Godkjent
NB

Øving 8

Ønsker tilbakemelding :)

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5.2

4) $f: A \to B$, there are $|B|^{|A|}$ functions from A to B, Since $|B| = 3$ we get:

$$3^{|A|} = 2187 = 3^7$$

So $\underline{\underline{|A| = 7}}$ ✓

20) If $A = \{1,2,3,4,5\}$
$f: A \to B$
Let $m = |B|$, then there are

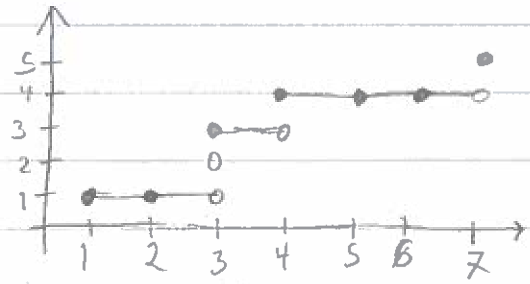
$$m \cdot (m-1) \cdot (m-2) \cdot (m-3) \cdot (m-4) = P(m,5)$$

injective functions from A to B
We have that $P(m,5) = 6720$
which gives $m = 8$
So $\underline{\underline{|B| = 8}}$ ✓

22) For $n \in \mathbb{Z}^+$, let $X_n = \{1,2,\ldots,n\}$
Given $m,n \in \mathbb{Z}^+$, $f: X_m \to X_n$

a) Domain: $X_7 = \{1,2,3,4,5,6,7\}$
Codomain: $X_5 = \{1,2,3,4,5\}$
$f: X_7 \to X_5$

Suppose f is a constant function with $f(1) = 1$. One way to make all the increasing monotone functions from this is to select four points from the domain (with repitition) and increase f by one at that point (and onwards). After doing this f might look like:



Select 3 twice and 4 and 7 once.

There are $\binom{7+4-1}{4} = 210$ ways to do this.

But all the 210 ways end with $f(7)=5$, because there's no option to "not select" a point. We can get this by adding one extra point to pick from. So the number of monotone increasing functions is $\binom{(7+1)+4-1}{4} = \underline{330}$

✓

b) With domain X_6 and codomain X_9, we must select 8 $(9-1)$ points with repitition from a set of 7 $(6+1)$ options. This gives $\binom{7+8-1}{8} = 3003$ increasing functions from X_6 to X_9.

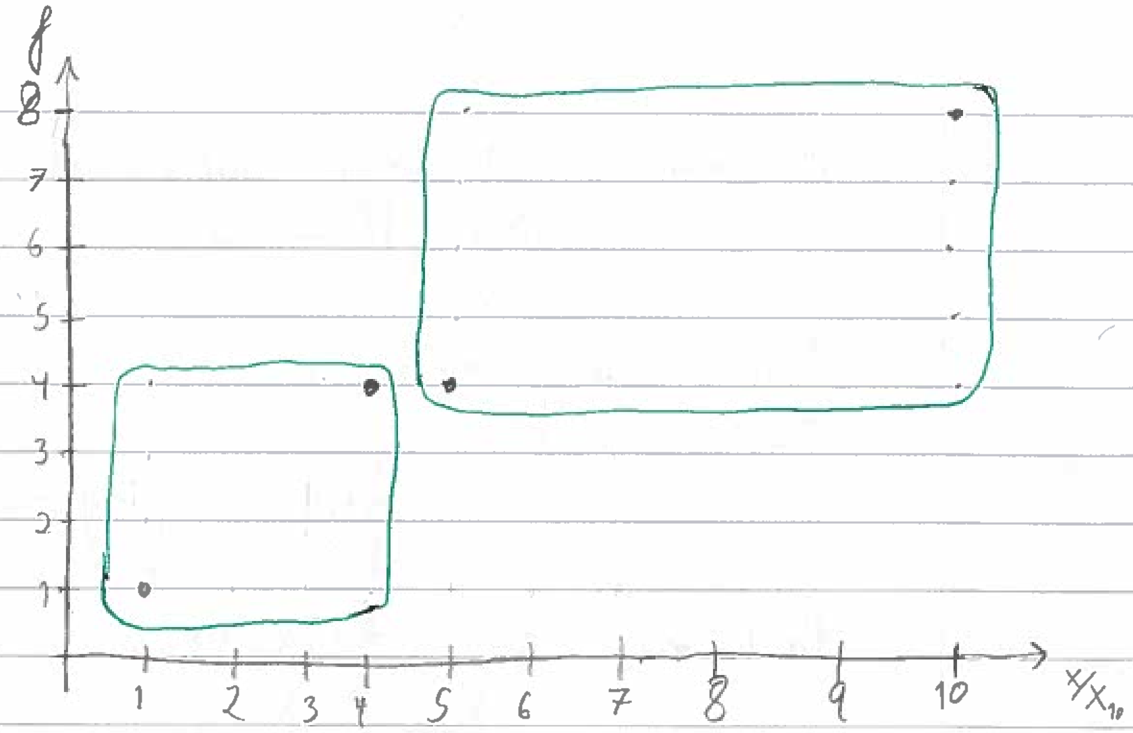
✓

c) From X_m to X_n there are

$$\binom{(m+1)+(n-1)-1}{n-1} = \binom{m+n-1}{n-1}$$ m

monotone increasing functions

X



d) $f : A \to B$

$f(4) = 4$

As the "graph" above illustrates, we can split the problem into two parts.

Part 1: Number of increasing functions from X_4 to X_4 when $f(4)=4$.
Select 3 points in X_4 with repitition.

$$\binom{4+3-1}{3} = \binom{6}{3}$$

Part 2: Number of increasing functions from $X_{10} \setminus X_4$ to $X_8 \setminus X_4$. By looking at the graph we can see that this the same num. of inc. funcs. from X_6 to X_5, we just shifted it so we can apply the formula from (c) and get

$$\binom{6+5-1}{5-1} = \binom{10}{4}$$

By rule of product there are then

$$\binom{6}{3}\cdot\binom{10}{4} = 4200$$ ✓

e) Same explanation as (d)

Part 1: $X_5 \to X_9$

$$\Rightarrow \binom{5+(9-1)-1}{9-1} = \binom{12}{8} = 4950$$ ✓

Part 2: $X_7 \setminus X_5 \to X_{12} \setminus X_9$

$$\Rightarrow X_{7-5} \to X_{12-(9-1)}$$

$$\Rightarrow X_2 \to X_4$$

SC LF

$$= \binom{2+4-1}{4-1} = \binom{6}{3} = 20$$

There are $\binom{12}{8}\cdot\binom{6}{3} = 9900$ increasing monotone functions from X_7 to X_{12} such that $f(5)=9$.

f) $f: X_m \to X_n$, $f(a_m) = b_n$, $a_m \in X_m$, $b_n \in X_n$

Part 1: $X_{a_m} \to X_{b_n} \Rightarrow \binom{a_m + (b_n-1) - 1}{b_n - 1}$

Part 2: $X_m \setminus X_{a_m} \to X_n \setminus X_{b_n}$

$$\Rightarrow X_{m-a_m} \to X_{n-(b_n-1)}$$

$$\Rightarrow \binom{m - a_m + n - (b_n-1) - 1}{n-(b_n-1)-1} = \binom{m-a_n+n-b_n}{n-b_n}$$

So in general the number of monotone inc. funcs. is:

$$\underline{\underline{\binom{a_m+b_n-2}{b_n-1}\cdot\binom{m-a_n+n-b_n}{n-b_n}}}$$ ✓

5.3 4) Let $A = \{1, 2, 3, 4\}$, $B = \{1, 2, 3, 4, 5, 6\}$

a) $|B|^{|A|} = 6^4 = 1296$

$P(|B|, |A|) = \frac{6!}{(6-4)!} = 6 \cdot 5 \cdot 4 \cdot 3 = 360$ ✓

There are 1296 functions from A to B. 360 of them are one-to-one and none of them are onto because $|B| > |A|$.

b) $|A|^{|B|} = 4^6 = 4096$ (total)

$P(4, 6) = 0$ (injections)

$4^6 - \binom{4}{3} 3^6 + \binom{4}{2} 2^6 - \binom{4}{1} \cdot 1^6$

$= 4^6 - 4 \cdot 3^6 + 6 \cdot 2^6 - 4$ ✓

$= 1560$ (surjections)

There are 4096 functions from B to A, 0 of them are injective and 1560 of them are surjective.

5.5

2) There are 7 different weekdays (7 pigeonholes)
8 people each born on 1 of the weekdays (8 pigeons)

By pigeonhole principle, at least two of them are born on the same day. ✓

12) $A \subseteq \{1, 2, 3, \dots, 25\}$, $|A| = 9$

Let $B \subset A$ and denote S_B as the sum of the elements in B.

There are $\binom{9}{5} = 126$ subsets of A with cardinality 5.
The sum of the subset satisfy this relation

$$1+2+3+4+5 \leq S_B \leq 21+22+23+24+25$$

$$\Leftrightarrow \quad 15 \leq S_B \leq 115$$

There are 126 subsets of A, and each one will have a subset sum between 15 and 115, so that's 100 subset sum possibilities.
By pigeonhole principle there must then be at least two distinct subsets $C, D \subseteq A$, $|C| = |D| = 5$ whose sum is identical. ▥

✓

5.6 15) $A = \{1,2,3,4,5\}$, $B = \{6,7,8,9,10,11,12\}$

$f: A \to B$

We want $f^{-1}(\{6,7,8\}) = \{1,2\}$.
There are $3^2 = 9$ ways to do this specifically.
The remaining part of A $\{3,4,5\}$ can then be
$B \setminus \{6,7,8\}$ mapped to anywhere in B, in $7^3 = 343$ ways.
By rule of product we get that the number
X of ways to map A to B such that
Se LF $f^{-1}(\{6,7,8\}) = \{1,2\}$ is

$$3^2 \cdot 7^3 = 9 \cdot 343 = \underline{\underline{3087}} \text{ ways.}$$

22) Since $|A| = |B| = 5$, f being one-to-one implies that f is also onto, and thus bijective and thus invertible, so we want to count number of one-to-one functions.

$$P(5,5) = 5! = 120.$$

There are 120 invertible functions from A to B if $|A| = |B| = 5$. ✓