Oppgave 1

a) $x_1 + x_2 + x_3 + x_4 + x_5 = 21$

How many solutions?

This is just combination with repitition.

$$\binom{21+5-1}{5-1} = \binom{25}{4} \text{ solutions.}$$

b) Make a change of variable, $x_2' = x_2 - 2 \geq 0$
$x_3' = x_3 - 3 \geq 0$

This gives $x_2 = x_2' + 2$, $x_3 = x_3' + 3$ which we substitute in in the equation.

$$x_1 + (x_2' + 2) + (x_3' + 3) + x_4 + x_5 = 21$$

$$\iff x_1 + x_2' + x_3' + x_4 + x_5 = 16, \quad x_2', x_3' \geq 0$$

which has $\binom{16+5-1}{5-1} = \binom{20}{4}$ solutions.

c) ~~The condition C_4: x_4~~

c) We create a condition c: $x_4 > 4$, and we want to find the number where c is not satisfied, that is $N(\bar{c})$. If N is the total number of solutions, then $N = N(c) + N(\bar{c})$.

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So $N(\overline{c}) = N - N(c)$.

We calculate $N(c)$ using the same technique as in (b).

$$x_4^* = x_4' - 4\,5 \quad (\geq 0)$$

Substitution: $x_1 + x_2 + x_3 + (x_4' + 5) + x_5 = 21$

$$\Leftrightarrow \quad x_1 + x_2 + x_3 + x_4' + x_5 = 16$$

$$\Rightarrow \quad \binom{16+5-1}{5-1} = \binom{20}{4} \text{ solutions}$$

So $N(c) = \binom{20}{4}$, $N = \binom{25}{4}$.

$$\Rightarrow \quad N(\overline{c}) = \binom{25}{4} - \binom{20}{4} = 12650 - 4845$$

$$= 7805$$

There are 7805 solutions of (a) where $x_4 \leq 4$.

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Oppgave 2

a) Consider $p = q = r = F_0$ (...).

Then $(*): ((p \to q) \land (r \to \neg q)) \to (p \land r)$

boils down to

$$((F_0 \to F_0) \land (F_0 \to T_0)) \to (F_0 \land F_0)$$

$$\Leftrightarrow \quad T_0 \land T_0 \to F_0$$

$$\Leftrightarrow \quad T_0 \to F_0$$

$$\Leftrightarrow \quad F_0$$

So $(*)$ is not a tautology.

b) Since $p \to q \Leftrightarrow \neg p \lor q$

DeMorgan, Double negation

we get $\neg(p \to q) \Leftrightarrow \neg(\neg p \lor q) \Leftrightarrow p \land \neg q$

Which we'll use below.

$\neg[((p \to q) \land (p \to \neg q)) \to (p \land r)]$

$\Leftrightarrow \quad (p \to q) \land (r \to \neg q) \land \neg(p \lor r)$ as explained above

$\Leftrightarrow \quad (p \to q) \land (r \to \neg q) \land (\neg p \land \neg r)$ DeMorgan.

So the negation is $(p \to q) \land (r \to \neg q) \land (\neg p \land \neg r)$
(a)

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c)

$p \to \neg q$	Premise 1 (p1)
$\neg r \vee q$	Premise 2 (p2)
r	Premise 3 (p3)
q	(p2)+(p3) (4)
$\neg(\neg q) \to \neg p$	Contrapositive of (p1) (5)
$q \to \neg q$	Double negation of (5), (6)
$\therefore \neg p$	Modus ponens, (4)+(6)

So it is a valid argument.

Oppgave 3

Let $r \in \mathbb{R}$, $r \neq 1$.

Let $p(n)$ be the proposition that $\sum_{i=0}^{n} r^i = \frac{1-r^{n+1}}{1-r}$

We want to prove that this is true for all $n \geq 0$.

Base case: $n=0$:

$$\sum_{i=0}^{0} r^i = r^0 = 1$$

$$\frac{1-r^{0+1}}{1-r} = \frac{1-r}{1-r} = 1 \quad \text{since } r \neq 1.$$

This shows that $p(0)$ is true.

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Induction hypothesis: Assume as an induction hypothesis (IH) that $p(k)$ is true for some $k \in \mathbb{Z}^+$.

Induction step: Want to show that $p(k) \Rightarrow p(k+1)$ using the IH.

We have by (IH) $\sum_{i=0}^{k} r^i = \frac{1-r^{k+1}}{1-r}$.

Investigating the case $k+1$ gives:

$$\sum_{i=0}^{k+1} r^i = \sum_{i=0}^{k} r^i + r^{k+1}$$

$$\overset{(IH)}{=} \frac{1-r^{k+1}}{1-r} + r^{k+1}$$

$$= \frac{1-r^{k+1} + (1-r)r^{k+1}}{1-r}$$

$$= \frac{1-r \cdot r^{k+1}}{1-r}$$

$$= \frac{1-r^{(k+1)+1}}{1-r}$$

which is the same as the formula gives so $p(k) \Rightarrow p(k+1)$

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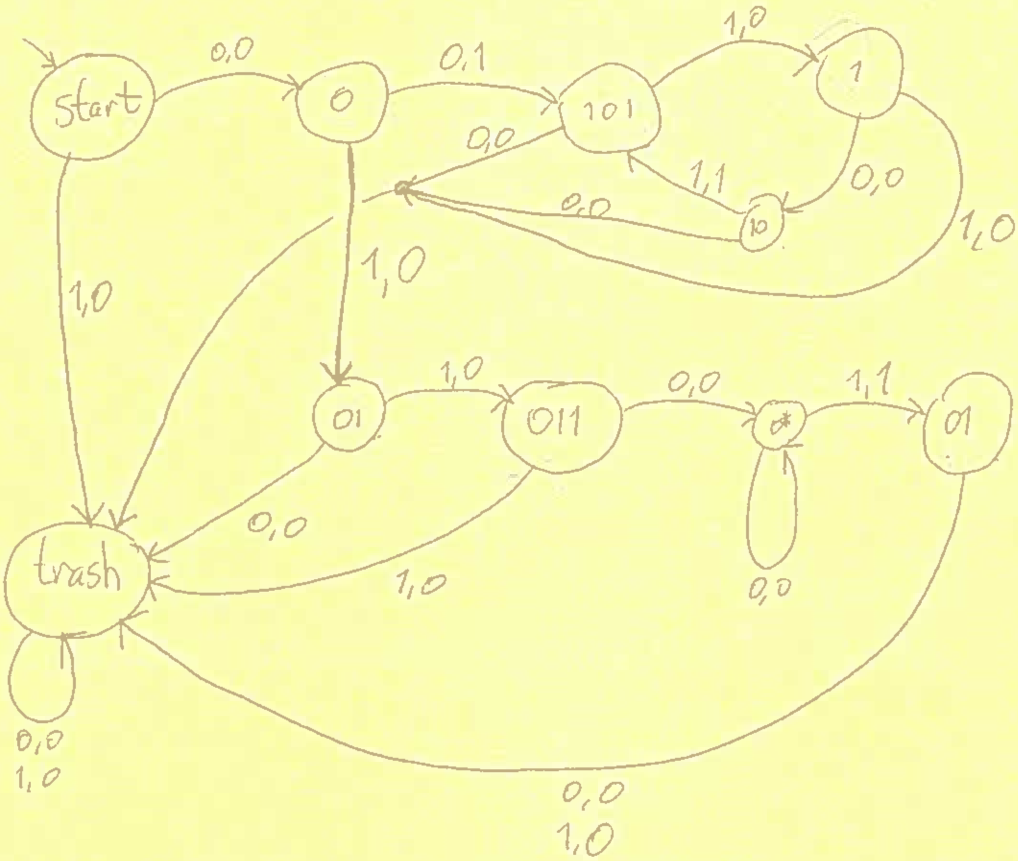
Since $p(0)$ is true and $p(k) \Rightarrow p(k+1)$, then by induction $p(n)$ is true for all $n \geq 0$ which is what we wanted to show.

Oppgave 4

a) 00, 00101, 00 [illegible]

b)



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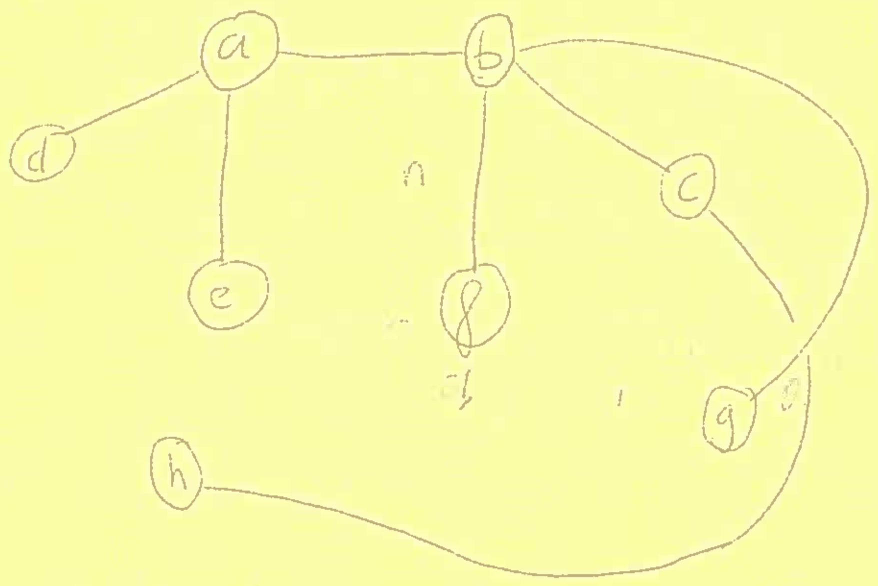
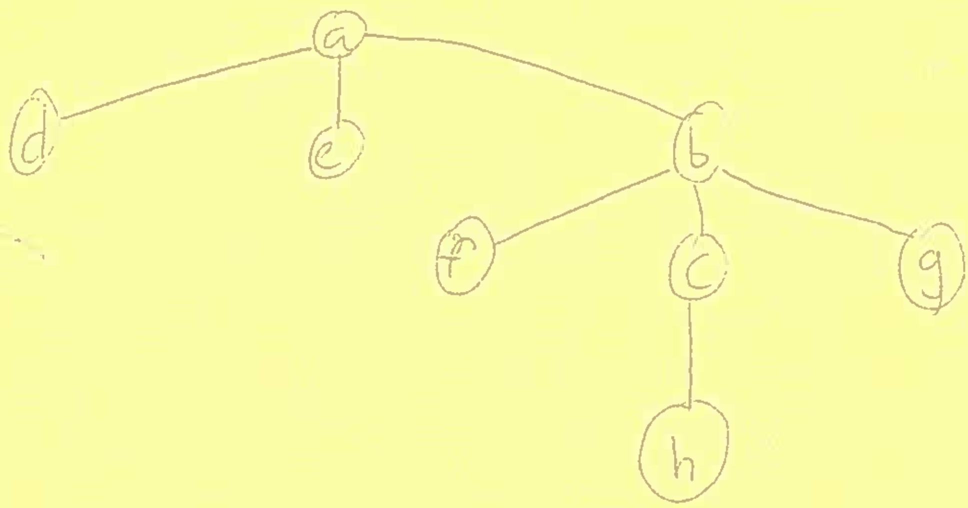
Oppgave 5

a)



Is a spanning tree

[illegible] a rooted tree:



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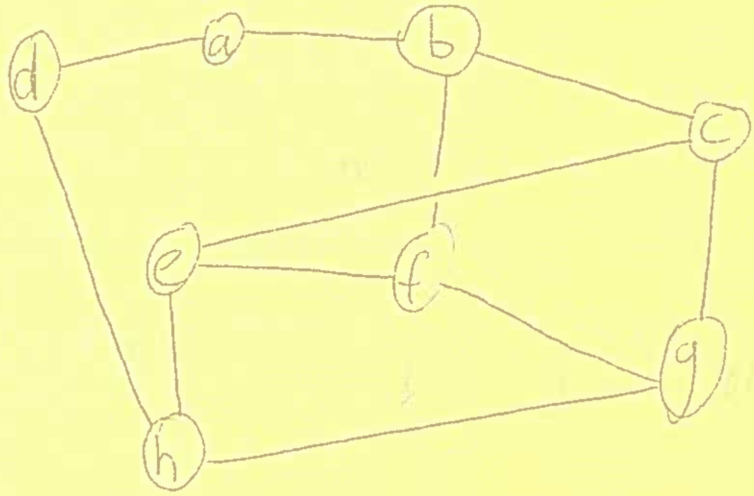
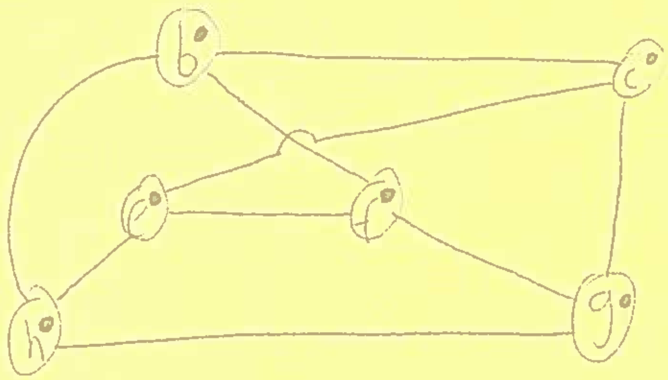
b) Consider the following subgraph:



which is homeomorphic to:



which is $K_{3,3}$.

We know that any graph which has a subgraph homeomorphic to K_5 or $K_{3,3}$ is non-planar, so the graph in figure 1 is non-planar.

c) A graph has a euler trail iff exactly two vertices have odd degrees.

d, a, e, h have odd degrees so there is no Euler trail.

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Oppgave 6

Let A be the set of functions from $\mathbb{Z}^+$ to $\{1, 2, 3\}$.

a) An equivalence relation is a relation that is symmetric, reflexive and transitive.

b) $f R_1 g \iff f(5) = g(5)$.

Reflexive: Since $f(5) = f(5)$, $f R_1 f$ is true so R_1 is reflexive.

Symmetry: Assume $f R_1 g$.

Then $f(5) = g(5)$ which implies that $g(5) = f(5)$ ~~which~~ means that $g R_1 f$. This shows that R_1 is symmetric.

Transitive: Assume $f R_1 g$ and $g R_1 h$.

Then $f(5) = g(5)$ and $g(5) = h(5)$

$\Rightarrow f(5) = h(5)$

$\Rightarrow f R_1 h$

So R_1 is transitive.

Since R_1 is reflexive, symmetric and transitive R_1 is an equivalence relation.

c) Define R_2 on A by $f R_2 g$ iff. there exists an $n \in \mathbb{Z}^+$ such that $f(n) = g(n)$.

Consider $f(1) = g(1)$ and $g(2) = h(2)$. Then we cannot by any means conclude that $f R_2 h$ ~~c~~ is an $n \in \mathbb{Z}^+$ such that $f(n) = h(n)$, so this is not a transitive relation and therefore it is not an equivalence relation.

For the sake of rigour here is a counterexample: $f(n) = 1$, $h(n) = 2$

$$g(n) = \begin{cases} 1, & n = 1 \\ 2, & \text{otherwise.} \end{cases}$$

Then the above considerations hold but $f(n) \neq h(n)$ for all $n \in \mathbb{Z}^+$