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English version

Exam in TTK4135

Optimization and Control

Optimalisering og regulering

Friday May 24, 2013

Time: 09:00 – 13:00

English 1

Norsk 7

Appendix 13

Combination of allowed help remedies:
D — No printed or hand-written notes.
Certifed calculator with empty memory.

In the Appendix potentially useful information is included.
The grades will be available by June 14.

1 QP (20 %)

Assume the following QP problem.

$$\begin{aligned}
\min_{x} \quad & f(x) = \frac{1}{2}x_1^2 + x_2^2 - x_1 x_2 - 2x_1 - 6x_2 \\
\text{s.t.} \quad & x_1 + x_2 \leq 2 \\
& -x_1 + 2x_2 \leq 2 \\
& x_1 \geq 0 \\
& x_2 \geq 0
\end{aligned}$$

a (5 %) Specify the problem in the standard form (A.7). Specify G, c, a_i, b_i, $\mathcal{E}$ and $\mathcal{I}$.

b (3 %) Is the problem convex? Explain.

c (3 %) Derive the KKT conditions for this QP problem.

d (9 %) The first and second inequality constraints are active at the solution. Compute the solution x^* and the Lagrange multipliers λ^*.

2 Optimization problem formulation (20 %)

We start by studying optimization problem (A.1), see the Appendix.

a (2 %) How many decision variables are there in (A.1)?

b (3 %) Is the following statement true? "If $c_i(x)$ is a nonlinear function when $i \in \mathcal{E}$, then (A.1) is always a non-convex problem". Please answer by yes or no.

c (3 %) Is the following statement true? "If $c_i(x)$ is a nonlinear function when $i \in \mathcal{I}$, then (A.1) is always a non-convex problem". Please answer by yes or no.

d (2 %) How many equality constraints and inequality constraints are there if $\mathcal{E} = \{1, 2\}$ and $\mathcal{I} = \emptyset$?

For each of the following five problems, classify the problem and suggest a suitable algorithm for solving problems of that class.

e (2 %)

$$\begin{aligned} \min_{x \in \mathbb{R}^2} \quad & x_1 + x_2 \\ \text{s.t.} \quad & 3x_1 - 2x_2 \leq 2 \\ & x_1 \geq 0 \end{aligned}$$

f (2 %)

$$\begin{aligned} \min_{x \in \mathbb{R}^2} \quad & 3x_1^2 - x_2 \\ \text{s.t.} \quad & x_1^2 + x_2^2 \leq 4 \end{aligned}$$

g (2 %)

$$\begin{aligned} \min_{x \in \mathbb{R}^2} \quad & x_1 \sin(x_2) \\ \text{s.t.} \quad & x_1 = x_2^2 \end{aligned}$$

h (2 %)

$$\begin{aligned} \min_{x \in \mathbb{R}^2} \quad & x_1^2 + x_2^2 + x_1 \\ \text{s.t.} \quad & x_1 = x_2 - 1 \\ & x_2 \geq 3 \end{aligned}$$

i (2 %)

$$\begin{aligned} \min_{x \in \mathbb{R}^2} \quad & x_2^2 + x_1 + 3x_2 \\ \text{s.t.} \quad & x_1 - 3x_2 = 1 \end{aligned}$$

3 Various topics (26 %)

Linear Independence Constraint Qualification (LICQ)

a (6 %) Consider

$$\begin{aligned} \min_{x \in \mathbb{R}^2} \quad & f(x) \\ \text{s.t.} \quad & (x_1 - 1)^2 + (x_2 - 1)^2 \leq 2 \\ & (x_1 - 1)^2 + (x_2 + 1)^2 \leq 2 \\ & x_1 \geq 0 \end{aligned}$$

Assume that $x^* = \begin{bmatrix} 0 & 0 \end{bmatrix}^\top$. Does the LICQ condition hold? Explain. (You do not have to calculate the constraint gradients to answer.)

Lagrange multipliers

Consider the two *convex* two-dimensional optimization problems in Figures 1 and 2 with the solution x^* indicated; the two constraint functions $c_1(x)$ and $c_2(x)$ are *inequality constraint* functions (there are no equality constraints).

b (3 %) For the problem illustrated in Figure 1 (page 5), what is the value of the Lagrange multipliers (specify if they are positive, zero or negative), and which of the inequality constraints are active?

c (3 %) For the problem illustrated in Figure 2 (page 5), what is the value of the Lagrange multipliers (specify if they are positive, zero or negative), and which of the inequality constraints are active? What is this situation called?

Nonlinear programming and SQP

This part is about the SQP algorithm on page 16 in the Appendix (Algorithm 18.3 in Nocedal and Wright).

d (3 %) ϕ_1, as used in the algorithm, is a merit function. Specify a suitable merit function for problem (A.1) when $\mathcal{E} = \{1\}$ and $\mathcal{I} = \emptyset$.

e (3 %) The parameter μ is a part of the merit function, and it is calculated at each iteration of the SQP algorithm. Will μ normally increase or decrease from one iteration k to the next iteration $k+1$? Explain briefly.

f (4 %) The merit function ϕ_1 is used in the line search part of the SQP algorithm. Will the merit function decrease from one iteration k to the next? Further, will the objective f decrease from one iteration point k to the next? Explain briefly.

g (4 %) Explain briefly the meaning of an *exact merit function*.

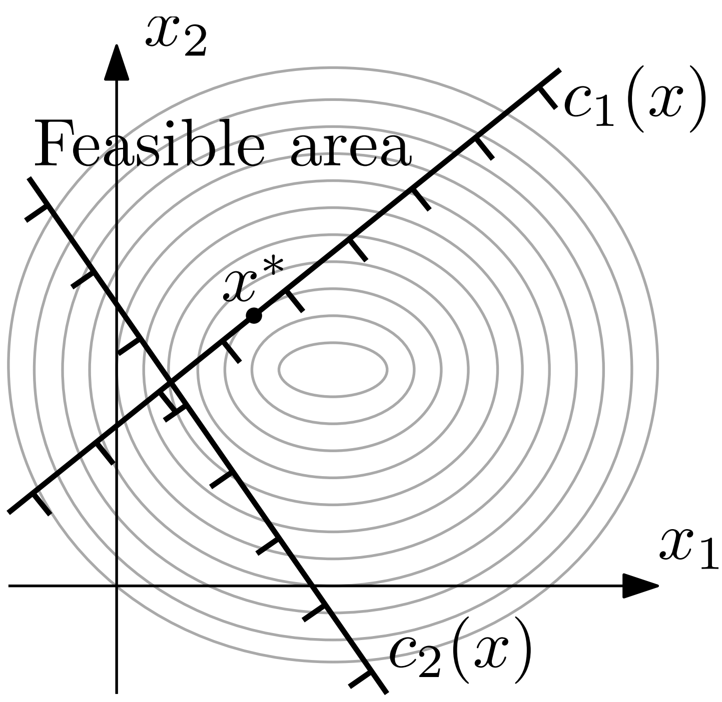

Figure 1: Illustration for Problem 3 b.

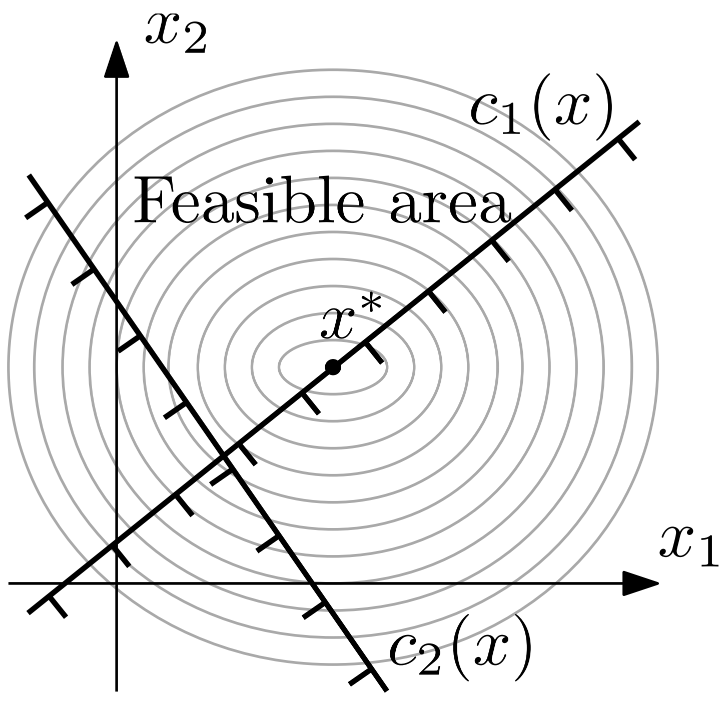

Figure 2: Illustration for Problem 3 c.

4 MPC and dynamic optimization (34 %)

For the questions below we consider the dynamic optimization problem (A.9) in the Appendix.

a (3 %) Explain briefly why (A.9) is called an open loop optimization problem.

b (3 %) What is the reason for including the inequality constraints (A.9f)?

c (2 %) Assume that $N = 15$, $n_x = 10$ and $n_u = 2$. What is the dimension of z in (A.9l) in this case?

d (4 %) The number of decision variables can be significantly reduced by eliminating $x_1, \dots, x_N$ from z in (A.9l) using the equality constraints (A.9b). What are the pros and cons of using the full space formulation, i.e., z in (A.9l), compared to a reduced space formulation with only $u_0, \dots, u_{N-1}$ as decision variables?

e (6 %) It is common to not measure all the states in x_t, but rather a subset $y_t = Cx_t$. Assume that the open loop optimization problem (A.9) is solved in an output feedback linear MPC controller where we only have access to y_t. Write down a suitable algorithm. (Please use similar format and level of detail as in the course handout "Merging Optimization and Control").

f (8 %) When (A.9) is used in linear MPC, the problem may be infeasible, i.e., there is no solution. Explain how (A.9) can be changed to avoid infeasibility. Please refer specifically to which equations that are changed and how they are changed.

g (2 %) Assume that we replace the linear model (A.9b) with a nonlinear discrete time model

$$x_{t+1} = g(x_t, u_t) \tag{1}$$

in an MPC controller. Suggest an optimization algorithm for solving the resulting open loop optimization problem.

h (6 %) Instead of using the nonlinear model (1) in the open loop optimization problem, we approximate the nonlinear model with a linear time varying (LTV) model

$$x_{t+1} = A_t x_t + B_t u_t \tag{2}$$

around a stationary point $\bar{x}_t, \bar{u}_t$. How can A_t and B_t be computed from $g(x_t, u_t)$? What type of smoothness condition must be placed on g with these formulas for A_t og B_t?

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Utgave/Utgåve: bokmål/nynorsk

Eksamen i TTK4135

Optimalisering og regulering

Optimization and Control

Fredag 24. mai 2013

Tid: 09:00 – 13:00

English **1**

Norsk **7**

Appendix **13**

Tillatte hjelpemidler / Tilletne hjelpemiddel:
D — Ingen trykte eller skrevne hjelpemidler. / Inga trykte eller skrevne hjelpemiddel.
Godkjent kalkulator med tomt minne. / Godkjend kalkulator med tomt minne.

Nyttig informasjon finnes i vedlegg. / Nyttig informasjon finns i vedlegg.
(Denne informasjonen er gitt på engelsk for å samsvare med pensumlitteraturen som den er hentet ifra.)
Sensur faller 14. juni. / Sensur fell 14. juni.

1 QP (20 %)

Gitt følgende QP problem.

$$
\begin{aligned}
\min_{x} \quad & f(x) = \frac{1}{2}x_1^2 + x_2^2 - x_1x_2 - 2x_1 - 6x_2 \\
\text{s.t.} \quad & x_1 + x_2 \leq 2 \\
& -x_1 + 2x_2 \leq 2 \\
& x_1 \geq 0 \\
& x_2 \geq 0
\end{aligned}
$$

a (5 %) Transformer QP-problemet til standardformen (A.7). Spesifiser G, c, a_i, b_i, $\mathcal{E}$ og $\mathcal{I}$.

b (3 %) Er problemet konvekst? Forklar.

c (3 %) Utled KKT-betingelsene for dette QP-problemet.

d (9 %) Den første og den andre ulikhetsbetingelsene er aktive i løsningspunktet. Beregn løsningen x^* og Lagrange-multiplikatorene λ^*.

2 Formulering av optimaliseringsproblemer (20 %)

Vi starter med optimaliseringsproblemet (A.1), se Appendiks.

a (2 %) Hvor mange beslutningsvariable er det i (A.1)?

b (3 %) Er følgende utsagn sant? "Dersom $c_i(x)$ er en ulineær funksjon når $i \in \mathcal{E}$, da er (A.1) alltid et ikke-konvekst problem". Vennligst svar med ja eller nei.

c (3 %) Er følgende utsagn sant? "Dersom $c_i(x)$ er en ulineær funksjon når $i \in \mathcal{I}$, da er (A.1) alltid et ikke-konvekst problem". Vennligst svar med ja eller nei.

d (2 %) Hvor mange likhetsbetingelser og ulikhetsbetinglser er det dersom $\mathcal{E} = \{1, 2\}$ og $\mathcal{I} = \emptyset$?

For hvert av de fem problemene nedenfor, spesifiser type optimaliseringsproblem og foreslå en egnet algoritme for å løse problemer av den typen.

e (2 %)

$$\begin{aligned} \min_{x \in \mathbb{R}^2} \quad & x_1 + x_2 \\ \text{s.t.} \quad & 3x_1 - 2x_2 \leq 2 \\ & x_1 \geq 0 \end{aligned}$$

f (2 %)

$$\begin{aligned} \min_{x \in \mathbb{R}^2} \quad & 3x_1^2 - x_2 \\ \text{s.t.} \quad & x_1^2 + x_2^2 \leq 4 \end{aligned}$$

g (2 %)

$$\begin{aligned} \min_{x \in \mathbb{R}^2} \quad & x_1 \sin(x_2) \\ \text{s.t.} \quad & x_1 = x_2^2 \end{aligned}$$

h (2 %)

$$\begin{aligned} \min_{x \in \mathbb{R}^2} \quad & x_1^2 + x_2^2 + x_1 \\ \text{s.t.} \quad & x_1 = x_2 - 1 \\ & x_2 \geq 3 \end{aligned}$$

i (2 %)

$$\begin{aligned} \min_{x \in \mathbb{R}^2} \quad & x_2^2 + x_1 + 3x_2 \\ \text{s.t.} \quad & x_1 - 3x_2 = 1 \end{aligned}$$

3 Diverse emner (26 %)

LICQ-betingelsen ("Linear Independence Constraint Qualification")

a (6 %) Gitt følgende problem:

$$\begin{aligned} \min_{x \in \mathbb{R}^2} \quad & f(x) \\ \text{s.t.} \quad & (x_1 - 1)^2 + (x_2 - 1)^2 \le 2 \\ & (x_1 - 1)^2 + (x_2 + 1)^2 \le 2 \\ & x_1 \ge 0 \end{aligned}$$

Anta at $x^* = \begin{bmatrix} 0 & 0 \end{bmatrix}^\top$. Er LICQ-betingelsen oppfyllt? Begrunn svaret. (Du trenger ikke å beregne gradienter for å svare på dette spørsmålet.)

Lagrange-multiplikatorer

Se de to *konvekse* to-dimensjonale optimeringsproblemene i Figurer 3 og 4 hvor løsningen x^* er inntegnet; de to betingelse-funksjonene $c_1(x)$ og $c_2(x)$ ("constraint functions") er *ulikhetsbetingelse*-funksjoner ("inequality constraint functions") (det er ingen likhetsbetingelser ("equality constraints")).

b (3 %) For problemet illustrert i Figur 3 (side 11), hva er verdien av Lagrange-multiplikatorene (spesifiser om de er positive, null eller negative) og hvilke ulikhetsbetinger er aktive?

c (3 %) For problemet illustrert i Figur 4 (side 11), hva er verdien av Lagrange-multiplikatorene (spesifiser om de er positive, null eller negative) og hvilke ulikhetsbetinger er aktive? Hva kalles denne situasjonen?

Ulineær programmering og SQP

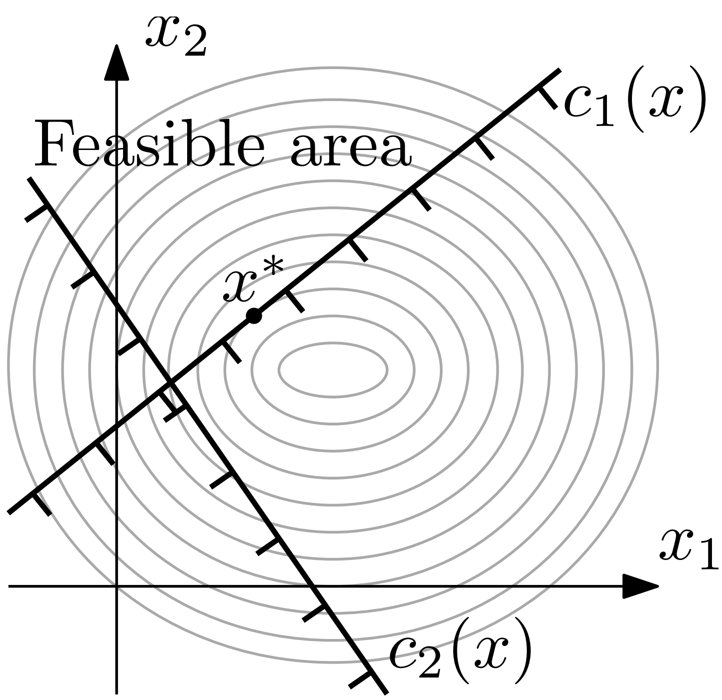
Denne delen omhandler SQP-algoritmen på side 16 i Appendix (Algoritme 18.3 i Nocedal og Wright).

d (3 %) ϕ_1 i algoritmen kalles en merit-funksjon. Spesifiser en passende merit-funksjon for (A.1) når $\mathcal{E} = \{1\}$ og $\mathcal{I} = \emptyset$.

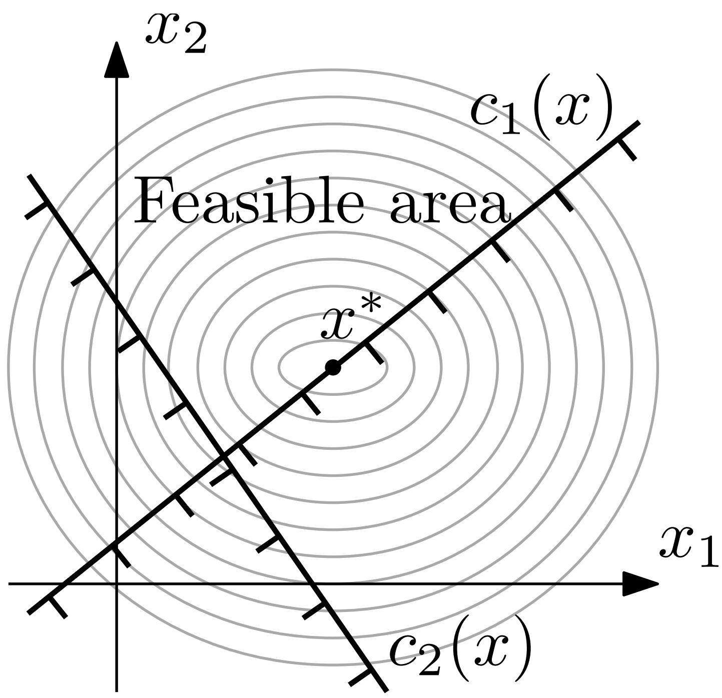
e (3 %) Parameteren μ tilhører merit-funksjonen, og den beregnes i hver iterasjon av SQP-algoritmen. Vil μ vanligvis øke eller avta fra en iterasjon k til neste iterasjon $k + 1$? Forklar kort.

f (4 %) Merit-funksjonen ϕ_1 brukes i linjesøksdelen av SQP-algoritmen. Avtar merit-funksjonen fra en iterasjon k til $k+1$? Avtar objekt-funksjonen fra en iterasjon k til $k + 1$? Forklar kort.

g (4 %) Hva vil det si at en merit-funksjon er en *eksakt merit-funksjon*?



Figur 3: Illustrasjon for Oppgave 3 b. “Feasible area” = “gyldig område”.



Figur 4: Illustrasjon for Oppgave 3 c. “Feasible area” = “gyldig område”.

4 MPC og dynamisk optimalisering (34 %)

I de påfølgende spørsmål tar vi utgangspunkt i det dynamiske optimaliseringsproblemet (A.9) i Appendix.

a (3 %) Forklar kort hvorfor (A.9) kalles et åpen sløyfe optimaliseringsproblem.

b (3 %) Hva er begrunnelsen for å ta med ulikhetsbetingelsen (A.9f)?

c (2 %) Anta at $N = 15$, $n_x = 10$ og $n_u = 2$. Hva er da dimensjonen av z i (A.9l)?

d (4 %) Antallet beslutningsvariable kan reduseres kraftig dersom $x_1, \dots, x_N$ elimineres ifra z i (A.9l), ved å bruke likhetsbetingelsene (A.9b). Hva er fordelene og ulempene med å bruke en "full space formulation", dvs. at z er gitt som i (A.9l), sammenliknet med en "reduced space formulation" hvor bare $u_0, \dots, u_{N-1}$ er beslutningsvariable?

e (6 %) Ofte måles ikke alle tilstandene i x_t, men kun et subsett $y_t = Cx_t$. Anta at et åpen sløyfe optimaliseringsproblem som i (A.9), benyttes i en lineær output feedback MPC-regulator der vi kun har tilgang på y_t. Spesifiser en passende algoritme. (Vennligst bruk tilsvarende format og detaljnivå som i kursnotatet om MPC og optimalisering).

f (8 %) Dersom (A.9) benyttes i lineær MPC, kan problemet bli "infeasible", dvs. at det ikke eksisterer noen løsning. Forklar hvordan (A.9) kan endres for å unngå dette problemet. Vennligst referer til hvilke ligninger som må endres og hvordan disse forandres.

g (2 %) Anta at den lineære modellen (A.9b) byttes ut med en diskret-tid ulineær modell

$$x_{t+1} = g(x_t, u_t) \tag{1}$$

i en MPC-regulator. Foreslå en passende optimaliseringsalgoritme for det resulterende åpen sløyfe optimaliseringsproblemet.

h (6 %) I stedet for å bruke den ulineære modellen (1) i åpen sløyfe optimaliseringsproblemet approksimerer vi modellen med en lineær tidvarierende (LTV) modell

$$x_{t+1} = A_t x_t + B_t u_t \tag{2}$$

rundt et stasjonært punkt $\bar{x}_t, \bar{u}_t$. Hvordan kan A_t og B_t beregnes fra $g(x_t, u_t)$? Hvilken glatthetsbetingelse må legges på g med disse formlene for A_t og B_t?

Appendix

Part 1 Optimization Problems and Optimality Conditions

A general formulation for constrained optimization problems is

$$\begin{aligned}
\min_{x\in\mathbb{R}^n} \quad & f(x) && && \text{(A.1a)}\\
\text{s.t.} \quad & c_i(x) = 0, && i \in \mathcal{E} && \text{(A.1b)}\\
& c_i(x) \geq 0, && i \in \mathcal{I} && \text{(A.1c)}
\end{aligned}$$

where f and the functions c_i are all smooth, differentiable, real-valued functions on a subset of $\mathbb{R}^n$, and $\mathcal{E}$ and $\mathcal{I}$ are two finite sets of indices.

The Lagrangean function for the general problem (A.1) is

$$\mathcal{L}(x,\lambda) = f(x) - \sum_{i\in\mathcal{E}\cup\mathcal{I}} \lambda_i c_i(x) \tag{A.2}$$

The KKT-conditions for (A.1) are given by:

$$\begin{aligned}
\nabla_x \mathcal{L}(x^*,\lambda^*) &= 0 && && \text{(A.3a)}\\
c_i(x^*) &= 0, && i \in \mathcal{E} && \text{(A.3b)}\\
c_i(x^*) &\geq 0, && i \in \mathcal{I} && \text{(A.3c)}\\
\lambda_i^* &\geq 0, && i \in \mathcal{I} && \text{(A.3d)}\\
\lambda_i^* c_i(x^*) &= 0, && i \in \mathcal{E}\cup\mathcal{I} && \text{(A.3e)}
\end{aligned}$$

2nd order (sufficient) conditions for (A.1) are given by:

$$w \in \mathcal{C}(x^*,\lambda^*) \Leftrightarrow \begin{cases} \nabla c_i(x^*)^\top w = 0 & \text{for all } i \in \mathcal{E} \\ \nabla c_i(x^*)^\top w = 0 & \text{for all } i \in \mathcal{A}(x^*)\cap\mathcal{I} \text{ with } \lambda_i^* > 0 \\ \nabla c_i(x^*)^\top w \geq 0 & \text{for all } i \in \mathcal{A}(x^*)\cap\mathcal{I} \text{ with } \lambda_i^* = 0 \end{cases} \tag{A.4}$$

Theorem 1: (Second-Order Sufficient Conditions) *Suppose that for some feasible point $x^* \in \mathbb{R}^n$ there is a Lagrange multiplier vector λ^* such that the KKT conditions* (A.3) *are satisfied. Suppose also that*

$$w^\top \nabla^2_{xx}\mathcal{L}(x^*,\lambda^*)w > 0, \qquad \text{for all } w \in \mathcal{C}(x^*,\lambda^*),\ w \neq 0. \tag{A.5}$$

Then x^* is a strict local solution for (A.1).

LP problem in standard form:

$$\min_x \quad f(x) = c^\top x \tag{A.6a}$$
$$\text{s.t.} \quad Ax = b \tag{A.6b}$$
$$x \geq 0 \tag{A.6c}$$

where $A \in \mathbb{R}^{m \times n}$ and $\operatorname{rank} A = m$.

QP problem in standard form:

$$\min_x \quad f(x) = \frac{1}{2} x^\top G x + x^\top c \tag{A.7a}$$
$$\text{s.t.} \quad a_i^\top x = b_i, \qquad i \in \mathcal{E} \tag{A.7b}$$
$$a_i^\top x \geq b_i, \qquad i \in \mathcal{I} \tag{A.7c}$$

where G is a symmetric $n \times n$ matrix, $\mathcal{E}$ and $\mathcal{I}$ are finite sets of indices and c, x and $\{a_i\}, i \in \mathcal{E} \cup \mathcal{I}$, are vectors in $\mathbb{R}^n$. Alternatively, the equalities can be written $Ax = b$, $A \in \mathbb{R}^{m \times n}$.

Iterative method:

$$x_{k+1} = x_k + \alpha_k p_k \tag{A.8a}$$
$$x_0 \text{ given} \tag{A.8b}$$
$$x_k, p_k \in \mathbb{R}^n, \; \alpha_k \in \mathbb{R} \tag{A.8c}$$

p_k is the search direction and α_k is the line search parameter.

Part 2 Optimal Control

A typical open-loop optimal control problem on the time horizon 0 to N is

$$\min_{z\in\mathbb{R}^n} f(z) = \sum_{t=0}^{N-1} \frac{1}{2}x_{t+1}^\top Q_{t+1}x_{t+1} + d_{xt+1}x_{t+1} + \frac{1}{2}u_t^\top R_t u_t + d_{ut}u_t \tag{A.9a}$$

subject to

$$\begin{aligned}
x_{t+1} &= A_t x_t + B_t u_t, & t &= 0,\ldots,N-1 & \text{(A.9b)}\\
x_0 &= \text{given} & & & \text{(A.9c)}\\
x^{\text{low}} &\le x_t \le x^{\text{high}}, & t &= 1,\ldots,N & \text{(A.9d)}\\
u^{\text{low}} &\le u_t \le u^{\text{high}}, & t &= 0,\ldots,N-1 & \text{(A.9e)}\\
-\Delta u^{\text{high}} &\le \Delta u_t \le \Delta u^{\text{high}}, & t &= 0,\ldots,N-1 & \text{(A.9f)}\\
Q_t &\succeq 0 & t &= 1,\ldots,N & \text{(A.9g)}\\
R_t &\succeq 0 & t &= 0,\ldots,N-1 & \text{(A.9h)}
\end{aligned}$$

where

$$\begin{aligned}
u_t &\in \mathbb{R}^{n_u} & \text{(A.9i)}\\
x_t &\in \mathbb{R}^{n_x} & \text{(A.9j)}\\
\Delta u_t &= u_t - u_{t-1} & \text{(A.9k)}\\
z^\top &= (x_1^\top,\ldots,x_N^\top,u_0^\top,\ldots,u_{N-1}^\top) & \text{(A.9l)}
\end{aligned}$$

The subscript t denotes discrete time sampling instants.

The optimization problem for linear quadratic control of discrete dynamic systems is given by

$$\min_{z\in\mathbb{R}^n} f(z) = \sum_{t=0}^{N-1} \frac{1}{2}x_{t+1}^\top Q_{t+1}x_{t+1} + \frac{1}{2}u_t^\top R_t u_t \tag{A.10a}$$

subject to

$$\begin{aligned}
x_{t+1} &= A_t x_t + B_t u_t & \text{(A.10b)}\\
x_0 &= \text{given} & \text{(A.10c)}
\end{aligned}$$

where

$$\begin{aligned}
u_t &\in \mathbb{R}^{n_u} & \text{(A.10d)}\\
x_t &\in \mathbb{R}^{n_x} & \text{(A.10e)}\\
z^\top &= (x_1^\top,\ldots,x_N^\top,u_0^\top\ldots,u_{N-1}^\top) & \text{(A.10f)}
\end{aligned}$$

Theorem 2: *The solution of (A.10) with $Q_t \succeq 0$ and $R_t \succ 0$ is given by*

$$u_t = -K_t x_t \tag{A.11a}$$

where the feedback gain matrix is derived by

$$K_t = R_t^{-1} B_t^\top P_{t+1}(I + B_t R_t^{-1} B_t^\top P_{t+1})^{-1} A_t, \qquad t = 0, \dots, N-1 \tag{A.11b}$$

$$P_t = Q_t + A_t^\top P_{t+1}(I + B_t R_t^{-1} B_t^\top P_{t+1})^{-1} A_t, \qquad t = 0, \dots, N-1 \tag{A.11c}$$

$$P_N = Q_N \tag{A.11d}$$

Part 3 Sequential quadratic programming (SQP)

Algorithm 18.3 (Line Search SQP Algorithm).

Choose parameters $\eta \in (0, 0.5)$, $\tau \in (0, 1)$, and an initial pair (x_0, λ_0);
Evaluate $f_0, \nabla f_0, c_0, A_0$;
If a quasi-Newton approximation is used, choose an initial $n \times n$ symmetric positive definite Hessian approximation B_0, otherwise compute $\nabla^2_{xx}\mathcal{L}_0$;
repeat until a convergence test is satisfied
    Compute p_k by solving (18.11); let $\hat{\lambda}$ be the corresponding multiplier;
    Set $p_\lambda \leftarrow \hat{\lambda} - \lambda_k$;
    Choose μ_k to satisfy (18.36) with $\sigma = 1$;
    Set $\alpha_k \leftarrow 1$;
    while $\phi_1(x_k + \alpha_k p_k; \mu_k) > \phi_1(x_k; \mu_k) + \eta \alpha_k D_1(\phi(x_k; \mu_k) p_k)$
        Reset $\alpha_k \leftarrow \tau_\alpha \alpha_k$ for some $\tau_\alpha \in (0, \tau]$;
    end (while)
    Set $x_{k+1} \leftarrow x_k + \alpha_k p_k$ and $\lambda_{k+1} \leftarrow \lambda_k + \alpha_k p_\lambda$;
    Evaluate $f_{k+1}, \nabla f_{k+1}, c_{k+1}, A_{k+1}$, (and possibly $\nabla^2_{xx}\mathcal{L}_{k+1}$);
    If a quasi-Newton approximation is used, set
        $s_k \leftarrow \alpha_k p_k$ and $y_k \leftarrow \nabla_x \mathcal{L}(x_{k+1}, \lambda_{k+1}) - \nabla_x \mathcal{L}(x_k, \lambda_{k+1})$,
    and obtain B_{k+1} by updating B_k using a quasi-Newton formula;
end (repeat)