Godkjent
AB

Øving 2

Rendell Cole, gruppe 2, mttK

2.3

8)

Steps	Reasons
1) $\neg s \wedge \neg u$	Premise
2) $\neg u$	Conjunctive Simplification (1)
3) $\neg u \to \neg t$	Premise
4) $\neg t$	Modus Ponens ~~with~~ (2) and (3)
5) $\neg s$	Conjunctive Simplifiation (1)
6) $\neg s \wedge \neg t$	Conjunction (4) and (5)
7) $r \to (s \vee t)$	Premise
8) $\neg(s \vee t) \to \neg r$	$P_1 \to P_2 \Leftrightarrow \neg P_2 \to \neg P_1$ (7)
9) $(\neg s \wedge \neg t) \to \neg r$	DM on (8)
10) $\neg r$	Modus Ponens on (6) and (9)
11) $(\neg p \vee q) \to r$	Premise
12) $\neg r \to \neg(\neg p \vee q)$	$P_1 \to P_2 \Leftrightarrow \neg P_2 \to \neg P_1$ (11)
13) $\neg r \to p \wedge \neg q$	DM on (12)
14) $p \wedge \neg q$	Modus Ponens on (10) and (13)
15) $\therefore p$	Conjunctive Simplification (14)

10 c) $p \to q$ Premise 1

$\neg q$ Premise 2

$\neg r$ Premise 3

$\therefore \neg(p \vee r)$

Steps		Reasons
1)	$p \to q$	Premise 1
2)	$\neg q$	Premise 2
3)	$\neg p$	Modus Tollens
4)	$\neg r$	Premise 3
5)	$\neg p \wedge \neg r$	Conjunction
6)	$\neg(p \vee r)$	DeMorgan

Utsagnet er bevist sant.

d) $p \to q$ Premise 1

$r \to \neg q$ Premise 2

$\neg r$ Premise 3

$\therefore \neg p$

Steps Reasons

2.3

10 a)

$p \to (q \to r)$	Premise 1
$p \vee s$	Premise 2
$t \to q$	Premise 3
$\neg s$	Premise 4
$\therefore \neg r \to \neg t$	

Steps	Reasons
1) $p \vee s$	Premise 2
2) $\neg s$	Premise 4
3) p	Disjunctive Syllogism (1) and (2)
4) $p \to (q \to r)$	Premise 1
5) $q \to r$	Modus Ponens (3) og (4)
6) $t \to q$	Premise 3
7) $t \to r$	Syllogism (5) and (6)
(8) $\neg r \to \neg t$	$P_1 \to P_2 \Leftrightarrow \neg P_2 \to \neg P_1$ (7)

Utsagnet er dermed logisk bevist.

10 h) $p \vee q$ Premise 1

$\neg p \vee r$ Premise 2

$\neg r$ Premise 3

$\therefore q$

	Steps	Reasons
1)	$\neg p \vee r$	Premise 2
2)	$\neg r$	Premise 3
3)	$\neg p$	Disjunctive Syllogism (1) and (2)
4)	$p \vee q$	Premise 1
5)	q	Disjunctive Syllogism (3) and (4)

Utsagnet er logisk bevist.

2.3 11)

c) $p: 1$, $q: 1$, $r: 1$, $s: 0$

$p \leftrightarrow q \Leftrightarrow T_0$

$q \rightarrow r \Leftrightarrow T_0$

$r \vee \neg s \Leftrightarrow T_0$

$\neg s \rightarrow q \Leftrightarrow T_0$

Alle premissene er sanne, men s er 0 (F_0) så Konklusjonen må være feil.

d) $p: 1$, $q: 1$, $r: 1$, $s: 0$

$p \Leftrightarrow T_0$

$p \rightarrow \neg r \Leftrightarrow T_0$

$p \rightarrow (q \vee \neg r) \Leftrightarrow T_0$

$\neg q \vee \neg s \Leftrightarrow T_0$

Alle premissene er sanne, men $s \Leftrightarrow F_0$ så Konklusjonen må være feil.

2.4

(a) $p(x,y): x^2 \geq y$, $q(x,y): x+2 < y$

a) $p(2,4): 2^2 \geq 4 \iff T_0$ Sann

b) $q(1,\pi): 1+2 < \pi \iff T_0$ Sann

c) $p(-3,8) \wedge q(1,3): ((-3)^2 \geq 8) \wedge (1+2 < 3)$

$\iff F_0$ usann

d) $p(\frac{1}{2},\frac{1}{3}) \vee \neg q(-2,-3): \left[\left(\frac{1}{2}\right)^2 \geq \frac{1}{3}\right] \vee \neg\left[-2+2 < -3\right]$

$\iff F_0 \vee \neg F_0$

$\iff T_0$ Sann

e) $p(2,2) \rightarrow q(1,1): [2^2 \geq 2] \rightarrow [1+2 < 1]$

$\iff T_0 \rightarrow F_0$

$\iff F_0$ Usann

f) $p(1,2) \Leftrightarrow \neg q(1,2): [1^2 \geq 2] \leftrightarrow \neg[1+2 < 2]$

$\iff F_0 \Leftrightarrow \neg F_0$

$\iff F_0$ Usann

2.4

18)

a) $\neg(\exists x[p(x) \vee q(x)])$

$\Leftrightarrow \forall x \neg[p(x) \vee q(x)]$

$\Leftrightarrow \forall x [\neg p(x) \wedge \neg q(x)]$

b) $\neg(\forall x[p(x) \wedge \neg q(x)])$

$\Leftrightarrow \exists x \neg[p(x) \wedge \neg q(x)]$

$\Leftrightarrow \exists x [\neg p(x) \vee q(x)] \Leftrightarrow \exists x [p(x) \rightarrow q(x)]$

c) $\neg(\forall x [p(x) \rightarrow q(x)])$

$\Leftrightarrow \exists x \neg[p(x) \rightarrow q(x)]$

$\Leftrightarrow \exists x \neg[\neg p(x) \vee q(x)]$

$\Leftrightarrow \exists x [p(x) \wedge \neg q(x)]$

d) $\neg(\exists x\, [(p(x) \vee q(x)) \to r(x)])$

$\iff \forall x\, \neg[(p(x) \vee q(x)) \to r(x)]$

$\iff \forall x\, \neg[\neg(p(x) \vee q(x)) \vee r(x)]$

$\iff \forall x\, [(p(x) \vee q(x)) \wedge \neg r(x)]$

21)

a) $\exists x \exists y\, [xy = 1] \iff T$ sann, (bevis: $(x=1, y=1) \Rightarrow xy = 1$)

b) $\exists x \forall y\, [xy = 1] \iff F$

Etter at x har blitt valgt må $y = \frac{1}{x}$, og dette gjelder ikke for alle y.

c) $\forall x \exists y\, [xy = 1] \iff T$

Etter x er valgt velg $y = \frac{1}{x}$ så blir utsagnet sant. Siden $x \neq 0$ gjelder dette for alle x.