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Exercise 3
TTK4130 Modeling and Simulation

Problem 1 (Euler's method)

In this problem we will study the system

$$\ddot{x} + c\dot{x} + g\left[1 - \left(\frac{1}{x}\right)^{\kappa}\right] = 0, \tag{1}$$

where $\kappa = 1.40$ and $g = 9.81$.

(a) Write the system on state-space form.

(b) Set up Euler's method for the system.

(c) Set up modified Euler's method for the system.

(d) Show that $x = 1$ is a stationary point, and linearize the system about this point.

(e) For which steplengths h will Euler's method be stable, if

i) $c = 0$?

ii) $c = 2\sqrt{g\kappa} = 7.412$?

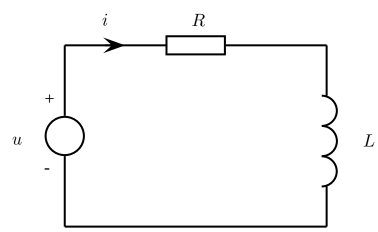

Figure 1: Electrical circuit

Problem 2 (Modeling, control and simulation of RL-circuit)

Figure 1 shows an electrical RL circuit where $L > 0$ is inductance, $R > 0$ is resistance, i is current and i_0 is initial current. The voltage u of the voltage source can be adjusted (is an input). The circuit is modeled by

$$L\frac{\mathrm{d}i}{\mathrm{d}t} + Ri = u, \quad t > 0 \tag{3a}$$

$$i(0) = i_0 \tag{3b}$$

(a) Let $u = 0$, and show that the system is (asymptotically) stable about the equilibrium $i^* = 0$, using energy-based methods. What happens with the current $i(t)$ when $t \to \infty$? Make a sketch.

(b) Now, assume we desire a given, constant current $i_{\text{ref}} > 0$. Design u such that $i(t) \to i_{\text{ref}}$ when $t \to \infty$, by considering the dynamics of $e(t) = i(t) - i_{\text{ref}}$ (what is $\frac{\mathrm{d}e}{\mathrm{d}t}$?), and use the energy function

$$E(t) = \frac{1}{2}Le^2(t) > 0.$$

(c) Let

$$i_0 = 1\ \text{A}$$
$$L = 1\ \text{H}$$
$$R = 2\ \Omega$$

Now, we want to simulate (3) from $t = 0$ to $t = 5$ by using the (classical) fourth order ERK-method, with constant step length $h = 0.01$ s.

Write a Matlab script that does the job, and enclose a code printout.

(d) Implement the model in Dymola. This could be done using components from the Modelica Standard Library, but instead write in the parameter definitions and equations. When you simulate, choose the solver Rkfix4 ('Simulation' $\rightarrow$ 'Setup'), which is the same solver you implemented in task (c). Set the step length ('Fixed integrator step') to 0.01. Compare with the solution of the Matlab implementation.

Problem 3 (Use of Matlab/Simulink's ODE solvers)

Download the file orbit.mdl from Blackboard, and experiment with it in MATLAB SIMULINK. This file models/simulates the restricted three-body problem from Ch. 14.1.3 in the book. The parameters in the file are for *Orbit 1* in Table 14.1.

(a) Choose the solver ode45 (this is probably the default solver), and check what *Relative Tolerance* is needed to simulate one, two and three rounds (simulate more rounds by increasing the stop time).

(b) Find the step length that gives an accurate solution for the five-stage explicit Runge-Kutta method ode5. Select the method by choosing *Simulation* $\rightarrow$ *Configuration parameters* $\rightarrow$ *Fixed step* and *ode5*.