Øving 5

1.1. 13)

$$\begin{aligned} x_1 \qquad - 3x_3 &= 8 \quad &(i)\\ 2x_1 + 2x_2 + 9x_3 &= 7 \quad &(ii)\\ x_2 + 5x_3 &= -2 \quad &(iii)\end{aligned}$$

$$\Updownarrow$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{array}\right) \quad R_2 \leftarrow R_2 - 2R_1$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 1 & 5 & -2 \end{array}\right) \quad R_3 \leftarrow R_3 - \frac{1}{2} R_2$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 0 & -\frac{5}{2} & \frac{5}{2} \end{array}\right) \quad \begin{aligned} R_3 &\leftarrow -\tfrac{2}{5} R_3 \\ R_2 &\leftarrow \tfrac{1}{2} R_2 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 15/2 & -\frac{9}{2} \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$\Rightarrow x_3 = -1$$

$$(iii) \Rightarrow x_2 + 5 \cdot (-1) = -2$$

$$x_2 = 3$$

$$(i) \Rightarrow x_1 - 3(-1) = 8$$

$$x_1 = 5$$

$\underline{\underline{x_1 = 5,\ x_2 = 3,\ x_3 = -1 \text{ løser systemet}}}$

14) $$x_1 - 3x_2 = 5 \quad \text{(i)}$$
$$-x_1 + x_2 + 5x_3 = 2 \quad \text{(ii)}$$
$$x_2 + x_3 = 0 \quad \text{(iii)}$$

$$\Updownarrow$$

$$\left(\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{array}\right) \quad R_2 \leftarrow R_2 + R_1$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & -2 & 5 & 7 \\ 0 & 1 & 1 & 0 \end{array}\right) \quad R_3 \leftarrow R_3 + \frac{1}{2} R_2$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & -2 & 5 & 7 \\ 0 & 0 & \frac{7}{2} & \frac{7}{2} \end{array}\right)$$

$$\Rightarrow x_3 = 1$$

(iii) $x_2 + 1 = 0$

$$\Rightarrow x_2 = -1$$

(i) $x_1 - 3(-1) = 5$

$$\Rightarrow x_1 = 2$$

$x_1 = 2$, $x_2 = -1$ og $x_3 = 1$ løser systemet

25) $\left(\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{array}\right)$ $R_3 \leftarrow R_3 + 2R_1$

$\Rightarrow \left(\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & -3 & 5 & k+2g \end{array}\right)$ $R_3 \leftarrow R_3 + R_2$

$\left(\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & 0 & 0 & k+2g+h \end{array}\right)$

Hvis $\underline{\underline{k+2g+h=0}}$ kan systemet løses

1.2

3) $\begin{array}{c} \text{pivot} \\ \left(\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{array}\right) \end{array}$ $R_2 \leftarrow R_2 - 4R_1$
$R_3 \leftarrow R_3 - 6R_1$

$\left(\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 0 & -5 & -10 & -15 \end{array}\right)$ $R_2 \leftarrow -\frac{1}{3}R_2$
$R_3 \leftarrow -\frac{1}{5}R_3$

$\left(\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{array}\right)$ $R_3 \leftarrow R_3 - R_2$
$R_1 \leftarrow R_1 - 2R_2$

Pivots $\left(\begin{array}{cccc} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$ Pivots $\left(\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{array}\right)$

pivot-columns

FINAL ORIGINAL

7) $\begin{pmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{pmatrix}$ $R_2 \leftarrow R_2 - 3R_1$

$\Rightarrow \begin{pmatrix} 1 & 3 & 4 & 7 \\ 0 & 0 & -5 & -15 \end{pmatrix}$ $R_2 \leftarrow -\frac{1}{5} R_2$

$\Rightarrow \begin{pmatrix} 1 & 3 & 4 & 7 \\ 0 & 0 & 1 & 3 \end{pmatrix}$ $R_1 \leftarrow R_1 - 4R_2$

$\Rightarrow \begin{pmatrix} 1 & 3 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{pmatrix}$

Som svarer til: $\begin{aligned} x_1 + 3x_2 &= -5 \\ x_3 &= 3 \end{aligned}$

Velger x_2 som fri variabel.
Generell løsning:

$$\begin{cases} x_1 = -5 - 3x_2 \\ x_2 \text{ fri} \\ x_3 = 3 \end{cases}$$

11) $\left(\begin{array}{ccc|c} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{array}\right)$ $R_2 \leftarrow R_2 + 3R_1$
$R_3 \leftarrow R_3 + 2R_1$

$\Rightarrow \begin{pmatrix} 3 & -4 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\Rightarrow \quad 3x_1 - 4x_2 + 2x_3 = 0$

x_2 og x_3 er frie variabler.
Generell løsning blir:

$$\begin{cases} x_1 = \frac{1}{3}(4x_2 - 2x_3) \\ x_2 \quad \text{fri} \\ x_3 \quad \text{fri} \end{cases}$$

17) $\begin{pmatrix} 2 & 3 & h \\ 4 & 6 & 7 \end{pmatrix}$ $R_2 \leftarrow R_2 - 2R_1$

$\Rightarrow \begin{pmatrix} 2 & 3 & h \\ 0 & 0 & 7-2h \end{pmatrix}$

For $7-2h = 0$ vil systemet være konsistent.

29) Et slikt underbestemt system vil ha en (eller flere) frie variabler. Frie variabler kan ha uendelig mange forskjellige verdier og hver verdi (verdi-kombinasjon) vil svare til én bestemt løsning. Så derfor er det uendelig mange løsninger.

1.3

5) $6x_1 - 3x_2 = 1$

$-x_1 + 4x_2 = -7$

$5x_1 = -5$

11) $x_1 \cdot \vec{a}_1 + x_2 \vec{a}_2 + x_3 \vec{a}_3 = \vec{b}$?

$\Leftrightarrow x_1 \cdot \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + x_3 \begin{pmatrix} 5 \\ -6 \\ 8 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix}$

$\Downarrow$

$$\left(\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{array}\right) \quad R_2 \leftarrow R_2 + 2R_1$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 2 & 8 & 6 \end{array}\right) \quad R_3 \leftarrow R_3 - 2R_2$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\Rightarrow \begin{cases} x_1 = 2 - 5x_3 \\ x_2 = 3 - 4x_3 \\ x_3 \text{ fri} \end{cases} \qquad (*)$$

$\vec{b}$ er en lineær kombinasjon av $\vec{a}_1, \vec{a}_2$ og $\vec{a}_3$

17) $\vec{a}_1 = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$, $\vec{a}_2 = \begin{pmatrix} -2 \\ -3 \\ 7 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 4 \\ 1 \\ h \end{pmatrix}$

$$\left(\begin{array}{cc|c} 1 & -2 & 4 \\ 4 & -3 & 1 \\ -2 & 7 & h \end{array}\right) \quad \begin{array}{l} R_2 \leftarrow R_2 - 4R_1 \\ R_3 \leftarrow R_3 + 2R_1 \end{array}$$

$$\Rightarrow \left(\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 5 & -15 \\ 0 & 3 & h-8 \end{array}\right) \quad R_2 \leftarrow \frac{1}{5} R_2$$

$$\Rightarrow \left(\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 3 & h-8 \end{array}\right) \quad R_3 \leftarrow R_3 - 3R_2$$

$$\Rightarrow \left(\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & h+1 \end{array}\right)$$

Må kreve $h+1 = 0$, altså $\underline{\underline{h = -1}}$

22) $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}$

$\vec{b} = \begin{pmatrix} 0 \\ 0 \\ h \end{pmatrix}$

Velg h slik at $A \cdot x \neq \vec{b}$.

Altså slik at systemet:

$$\left(\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 \\ 1 & 3 & 5 & h \end{array}\right)$$

ikke er konsistent.

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 \\ 1 & 3 & 5 & h \end{array}\right) \quad \begin{array}{l} R_2 \leftarrow R_2 - R_1 \\ R_3 \leftarrow R_3 - R_1 \end{array}$$

$$\Leftrightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 2 & 4 & h \end{array}\right) \quad R_3 \leftarrow R_3 - 2R_2$$

$$\Leftrightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & h \end{array}\right)$$

Hvis $h=0$ er systemet konsistent, men ønsker det motsatte så setter $h=1$. Da vil $\vec{b}$ ikke være i span$\{A\}$.

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

1.4

1) Produktet er udefinert fordi $\vec{x} = \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix}$ må ha like mangerader som $A = \begin{pmatrix} -4 & 2 \\ -1 & 6 \\ 0 & 1 \end{pmatrix}$ har kollonner.

8) $\vec{x} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}, \quad A = \begin{pmatrix} 4 & -4 & -5 & 3 \\ -2 & 5 & 4 & 0 \end{pmatrix}$

$$\vec{b} = \begin{pmatrix} 4 \\ 13 \end{pmatrix}$$

$$A\vec{x} = \vec{b}$$

$$\Updownarrow$$

$$\begin{pmatrix} 4 & -4 & -5 & 3 \\ -2 & 5 & 4 & 0 \end{pmatrix} \cdot \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} = \begin{pmatrix} 4 \\ 13 \end{pmatrix}$$

10) Vektor: $x_1 \cdot \begin{pmatrix} 8 \\ 5 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} -1 \\ 4 \\ -3 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$

Matrise: $\begin{pmatrix} 8 & -1 \\ 5 & 4 \\ 1 & -3 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$

17) Hvis A har en pivot i hver rad har $A\vec{x} = \vec{b}$ en løsning for alle $\vec{b} \in \mathbb{R}^4$.

$$A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix} \quad \begin{matrix} \\ R_2 \leftarrow R_2 + R_1 \\ R_3 \leftarrow \frac{1}{2} R_3 \\ R_4 \leftarrow R_4 - 2R_1 \end{matrix}$$

$$\Leftrightarrow \begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ 0 & -2 & 1 & -4 \\ 0 & -6 & 3 & -7 \end{bmatrix} \quad \begin{matrix} \\ \\ R_3 \leftarrow R_3 + R_2 \\ R_4 \leftarrow R_4 + 3R_2 \end{matrix}$$

$$\begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

A har 3 pivot-posisjoner og $A\vec{x} = \vec{b}$ har ikke en løsning for alle $\vec{b} \in \mathbb{R}^4$.

32) Nei, fordi hvis den kan det hadde den hatt en pivot i hver rad, og med 3 vektorer er det umulig.

$$\begin{bmatrix} \blacksquare & * & * \\ 0 & \blacksquare & * \\ 0 & 0 & \blacksquare \\ 0 & 0 & 0 \end{bmatrix}$$