Øving 5

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Oppgave 1

a) A – 03 D – 01

B – 02 E – 05

C – 04 F – 06

b) Siden t_1 og t_2 skjer ved topper kan vi si at

$$\frac{|h(t_1)|}{|h(t_2)|} = \frac{|0{,}8|}{|-0{,}25|} = \frac{0{,}8}{0{,}25}$$

$$= \frac{e^{-\alpha t_1}}{e^{-\alpha t_2}} = e^{\alpha(t_2 - t_1)} = e^{\alpha \frac{\pi}{\beta}} \quad (*)$$

Ønsker å finne $\zeta = \sin\varphi = \sin\left(\tan^{-1}\left(\frac{\alpha}{\beta}\right)\right)$

$$(*) \Rightarrow \frac{\alpha}{\beta}\pi = \ln\left(\frac{0{,}8}{0{,}25}\right)$$

$$\Rightarrow \zeta = \sin\left(\tan^{-1}\left(\frac{\ln(0{,}8)/\ln(0{,}25)}{\pi}\right)\right)$$

$$= 0{,}35 \qquad \underline{\underline{B}}$$

Oppgave 2

a)
$$\dot{x}_1 = \frac{1}{m_1 C_1}\left(u_1 - g_1(x_1 - x_2)\right)$$

$$\dot{x}_2 = \frac{1}{m_2 C_2}\left(g_1(x_1 - x_2) + q\rho c_2 v_1 - q\rho c_2 x_2 - g_2(x_2 - v_2)\right)$$

Hvis vi ser bort fra dynamikken i varmeelementet betyr det at

$$\dot{x}_1 = 0$$

$$\Rightarrow \quad u_1 = g_1(x_1 - x_2)$$

$$\Leftrightarrow \quad x_1 = x_2 + \frac{u_1}{g_1}$$

Det gir

$$\dot{x}_2 = \frac{1}{m_2 C_2}\left(u_1 + q\rho c_2(v_1 - x_2) - g_2(x_2 - v_2)\right)$$

$$= \underbrace{-\frac{1}{m_2 C_2}(q\rho c_2 + g_2)}_{= -\frac{1}{T}} x_2 + \frac{1}{m_2 C_2} u_1 + \frac{1}{m_2 C_2} q\rho c_2 v_1 + \frac{g_2}{m_2 C_2} v_2$$

$$\Rightarrow \quad \underline{\underline{T = \frac{m_2 C_2}{q\rho c_2 + g_2}}}$$

b) Finner K ved å ignorere forstyrrelsene og se på steady state:

$$\Rightarrow \quad 0 = -\frac{1}{\cancel{m_2 c_2}}\left(q\rho c_2 + g_2\right) + \frac{1}{\cancel{m_2 c_2}} u_1$$

$$\Leftrightarrow \quad x_2 = \frac{1}{q\rho c_2 + g_2} u_1 = K u_1$$

$$\Rightarrow \quad \underline{\underline{K = \frac{1}{q\rho c_2 + g_2}}}$$

c) Forstyrrelsesleddene fra forrige side er $\frac{v}{m_2 c_2}$ så

$$V = \cancel{m_2 c_2} \cdot \left(\frac{q\rho c_2 v_1}{\cancel{m_2 c_2}} + \frac{g_2 v_2}{\cancel{m_2 c_2}}\right)$$

$$= q\rho c_2 v_1 + g_2 v_2$$

Oppgave 3

a) $$k_1(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}\cdot h_1(s)\right\}$$
$$= \mathcal{L}^{-1}\left\{\frac{e^{-\tau s}}{s}\right\}$$
$$= \mu_1(t-\tau)$$

b) $$k_2(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}\, h_2(s)\right\}$$

$$\frac{1-\frac{\tau}{2}s+\frac{\tau^2}{8}s^2}{s\left(1+\frac{\tau}{2}s+\frac{\tau^2}{8}s^2\right)} = \frac{A}{s} + \frac{Bs+C}{1+\frac{\tau}{2}s+\frac{\tau^2}{8}s^2}$$

$$\Rightarrow 1-\frac{\tau}{2}s+\frac{\tau^2}{8}s^2 = A\left(1+\frac{\tau}{2}s+\frac{\tau^2}{8}s^2\right)+Bs^2+Cs$$

$$\Rightarrow s^2: \quad \frac{\tau^2}{8} = \frac{\tau^2}{8}A+B$$

$$s^1: \quad -\frac{\tau}{2} = A\frac{\tau}{2}+C$$

$$s^0: \quad 1 = A$$

$$\Rightarrow A=1,\ B=0,\ C=-\tau$$

$$\Rightarrow \frac{1}{s} h_2(s) = \frac{1}{s} - \frac{\tau}{1 + \frac{\tau}{2}s + \frac{\tau^2}{8}s^2}$$

$$= \frac{1}{s} - \frac{\frac{8}{\tau}}{\underbrace{\frac{8}{\tau^2} + \frac{4}{\tau}s + s^2}_{s^2 + 2\zeta\omega_0 s + \omega_0^2}}$$

$$\Rightarrow \omega_0 = \sqrt{\frac{8}{\tau^2}} = \frac{2\sqrt{2}}{\tau}$$

$$\zeta = \frac{4}{\tau} \cdot \frac{1}{2 \cdot \omega_0} = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Bruker formell 11 i app. B.

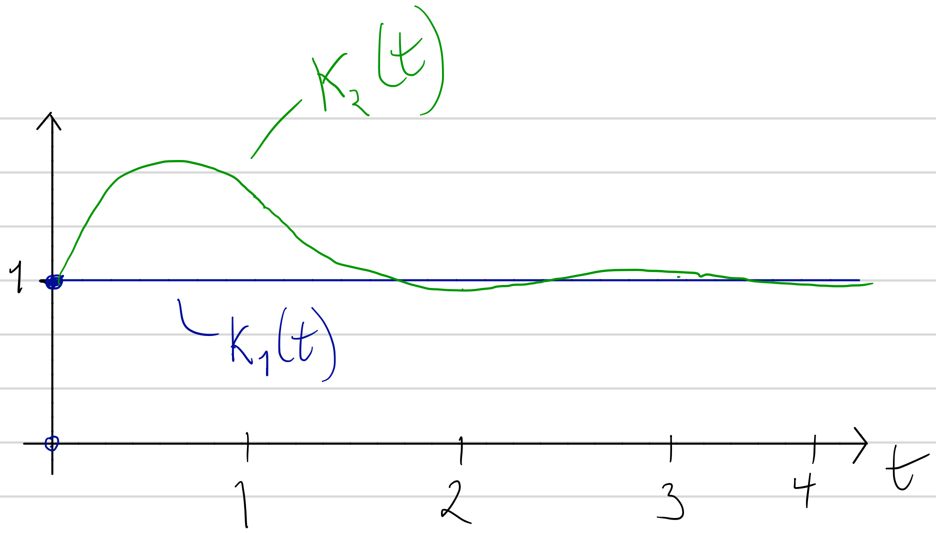
$$\mathcal{L}^{-1}\left\{\frac{1}{s^2 + 2\zeta\omega_0 s + \omega_0^2}\right\} = \frac{1}{\omega_0\sqrt{1-\zeta^2}} e^{-\zeta\omega_0 t} \sin\left(\sqrt{1-\zeta^2}\,\omega_0 t\right)$$

$$= \frac{\tau}{2} e^{-\frac{2t}{\tau}} \sin\left(\frac{2}{\tau}t\right)$$

Siden $\mathcal{L}^{-1}\{1/s\} = \mu_1(t)$ får vi

$$k_2(t) = \mu_1(t) - \frac{8}{\tau} \cdot \frac{\tau}{2} e^{-\frac{2t}{\tau}} \sin\left(\frac{2}{\tau}t\right)$$

$$= \mu_1(t) - 4e^{-\frac{2}{\tau}t} \sin\left(\frac{2}{\tau}t\right)$$



$K_2(t)$
1
$K_1(t)$
1
2
3
4
t