Chapter 16

# Waves - I

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# 16-1 Transverse Waves

## Learning Objectives

**16.01** Identify the three main types of waves.

**16.02** Distinguish between transverse waves and longitudinal waves.

**16.03** Given a displacement function for a traverse wave, determine amplitude $y_m$, angular wave number $k$, angular frequency $\omega$, phase constant $\phi$, and direction of travel, and calculate the phase $kx \pm t + \phi$ and the displacement at any given time and position

**16.04** Given a displacement function for a traverse wave, calculate the time between two given displacements.

**16.05** Sketch a graph of a transverse wave as a function of position, identifying amplitude $y_m$ wavelength $\lambda$, where the slope is greatest, where it is zero, and where the string elements have positive velocity, negative velocity, and zero velocity.

**16.06** Given a graph of displacement versus time for a transverse wave, determine amplitude $y_m$ and period $T$.

**16.07** Describe the effect on a transverse wave of changing phase constant $\phi$.

**16.08** Apply the relation between the wave speed $v$, the distance traveled by the wave, and the time required for that travel.

**16.09** Apply the relationships between wave speed $v$, angular frequency $\omega$, angular wave number $k$, wavelength $\lambda$, period $T$, and frequency $f$.

**16.10** Describe the motion of a string element as a transverse wave moves through its location, and identify when its transverse speed is zero and when it is maximum.

**16.11** Calculate the transverse velocity $u(t)$ of a string element as a transverse wave moves through its location.

**16.12** Calculate the transverse acceleration $a(t)$ of a string element as a transverse wave moves through its location.

**16.13** Given a graph of displacement, transverse velocity, or transverse acceleration, determine the phase constant $\phi$.

## Types of Waves

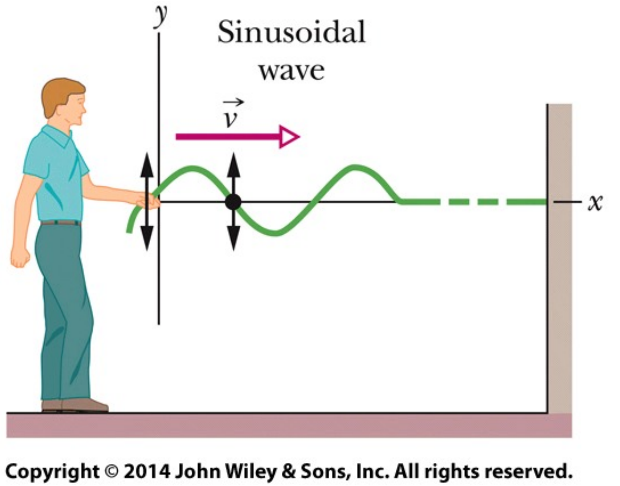
1. **Mechanical Waves:** They are governed by Newton's laws, and they can exist only within a material medium, such as water, air, and rock. Examples: water waves, sound waves, and seismic waves.

2. **Electromagnetic waves:** These waves require no material medium to exist. Light waves from stars, for example, travel through the vacuum of space to reach us. All electromagnetic waves travel through a vacuum at the same speed c = 299 792 458 m/s.

3. **Matter waves**: These waves are associated with electrons, protons, and other fundamental particles, and even atoms and molecules. Because we commonly think of these particles as constituting matter, such waves are called matter waves.

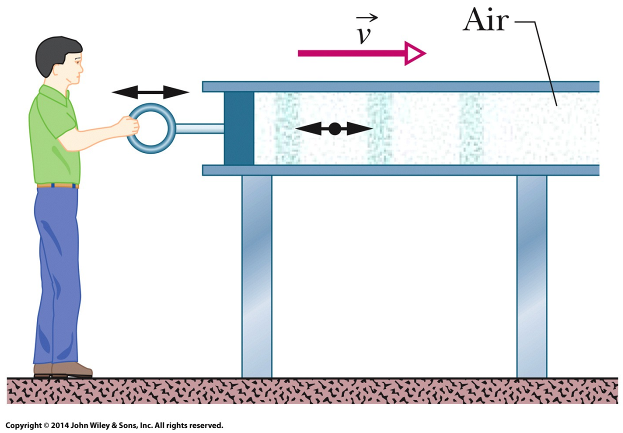
## Transverse and Longitudinal Waves

A sinusoidal wave is sent along the string (Figure (a)). A typical string element moves up and down continuously as the wave passes. This is **transverse wave**.




(a) Transverse Wave

A sound wave is set up in an air- filled pipe by moving a piston back and forth (Figure (b)). Because the oscillations of an element of the air (represented by the dot) are parallel to the direction in which the wave travels, the wave is a **longitudinal wave**.




(b) Longitudinal Wave

# Sinusoidal Function

Five “snapshots” (y *vs* x each at a constant time) of a string wave traveling in the positive direction along an *x* axis. The amplitude $y_m$ is indicated. A typical wavelength $\lambda$, measured from an arbitrary position $x_1$, is also indicated.

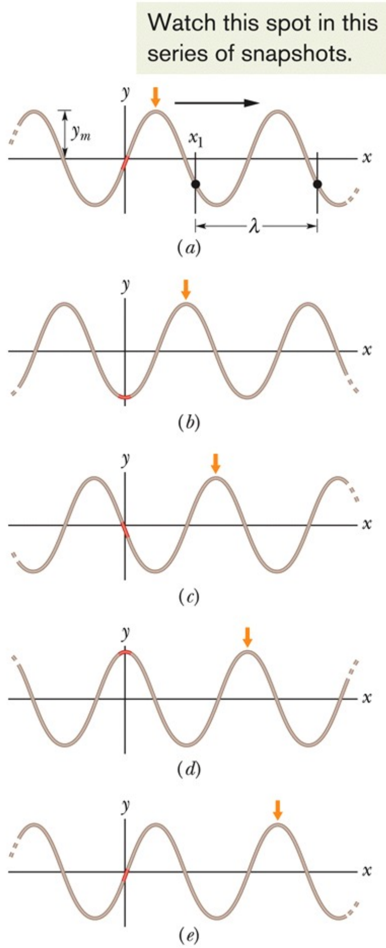
Amplitude
Oscillating term
Displacement
Phase

$$y(x,t) = y_m \sin(kx - \omega t)$$

Angular wave number
Position
Time
Angular frequency



*The sine function describes the shape of the wave*

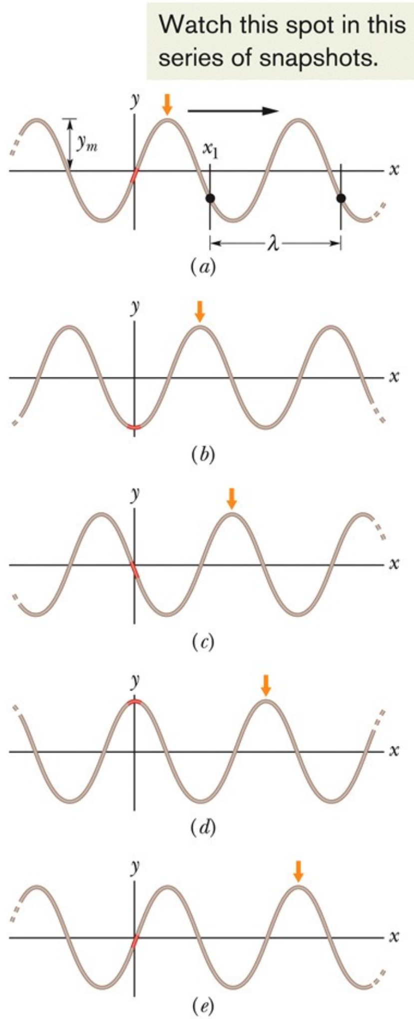


# Period, Wave Number, Angular Frequency and Frequency

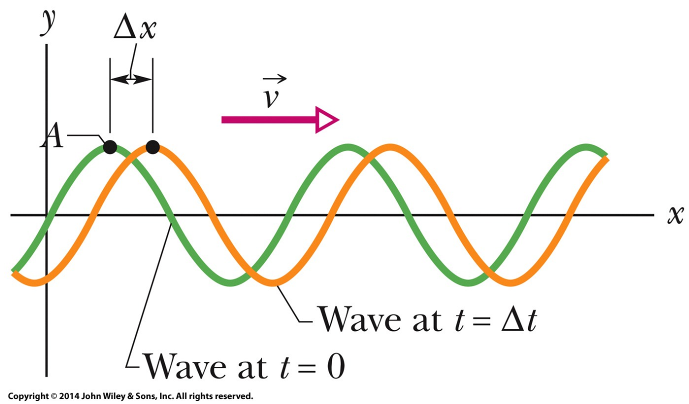
$$k = \frac{2\pi}{\lambda} \quad \text{(angular wave number).}$$

$$\omega = \frac{2\pi}{T} \quad \text{(angular frequency).}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \quad \text{(frequency).}$$



## The Speed of a Traveling Wave




Two snapshots of the wave: at time *t=0,* and then at time *t=Δt*. As the wave moves to the right at velocity *v,* the entire curve shifts a distance *Δx* during *Δt.*

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f \quad \text{(wave speed).}$$

$$y(x, t) = y_m \sin(kx + \omega t).$$

# 16-2 Wave Speed on a Stretched String

## Learning Objectives

**16.14** Calculate the linear density $\mu$ of a uniform string in terms of the total mass and total length.

**16.15** Apply the relationship between wave speed $v$, tension $\tau$, and linear density $\mu$.

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# 16-2 Wave Speed on a Stretched String

## Learning Objectives

The speed of a wave on a stretched string is set by properties of the string (i.e. linear density), not properties of the wave such as frequency or amplitude.  Tau is the tension (in N) in the string.

$$\mu = \frac{m}{l} \qquad \text{(linear density)}$$

$$v = \sqrt{\frac{\tau}{\mu}} \quad \text{(speed)},$$

# 16-3 Energy and Power of a Wave Traveling along a String

## Learning Objective

**16.16** Calculate the average rate at which energy is transported by a transverse wave.

# 16-3 Energy and Power of a Wave Traveling along a String

- When we set up a wave on a stretched string, we provide energy for the motion of the string. As the wave moves away from us, it transports that energy as both kinetic energy and elastic potential energy.
- **The Rate of Energy Transmission** The kinetic energy *dK* associated with a string element of mass *dm* is given by

$$dK = \tfrac{1}{2}\, dm\, u^2,$$

where *u* is the trar … d of the oscillating string element

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# 16-3 Energy and Power of a Wave Traveling along a String

- The **average power** of, or average rate at which energy is transmitted by, a sinusoidal wave on a stretched string is given by

$$P_{avg} = \tfrac{1}{2}\mu v \omega^2 y_m^2.$$

The factors *μ* and *v* in this equation depend on the material and tension of the string. The factors *ω* and $y_m$ depend on the process that generates the wave.

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# **16-4** The Wave Equation

## Learning Objective

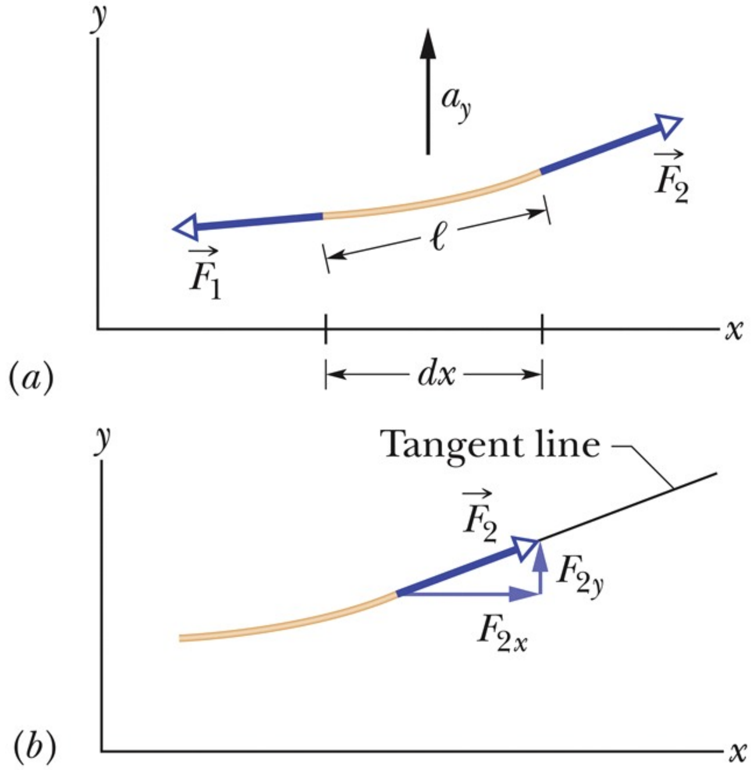
**16.17** For the equation giving a string-element displacement as a function of position *x* and time *t*, apply the relationship between the second derivative with respect to *x* and the second derivative with respect to *t*.

# 16-4 The Wave Equation

By applying Newton's second law to the element's motion, we can derive a general differential equation, called the wave equation, that governs the travel of waves of any type.

(a) A string element as a sinusoidal transverse wave travels on a stretched string. Forces $\vec{F}_1$ and $\vec{F}_2$ act at the left and right ends, producing acceleration with a vertical component $a_y$ .

(b) The force at the element's right end is directed along a tangent to the element's right side.



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## 16-4 The Wave Equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \quad \text{(wave equation).}$$

This is the general differential equation that governs the travel of waves of all types. Here the waves travel along an $x$ axis and oscillate parallel to the $y$ axis, and they move with speed $v$, in either the positive x direction or the negative $x$ direction.

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# 16-5 Interference of Waves

## Learning Objectives

**16.18** Apply the principle of superposition to show that two overlapping waves add algebraically to give a resultant (or net) wave.

**16.19** For two transverse waves with the same amplitude and wavelength and that travel together, find the displacement equation for the resultant wave and calculate the amplitude in terms of the individual wave amplitude and the phase difference.

**16.20** Describe how the phase difference between two transverse waves (with the same amplitude and wavelength) can result in fully constructive interference, fully destructive interference, and intermediate interference.

**16.21** With the phase difference between two interfering waves expressed in terms of wavelengths, quickly determine the type of interference the waves have.

# 16-5 Interference of Waves

## Principle of Superposition of waves

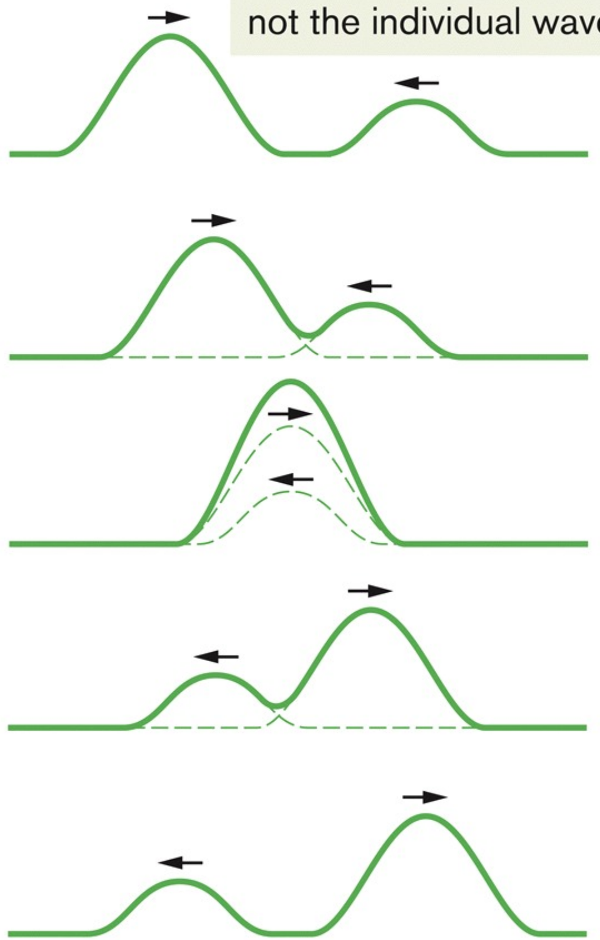
Let $y_1(x, t)$ and $y_2(x, t)$ be the displacements that the string would experience if each wave traveled alone. The displacement of the string when the waves overlap is then the algebraic sum

$$y'(x,t) = y_1(x,t) + y_2(x,t).$$

This summation of displacements along the string means that

- Overlapping waves algebraically add to produce a **resultant wave** (or **net wave**).
- Overlapping waves do not in any way alter the travel of each other.

When two waves overlap, we see the resultant wave, not the individual waves.

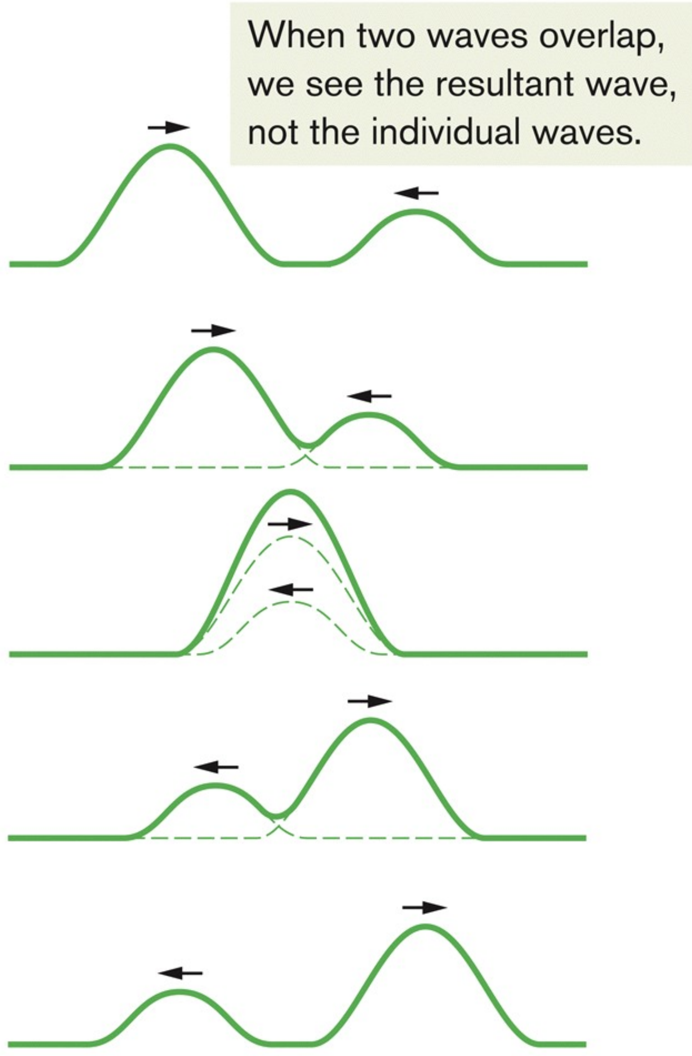


# 16-5 Interference of Waves

The resultant wave due to the interference of two sinusoidal transverse waves, is also a sinusoidal transverse wave, with an amplitude and an oscillating term.

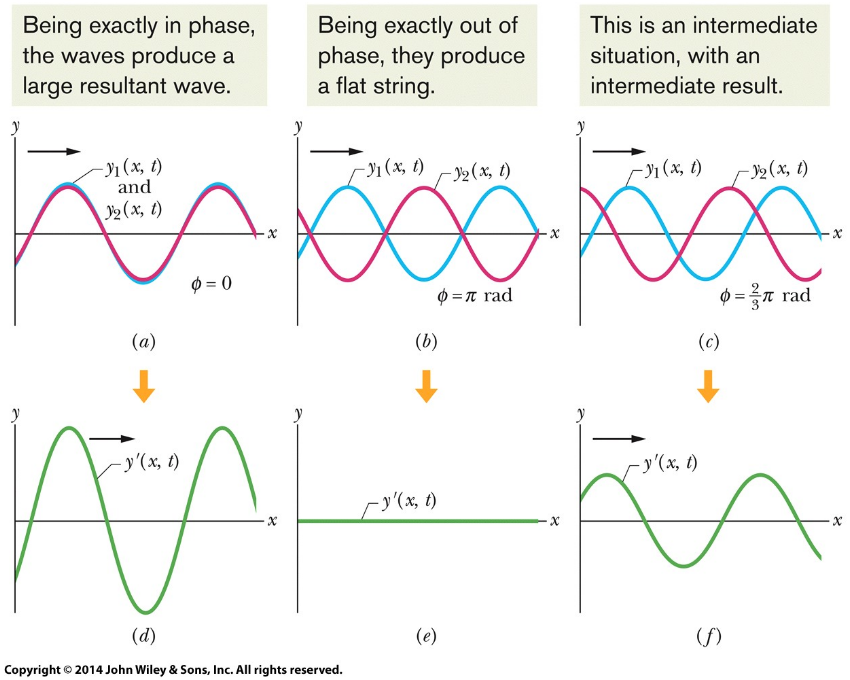
$$\overbrace{y'(x,t)}^{\text{Displacement}} = \underbrace{[2y_m \cos \tfrac{1}{2}\phi]}_{\substack{\text{Magnitude}\\ \text{gives}\\ \text{amplitude}}} \underbrace{\sin(kx - \omega t + \tfrac{1}{2}\phi)}_{\substack{\text{Oscillating}\\ \text{term}}}$$





# Constructive and Destructive Interference

$$y'(x, t) = y_1(x, t) + y_2(x, t).$$




Two identical sinusoidal waves, $y_1(x, t)$ and $y_2(x, t)$, travel along a string in the positive direction of an *x* axis. They interfere to give a resultant wave *y'(x, t)*. The resultant wave is what is actually seen on the string. The phase difference $\Phi$ between the two interfering waves is (a) $0$ rad or $0^o$, (b) $\pi$ rad or $180^o$, and (c) $2/3\ \pi$ rad or $120^o$. The corresponding resultant waves are shown in (d), (e), and ( f ).

# 16-6 Phasors

## Learning Objectives

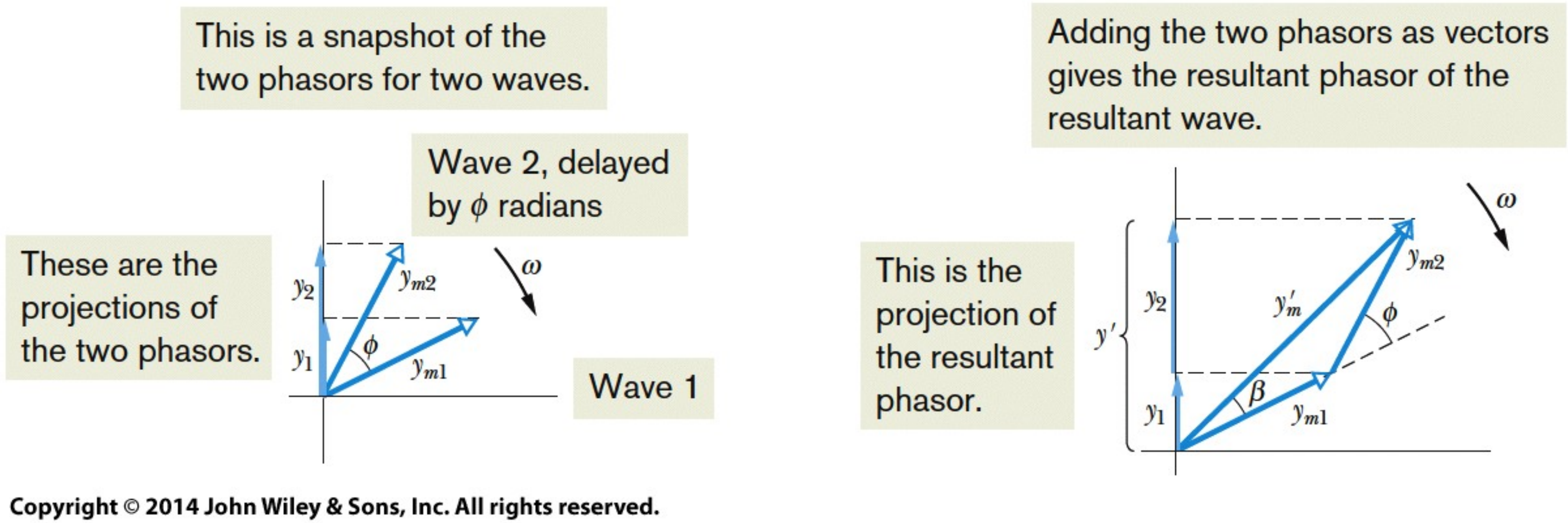
**16.22** Using sketches, explain how a phasor can represent the oscillations of a string element as a wave travels through its location.

**16.23** Sketch a phasor diagram for two overlapping waves traveling together on a string, indicating their amplitudes and phase difference on the sketch.

**16.24** By using phasors, find the resultant wave of two transverse waves traveling together along a string, calculating the amplitude and phase and writing out the displacement equation, and then displaying all three phasors in a phasor diagram that shows the amplitudes, the leading or lagging, and the relative phases.

# 16-6 Phasors

A phasor is a vector that rotates around its tail, which is pivoted at the origin of a coordinate system. The magnitude of the vector is equal to the amplitude $y_m$ of the wave that it




(a) (b)

(a) A second phasor, also of angular speed $\omega$ but of magnitude $y_{m2}$ and rotating at a constant angle $\beta$ from the first phasor, represents a second wave, with a phase constant $\Phi$. (b) The resultant wave is represented by the vector sum $y'_m$ of the two phasors.

# 16-7 Standing Waves and Resonance

## Learning Objectives

**16.25** For two overlapping waves (same amplitude and wavelength) that are traveling in opposite directions, sketch snapshots of the resultant wave, indicating nodes and antinodes.

**16.26** For two overlapping waves (same amplitude and wavelength) that are traveling in opposite directions, find the displacement equation for the resultant wave and calculate the amplitude in terms of the individual wave amplitude.

**16.27** Describe the SHM of a string element at an antinode of a standing wave.

**16.28** For a string element at an antinode of a standing wave, write equations for the displacement, transverse velocity, and transverse acceleration as functions of time.

**16.29** Distinguish between “hard” and “soft” reflections of string waves at a boundary.

**16.30** Describe resonance on a string tied taut between two supports, and sketch the first several standing wave patterns, indicating nodes and antinodes.

**16.31** In terms of string length, determine the wavelengths required for the first several harmonics on a string under tension.

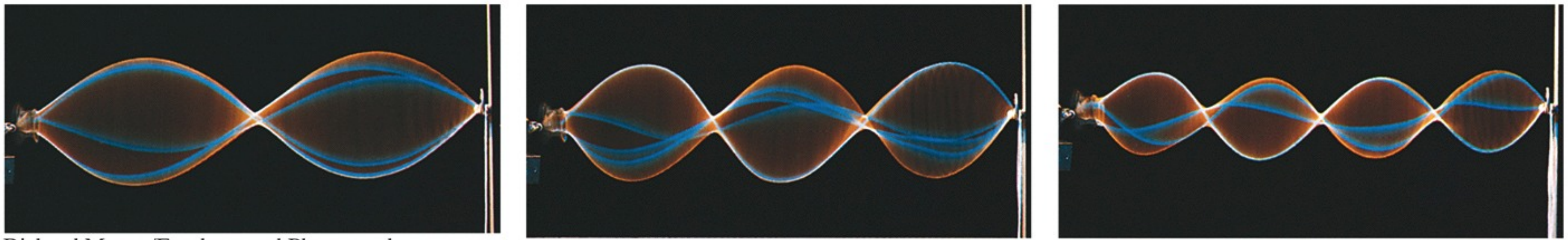
**16.32** For any given harmonic, apply the relationship between frequency, wave speed, and string length.

# Standing Waves

The interference of two identical sinusoidal waves moving in opposite directions produces standing waves. For a string with fixed ends, the standing wave is given by

$$\overbrace{y'(x,t)}^{\text{Displacement}} = \underbrace{[2y_m \sin kx]}_{\substack{\text{Magnitude}\\ \text{gives}\\ \text{amplitude}\\ \text{at position } x}} \underbrace{\cos \omega t}_{\substack{\text{Oscillating}\\ \text{term}}}$$





Richard Megna/Fundamental Photographs

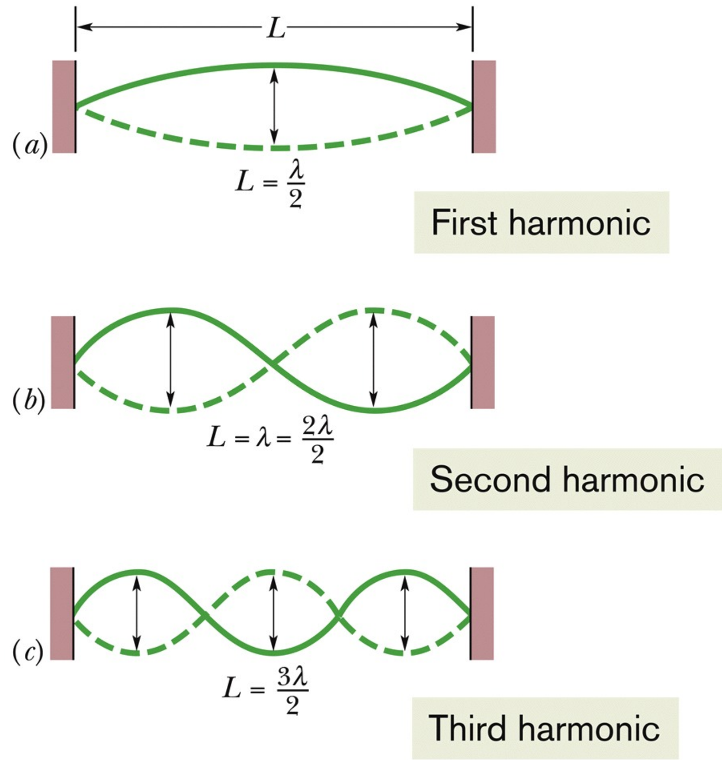
Stroboscopic photographs reveal (imperfect) standing wave patterns on a string being made to oscillate by an oscillator at the left end. The patterns occur at certain frequencies of oscillation.

# 16-7 Standing Waves and Resonance

## Harmonics

Standing waves on a string can be set up by reflection of traveling waves from the ends of the string. If an end is fixed, it must be the position of a node. This limits the frequencies at which standing waves will occur on a given string. Each possible frequency is a **resonant frequency**, and the corresponding standing wave pattern is an oscillation mode. For a stretched string of length L with fixed ends, the resonant frequencies are

$$f = \frac{v}{\lambda} = n\frac{v}{2L}, \quad \text{for } n = 1, 2, 3, \ldots.$$



# 16 Summary

## Waves

- Transverse Waves
- Longitudinal Waves

## Wave Speed

- Angular velocity/ Angular wave number

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f.$$ **Eq. (16-13)**

## Sinusoidal Waves

- Wave moving in positive direction (vector)

$$y(x, t) = y_m \sin(kx - \omega t)$$ **Eq. (16-2)**

## Traveling Waves

- A functional form for traveling waves

$$y(x, t) = h(kx \pm \omega t)$$ **Eq. (16-17)**

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# 16 Summary

## Powers

- Average Power is given by

$$P_{avg} = \tfrac{1}{2}\mu v \omega^2 y_m^2$$ **Eq. (16-33)**

## Interference of Waves

- Two sinusoidal waves on the same string exhibit interference

$$y'(x,t) = [2y_m \cos \tfrac{1}{2}\phi] \sin(kx - \omega t + \tfrac{1}{2}\phi).$$

**Eq. (16-51)**

## Standing Waves

- The interference of two identical sinusoidal waves moving in opposite directions produces standing waves.

$$y'(x,t) = [2y_m \sin kx] \cos \omega t.$$ **Eq. (16-60)**

## Resonance

- For a stretched string of length L with fixed ends, the resonant frequencies are

$$f = \frac{v}{\lambda} = n\frac{v}{2L}, \quad \text{for } n = 1, 2, 3, \ldots$$

**Eq. (16-66)**