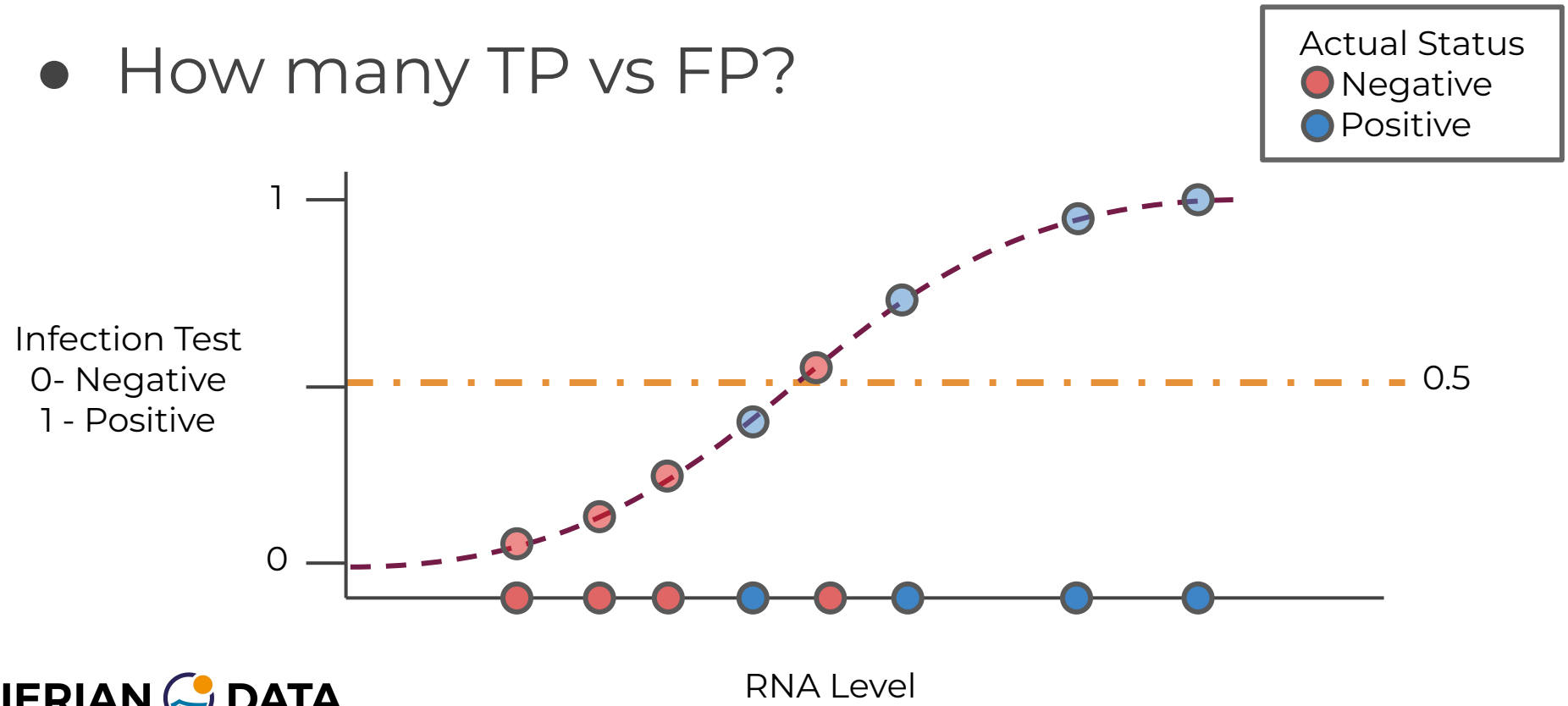
Classification Metrics

- Default is to choose 0.5 as cut-off.



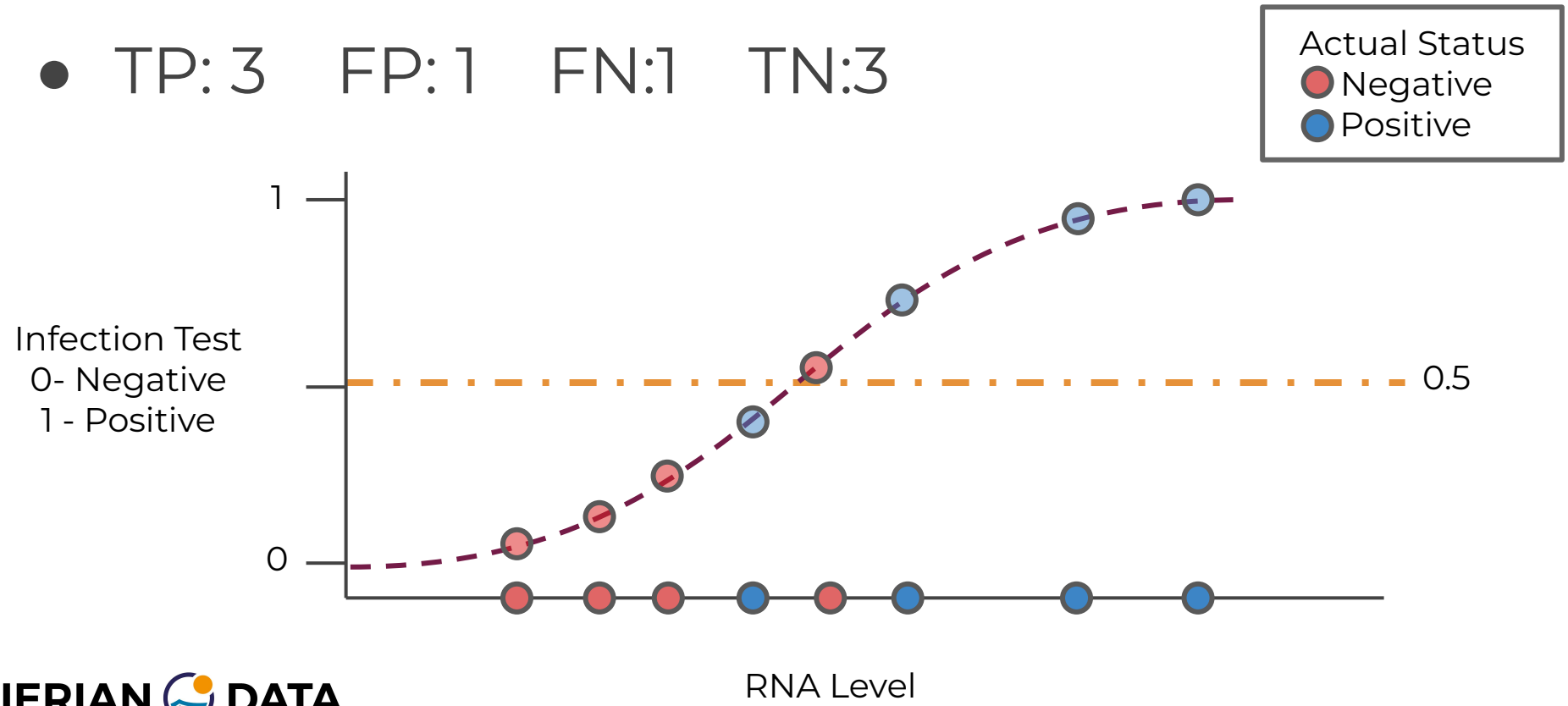
Classification Metrics

- How many TP vs FP?



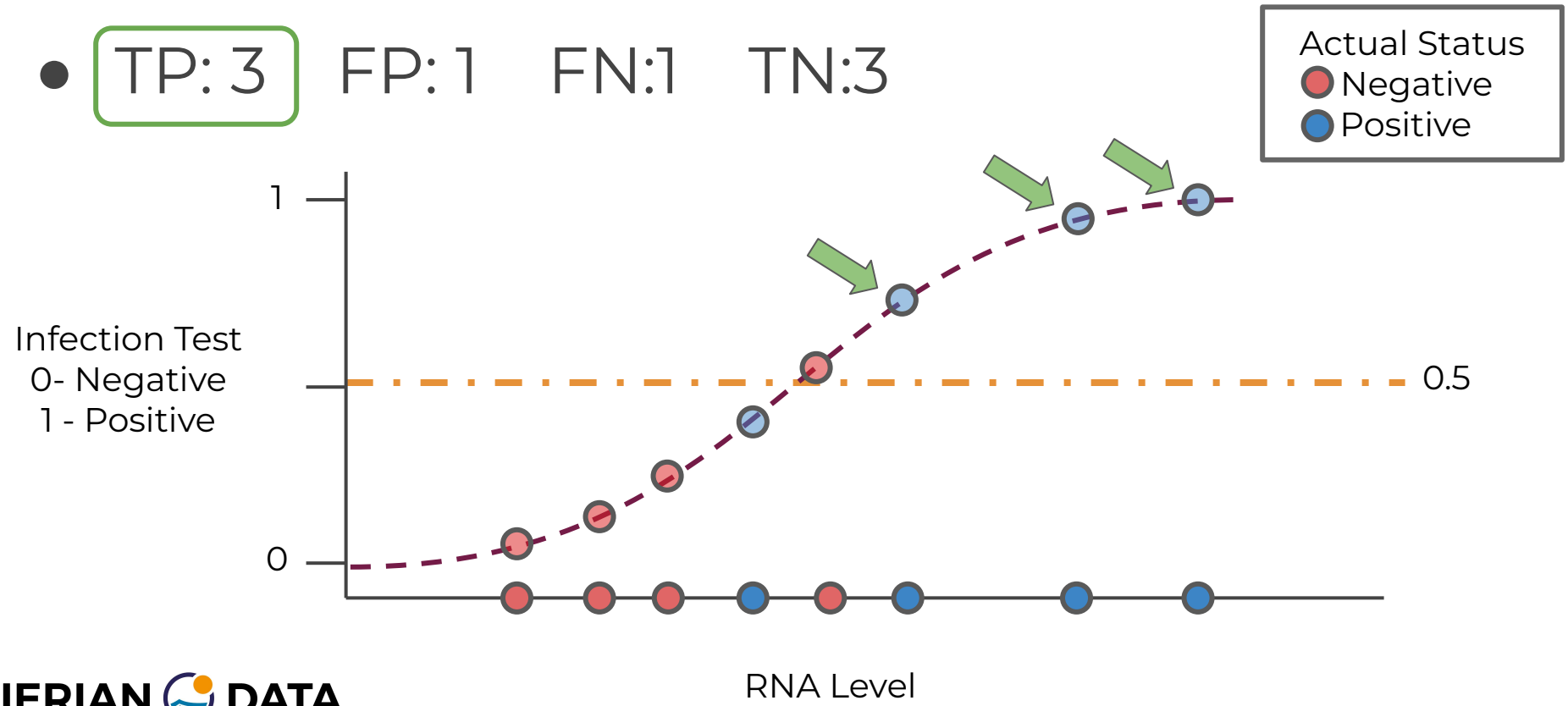
Classification Metrics

- TP: 3 FP: 1 FN:1 TN:3



Classification Metrics

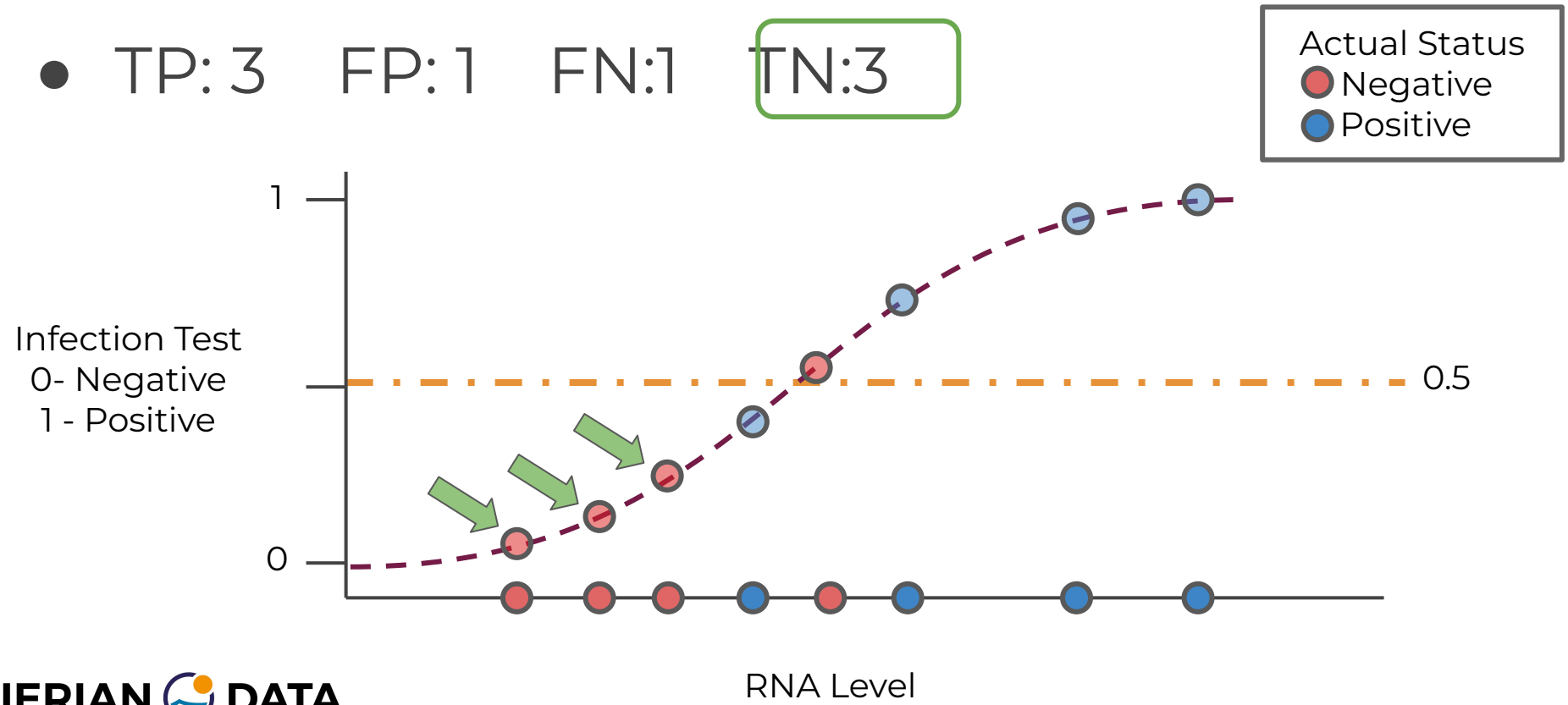
- TP: 3 FP: 1 FN:1 TN:3



PIERIAN DATA

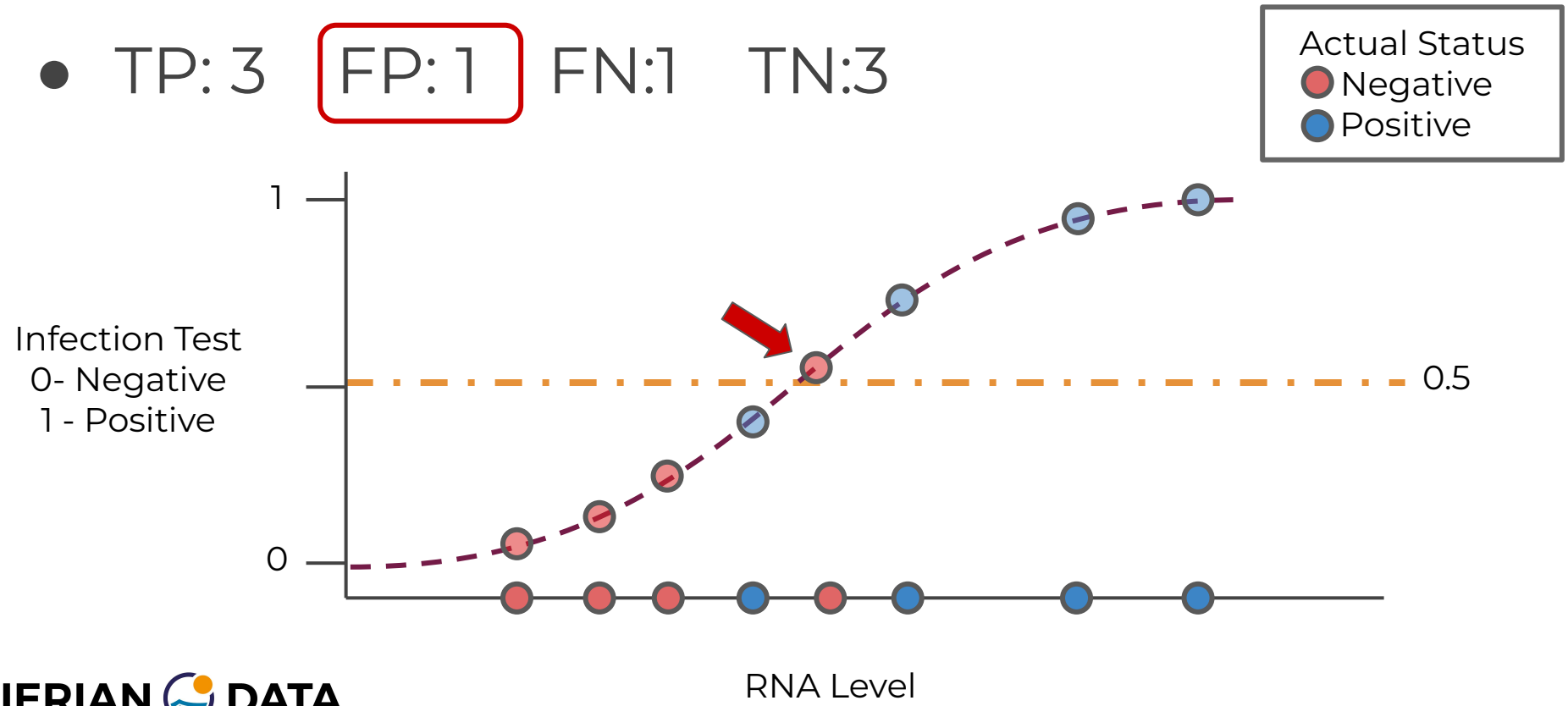
Classification Metrics

- TP: 3 FP: 1 FN:1 TN:3



Classification Metrics

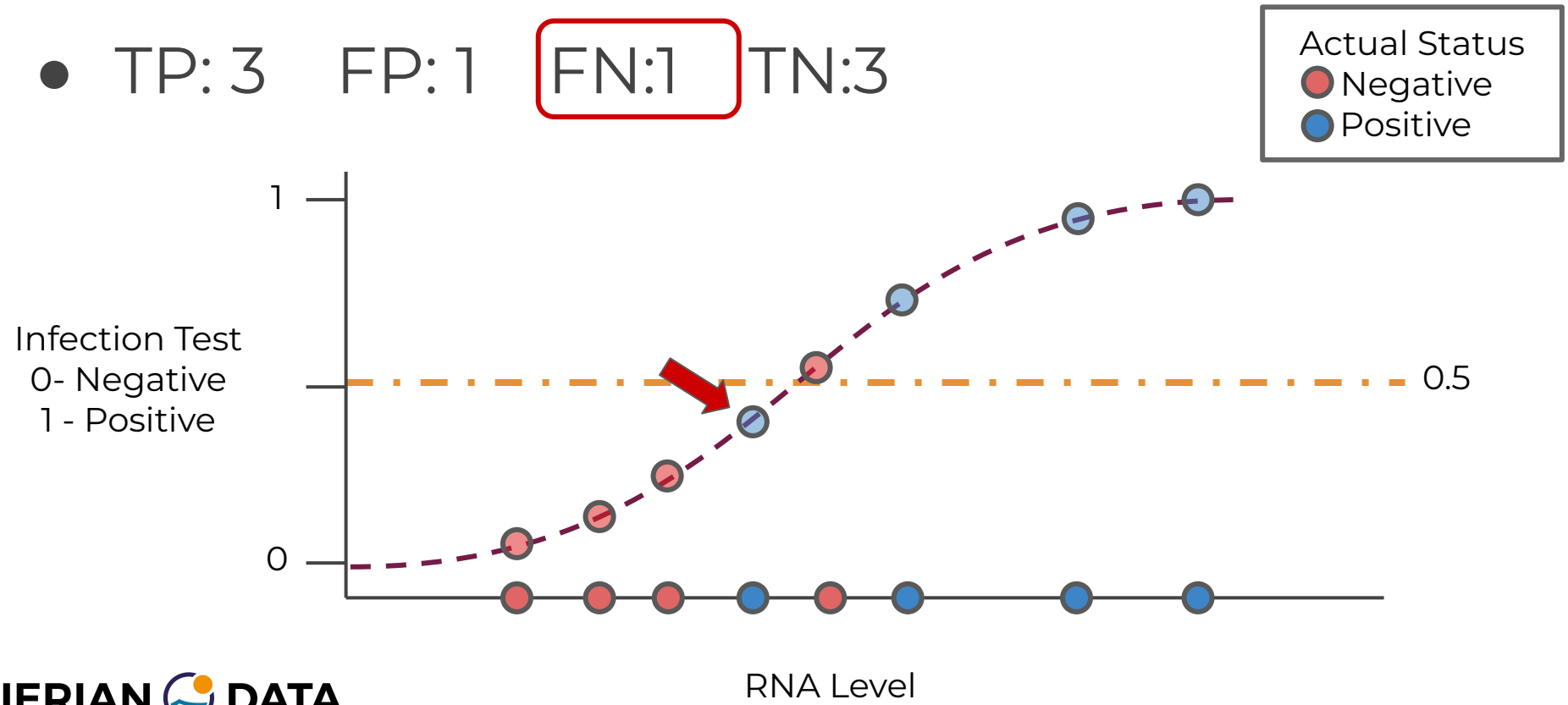
- TP: 3 FP: 1 FN:1 TN:3



PIERIAN DATA

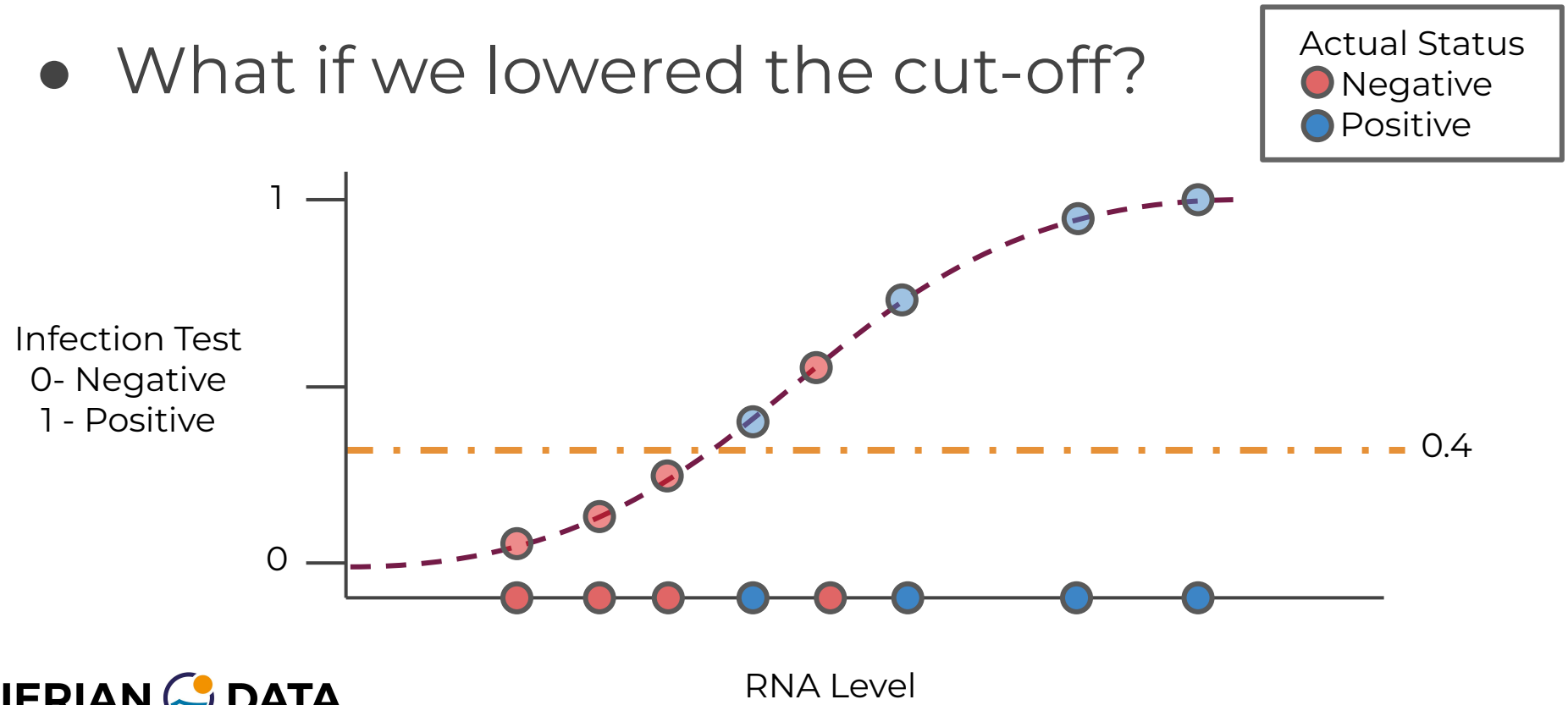
Classification Metrics

- TP: 3 FP: 1 FN:1 TN:3



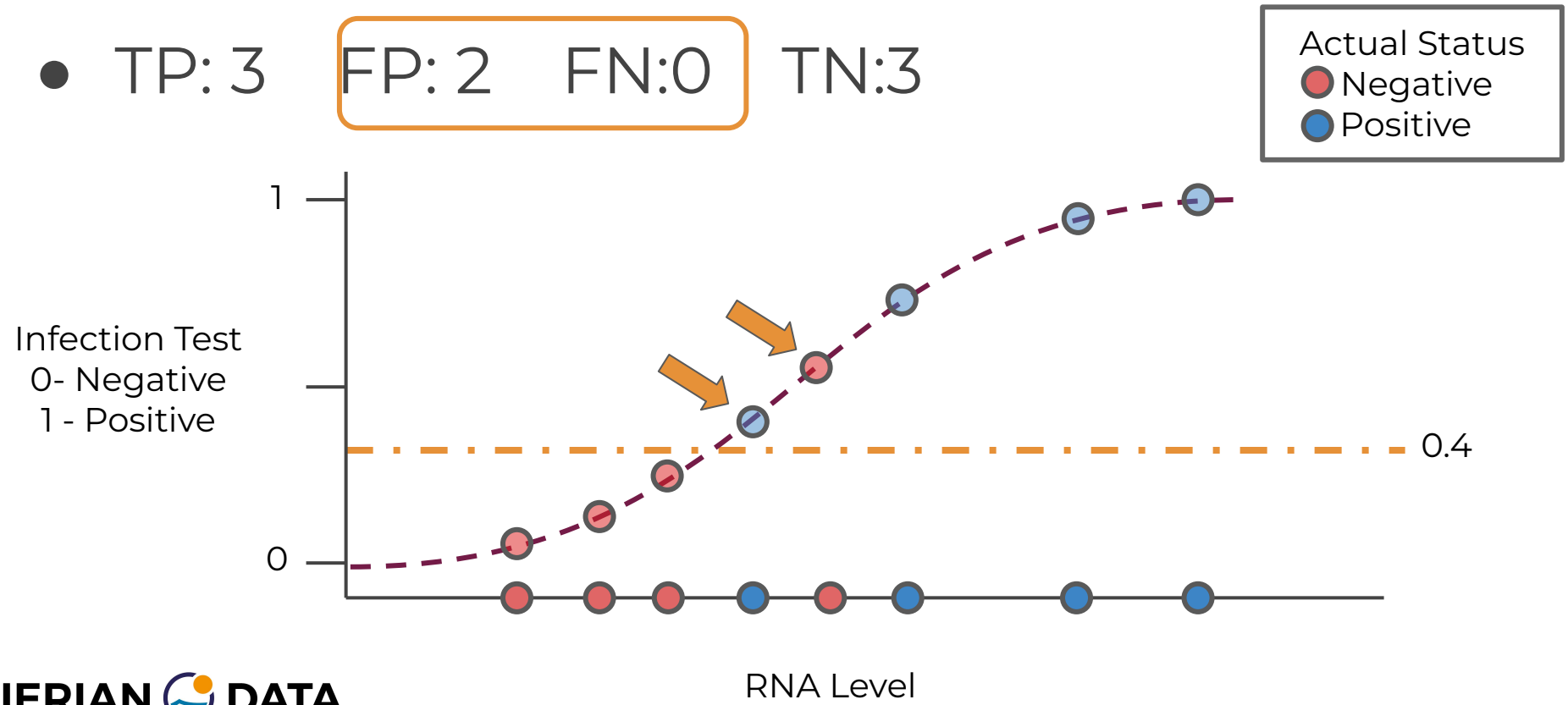
Classification Metrics

- What if we lowered the cut-off?



Classification Metrics

- TP: 3 FP: 2 FN:0 TN:3

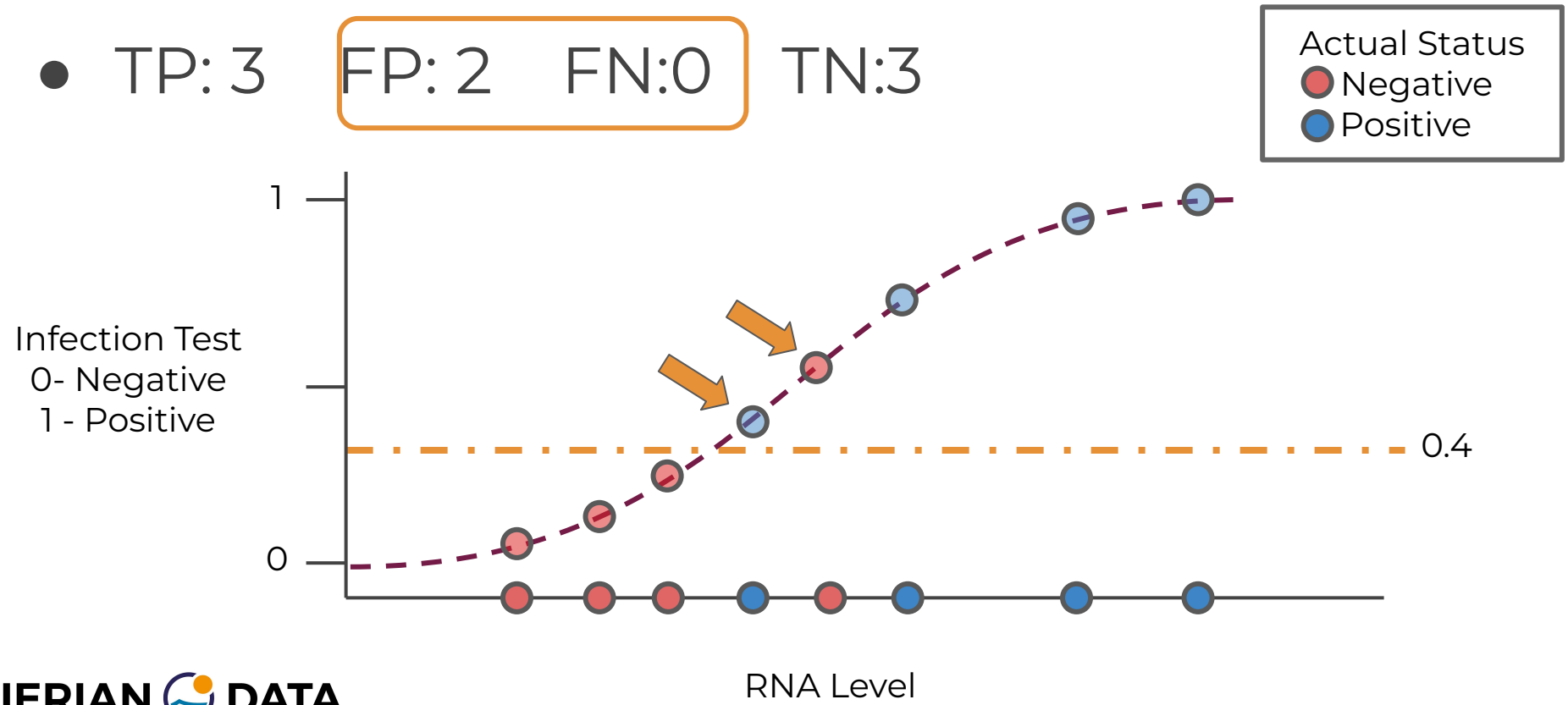


Classification Metrics

- In certain situations, we gladly accept more false positives to reduce false negatives.
- Imagine a dangerous virus test, we would much rather produce false positives and later do more stringent examination than accidentally release a false negative!

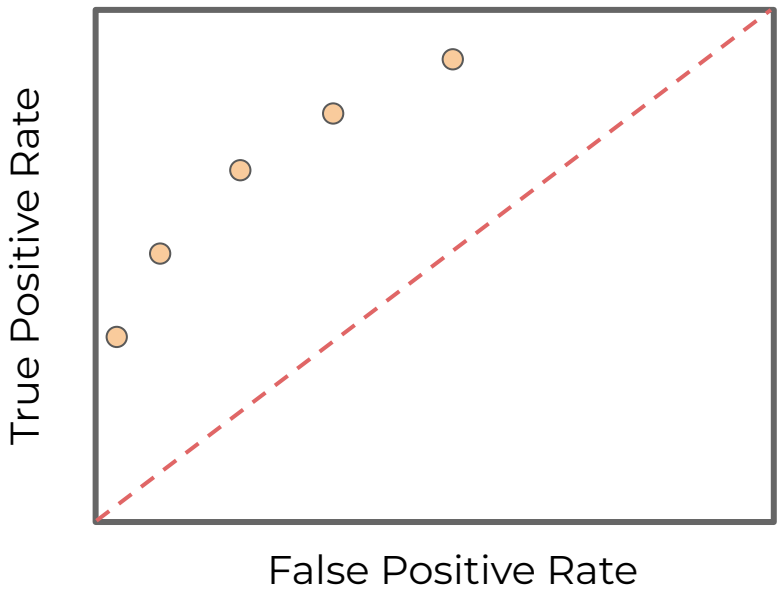
Classification Metrics

- TP: 3 FP: 2 FN:0 TN:3



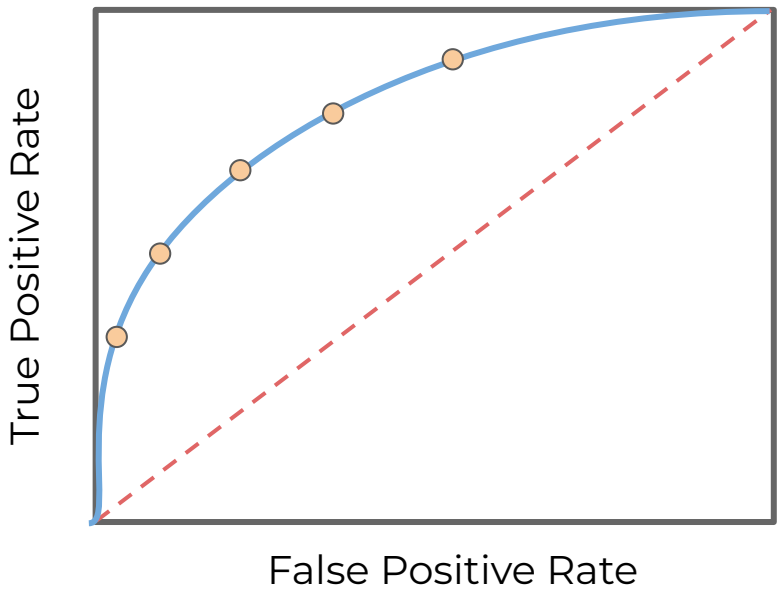
Classification Metrics

- Chart the True vs. False positives for various cut-offs for the ROC curve.



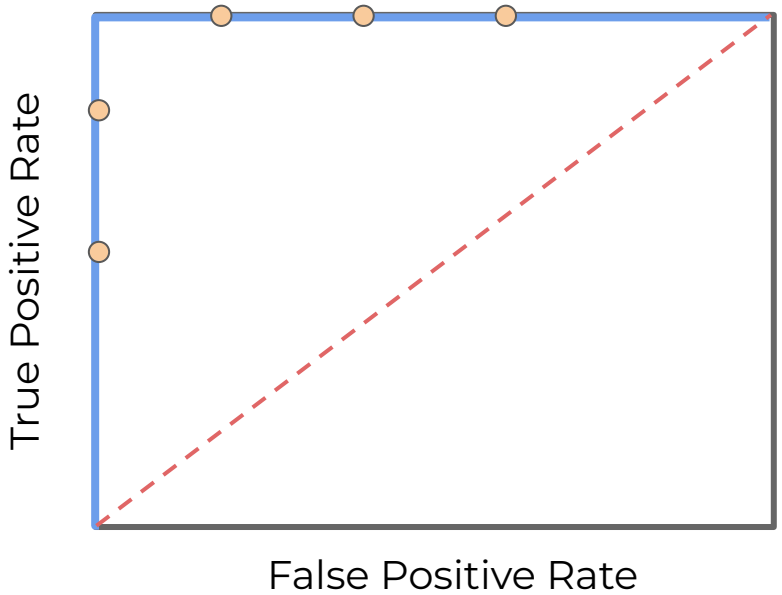
Classification Metrics

- By changing the cut-off limit, we can adjust our True vs. False Positives!



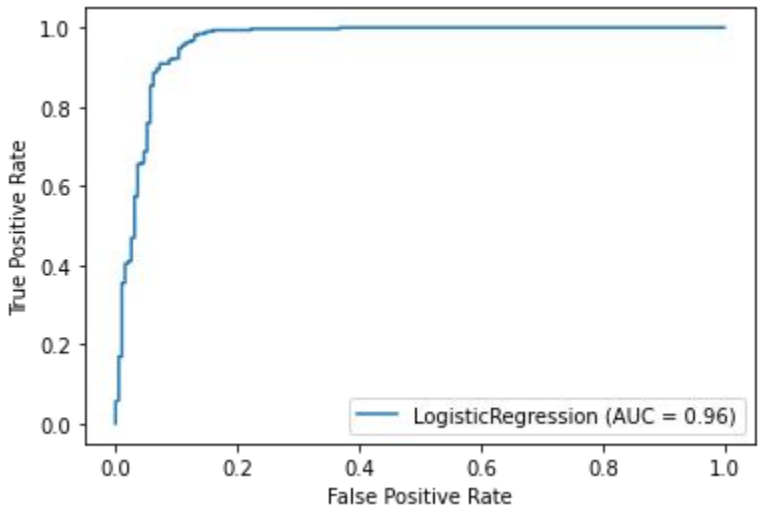
Classification Metrics

- A perfect model would have a zero FPR.
- Random guessing is the red line.



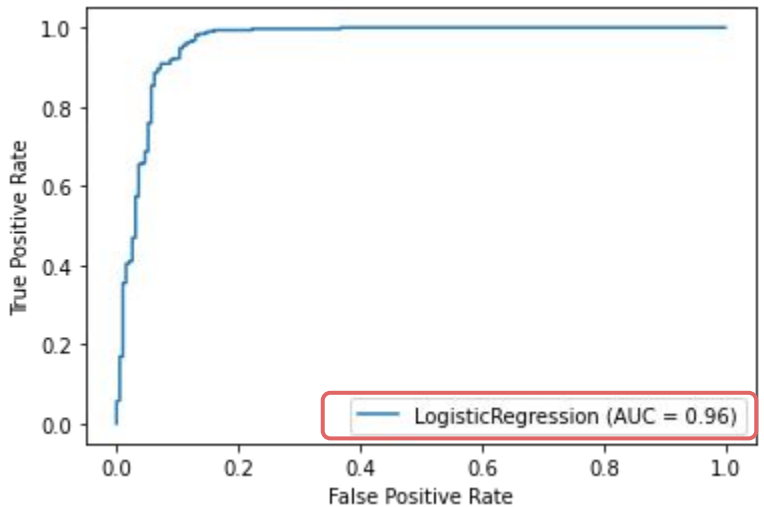
Classification Metrics

- Realistically with smaller data sets the ROC curves are not as smooth.



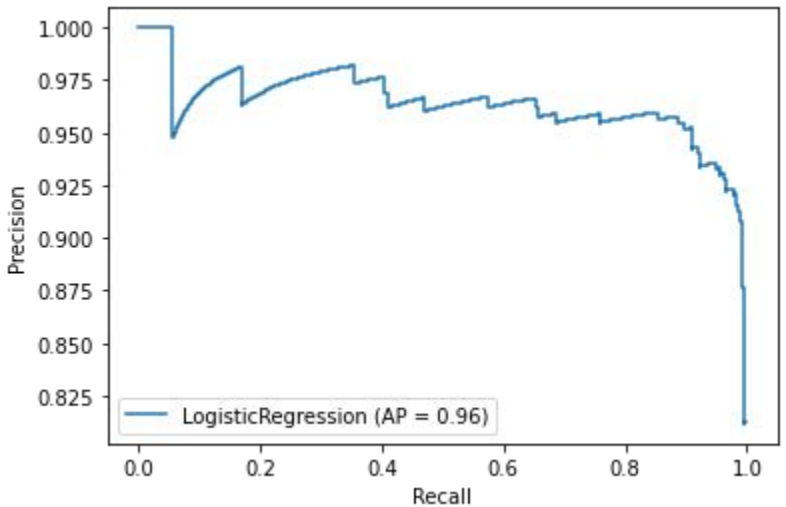
Classification Metrics

- AUC - Area Under the Curve , allows us to compare ROCs for different models.



Classification Metrics

- Can also create precision vs. recall curves:



Logistic Regression with Scikit-Learn

Part Three: Performance Metrics

PIERIAN DATA

Logistic Regression Multi-Class Problems

Part One: Data and Model

PIERIAN DATA

Logistic Regression Multi-Class Problems

Part Two: Training and Performance Evaluation

PIERIAN DATA

Logistic Regression Exercise Overview

Logistic Regression Exercise Solutions