# Øving 5

Ønsker tilbakemelding :)

Bra!

Se kommentere

Rendell Cale, gruppe 2, mttk

4.1 2)

a) $S(n): \sum_{i=1}^{n} 2^{i-1} = 2^n - 1$

Base case: $S(1)$

$$\sum_{i=1}^{1} 2^{i-1} = 2^1 - 1$$
$$\downarrow \qquad \downarrow$$
$$2^{1-1} = 1$$
$$\downarrow \qquad \downarrow$$
$$1 = 1 \quad \checkmark$$

S(1) is true.

NB! BLOD unnskyld

Induction step: Assume as an induction hypothesis that $S(k)$ is true for $k \in \mathbb{Z}^+$,

~~so~~ $\sum_{i=1}^{k} 2^{i-1} = 2^k - 1$.

Blodig alvor!

$$S(k+1): \sum_{i=1}^{k+1} 2^{i-1}$$
$$= \underbrace{\sum_{i=1}^{k} 2^{i-1}}_{} + 2^{(k+1)-1}$$
$$\overset{IH}{=} 2^k - 1 + 2^k$$
$$= 2 \cdot 2^k - 1$$
$$= 2^{k+1} - 1$$

So since $S(K) \Rightarrow S(K+1)$ and $S(1)$ is true, $S(n)$ is true for all $n$ by induction. ∎

b) $S(n): \sum_{i=1}^{n} i \cdot 2^i = 2+(n-1)2^{n+1}$

Base case: $n=1$:

$$S(1): \sum_{i=1}^{1} i2^i = 2+(1-1)2^{n+1}$$

$$1 \cdot 2^1 = 2$$

$$2 = 2 \quad \checkmark$$

$S(1)$ is true.

Induction step: Let $K \in \mathbb{Z}^+$ and assume that $S(K)$ is true as an induction hypothesis. We then have that

$$\sum_{i=1}^{K} i2^i = 2+(K-1) \cdot 2^{K+1}$$

$$S(K+1): \sum_{i=1}^{K+1} i2^i = \underbrace{\sum_{i=1}^{K} i2^i}_{} + (K+1)2^{K+1}$$

$$\overset{IH}{=} 2+(K-1)2^{K+1} + (K+1)2^{K+1}$$

$$= 2+(K-1+K+1)2^{K+1}$$

$$= 2+2K \cdot 2^{K+1} = 2+K \cdot 2^{K+2}$$

To make it clear:

$$2 + k2^{k+2} = 2 + ((k+1)-1)\,2^{(k+1)+1}$$

So $S(k) \Rightarrow S(k+1)$ and since $S(1)$ is true, then by induction, $S(n)$ is true for all $n$.

c) $S(n)$: $\sum_{i=1}^{n} (i)(i!) = (n+1)! - 1$

Base case: $n=1$

$$S(1): \sum_{i=1}^{1} (i)(i!) = (1+1)! - 1$$

$$1 \cdot 1! = 2! - 1$$

$$1 = 1 \checkmark$$

So $S(1)$ is true

Induction step: Let $k \in \mathbb{Z}^+$ and assume as an induction hypothesis that $S(k)$ holds. So:

$$\sum_{i=1}^{k} (i \cdot (i!)) = (k+1)! - 1$$

$$S(k+1): \sum_{i=1}^{k+1} (i) \cdot (i!) = \underbrace{\sum_{i=1}^{k}}_{} + (k+1)(k+1)!$$

$$\overset{IH}{=} (k+1)! - 1 + (k+1)(k+1)!$$

$$= (k+1)!\,(1 + k + 1) - 1$$

$$= (k+1)! \cdot (k+2) - 1$$

Since $(k+1)! \cdot (k+2) = (k+2)!$ we get

$$S(k+1): \sum_{i=1}^{k+1} (i)(i!) = (k+2)! - 1$$

$$= ((k+1)+1)! - 1 \quad R$$

This shows that $S(k) \Rightarrow S(k+1)$ and since $S(1)$ is true then $S(n)$ is, by induction, true for all $n$.

14) $n \in \mathbb{Z}^+,\ n \geq 4$

$$S(n): 2^n < n!$$

Base case: $n=4$:

$$S(4): \quad 2^4 \quad < \quad 4!$$

$$16 < 24 \quad \checkmark$$

$S(4)$ is true.

Induction step: Let $k \in \mathbb{Z}^+$, $k \geq 4$ and assume $S(k)$ holds as an induction hypothesis. So we can write:

$$S(k): 2^k < k!$$

$S(k+1)$: $2^{k+1} = 2 \cdot 2^k$

By IH $2^k < k!$ so

$$2^{k+1} < 2 \cdot k!$$

Since $k > 2$, $2 \cdot k! < (k+1)!$ so we can write

$$2^{k+1} < 2k! < (k+1)!$$

This shows that $S(k) \Rightarrow S(k+1)$ and since $S(4)$ is true, $S(n)$ holds for all $n \geq 4$ (by induction. It doesn't say anything about the case of $n \leq 3$ so we say $S(n)$ is true there as well. So $S(n)$ is true for all $n$.

Bra!

24)

$a_1 = 1,\ a_2 = 2,\ a_n = a_{n-1} + a_{n-2},\ n \geq 3$

a) $a_3 = a_2 + a_1 = 2+1 = 3$

$a_4 = a_3 + a_2 = 3+2 = 5$

$a_5 = a_4 + a_3 = 5+3 = 8$

$a_6 = a_5 + a_4 = 8+5 = 13$

$a_7 = a_6 + a_5 = 13+8 = 21$

b) $S(n): a_n < \left(\frac{7}{4}\right)^n$

Base cases: $a_1 = 1 < \left(\frac{7}{4}\right)^1$ $S(1)$ ✓

$a_2 = 2 < \left(\frac{7}{4}\right)^2 \approx 3$ $S(2)$ ✓

Let $K \in \mathbb{Z}^+$ and assume as an induction hypothesis that $S(K)$ and $S(K-1)$ are true:

$$a_{K-1} < \left(\frac{7}{4}\right)^{K-1},\quad a_K < \left(\frac{7}{4}\right)^K$$

$S(k+1)$: $a_{K+1} = a_{K-1} + a_K$

By the IH:

$$a_{K+1} < \left(\frac{7}{4}\right)^{K-1} + \left(\frac{7}{4}\right)^K$$

$$< \left(1 + \frac{7}{4}\right) \cdot \left(\frac{7}{4}\right)^{K-1}$$

$$< \left(\frac{7}{4}\right)^2 \cdot \left(\frac{7}{4}\right)^{K-1}$$

$$< \left(\frac{7}{4}\right)^{K+1}$$

So $S(K-1)$ and $S(K)$ implies $S(K+1)$ and since $S(1)$ and $S(2)$ are true then, by induction, $S(n)$ is true for all $n$

4.2 1)

c) $\underline{c_n = 3n + 7}$

$$c_1 = 3 \cdot 1 + 7 = 10$$
$$c_{n+1} = c_n + 3$$

e) $\underline{c_n = n^2}$

$$c_1 = 1$$
$$c_{n+1} = c_n + 2n - 1$$

f) $\underline{c_n = 2 - (-1)^n}$ ← denne er bra!

$$c_1 = 2 - (-1)^1 = 3$$
$$c_{n+1} = 4 - c_n$$

Nesten, men ikke helt veldefinert, f.eks.

Blir riktig med 2.

12) $F_0 = 0$ , $F_1 = 1$

$F_n = F_{n-2} + F_{n-1}$ , $n \geq 0$

Base case: $n = 0$

$S(0): F_0 = F_2 - 1$

$0 = 1 - 1$

$0 = 1$ ✓

So $S(0)$ is true.

Assume as an induction hypothesis that $S(k)$ holds.

So: $F_0 + F_1 + \dots + F_k = F_{k+2} - 1$

$S(k+1): \underbrace{F_0 + F_1 + \dots + F_k} + F_{k+1}$

$\overset{IH}{=} F_{k+2} - 1 + F_{k+1}$

$= F_{k+3} - 1 = F_{(k+1)+2} - 1$

So $S(k) \Rightarrow S(k+1)$ and since $S(0)$ is true, $S(n)$ holds for all $n \in \mathbb{Z}^+$ (by induction).