NORWEGIAN UNIVERSITY
OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF ELECTRONICS AND TELECOMMUNICATIONS

Contact during examination:
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EXAMINATION IN COURSE
TTT4120 DIGITAL SIGNAL PROCESSING

Date: Monday, 7 Desember 2009
Time: 09.00 - 13.00

Permitted aids: D–No printed or hand-written material allowed.
Specified, simple calculator allowed.

INFORMATION

- The examination consists of 4 problems:
  - Problem 1 concerns analysis of digital filters.
  - Problem 2 concerns finite word length effects in filter structures.
  - Problem 3 concerns stocastical processes.
  - Problem 4 concerns multirate systems.

  The weight of each subproblem is given in parenthesis. Total number of points is 50.
- **The solution steps should be evident and all answers should be justified!**
- Some important formulas can be found in the appendix.
- The teacher will go around twice, around 10.00 and around 11.45.
- Grades will be announced by January 7th.

Good luck!

Problem 1 (2+2+2+1+1+1+2=11 points)

Figure 1 shows an implementation of a digital filter.

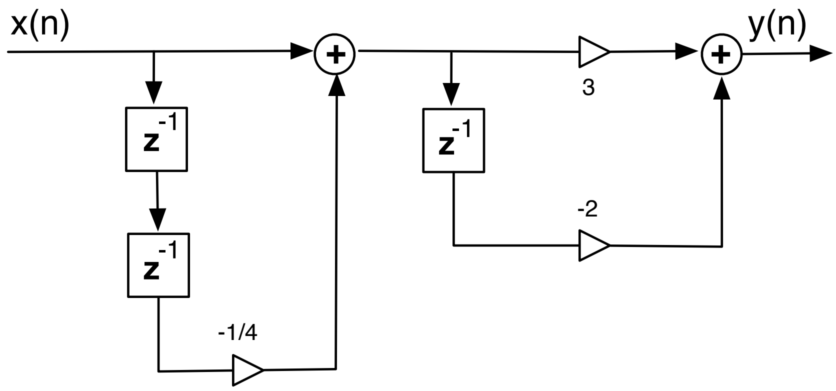


Figure 1: Implementation of a digital filter

1a) Write a difference equation that shows the relationship between input signal $x(n)$ and output signal $y(n)$.

1b) Find the transfer function of the filter.

1c) Find the unit sample response of the filter.

1d) Is this an FIR or an IIR filter? Justify your answer.

1e) Does the filter have a linear phase response? Justify your answer.

1f) Does the filter have a causal and stable inverse filter? Justify your answer.

1g) Find the unit sample response of the stable inverse filter.

Problem 2 (2+3+4+4=13 points)

All internal and external signals in the filter structure in Figure 1 are represented by binary fixed-comma representation with 8 bits and the dynamic range $[-1, 1)$.

2a) Find the value of the rounding error power σ_e^2 after a multiplication.

2b) Show that the signal power at the output of an LTI filter with unit sample response $h(n)$ is equal to

$$\sigma_q^2 = \sigma_e^2 \, r_{hh}(0)$$

when white noise $e(n)$ with power σ_e^2 is sent at the input.

2c) Find the total rounding error power at the output of the filter in Figure 1.

2d) Find the necessary scaling factor at the input of the filter in Figure 1 such that we are guaranteed to avoid overflow.

Problem 3 (1+4+4+2=11 points)

A stocastical process $x(n)$ is given by

$$x(n) = w(n) - 0.5w(n-1),$$

where $w(n)$ is white Gaussian noise with varianse $\sigma_w^2 = 1$.

3a) Which type of prosess is $x(n)$?

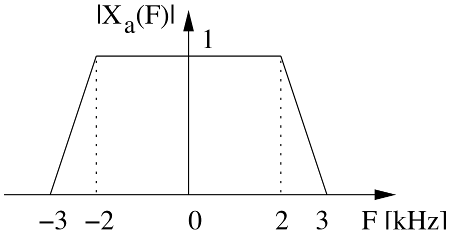
3b) Find autocorrelation function $\gamma_{xx}(l)$ and power spectral density $\Gamma_{xx}(f)$ of the process.

3c) Derive an expression for the optimal prediction coefficient of the first order linear predictor of the process $x(n)$ and find its value. Find also the corresponding prediction error power.

3d) Write an expression for an estimator of the power spectral density $\Gamma_{xx}(f)$ based on the best AR(1)-model of the process $x(n)$.

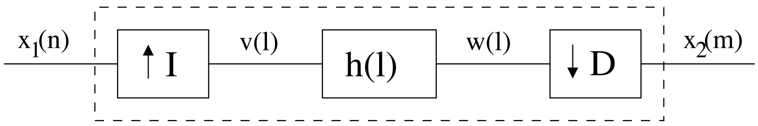
Problem 4 (6+1+6+2=15 points)

Let $x_a(t)$ be an analog signal with magnitude spectrum $|X_a(F)|$ shown in the figure below.



A discrete-time signal $x_1(n)$ was generated by sampling the signal $x_a(t)$ with sampling frequency $F_{s1} = 8$ kHz.

We wish to design a digital system that increases the sampling frequency of the signal $x_1(n)$ to $F_{s2} = 12$ kHz. The block diagram of the system is shown in the figure below.



4a) Explain the function of each block in the system and write expressions that describe the relationship between

- $v(l)$ and $x_1(n)$
- $w(l)$ and $v(l)$
- $x_2(m)$ and $w(l)$

4b) Find the minimal values of I and D needed to achieve the desired rate conversion.

4c) All the graphs in this subproblem should cover the frequency range that corresponds at least to $f \in [-1, 1]$, where f is the digital frequency. Write the value of the corresponding sampling frequency for each graph.

Sketch the magnitude spectra of the signals $x_1(n)$, $v(l)$, $w(l)$ and $x_2(m)$. Give a short justification of each graph.

Find the necessary specifications for the filter, i.e. the filter type and the limit frequencies of the passband and stopband. Justify your answer.

Sketch the magnitude response of a filter that fulfills the specifications.

4d) Is the order of upsampling and downsampling in the above block diagram important, i.e. can we achieve the same result by exchanging the two blocks and possible change of the filter specifications? Justify your answer.