TMA4100, Innlevering 3

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oppg 1

a) $f(x) = \sin(x)$ rundt $x = a = 0$

$f(x) = \sin(x)$	$f(0) = 0$
$f'(x) = \cos(x)$	$f'(0) = 1$
$f''(x) = -\sin(x)$	$f''(0) = 0$
$f'''(x) = -\cos(x)$	$f'''(0) = -1$
$f^{(4)}(x) = \sin(x)$	$f^{(4)}(0) = 0$
$f^{(5)}(x) = \cos(x)$	$f^{(5)}(0) = 1$

$$P_1(x) = f(0) + f'(0)x = x$$

$$P_3(x) = P_1 + \frac{f'''(0)}{3!}x^3 = x - \frac{1}{6}x^3$$

$$P_5(x) = P_3 + \frac{f^{(5)}(0)}{5!}x^5 = x - \frac{1}{6}x^3 + \frac{1}{120}x^5$$

~~b) Observer at $P_5(x) = P_6(x)$ fordi $f^{(6)}(0) = 0$~~

$$\cancel{P_6(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5}$$

~~Feilen da blir ifølge Taylors formel~~

$$\cancel{E(x) = \frac{f^{(7)}(s)}{7!}}$$

b) $$|\sin\pi - P_n(\pi)| = |E_n(\pi)| \leq \frac{1}{1000}$$

$$|E_n(\pi)| = \frac{|f^{(n+1)}(s)|}{(n+1)!}\pi^{n+1} \quad , 0 < s < \pi$$

fordi $f(x) = \sin(x)$ vil $f^{(n+1)}(x)$ enten være en cosinus eller sinus-funksjon.
$|f^{(n+1)}(s)|$ vil derfor aldri bli høyere enn 1.

$$\Rightarrow |E_n(\pi)| = \frac{|f^{(n+1)}(s)|}{(n+1)!}\pi^{n+1} \leq \frac{1}{(n+1)!}\pi^{n+1}$$

$$\frac{1}{1000} \leq \frac{1}{(n+1)!}\pi^{n+1}$$

$$\frac{(n+1)!}{\pi^{n+1}} \geq 1000$$

$\Rightarrow$ Når $n \geq 12$ vil ulikheten over stemme feilen vil garantert være mindre enn $1/1000$, men vet at $P_{12}(x) = P_{11}(x)$ så et Taylor polynom av 11. grad vil faktisk være nok.

$$\Rightarrow \underline{\underline{n \geq 11}}$$

oppg 2

a) $\int_0^1 x^2 e^x \, dx$

Bruker delvis integrasjon

$u = x^2 \rightarrow u' = 2x$
$v' = e^x \rightarrow v = e^x$

$\int_0^1 x^2 e^x dx = x^2 \cdot e^x - \int_0^1 2x e^x \, dx$

Bruker delvis integrasjon igjen

$\int_0^1 2xe^x dx$: $u = x \rightarrow u' = 1$
$v' = e^x \rightarrow v = e^x$

$\int 2xe^x dx = 2 \cdot \int xe^x dx = 2xe^x - 2\int e^x dx$

$= 2xe^x - 2e^x + C$

$\Rightarrow \int_0^1 x^2 e^x dx = (x^2 e^x - 2xe^x + 2e^x + C) \Big|_0^1$

$= e^x(x^2 - 2x + 2 + C) \Big|_0^1$

$= e(1 - 2 + 2 + C) - e^0(+2 + C)$

$\int_0^1 x^2 e^x dx = e - 2$

b) $\int_0^{\frac{\pi}{2}} \sin^5(x)\,dx$

$$\sin^5(x) = \sin^4(x)\sin(x)$$

$$= [1-\cos^2(x)]^2 \sin(x)$$

$$= [1-2\cos^2(x)+\cos^4(x)]\sin(x)$$

La $u = \cos(x) \Rightarrow \frac{du}{dx} = -\sin(x)$

$$\Leftrightarrow dx = -\frac{du}{\sin(x)}$$

$$\int \sin^5(x)\,dx = \int [1-2\cos^2(x)+\cos^4(x)]\sin(x)\,dx$$

$$= \int [1-2u^2+u^4]\cancel{\sin x}\cdot\frac{-du}{\cancel{\sin x}}$$

$$= \int (-u^4+2u^2-1)\,du$$

$$= -\frac{1}{5}u^5+\frac{2}{3}u^3-u+C$$

$$= -\frac{1}{5}\cos^5(x)+\frac{2}{3}\cos^3(x)-\cos(x)+C$$

$$\int_0^{\frac{\pi}{2}} \sin^5(x)\,dx = \left[-\frac{1}{5}\cos^5 x+\frac{2}{3}\cos^3(x)-\cos(x)\right]\Big|_0^{\pi/2}$$

$$= \left(-\frac{1}{5}+\frac{2}{3}-1\right)-(0) \quad = -\frac{1}{5}+\frac{2}{3}-1$$

$$= -\frac{3-10+15}{15} = \underline{\underline{\frac{8}{15}}}$$

oppg 3

$$f(x) = \frac{x^3}{1-x^2}$$

a) f er strengt voksende når $f' > 0$

— 11 — synkende når $f' < 0$

$$f'(x) = \frac{(1-x^2)3x^2 - (-2x)x^3}{(1-x^2)^2}$$

$$= \frac{3x^2 - 3x^4 + 2x^4}{(1-x^2)^2} = \frac{x^2(3-x^2)}{(1-x^2)^2}$$

1: $x^2 = 0 : x = 0$

2: $3 - x^2 = 0 : |x| = \sqrt{3}$

3: $1 - x^2 = 0 : |x| = 1$

	$-\sqrt{3}$ -1 0 1 $\sqrt{3}$
x^2	+ + + + + + + + + 0 + + + + + + + + + +
$3-x^2$	− − 0 + + + + + + + + + + + 0 − −
$(1-x^2)^2$	+ + + + + 0 + + + + + 0 + + + + +
$f'(x)$	− − 0 + + udef + + 0 + + udef + + + 0 − −

f voksende på $x \in (\quad , -1) \cup (-1, 0) \cup (0, 1) \cup (1, \sqrt{3})$

f synkende på $x \in (-\infty, -\sqrt{3}) \cup (\sqrt{3}, +\infty)$

b) $f(x)+x = \dfrac{x^3}{1-x^2} + x$

$$= x\left(\frac{x^2}{1-x^2} + 1\right)$$

$$= x\left(\frac{x^2}{1-x^2} + \frac{1-x^2}{1-x^2}\right)$$

$$= x\left(\frac{x^2+1-x^2}{1-x^2}\right)$$

$$= \frac{x}{1-x^2}$$

$$\lim_{x\to\pm\infty}\left(\frac{x}{1-x^2}\right) = 0 \Rightarrow \lim_{x\to\pm\infty}\left(f(x)+x\right) = 0$$

c) Vertikale asymptoter hvor f' er udefineret.
Altså: $x=-1$ og $x=1$

$$f(x) = x^3 : (1-x^2) = x^3 : (-x^2+1) = -x + \frac{x}{1-x^2}$$

$$\frac{x^3}{0-x}$$

når $x \to \pm\infty$ vil $\dfrac{x}{1-x^2} \to 0$

$\Rightarrow f(x) \to -x$

$\Rightarrow$ Diagonal asymptote er: $y=-x$

d) $f'(x) = 0 \Rightarrow \quad x_1 = -\sqrt{3} \approx -1{,}7 \quad , \quad f(-\sqrt{3}) \approx 2{,}6$

$x_2 = 0 \quad , \quad f(0) \approx 0$

$x_3 = \sqrt{3} \approx 1{,}7 \quad , \quad f(\sqrt{3}) \approx -2{,}6$