Dynamic Contracting under Positive Commitment

Ilan Lobel and Renato Paes Leme

NYU Stern and Google

AAAI 2019

No Commitment

- Only static auctions are feasible
- Revenue is at most Myerson
- Traditional spot market
- Overly pessimistic
- Doesn't explore time dynamics

Positive Commitment

- Middle ground achieving better than Myerson revenue but not quite full surplus
- Seller can commit to some future actions that are mutually beneficial
- Smart contracts in the exchange

Full Commitment

- Full power of dynamic auctions
- Optimal obtains all surplus
- Guaranteed contracts
- Overly optimistic
- Assumes non-self-enforcible threats

No Commitment

- Only static auctions are feasible
- Revenue is at most Myerson
- Traditional spot market
- Overly pessimistic
- Doesn't explore time dynamics

Positive Commitment

- Middle ground achieving better than Myerson revenue but not quite full surplus
- Seller can commit to some future actions that are mutually beneficial
- Smart contracts in the exchange

Full Commitment

- Full power of dynamic auctions
- Optimal obtains all surplus
- Guaranteed contracts
- Overly optimistic
- Assumes non-self-enforcible threats

No Commitment

- Only static auctions are feasible
- Revenue is at most Myerson
- Traditional spot market
- Overly pessimistic
- Doesn't explore time dynamics

Positive Commitment

- Middle ground achieving better than Myerson revenue but not quite full surplus
- Seller can commit to some future actions that are mutually beneficial
- Smart contracts in the exchange

Full Commitment

- Full power of dynamic auctions
- Optimal obtains all surplus
- Guaranteed contracts
- Overly optimistic
- Assumes non-self-enforcible threats

How Should I Sell a Stream of Goods?

One seller, one buyer.

The seller has one good for sale at each period, $t = 1, 2, ..., T$.

Good t becomes obsolete at the end of period t.

The buyer's valuation is i.i.d., with $v_t \sim U[0, 1]$. The buyer learns v_t at time t.

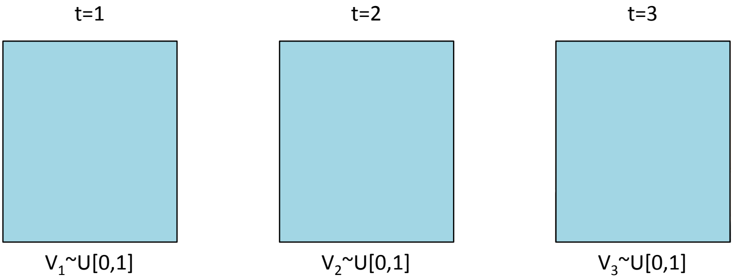
It is now $t = 0$. How should the seller sell this stream of goods?

Application: Online Advertisement

The seller is an online publisher.

The buyer is an advertiser.

The products are impressions.



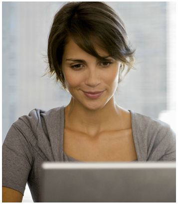
Application: Online Advertisement

The seller is an online publisher.

The buyer is an advertiser.

The products are impressions.

t=1 t=2 t=3



V_1=0.6 V_2~U[0,1] V_3~U[0,1]

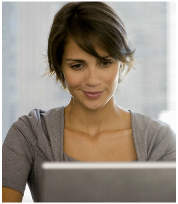
Application: Online Advertisement

The seller is an online publisher.

The buyer is an advertiser.

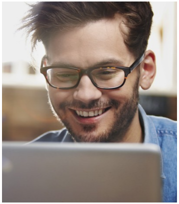
The products are impressions.

t=1



V_1=0.6

t=2



V_2=0.3

t=3

V_3~U[0,1]

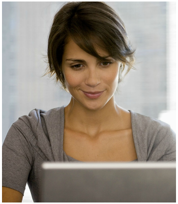
Application: Online Advertisement

The seller is an online publisher.

The buyer is an advertiser.

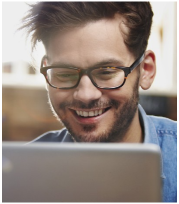
The products are impressions.

t=1



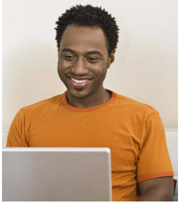
V_1=0.6

t=2



V_2=0.3

t=3



V_3=0.8

No Commitment

Without commitment: use the Myerson (1981) auction in every period.

With a single buyer, this reduces to using the optimal price $p^* = 0.5$.

In expectation, half the goods are sold.

Expected revenue per good is $1/4$.

Full Commitment

With commitment: use dynamic mechanism design.

Make a single take-it-or-leave-it offer at $t = 0$ at price $T/2$.

In equilibrium, buyer accepts the offer.

Expected revenue per good is $1/2$.

In practice, could we double our revenues?

Level of Commitment

No commitment is clearly too pessimistic.

In this work, we argue that full commitment is too optimistic.

We propose an intermediate model that we believe is more realistic:

positive commitment.

Under positive commitment, the seller's expected revenue per good is $3/8$.

Level of Commitment

No commitment is clearly too pessimistic.

In this work, we argue that full commitment is too optimistic.

We propose an intermediate model that we believe is more realistic:

positive commitment.

Under positive commitment, the seller's expected revenue per good is $3/8$.

What’s Unreasonable about Full Commitment?

What’s unreasonable about making a take-it-or-leave-it offer?

Sellers make “take-it-or-leave-it” offers all the time. If the buyer says yes, they can sign a futures contract allocating all units to the buyer.

However, the leave-it part is not really realistic.

Suppose the buyer says no. Can the seller really commit not to trade with him next period?

Not All Commitments Are Created Equal

It is easy for firms to make positive commitments about the future. Think supply chain contracts.

It is difficult to enforce a negative commitment. I cannot sign an enforceable contract with my future self promising not to trade with a buyer.

Furthermore, threats, which are based on negative commitments, are often undesirable business strategies.

In this work, we propose a framework where firms can make positive but not negative commitments.

Most Relevant Papers

Skreta (2005, 2015): Revelation principle does not apply. Instead, study the Perfect Bayesian equilibrium (PBE) of game with a general messaging space.

Deb and Said (2015): Apply Skreta's technique for dynamic screening.

The Model

One seller, one buyer. The seller has one good for sale at each period.

State of the game: $X_t \subseteq \{t, t+1, ...T\}$. Initial state: $X_0 = \{1, 2, ..., T\}$. The set of possible states at period t is denoted by $\mathcal{X}_t$.

The buyer's valuation is i.i.d., with $v_t \sim F(\cdot)$ with domain $[0, 1]$ and mean μ, where $F(\cdot)$ has an increasing virtual value.

Allowable Mechanisms

At every period t, the seller offers mechanism $M_t = (A_t, y_t, r_t)$. The set of available mechanisms at time t is denoted by $\mathcal{M}_t$.

The set A_t is an arbitrary set of actions. Without the revelation principle, we cannot focus on direct mechanisms.

The function $y_t : A_t \to 2^{\{t,t+1,\dots,T\}}$ determines goods allocated to the buyer. It must satisfy $y_t(a_t) \subseteq X_t$ for all $a_t \in A_t$.

The transition dynamics are given by $X_{t+1} = X_t \setminus (\{t\} \cup y_t(a_t))$.

The function $r_t : A_t \to \mathbb{R}$ determines the buyer's payment at time t.

Non-participation is an option: $\exists\, a_\emptyset \in A_t$ such that $y_t(a_\emptyset) = \emptyset$ and $r_t(a_\emptyset) = 0$.

Perfect Bayesian Equilibrium

The seller's utility is given by $U^s = \sum_{t=0}^{T} r_t(a_t)$.

The buyer's purchase set is $P = \cup_{t=0}^{T} y_t(a_t)$. His utility is $U^b = \sum_{t \in P} v_t - U^s$.

A seller's strategy is $\sigma^s = \{\sigma_t^s\}_{t=0,\dots,T}$ where $\sigma_t^s : \mathcal{X}_t \to M_t$.

A buyer's strategy is $\sigma^b = \{\sigma_t^b\}_{t=0,\dots,T}$ where $\sigma_t^b : \mathcal{X}_t \times [0,1] \to A_t$ for $t > 0$ and $\sigma_0^b : \mathcal{X}_0 \to A_0$.

A strategy profile $\sigma = (\sigma^s, \sigma^b)$ is a perfect Bayesian equilibrium (PBE) if it is sequentially rational. Every action must be chosen to maximize the expected utility of the player given the player's information set.

The Mechanism Design Problem

The seller's problem is given by

$$\begin{array}{ll} \sup\limits_{\sigma} & E[U^s] \\ s.t. & \sigma \text{ is a PBE} \end{array}$$

Mechanism Design without the Revelation Principle

- $q^{\sigma}_{t,r}(X_t, v)$: indicator of whether good r is allocated at period $t \leq r$ given state X_t in equilibrium σ assuming $v_t = v$.
- $p^{\sigma}_t(X_t, v)$: payment at period t given state X_t in equilibrium σ assuming $v_t = v$.
- $X^{\sigma}_{t+1}(X_t, v)$: state at period $t+1$ assuming state at period t is X_t in equilibrium σ assuming $v_t = v$.
- $U^{b,\sigma}_t(X_t, v, z)$: value-to-go of the buyer at time t given state X_t in equilibrium σ assuming $v_t = v$, but buyer acts as if $v_t = z$.

$$U^{b,\sigma}_t(X_t, v, z) = v \cdot q^{\sigma}_{t,t}(X_t, z) + \mu \sum_{r \in X_t \setminus \{t\}} q^{\sigma}_{t,r}(X_t, z) - p^{\sigma}_t(X_t, z) + V^{b,\sigma}_{t+1}\left(X^{\sigma}_{t+1}(X_t, z)\right)$$

$$\text{where} \quad V^{b,\sigma}_t(X_t) = E_{v_t}[U^{b,\sigma}_t(X_t, v_t, v_t)].$$

Incentive Constraints

Consider the subgame starting at time t and state X_t.

The equilibrium of the mechanism proposed by the seller must withstand one-period deviations from the buyer at time t.

Therefore, the mechanism offered at time t must satisfy incentive compatibility (IC) and individual rationality (IR) constraints.

$$U_t^{b,\sigma}(X_t, v, v) \geq U_t^{b,\sigma}(X_t, v, z) \qquad \text{for all v, z} \in [0, 1]$$
$$U_t^{b,\sigma}(X_t, v, v) \geq V_{t+1}^{b,\sigma}(X_t \setminus \{t\}) \qquad \text{for all v} \in [0, 1]$$

One-Period Deviations

For each period t subgame and state X_t, we replace the PBE constraint with IC and IR. We also relax the feasibility constraints.

$$
\begin{aligned}
V_t^{s,\sigma}(X_t) = \sup_{q_{t,t}^{\sigma}(\cdot),\ldots,q_{t,T}^{\sigma}(\cdot),p_t^{\sigma}(\cdot)} \quad & E_{v_t}\left[p_t^{\sigma}(X_t, v_t) + V_{t+1}^{s,\sigma}(X_{t+1}^{\sigma}(X_t, v_t))\right] & \\
s.t. \quad & U_t^{b,\sigma}(X_t, v, v) \geq U_t^{b,\sigma}(X_t, v, z) & \forall v, z \in [0, 1] \\
& U_t^{b,\sigma}(X_t, v, v) \geq V_{t+1}^{b,\sigma}(X_t \setminus \{t\}) & \forall v \in [0, 1] \\
& 0 \leq q_{t,r}^{\sigma}(X_t, \cdot) \leq 1 & \forall r \in X_t \\
& q_{t,r}^{\sigma}(X_t, \cdot) = 0 & \forall r \notin X_t
\end{aligned}
$$

Using the Myersonian Approach

$$V_t^{s,\sigma}(X_t) =$$

$$\sup_{q^\sigma_{t,t}(\cdot),\ldots,q^\sigma_{t,T}(\cdot)} E_{v_t}\Bigg[v_t \cdot q^\sigma_{t,t}(X_t, v_t) - \int_0^{v_t} q^\sigma_{t,t}(X_t, x)dx - V^{b,\sigma}_{t+1}(X_t \setminus \{t\})$$

$$+ \mu \sum_{r \in X_t \setminus \{t\}} q^\sigma_{t,r}(X_t, v_t) + V^{b,\sigma}_{t+1}\left(X^\sigma_{t+1}(X_t, v_t)\right) + V^{s,\sigma}_{t+1}(X^\sigma_{t+1}(X_t, v_t))\Bigg]$$

$$\begin{aligned} s.t. \quad & q^\sigma_{t,t}(X_t, \cdot) \text{ is a nondecreasing function} & \\ & 0 \le q^\sigma_{t,r}(X_t, \cdot) \le 1 & \forall r \in X_t \\ & q^\sigma_{t,r}(X_t, \cdot) = 0 & \forall r \notin X_t \end{aligned}$$

The present and future problems decouple.

The Present Problem

$$
\begin{aligned}
\sup_{q^{\sigma}_{t,t}(\cdot)} \; & E_{v_t}\left[v_t \cdot q^{\sigma}_{t,t}(X_t, v_t) - \int_0^{v_t} q^{\sigma}_{t,t}(X_t, x)dx \right] \\
s.t. \quad & q^{\sigma}_{t,t}(X_t, \cdot) \text{ is a nondecreasing function} \\
& 0 \leq q^{\sigma}_{t,t}(X_t, \cdot) \leq 1
\end{aligned}
$$

Assuming $t \in X_t$, it's identical to the Myerson problem.

Due to increasing virtual values, offer today's good at the monopoly price p^*.

The Future Problem

We can suppress v_t since it does not affect the decision-making.

$$
\begin{aligned}
\sup_{q^\sigma_{t,t+1}(\cdot),\ldots,q^\sigma_{t,T}(\cdot)} \quad & - V^{b,\sigma}_{t+1}(X_t \setminus \{t\}) + \mu \sum_{r \in X_t \setminus \{t\}} q^\sigma_{t,r}(X_t) \\
& + V^{b,\sigma}_{t+1}\left(X^\sigma_{t+1}(X_t)\right) + V^{s,\sigma}_{t+1}(X^\sigma_{t+1}(X_t)) \\
s.t. \quad & 0 \leq q^\sigma_{t,r}(X_t) \leq 1 \qquad \forall r \in X_t \\
& q^\sigma_{t,r}(X_t) = 0 \qquad \forall r \notin X_t
\end{aligned}
$$

Solving the Future Problem

We must have that

$$\mu \sum_{r \in X_t \setminus \{t\}} q^{\sigma}_{t,r}(X_t) + V^{b,\sigma}_{t}(X_t) + V^{s,\sigma}_{t}(X_t) \leq \mu \cdot |X_t|$$

since the maximum the seller and buyer can jointly obtain from a good is μ.
The future problem:

$$\begin{aligned}
\sup_{q^{\sigma}_{t,t+1}(\cdot),\ldots,q^{\sigma}_{t,T}(\cdot)} \quad & -V^{b,\sigma}_{t+1}(X_t \setminus \{t\}) + \mu \sum_{r \in X_t \setminus \{t\}} q^{\sigma}_{t,r}(X_t) \\
& + V^{b,\sigma}_{t+1}\left(X^{\sigma}_{t+1}(X_t)\right) + V^{s,\sigma}_{t+1}(X^{\sigma}_{t+1}(X_t)) \\
s.t. \quad & 0 \leq q^{\sigma}_{t,r}(X_t) \leq 1 \qquad \forall r \in X_t \\
& q^{\sigma}_{t,r}(X_t) = 0 \qquad \forall r \notin X_t
\end{aligned}$$

The solution to the future problem is to let $q^{\sigma}_{t,r}(X_t) = 1$ for all $r \in X_t \setminus \{t\}$.

Payments?

We have now determined the allocation. Today's good is offered at p^* and all future goods are allocated to the buyer.

We still need to determine how much the buyer pays for future goods.

Let M^s and M^b be the seller's and buyer's utilities under Myerson.

Under the optimal allocation, we know from the solution of the seller's optimization problem that his total profits are given by

$$V_t^{s,\sigma}(X_t) = M^s \cdot I_{\{t \in X_t\}} + \mu \cdot |X_t \setminus \{t\}| - V_{t+1}^{b,\sigma}(X_t \setminus \{t\}).$$

We can solve this problem by induction.

Induction

Inductive hypothesis: $V_t^{b,\sigma}(X_t) = M^b \cdot |X_t|$.

We can prove this statement directly for period T.

Assuming the inductive hypothesis is true for $t+1$, then we obtain for t:

$$V_t^{s,\sigma}(X_t) = M^s \cdot I_{\{t \in X_t\}} + \mu \cdot |X_t \setminus \{t\}| - M^b \cdot |X_t \setminus \{t\}|.$$

We also know the sum of the seller's and the buyer's utility must satisfy:

$$V_t^{s,\sigma}(X_t) + V_t^{b,\sigma}(X_t) = (M^s + M^b) \cdot I_{\{t \in X_t\}} + \mu \cdot |X_t \setminus \{t\}|.$$

Subtracting the two, we obtain

$$V_t^{b,\sigma}(X_t) = M^b \cdot I_{\{t \in X_t\}} + M^b \cdot |X_t \setminus \{t\}| = M^b \cdot |X_t|.$$

The Single Agent Solution

Definition

We define $\hat{p} = \mu - M^b$ to be the *positive commitment price*.

Theorem

At every stage of the game, the seller offers the current product at price p^* and future products at price $\hat{p}$.

The buyer buys good t at time t if $v_t \geq p^*$ and buyers future goods with probability 1.

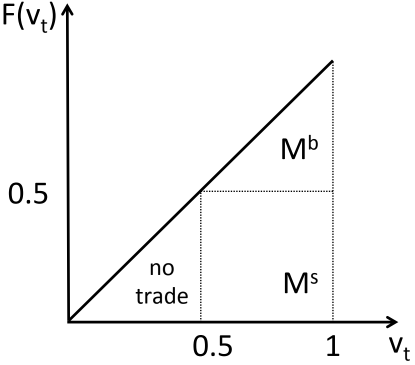
The seller must give M^b utility to the buyer for every good, not just the period T one.

Positive Commitment Removes Inefficiency

If valuations are U[0,1], then

$$\hat{p} = \frac{1}{2} - \frac{1}{8} = \frac{3}{8}.$$

Under positive commitment PBE, the seller extracts the “no trade” inefficiency.



Closer to Full or No Commitment Revenue?

Is $\hat{p}$ closer to the Myerson revenue M^s or the full commitment revenue μ?

It can go both ways. It depends on how inefficient is Myerson.

Efficient Myerson case:

$$f(x) = \left(\frac{L}{L-1}\right)\frac{1}{x^2} \quad \text{for } x \in [1, L],$$

$M^s = 1$ and $\mu \approx \log(L)$. We have $p^* = 1$ (efficient) and $\hat{p} = 1$.

Inefficient Myerson case:

$$f(x) = \left(\frac{L+1}{L}\right)\frac{1}{(x+1)^2} \quad \text{for } x \in [0, L],$$

$M^s \approx 1$ and $\mu \approx \log(L)$. We have $p^* \approx \sqrt{L}$ (inefficient) and $\hat{p} \approx \log(L)/2$.

Approximation

Proposition

Assume the hazard rate of $F(\cdot)$ is non-decreasing. Then,

$$\hat{p} \leq (e-1)M^s.$$

With a light-tailed distribution, $\hat{p}$ is not much larger than M^s.

Conclusions

We consider a model where the seller can make positive promises, but not negative ones.

The seller sells present-day items for p^* and future items for the positive commitment price $\hat{p} = \mu - M^b$.