Assignment 5, ttk4215

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Problem 4.9

Using the same setup as in Assignment 4, which was

$$\theta = \begin{bmatrix} m \\ \beta \\ k \end{bmatrix}$$

$$\phi = \frac{\begin{bmatrix} s^2 \\ s \\ s \end{bmatrix}}{s^2 + \lambda_1 s + \lambda_2} y$$

$$z = \frac{u}{s^2 + \lambda_1 s + \lambda_2}$$

For both d and e, I tried all three least square algorithms. I didn't change the design parameters much between each version this might not be a fair comparison. For all the given plots, I've used $\alpha = 1$, $\beta = 1$ (forgetting factor), $\rho_0 = 100$, $\rho_1 = 10$, $P_0 = \rho_0 I$, and $R_0 = 1$.

d

With constant $\theta^* = \begin{bmatrix} 20 \\ 0.1 \\ 5 \end{bmatrix}$, we get the following plots.

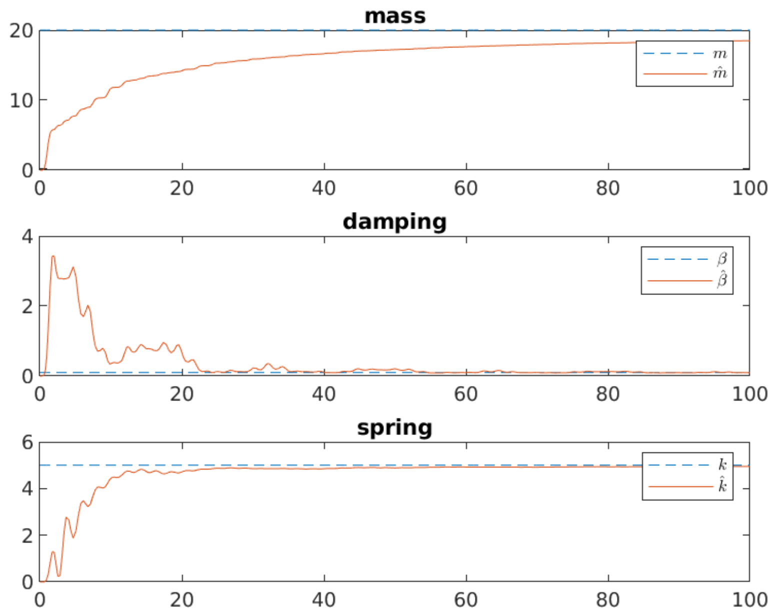

Figure 1: Problem 4.9 d: Pure Least-squares

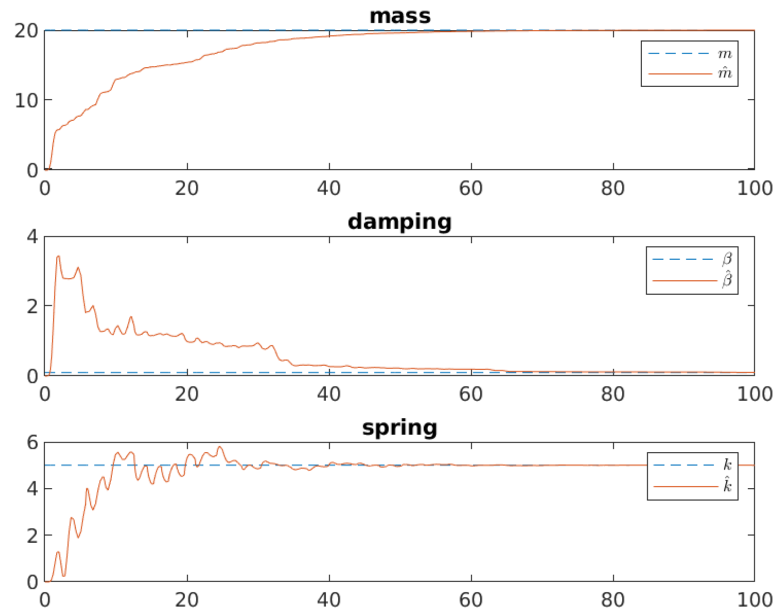

Figure 2: Problem 4.9 d: Least-squares with covariance resetting

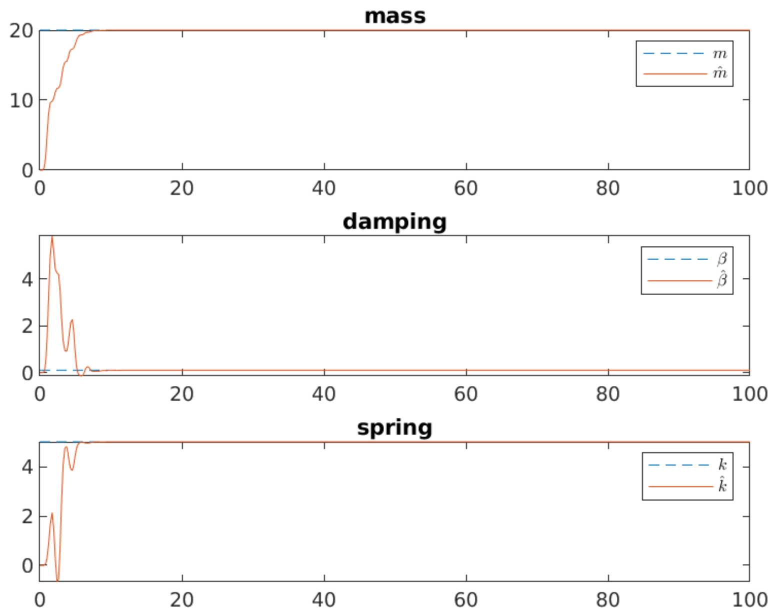

Figure 3: Problem 4.9 d: Least-squares with forgetting

e

With θ^* time varying as given in the text, we get the following plots.

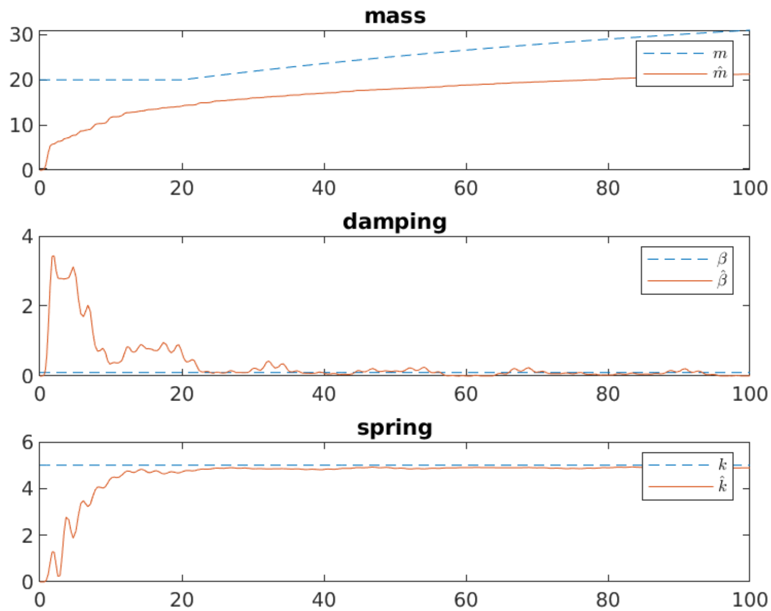

Figure 4: Problem 4.9 e: Pure Least-squares

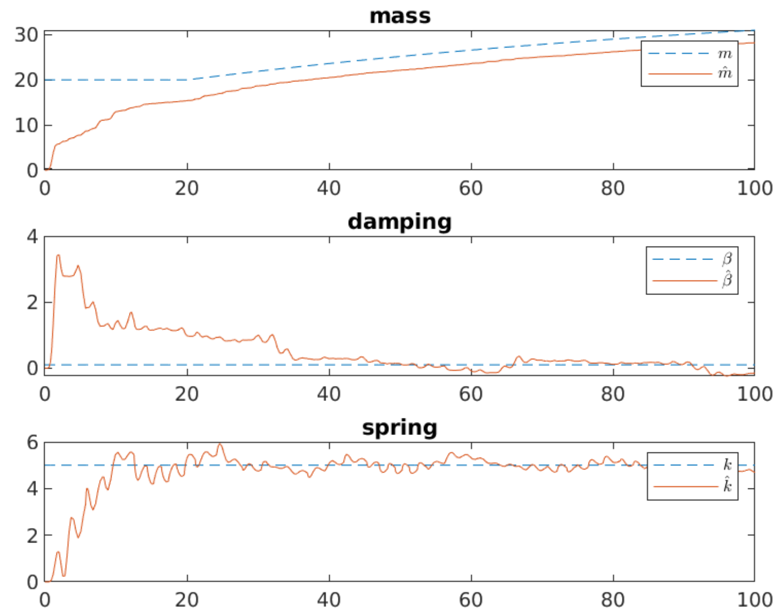

Figure 5: Problem 4.9 e: Least-squares with covariance resetting

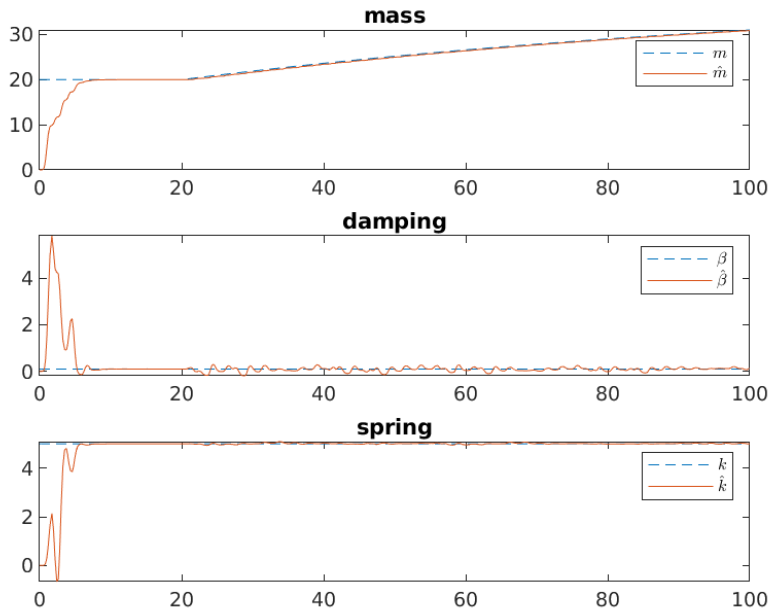

Figure 6: Problem 4.9 e: Least-squares with forgetting

Problem 4.10

a

Spring is stretched out a distance $d = y_1 - y_2$. Hooks law gives that the force from the spring is $F_k = kd = k(y_1 - y_2)$. Damping is proportional to speed of mass $F_d = \beta \dot{y_2}$. Newtons law then gives

$$m\ddot{y_2} = -F_d + F_k = -\beta\dot{y_2} + k(y_1 - y_2),$$

which is equivalent to

$$k(y_1 - y_2) = m\ddot{y_2} + \beta\dot{y_2}.$$

u applies a force to the spring, and balancing that force with the spring force we get

$$F_k = u \Leftrightarrow k(y_1 - y_2) = u.$$

b

Writing the equations in Laplace form.

$$s^2 m y_2 + s\beta y_2 = u$$

$$k(y_1 - y_2) = u$$

We want to estimate $\theta^* = \begin{bmatrix} m \\ \beta \\ k \end{bmatrix}$. By writing

$$\phi = \frac{\begin{bmatrix} s^2 y_2 \\ s y_2 \\ y_1 - y_2 \end{bmatrix}}{\Lambda(s)}$$

where $\Lambda(s)$ is Hurwitz, we get

$$z = {\theta^*}^T \phi = \frac{s^2 m y_2 + s\beta y_2}{\Lambda(s)} + \frac{k(y_1 - y_2)}{\Lambda(s)} = \frac{u}{\Lambda(s)} + \frac{u}{\Lambda(s)} = \frac{2u}{\Lambda(s)}$$

So using z, ϕ, and θ as above, we have written the system in linear parametric form.

Now we can apply any of the standard on-line estimators, for instance SPR-Lyapunov design with $W(s) = 1$.

Estimation model: $\hat{z} = \theta^T \phi$

Normalized estimation error: $\epsilon = \frac{z - \hat{z}}{m^2}$

Adaptive law: $\dot{\theta} = \Gamma \epsilon \phi$

Design parameters: $m^2 = 1 + n_s^2$ such that $\frac{\phi}{m} \in \mathcal{L}_\infty$, $\Lambda(s) = s^2 + \lambda_1 s + \lambda_2$ with $\lambda_1, \lambda_2 > 0$.