Assignment 7, ttk4215

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Problem 4.13

First we rewrite the system to the bilinear form based on signals u and y.

$$y = \rho^* \left(u - m\ddot{y} - \beta\dot{y}\right) \tag{1}$$

$$\frac{y}{\Lambda(s)} = \rho^* \left(-m\frac{s^2 y}{\Lambda(s)} - \beta\frac{sy}{\Lambda(s)} + \frac{u}{\Lambda(s)}\right) \tag{2}$$

$$z = \rho^* \left({\theta^*}^T \phi + z_1\right) \tag{3}$$

So we have a bilinear parametric model with

$$\theta^* = \begin{bmatrix} m & \beta \end{bmatrix}^T \tag{4}$$

$$\rho^* = 1/k \tag{5}$$

$$\phi = \begin{bmatrix} \frac{-s^2 y}{\Lambda(s)} & \frac{-sy}{\Lambda(s)} \end{bmatrix}^T \tag{6}$$

$$z_1 = \frac{u}{\Lambda(s)} \tag{7}$$

$$z = \frac{y}{\Lambda(s)} \tag{8}$$

$$\Lambda(s) = (s - \lambda_1)(s - \lambda_2) \tag{9}$$

$$\lambda_1, \lambda_2 < 0 \tag{10}$$

We know that $\rho^* > 0$, but we don't have a non-zero lower bound. Will use gradient algorithm for bilinear parametrizations to estimate.

$$\hat{z} = \rho\left(\theta^T\phi + z_1\right) = \rho\xi \tag{11}$$

$$\epsilon = \frac{z - \hat{z}}{m^2} \tag{12}$$

$$m^2 = 1 + n_s^2 = 1 + \alpha(\phi^T\phi + z_1^2) \tag{13}$$

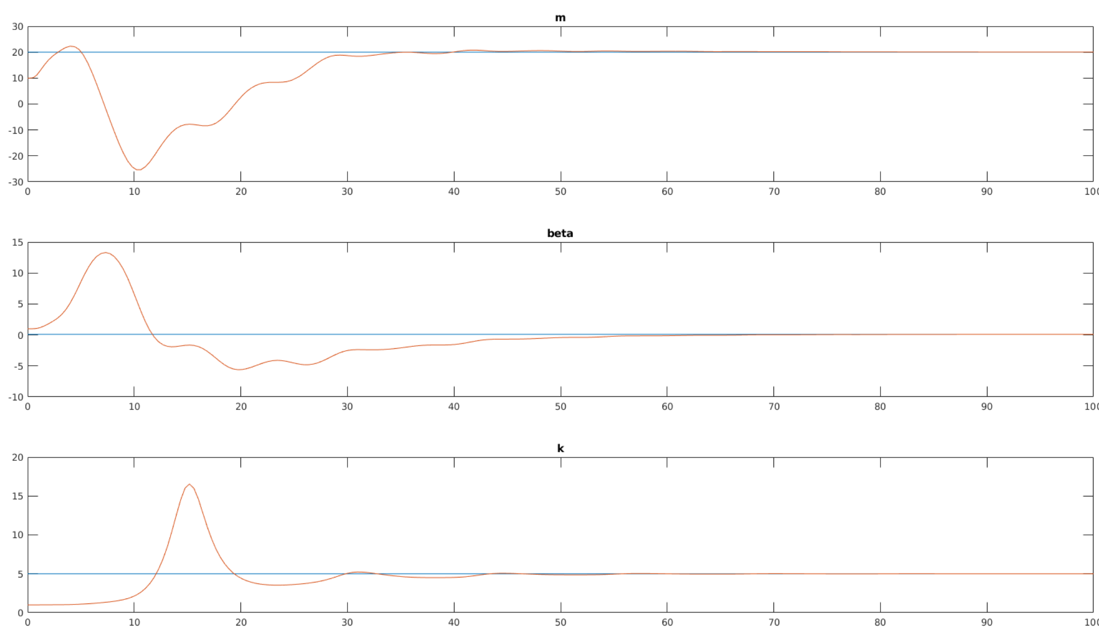
$$\xi = \theta^T\phi + z_1 \tag{14}$$

$$\dot{\theta} = \Gamma\epsilon\phi\mathrm{sgn}(\rho^*) \tag{15}$$

$$\dot{\rho} = \gamma\epsilon\xi \tag{16}$$

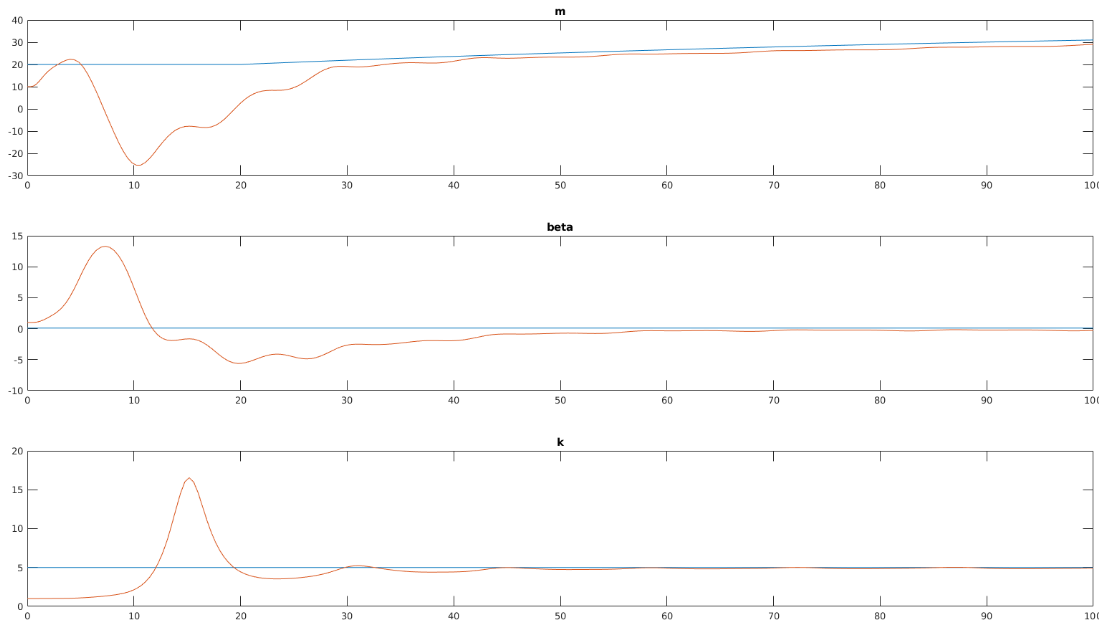
We first simulate with $m = 20$, $\beta = 0.1$, and $k = 5$, which means $\rho^* = 0.2$, $\theta^* = \begin{bmatrix}20 & 0.1\end{bmatrix}^T$.

As input we use $u(t) = \sin(0.1t) + \sin(5t) + 20$.



The estimates do converge on the actual value, but it does take some time. Interestingly both m and β go into the negative numbers. From a physical standpoint this doesn't make sense so we could probably improve the simulation with a projection to remove this.

I wasn't able to get good convergence in less than 20 seconds, so adding time varying mass might not be that meaningful, but here we go.



It looks like β and ρ needs more time to converge now with the changing mass. Actually the estimate for β is quite bad since it is negative, but the estimate for ρ is close to the actual value. The mass estimate isn't able to track the actual mass closely, but they follow a similar curve, and the estimate seems to improve over time. Again we have the problem that m and β estimates are negative, which we could account for by using a projection.