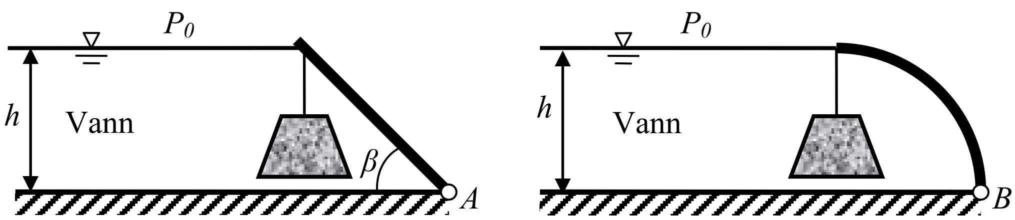
**Oppgave 1**



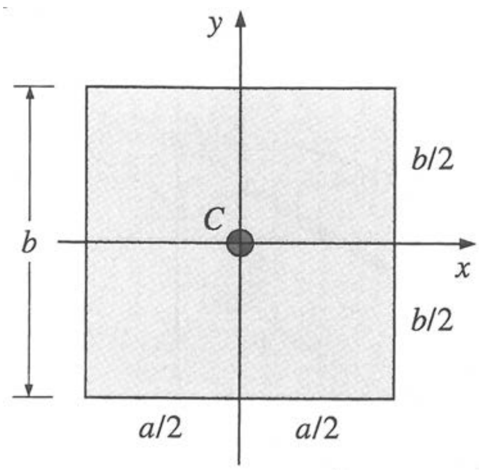

En 1m bred luke med neglisjerbar vekt holder tilbake vann med dybde $h$ = 3m. Til venstre i figuren er luka en plan flate med en helning på $\beta$ = 45º som er hengslet i nedre kant $A$, og til høyre i figuren er luken en kvartsirkel, også den hengslet i nedre kant $B$. En sementblokk (tetthet $\rho_{\text{sement}} = 2400$ kg/m$^3$) henger fra toppen av luka og er helt nedsenket i vann (tetthet $\rho_{\text{vann}} = 1000$ kg/m$^3$). Tyngdens akselerasjon er $g = 10$ m/s$^2$.

a) Finn kraften (størrelse og retning) fra vannet på den venstre luken.

b) Vekten av sementblokka er akkurat nok til at luka til venstre er i ro i den viste posisjonen. Finn volumet til den venstre sementblokka.

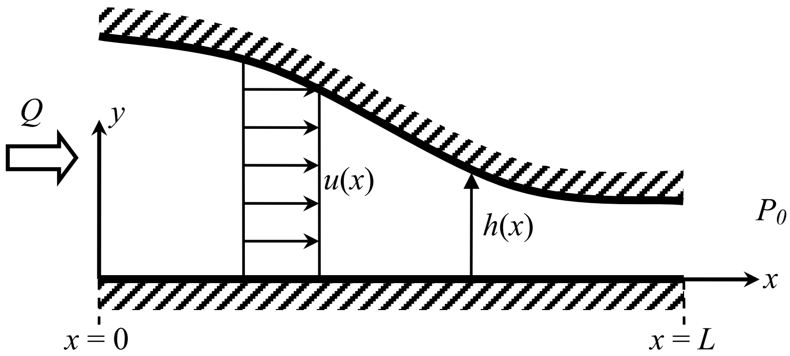
c) Finn kraften (horisontal og vertikal komponent) fra vannet på den høyre luka.

d) Hva må volumet til sementblokka som henger fra den høyre luka være for at denne skal være i ro i den viste posisjonen?




$A = ab,\ I_{xx,C} = ab^3/12$

**Oppgave 2**




Figuren viser en todimensjonal friksjonsfri strømning mellom to faste flater, som er åpen mot atmosfæren ved $x = L$ der trykket er $P_0$. Nedre flate er planet $y = 0$, mens øvre flate har høyden $h(x)$. Strømningen er inkompressibel, og volumstrømmen mellom flatene er oppgitt til å være $Q$. Hastigheten i $x$-retning er gitt som

$$u(x) = \frac{Q}{H}\left(2 - \cos\frac{\pi x}{L}\right)$$ der $H$ og $L$ er konstanter.

Vi betrakter bare strekningen $0 \le x \le L$. Se bort fra tyngden. Bruk én lengdeenhet i $z$-retning, dvs. vinkelrett på papirplanet.

a) Finn $h = h(x)$, og finn $y$-komponenten av hastigheten, $v = v(x,y)$.

b) Når kan vi bruke strømfunksjonen? Finn strømfunksjonen $\psi$. Finn et uttrykk for strømlinjene. Vis at $h(x)$ er en strømlinje.

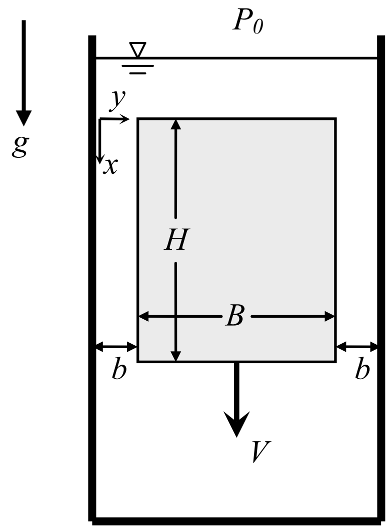
I resten av oppgaven antar vi at $H \ll L$. I dette tilfellet kan hastighetskomponenten $v$ neglisjeres i forhold til $u$, og trykket kan antas å være en funksjon bare av $x$.

c) Finn trykket $P = P(x)$.

d) Finn kraften i $x$-retning fra strømningen på flatene.

**Oppgave 3**

En massiv kloss med høyde $H$ og bredde $B$ synker med konstant hastighet $V$ ned i et kar med væske som er åpen mot atmosfære-trykket $P_0$. Væsken er inkompressibel med tetthet $\rho$ og viskositet $\mu$. Når klossen synker, passerer væske laminært og stasjonært på hver side av klossen. Spaltebredden $b$ er mye mindre enn spaltehøyden $H$ slik at væsken i spalten kan antas å strømme kun i $x$-retning som vist i figuren. Tyngdens akselerasjon er $g$. Strømningen antas 2-dimensjonal, så det er ingen bevegelse vinkelrett på papirplanet.




a) Finn volumstrømmen $Q$ (pr. lengdeenhet vinkelrett på papirplanet) i hver av spaltene.

b) Vis at bevegelseslikningen for strømningen i én spalte kan reduseres til

$$\frac{d^2u}{dy^2} = K \quad \text{der } K \text{ er en konstant.}$$

c) Finn $u(y)$ i spalten uttrykt ved $V$, $K$ og $b$.

d) Finn $K$ uttrykt ved $V$, $B$ og $b$. Tegn en skisse av hastighetsprofilet $u(y)$.

e) Gi en skisse av et Matlab-program (riktig syntaks kreves ikke) som plotter hastighetsprofilet gitt ved

$$u = C_1 y^2 + C_2 y + C_3, \quad 0 \le y \le b$$

der $C_1$, $C_2$ og $C_3$ er kjente konstanter. Programmet skal også regne ut Reynoldstallet basert på spaltebredden $b$ og gjennomsnittshastigheten.

# Formulae, TEP4100 Fluidmekanikk

Ideal gas law:

$$P = \rho R T; \quad R_{\text{air}} = 287\,{}^{\text{Pa m}^3}\!/_{\text{kg K}}$$

Reynolds' number:

$$\text{Re} = \frac{UL}{\nu}.$$

Kinematic viscosity

$$\nu = \frac{\mu}{\rho}.$$

Shear stress

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right).$$

Surface tension (soap bubble)

$$\Delta P = \sigma_s\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Hydrostatic pressure distribution

$$\frac{\mathrm{d}P}{\mathrm{d}z} = -\rho g.$$

Hydrostatic force on plane submerged surface

$$F_{\text{R}} = (P_0 + \rho g h_{\text{C}})A = P_{\text{C}}A.$$

Centre of pressure

$$y_{\text{P}} = y_{\text{C}} + \frac{I_{xx,\text{C}}}{\left[y_{\text{C}} + P_0/(\rho g \sin\theta)\right]A}; \quad y_{\text{P}} = y_{\text{C}} + \frac{I_{xx,\text{C}}}{y_{\text{C}}A}.$$

Pressure distribution in rigid body motion

$$\vec{\nabla} P + \rho g \vec{k} = -\rho \vec{a}.$$

Pressure distribution in rigid body rotation

$$P = P_0 + \frac{1}{2}\rho\omega^2 r^2 - \rho g z.$$

Acceleration (Cartesian)

$$\frac{\mathrm{D}\vec{V}}{\mathrm{D}t} = \frac{\partial \vec{V}}{\partial t} + (\vec{V}\cdot\nabla)\vec{V} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}.$$

Along a streamline

$$\frac{\mathrm{d}r}{V} = \frac{\mathrm{d}x}{u} = \frac{\mathrm{d}y}{v} = \frac{\mathrm{d}z}{w}.$$

Vorticity

$$\vec{\zeta} = 2\vec{\omega} = \vec{\nabla}\times\vec{V}.$$

Reynolds transport theorem

$$\frac{\mathrm{d}}{\mathrm{d}t}B_{\text{sys}} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{\text{CV}} \rho b\,\mathrm{d}\mathcal{V}\right) + \oint_{\text{CS}} \rho b\left(\vec{V}_{\text{r}}\cdot\vec{n}\right)\mathrm{d}A.$$

Volume flow rate through cross section $A_{\text{c}}$

$$\dot{\mathcal{V}} = \int_{A_{\text{c}}} \vec{V}_{\text{r}}\cdot\vec{n}\,\mathrm{d}A.$$

Conservation of mass

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\text{CV}} \rho\,\mathrm{d}\mathcal{V} + \oint_{\text{CS}} \rho\left(\vec{V}_{\text{r}}\cdot\vec{n}\right)\mathrm{d}A = 0.$$

Bernoulli equation along streamline, unsteady compressible flow

$$\int\frac{\mathrm{d}P}{\rho} + \int\frac{\partial V}{\partial t}\mathrm{d}s + \frac{V^2}{2} + gz = \text{constant}.$$

Bernoulli equation along streamline, steady incompressible flow

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant}.$$

Energy equation

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{\mathrm{d}}{\mathrm{d}t}\int_{\text{CV}} e\rho\,\mathrm{d}\mathcal{V} + \oint_{\text{CS}}\left(\frac{P}{\rho} + e\right)\rho\left(\vec{V}_{\text{r}}\cdot\vec{n}\right)\mathrm{d}A,$$

where total energy per unit mass is

$$e = u + \frac{V^2}{2} + gz.$$

Energy equation for steady flow with one inlet and one outlet

$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 + h_{\text{pump,u}} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 + h_{\text{turbine,e}} + h_{\text{L}}.$$

Linear momentum equation

$$\sum \vec{F} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{\text{CV}} \rho\vec{V}\,\mathrm{d}\mathcal{V}\right) + \oint_{\text{CS}} \rho\vec{V}\left(\vec{V}_{\text{r}}\cdot\vec{n}\right)\mathrm{d}A.$$

Net pressure force on closed CS

$$\vec{F}_{\text{press}} = -\oint_{\text{CS}} P_{\text{gage}}\,\vec{n}\,\mathrm{d}A$$

Angular momentum equation

$$\sum \vec{M} = \frac{\mathrm{d}}{\mathrm{d}t}\int_{\text{CV}}\left(\vec{r}\times\vec{V}\right)\rho\,\mathrm{d}\mathcal{V} + \oint_{\text{CS}}\left(\vec{r}\times\vec{V}\right)\rho\left(\vec{V}_{\text{r}}\cdot\vec{n}\right)\mathrm{d}A.$$

Critical Reynolds number, pipe flow

$$\text{Re}_{\text{crit}} \approx 2300.$$

Entry length

$$\text{laminar} \qquad \frac{L_{h,\text{ laminar}}}{D} \approx 0.05\text{Re},$$

$$\text{turbulent} \qquad \frac{L_{h,\text{ turbulent}}}{D} \approx 1.359\text{Re}^{1/4}.$$

Darcy friction factor for laminar pipe flow

$$f = \frac{64}{\text{Re}}.$$

Pipe head loss

$$h_{\text{L}} = f\frac{L}{D}\frac{V_{\text{avg}}^2}{2g}.$$

Colebrook's formula

$$\frac{1}{\sqrt{f}} = -2.0\log\left[\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right].$$

Haaland's formula

$$\frac{1}{\sqrt{f}} \simeq -1.8\log\left[\frac{6.9}{\mathrm{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right].$$

Prandtl's approximation, turbulent pipe flow

$$\frac{1}{\sqrt{f}} = 2.0\log\left(\mathrm{Re}\sqrt{f}\right) - 0.8.$$

Total head loss, constant cross-sectional area

$$h_{\mathrm{L,\ total}} = h_{\mathrm{L,\ major}} + h_{\mathrm{L,\ minor}} = \left(f\frac{L}{d} + \sum K_{\mathrm{L}}\right)\frac{V^2}{2g}$$

Continuity equation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\left(\rho\vec{V}\right) = 0.$$

Continuity equation in cylindrical coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(r\rho u_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho u_\theta)}{\partial \theta} + \frac{\partial(\rho u_z)}{\partial z} = 0.$$

Incompressible continuity equation

$$\vec{\nabla}\cdot\vec{V} = 0.$$

Incompressible stream function $\psi$

$$\begin{aligned}
&(\text{Cartesian})\ u = \frac{\partial\psi}{\partial y}; \quad v = -\frac{\partial\psi}{\partial x};\\
&(\text{Cylindrical})\ u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta}; \quad u_\theta = -\frac{\partial\psi}{\partial r};\\
&(\text{Axisymmetric flow})\ u_r = -\frac{1}{r}\frac{\partial\psi}{\partial z}; \quad u_z = \frac{1}{r}\frac{\partial\psi}{\partial r}.
\end{aligned}$$

Incompressible Navier-Stokes equation

$$\rho\frac{\partial\vec{V}}{\partial t} + \rho\left(\vec{V}\cdot\vec{\nabla}\right)\vec{V} = -\vec{\nabla}P + \rho\vec{g} + \mu\nabla^2\vec{V}.$$

Velocity potential

$$\vec{V} = \vec{\nabla}\phi.$$

2D irrotational flow

$$\nabla^2\phi = \nabla^2\psi = 0.$$

Plane flow

$$\begin{aligned}
&(\text{uniform stream}) & \psi &= Vy; & \phi &= Vx;\\
&(\text{source/sink}) & \psi &= \frac{\dot{\mathcal{V}}/L}{2\pi}\theta; & \phi &= \frac{\dot{\mathcal{V}}/L}{2\pi}\ln r;\\
&(\text{line vortex}) & \psi &= -\frac{\Gamma}{2\pi}\ln r; & \phi &= \frac{\Gamma}{2\pi}\theta;\\
&(\text{doublet}) & \psi &= -K\frac{\sin\theta}{r}; & \phi &= K\frac{\cos\theta}{r}.
\end{aligned}$$

Displacement thickness

$$\delta^* = \int_0^\infty\left(1 - \frac{u}{U}\right)dy.$$

Momentum thickness

$$\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right)dy.$$

Flat plate boundary layer thickness

$$\frac{\delta}{x} \approx \begin{cases} \frac{4.91}{\mathrm{Re}_x^{1/2}}, & \text{laminar}\\ \frac{0.16}{\mathrm{Re}_x^{1/7}}, & \text{turbulent}\end{cases}$$

Local skin friction coefficient

$$C_{\mathrm{f,x}} = \frac{\tau_{\mathrm{w}}}{\frac{1}{2}\rho U^2}.$$

Drag and lift coefficients

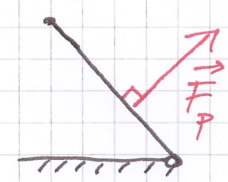
$$C_{\mathrm{D}} = \frac{F_{\mathrm{D}}}{\frac{1}{2}\rho V^2 A}; \quad C_{\mathrm{L}} = \frac{F_{\mathrm{L}}}{\frac{1}{2}\rho V^2 A}.$$

## Problem 1

1a) Magnitude of force is $F_p = p_{CG} \cdot A$ where $A = \sqrt{2}h$ (per unit length into paper plane) and

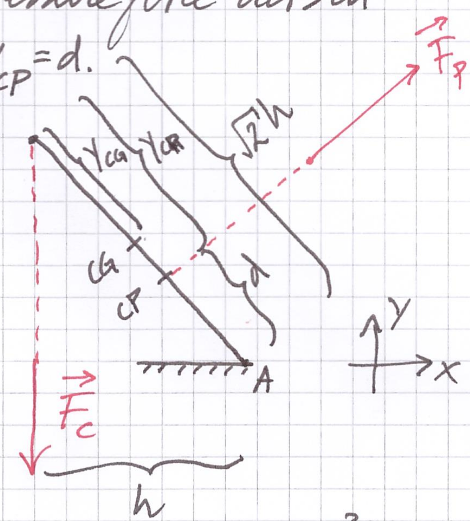
$$p_{CG} = \rho_w g h_{CG} = \frac{1}{2}\rho_w g h \,, \quad \text{so}$$

$$\underline{F_p = \frac{1}{\sqrt{2}}\rho_w g h^2} \quad = 63.6 \text{ kN/m}$$




The direction is normal to the gate, i.e. 45° upwards & to the right.

1b) Need to have zero total torque about A for stationary system. The pressure force acts in CP and has arm $\sqrt{2}h - Y_{CP} = d$.
The force from the concrete block has arm $h$.




Must find $Y_{CP}$:

$$Y_{CP} = Y_{CG} + \frac{I_{xx,C}}{Y_{CG} \cdot A}$$

$$= \sqrt{2}h\left(\frac{1}{2} + \frac{1}{6}\right) = \underline{\sqrt{2}h \cdot \frac{2}{3}}$$

$$I_{xx,C} = \frac{(\sqrt{2}h)^3}{12}$$

$$A = \sqrt{2}h$$

$$Y_{CG} = \frac{1}{2}\sqrt{2}h = \frac{1}{\sqrt{2}}h$$

So: $d = \sqrt{2}h - Y_{CP} = \underline{\frac{1}{3}\sqrt{2}h}$

Torque from water is then

$$\vec{M}_p = -F_p d \vec{k} = \underline{-\frac{1}{3}\rho_w g h^3}$$

Torque from concrete block is from weight of the block minus the buoyancy force on the block. The force is thus:

$$\vec{F}_c = -(\underset{\uparrow \text{ weight}}{W_c} - \underset{\uparrow \text{ buoyancy}}{F_B})\vec{j} = (-g\underset{\uparrow \text{ density of concrete}}{\rho_c} V + g\rho_w V)\vec{j}$$

$$= -gV(\rho_c - \rho_w)\vec{j}$$

The torque from the concrete, thus:

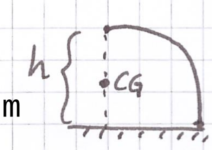
$$\vec{M}_c = gV(\rho_c - \rho_w)h\,\vec{k}$$

Total torque is zero: $\vec{M}_p + \vec{M}_c = 0$, i.e.,

$$\underline{\underline{V = \frac{\rho_w}{3(\rho_c - \rho_w)} h^2}} \quad = 2.14 \text{ m\^{}2}$$

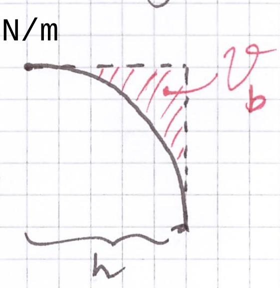
1c) The horizontal force component equals that on the vertical projection:

$$F_H = p_{CG} \cdot h = \underline{\frac{1}{2}\rho_w g h^2} \quad = 45.0 \text{ kN/m}$$

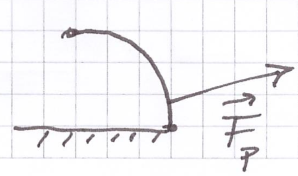



The vertical force is the buoyancy force acting on the "missing" water in volume $V_b$ above the gate:

$$F_V = \rho_w g V_b = \underline{\rho_w g h^2\left(1 - \frac{\pi}{4}\right)} \quad = 19.3 \text{ kN/m}$$

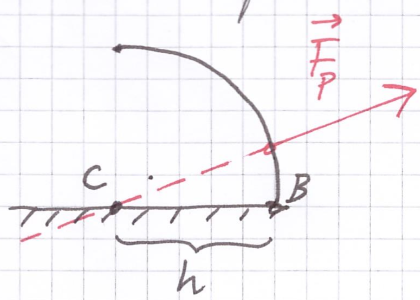



The force $\vec{F}_P = F_H\vec{i} + F_V\vec{j}$ points up and to the right.



1d) Must again find the torque from the water about the axis B. We use a standard trick for gates which have circular shape. Since pressure force is always locally normal to the surface, we know that the extension line of the force $\vec{F}_P$ must go through the centre of the circle, point C. The torque is

$$\vec{M}_P = \vec{r} \times \vec{F}_P$$




where $\vec{r}$ points from B to <u>any</u> point on the extension line, for example C. Letting $\vec{r}$ point from $B \to C$, we get:

$$\vec{M}_P = \vec{r} \times \vec{F}_P = (-h, 0) \times (F_H, F_V)$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -h & 0 & 0 \\ F_H & F_V & 0 \end{vmatrix} = \underline{-hF_V \vec{k}}$$

Just like in 1b the torque from the concrete block is $\vec{M}_c = gV(\rho_c - \rho_w)h\vec{k}$

so requiring $\vec{M}_P + \vec{M}_c = 0$ gives

$$V = \frac{F_V}{g(\rho_c - \rho_w)} = \underline{\underline{\frac{(1-\frac{\pi}{4})\rho_w}{\rho_c - \rho_w} h^2}} \text{ =1.38 m^2}$$

This is only 64% of the volume needed in 1b.

# Exercise 2

a) The volume flow $Q = h(x)u(x)$ must be the same through all vertical sections, so

$$h(x) = \frac{Q}{u(x)} = \underline{\underline{\frac{H}{2-\cos\frac{\pi x}{L}}}}$$

Continuity equation $\nabla\cdot\vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, so

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = -\frac{\pi Q}{HL}\sin\frac{\pi x}{L}$$

undetermined

Integrating: $v(x,y) = -\frac{\pi Q}{HL}\,y\cdot\sin\frac{\pi x}{L} + f(x)$

Since we must have $v(x,0) = 0$, we must have $f(x) = 0$, i.e.

$$\underline{\underline{v(x,y) = -\frac{\pi Q}{HL}\,y\sin\frac{\pi x}{L}}}$$

2b : The stream function $\psi$ can be used for flow which is 2D (In our course we only use it for incompressible flow)

Have by definition: $u = \frac{\partial \psi}{\partial y}$ ; $v = -\frac{\partial \psi}{\partial x}$

So:
$$\psi = \int u\,dy = \frac{Q}{H}\left(2-\cos\frac{\pi x}{L}\right)y + f(x)$$

undetermined functions

and
$$\psi = \int v\,dx = -\frac{Q}{H}\cos\left(\frac{\pi x}{L}\right)y + g(y)$$

By comparison:
$$\psi = \underline{\underline{\frac{Q}{H}\left(2-\cos\frac{\pi x}{L}\right)y}} + \text{constant}$$

We can choose the constant to be zero.

Streamlines can be found as solutions $\psi$ = constant (alternatively we can solve equation $\frac{dx}{u} = \frac{dy}{v}$).

This gives
$$y = \frac{(H/Q)\cdot K}{2-\cos\frac{\pi x}{L}}$$

constant

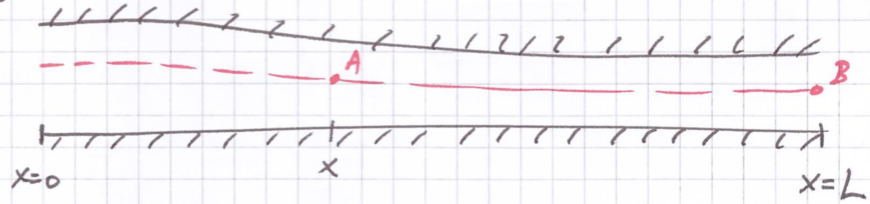
or simply

another constant.

$$\underline{\underline{y = \frac{G}{2-\cos\frac{\pi x}{L}}}} \quad \text{along streamlines.}$$

Since $h(x) = \dfrac{H}{2-\cos\frac{\pi x}{L}}$ we see that $h(x)$ is the streamline defined by $C = H$.

2c) Bernoulli's equation along a streamline (neglecting gravity):

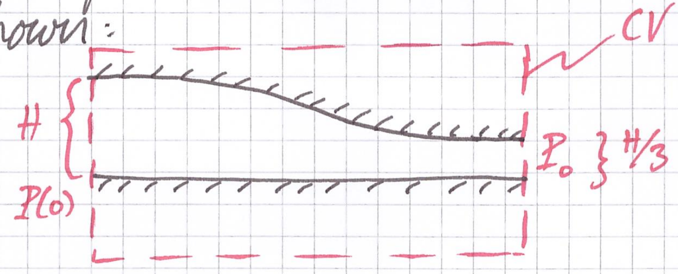



$$P(x) + \tfrac{1}{2}\rho U_A^2 = P_0 + \tfrac{1}{2}\rho U_B^2$$

$$\Rightarrow \quad P(x) = P_0 + \tfrac{1}{2}\rho\left[U(L)^2 - U(x)^2\right]$$

$$= \underline{\underline{P_0 + \tfrac{1}{2}\rho\frac{Q^2}{H^2}\left[9 - \left(2-\cos\tfrac{\pi x}{L}\right)^2\right]}}$$

2d) To find the force we use the momentum equation for control volumes and choose CV as shown:




External forces on CV are the net pressure force and an unknown contact force required to keep the system stationary:

$$\sum F_{x,ext} = F_{P,net} + \underset{\text{contact force}}{F_C} = P(0)H - P_0 \cdot H + F_C$$

$$= \frac{4Q^2\rho}{H} + F_c$$

(Alternatively: use gage pressure)

Next, by the momentum equation: (stationary flow)

$$\sum F_{x,ext} = \oint_{CV} \rho u (\vec{V}\cdot\vec{n})\,dA = -\rho u(0)^2 H + \rho u(L)^2 \frac{H}{3}$$

$\vec{V}\cdot\vec{n} = -u(0)$ at inlet | $\vec{V}\cdot\vec{n} = +u(L)$ at outlet

$$= -\rho H\left(\frac{Q}{H}\right)^2 + \rho\frac{H}{3}\left(\frac{3Q}{H}\right)^2$$

$$= 2\rho\frac{Q^2}{H}.$$

The two expressions for $\sum F_{x,ext}$ are equal, so

$$F_c = -2\rho\frac{Q^2}{H}$$

The force from the flow to the walls must be equal but of opposite direction:

$$\underline{\underline{F_{walls} = 2\rho\frac{Q^2}{H}}}$$

3a: The total volume flow, $2Q$, must equal the volume captured by the sinking block per time unit, i.e.:

$$2Q = V \cdot B$$

$$\Rightarrow \quad \underline{\underline{Q = \tfrac{1}{2} V B}}$$

3b: Inside the gaps we assume $v = 0$. From continuity it then follows that $\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y} = 0$, so $u$ only depends on $y$, not on $x$.

Regard x-component of Navier-Stokes:

$$\underset{\substack{=0 \\ \text{(stationary)}}}{\cancel{\frac{\partial u}{\partial t}}} + \underset{\frac{\partial u}{\partial x}=0}{\cancel{u \frac{\partial u}{\partial x}}} + \underset{v=0}{\cancel{v \frac{\partial u}{\partial y}}} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + g + \underbrace{\frac{\mu}{\rho}}_{=\nu}\left(\underset{\frac{\partial^2 u}{\partial x^2}=0}{\cancel{\frac{\partial^2 u}{\partial x^2}}} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\Rightarrow \quad \frac{\partial^2 u}{\partial y^2} = \frac{1}{\nu}\left(\frac{1}{\rho}\frac{\partial p}{\partial x} - g\right)$$

Must now find out about $\frac{\partial p}{\partial x}$. Regard N-S in y-direction:

$$\underset{v=0}{\cancel{\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}}} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \cancel{\nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)} \; v=0$$

$$\Rightarrow \quad \frac{\partial p}{\partial y} = 0$$

This means that $p = p(x)$ (depends only on $x$, not $y$).

But that means that

$$\underbrace{\frac{\partial^2 u}{\partial y^2}}_{\text{depends only on } y} = \underbrace{\frac{1}{\nu}\left(\frac{1}{\rho}\frac{\partial p}{\partial x} - g\right)}_{\text{depends only on } x}$$

This can only be possible if both sides of the equation is constant and depends neither on $x$ nor $y$!

Thus:

$$\frac{d^2 u}{dy^2} = K$$

3c) Now integrate twice and get

integration constants

$$u(y) = \frac{1}{2}Ky^2 + C_1 y + C_2.$$

We have viscous flow so no-slip at $y=0$ and $y=b$ gives $u(0) = 0$ and $u(b) = V$

Thus: $C_2 = 0$ and

$$\frac{1}{2}Kb^2 + C_1 b = V$$

$$\Rightarrow \quad C_1 = \frac{V}{b} - \frac{1}{2}Kb$$

thus

$$u(y) = \frac{1}{2}Ky^2 + \left(\frac{V}{b} - \frac{1}{2}Kb\right)y$$

$$= \underline{\underline{\frac{1}{2}Ky(y-b) + V\frac{y}{b}}}$$

3d) The volume flow $Q$ was found before to be $\frac{1}{2}VB$.
Moreover the volume flow can be found from $u(y)$:

$$Q = -\int_0^b u(y)\,dy = -\int_0^b \left[\frac{K}{2}y^2 + \left(\frac{V}{b} - \frac{Kb}{2}\right)y\right]dy$$

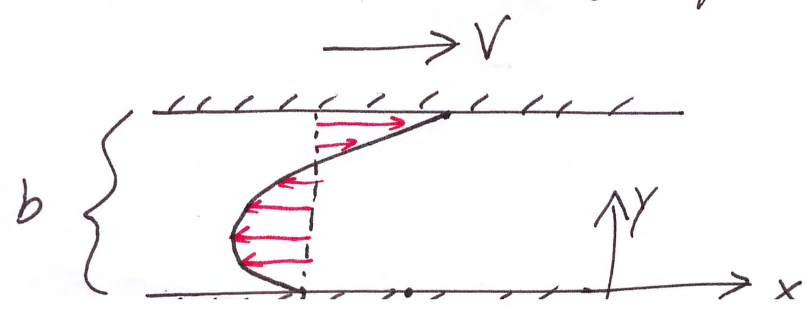
Net flow towards negative x

$$= -\frac{K}{6}b^3 - \left(\frac{V}{b} - \frac{Kb}{2}\right)\frac{1}{2}b^2$$

$$= -\frac{1}{2}Vb + \frac{Kb^3}{12} \overset{3a}{=} \frac{1}{2}VB$$

$$\Rightarrow \quad K = \frac{12}{b^3}\left(\frac{1}{2}VB + \frac{1}{2}Vb\right) = \underline{\underline{\frac{6}{b^3}V(B+b)}}$$

If we assume (as indicated by the figure) that $B > b$, then $K < 0$ and the velocity profile looks something like



```
% Assuming the following quantities to be known and previously defined:
%
% mu (viscosity of water)
% rho (density of water)
% b  (width of channel)
% C1, C2, C3 (constants)


% Vector of values of y, for example in 100 steps from 0 to b.
y = (0:.01:1)*b;
u = C1*y.^2 + C2*y + C3;

% Now plot the velocity profile. It is costumary to have the y-axis
% upward and the velocity axis towards the right (but this is not
% important).

plot(u,y);

%Calculating the average velocity. There are many ways to do this,
%for example:
%

uavg = trapz(y,x)/b;  %integral by trapezoid approximation

%... or ... (slight overkill)
%
% uavg = integral(@(y)(C1*y.^2 + C2*y + C3), 0, b);
%
% or you could even invent your own little method. It is also
% possible to calculate the average value by hand and get the
% exact answer
%
% uavg = C1*b^2/3 + C2*b/2 + C3;

% Now calculate the Reynolds number using uavg:

Re = uavg*b*rho/mu;
```