Matte 4K, øving 7

Rendell Cale, gruppe 2

Ønsker tilbakemelding :) ok

124:

13) $u_{xx} + 5u_{xy} + 4u_{yy} = 0 \qquad (*)$

$AC - B^2 = 4 - \frac{25}{4} < 0$

So $(*)$ is a wave equation

Have to solve

$$(y')^2 - 5y' + 4 = 0$$

$$\Rightarrow \quad y' = \frac{5 \pm \sqrt{25 - 4 \cdot 4}}{2 \cdot 1}$$

$$= \frac{5 \pm 3}{2}$$

$$= 1 \text{ and } 4$$

$$\Rightarrow \begin{cases} y = x + C & (\int 1\, dx) \\ y = 4x + C & (\int 4\, dx) \end{cases}$$

$$\Rightarrow \begin{cases} y - x = \phi \\ y - 4x = \phi \end{cases}$$

$$\Rightarrow u(x,t) = f_1(y - x) + f_2(y - 4x) \quad R$$

12.7:

2) $u_t = c^2 u_{xx}$

$$u(x,0) = f(x) = \begin{cases} 1, & |x|<a \\ 0 & \text{otherwise} \end{cases}$$

$$u(x,t) = \int_0^\infty A(p)\cos(px)\, e^{-c^2p^2t}\, dp$$

Since $f(x)$ is even

$$A(p) = \frac{2}{\pi}\int_0^a \cos(px)\, dp$$

$$= \frac{2}{\pi p}\sin(px)\Big|_0^a$$

sin(0) = 0

$$= \frac{2\sin(ax)}{\pi a} - \cancel{\frac{2}{\pi}}$$

$$= \frac{2(\sin(ax) - \cancel{ax})}{\pi a}$$

ønsker svar på integralform

$$\Rightarrow u(x,t) = \frac{2}{\pi a}\underline{(\sin(ax) - \cancel{ax})\int_0^\infty \cos(px) e^{-c^2p^2t}\, dp} \quad R$$

$$= \frac{2}{\pi a}(\sin(ax) - \cancel{ax})\frac{\sqrt{\pi}}{2} e^{-\frac{x^2}{4c^2t}}$$

$$= \frac{(\sin(ax) - \cancel{ax})}{a\sqrt{\pi}} e^{-\frac{x^2}{4c^2t}}$$

~~Ok~~

13) $f(x) = \begin{cases} 1 , & x>0 \\ 0 , & x<0 \end{cases}$

(12) gives $u(x,t) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} f(x+2cw\sqrt{t})e^{-w^2}\,dw$

$= \frac{1}{\sqrt{\pi}} \int_{-\frac{x}{2c\sqrt{t}}}^{\infty} e^{-w^2}\,dw$

w > this gives f = 1

$= \frac{1}{2}\left(\frac{2}{\sqrt{\pi}} \int_{-\frac{x}{2c\sqrt{t}}}^{\infty} e^{-w^2}\,dw\right)$

$= \frac{1}{2}\left(\underbrace{1}_{\mathrm{erf}(\infty)} - \mathrm{erf}\left(-\frac{x}{2c\sqrt{t}}\right)\right)$

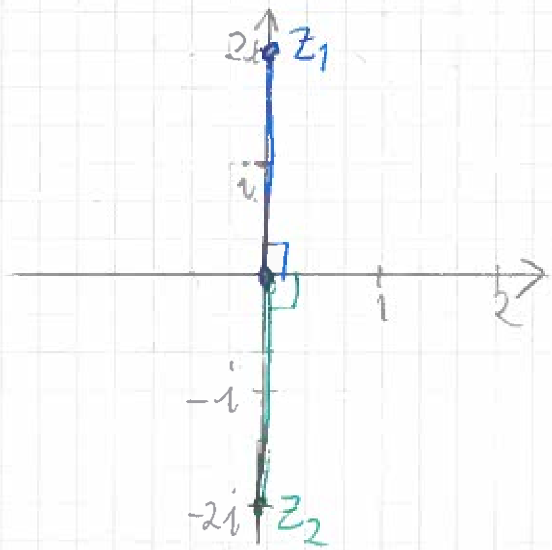
$= \frac{1}{2} - \frac{1}{2}\mathrm{erf}\left(-\frac{x}{2c\sqrt{t}}\right) \quad , t>0$

R

13,3:

3) $z_1 = 2i = 2e^{i\frac{\pi}{2}}$

$z_2 = -2i = 2e^{-i\frac{\pi}{2}}$



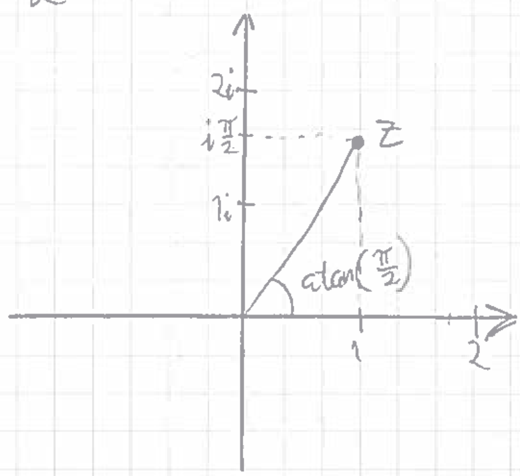
g

7) $z = 1 + \frac{\pi}{2}i$

$|z| = \sqrt{1 + \frac{\pi^2}{4}} = \frac{1}{2}\sqrt{4+\pi^2} \approx 1{,}86$

$\arg z = \operatorname{atan}\left(\frac{\pi/2}{1}\right) \approx 1{,}00$

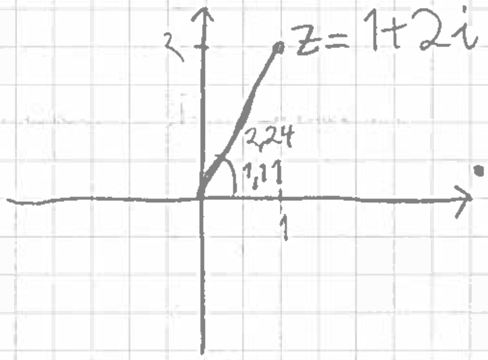
$\Rightarrow z = \frac{1}{2}\sqrt{4+\pi^2}\, e^{i\operatorname{atan}(\frac{\pi}{2})} \approx 1{,}86e^{i}$



8)

$$z = \frac{7+4i}{3-2i} = \frac{\cancel{7}\cdot 3-2\cdot 4}{3^2+2^2} + i\,\frac{7\cdot 2i + 3\cdot 4i}{3^2+2^2} = 1+2i$$

$$\cancel{= \frac{\sqrt{7^2+4^2}}{\sqrt{3^2+2^2}}\, e^{i\left[\operatorname{atan}\left(\frac{4}{7}\right)-\operatorname{atan}\left(\frac{-2}{3}\right)\right]} \approx \sqrt{5}\, e^{i(0{,}52-(-0{,}59))} \approx 2{,}24\, e^{1{,}11 i}}$$



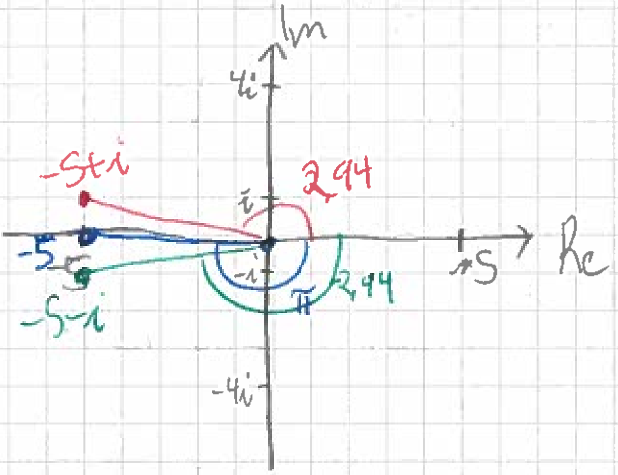
$$z = \sqrt{5}\, e^{i\tan^{-1}(2)} \approx 2{,}24\, e^{i\,1{,}11}$$

10)

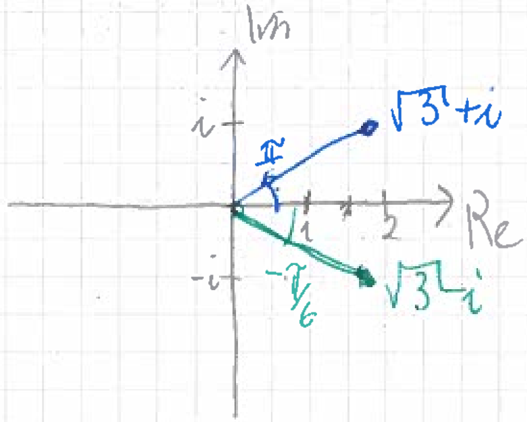
$$\arg(-5) = \pi$$

$$\arg(-5-i) = \operatorname{atan}\left(\frac{-1}{-5}\right) - \pi \approx -2{,}94$$

$$\arg(-5+i) = \operatorname{atan}\left(\frac{1}{-5}\right) + \pi \approx 2{,}94$$

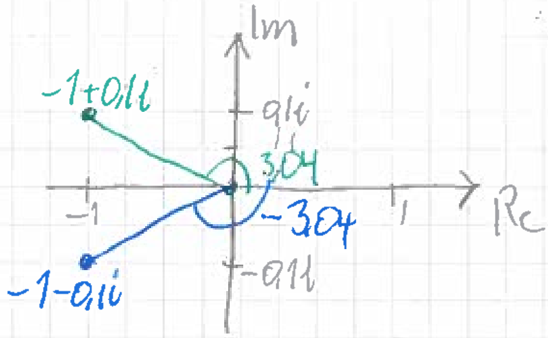


11) $\arg(\sqrt{3} \pm i) = \pm\frac{\pi}{6}$



14) $\arg(-1+0{,}1i) + \pi \approx 3{,}04$

$\arg(-1 - 0{,}1i) \approx -3{,}04$



21) Want to solve $z^3 = 1-i$

$$1-i = \sqrt{2}\,e^{-i\pi/4}$$

$$\Leftrightarrow |z^3| = \sqrt{2}$$

$$\Rightarrow |z| = \sqrt[6]{2}$$

$$\arg(z^3) = -\frac{\pi}{4} + 2\pi n, \quad n = 0, \pm 1, \pm 2, \ldots$$

$$\arg(z) = \frac{1}{3}\arg(z^3)$$

$$\stackrel{!}{=} -\frac{\pi}{12} + \frac{2\pi}{3} n$$

Need three z's since this is a 3'rd degree polynomial. Pick $n=-1,0,1$

$$\left.\begin{aligned} z_1 &= \sqrt[6]{2}\, e^{-i\frac{3}{4}\pi} \\ z_2 &= \sqrt[6]{2}\, e^{-i/_{12}\pi} \\ z_3 &= \sqrt[6]{2}\, e^{i\frac{7}{12}\pi} \end{aligned}\right\} \text{solve } z^3 = \sqrt{1-i}$$

23) Want to solve $z^3 = 343$.

$$|z| = \sqrt[3]{343} = 7 \quad \text{because } |z| \in \mathbb{R}$$

$$\arg z = \frac{1}{3}(0 + 2\pi n) = \frac{2\pi}{3} n$$

Same procedure, $n=-1,0,1$ gives

$$\left.\begin{aligned} z_1 &= 7\, e^{-\frac{2\pi}{3} i} \\ z_2 &= 7 \\ z_3 &= 7\, e^{\frac{2\pi}{3} i} \end{aligned}\right\} \text{Solve } z^3 = 343$$

Sup. P) Tror du har gjort oppgava fra feil kapittel. Du skulle

$f(x) = \pi x - x^2, \quad 0 < x \leq \pi$ gjort 13.3,

$f(x) = \sum_{n=1}^{\infty} B_n \sin(n\pi x)$ ikke 13.2.

$$B_n = \frac{2}{\pi} \int_0^{\pi} (\pi x - x^2) \sin(nx)\, dx$$

$$= \frac{2}{\pi n} \int_0^{\pi} (\pi - 2x) \cos(nx)\, dx$$

$$\Leftrightarrow \quad B_n = \frac{2}{\pi n^2}(\pi - 2x)\sin(nx)\Big|_0^{\pi} - \frac{2}{\pi n^2}\int_0^{\pi}(-2)\sin(nx)\,dx$$

$$= -\frac{4}{\pi n^3}\left(\cos(n\pi) - 1\right)$$

$$= \frac{4}{\pi n^3}\left(1-(-1)^n\right)$$

$$= \frac{8}{\pi n^3} \text{ for } n = 1, 3, 5, \ldots \quad \boxed{R}$$

So $f(x) = \frac{8}{\pi}\sum_{k=0}^{\infty}\frac{1}{(2k+1)^3}\sin((2k+1)x)$

b) Want to solve

$$\begin{cases} u_t = u_{xx} - 2u_x \,, & 0 < x < \pi,\ t > 0 \quad (1) \\ u(0,t) = 0 = u(\pi,t)\,, & t > 0 \quad (2). \end{cases}$$

Assume $u(x,t) = F(x)G(t)$, then

$$u_t = F(x)\dot{G}(t),$$
$$u_{xx} - 2u_x = F''(x)G(t) - 2F'(x)G(t)$$

(1) then gives $F(x)\dot{G}(t) = F''(x)G(t) - 2F'(x)G(t)$

$\Leftrightarrow \quad F''(x)G(t) - 2G(t)F'(x) - F(x)\dot{G}(t) = 0$

$G(t) \neq 0$: $\quad F''(x) - 2F'(x) - F(x)\frac{\dot{G}(t)}{G(t)} = 0 \quad (3)$

Lets go back to (1):

$$F \cdot \dot{G} = F''G - 2F'G$$

$$\Leftrightarrow \frac{\dot{G}}{G} = \frac{F'' - 2F'}{F} = \text{const}$$

So $\frac{\dot{G}(t)}{G(t)} = K$ for some K

(3) $F'' - 2F' - KF = 0$

$$\Rightarrow \lambda^2 - 2\lambda - K = 0$$

$$\Rightarrow \lambda = \frac{2 \pm \sqrt{4 - 4K}}{2}$$

$$= 1 \pm \sqrt{1-K} = 1 \pm i\sqrt{K-1}$$

$$\Rightarrow F(x) = Ae^x \cos(\sqrt{K-1}\,x) + Be^x \sin(\sqrt{K-1}\,x)$$

Since $u(0,t) = 0 = F(0)G(t)$, we must have $F(0) = 0 \Rightarrow A = 0$ so

$$F(x) = Be^x \sin(\sqrt{K-1}\,x)$$

Since $u(\pi, t) = 0 = F(\pi)G(t)$, we must have

$F(\pi) = 0 \Rightarrow \sin(\sqrt{K-1}\,\pi) = 0$

$\Rightarrow \sqrt{K-1}$ is an integer, call it n.

Then we have

$$\underline{\underline{F_n(x) = B_n e^x \sin(nx)}}$$

R

c) $u(x,0) = e^x f(x)$, $0<x<\pi$ $\quad (*)$

$\sqrt{K-1} = n \Leftrightarrow K = n^2+1$

Since $\frac{\dot G}{G} = k$ we get

$$\dot G - kG = 0$$

$$\Rightarrow G(t) = Ce^{-kt}$$

$$\Leftrightarrow G_n(t) = C_n e^{-(n^2+1)t} \quad R$$

So $u(x,t) = F(x)G(t)$

$$\Rightarrow u_n(x,t) = D_n e^x \sin(nx) e^{-(n^2+1)t}, \quad D_n = C \cdot B$$

Since $u(x,t) = \sum u_n(x,t)$ we get

$$u(x,t) = \sum_{n=1}^{\infty} D_n e^x \sin(nx) e^{-(n^2+1)t}$$

Want $u(x,0) = e^x f(x)$

$$\Leftrightarrow \sum_{n=1}^{\infty} D_n e^x \sin(nx) = \frac{8}{\pi}\sum_{k=0}^{\infty} \frac{1}{(2k+1)^3} e^x \sin(nx)$$

So $D_n = B_n$ from (a)

$$\Rightarrow D_{2k+1} = \frac{8/\pi}{(2k+1)^3} \quad R$$

Using this we could compute the general solution, but since we are only after "a" solution, we pick $\underline{\underline{u = u_1 = \frac{8}{\pi} e^x \sin(x) e^{-2t}}}$ g

Dette tilfredsstiller ikke initial-betingelsen $u(x,0) = e^x f(x)$!!

Q) $z_0 = 1$

(i) $f(z) = z\,\mathrm{Re}(z)$

$$\lim_{\Delta \to 0} \underbrace{\frac{f(z_0+\Delta) - f(z_0)}{\Delta}}$$

$$= \frac{(z_0+\Delta)\mathrm{Re}(z_0) - z_0\mathrm{Re}(z_0)}{\Delta}$$

$$= \frac{1+\cancel{\Delta}-1}{\cancel{\Delta}}$$

$$= 0$$

Se LF

$f'(z_0)$ is defined so f is analytic at z_0. g

ii) $f(z) = z^2$ is a polynomial so it is analytic at all z.

iii) $f(z) = \frac{1}{z}$

$$\frac{f(z_0+\Delta) - f(z_0)}{\Delta} = \frac{\frac{1}{1+\Delta} - 1}{\Delta}$$

$$= \frac{1}{\Delta(1+\Delta)} - \frac{1}{\Delta}$$

$$= \frac{1 - 1 - \Delta}{\Delta(1+\Delta)}$$

$$= \frac{-1}{1+\Delta}$$

$= -1$ when $\Delta \to 0$

$f'(z_0) = -1$ is defined so it is an analytic function R