Industriell elektroteknikk, Øving 3

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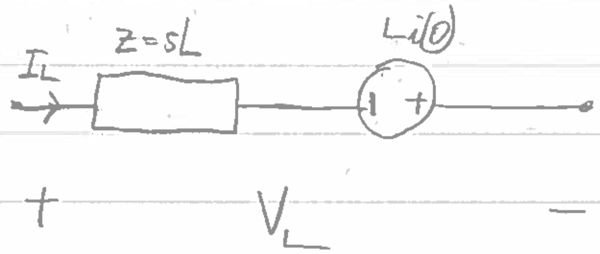
Ønsker tilbakemelding :)

Supert! Godkjent! Ahm 24/9-2016

Oppgave 1

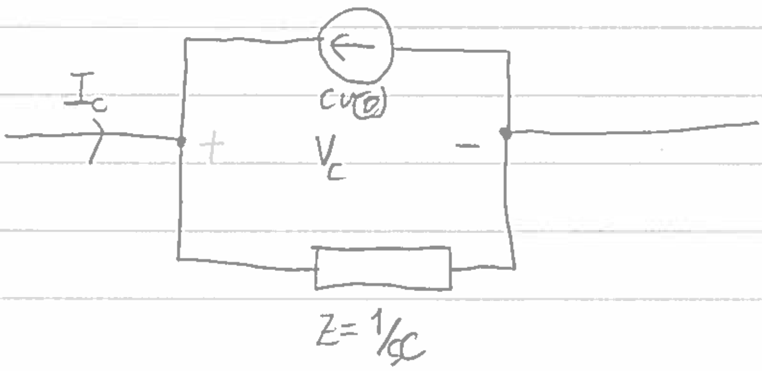
a) $V_L = \underbrace{sL}_{Z \text{ (motstand)}} I_L - L\,i(0)$

Dette er en spg. deler hvor spg. V_L ligger over en motstand Z og en annen spg. kilde. Strømmen gjennom er I_L så vi får



$I_C = sCV_C - C\,V_C(0)$

Strømmen I_C blir delt på to elementer så dette er en parallellkobling. Den ene delen får en strøm sCV_C gjennom seg og har spg V_C over seg. Dette er altså en motstand med $Z = \frac{1}{sC}$. $C\,V(0)$ svarer til en strømkilde motsatt rettet av I_C, altså:

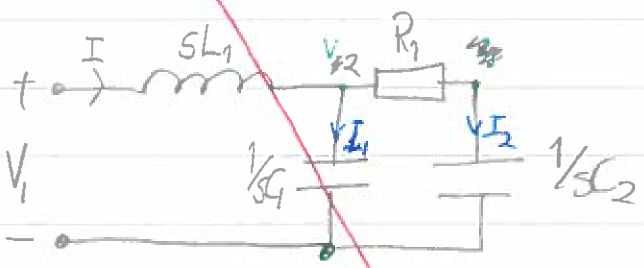


b) Spole : $Z_L = sL$

R

Kondensator : $Z_C = 1/sC$

c) $Z(s) = \frac{V(s)}{I(s)}$



KCL V_1 : $I = I_1 + I_2$

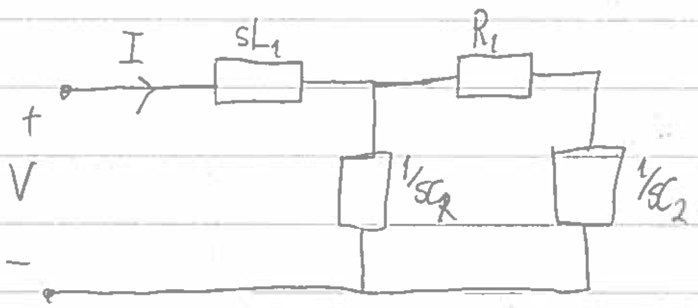
$$\frac{V_1 - V_2}{sL_1} = V_2 sC_2 + \frac{V_2}{R_1 + 1/sC_2}$$

$$\frac{V_1}{sL_1} = V_2\left(\frac{1}{sL_1} + sC_2 + \frac{1}{R_1 + 1/sC_2}\right)$$

$$V_1 = V_2\left(1 + s^2 L_1 C_2 + \frac{sC_2}{sR_1C_2 + 1} \cdot sL_1\right)$$

$$= V_2\left(s^2\left(L_1C_2 + \frac{1}{sR_1C_2+1}\right) + 1\right)$$

c) $Z(s) = \dfrac{V(s)}{I(s)}$



$$\rightsquigarrow \quad Z_{eq,1} = R_1 + 1/sC_2 = \frac{sR_1C_2 + 1}{sC_2}$$

$$Z_{eq,2} = \left(\frac{1}{1/sC_1} + \frac{1}{Z_{eq,1}}\right)^{-1}$$

$$= \left(sC_1 + \frac{sC_2}{sR_1C_2 + 1}\right)^{-1}$$

$$= \left(\frac{s^2R_1C_1C_2 + s(C_1 + C_2)}{sR_1C_2 + 1}\right)^{-1}$$

$$= \frac{sR_1C_2 + 1}{s^2R_1C_1C_2 + s(C_1 + C_2)}$$

$$Z(s) = sL_1 + Z_{eq2} = sL_1 + \frac{sR_1C_2 + 1}{s^2R_1C_1C_2 + s(C_1 + C_2)}$$

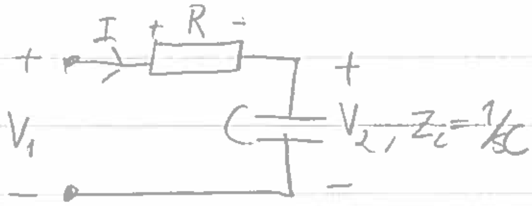
R

d) $L_1 = 1H, \quad R = 1\Omega, \quad C_1 = C_2 = 2F$

$$\Rightarrow Z(s) = s + \frac{2s+1}{4s^2+4s}$$

$$\Rightarrow Z(j\omega) = j\omega + \frac{2j\omega+1}{-4\omega^2+4j\omega}$$ R

e) $$H(s) = \frac{V_2(s)}{V_1(s)}$$



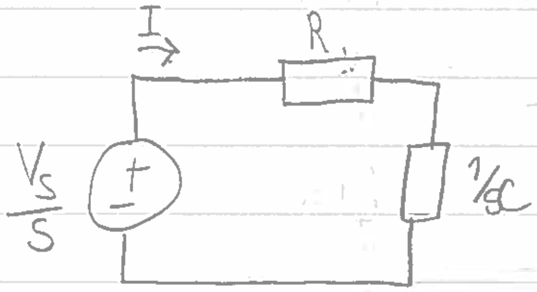
$$\frac{V_1 - V_2}{R} = \frac{V_2}{1/sC} \Leftrightarrow \frac{V_1}{R} = V_2\left(\frac{1}{R} + sC\right)$$

$$\Leftrightarrow \frac{V_2}{V_1} = \frac{1}{1+sRC} = H(s)$$

$$H(j\omega) = \frac{1}{1+j\omega RC}$$ R

Oppgave 2

a) Tegner kretsen i s-rommet.



Spenningsdeler: $I = \dfrac{V_s/s}{R + 1/sC}$

$$= \frac{V_s}{Rs + 1/C}$$

$$= \frac{V_s}{R} \cdot \frac{1}{s + 1/RC}$$

$$\Rightarrow \quad i = \frac{V_s}{R} e^{-\frac{1}{RC}t} u(t)$$

$$v_c(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau = \frac{V_s}{RC} \cdot (-RC)\, e^{-\frac{1}{RC}\tau}\Big|_{\tau=0}^{\tau=t}$$

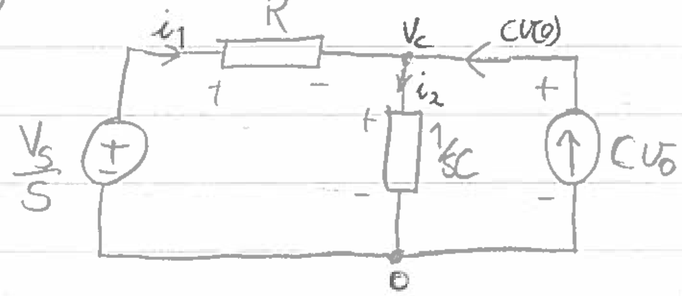
$$= -V_s\left(e^{-\frac{1}{RC}t} - 1\right)$$

$$= V_s - V_s e^{-\frac{1}{RC}t}$$

$$= 15 - 15e^{-t} \quad , \; t \geq 0 \qquad v_c(t<0) = 0$$

b) $v_C(0) = v_0 = 5V$

Tegner kretsen i s-domenet.



KCL i v: $i_1 + Cv_0 = i_2$

$$\Leftrightarrow \frac{\frac{V_s}{s} - V_c}{R} + Cv_0 = sV_cC$$

$$\Leftrightarrow \frac{V_s}{sR} + Cv_0 = V_c(sC + 1/R)$$

$$\Leftrightarrow V_c = \frac{V_s/sR}{sC + 1/R} + \frac{Cv_0}{sC + 1/R}$$

$$= \frac{V_s/RC}{s(s + 1/RC)} + \frac{v_0}{s + 1/RC}$$

Delbrøksoppspaltning:

$$\frac{V_s}{RC} = A\left(s + \frac{1}{RC}\right) + Bs$$

$\Rightarrow \quad A = V_S \;, B = -V_S$

Det gir

$$V_C = \frac{V_S}{s} - \frac{V_S}{s + 1/RC} + \frac{v_0}{s + 1/RC}$$

$$\Rightarrow v_C(t) = V_S - V_S e^{-\frac{1}{RC}t} + v_0 e^{-\frac{1}{RC}t} \;, t \geq 0$$

$$= V_S - (V_S - v_0) e^{-\frac{1}{RC}t}$$

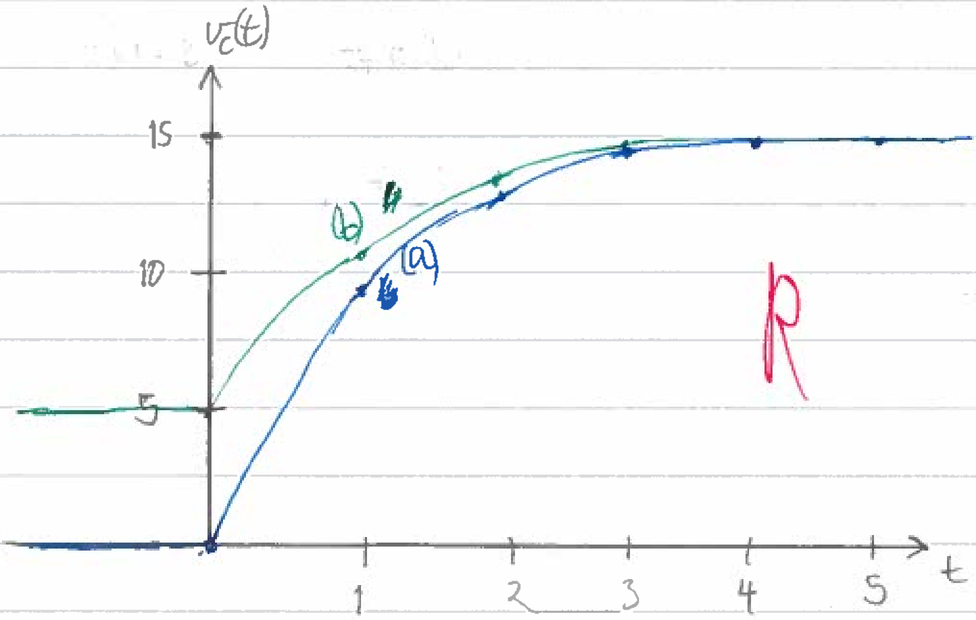
$$= 15 - (15 - 5) e^{-t}$$

$$= 15 - 10 e^{-t} \;, t \geq 0 \;; \quad v_C(t<0) = 5 \;;$$

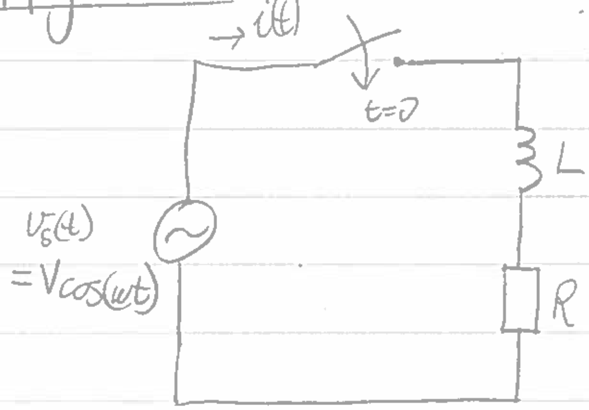
R

a) Tidskonstanten τ i (a) er $RC = 1$ R

c)



Oppgave 3



$V = 10$ V
$R = 1\ \Omega$
$L = 5$ mH
$\omega = 2\pi \cdot 50$ rad/s
$i_L(0) = 0$

a) I s-domenet vil $Z_L = sL$ og $\mathcal{L}\{V_S\} = \frac{Vs}{\omega^2 + s^2}$

Det gir $I(s) = \frac{V \cdot \frac{s}{s^2+\omega^2}}{sL + R}$

$= \frac{Vs}{\omega^2+s^2} \cdot \frac{1}{sL+R}$

b) Delbrøksoppspalting:

$$V \cdot s = (As+B)(sL+R) + C(\omega^2+s^2)$$

$\Rightarrow$ (s^2 terms): $0 = A \cdot L + C$ (1)

(s^1 terms): $V = AR + BL$ (2)

(Constants): $0 = BR + C\omega^2$ (3)

$$(1) \Leftrightarrow \quad C = -A\cdot L \quad \Leftrightarrow \quad A = -\frac{C}{L}$$

$$(3) \Leftrightarrow \quad B = -\frac{C\omega^2}{R}$$

$$\overset{(1)\,\&\,(3)}{\Longrightarrow} \quad V = \left(-\frac{C}{L}\right)R - \frac{C\omega^2 L}{R}$$

$$\Leftrightarrow \quad V = -C\left[\frac{R}{L} + \frac{\omega^2 L}{R}\right]$$

$$\Leftrightarrow \quad C = \frac{-V}{\frac{R}{L} + \frac{\omega^2 L}{R}} = -\frac{VRL}{R^2+(\omega L)^2}$$

$$\overset{(1)}{\Longrightarrow} \quad A = \frac{VR}{R^2+(\omega L)^2}$$

$$\overset{(3)}{\Longrightarrow} \quad B = +\frac{V\cancel{R}L}{R^2+(\omega L)^2}\cdot\frac{\omega^2}{\cancel{R}} = \frac{VL\omega^2}{R^2+(\omega L)^2}$$

Så

$$I(s) = \frac{As+B}{\omega^2+s^2} + \frac{C}{sL+R}$$, der A,B,C er som over.

$$I(s) = A\frac{s}{\omega^2+s^2} + \frac{B}{\omega}\frac{\omega}{\omega^2+s^2} + \frac{C/L}{s+R/L}$$

$$\Leftrightarrow i(t) = A\cos(\omega t) + \frac{B}{\omega}\sin(\omega t) + \frac{C}{L}e^{-\frac{R}{L}t}$$

Regner ut $A, \frac{B}{\omega}, \frac{C}{L}$

$$A = \frac{10\cdot 1}{1^2+(2\pi\cdot 50\cdot 5\cdot 10^{-3})^2} \approx 2{,}884$$

$$\frac{B}{\omega} = \frac{1423{,}20}{\omega} = 4{,}530$$

$$\frac{C}{L} = \frac{-0{,}014}{L} = -2{,}884 \quad (= -A)$$

Vi har da

$$i(t) = 2{,}884\cos(100\pi\cdot t) + 4{,}530\cdot\sin(100\pi t)$$
$$-2{,}884e^{-200t}$$

R

c) De stasjonære verdiene/leddene er de som ikke går mot 0 når $t\to\infty$. Altså:

$$i_{stasj}(t) = 2{,}884\cos(\omega t) + 4{,}530\sin(\omega t)$$

Den samlede amplituden blir $A' = \sqrt{2{,}884^2 + 4{,}530^2}$

$\approx 5{,}370.$

Vi må forskyve med en vinkel $\alpha = \tan^{-1}\left(\frac{4{,}530}{2{,}884}\right) = 57{,}52^\circ$

Så

$$i_{støj}(t) = 5{,}370 \cdot \cos(\omega t - 57{,}52^\circ) \text{ A}$$

e) $H(s) = \frac{V(s)}{I(s)}$

$$V(s) = \mathcal{L}\{V\cos(\omega t)\} = V \frac{s}{\omega^2 + s^2}$$

$$I(s) = \frac{Vs}{\omega^2 + s^2} \cdot \frac{1}{sL + R}$$

$$\Rightarrow H(s) = \frac{V \cdot s / (\omega^2 + s^2)}{V \cdot s / (\omega^2 + s^2) \cdot 1/(sL+R)} = sL + R$$

$$\Rightarrow H(j\omega) = R + j\omega L$$

Setter inn tallverdier:

$$H(j\omega) = 1 + 1{,}57j$$

Siden $H(j\omega) = 1 + 1{,}57j$ vil

$$|H(j\omega)| = \sqrt{1^2 + 1{,}57^2} = 1{,}86$$

og

$$\angle H(j\omega) = \tan^{-1}\left(\frac{1{,}57}{1}\right) = 57{,}51^\circ$$

f) $v_s(t) = 10\cos(\omega t)$

$i_{stasj}(t) = 5{,}37\cos(\omega t - 57{,}52^\circ)$

Forholdet mellom amplitudene blir $\frac{10}{5{,}37} = 1{,}86$

som er det samme som $|H(j\omega)|$.

Faseuinkelforskjellen mellom v_s og i_{stasj} er $57{,}52^\circ$, som er en avrundingsfeil unna å være lik $\angle H(j\omega) = 57{,}52^\circ$

Oppgave 4

$$H_1(s) = \frac{\Omega(s)}{V_m(s)} = \frac{K}{\tau s+1}$$

Ω_m: rotasjonshastighet
v_m: spenning

a)

θ_m: vinkelposisjon til motor

Vi vet at $\Theta_m(t) - \theta(0) = \int_0^t \Omega_m(\tau)\, d\tau$

Antar $\theta(0)=0$ og får $\theta_m(t) = \int_0^t \Omega_m(\tau)\, d\tau$

Det betyr at $\mathcal{L}\{\theta_m\} = \frac{1}{s}\cdot \mathcal{L}\{\Omega_m\} = \frac{\Omega_m(s)}{s}$

Setter dette inn i H_2:

$$H_2(s) = \frac{\Theta(s)}{V_m(s)} = \frac{\frac{1}{s}\cdot \Omega_m(s)}{V_m(s)}$$

$$= \frac{1}{s}\cdot \underbrace{\frac{\Omega_m(s)}{V_m(s)}}_{H_1} = \frac{1}{s}\cdot H_1(s)$$

$$= \frac{K}{s(\tau s+1)}$$ R

b) $v_m(t) = u(t)$, $K = 23$, $\tau = 0{,}13$

Ønsker å finne $\Omega_m(t)$ og $\theta_m(t)$.

Vel at

$$\Omega_m(s) = V_m(s) \cdot H_1(s)$$

og $$\theta_m(s) = V_m(s) \cdot H_2(s)$$

$$V_m(s) = \mathcal{L}\{u(t)\} = \frac{1}{s}$$

$$H_1(s) = \frac{K}{\tau s + 1}, \quad H_2(s) = \frac{K}{s(\tau s + 1)}$$

Dette gir:

(1) $$\Omega_m(s) = \frac{1}{s} \cdot \frac{K}{\tau s + 1}$$

$$\Leftrightarrow \quad \frac{A}{s} + \frac{B}{\tau s + 1} = \frac{K}{s(\tau s + 1)}$$

$$\Leftrightarrow \quad A(\tau s + 1) + Bs = K$$

$$\Rightarrow A = K$$

$$A\tau + B = 0 \Leftrightarrow B = -K\tau$$

$$\Leftrightarrow \quad \Omega_m(s) = \frac{K}{s} - \frac{K\tau}{\tau s + 1}$$

$$= \frac{K}{s} - \frac{K}{s + 1/\tau}$$

(2) $\Theta_m(s) = \frac{1}{s} \cdot \frac{K}{s(\tau s+1)}$

$$\Leftrightarrow \frac{A}{s} + \frac{B}{s^2} + \frac{C}{\tau s+1} = \frac{K}{s^2(\tau s+1)}$$

$$\Leftrightarrow As(\tau s+1) + B(\tau s+1) + Cs^2 = K$$

(s^2): $A\tau + C = 0$

(s^1): $A + B\tau = 0$

(1): $B = K$

$$B = K \overset{(s^1)}{\Longrightarrow} A = -K\tau$$

$$\overset{(s^2)}{\Longrightarrow} C = -A\tau = K\tau^2$$

Så $\Theta_m(s) = -\frac{K\tau}{s} + \frac{K}{s^2} + \frac{K\tau^2}{\tau s+1}$

$$= -\frac{K\tau}{s} + \frac{K}{s^2} + \frac{K\tau}{s+1/\tau}$$

Tar invers transformen av uttrykkene og får

$$\Omega_m(t) = K u(t) - K e^{-\frac{1}{\tau}t}$$

$$\Theta_m(t) = -K\tau \cdot u(t) - Kt + K\tau e^{-\frac{1}{\tau}t}$$

Setter inn $K=23$ og $\tau = 0{,}13$:

$$\Omega_m(t) = 23\left(1 - e^{-7{,}69t}\right) u(t)$$

$$\Theta_m(t) = \left(-2{,}99\left(1 - e^{-7{,}69t}\right) + 23t\right) u(t)$$

c)