Solution Suggestion

Exam - TTK4115 Linear System Theory

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EIG (2009-12-14)

Problem 1

a) The proof is similar to the proof of Theorem 6.2 in Chen. The controllability matrix of $\bar{\mathcal{C}}$ of the transformed system is:

$$\begin{aligned}
\bar{\mathcal{C}} &= \begin{bmatrix}\bar{B} & \bar{A}\bar{B} & \cdots & \bar{A}^{n-1}\bar{B}\end{bmatrix} \\
&= \begin{bmatrix}PB & PAP^{-1}PB & \cdots & \left(PAP^{-1}\right)^{n-1}PB\end{bmatrix} \\
&= \begin{bmatrix}PB & PAP^{-1}PB & \cdots & PA^{n-1}P^{-1}PB\end{bmatrix} \\
&= P\begin{bmatrix}B & AB & \cdots & A^{n-1}B\end{bmatrix} \\
&= P\mathcal{C}
\end{aligned}$$

Since the rank of a matrix will not change after pre- or postmultiplication by a nonsingular matrix, and P is nonsingular, the rank of $\bar{\mathcal{C}}$ is equal to the rank of $\mathcal{C}$. Since $\mathcal{C}$ was assumed to have full rank, so will $\bar{\mathcal{C}}$.

b) To find the eigenvalues of A, we solve the equation $\det(\lambda I - A) = 0$. We find that

$$\begin{aligned}
\det(\lambda I - A) &= \begin{vmatrix}\lambda + 1 & 1 \\ 4 & \lambda - 2\end{vmatrix} \\
&= \lambda^2 - \lambda - 6 \\
&= (\lambda + 2)(\lambda - 3)
\end{aligned}$$

The eigenvalues are therefore $\lambda_1 = -2$ and $\lambda_2 = 3$.The eigenvector $q_1 = (q_{11}, q_{12})^\top$ corresponding to λ_1 is found by solving $(A - \lambda_1 I)\, q_1 = 0$. Since

$$\begin{bmatrix}1 & -1 \\ -4 & 4\end{bmatrix}\begin{bmatrix}q_{11} \\ q_{12}\end{bmatrix} = \begin{bmatrix}q_{11} - q_{12} \\ -4q_{11} + 4q_{12}\end{bmatrix},$$

we can pick $q_1 = (1, 1)^\top$. In a similar manner we find the eigenvector $q_2 = (1/4, -1)^\top$, corresponding to λ_2.

c) As announced on the exam D should be taken as $D = 0$. The transformed system can be written as

$$\begin{aligned}
\dot{\bar{x}} &= \bar{A}\bar{x} + \bar{B}u \\
y &= \bar{C}\bar{x} + \bar{D}u
\end{aligned}$$

where $\bar{A} = PAP^{-1}$, $\bar{B} = PB$, $\bar{C} = CP^{-1}$ and $\bar{D} = D$. We define $Q := \begin{bmatrix}q_1 & q_2\end{bmatrix}$ and take $P = Q^{-1}$. We find that

$$P = \begin{bmatrix}\frac{4}{5} & \frac{1}{5} \\ \frac{4}{5} & -\frac{4}{5}\end{bmatrix}.$$

Performing the matrix calculations gives

$$\begin{aligned}\dot{\bar{x}} &= \begin{bmatrix}-2 & 0\\ 0 & 3\end{bmatrix}\bar{x} + \begin{bmatrix}1/5\\ -4/5\end{bmatrix} u\\ y &= \begin{bmatrix}1 & \frac{1}{4}\end{bmatrix}\bar{x}\end{aligned}$$

d) Using the formula in the appendix, we find that

$$\begin{aligned}\bar{x}(t) &= \begin{bmatrix}e^{-2t} & 0\\ 0 & e^{3t}\end{bmatrix}\bar{x}(0) + \int_0^t \begin{bmatrix}e^{-2(t-\tau)} & 0\\ 0 & e^{3(t-\tau)}\end{bmatrix}\begin{bmatrix}1/5\\ -4/5\end{bmatrix} u(\tau)\, d\tau\\ &= \begin{bmatrix}\bar{x}_1(0)\, e^{-2t}\\ \bar{x}_2(0)\, e^{3t}\end{bmatrix} + \int_0^t \begin{bmatrix}1/5 e^{-2(t-\tau)}\\ -4/5 e^{3(t-\tau)}\end{bmatrix} u(\tau)\, d\tau\end{aligned}$$

Since $y(t) = (1, 1/4)\,\bar{x}(t)$, and $u \equiv 1$, we get that for $x_0 = (x_{10}, x_{20})^\top$,

$$\begin{aligned}y(1) &= \bar{x}_1(0)\, e^{-2} + 1/4\bar{x}_2(0)\, e^{3} + \frac{1}{5}\int_0^1 e^{-2(1-\tau)} d\tau - \frac{1}{5}\int_0^1 e^{3(1-\tau)} d\tau\\ &= \bar{x}_1(0)\, e^{-2} + \frac{1}{4}\bar{x}_2(0)\, e^{3} + \frac{1}{5}\int_0^1 e^{-2(1-\tau)} d\tau - \frac{1}{5}\int_0^1 e^{3(1-\tau)} d\tau\\ &= \left(\frac{4}{5}x_{10} + \frac{1}{5}x_{20}\right)e^{-2} + \frac{1}{4}\left(\frac{4}{5}x_{10} - \frac{4}{5}x_{20}\right)e^{3} + \frac{1}{5}\left(\frac{1}{2} - \frac{1}{2}e^{-2}\right) - \frac{1}{5}\left(\frac{1}{3}e^{3} - \frac{1}{3}\right)\\ &= \left(\frac{4}{5}e^{-2} + \frac{1}{5}e^{3}\right)x_{10} + \frac{1}{5}\left(e^{-2} - e^{3}\right)x_{20} - \frac{1}{10}e^{-2} - \frac{1}{15}e^{3} + \frac{1}{6}\end{aligned}$$

Problem 2

a) With output $y = x_1$, the system has transferfunction

$$G(s) = \frac{1}{s^2},$$

that is, two poles in the origin. By Theorem 5.3 in Chen, the system is not BIBO stable. The system is controllable, since the controllability matrix

$$\mathcal{C} = \begin{bmatrix}B & AB\end{bmatrix} = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$

has full rank.

b) By comparing the terms we find that

$$Q = \begin{bmatrix}\alpha & 0\\ 0 & \beta\end{bmatrix}, \quad R = \gamma.$$

Increasing α and β penalizes the state varialbles x_1 and x_2, respectively. Increasing γ penalizes the control efforts.

c) The resulting controll will be timevarying, since t_e is finite.

d) We find that

$$A = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}, \quad B = \begin{bmatrix}0\\ 1\end{bmatrix}, \quad R = 1, \quad Q = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Inserting into the Ricatti equation, we find that.

$$\begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}+\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}^T\begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}+\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}-\begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}^T\begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}$$

$$\begin{bmatrix} 0 & p_{11} \\ 0 & p_{12} \end{bmatrix}+\begin{bmatrix} 0 & 0 \\ p_{11} & p_{12} \end{bmatrix}+\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}-\begin{bmatrix} p_{12}^2 & p_{12}p_{22} \\ p_{12}p_{22} & p_{22}^2 \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

or

$$\begin{bmatrix} 1-p_{12}^2 & p_{11}-p_{12}p_{22} \\ p_{11}-p_{12}p_{22} & -p_{22}^2+2p_{12}+1 \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

We have the following equation

$$\begin{aligned} 1-p_{12}^2 &= 0 \\ -p_{22}^2+2p_{12}+1 &= 0 \\ p_{11}-p_{12}p_{22} &= 0 \end{aligned}$$

The first equation implies that $p_{12} \pm 1$. If $p_{12}=-1$, we find from the second equation that $p_{22}=\pm i$. We are only interested in real solutions, so $p_{12}=1$. Inserting $p_{12}=1$ into the second equation, we find that $p_{22}=\pm\sqrt{3}$. For $p_{22}=\sqrt{3}$, we get from the last equation that $p_{11}=\sqrt{3}$, were as for $p_{22}=-\sqrt{3}$, we get that $p_{11}=-\sqrt{3}$. Possible solutions are therefore

$$P=\begin{bmatrix} \sqrt{3} & 1 \\ 1 & \sqrt{3} \end{bmatrix} \quad \text{or} \quad P=\begin{bmatrix} -\sqrt{3} & 1 \\ 1 & -\sqrt{3} \end{bmatrix}.$$

We are only interested in the positive (semi-) definite solution, so

$$P=\begin{bmatrix} \sqrt{3} & 1 \\ 1 & \sqrt{3} \end{bmatrix}$$

e) The optimal control is

$$u=-\begin{bmatrix} 1 & \sqrt{3} \end{bmatrix}x.$$

The closed-loop system is

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}x-\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & \sqrt{3} \end{bmatrix}x \\ &= \begin{bmatrix} 0 & 1 \\ -1 & -\sqrt{3} \end{bmatrix}x \end{aligned}$$

The characteristic polynomical of the closed-loop system is $\Delta(\lambda)=\lambda\left(\lambda+\sqrt{3}\right)+1$. By solving $\Delta(\lambda)=0$, we find that $\lambda=-\frac{1}{2}\sqrt{3}\pm\frac{1}{2}i$. The eigenvalues are in the left hand plane, and therefore the system is asymptotically stable.

Problem 3

a) Realizing that the process signal is on the form

$$R_s(\tau)=\sigma^2 e^{-\beta|\tau|}$$

with mean-square value $\sigma^2=8/3$ and time constant $1/\beta=4/3$, we recongnize the process as a Gauss-Markov process, which power spectral desity function is

$$S_s(s)=\frac{2\sigma^2\beta}{-s^2+\beta^2}.$$

The noise signal should be recognized as white noise, with spectral amplitude $A = 4$. The corresponding power spectral function is

$$S_n(s) = A.$$

Spectral factorization gives

$$\begin{aligned} S_s(s) &= S^+(s) \cdot S^-(s) \\ &= \frac{\sqrt{2\frac{8}{3}\frac{3}{4}}}{s+\frac{3}{4}} \cdot \frac{\sqrt{2\frac{8}{3}\frac{3}{4}}}{-s+\frac{3}{4}} \\ &= \frac{2}{s+\frac{3}{4}} \cdot \frac{2}{-s+\frac{3}{4}} \end{aligned}$$

$S^+(s)$ is the shaping filter from the unity white noise input u to the state x, so

$$\frac{X(s)}{U(s)} = \frac{2}{s+\frac{3}{4}}$$

or

$$X\left(s + \frac{3}{4}\right) = 2U,$$

which it time-domain becomes

$$\dot{x} = -\frac{3}{4}x + 2u$$

Since the additve measurement is white, no augmentation of the state vector is required, and the measurement is

$$z = x + v.$$

Since u is unity white noise, $Q = 1$ and since v is white noise of amplitude 4, $R = 4$.

b) Comparing the state model with the model in the appendix, we recognize that $G = 2$, $H = 1$ and $F = -\frac{3}{4}$. Inserting for H, F, Q and R into the differential Ricatti equation gives:

$$\dot{P} = -\frac{1}{4}P^2 - \frac{3}{2}P + 4.$$

c) We rewrite

$$\begin{aligned} \dot{P} &= -\frac{1}{4}\left(P^2 + 6P - 16\right) \\ &= -\frac{1}{4}(P+8)(P-2). \end{aligned}$$

We write $\dot{P} = dP/dt$, and spearate the variables:

$$\frac{dP}{(P+8)(P-2)} = -\frac{1}{4}dt, \tag{1}$$

Note that,

$$\begin{aligned} \frac{dP}{(P+8)(P-2)} &= \frac{1}{10}\left(\frac{dP(P+8)}{(P+8)(P-2)} - \frac{dP(P-2)}{(P+8)(P-2)}\right) \\ &= \frac{1}{10}\left(\frac{dP}{(P-2)} - \frac{dP}{(P+8)}\right) \end{aligned}$$

Inserting into (1), rearranging terms and taking the integral of both sides, gives

$$\int_{P(t_0)}^{P(t)} \frac{dP}{(P-2)} - \int_{P(t_0)}^{P(t)} \frac{dP}{(P+8)} = -\frac{5}{2}\int_{t_0}^{t} 1dt$$

Calculating the integrals, gives

$$\begin{aligned} \ln\left(P\left(t\right)-2\right)-\ln\left(P\left(t_0\right)-2\right)-\ln\left(P\left(t\right)+8\right)+\ln\left(P\left(t_0\right)-8\right) &= -\frac{5}{2}\left(t-t_0\right), \\ \implies \ln\frac{P\left(t\right)-2}{P\left(t\right)+8}\frac{P\left(t_0\right)+8}{P\left(t_0\right)-2} &= -\frac{5}{2}\left(t-t_0\right), \\ \implies \frac{P\left(t\right)-2}{P\left(t\right)+8}\frac{P\left(t_0\right)+8}{P\left(t_0\right)-2} &= e^{-\frac{5}{2}\left(t-t_0\right)}. \end{aligned}$$

Solving for $P\left(t\right)$ gives.

$$P\left(t\right) = \frac{2\left(\frac{P(t_0)+8}{P(t_0)-2} + 4e^{\frac{-5}{2}\left(t-t_0\right)}\right)}{\frac{P(t_0)+8}{P(t_0)-2} - e^{\frac{-5}{2}\left(t-t_0\right)}}$$

Inserting $P\left(t_0\right) = P\left(0\right) = 0$, gives

$$\begin{aligned} P\left(t\right) &= \frac{2\left(-4+4e^{\frac{-5}{2}t}\right)}{-4-e^{\frac{-5}{2}t}} \\ &= 8\frac{\left(1-e^{\frac{-5}{2}t}\right)}{4+e^{\frac{-5}{2}t}}. \end{aligned} \tag{2}$$

Another possibility is to transform the nonlinear differential Ricatti equation into a system of linear equations. Assume that P can be written on product form $P = XZ^{-1}$ with $Z\left(0\right) = I$. Then the X and Z satisfy

$$\begin{aligned} \dot{X} &= FX + GQG^T Z, \qquad X\left(0\right) = P_0 \\ \dot{Z} &= H^T R^{-1} HX - F^\top Z, \quad Z\left(0\right) = I. \end{aligned}$$

Inserting for F, G, Q, H and R we get

$$\begin{aligned} \dot{X} &= -\frac{3}{4}X + 4Z \\ \dot{Z} &= \frac{1}{4}X + \frac{3}{4}Z \end{aligned}$$

or by Laplace transform

$$\begin{aligned} X\left(s\right)\cdot s - P_0 + \frac{3}{4}X\left(s\right) &= 4Z\left(s\right) \implies X\left(s\right) = \frac{4Z\left(s\right)+P_0}{s+\frac{3}{4}} \\ Z\left(s\right)\cdot s - 1 - \frac{3}{4}Z\left(s\right) &= \frac{1}{4}X\left(s\right) \implies Z\left(s\right) = \frac{1+\frac{1}{4}X\left(s\right)}{s-\frac{3}{4}} \end{aligned}$$

Solving for $X\left(s\right)$ and $Z\left(s\right)$, we get after some intermediate calculations that:

$$\begin{aligned} X\left(s\right) &= \frac{P_0 s + 4 - P_0\frac{3}{4}}{s^2-\left(\frac{5}{4}\right)^2} \\ Z\left(s\right) &= \frac{s+\frac{1}{4}\left(P_0+3\right)}{s^2-\left(\frac{5}{4}\right)^2} \end{aligned}$$

To use the Laplace table in the appendix we rewrite

$$\begin{aligned} X(s) &= \frac{P_0 s + \frac{5i}{4}\frac{4}{5i}\left(4 - P_0\frac{3}{4}\right)}{s^2 + \left(\frac{5}{4}i\right)^2} \\ Z(s) &= \frac{s + \frac{5}{4i}\frac{4}{20i}\left(P_0 + 3\right)}{\left(s^2 + \left(\frac{5}{4}i\right)^2\right)} \end{aligned}$$

and find that

$$\begin{aligned} x(t) &= P_0 \cos\frac{5}{4}it + \frac{4}{5i}\left(4 - P_0\frac{3}{4}\right)\sin\frac{5}{4}it \\ &= \frac{P_0}{2}\left(e^{\frac{5}{4}t} + e^{-\frac{5}{4}t}\right) + \frac{2}{5}\left(4 - P_0\frac{3}{4}\right)\left(e^{\frac{5}{4}t} - e^{-\frac{5}{4}t}\right) \\ &= \frac{1}{20}e^{\frac{5}{4}t}\left(4P_0 + 32\right) + \frac{1}{5}e^{-\frac{5}{4}t}\left(4P_0 - 8\right) \end{aligned}$$

and that

$$\begin{aligned} z(t) &= \cos\frac{5}{4}it + \frac{4}{20i}\left(P_0 + 3\right)\sin\frac{5}{4}it \\ &= \frac{1}{2}\left(e^{\frac{5}{4}t} + e^{-\frac{5}{4}t}\right) + \frac{1}{10}\left(P_0 + 3\right)\left(e^{\frac{5}{4}t} - e^{-\frac{5}{4}t}\right) \\ &= \frac{1}{10}\left(P_0 + 8\right)e^{\frac{5}{4}t} - \frac{1}{10}\left(P_0 - 2\right)e^{-\frac{5}{4}t} \end{aligned}$$

Using that $P = XZ^{-1}$, and $P(0) = 0$, we get that

$$\begin{aligned} P(t) &= \frac{\frac{1}{20}e^{\frac{5}{4}t}\left(4P_0 + 32\right) + \frac{1}{5}e^{-\frac{5}{4}t}\left(4P_0 - 8\right)}{\frac{1}{10}\left(P_0 + 8\right)e^{\frac{5}{4}t} - \frac{1}{10}\left(P_0 - 2\right)e^{-\frac{5}{4}t}} \\ &= \frac{8\left(e^{\frac{5}{4}t} - e^{-\frac{5}{4}t}\right)}{\left(4e^{\frac{5}{4}t} + e^{-\frac{5}{4}t}\right)}\frac{e^{-\frac{5}{4}t}}{e^{-\frac{5}{4}t}} \\ &= \frac{8\left(1 - e^{-\frac{5}{2}t}\right)}{\left(4 + e^{-\frac{5}{2}t}\right)} \end{aligned}$$

(The introduction of $\exp(-5/4t)/\exp(5/4t)$ in the second to last equation were made to get $P(t)$ at the same form as in (2).

d) The Kalman gain in the stationary case is $K_\infty = P_\infty H^\top R^{-1}$, where $P_\infty = \lim_{t\to\infty} P(t)$ is found using the answer in the previous exercise or by solving the algebraic Ricatti equation:

$$0 = -\frac{1}{4}\left(P_\infty + 8\right)\left(P_\infty - 2\right).$$

The positive solution $P_\infty = 2$, gives $K_\infty = 1/2$.

Problem 4

a) The transfer function matrix is given as

$$G(s) = C\underbrace{(sI - A)^{-1}}_{\Omega}B + D.$$

Since D is the zero matrix, and since most of the elements of B and C are zero, we only need to calculate some of the elments of Ω to find $G(s)$:

$$\begin{aligned} G(s) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{21} & \omega_{22} & \omega_{23} \\ \omega_{31} & \omega_{32} & \omega_{33} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} \omega_{12} & \omega_{11} \\ \omega_{22} & \omega_{21} \end{bmatrix} \end{aligned}$$

Using the formula in the appendix

$$\omega_{ij} = \frac{c_{ji}}{\det(sI - A)} = \frac{(-1)^{i+j} \det [sI - A]_{ji}}{|(sI - A)|}$$

where $[sI - A]_{ji}$ is the submatrix you get by removing row j and column i of $sI - A$. First we find that

$$\begin{aligned} sI - A &= \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ -4 & -4 & 0 \\ 2 & 0 & 3 \end{bmatrix} \\ &= \begin{bmatrix} s & -1 & 0 \\ 4 & s+4 & 0 \\ -2 & 0 & s-3 \end{bmatrix} \end{aligned}$$

and that $\det(sI - A) = s(s+4)(s-3) + 4(s-3) = (s-3)(s+2)^2$. Furthermore,

$$\begin{aligned} \omega_{12} &= \frac{(-1)^3 \begin{vmatrix} -1 & 0 \\ 0 & s-3 \end{vmatrix}}{(s-3)(s+2)^2} \\ &= \frac{s-3}{(s-3)(s+2)^2} \\ &= \frac{1}{(s+2)^2}, \end{aligned}$$

$$\begin{aligned} \omega_{11} &= \frac{(-1)^2 \begin{vmatrix} s+4 & 0 \\ 0 & s-3 \end{vmatrix}}{(s-3)(s+2)^2} \\ &= \frac{s+4}{(s+2)^2} \end{aligned}$$

$$\begin{aligned} \omega_{22} &= \frac{(-1)^4 \begin{vmatrix} s & 0 \\ -2 & s-3 \end{vmatrix}}{(s-3)(s+2)^2} \\ &= \frac{s}{(s+2)^2} \end{aligned}$$

$$\begin{aligned} \omega_{21} &= \frac{(-1)^3 \begin{vmatrix} 4 & 0 \\ -2 & s-3 \end{vmatrix}}{(s-3)(s+2)^2} \\ &= \frac{-4}{(s+2)^2} \end{aligned}$$

Hence,

$$G(s) = \frac{1}{(s+2)^2}\begin{bmatrix} 1 & s+4 \\ s & -4 \end{bmatrix}$$

b) To keep the notation consistent with the one in the appendix, we define $\hat{G}(s) = G(s)$. We find that

$$d(s) = (s+2)^2 = s^2 + 4s + 4.$$

Comparing with the expression in the appendix, $d(s) = s^r + \alpha_1 s^{r-1} + \cdots + \alpha_{r-1}s + \alpha_r$, we find that $r = 2$, $\alpha_1 = 4$ and $\alpha_2 = 4$. $\hat{G}(s)$ is strictly proper, hence $\hat{G}_\infty(s) = 0$, and $\hat{G}_{sp}(s) = \hat{G}(s)$. We can express $\hat{G}_{sp}(s)$ as

$$\hat{G}_{sp}(s) = \frac{1}{d(s)}\left[\underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{N_1} s + \underbrace{\begin{bmatrix} 1 & 4 \\ 0 & -4 \end{bmatrix}}_{N_2}\right].$$

We end up with the following realization using the equations in the appendix:

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -4 & 0 & -4 & 0 \\ 0 & -4 & 0 & -4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} u \\ y &= \begin{bmatrix} 0 & 1 & 1 & 4 \\ 1 & 0 & 0 & -4 \end{bmatrix} x \end{aligned}$$

c) No, $(A_{m_1}, B_{m_1}, C_{m_1}, D_{m_1})$ is no realization of $G(s)$, which can be seen from these calculations:

$$\begin{aligned} G_{m_1}(s) &= C_{m_1}(sI - A_{m_1})^{-1} B_{m_1} + D_{m_1} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} s & -1 \\ 4 & s+4 \end{bmatrix}^{-1}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \frac{1}{s^2+4s+4}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} s+4 & 1 \\ 4 & s \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \frac{1}{s^2+4s+4}\begin{bmatrix} 1 & s+4 \\ s & 4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \\ &= G(s) + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \\ &\neq G(s) \end{aligned}$$