Out: March 07, 2018
Recommended completion: April 10, 2018

NTNU – Trondheim
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Exercise 9

TTK4130 Modeling and Simulation

Problem 1 (Sliding stick (Exam 2010))

Consider a stick of length ℓ with uniformly distributed mass m. It has center of mass/gravity C, about which it has a moment of inertia I_z. The stick is in contact with a frictionless horizontal surface, and moves due to the influence of gravity. See Figure 1.

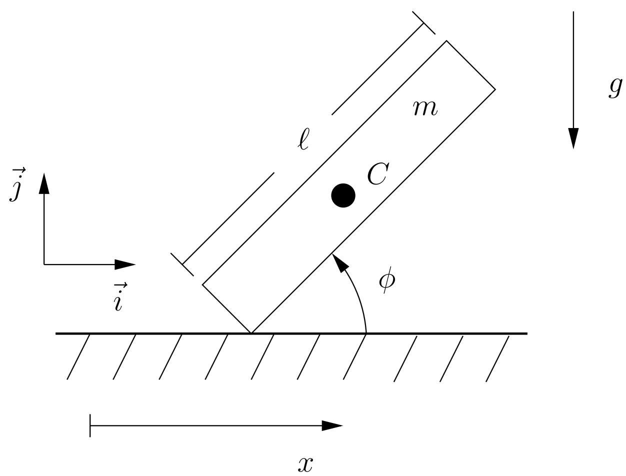

Figure 1: Stick sliding on frictionless surface

(a) Choose appropriate generalized coordinates (the figure should give you some hints). What are the corresponding generalized (actuator) forces?

(b) What are the position, velocity, and angular velocity of the center of mass, as function of your chosen generalized coordinates (and/or their derivatives)?

(c) Write up the kinetic and potential energy of the stick, as function of your chosen generalized coordinates (and/or their derivatives).

(d) Derive the equations of motion for the stick.

Problem 2 (Double inverted pendulum)

The double inverted pendulum on a cart (DIPC) poses a The double inverted pendulum on a cart (DIPC) poses a challenging control problem. In a DIPC system, two rods are connected together on a moving cart as shown in Figure 2. The rod is located above the centre of mass of the cart. The length of the first rod is denoted by l_1 and the length of the second rod by l_2. The mass of the cart is denoted by m_0, its length by l_0 and its width by b_0. The height of the cart is denoted by h_0. Both rods have a mass, which are denoted by m_1 and m_2. All masses are assumed to be concentrated into the centre of mass. The moments of inertia are denoted by I_i. Furthermore, the force τ is acting on the cart.

(a) Find the position of cart and the two rods.

(b) Find the kinetic energy of the system. $\left(\textit{Hint: } \cos(x-y) = \cos x \cos y + \sin x \sin y\right)$

(c) Find the potential energy of the system.

(d) Find the equation of motion of the system.

(e) The moment of inertia is defined as

$$I = \int_Q r^2 dm \tag{18}$$

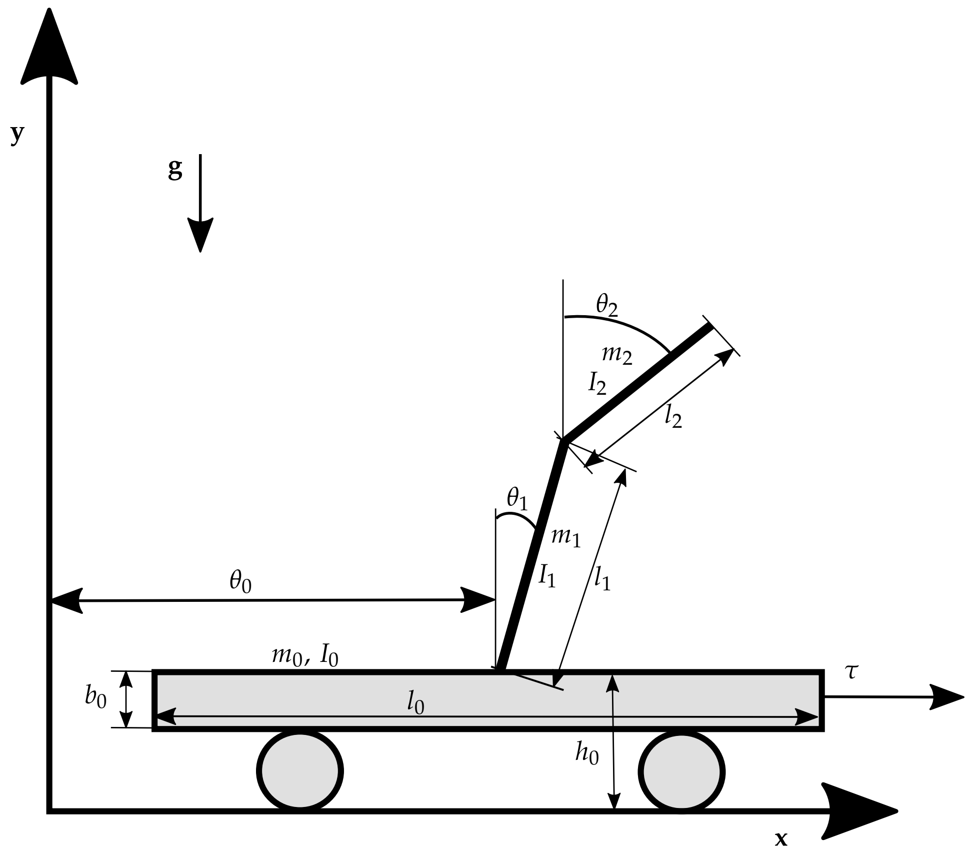

Figure 2: Double inverted pendulum on a cart

where r is the distance from each point to the axis of rotation and Q is the entire mass. Derive the moment of inertia for the rectangular plate with the length l and height h for the centre of mass of the plate (Fig. 3). The axis of rotation is the z-axis (perpendicular to the plate). Assume that the plate has a constant width b and a homogeneous density ρ.

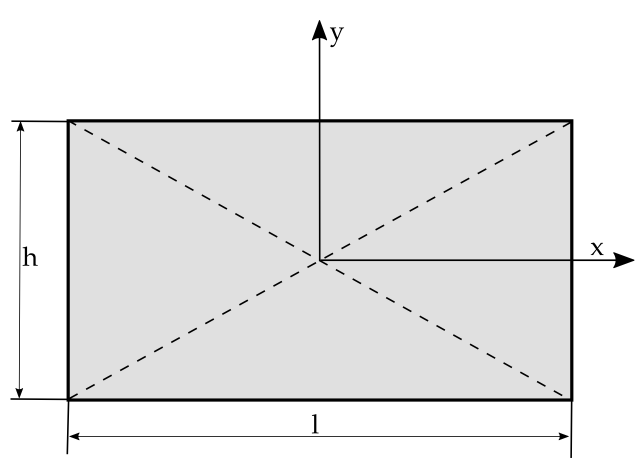

Figure 3: Rectangular plate

(f) Use the parallel axes theorem to change the point of rotation from the centre of mass to point **A** given in Fig. 5.
(*Help: If you were not able to solve the previous task, use as moment of inertia* $I = \frac{1}{4}m(l^2 + h^2)$ *Please state explicitly, if you use this for further calculations.*)

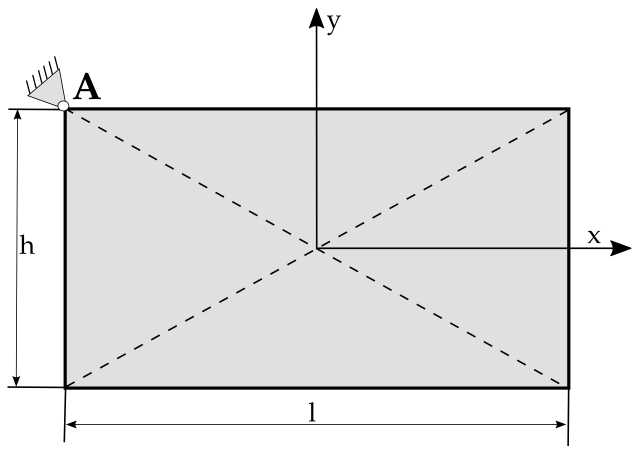

Figure 5: Rectangular plate with attachment on one corner

(g) Make an appropriate simplification for the moment of inertia of the rods used in the DIPC system. Justify your decision!