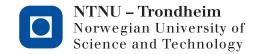
Recommended completion: February 13, 2018



# Exercise 5 TTK4130 Modeling and Simulation

### Problem 1 (Lotka-Volterra predator-prey)

An example of the Volterra-Lotka predator-prey model is

$$\dot{u} = u \left( v - 3 \right) \tag{1a}$$

$$\dot{v} = v(2 - u) \tag{1b}$$

The equations can be used to describe the dynamics of biological systems in which two species interact, one a predator and one its prey. Here, u may represent the number of predators (in thousands), for instance foxes, and v number of preys in thousands, for instance rabbits.

Consider an island where foxes are the only predator, and rabbits the only prey. When the number of rabbits gets large (v > 3), the number of foxes will grow. But when the number of foxes becomes large (u > 2), the number of rabbits will decrease, which in turn make the number of foxes decrease. This leads to large, periodic variations in the two populations, with a "phase shift" between the number of foxes and rabbits.

Consider the "energy-like" function<sup>1</sup>

$$V = u - 2\ln u + v - 3\ln v \tag{2}$$

(a) Calculate

$$\dot{V} = \frac{\partial V}{\partial u}\dot{u} + \frac{\partial V}{\partial v}\dot{v} \tag{3}$$

and show that V is constant for solutions of the Lotka-Volterra system (1). Make an attempt at an interpretation.

- (b) Implement the system in Dymola, and simulate over 20 time units. Use u = 1 and v = 4 as initial conditions. Comment on the trajectories of u and v, and verify that V is constant. (That is, in addition to implementation of the two Lotka-Volterra equations, implement the equation for V.)
- (c) Linearize the system about the equilibrium  $\{u = u^* = 2, v = v^* = 3\}$ , and calculate eigenvalues. Simulate the system at this equilibrium, and with small deviations from it, and comment on connection between eigenvalues and periods/frequencies.
- (d) Simulate this model (initial conditions and horizon as in (b)) in Matlab using Euler, implicit Euler, and the implicit midpoint rule. Re-use code from Exercise 4. Make plots of u and v (for instance in phase plots) and V for the different numerical solutions. Use h=0.1 (for instance, but experiment). Comment!

#### Problem 2 (Lobatto IIIA)

- (a) Show that Lobatto IIIA is A-stable, by showing that the conditions for A-stability is fulfilled.
- (b) Is Lobatto IIIA L-stable? Calculations are required.

## Problem 3 (Linear stability of Runge-Kutta methods)

Given the Runge-Kutta method

$$\begin{array}{c|cccc}
0 & \frac{1}{4} & -\frac{1}{4} \\
\frac{2}{3} & \frac{1}{4} & \frac{5}{12} \\
& \frac{1}{4} & \frac{3}{4}
\end{array}$$

<sup>&</sup>lt;sup>1</sup>Strictly speaking this function is not positive definite. However, we can show that the states of the Lotka-Volterra system will remain positive if initialized positive, and for positive u and v, V will be positive (in general, after addition of a constant). Therefore, think of V as measure of "energy" (total population) in the system.

- (a) Is this an explicit or implicit method? Why?
- (b) Find the stability function for the method. Is this a Padé-approximation?
- (c) Is this method A-stable? L-stable? Justify your answers, not necessarily with computations.

## Problem 4 (Index analysis of DAE-systems)

Consider the following DAE-system

$$\dot{x}_1 = x_3 \tag{5}$$

$$\dot{x}_2 = x_1 \tag{6}$$

$$0 = x_1 - u, \tag{7}$$

where u is the input to the system.

- (a) Determine the differential and algebraic variables.
- (b) Find the degrees of freedom of the considered system.
- (c) Determine the index of the DAE-system.
- (d) Set up the system such that it has index 1 by using the maximum amount of algebraic equations and minimal amount of differential equations.