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EKSAMEN I FAG TEP4100 FLUIDMEKANIKK

Bokmål

Tirsdag 7. juni 2011
Tid: kl. 09.00 - 13.00

Hjelpemidler C: Bestemt enkel kalkulator
K. Rottmann: "Matematisk formelsamling"
Sensuren faller i uke nr 26

Oppgave 1

a) Bruk formelen

$$\nabla p = \rho(\mathbf{g} - \mathbf{a})$$

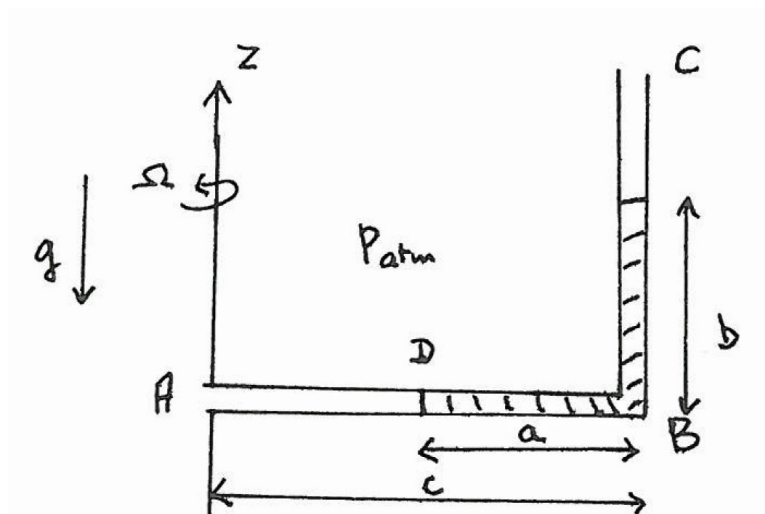
fra formelarket til å gi en kort utledning av uttrykket for trykket $p = p(r, z)$ i et kar som roterer med konstant vinkelhastighet Ω omkring z -aksen,

$$p = p_0 - \gamma z + \frac{1}{2}\rho r^2 \Omega^2.$$

Her er g tyngdens akselerasjon, $\gamma = \rho g$ hvor ρ er tettheten, og p_0 er en konstant.

b) Et tynt sirkulært rør ABC bøyd i rett vinkel er åpent mot atmosfæren i begge ender A og C; se figur 1(a). Atmosfæretrykket er p_{atm} . Røret roterer i tyngdefeltet med konstant vinkelhastighet Ω omkring z -aksen. Avstanden AB er lik c . Røret inneholder en inkompressibel væske. Den totale lengden av væskesøylen er $L = a + b$, hvor a og b er lengdene av henholdsvis den horisontale og den vertikale gren.

Gjør bruk av grensebetingelsene ved søylens endeflater til å beregne for hvilken verdi av Ω de to grenene blir like lange, $a = b = L/2$. Uttrykk svaret ved g, c og L .

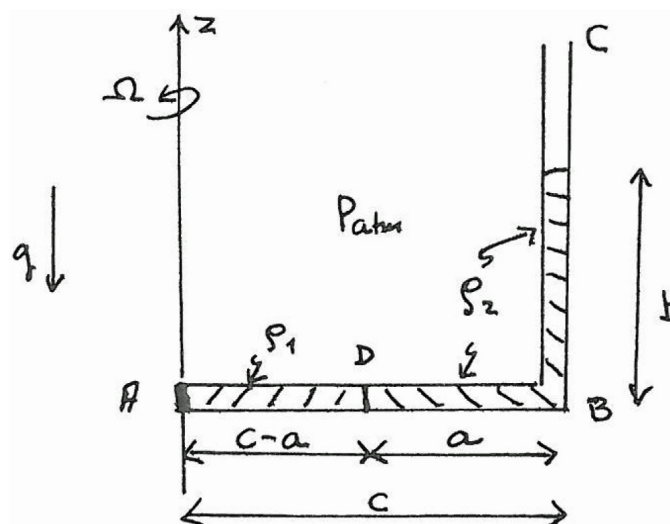


Figur 1(a)

c) Figur 1(b) viser samme situasjon, men hvor det er satt inn en sylindrisk plugg i A slik at denne åpningen er lukket. Røret inneholder nå to inkompressible væsker, den ene med tetthet ρ_1 , den andre med tetthet ρ_2 . Væskesøylenes lengder er henholdsvis $(c - a)$ og $(a + b)$, hvor c, a og b er gitte størrelser (forholdet mellom b og a er nå ikke spesifisert). Røret roterer med konstant vinkelhastighet Ω , som før.

Trykket må være kontinuerlig over grenseflaten (punkt D) mellom væskene 1 og 2. Hvorfor? Bruk dette til å finne trykket p_A i punkt A.

[Hint: Finn trykket i punkt D uttrykt ved størrelser i væske 1, og trykket i samme punkt uttrykt ved tilsvarende størrelser i væske 2. Sett så trykkene lik hverandre.]

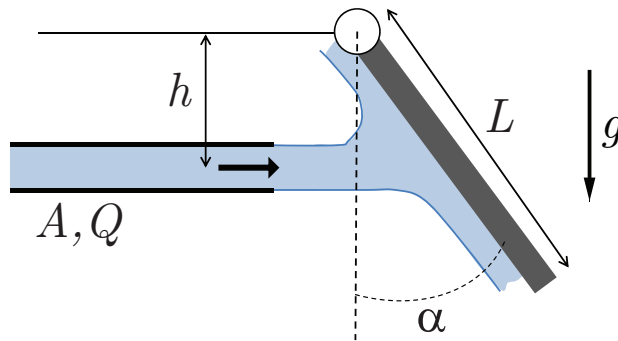


Figur 1(b)

Oppgave 2

DEL I

Figur 2(a) viser et design for en veldig enkel volumstrømmåler. Måleren består av en luke med lengde L og vekt W og kan rotere uten friksjon rundt hengselet på toppen. En væskestråle (tetthet ρ , ikke-viskøs) kommer fra et rør med tverrsnitt A og kolliderer med luken en avstand h under hengselet. Man vil bruke dette utstyret til å måle volumstrømmen Q som kommer fra røret ved å måle vinkelen α .

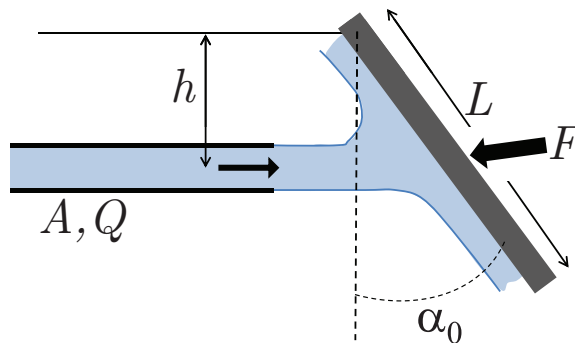


Figur 2(a).

- a) Bruk dreiemomentbalanse til å finne uttrykket for Q som funksjon av α . Se bort fra væskevekta. (Hint: krefter som virker parallelt med luka påvirker ikke dreiemomentet om hengselet).

DEL II

- b) Vi ønsker nå at luka skal danne en gitt vinkel α_0 (se Figur 2(b)). Se bort fra lukevekta og vekta til væsken. Finn et uttrykk for kraften \vec{F} som trengs for å holde luka fast i denne vinkelen. Anta ikke-viskøs væske og at den innkommende strålen deler seg opp i to identiske stråler etter kollisjonen med luka. Bestem kraften \vec{F} for de følgende verdiene: $Q = 288 \text{ m}^3/\text{h}$, $A = 0.008 \text{ m}^2$, $\alpha_0 = 30^\circ$, $\rho = 1000 \text{ kg/m}^3$.



Figur 2(b).

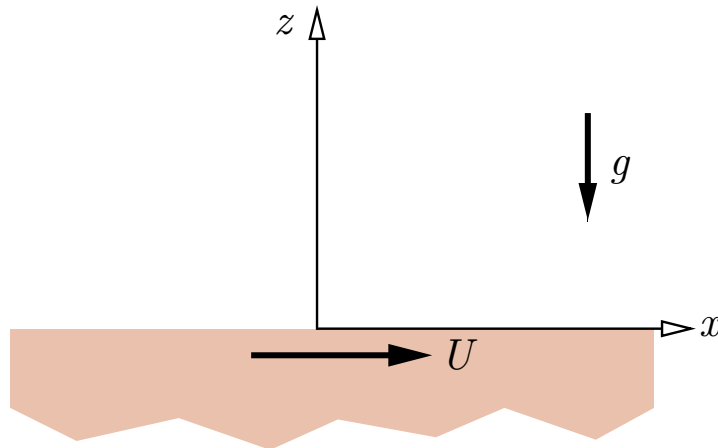
DEL III

Anta nå at væsken som kommer fra røret er viskøs, og at viskositeten avhenger av temperaturen til væsken (viskositeten blir mindre ved høyere temperatur), men tettheten er konstant. Væsken har omgivelsens temperatur ved innløpet til røret og varmes opp langs røret. Reynoldstallet Re_d ved inngangen av røret er lav ($Re_d \approx 1$) og øker langs røret på grunn av den økende temperaturen inntil strømmingen blir turbulent ved utløpet.

- c) Se bort fra inn- og utløpseffekter. Lag en kvalitativ skisse av hastighetsprofilen ved inngangen og ved utgangen av røret.
- d) Tapshøyde i et rør er avhengig av endringer i stedshøyden og trykkfallet langs røret på følgende måte: $h = \Delta z + \Delta p/(\rho g)$. Hva er sammenhengen mellom tapshøyde og Darcys friksjonsfaktor som brukes til å beregne friksjonstap i et rør? Tegn kvalitativt $\log_{10}(-dp/dx)$ versus $\log_{10}(Re_d)$ langs røret, hvor x er posisjonen langs røraksen. Betrakt både et glatt og et ru rør. Anta at $1 < Re_d < 10^6$.

Oppgave 3

Vi vil undersøke hva som skjer når havbunnen eller jordoverflaten plutselig beveger seg i en horisontal retning som vi velger som x -retning. Fordi vi kun betrakter en relativt liten flate i forhold til jordas overflate og ser bort fra endeeffekter, antar vi at strømmingen av vann eller luft er todimensjonal, dvs. y -hastighetskomponent $v = 0$ og $\frac{\partial}{\partial y} = 0$ der y -retningen ligger i flatens plan normal til x -retningen. Dessuten antar vi at x -hastighetskomponent u , z -hastighetskomponent w og trykk p kun avhenger av tiden t og avstanden z fra flaten. Trykket på flaten antas være konstant. Tyngdens akselerasjon g virker i negativ z -retning, se figuren. Vann/luft antas være et Newtonsk fluid med konstant dynamisk viskositet μ og konstant tetthet ρ . Vi vil bruke differensialligningene for inkompressibel og todimensjonal strømning for å regne ut strømmingen som oppstår etter at fluidet ved $t = 0$ har vært i ro og at flaten ved $t = 0$ plutselig settes i bevegelse med konstant hastighet U i x -retning.

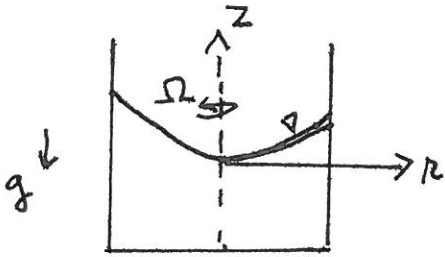


- Bruk kontinuitetsligningen til å vise at z -hastighetskomponent w er null.
- Bestem trykket p .
- Forenkler x -komponenten av Navier-Stokesligningene og gi begrunnelse for alle leddene som er null.

Vis at hvis x -hastighetskomponenten har formen $u(z, t) = U \left(1 - \operatorname{erf} \left(\frac{z}{2\sqrt{\nu t}} \right) \right)$ for $t > 0$, er den en løsning av den forenklete x -komponenten av Navier-Stokesligningene som du fant, og oppfyller heftbetingelsen. $\nu = \frac{\mu}{\rho}$ er den kinematiske viskositeten til fluidet, og $\operatorname{erf}(\zeta) = \frac{2}{\sqrt{\pi}} \int_0^\zeta \exp(-\eta^2) d\eta$ kalles feilfunksjonen.

- Bestem skjærspenningen $\tau_{xz} = \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)$ for $t > 0$. Diskuter dens oppførsel for $t \rightarrow \infty$ og for $z \rightarrow \infty$.

Bestem effekten som man må yte for å bevege en flate med lengde L i x -retning og bredde b i y -retning med konstant hastighet U i x -retning. Se bort fra endeeffekter. Hva vil forholdet være mellom effekten i vann og effekten i luft? Bruk $\mu_{\text{vann}} = 10^{-3} \frac{\text{kg}}{\text{m s}}$, $\rho_{\text{vann}} = 10^3 \frac{\text{kg}}{\text{m}^3}$, $\mu_{\text{luft}} = 1.8 \cdot 10^{-5} \frac{\text{kg}}{\text{m s}}$, $\rho_{\text{luft}} = 1.2 \frac{\text{kg}}{\text{m}^3}$.

Solution Problem 1a)

Start with

$$\nabla p = \rho(\vec{g} - \vec{a}) \quad (1) \quad (\text{Formula sheet})$$

Here \vec{a} is the centripetal acceleration.The r -component of (1):

$$\frac{\partial p}{\partial r} = -\rho a_r = \rho r \Omega^2$$

Integration: $p = \frac{1}{2} \rho r^2 \Omega^2 + f(z)$, $f(z)$ arbitrary functionDifferentiate with respect to z :

$$\frac{\partial p}{\partial z} = f'(z)$$

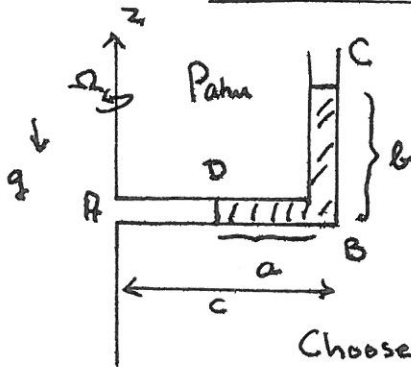
Comparison with the z -component of (1),

$$\frac{\partial p}{\partial z} = \rho g_z = -\rho g, \text{ yields } f'(z) = -\rho g = -\gamma.$$

Thus $f(z) = -\gamma z + p_0$, where p_0 is a constant.

$$\Rightarrow \underline{p = -\gamma z + \frac{1}{2} \rho r^2 \Omega^2 + p_0}$$

Thus p_0 is the pressure at $r = z = 0$.

Solution Problem 1b

Boundary conditions: $P = P_{atm}$ at the end
Surfaces of the column.

Suppose density ρ , with $\gamma = \rho g$.

Start with formula $P = P_0 - \gamma z + \frac{1}{2} \rho \Omega^2 r^2$

Choose $z = 0$ in level AB.

Left end surface D:

$$P_{atm} = P_0 + \frac{1}{2} \rho (c-a)^2 \Omega^2$$

Upper end surface C:

$$P_{atm} = P_0 - \gamma b + \frac{1}{2} \rho c^2 \Omega^2$$

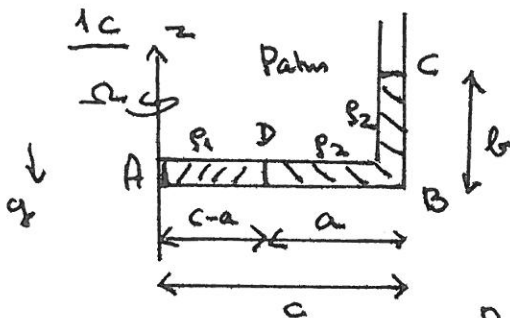
$$\left. \begin{array}{l} P_{atm} = P_0 + \frac{1}{2} \rho (c-a)^2 \Omega^2 \\ P_{atm} = P_0 - \gamma b + \frac{1}{2} \rho c^2 \Omega^2 \end{array} \right\} \Rightarrow \frac{1}{2} \rho (c-a)^2 \Omega^2 = -\gamma b + \frac{1}{2} \rho c^2 \Omega^2$$

Thus $\gamma b = \frac{1}{2} \Omega^2 [c - (c-a)^2]$, which gives

$$\Omega^2 = \frac{2 \gamma b}{c^2 - (c-a)^2} \stackrel{b=a}{=} \frac{2 \gamma a}{c^2 - (c-a)^2} = \frac{2 \gamma}{2c - L/2}$$

$$\Omega = \sqrt{\frac{4 \gamma}{4c - L}}$$

Independent of the density ρ .



The pressure at the boundary D between
the liquids has to be the same; otherwise the
liquid would accelerate.

Seen from liquid 1:

$$P_D = P_A + \frac{1}{2} \rho_1 (c-a)^2 \Omega^2$$

Seen from liquid 2:

$$P_D + \left[\frac{1}{2} \rho_2 c^2 - \frac{1}{2} \rho_2 (c-a)^2 \right] \Omega^2 = P_B = \underbrace{P_{atm} + \rho_2 g b}_{\text{Static pressure in } P_B}$$

Set the two expressions for P_D equal to each other:

$$P_A + \frac{1}{2} \rho_1 (c-a)^2 \Omega^2 = P_{atm} + \rho_2 g b - \left[\frac{1}{2} \rho_2 c^2 - \frac{1}{2} \rho_2 (c-a)^2 \right] \Omega^2$$

\Rightarrow

$$\underline{P_A = P_{atm} + \rho_2 g b - \frac{1}{2} (\rho_1 - \rho_2) (c-a)^2 \Omega^2 - \frac{1}{2} \rho_2 c^2 \Omega^2}$$

Problem 2

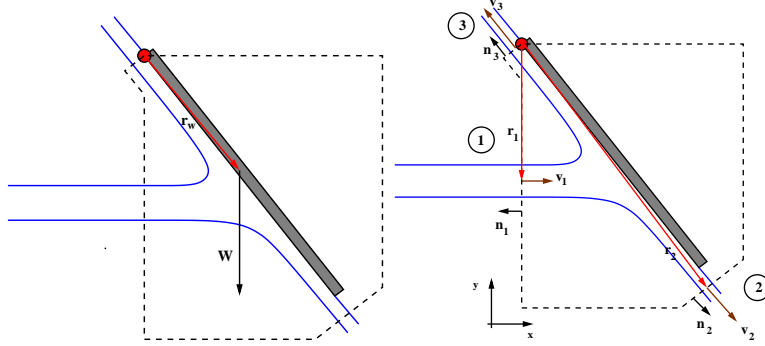
Part I

(a)

For a nondeformable inertial control volume CV, the conservation of angular momentum (c.f. WHITE, equation (3.55)) gives us the following relation between torque and change of angular momentum in CV:

$$\sum \vec{M} = \frac{d}{dt} \left[\int_{CV} \rho (\vec{r} \times \vec{v}) dV \right] + \int_{CS} \rho (\vec{r} \times \vec{v}) (\vec{v} \cdot \vec{n}) dA, \quad (1)$$

where \vec{v} is the velocity of the fluid, \vec{r} is the position vector and \vec{n} the normal vector. For the present problem a good choice of a control volume would be the following:



In these figures a red color is used for the position vectors, whereas the velocity vectors are in brown, the normal vectors and the weight vector of the plate in black. The control volume is marked by a dashed line. In the left figure we sketched the position vector \vec{r}_w of the center of gravity where the weight \vec{W} of the plate acts. The resulting torque $\vec{r}_w \times \vec{W}$ around the hinge (marked by a red dot) is the only contribution to the sum on the left hand side of equation (1). In order to compute the right hand side of equation (1), see figure to the right, we remark, that the first term, the unsteady term, vanishes, since we are dealing with a steady system. It remains thus to compute the following surface integration over the control surface:

$$\int_{CS} \rho (\vec{r} \times \vec{v}) (\vec{v} \cdot \vec{n}) dA. \quad (2)$$

We proceed by splitting up the integration into three contributions:

$$\int_{CS} = \int_{CS_1} + \int_{CS_2} + \int_{CS_3}, \quad (3)$$

where the sides CS_i are marked by their respective number in the right hand figure. Since \vec{r}_2 is approximately parallel to \vec{v}_2 , the cross product $\vec{r}_2 \times \vec{v}_2 \approx \vec{0}$ vanishes:

$$\int_{CS_2} \rho (\vec{r}_2 \times \vec{v}_2) (\vec{v}_2 \cdot \vec{n}_2) dA = 0. \quad (4)$$

For the side CS_3 , we have that $\vec{r}_3 \approx \vec{0}$ and therefore also the surface integral on this side vanishes:

$$\int_{CS_3} \rho (\vec{r}_3 \times \vec{v}_3) (\vec{v}_3 \cdot \vec{n}_3) dA = 0. \quad (5)$$

The only contribution we are left with is the integral on CS_1 :

$$\int_{CS} \rho (\vec{r} \times \vec{v}) (\vec{v} \cdot \vec{n}) dA = \int_{CS_1} \rho (\vec{r}_1 \times \vec{v}_1) (\vec{v}_1 \cdot \vec{n}_1) dA = -\rho v_1^2 h A \vec{e}_z = -\rho Q^2 \frac{h}{A} \vec{e}_z, \quad (6)$$

where \vec{e}_z is the unit vector in z -direction (pointing out of the paper). As mentioned above, the torque on the left hand side of (1) is given by:

$$\sum \vec{M} = \vec{r}_w \times \vec{W} = -\frac{L}{2} W \sin \alpha \vec{e}_z. \quad (7)$$

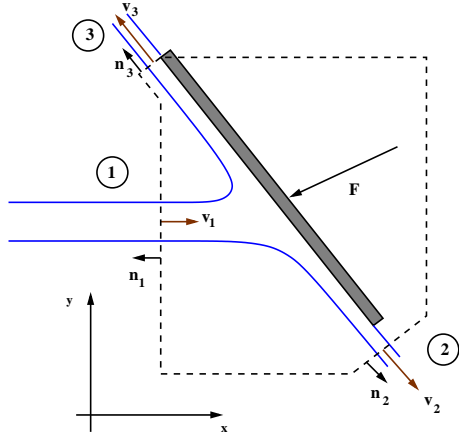
Equations (6) and (7) allow us to find a relation between α and Q :

$$Q = \sqrt{\frac{W L A \sin \alpha}{2 \rho h}}. \quad (8)$$

Part II

(b)

For this exercise we can use the same control volume as in part (a):



The linear momentum equation for a fixed control volume is given by (WHITE (3.37)):

$$\vec{F} = \frac{d}{dt} \left[\int_{CV} \rho \vec{v} dV \right] + \int_{CS} \rho \vec{v} (\vec{v} \cdot \vec{n}) dA. \quad (9)$$

Since the system is steady we only need to compute the momentum flux through the boundaries of the control volume:

$$\int_{CS} \rho \vec{v} (\vec{v} \cdot \vec{n}) dA. \quad (10)$$

The jet splits up into two identical jets after impinging on the plate. Therefore, we have $\vec{v}_2 = -\vec{v}_3 = v_2 \vec{n}_2$ and $A_2 = A_3$. The momentum flux through CS_2 can then be computed by:

$$\int_{CS_2} \rho \vec{v} (\vec{v} \cdot \vec{n}) dA = \int_{CS_2} \rho \vec{v}_2 (\vec{v}_2 \cdot \vec{n}_2) dA = \rho v_2^2 A_2 \vec{n}_2. \quad (11)$$

On CS_3 , the calculation is similar:

$$\int_{CS_3} \rho \vec{v} (\vec{v} \cdot \vec{n}) dA = \int_{CS_3} \rho \vec{v}_3 (\vec{v}_3 \cdot \vec{n}_3) dA = -\rho v_2^2 A_2 \vec{n}_2. \quad (12)$$

The contributions on CS_2 , equation (11), and on CS_3 , equation (12), cancel. The velocity \vec{v}_1 on CS_1 can be written as

$$\vec{v}_1 = v_1 \vec{e}_x. \quad (13)$$

The momentum flux is then

$$\int_{CS_1} \rho \vec{v} (\vec{v} \cdot \vec{n}) dA = \int_{CS_1} \rho \vec{v}_1 (\vec{v}_1 \cdot \vec{n}_1) dA = -\rho v_1^2 A_1 \vec{e}_x. \quad (14)$$

Neglecting the weight of the plate and the water and knowing that the pressure force sums to zero, since we have only atmospheric pressure on all sides, the sum on the right hand side of (9) consists only of the force \vec{F} acting on the plate. Equation (9) becomes therefore:

$$\vec{F} = -\rho v_1^2 A_1 \vec{e}_x = -\rho \frac{Q^2}{A_1} \vec{e}_x. \quad (15)$$

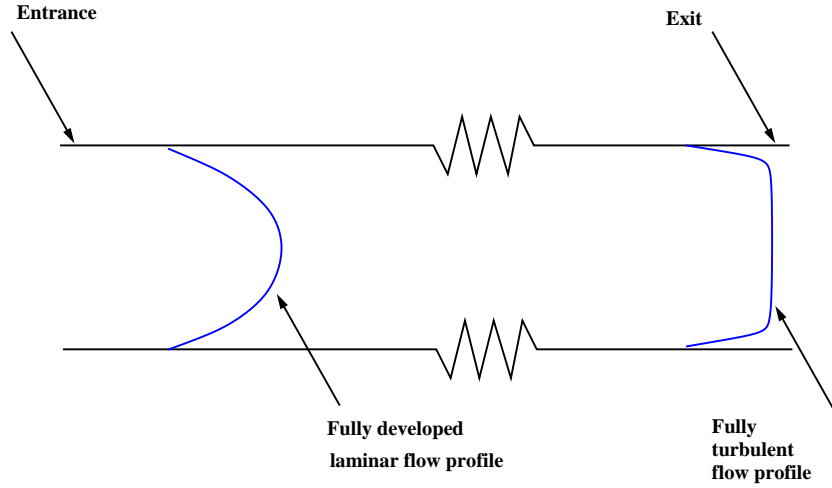
The force is thus aligned with the x -axis. Using the given data we can compute the value of F_x :

$$F_x = -\frac{10^3 \frac{\text{kg}}{\text{m}^3} 288^2 \frac{\text{m}^6}{\text{s}^2}}{(3.6 \cdot 10^3)^2 \cdot 8 \cdot 10^{-3} \text{m}^2} = -7.97 \cdot 10^2 \text{N}. \quad (16)$$

Part III

(c)

Average flow profiles:



(d)

The head loss in the pipe is related to the change in height and the pressure drop in the following way:

$$h = \Delta z + \frac{\Delta p}{\rho g}. \quad (17)$$

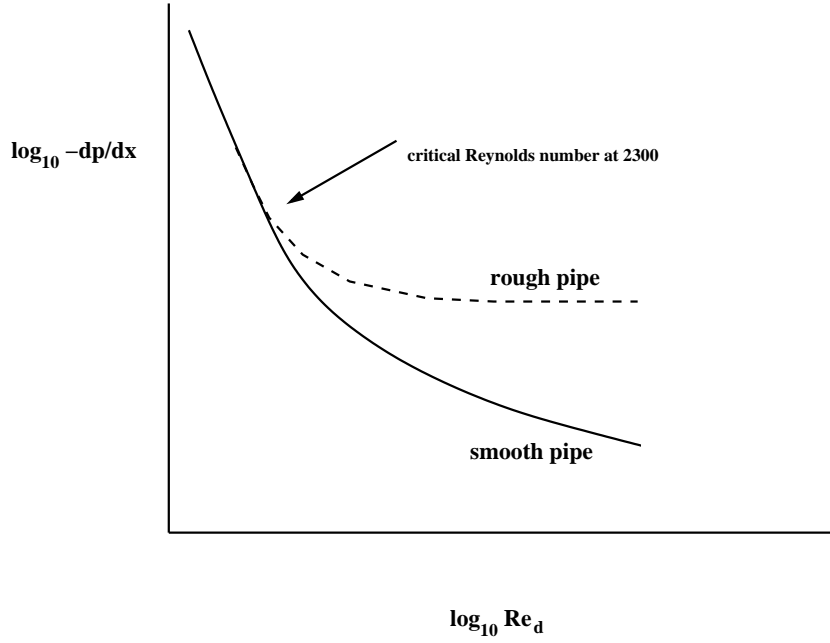
Since the pipe is horizontal, we have $\Delta z = 0$. For the head loss we are on the other hand given that:

$$h = f \frac{L}{d} \frac{V^2}{2g}. \quad (18)$$

We can therefore write:

$$-\frac{dp}{dx} \approx \frac{\Delta p}{L} = f \frac{\rho}{d} \frac{V^2}{2}. \quad (19)$$

Assuming that ρ does not change significantly with temperature, the velocity stays constant due to volume conservation. The only variation of $\frac{dp}{dx}$ is therefore due to f , the friction factor. The Moody diagram tells us how f changes with the Reynolds number Re_d for smooth and rough pipes. Thus the final sketch is:



Problem 3

(a)

The continuity equation for incompressible flow is given on the formula sheet:

$$\nabla \cdot \mathbf{u} = 0 . \quad (20)$$

In Cartesian coordinates, it reads:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 . \quad (21)$$

It is given in the problem description that $v = 0$. In addition it is given that the x -component u of the velocity only depends on z and t , thus $\frac{\partial u}{\partial x} = 0$. The continuity equation (21) reduces therefore to:

$$\frac{\partial w}{\partial z} = 0 . \quad (22)$$

Integrating (22) with respect to z gives $w(z, t) = C_1(t)$, where $C_1(t)$ is an integration constant which can be determined from the boundary condition. The normal velocity at the wall is zero, i.e., $w(z = 0, t) = 0$, which gives $C_1(t) = 0$ and therefore

$$w = 0 . \quad (23)$$

(b)

From the problem description we know that the pressure only depends on z and t . To find the pressure we use the z -component of the Navier-Stokes equations:

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - g . \quad (24)$$

With $w = 0$ (24) reduces to $\frac{\partial p}{\partial z} = -\rho g$, which is integrated with respect to z leading to $p(z, t) = -\rho g z + C_2(t)$. Using the pressure boundary condition $p(z = 0, t) = p_{\text{wall}}$, we can determine the integration constant as $C_2(t) = p_{\text{wall}}$. Finally the pressure is:

$$p(z) = -\rho g z + p_{\text{wall}} . \quad (25)$$

(c)

The x -component of the incompressible Navier-Stokes equations reads:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) - g_x . \quad (26)$$

Since u only depends on z and t , we have

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2} = 0 . \quad (27)$$

Since we showed in question a) that $w = 0$ and in question b) that $p = p(z)$, we have $w \frac{\partial u}{\partial z} = \frac{\partial p}{\partial x} = 0$. There is no external acceleration in x -direction which gives $g_x = 0$. Consequently (26) reduces to

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial z^2} . \quad (28)$$

Differentiating the given velocity profile by applying the fundamental theorem of calculus

$$\frac{d \operatorname{erf}(\zeta)}{d\zeta} = \frac{2}{\sqrt{\pi}} \frac{d}{d\zeta} \left(\int_0^\zeta \exp(-\eta^2) d\eta \right) = \frac{2}{\sqrt{\pi}} \exp(-\zeta^2) , \quad (29)$$

as well as the chain rule;

$$\frac{\partial \operatorname{erf}(\zeta(z, t))}{\partial z} = \frac{d \operatorname{erf}(\zeta(z, t))}{d\zeta} \frac{\partial \zeta(z, t)}{\partial z} = \frac{2}{\sqrt{\pi}} \exp\left(-\frac{z^2}{4\nu t}\right) \frac{1}{2\sqrt{\nu t}} \quad (30)$$

and

$$\frac{\partial \operatorname{erf}(\zeta(z, t))}{\partial t} = \frac{d \operatorname{erf}(\zeta(z, t))}{d\zeta} \frac{\partial \zeta(z, t)}{\partial t} = \frac{2}{\sqrt{\pi}} \exp\left(-\frac{z^2}{4\nu t}\right) \left(-\frac{z}{4\sqrt{\nu t^3}}\right) , \quad (31)$$

with $\zeta(x, t) = \frac{z}{2\sqrt{\nu t}}$, we get:

$$\frac{\partial u}{\partial t} = \frac{Uz}{2\sqrt{\pi\nu t^3}} \exp\left(-\frac{z^2}{4\nu t}\right) \quad (32)$$

$$\frac{\partial u}{\partial z} = -\frac{U}{\sqrt{\pi\nu t}} \exp\left(-\frac{z^2}{4\nu t}\right) \quad (33)$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{Uz}{2\sqrt{\pi\nu^3 t^3}} \exp\left(-\frac{z^2}{4\nu t}\right) . \quad (34)$$

To check whether the given velocity profile $u(z, t)$ is a solution of the simplified x -component of the Navier-Stokes equations, we insert (32) and (34) into (28)

$$\frac{Uz}{2\sqrt{\pi\nu t^3}} \exp\left(-\frac{z^2}{4\nu t}\right) = \nu \frac{Uz}{2\sqrt{\pi\nu^3 t^3}} \exp\left(-\frac{z^2}{4\nu t}\right) . \quad (35)$$

Dividing the numerator and denominator of the right hand side of (35) by ν , we see that (35) indeed is an equality.

The no-slip condition requires that $u(z = 0, t) = U$ for $t > 0$. The given velocity profile satisfies the boundary condition since $\text{erf}(0) = 0$.

(d)

We know from question a) that $w = 0$ and therefore $\frac{\partial w}{\partial x} = 0$. The derivative of the velocity in z -direction is given in (33), leading to the following expression for the shear stress:

$$\tau_{xz} = -\frac{\mu U}{\sqrt{\pi\nu t}} \exp\left(-\frac{z^2}{4\nu t}\right) . \quad (36)$$

As $t \rightarrow \infty$ we have

$$\tau_{xz} = -\underbrace{\frac{\mu U}{\sqrt{\pi\nu t}}}_{\rightarrow 0} \exp\left(\underbrace{-\frac{z^2}{4\nu t}}_{\rightarrow 0}\right) \rightarrow 0 . \quad (37)$$

After a sufficient long time the fluid will have the velocity $u = U$ everywhere and the shear stress will become zero. Similarly for $z \rightarrow \infty$,

$$\tau_{xz} = -\frac{\mu U}{\sqrt{\pi\nu t}} \exp\left(\underbrace{-\frac{z^2}{4\nu t}}_{\rightarrow -\infty}\right) \rightarrow 0 . \quad (38)$$

Far away from the surface the fluid will not be aware of the moving surface since the momentum introduced by the wall is not yet propagated far enough into the fluid, therefore the fluid velocity is still $u = 0$, thus there will be no shear stress.

The friction force at the surface acting on the fluid in x -direction is given by the area of the surface multiplied by the negative shear stress at the wall $\tau_{xz}(z=0)$ (for $\boldsymbol{\tau} \cdot \mathbf{n} = -\tau_{xz}\mathbf{e}_x$, with $\mathbf{n} = -\mathbf{e}_z$)

$$F(t) = \frac{\mu L b U}{\sqrt{\pi \nu t}} = \sqrt{\frac{\mu \rho}{\pi t}} L b U . \quad (39)$$

The power required to move the surface is obtained by multiplying the friction force by the velocity

$$P(t) = \sqrt{\frac{\mu \rho}{\pi t}} L b U^2 . \quad (40)$$

Finally the ratio between the power in water and air can be computed as:

$$\frac{P_{\text{water}}}{P_{\text{air}}} = \sqrt{\frac{\mu_{\text{water}} \rho_{\text{water}}}{\mu_{\text{air}} \rho_{\text{air}}}} = 215.16 . \quad (41)$$