Artificial Intelligence Methods, exercise 4

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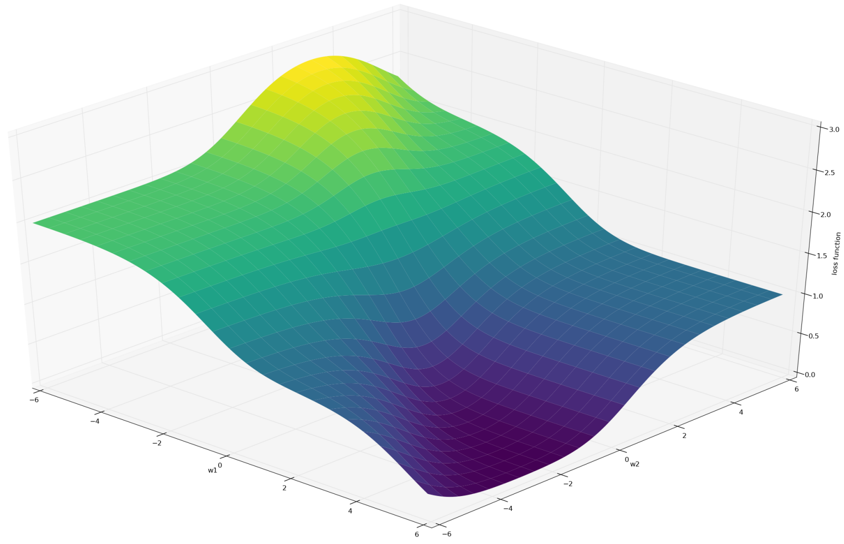
Part A



From the plot it does not seem that there is a definite lowest point, but rather that the function decreases monotonically toward zero in the as $w = (w_1, w_2) = (w, -w)$ and $w \to \infty$.

Part A.1

Part A.2

Using the definitions for $\sigma(w, x)$ and $L_{simple}(w)$,

$$\sigma(w, x) = \frac{1}{1 + e^{-w^T x}} \tag{1}$$

$$L_{simple} = [\sigma(w, [1, 0]) - 1]^2 + \sigma(w, [0, 1])^2 + [\sigma(w, [1, 1]) - 1]^2 , \tag{2}$$

we can compute the gradient of L_{simple}, $\nabla_w L_{simple}$.

We begin by writing out the expression using $w = [w_1, w_2]^T$.

$$L_{simple} = \left[\frac{1}{1+e^{-w_1}} - 1\right]^2 + \left[\frac{1}{1+e^{-w_2}}\right]^2 + \left[\frac{1}{1+e^{-w_1-w_2}} - 1\right]^2 \tag{3}$$

$$= \left[\sigma(w_1) - 1\right]^2 + \sigma(w_2)^2 + \left[\sigma(w_1+w_2) - 1\right]^2. \tag{4}$$

It is quite well known that

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x)(1-\sigma(x)) \tag{5}$$

Using this and the chain rule, we find that

$$\frac{\partial L_{simple}}{\partial w_1}(w) = 2\left[\sigma(w_1) - 1\right]\frac{\partial \sigma(w_1)}{\partial w_1} + 2\left[\sigma(w_1+w_2) - 1\right]\frac{\partial \sigma(w_1+w_2)}{\partial w_1} \tag{6}$$

$$= 2(\sigma(w_1)-1)\sigma(w_1)(1-\sigma(w_1)) + 2(\sigma(w_1+w_2)-1)\sigma(w_1+w_2)(1-\sigma(w_1+w_2)) \tag{7}$$

$$= -2\left[\sigma(w_1) - 1\right]^2\sigma(w_1) - 2\left[\sigma(w_1+w_2) - 1\right]^2\sigma(w_1+w_2). \tag{8}$$

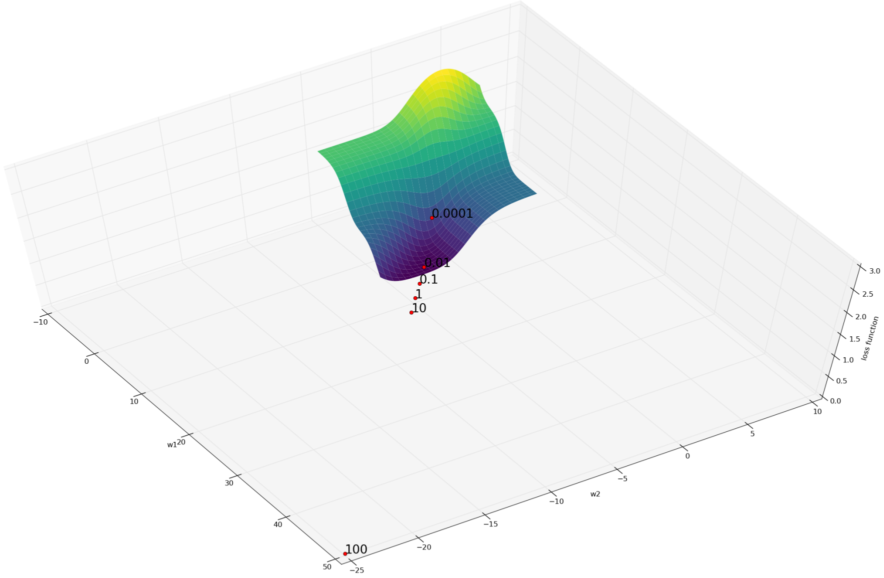
For w_2 we get

$$\frac{\partial L_{simple}}{\partial w_2}(w) = 2\sigma(w_2)\sigma(w_2)(1-\sigma(w_2)) + 2\left[\sigma(w_1+w_2) - 1\right]\sigma(w_1+w_2)(1-\sigma(w_1+w_2)) \tag{9}$$

$$= 2\sigma(w_2)^2(1-\sigma(w_2)) - 2\sigma(w_1+w_2)\left[\sigma(w_1+w_2) - 1\right]^2. \tag{10}$$

Part A.3

After implementing the update rule in Python, starting the weights at $(w_1, w_2) = (0, 0)$, running it for 10000 iterations and plotting the surface along with the search results for different η, I get:



Notice in the plot that there is quite a big difference between the different choices of η. With $\eta = 100$ the weights ended up quite far away from the others, but actually the difference in the loss function values of the resulting points isn't that big when we compare $\eta = 100$ to $\eta = 0.01$. The function does decrease in the direction of $(w_1, w_2) = (\infty, -\infty)$, but is essentially constantly zero from quite early on.

Another interesting thing we can see from this plot is that for $\eta = 0.0001$, the loss function is significantly larger. This is probably since the sequence converges slower for small values of η.