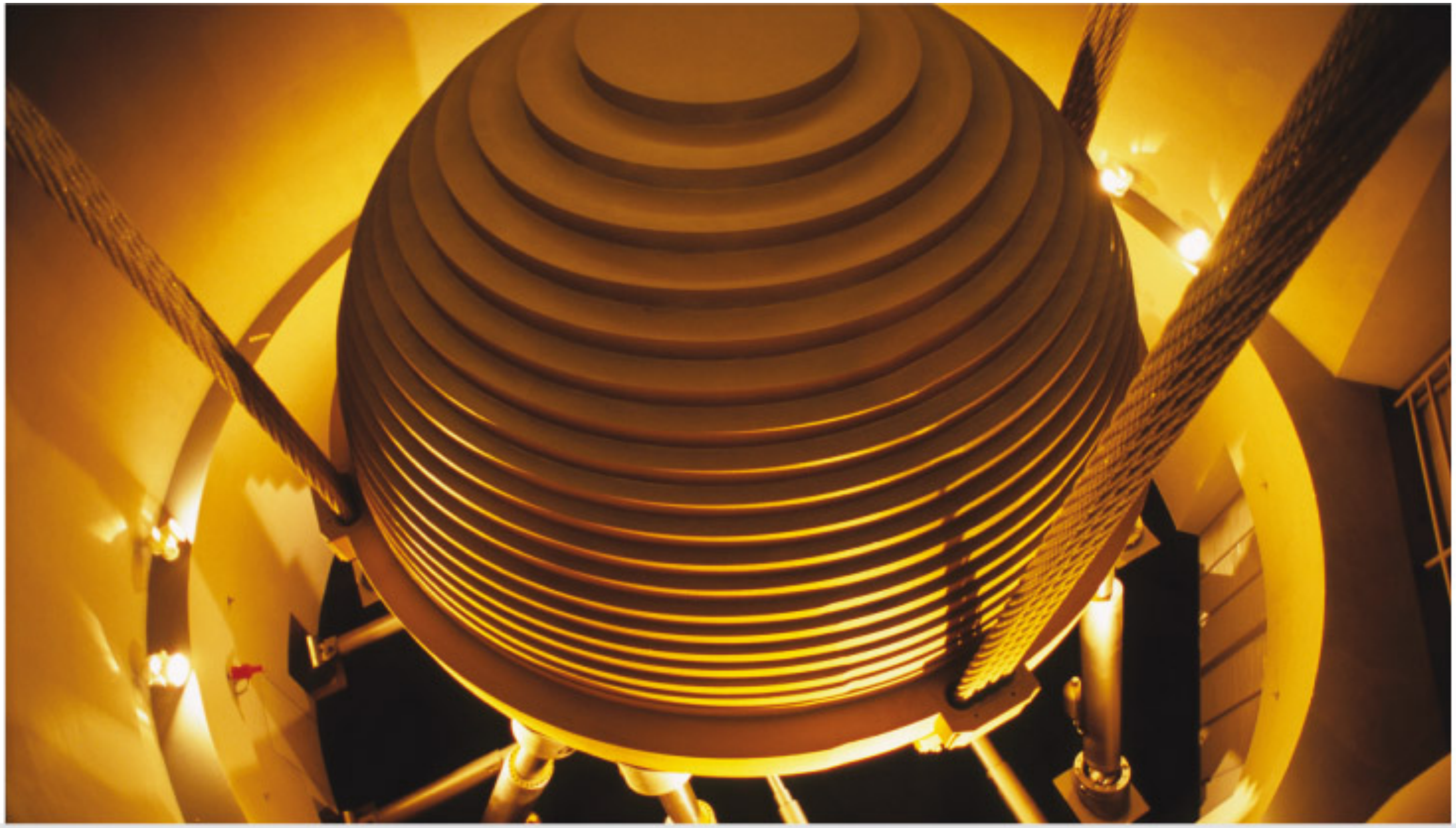
THE UNIVERSITY OF
NEW SOUTH WALES

Chapter 15

Oscillatory Motion

Professor Michael Burton
School of Physics, UNSW

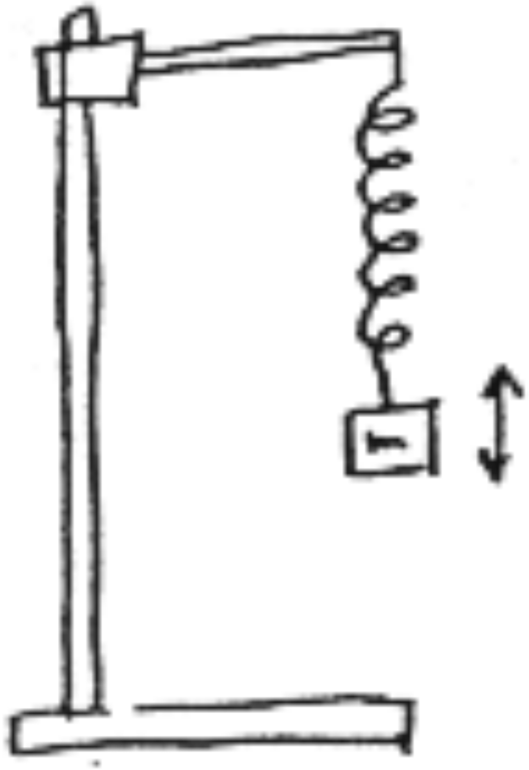
Taipei 101



Ranjit Doroszkeiwicz/Alamny

Mh1: Simple Harmonic Motion

Simple block and spring



Example: the tides,

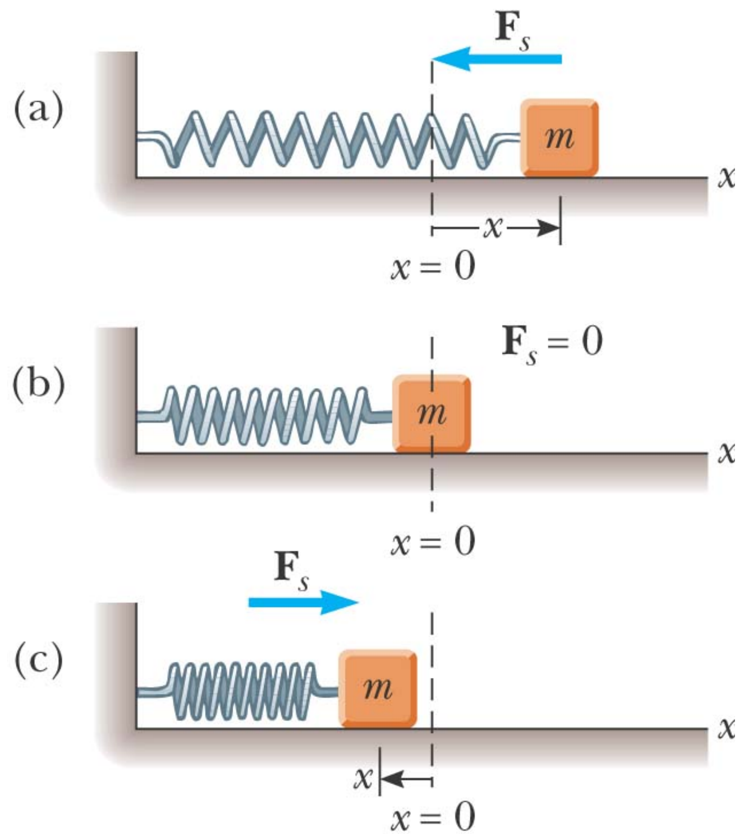
a swing

Periodic Motion

- ***Periodic motion*** is motion of an object that regularly repeats
  - Object returns to a given position after fixed time interval
- ***Simple Harmonic Motion***
  - When the force acting on the object is proportional to the position of the object relative to some equilibrium position, and is directed towards it
  - $F = - k \cdot x$

Motion of a Spring-Mass System

- *Active Figure 15.02*
- Block of mass *m* is attached to a spring, and free to move on a frictionless horizontal surface
- When the spring is neither stretched nor compressed, the block is at the ***equilibrium position***
  - $x = 0$



Hooke's Law

- *Active Figure 15.01*
- Hooke's Law for the stretched spring

  $F_s = - kx$
  - F_s is the restoring force
    - It is always directed toward the equilibrium position
    - Thus it is always opposite the displacement from equilibrium
  - k is the force (spring) constant
  - x is the displacement from the equilibrium position

Acceleration

- The force described by Hooke's Law is the net force in Newton's Second Law

$$F_{\text{Hooke}} = F_{\text{Newton}}$$

$$-kx = ma_x$$

$$a_x = -\frac{k}{m}x$$

$$a = -\frac{k}{m}x$$

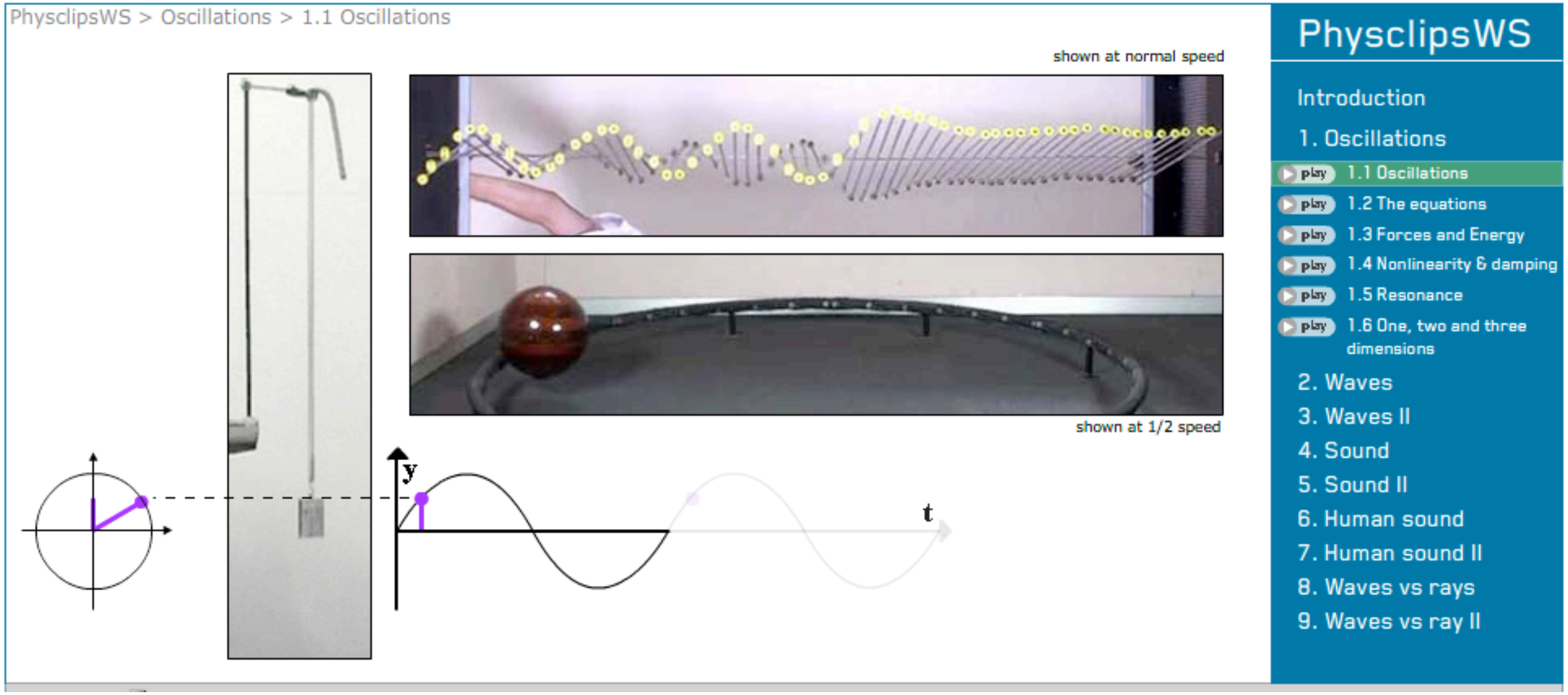
The Acceleration is

- Proportional to the displacement of the block
- Has direction opposite the direction of the displacement from equilibrium
- Thus, this is *Simple Harmonic Motion*
- It is ***not*** constant
  - Thus, the kinematic equations (i.e. *v=u+at* etc.) for constant acceleration <u>cannot</u> be applied

A block on the end of a spring is pulled to position $x = A$ and released from rest. In one full cycle of its motion, through what total distance does it travel?

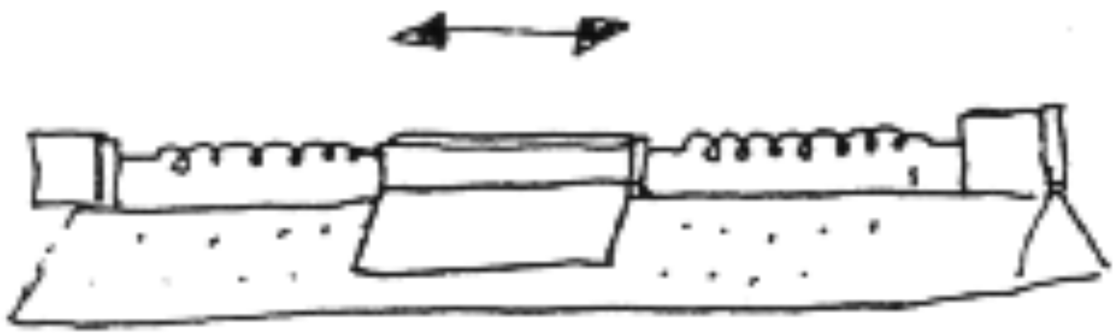
1. $A/2$
2. A
3. $2A$
4. $4A$

PHYSCLIPS 2: Waves and Sound
1.1 Oscillations



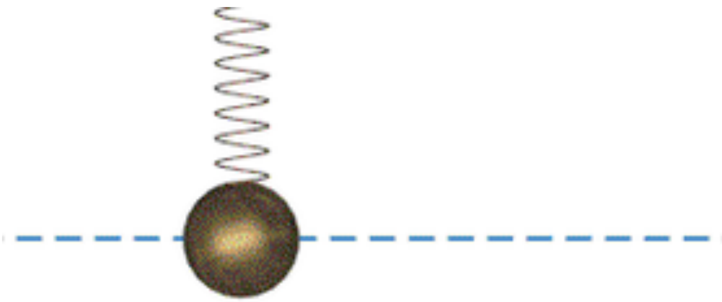
Mh11: Simple Harmonic Motion – air track & springs

No friction!



Vertical Orientation

- When the block is hung from a vertical spring, its weight will cause the spring to stretch
- If the resting position of the spring is defined as $y = 0$, the same analysis as was done for the horizontal spring will apply to the vertical spring-block system
  - Here $y = x - x_{equilibrium}$
    where $mg = k\,x_{equilibrium}$
- *See Movie HMM03AN1*
  - *SHM with vertical spring*
    - 19sec



Simple Harmonic Motion – Mathematical Representation

- Model the block as a particle
- Choose *x* as the axis along which the oscillation occurs
- Apply Newton's 2nd Law:
- Acceleration $a = \frac{d^2x}{dt^2} = -\frac{k}{m}x$
- We let $\omega^2 = \frac{k}{m}$
- Then $a = -\omega^2 x$

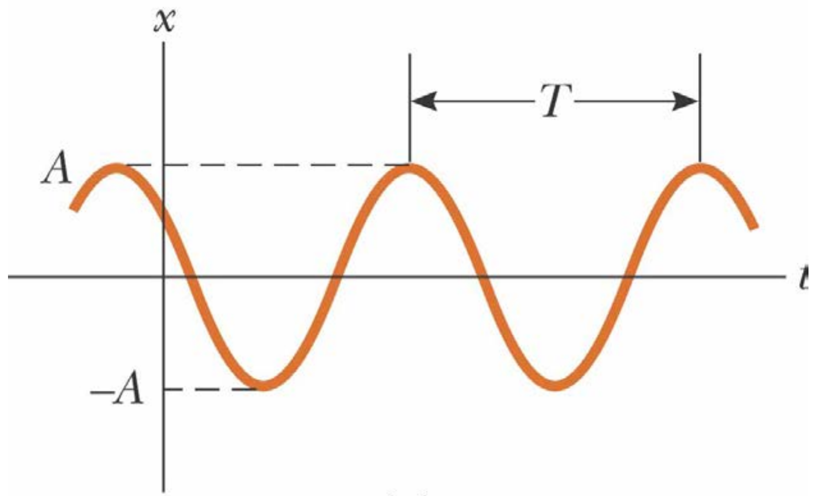
Simple Harmonic Motion – Graphical Representation

- A solution is $x(t) = A \cos(\omega t + \phi)$
  - A, ω, ϕ are all constants
- Differentiate, to yield:

  $v = -\omega A \sin(\omega t + \phi)$

  $a = -\omega^2 A \cos(\omega t + \phi)$
- So that

  $a = -\omega^2 x$ as for SHM



(a)

- Similarly $x(t) = A \sin(\omega t + \phi)$ is also a solution

Simple Harmonic Motion – the constants of motion

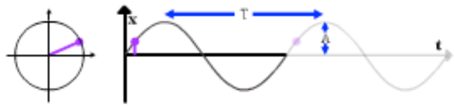
- *A* is the amplitude of the motion
  - This is the maximum position of the particle in either the positive or negative direction
- ω is called the angular frequency
  - Units are radian/s
- ϕ is the phase constant or the initial phase angle

Simple Harmonic Motion, cont

- Since $x(t) = A \cos(\omega t + \phi)$, then A and ϕ are determined uniquely by the position and velocity of the particle at $t = 0$
  - e.g. If the particle is at $x = A$ at $t = 0$, then $\phi = 0$
- The ***phase*** of the motion is $(\omega t + \phi)$
- $x(t)$ is periodic and its value is the same each time ωt increases by 2π radians

PHYSCLIPS 2: Waves and Oscillations
1.2 The Equations

PhysclipsWS > Oscillations > 1.2 The equations



$$ma = F = -kx \quad (1)$$

$$a = \frac{dv}{dt} \qquad a = \text{rate of change of } v$$

$$v = \frac{dx}{dt} \qquad v = \text{rate of change of } x$$

$$a = \frac{d^2x}{dt^2} \quad \text{so} \quad a = \text{rate of change of rate of change of } x$$

$$(1) \rightarrow \quad \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Solve this equation by finding x(t)

Links: Solving differential equations

Solution:

$$x = A\sin(\omega t + \phi) \quad \text{where} \quad \omega = \sqrt{\frac{k}{m}}$$

$$\frac{dx}{dt} = \omega A\cos(\omega t + \phi)$$

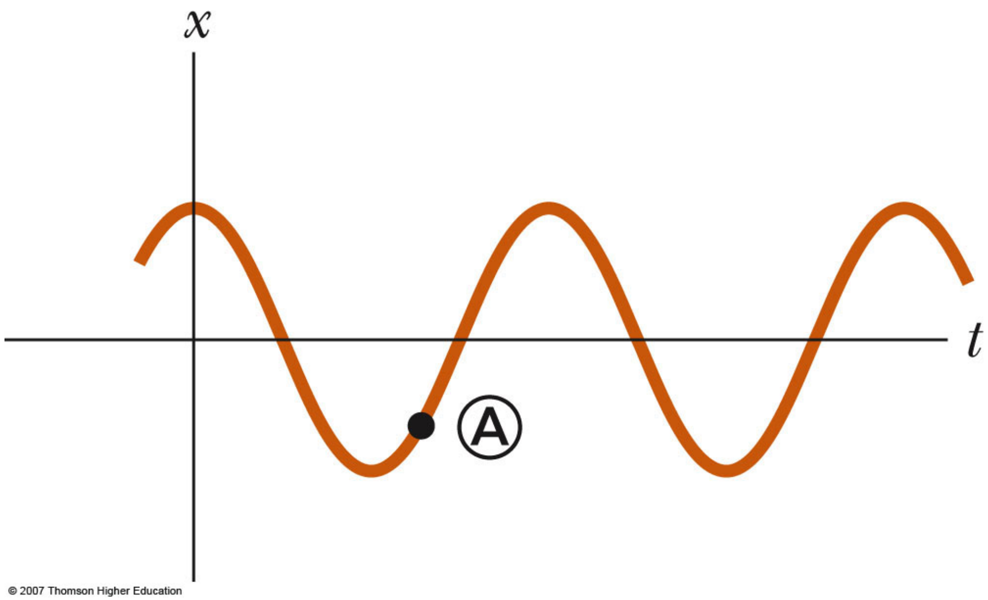
$$\frac{d^2x}{dt^2} = -\omega^2 A\sin(\omega t + \phi)$$

$$= -\frac{k}{m}A\sin(\omega t + \phi) = -\frac{k}{m}x$$

PhysclipsWS

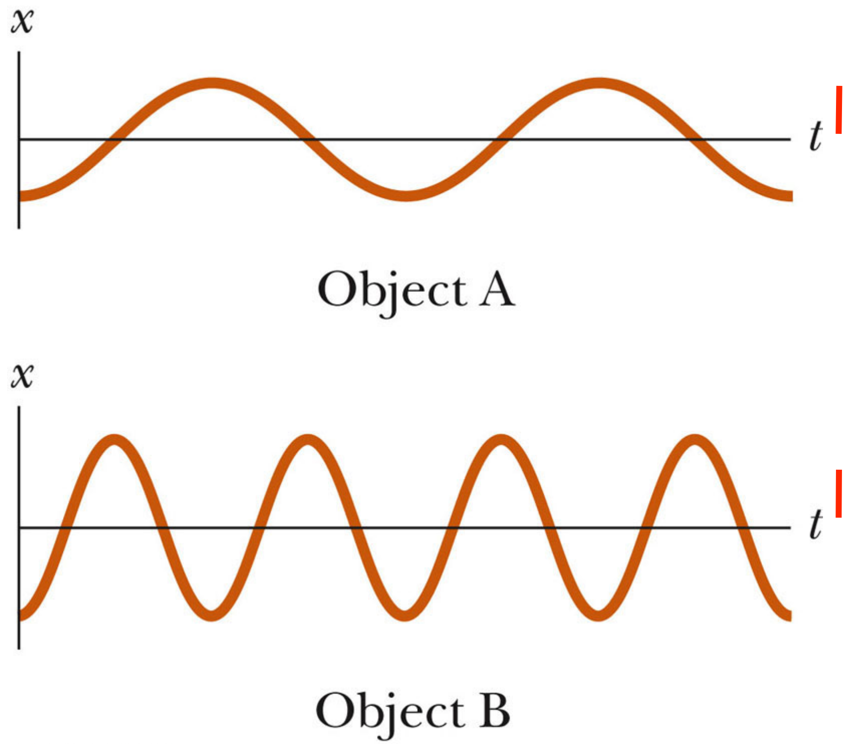
Consider a graphical representation of simple harmonic motion. When the object is at point **A** on the graph, what can you say about its position and velocity?

1. The position and velocity are both positive.
2. The position and velocity are both negative.
3. The position is positive, and its velocity is zero.
4. The position is negative, and its velocity is zero.
5. The position is positive, and its velocity is negative.
6. The position is negative, and its velocity is positive.



The figure shows two curves representing objects undergoing simple harmonic motion. The correct description of these two motions is that the simple harmonic motion of object B is:

1. of larger angular frequency and larger amplitude than that of object A.
2. of larger angular frequency and smaller amplitude than that of object A.
3. of smaller angular frequency and larger amplitude than that of object A.
4. of smaller angular frequency and smaller amplitude than that of object A.



Period and Frequency

- The ***period***, T, is the time interval required for the particle to go through one full cycle of its motion
  - i.e. $x(t)=x(t+T)$ and $v(t)=v(t+T)$ with $T=2\pi/\omega$
- The inverse of the period is called the ***frequency***
  - It represents the number of oscillations the particle undergoes per unit time interval
  - $f = 1/T = \omega/2\pi$
  - Units are cycles per second = hertz (Hz)
- Since $\omega^2 = \dfrac{k}{m}$ then $T = 2\pi\sqrt{\dfrac{m}{k}}$ $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$

Period and Frequency: II

- Frequency and period depend only on the mass of the particle and the force constant of the spring
- *See Active Figure 15.06*
- Frequency is larger for a stiffer spring (large values of *k*) and decreases with increasing mass of the particle

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Summary: Motion Equations for Simple Harmonic Motion

$$x(t) = A\cos(\omega t + \phi)$$

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

$$a = \frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \phi)$$

- Remember, simple harmonic motion is **not** uniformly accelerated motion

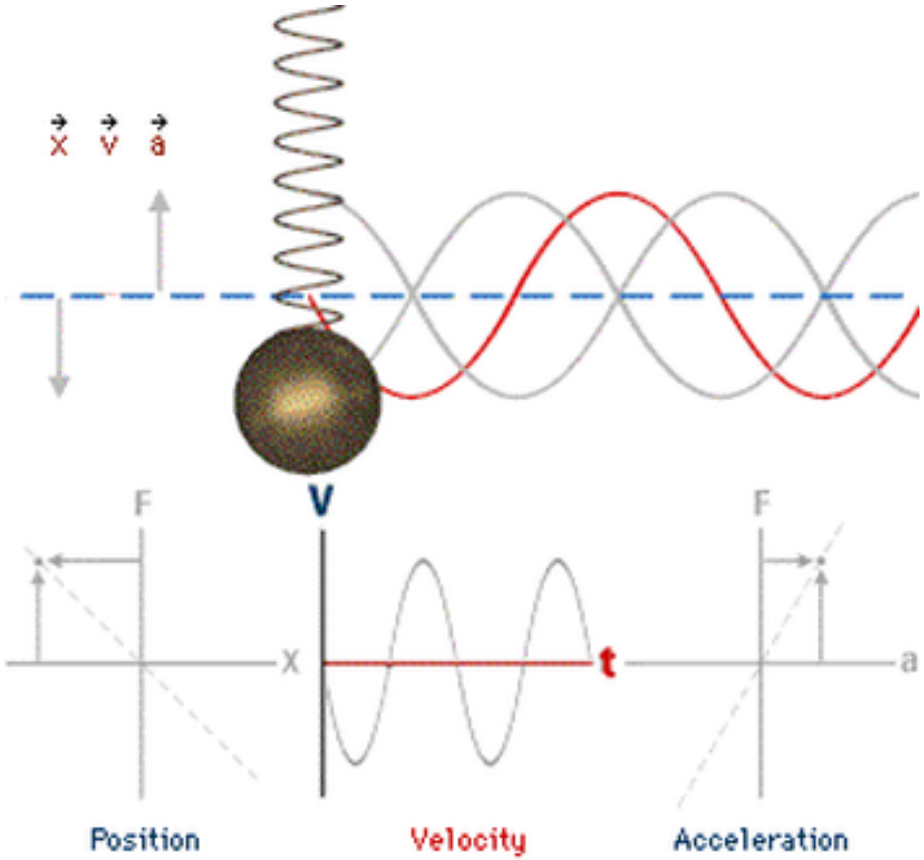
Maximum Values of *v* and *a*

- Because the sine and cosine functions oscillate between ±1, then the maximum values of velocity and acceleration for an object in SHM are

$$v_{\max} = \omega A = \sqrt{\frac{k}{m}} A$$

$$a_{\max} = \omega^2 A = \frac{k}{m} A$$

HMA03AN3: SHM for *x, v, a*

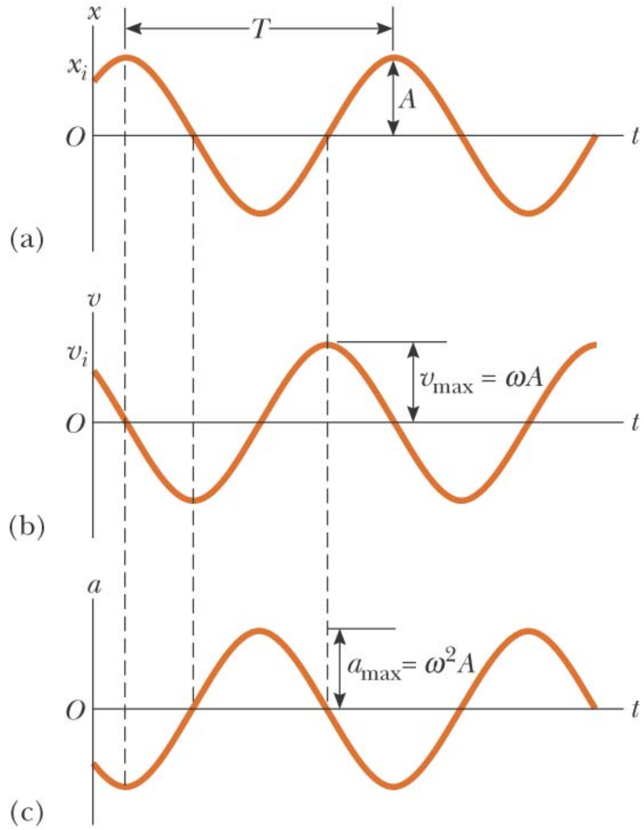


All three together

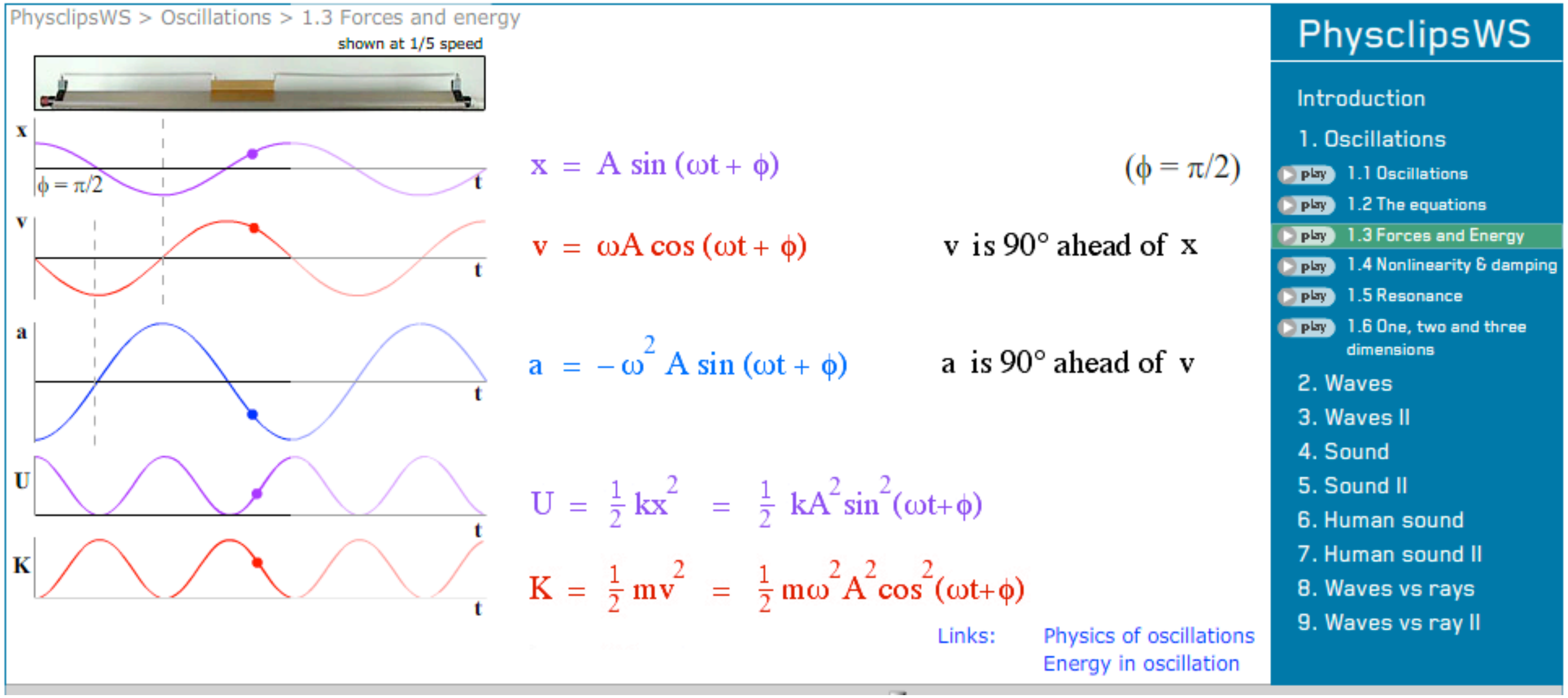
32 seconds

Phase of *x, v, a*

- The graphs show:
  - (a) displacement as a function of time
  - (b) velocity as a function of time
  - (c) acceleration as a function of time
- Velocity is 90° out of phase with the displacement
- Acceleration is 180° out of phase with the displacement



PHYSCLIPS 2: Waves and Oscillations 1.3 Forces & Energy

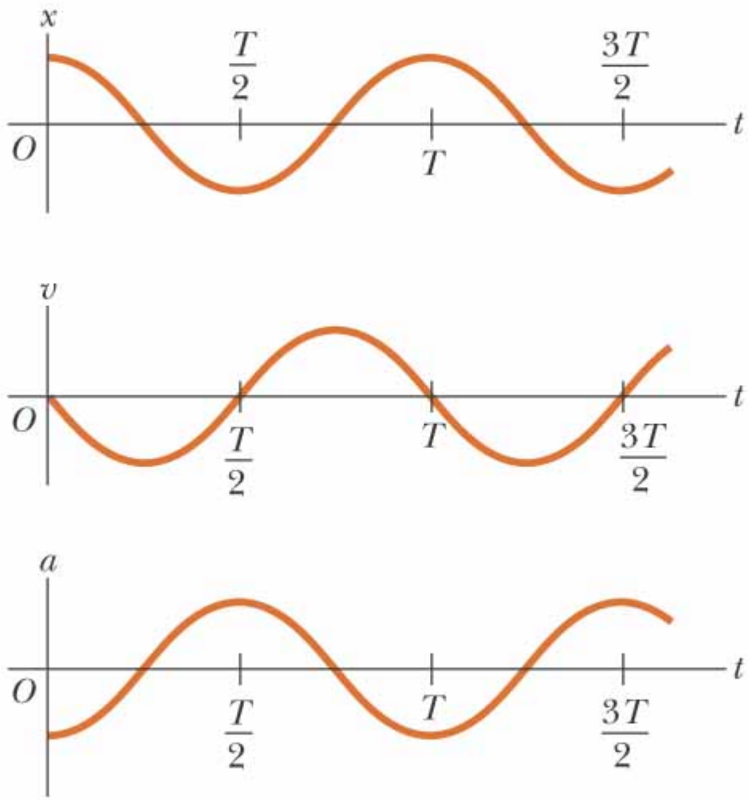


An object of mass *m* is hung from a spring and set into oscillation. The period of the oscillation is measured and recorded as *T*. The object of mass *m* is removed and replaced with an object of mass 2*m*. When this object is set into oscillation, what is the period of the motion?

1. $2T$
2. $\sqrt{2}T$
3. T
4. $T/\sqrt{2}$
5. $T/2$

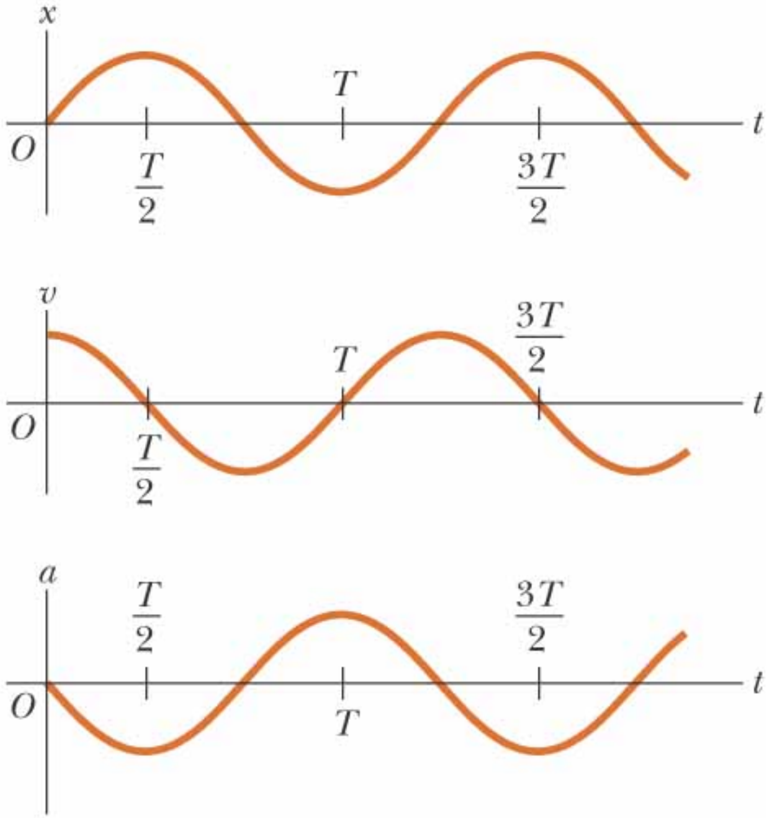
SHM Example 1

- $x(t) = A \cos(\omega t + \phi)$
- Initial conditions at $t = 0$:
  - $x(0) = A$
  - $v(0) = 0$
- Thus $\phi = 0$
- The acceleration reaches extremes of $\pm\, \omega^2 A$
- The velocity reaches extremes of $\pm\, \omega A$



SHM Example 2

- $x(t) = A \cos(\omega t + \phi)$
- Initial conditions at $t = 0$:
  - $x(0)=0$
  - $v(0) = v_{max}$
- Thus $\phi = -\pi/2$
  - i.e. $x(t) = A \sin(\omega t)$
- Graph is shifted one-quarter cycle to the right compared to graph when $x(0) = A$

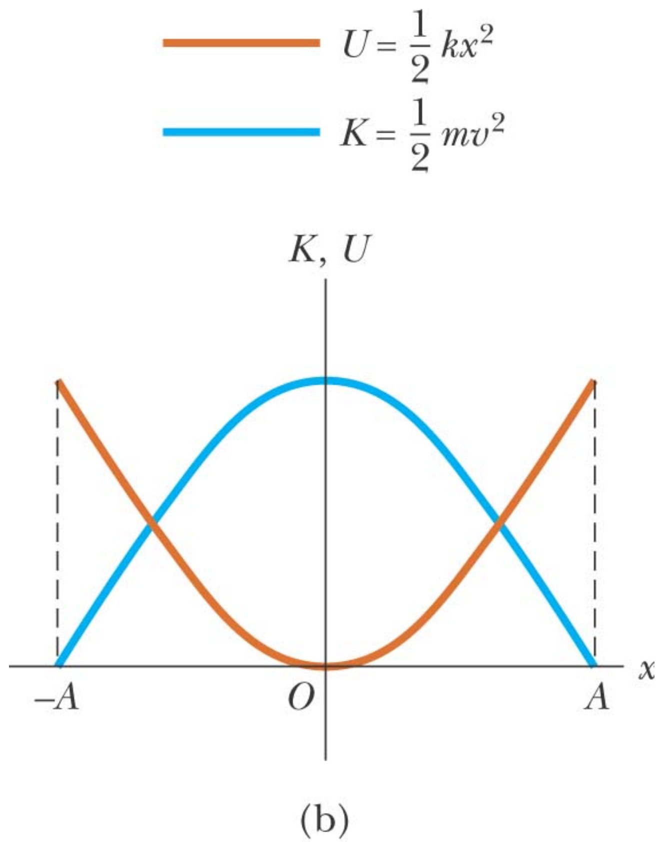


Energy of the SHM Oscillator: I

- Assume a spring-mass system is moving on a frictionless surface
  - This tells us the total energy is constant
- The kinetic energy can be found by
  - $K = \frac{1}{2} mv^2 = \frac{1}{2} m\omega^2 A^2 \sin^2(\omega t + \phi)$
  - i.e. $K = \frac{1}{2} k A^2 \sin^2(\omega t + \phi)$ as $\omega^2 = k/m$
- The elastic potential energy can be found by
  - $U = \int F dx = \int kx dx = \frac{1}{2} kx^2$
  - *i.e.* $U = \frac{1}{2} kx^2 = \frac{1}{2} kA^2 \cos^2(\omega t + \phi)$
- Thus, the total energy is $K + U = \frac{1}{2} kA^2$

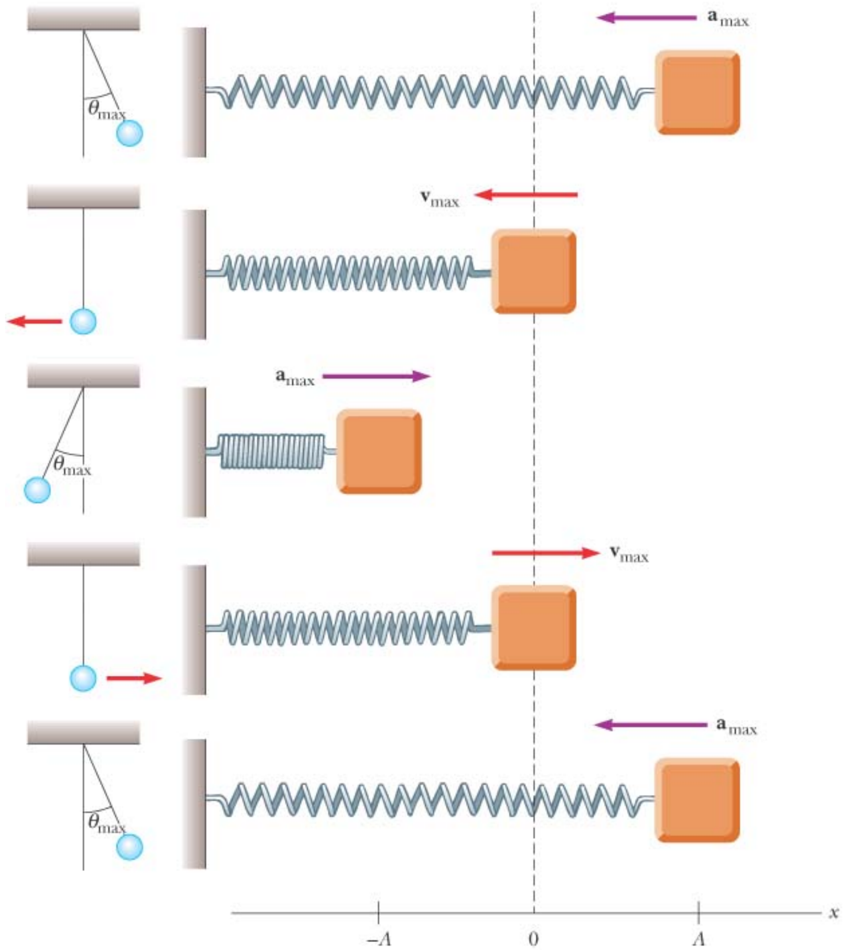
Energy of the SHM Oscillator: II

- *See Active Figure 15.09*
- The total mechanical energy is constant = ½ kA^2
- The total mechanical energy is proportional to the square of the amplitude
- Energy is continuously being transferred between potential energy stored in the spring and the kinetic energy of the block



Summary of Energy in SHM

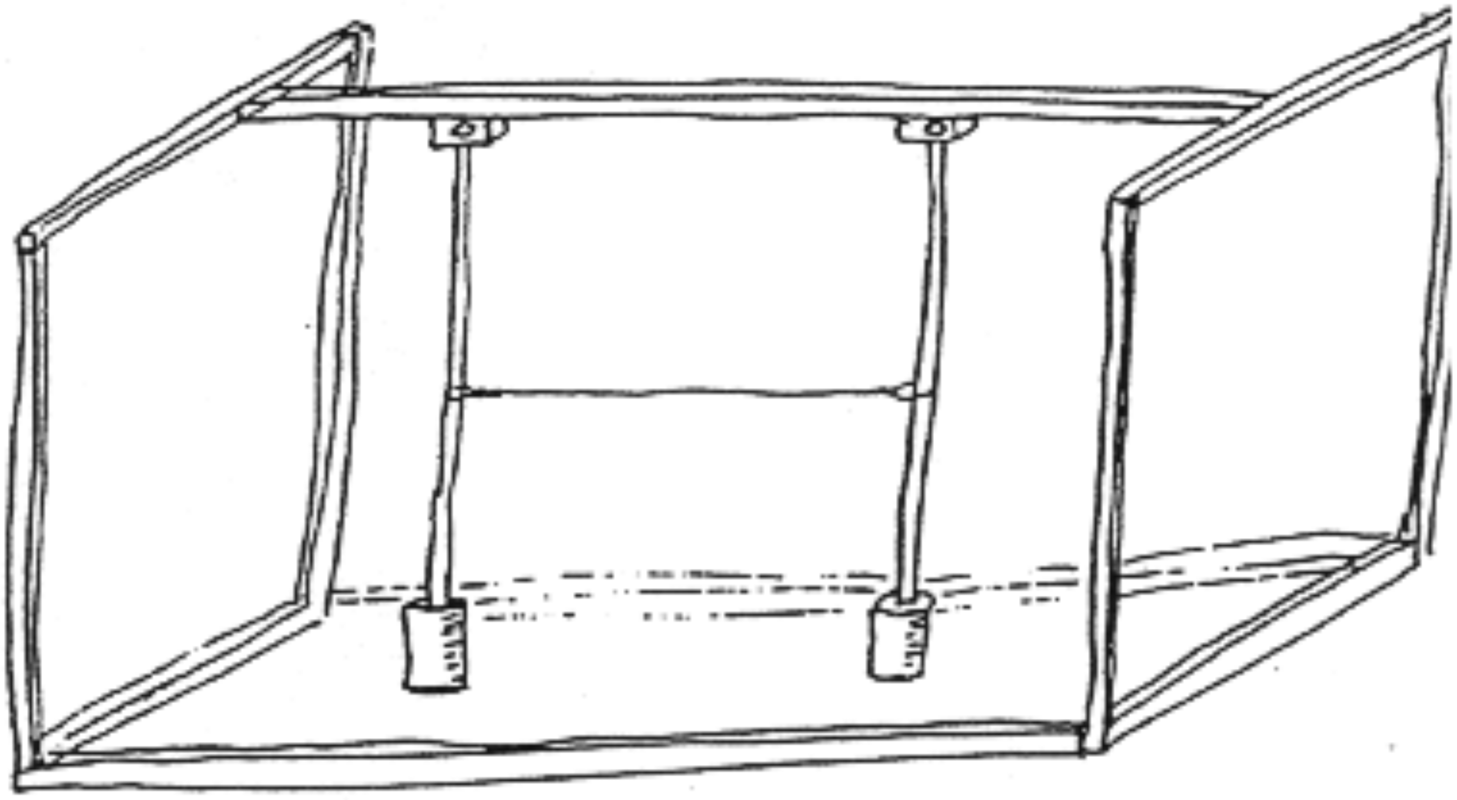
Active Figure 15.10



t	x	v	a	K	U
0	A	0	$-\omega^2 A$	0	$\frac{1}{2}kA^2$
$T/4$	0	$-\omega A$	0	$\frac{1}{2}kA^2$	0
$T/2$	$-A$	0	$\omega^2 A$	0	$\frac{1}{2}kA^2$
$3T/4$	0	ωA	0	$\frac{1}{2}kA^2$	0
T	A	0	$-\omega^2 A$	0	$\frac{1}{2}kA^2$

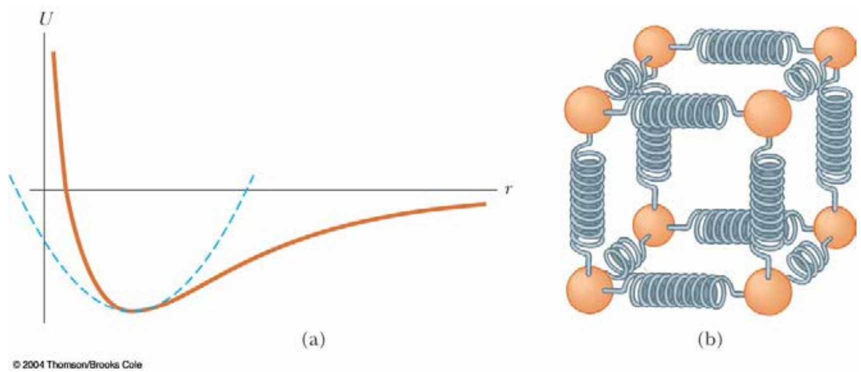
Mh7: Transfer of Energy

Coupled pendula



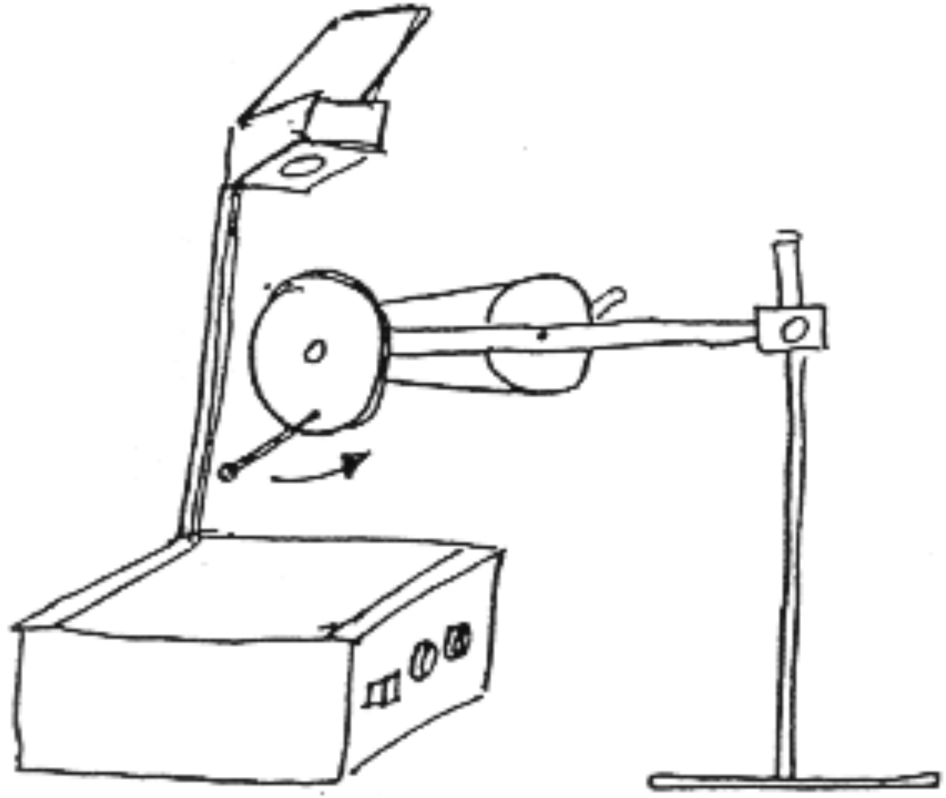
Molecular Model of SHM

- If the atoms in the molecule do not move too far, the force between them can be modeled as if there were springs between the atoms
- The potential energy acts similar to that of the SHM oscillator



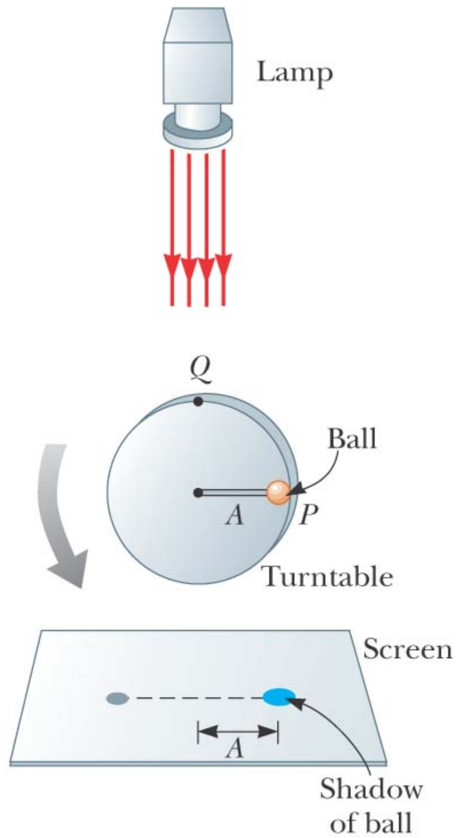
Mh3: Simple Harmonic Motion and Circular Motion

Connection between circular motion and SHM can be seen on the overhead projector



SHM and Circular Motion: I

- *See Active Figure 15.13*
  - An overhead view which shows the relationship between SHM and circular motion
- As the ball rotates with constant angular velocity, its shadow moves back and forth in simple harmonic motion

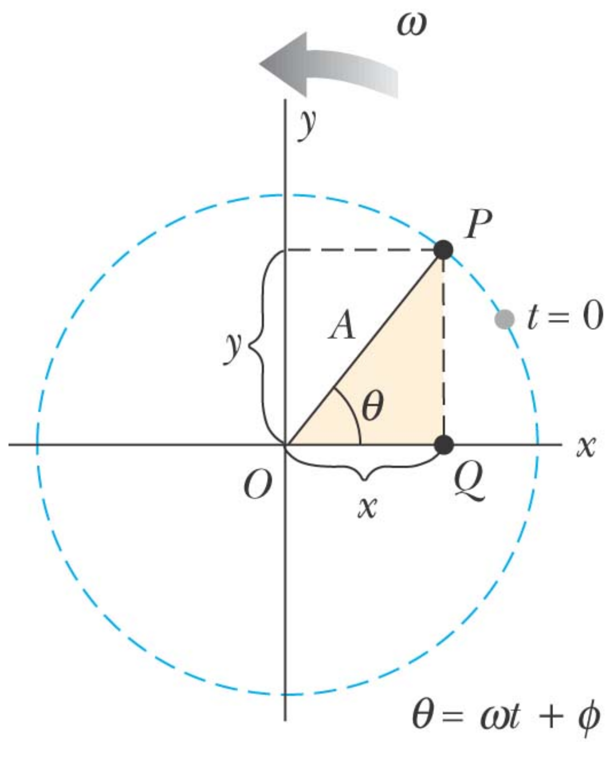


SHM and Circular Motion: II

- The particle moves along the circle with constant angular velocity ω
- OP makes an angle θ with the x axis
- At time t the angle between OP and the x axis will be $\theta = \omega t + \phi$
- So $x(t) = A \cos(\omega t + \phi)$

  and $v = -\omega A \sin(\omega t + \phi)$,

  $a = -\omega^2 A \cos(\omega t + \phi)$
- Thus $a = -\omega^2 x$ i.e. SHM!



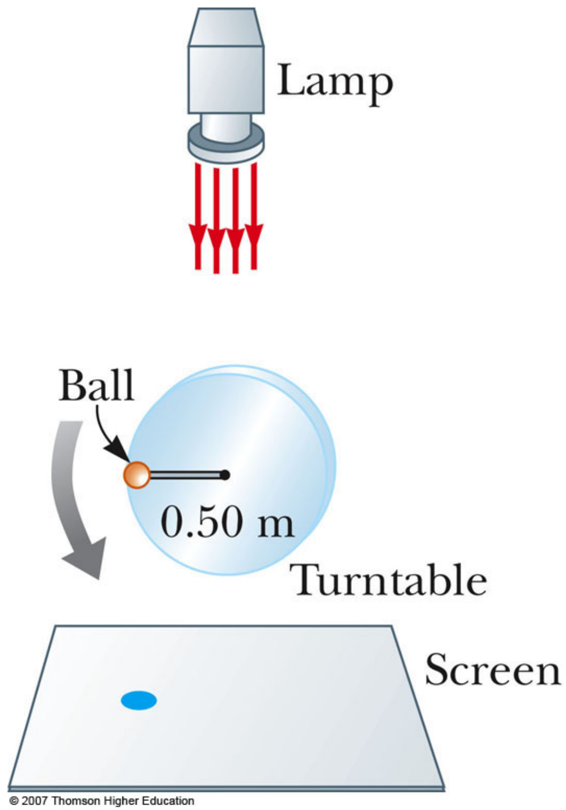
(b)

SHM and Circular Motion: III

- Simple Harmonic Motion along a straight line can be represented by the projection of uniform circular motion along the diameter of a reference circle
- Uniform circular motion can be considered a combination of two simple harmonic motions
  - One along the *x*-axis
  - The other along the *y*-axis
  - The two differ in phase by 90^{o}

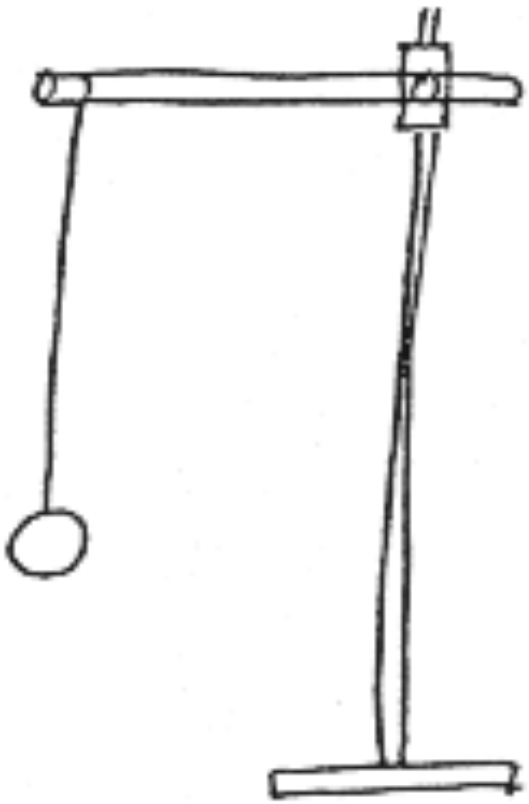
The figure shows the position of an object in uniform circular motion at $t = 0$. A light shines from above and projects a shadow of the object on a screen below the circular motion. What are the correct values for the *amplitude* and *phase constant* (relative to an x axis to the right) of the simple harmonic motion of the shadow?

1. 0.50 m and 0
2. 1.00 m and 0
3. 0.50 m and π
4. 1.00 m and π



Mh5: Simple Harmonic Motion – the simple pendulum

Simple, but a classic!

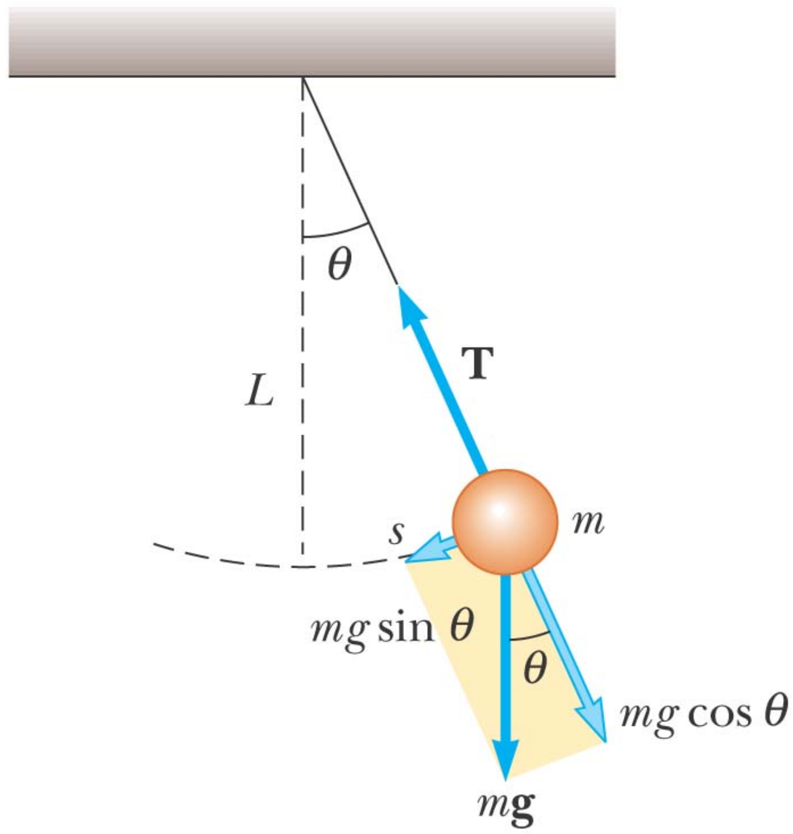


The Simple Pendulum

- *See Active Figure 15.16*
  - Exhibiting periodic motion
- The motion occurs in the vertical plane and is driven by the gravitational force
- The motion is very close to that of the SHM oscillator for small angles

Simple Pendulum II

- The forces acting on the bob are **T** and $m\mathbf{g}$
  - **T** is the force exerted on the bob by the string
  - $m\mathbf{g}$ is the gravitational force
- The tangential component of the gravitational force is a restoring force
  - This is $-mg\sin(\theta)$
- The arc length is $s = L\,\theta$
  - So $\dfrac{ds}{dt} = L\dfrac{d\theta}{dt}$ and $\dfrac{d^2s}{dt^2} = L\dfrac{d^2\theta}{dt^2}$



Simple Pendulum III

- In the tangential direction,

$$F_t = -mg\sin\theta = m\frac{d^2s}{dt^2} = mL\frac{d^2\theta}{dt^2}$$

- The length, *L*, of the pendulum is constant, and so, for small values of θ

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta = -\frac{g}{L}\theta$$

- Thus the motion is SHM with $\omega^2 = g/L$

Simple Pendulum IV

- The function θ can be written as

  $\theta = \theta_{max} \cos(\omega t + \phi)$

- The angular frequency is

$$\omega = \sqrt{\frac{g}{L}}$$

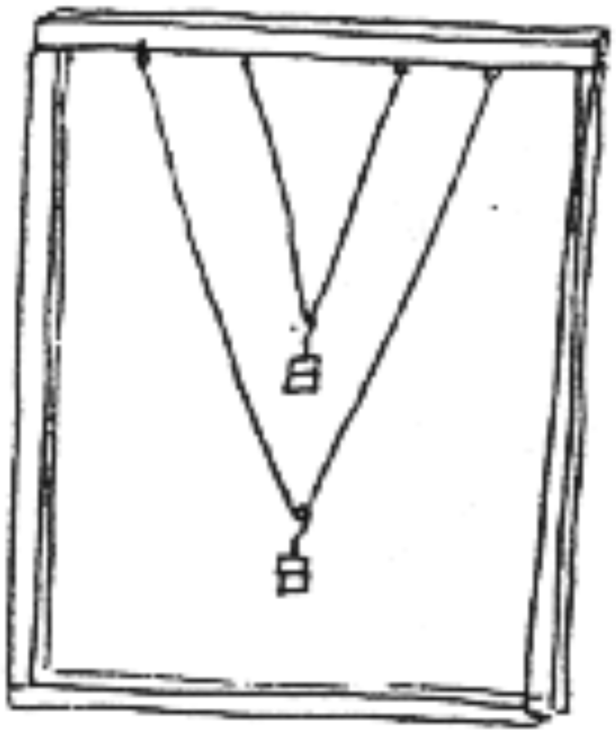
- The period is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

Mh12: The Pendulum – period and length

Half class time top pendulum, half class time bottom pendulum.

Is $T \propto \sqrt{L}$?

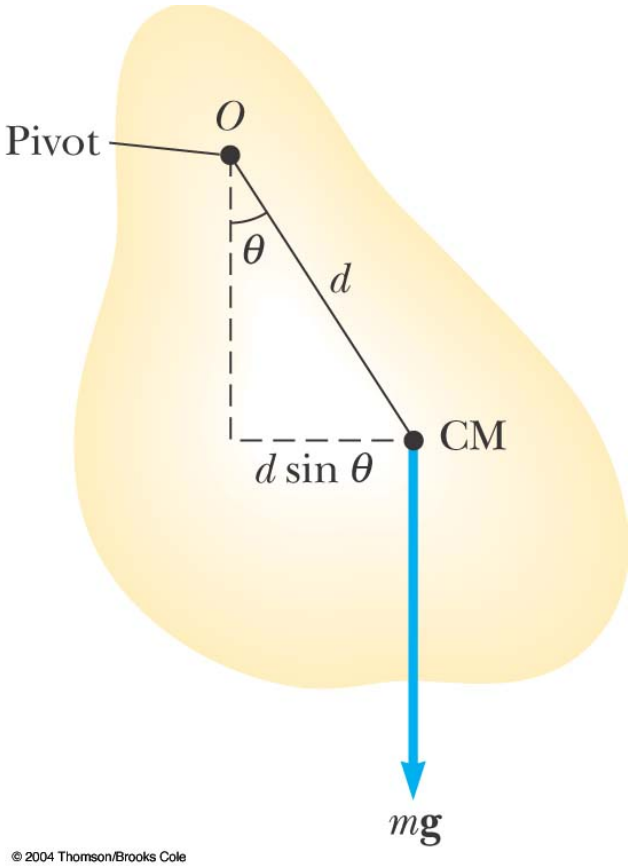


Physical Pendulum

- The gravitational force provides a torque about an axis through *O*
- The magnitude of the torque is $mgd \sin\theta$
- I is the moment of inertia about the axis through *O*
- We obtain Period $T = 2\pi\sqrt{\frac{I}{mgd}}$
- Also, make use of the Parallel Axes-theorem for the moment inertia about an axis *D* away from the Centre of Mass:

$$I = I_{CM} + MD^2$$

[See Halliday, Resnick & Walker Ch 10.5]



A grandfather clock depends on the period of a pendulum to keep correct time. Suppose a grandfather clock is calibrated correctly and then a mischievous child slides the bob of the pendulum downward on the oscillating rod. The grandfather clock will run:

1. slow.
2. fast.
3. correctly.

A grandfather clock depends on the period of a pendulum to keep correct time. Suppose a grandfather clock is calibrated correctly at sea level and is then taken to the top of a very tall mountain. The grandfather clock will now run:

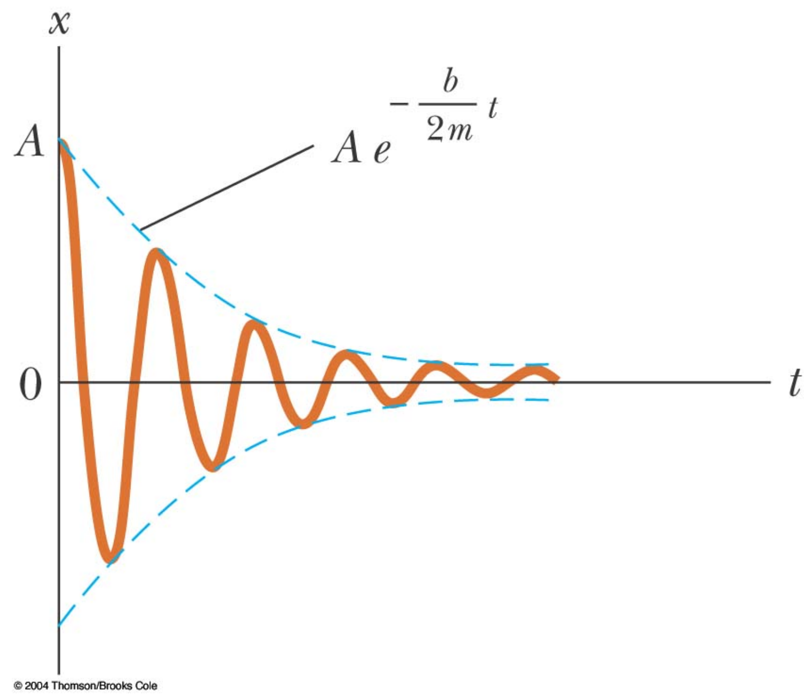
1. slow.
2. fast.
3. correctly.

Damped Oscillations: I

- *See Active Figure 15.21*
- In many real systems, friction is significant.
- In this case, the mechanical energy of the system diminishes in time, the motion is said to be ***damped***

Damped Oscillations: II

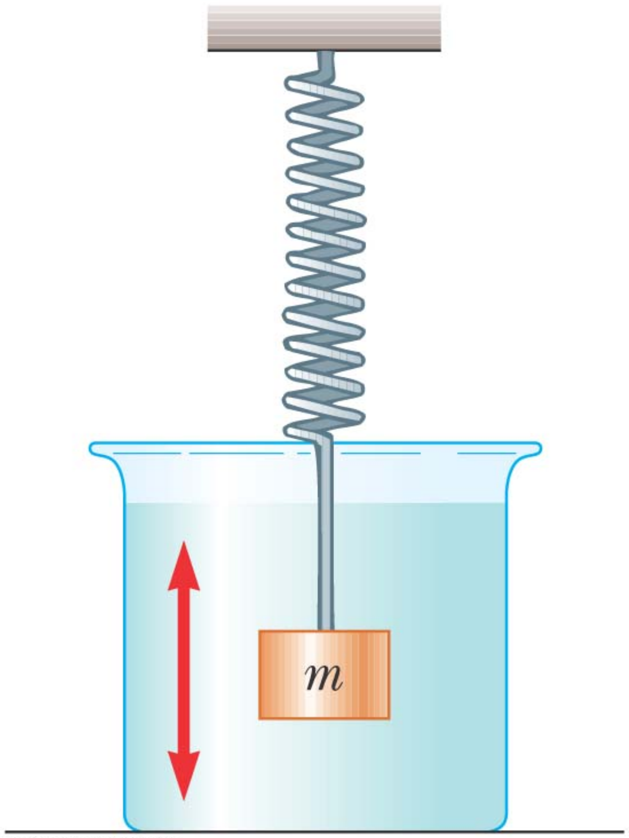
- A graph for a damped oscillation
- The amplitude decreases with time
- The blue dashed lines represent the ***envelope*** of the motion



Damped Oscillation, Example

- One example of damped motion occurs when an object is attached to a spring and submerged in a viscous liquid
- The retarding force can be expressed as $\mathbf{R} = - b\,\mathbf{v}$ where b is a positive constant
  - b is called the ***damping coefficient***
- From Newton's Second Law the equation of motion becomes

$$F_x = -k\,x - bv_x = ma_x$$



$$\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$$

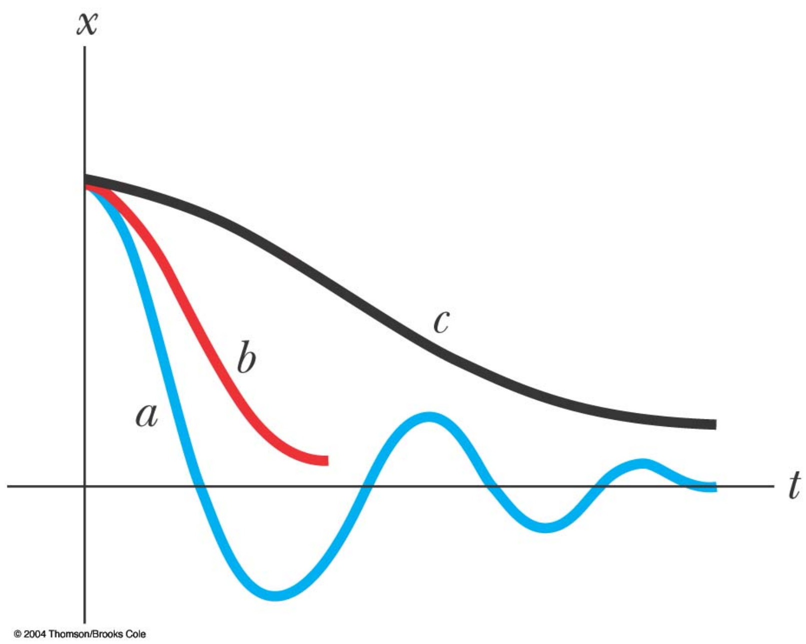
Types of Damping

- $\omega_0 = \sqrt{\frac{k}{m}}$ is also called the ***natural frequency*** of the system
- If $R_{max} = bv_{max} < kA$, the system is said to be ***underdamped***

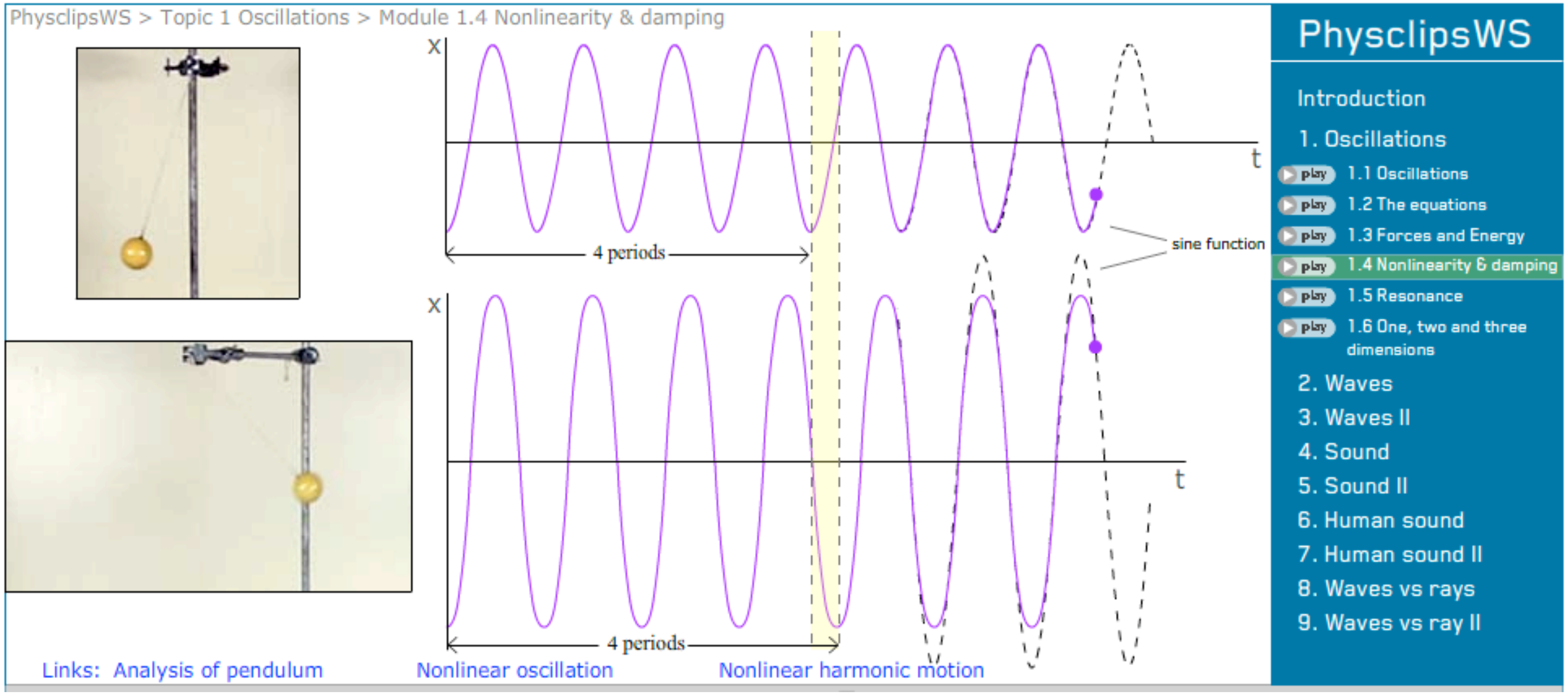
  [*kA* is the maximum restoring force for the spring]
- When b reaches a critical value b_c such that $b_c / 2\text{ m} = \omega_0$, the system will not oscillate
  - The system is said to be ***critically damped***
- If $R_{max} = bv_{max} > kA$ and $b/2m > \omega_0$, the system is said to be ***overdamped***

Types of Damping: II

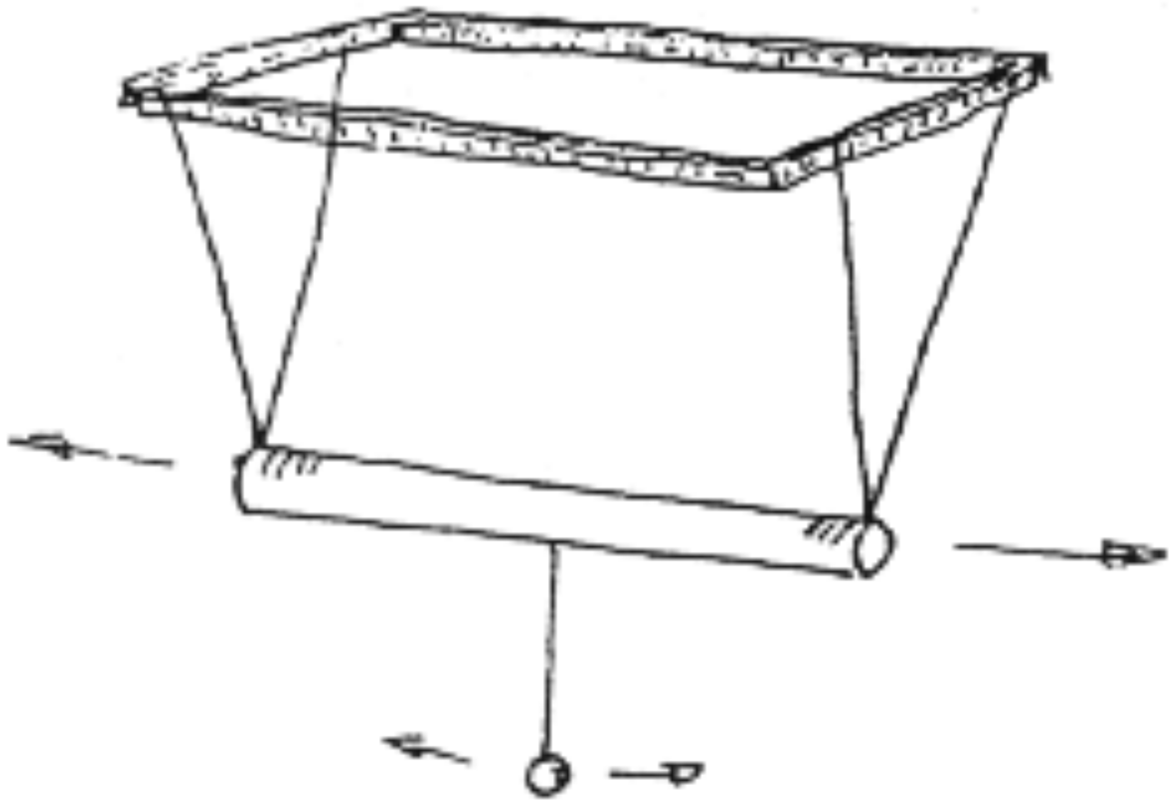
- Graphs of position versus time for
  - (a) an underdamped oscillator
  - (b) a critically damped oscillator
  - (c) an overdamped oscillator
- For critically damped and overdamped there is no angular frequency



PHYSCLIPS 2: Sound and Waves 1.4 Non-linearity and Damping



Mh13: Double Pendulum – Forced Oscillations



Forced Oscillations: I

- It is possible to compensate for the loss of energy in a damped system by applying an external force
  - e.g. pushing child on a swing at angular frequency ω
  - In general $F = F_0 \sin(\omega t) - bv - kx$
- The amplitude of the motion remains constant if the energy input per cycle exactly equals the decrease in mechanical energy in each cycle that results from resistive forces

Forced Oscillations: II

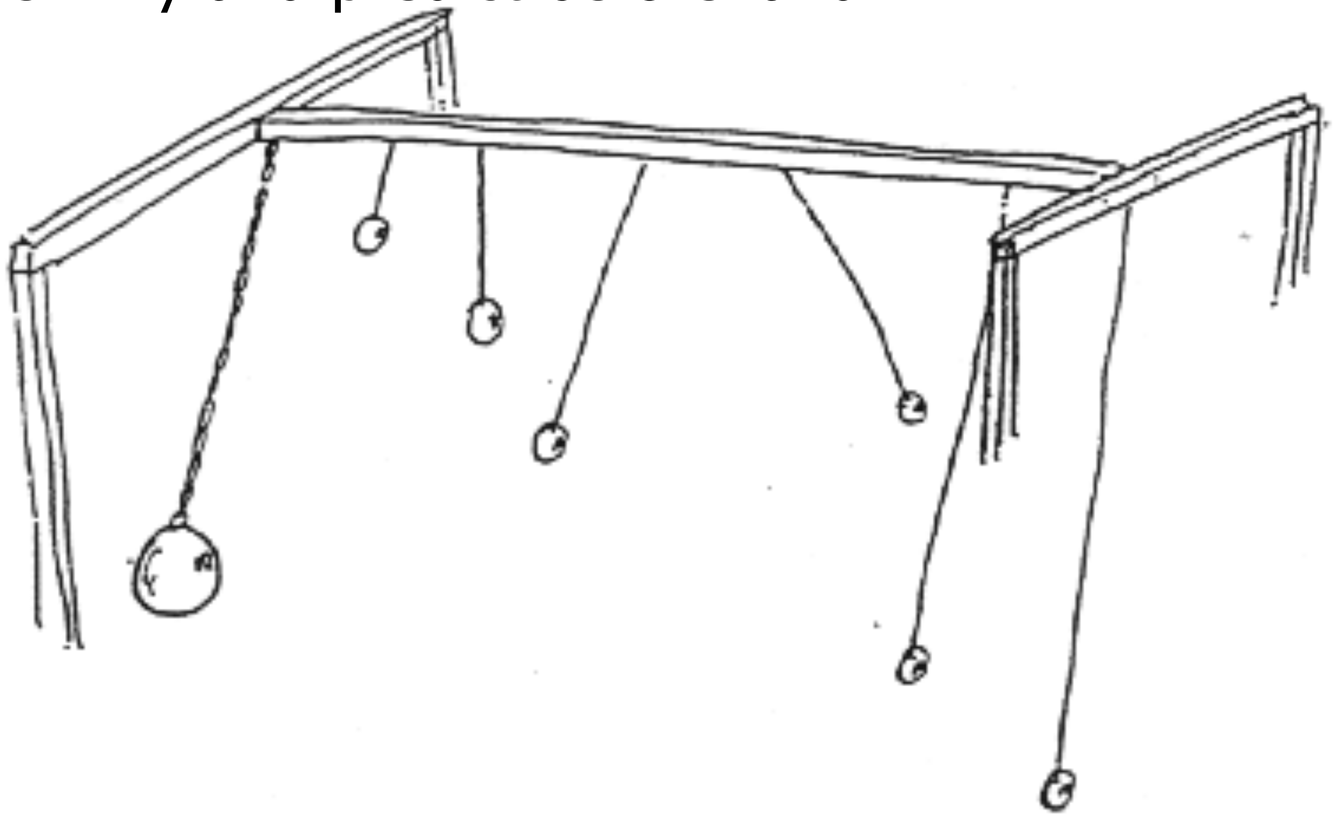
- Motion is $x(t) = A\cos(\omega t + \phi)$ where the amplitude of the driven oscillation is

$$A = \frac{F_0/m}{\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + \left(\frac{b\omega}{m}\right)^2}}$$

  - ω_0 is the natural frequency of the undamped oscillator

Mh2: Resonance Pendulum

Large ball will cause all balls to oscillate but only the ball of the same natural frequency will oscillate at large amplitude. Try and predict beforehand.

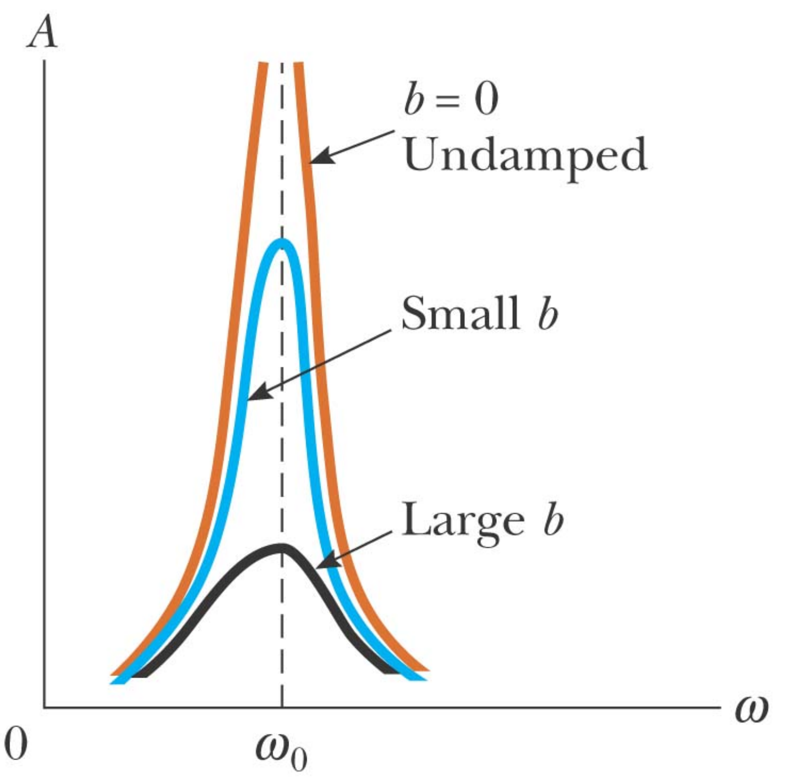


Resonance: I

- When the frequency of the driving force is near the natural frequency ($\omega \approx \omega_0$) an increase in amplitude occurs
- This dramatic increase in the amplitude is called ***resonance***
- The natural frequency ω_0 is also called the resonance frequency of the system

Resonance: II

- Resonance (maximum peak) occurs when the driving frequency equals the natural frequency
- The amplitude increases with decreased damping
- The curve broadens as the damping increases
- The shape of the resonance curve depends on *b*



PHYSCLIPS 2: Waves and Sound 1.5 Resonance

PhysclipsWS > Oscillations > 1.5 Resonance

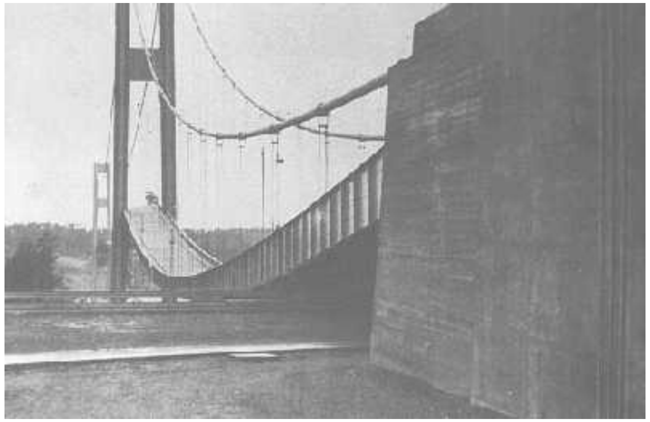
Links: Forced oscillation & resonance

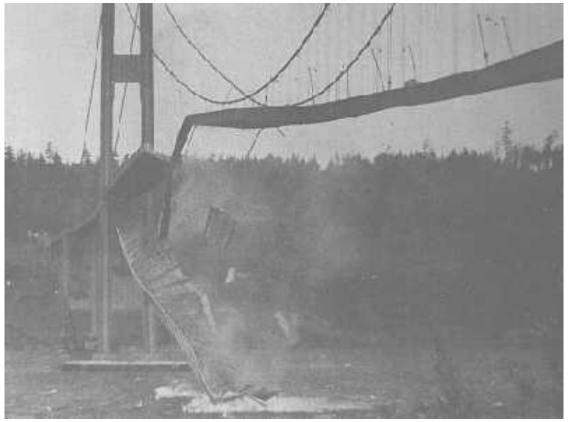
PhysclipsWS

Introduction

1. Oscillations
   - play 1.1 Oscillations
   - play 1.2 The equations
   - play 1.3 Forces and Energy
   - play 1.4 Nonlinearity & damping
   - play 1.5 Resonance
   - play 1.6 One, two and three dimensions
2. Waves
3. Waves II
4. Sound
5. Sound II
6. Human sound
7. Human sound II
8. Waves vs rays
9. Waves vs ray II

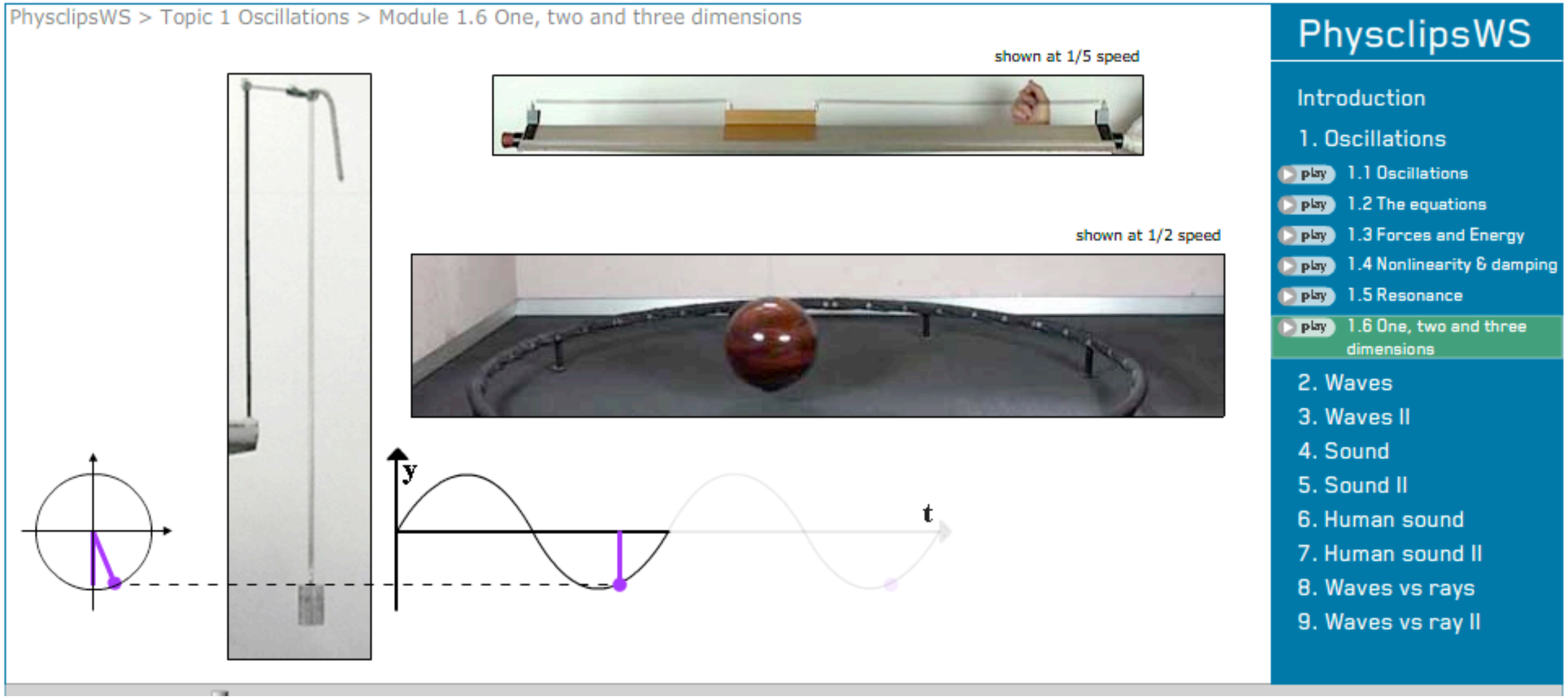
Resonance: Tacoma Narrows Bridge





November 7, 1940, Washington State, USA

PHYSCLIPS : Waves and Sound
1.6 1+2+3 Dimensions



Ranjit Doroszkeiwicz/Alamy

We began this chapter with an image of a pendulum that is part of the damping system for Taipei 101, and asked what the purpose of it is and how it works. The damping system of Taipei 101 is designed specifically to prevent oscillations of the building, a tower over 500 m tall, from becoming too large. Strong winds could cause the tower to oscillate, as in the case of the Tacoma Narrows Bridge.

Taipei is also subject to earthquakes, the shaking from which is periodic in nature, as we shall see in the next chapter. It is possible that a building may be shaken at its resonant frequency by an earthquake, setting up extremely large vibrations in the building and doing a great deal of damage. Hence the need for the damping system. The gigantic pendulum oscillates in the opposite direction to the building, damping any vibrations due to wind or movement of the ground, and so preventing the building itself from oscillating substantially. This is similar to the damping you did with your legs on the swing if you did the *Try this* example.