Godkjent
B

Øving 3

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2.5 6)

a) "All mail carriers carry a can of mace" (1)
"Mrs. Bacon is a mail carrier" (2)

(1): $\forall x \, [m(x) \to c(x)]$
(2): $m(b)$

where $m(x)$: "x is a mail carrier"
$c(x)$: "x carries a can of mace"
b : Mrs. Bacon

(1) and (2) gives:

$$\begin{array}{l} m(b) \to c(b) \\ \underline{m(b) \qquad} \\ \therefore c(b) \end{array}$$

(modus ponens)

So Mrs Bacon carries a can of mace.

b) "All law abiding citizens pay their taxes" (1)
"Mr. Pelosi pays his taxes" (2)

$L(x)$: x is a law abiding citizen
$T(x)$: x pays taxes
p = Mr. Pelosi

(1): $\forall x\, [L(x) \to T(x)]$
(2): $T(p)$

$T(p)$ does not imply (or deny) the possibility that $L(p)$ is true. Thus we don't know and the argument is <u>invalid</u>.

c) "All people who are concerned with the environment, recycle their plastic containers." (1)
"Margarita is not concerned about the environment." (2)

$e(x)$: x is concerned about the environment
$p(x)$: x recycles plastic containers
M = Margarita

(1): $\forall x\, [e(x) \to p(x)]$
$\neg p(M)$

$e(M)$ may or may not be true, thus the conclusion (argument) is <u>invalid</u>.

8)

a) (1) Steps | Reasons

Steps	Reasons
$\forall x\ p(x)$	Premise
$p(a)$	Universal specification
$p(a) \vee q(a)$	Disjunctive Amplification
$\therefore \forall x\ [p(x) \vee q(x)]$	Universal generalization

(2)

Steps	Reasons
$\forall x\ q(x)$	Premise
$q(a)$	Universal specification
$p(a) \vee q(a)$	Disjunctive amplificatio
$\therefore \forall x\ p(x) \vee q(x)$	Universal generalization

(3)

Steps	Reasons
$\forall x\ p(x) \rightarrow \forall x\ [p(x) \vee q(x)]$	(1)
$\forall x\ q(x) \rightarrow \forall x\ [p(x) \vee q(x)]$	(2)

$\therefore [\forall x p(x) \vee \forall x q(x)] \rightarrow \forall x [p(x) \vee q(x)]$ proof by cases

$\forall x p(x) \vee \forall x q(x) \Rightarrow \forall x\ [p(x) \vee q(x)]$

b) $\mathcal{U}$: integers , $p(x)$: "x is even"
$q(x)$: "x is odd"

We have then that $\forall x\ p(x) \Leftrightarrow F_0$
and $\forall x\ q(x) \Leftrightarrow F_0$
and $\forall x\ [p(x) \vee q(x)] \Leftrightarrow T_0$

9)

1) Premise
2) Premise
3) Universal specification (1)
4) Universal specification (2)
5) Conjunctive Simplification (4)
6) Modus ponens (5) and (3)
7) Conjunctive simplification (6)
8) —— || ——— (4)
9) Conjunction (7) and (8)
10) Universal generalization

1.2 20)

a) D A T G R M
A
A

$$\frac{8!}{3!} = 6720$$

b) D (AAA) T G R M

$6! = 720$

1.3 8) a) There are $\binom{13}{5}$ ways for one suit and 4 suits, so the answer is:

$$\binom{13}{5} \cdot 4$$

b) After picking 4 aces there are 48 cards left in the deck, so the answer is

48

c) 13 ways to pick four of a kind, 48 cards remaining.

$13 \cdot 48 = 624$

d) Pick 3 aces out of 4: $\binom{4}{3}$
Pick 2 jack out of 4: $\binom{4}{2}$

$$\binom{4}{3} \cdot \binom{4}{2} = 4 \cdot 6 = \underline{\underline{24}}$$

e) Pick 3 aces: $\binom{4}{3}$
12 remaining pairs with $\binom{4}{2}$ ways per pair:

$$\binom{4}{3} \cdot 12 \cdot \binom{4}{2}$$
$$= 4 \cdot 12 \cdot 6$$
$$= \underline{\underline{288}}$$

f) 13 ways to pick three of a kind with $\binom{4}{3}$ per kind
12 remaining pairs with $\binom{4}{2}$ per pair

$$\underline{\underline{13 \cdot \binom{4}{3} \cdot 12 \cdot \binom{4}{2}}}$$

g) 13 ways to pick three of a kind with $\binom{4}{3}$ per kid
Pick 2 out of the remaining cards.

$$\underline{13 \cdot \binom{4}{3} \cdot \binom{49}{2}}$$

But this includes picking "four of a kind." If we want to exclude that, then we swap the 49 for a 48

$$\underline{13 \cdot \binom{4}{3} \cdot \binom{48}{2}}$$

h) $13 \cdot \binom{4}{2} \cdot \binom{50}{3}$ - includes three/four of a kind

$13 \cdot \binom{4}{2} \cdot \binom{48}{3}$ - excludes — " —

16)

a) $\sum_{i=1}^{6} (i^2+1)$

$= (1^2+1) + (2^2+1) + (3^2+1) + (4^2+1) + (5^2+1) + (6^2+1)$

$= 1+4+9+16+25+36+6$

$= 5+50+42+$

$= \underline{97}$

c) $\sum_{i=0}^{10} [1+(-1)^i]$

$= (1+1) + (1+(-1)^1) + (1+(-1)^2) + (1+(-1)^3) + (1+(-1)^4)$

$+ (1+(-1)^5) + (1+(-1)^6) + (1+(-1)^7) + (1+(-1)^8)$

$+ (1+(-1)^9) + (1+(-1)^{10})$

$= 2 \cdot 6 + 0 \cdot 5 = \underline{12}$

e) $\sum_{i=1}^{6} i(-1)^i$

$= \cancel{-1} + 2 \cancel{-3} \cancel{+4} - 5 + 6$

$= \underline{3}$

1.4

8)
$$\left.\begin{aligned} x_1 + x_2 + x_3 &= 8 \\ y_1 + y_2 + y_3 &= 7 \end{aligned}\right\} x_i, y_i \geq 1$$

$$\Leftrightarrow \left.\begin{aligned} x_1' + x_2' + x_3' &= 8-3=5 \\ y_1' + y_2' + y_3' &= 7-3=4 \end{aligned}\right\} x_i, y_i \geq 0$$

$$\binom{5+3-1}{3} \cdot \binom{4+3-1}{3}$$

$$= \underline{\underline{\binom{7}{3}\binom{6}{3}}}$$

14)

a) $(3v + 2w + x + y + z)^8$

The coefficients are of the form $\binom{8}{n_1\ n_2\ n_3\ n_4\ n_5} \cdot (3v)^{n_1} (2w)^{n_2} x^{n_3} y^{n_4}$

In $v^2 w^4 x z$: $n_1 = 2,\ n_2 = 4,\ n_3 = 1,\ n_4 = 0,\ n_5 = 1$

$$\binom{8}{2\ 4\ 1\ 0\ 1} \cdot (3v)^2 \cdot (2w)^4 \cdot x \cdot y^0 \cdot z^1$$

$$= \frac{8!}{2!\,4!} \cdot 3^2 \cdot 2^4\, v^2 w^4 x z$$

$$= \underline{\underline{8! \cdot 3 \cdot v^2 w^4 x z}}$$

b) $n_1 + n_2 + n_3 + n_4 + n_5 = 8 \quad , n_i \geqslant 0$

$$\binom{8+5-1}{5} = \binom{12}{5} \text{ ways}$$