

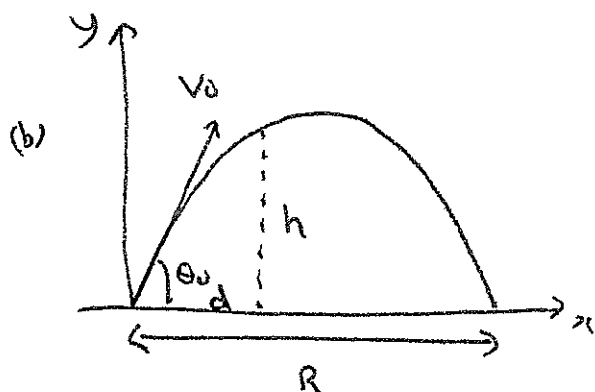
①
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(a)



$$\begin{aligned}
 \text{Mean Velocity} &= \frac{\text{Change in Displacement}}{\text{Time Taken}} \\
 &= \frac{-12\text{cm} - (+12\text{cm})}{30\text{ min}} \\
 &= \frac{-24 \times 10^{-3}}{30 \times 60} \text{ m/s} \\
 &= -1.33 \times 10^{-4} \text{ m/s}
 \end{aligned}$$

ie Speed is $1.3 \times 10^{-4} \text{ m/s} = 0.13 \text{ mm/s}$ to 2 SF
Direction is downwards



Initially Velocity components
 $V_x = V_0 \cos \theta_0$
 $V_y = V_0 \sin \theta_0$

(i) x-motion given by $x - x_0 = V_x t$ since no acceleration

$$\text{ie } \underline{d = V_0 \cos \theta_0 t}$$

(ii) In y-direction there is uniform acceleration $-g$

So, applying " $S = ut + \frac{1}{2}at^2$ " we have

$$h = y - y_0 = V_y \sin \theta_0 t - \frac{1}{2}gt^2$$

$$\text{ie } \underline{h = V_0 \sin \theta_0 t - \frac{1}{2}gt^2}$$

(iii) Range R given by the time t when returns to ground

$$\text{ie when } h=0 \Rightarrow V_0 \sin \theta_0 t - \frac{1}{2}gt^2 = 0$$

$$\text{So } t=0 \text{ [initial condition] or } \underline{t = \frac{2 V_0 \sin \theta_0}{g}} \text{ launch}$$

(iv) Substituting into the eqn for the x-distance

$$R = V_0 \cos \theta_0 t \quad \text{with} \quad t = 2 V_0 \sin \theta_0 / g$$

$$= 2 V_0^2 \sin \theta_0 \cos \theta_0 / g$$

$$\underline{R = V_0^2 \sin 2\theta_0 / g} \quad \text{where} \quad \sin 2\theta_0 = 2 \sin \theta_0 \cos \theta_0$$

(c) The range is max when R is at its greatest

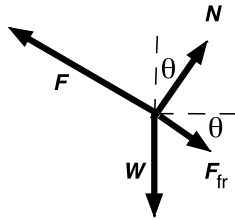
$$\text{i.e. } \sin 2\theta_0 = 1 \quad \text{or} \quad 2\theta_0 = 90^\circ \Rightarrow \theta_0 = 45^\circ$$

$$\text{Thus } R_{\max} = \frac{V_0^2}{g} = \frac{9.50^2}{9.8} = 9.21 \text{ m}$$

World Record is 8.95 m

$$\text{Hence it is } 9.21 - 8.95 = \underline{26 \text{ cm shorter}}$$

Question 2



a) (no marks explicitly for the FBD)

i) No acceleration in the normal direction:

$$N - mg \cos \theta = 0 \quad \text{so} \quad N = mg \cos \theta.$$

In the direction up the plane, apply Newton's second law.

$$ma = F - F_{fr} - mg \sin \theta$$

$$ma = F - \mu_k N - mg \sin \theta$$

$$= F - \mu_k mg \cos \theta - mg \sin \theta$$

$$F = m(a + g(\mu_k \cos \theta + \sin \theta))$$

ii) To maximise F with respect to θ , take the derivative:

$$dF/d\theta = mg(\cos \theta - \mu_k \sin \theta) = 0$$

$$\text{Rearrange} \quad \cos \theta = \mu_k \sin \theta$$

$$\theta = \tan^{-1}(1/\mu_k)$$

iii) As θ increases, the force up the plane is opposed by an increasing component of mg . However, the force up the plane is also opposed by friction and, as θ increases, the normal force and therefore the friction decrease.

b)

i) At maximal compression, the spring is neither compressing nor lengthening, so the relative velocity is zero.

ii) Let v be the speed at maximum compression.

Here, no non-conservative forces act, so mechanical energy is conserved.

$$U_i + K_i = U_f + K_f$$

$$0 + \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}kx^2 + \frac{1}{2}(m + m)v^2$$

$$kx^2 = mv_1^2 + mv_2^2 - 2mv^2 = m(v_1^2 + v_2^2 - 2v^2)$$

External forces in the x direction are negligible, so momentum in x direction is conserved.

Momentum before collision = momentum at maximum compression

$$mv_1 + mv_2 = 2mv$$

$$v = (v_1 + v_2)/2$$

Combine the two results

$$kx^2 = m(v_1^2 + v_2^2 - \frac{1}{2}(v_1 + v_2)^2)$$

$$= \frac{1}{2} m(2v_1^2 + 2v_2^2 - v_1^2 - 2v_1v_2 - v_2^2)$$

$$= \frac{1}{2} m(v_1^2 + v_2^2 - 2v_1v_2)$$

$$= \frac{1}{2} m(v_1 - v_2)^2$$

So, at maximum compression, $x = \sqrt{\frac{m}{2k}}(v_1 - v_2)$

Question 3

- a) i) $F_g = GMm/R^2$.
ii) $a = R\omega^2 = R(2\pi/T)^2$.
iii) Newton's second law: $F = ma$

$$GMm/R^2 = mR(2\pi/T)^2$$

so $GMT^2 = 4\pi^2 R^3$

so $R = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \dots = 42\,100 \text{ km}$

- iv) Total mechanical energy of an orbit, with respect to zero at infinite distance, is $-GMm/2r$. On the Earth, and neglecting rotation of the Earth, the spacecraft has only potential energy $-GMm/R_e$.

Easy method

Work energy theorem: total work done by all forces = $\Delta K = \frac{1}{2} m(R\omega)^2$
 $= \frac{1}{2} (145 \text{ kg})(42\,100 \text{ km} \cdot 2\pi/23.9 \text{ hr})^2$
 $= 685 \text{ MJ} \text{ or } 6.85 \times 10^8 \text{ J}$

Alternative method:

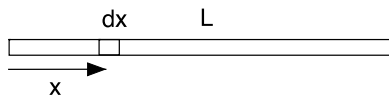
ΔE = total energy of the orbit – (only potential) energy on earth

But this energy includes the gravitational potential energy ΔU_g . So the total work done by all forces, including gravity, is

Work done = total energy of the orbit – (only potential) energy on earth – ΔU_g ,
 $= -GMm/2R - (-GMm/R_e) - (-GMm/R - (-GMm/R_e))$ *
 $= GMm/2R = 685 \text{ MJ} \text{ or } 6.85 \times 10^8 \text{ J}$

(Comment only: In practice, however, the work done is by the rockets is much higher because much energy is wasted lifting and accelerating fuel and rocket stages.)

- b) i) Moment of inertia $I = \int_{body} r^2 dm$ where r is the perpendicular distance from the axis of mass element dm .
 ii)



Let the linear mass density by $\lambda = m/L$, so

$$dm = \lambda dx$$

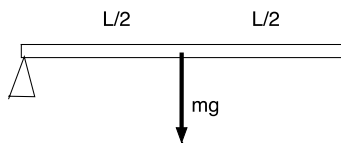
$$I = \int_{body} r^2 dm = \int_{x=0}^{x=L} x^2 dm$$

$$= \int_{x=0}^{x=L} x^2 \lambda dx$$

$$= \frac{\lambda}{3} [x^3]_{x=0}^{x=L}$$

$$= \frac{\lambda}{3} L^3, \quad \text{but by definition } \lambda L = m, \text{ so}$$

$$I = \frac{1}{3} mL^2.$$



Newton's second law for rotation:

$$\Sigma \tau = I\alpha$$

Consider rotation about the point of the wedge. The weight provides the only torque, so

$$mg(L/2) = (mL^2/3)\alpha$$

$$\alpha = (3/2)(g/L)$$

Acceleration at the end of the rod: $a = \alpha L = 3g/2$

1131 Solutions Question 4

a) i)

$$T_A = \frac{P_A V_A}{nR} = \frac{5 \times 1.01 \times 10^5 \times 16.46 \times 10^{-3}}{5 \times 8.314} = 200K$$

So as this is a diatomic molecule at 200K it has five degrees of freedom

ii) Three of these degrees of freedom are translational, corresponding to movement in the x, y and z direction.

Two of these degrees of freedom are rotational, corresponding to rotations about two axes.

iii)

$$\begin{aligned} W &= - \int P dV \\ &= -5 \times 1.01 \times 10^5 \times (32 - 16.46) \times 10^{-3} \\ &= -7848J \\ &= -7850J \text{ (3 sig fig)} \end{aligned}$$

iv) 0 J it's isothermal

v)

$$\Delta E_{int} = Q + W$$

$Q = 0$ it's adiabatic

Two methods to calculate ΔE_{int}

method 1 :

$$\Delta E_{int} = \frac{f}{2} nR(T_A - T_D)$$

$$T_D = \frac{P_D V_D}{nR} = 153.9K$$

$$\Rightarrow \Delta E_{int} = \frac{5}{2} \times 5 \times 8.314 \times (200 - 153.9) = 4792J$$

method 2:

As $A \rightarrow B \rightarrow C \rightarrow D \rightarrow A$ is cyclic, total $\Delta E_{int} = 0$

$$\Delta E_{int A \rightarrow B} = -7.850 + 27.5 = 19.65kJ$$

$$\Delta E_{int C \rightarrow D} = -34.2 - 2 \times 1.01 \times 10^5 \times (31.67 - 80) \times 10^{-3} = -24.44kJ$$

$$\text{so } \Delta E_{int D \rightarrow A} = -(-24.44 + 19.65) = 4.79kJ$$

As $\Delta E_{int} = W$

$$\Rightarrow W = 4.79kJ$$

Or it is possible to calculate it directly by integrating PdV using $PV^{1.4} = \text{constant}$, but this is harder.

$$\begin{aligned}
 \text{b) i)} \quad P &= kA \frac{\Delta T}{\Delta x} \\
 &= 50.2 \times 100.0 \times 10^{-4} \times \frac{30.00}{0.020} \\
 &= 753W
 \end{aligned}$$

$$\begin{aligned}
 \text{ii)} \quad Q &= Pt = mL \\
 m &= \frac{Pt}{L} = \frac{753 \times 60}{3.33 \times 10^5} \\
 &= 0.136kg
 \end{aligned}$$

iii)

$$\begin{aligned}
 \text{Energy needed to melt ice} &= m_i L_{fus} \\
 &= 0.500 \times 3.33 \times 10^5 \\
 &= 166500J
 \end{aligned}$$

$$\begin{aligned}
 \text{energy that water can lose as it goes to } 0^\circ C &= m_w c_w \Delta T \\
 &= 1.000 \times 4186 \times 30 = 125580J
 \end{aligned}$$

$$\begin{aligned}
 \text{energy that steel can lose as it goes to } 0^\circ C &= m_{st} c_{st} \Delta T \\
 &= 1.56 \times 456 \times 15 = 10670J
 \end{aligned}$$

As the water at the top and steel combined can not lose enough energy to melt all the ice the system has a final temperature of $0^\circ C$.

iv)

$$Q = \int_0^{60} P(t) dt = m_w c_w (T_h - T - f) + m_{st} c_{st} \left(\frac{T_h}{2} - \frac{T_f}{2} \right)$$

Note that P and T_f are functions of time.

Differentiate with respect to t .

$$\Rightarrow P = -m_w c_w \frac{dT_f}{dt} - \frac{m_{st} c_{st}}{2} \frac{dT_f}{dt}$$

$$\text{Now: } P = \frac{kAT_f}{d}$$

$$\Rightarrow \int_{T_h}^{T_f} \frac{dT_f}{T_f} = \int_0^{60} \frac{-kA}{d(m_w c_w + m_{st} c_{st}/2)} dt$$

$$\ln\left(\frac{T_f}{T_h}\right) = \frac{-60kA}{d(m_w c_w + m_{st} c_{st}/2)}$$

$$\Rightarrow T_f = T_h e^{\frac{-60kA}{d(m_w c_w + m_{st} c_{st}/2)}}$$

$$\frac{-60kA}{d(m_w c_w + m_{st} c_{st}/2)} = \frac{50.2 \times 100 \times 10^{-4} \times 60}{0.02 \times (4186 \times 1 + 1.56 \times 486/2)} = 0.331595$$

$$T_f = 30.0 \times e^{-0.331595} = 21.533^\circ C$$

$$Q = 1 \times 4186 \times (30 - 21.533) + \frac{1.56 \times 456}{2} \times (30.0 - 21.533)$$

$$= 38452.9J$$

$$= 38.5kJ \text{ (3 sig. fig.)}$$

Question 5 T1 2014

a)

For planet GJ832c:

$$g = \frac{GM_{\text{planet}}}{R^2} = 13.22 \frac{\text{m}}{\text{s}^2}$$

Simple pendulum:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Find L:

$$T^2 = \frac{4\pi^2 L}{g}$$

$$L = \frac{T^2 g}{4\pi^2} = 33.49 \text{ m} \rightarrow 33.5 \text{ m (3 sig. figs)}$$

b)

1st harmonic $\rightarrow \lambda = 2L$

$$v = \sqrt{\frac{T}{\mu}} = f\lambda \text{ and } T = mg$$

From a): $L = 33.5 \text{ m}$

Solve for μ :

$$\sqrt{\frac{T}{\mu}} = 2Lf$$

$$\frac{T}{\mu} = 4L^2 f^2$$

$$\mu = \frac{T}{4L^2 f^2} = \frac{mg}{4L^2 f^2} \text{ (} L = 33.5 \text{ m, } g = 13.22 \frac{\text{m}}{\text{s}^2}, f = 4.40 \text{ Hz, } m = 66 \text{ kg)}$$

$$\mu = 0.0100 \frac{\text{kg}}{\text{m}} \text{ (3 sig. figs)}$$

c)

After rope splits: $\mu' = \frac{2}{3}\mu$

1st harmonic: $f_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}} \rightarrow f_1 = \frac{1}{2L} \sqrt{\frac{T}{\mu}}$ and $T = mg$

$$f_1 = \frac{1}{2L} \sqrt{\frac{mg \cdot 3}{2\mu}}, \mu = 0.0100 \frac{\text{kg}}{\text{m}} \text{ from part b)}$$

$$f_1 = \frac{1}{2(33.5)} \sqrt{\frac{66 \cdot 13.22 \cdot 3}{2 \cdot 0.0100}} \text{ Hz}$$

$$f_1 = 5.40 \text{ Hz}$$

d)

Speed at time t: $v = gt = 13.2t$

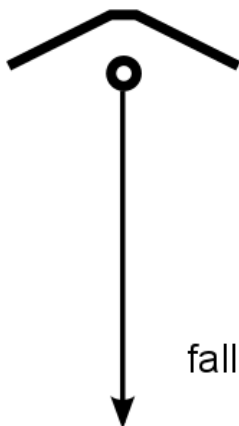
Doppler: $v_{\text{source}} = 13.2t$, observer stationary

$$f' = \left(\frac{v - v_o}{v + v_s} \right) f, f = 440.0 \text{ Hz}, v = 380.0 \frac{\text{m}}{\text{s}}$$

$$f' = \left(\frac{380 - 0}{380 + 13.2t} \right) f$$

$$f' = \frac{167200}{380 + 13.2t} \text{ Hz}$$

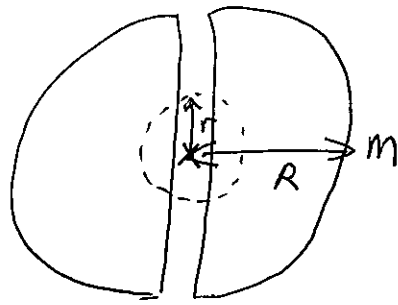
stationary observer $v_{\text{observer}} = 0$



falling $v = 13.2 t$

e)

Take center of the Earth as $r=0$.
As the particle undergoes SHM it will oscillate between $+R$ and $-R$.



We need to work out force acting on the particle. The force is the gravitational force so:

$$F = \frac{G m_1 m_2}{r^2}$$

However, only the mass inside the radius 'r' will contribute so:

$m_1 = m_0 =$ mass of object.

$$m_2 = \rho V = \frac{M}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi r^3 = \frac{Mr^3}{R^3}$$

$$\Rightarrow F = \frac{G m_0 M r^3}{r^2 R^3} = \frac{G m_0 M r}{R^3} = m_0 a$$

Now for SHM $a = -\omega^2 r$ where -ve sign shows acceleration/force and displacement are in the opposite direction to each other so

$$|a| = \omega^2 r = \frac{G M r}{R^3}$$

$$\Rightarrow \omega^2 = \frac{G M}{R^3}$$

$$\omega = \sqrt{\frac{G M}{R^3}}$$