Assignment 6, ttk4215

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Problem 4.10

c

We will modify the gradient algorithm to include projection. Before adding projection, we have the adaptive law

$$\dot{\theta} = \Gamma \epsilon \phi \tag{1}$$

To apply projection we need a function $g(\theta)$ such that $g(\theta) \leq 0$ when $0 \leq \beta \leq 1$, $k \geq 0.1$, and $m \geq 10$.

I wasn't able to find a single function that defines the area we want.

To constrain m we can use $g_1(m) = 10 - m$, giving $\nabla g_1(m) = -1$. For β, we can use that $g_2(\beta) = \beta(\beta - 1) \leq 0$ in the are we are interested in. We then have $\nabla g_2(\beta) = 2\beta - 1$. To constrain k we can use $g_3(k) = 0.1 - k$, giving $\nabla g_3(k) = -1$.

Will try to use $g(\theta) = g_1 + g_2 + g_3 = 10 - m + \beta(\beta - 1) + 0.1 - k$, which is negative on the when $\theta \in S$, but is not positive everywhere outside so it doesn't satisfy the definition in (4.4.3).

We then have

$$\nabla g = \begin{bmatrix} -1 & 2\beta - 1 & -1 \end{bmatrix}$$

We can then update the adaptive law to use this information.

$$\dot{\theta} = \Pr(\Gamma \epsilon \phi) \tag{2}$$

d

Problem 4.11

a

To create an on-line estimation scheme, I will develop a linear parametric form.

We have $(r-\theta_p)G_0(s)=\theta_p$. This gives

$$
\begin{aligned}
(r-\theta_p)k_0\omega_0^2 &= (s^2+2\xi_0\omega_0 s+\omega_0^2(1-k_0))\theta_p && (3)\\
rk_0\omega_0^2 &= (s^2+2\xi_0\omega_0 s+\omega_0^2)\theta_p && (4)\\
\frac{r}{\Lambda_1} &= \left(\frac{1}{k_0\omega_0^2}s^2+\frac{2\xi_0\omega_0}{k_0\omega_0^2}s+\frac{1}{k_0}\right)\frac{\theta_p}{\Lambda_1} && (5)\\
\frac{r}{\Lambda_1} &= \left(\theta_1 s^2+\theta_2 s+\theta_3\right)\frac{\theta_p}{\Lambda_1} && (6)
\end{aligned}
$$

where $\Lambda_1(s)$ is Hurwitz and at least of order 2.

Using $z_1=\frac{r(s)}{\Lambda_1(s)}$, $\bar{\theta}_1=\begin{bmatrix}\theta_1 & \theta_2 & \theta_3\end{bmatrix}^T$, $\phi_1=\begin{bmatrix}s^2 & s & 1\end{bmatrix}^T\frac{\theta_p}{\Lambda_1}=\frac{\alpha_2(s)^T\theta_p}{\Lambda_1}$, we have written the system in linear parametric form. To convert $\bar{\theta}_1$ to k_0, ω_0, and ξ_0, we would use

$$
\begin{aligned}
k_0 &= \theta_3^{-1} && (7)\\
\omega_0 &= \sqrt{\frac{\theta_3}{\theta_1}} && (8)\\
\xi_0 &= \frac{1}{2}\frac{\theta_2}{\sqrt{\theta_1\theta_3}} && (9)
\end{aligned}
$$

To estimate the other set of parameters, we use $\theta_p G_1(s)=\dot{\theta}$. This gives

$$
\theta_p k_1\omega_1^2=(s^2+2\xi_1\omega_1 s+\omega_1^2)\dot{\theta} \qquad (10)
$$

Which is essentially the same as eq. (4), so we can write in linear parametric form using $z_2=\frac{\theta_p}{\Lambda_2}$, $\bar{\theta}_2=\begin{bmatrix}\theta_4 & \theta_5 & \theta_6\end{bmatrix}^T$, $\phi_2=\frac{\alpha_2(s)^T\dot{\theta}}{\Lambda_2}$.

To convert to the system parameters, we use

$$k_1 = \theta_6^{-1} \tag{11}$$

$$\omega_1 = \sqrt{\frac{\theta_6}{\theta_4}} \tag{12}$$

$$\xi_1 = \frac{1}{2}\frac{\theta_5}{\sqrt{\theta_4\theta_6}} \tag{13}$$

To write the entire system in linear parametric form, we look at $z = z_1 + z_2$.

$$z = \frac{r}{\Lambda_1} + \frac{\theta_p}{\Lambda_2} \tag{14}$$

$$= \bar{\theta}_1^T\phi_1 + \bar{\theta}_2^T\phi_2 = \begin{bmatrix}\bar{\theta}_1 \\ \bar{\theta}_2\end{bmatrix}^T \begin{bmatrix}\phi_1 \\ \phi_2\end{bmatrix} = \begin{bmatrix}\theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6\end{bmatrix}^T \begin{bmatrix}\frac{s^2}{\Lambda_1}\theta_p \\ \frac{s}{\Lambda_1}\theta_p \\ \frac{1}{\Lambda_1}\theta_p \\ \frac{s^2}{\Lambda_2}\dot{\theta} \\ \frac{s}{\Lambda_2}\dot{\theta} \\ \frac{1}{\Lambda_2}\dot{\theta}\end{bmatrix} = \theta^T\phi \tag{15}$$

To estimate θ, we will use least-squares with forgetting factor, given below.

$$\hat{z} = \theta^T\phi \tag{16}$$

$$\epsilon = (z - \hat{z})/m^2 \tag{17}$$

$$\dot{\theta} = P\epsilon\phi \tag{18}$$

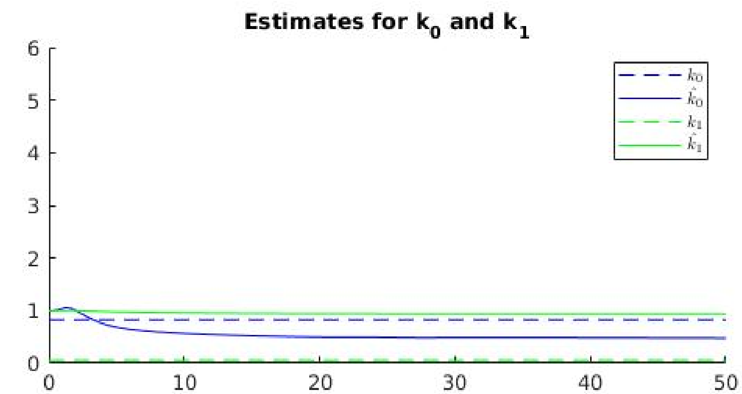
$$\dot{P} = \begin{cases} \beta P - P\frac{\phi\phi^T}{m^2}P & \text{if } ||P|| \leq R_0 \\ 0 \quad \text{otherwise} \end{cases} \tag{19}$$

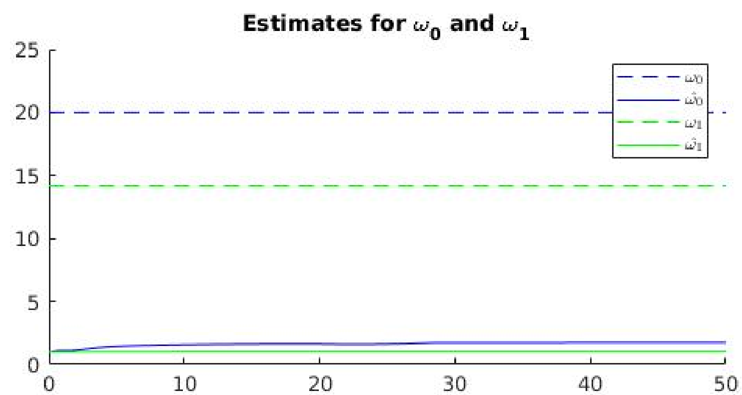
$$P(0) = P_0, \quad ||P_0|| \leq R_0 \tag{20}$$

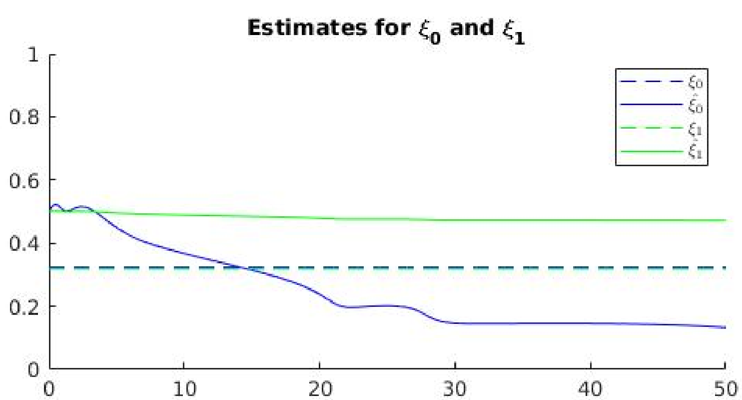
After experimenting a couple of hours with this, I was unable to get good convergence using this estimator.

i

With $r = 10\sin(0.2t) + 8$ and $V = 20$mph, the best I got was this



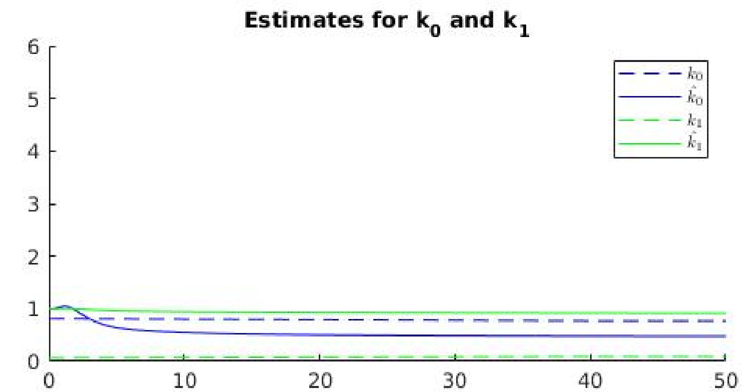


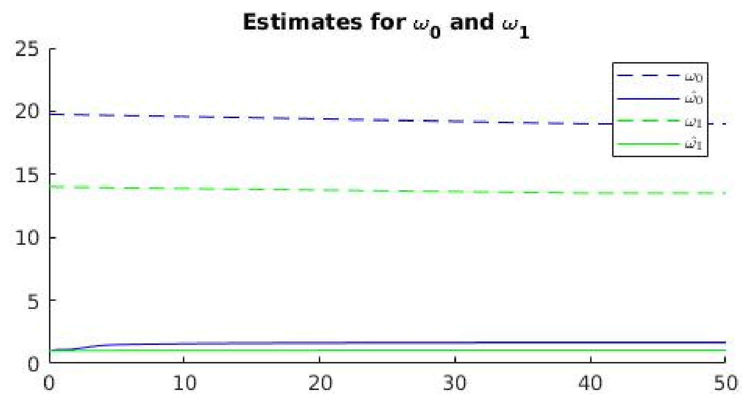


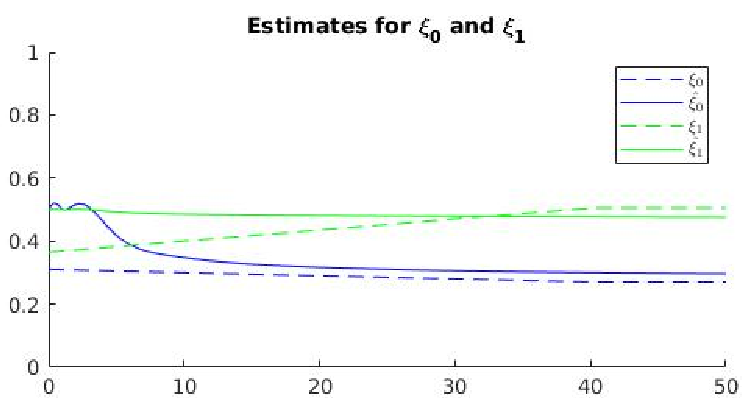
Unfortunately, the estimates don't seem to converge to the true value on any of the parameters.

ii

With $r = 5$ and V increasing from 30 to 60 in 40 seconds, I got this







The estimates seem to converge on k_i and ξ_i, but for ω_i the estimates are really off.