# Øving 4

4.5

1) $$y'' + 3y' + 2y = 4e^{-3t} \qquad (*)$$

$$y_p = ae^{-3t}$$
$$\Rightarrow y_p' = -3ae^{-3t}$$
$$\Rightarrow y_p'' = 9ae^{-3t}$$

Inserted into $(*)$:

$$\cancel{9ae^{-3t} + 3(-3ae^{-3t})} + 2a\cancel{e^{-3t}} = 4\cancel{e^{-3t}}$$
$$\Leftrightarrow \quad 2a = 4$$
$$\Leftrightarrow \quad a = 2$$

$$\underline{\underline{y_p = 2e^{-3t}}}$$

2) $$y'' + 6y' + 8y = -3e^{-t} \qquad (*)$$

$$y_p = ae^{-t}$$
$$\Rightarrow y_p' = -ae^{-t}$$
$$\Rightarrow y_p'' = ae^{-t}$$

Inserted into $(*)$:

$$a\cancel{e^{-t}} + 6(-a\cancel{e^{-t}}) + 8a\cancel{e^{-t}} = -3\cancel{e^{-t}}$$
$$3a = -3$$
$$a = -1$$

$$\underline{\underline{y_p = -e^{-t}}}$$

5) $y'' + 4y = \cos 3t \quad (*)$

$y_p = A\cos 3t + B\sin 3t$

$\Rightarrow y_p' = -3A\sin 3t + 3B\sin 3t$

$\Rightarrow y_p'' = -9A\sin(3t) + 9B\cos(3t)$

$(*)$: $-9A\sin(3t) + 9B\cos(3t) + 4A\cos(3t) + 4B\sin(3t) = \cos 3t$

$\Rightarrow 9B + 4A = 1 \quad (1)$

and $-9A + 4B = 0 \quad (2)$

$(2) \Leftrightarrow B = \frac{9}{4}A$

$(1) \Leftrightarrow 9 \cdot \frac{9}{4}A + 4A = 1$

$A\left(\frac{81}{4} + 4\right) = 1$

$A = 1 \cdot \frac{4}{97} = \frac{4}{97}$

$\Rightarrow B = \frac{9}{4} \cdot \frac{4}{97} = \frac{9}{97}$

$\therefore y_p = \frac{4}{97}\cos(3t) + \frac{9}{97}\sin(3t)$

10) $y'' + 4y = \cos(3t)$ $(*)$

$y = Re(z)$ where $z'' + 4z = e^{i3t}$

$z'' + 4z = e^{i3t}$. $(**)$

$z_p = a e^{i3t}$

$\Rightarrow z_p' = 3ia e^{i3t}$

$\Rightarrow z_p'' = -9a e^{i3t}$

So $(**)$ becomes:

$-9a e^{i3t} + 4a e^{i3t} = e^{i3t}$

$-5a = 1$

$a = -\frac{1}{5}$

$\Rightarrow z_p = -\frac{1}{5} e^{i3t}$

$= -\frac{1}{5}\cos 3t - \frac{1}{5} i \sin(3t)$

$\underline{\underline{y_p = Re(z_p) = -\frac{1}{5}\cos 3t}}$

11) $y'' + 9y = \sin(2t)$

$y = \operatorname{Im}(z)$, $z'' + 9z = e^{i2t}$

$z'' + 9z = e^{i2t}$

$z_p = a e^{i2t}$

$z_p' = 2ai e^{i2t}$

$z_p'' = -4a e^{i2t}$

$\Rightarrow -4a\cancel{e^{i2t}} + 9a\cancel{e^{i2t}} = \cancel{e^{i2t}}$

$5a = 1$

$a = \frac{1}{5}$

$\Rightarrow z_p = \frac{1}{5} e^{i2t}$

$\underline{y_p = \operatorname{Im}(z_p) = \frac{1}{5}\sin(2t)}$

3a) $z'' + pz' + qz$

$$= (\alpha y_f + \beta y_g)'' + p(\alpha y_f + \beta y_g)' + q(\alpha y_f + \beta y_g)$$

$$= \alpha y_f'' + p\alpha y_f' + q\alpha y_f + \beta y_g'' + \beta p y_g' + \beta q y_g$$

$$= \alpha\underbrace{[y_f'' + py_f' + qy_f]}_{f(t)} + \beta\underbrace{[y_g'' + py_g' + qy_g]}_{g(t)}$$

$$= \alpha f(t) + \beta g(t) \quad \blacksquare$$

3b) $y'' + 2y' + 2y = 3\cos t - \sin t \quad (*)$

$$y_1'' + 2y_1' + 2y_1 = 3\cos t \quad (1)$$

and $y_2'' + 2y_2' + 2y_2 = -\sin t \quad (2)$

(1): $y_{1p} = A\cos t + B\sin t$

$y_{1p}' = -A\sin t + B\cos t$

$y_{1p}'' = -A\cos t - B\sin t$

$(1) \Rightarrow -A\cos t - B\sin t - 2A\sin t + 2B\cos t + 2A\cos t + 2B\sin t = 3\cos t$

$\Rightarrow -A + 2B + 2A = 3$

$2B + A = 3 \quad (i)$

$-B - 2A + 2B = 0$

$B - 2A = 0 \quad (ii)$

$(i) - (ii)\cdot 2 = A - (-2A)\cdot 2 = 3$

$5A = 3$

$A = \frac{3}{5}$

$\Rightarrow B = 2\cdot\frac{3}{5} = \frac{6}{5}$

We have then that

$$y_{1p} = \frac{3}{5}\cos t + \frac{6}{5}\sin t$$

$$y_{2p} = A\cos t + B\sin t$$
$$y_{2p}' = -A\sin t + B\cos t$$
$$y_{2p}'' = -A\cos t - B\sin t$$

$$(2) \Rightarrow -A\cos t - B\sin t - 2A\sin t + 2B\cos t$$
$$+ 2A\cos t + 2B\sin t = -\sin t$$

$$\Rightarrow -A + 2A + 2B = 0$$
$$2B + A = 0 \quad (i)$$
$$-B + 2B - 2A = -1$$
$$B - 2A = -1 \quad (ii)$$
$$(ii) - \frac{1}{2}(i) = -2A - \frac{1}{2}A = -1$$

$$\frac{5}{2}A = 1$$
$$A = \frac{2}{5}$$
$$B = -1 + 2A$$
$$= -1 + \frac{4}{5} = -\frac{1}{5}$$

This gives $y_{2p} = \frac{2}{5}\cos t - \frac{1}{5}\sin t$

$$y_p = y_{1p} + y_{2p}$$
$$= \frac{3}{5}\cos t + \frac{6}{5}\sin t + \frac{2}{5}\cos t - \frac{1}{5}\sin t$$

$$\underline{\underline{y_p = \cos t + \sin t}}$$

37) $y'' + 4y' + 4y = e^{-2t} + \sin(2t)$

(1) $y_1'' + 4y_1' + 4y_1 = e^{-2t}$

(2) $y_2'' + 4y_2' + 4y_2 = \sin 2t$

Finner $y_{1p}$ først:

$$y_{1p} = Ae^{-2t}$$
$$y_{1p}' = -2Ae^{-2t}$$
$$y_{1p}'' = 4Ae^{-2t}$$

Innsatt i (1):

$$4Ae^{-2t} + 4(-2Ae^{-2t}) + 4Ae^{-2t} = e^{-2t}$$
$$8A - 8A = 1$$
$$0 \neq 1$$

Må prøve $y_{1p} = Ate^{-2t}$

$$\Rightarrow y_{1p}' = Ae^{-2t} - 2Ate^{-2t}$$
$$\Rightarrow y_{1p}'' = -2Ae^{-2t} - 2Ae^{-2t} + 4Ate^{-2t}$$
$$= -4Ae^{-2t} + 4Ate^{-2t}$$

Innsatt i (1):

$$\cancel{-4Ae^{-2t}} + \cancel{4Ate^{-2t}} + \cancel{4Ae^{-2t}} - 8Ate^{-2t} + \cancel{4Ate^{-2t}} = e^{-2t}$$

$\Leftrightarrow$ $0 \neq 1$ X

Må derfor bruke $y_{1p} = At^2e^{-2t}$

$$y_{1p} = At^2e^{-2t}$$

$$\Rightarrow y_{1p}' = 2Ate^{-2t} - 2At^2e^{-2t}$$

$$y_{1p}'' = 2Ae^{-2t} - 4Ate^{-2t} - 2A\cdot 2te^{-2t}$$
$$-2At^2e^{-2t}(-2)$$
$$= 4At^2e^{-2t} - 8Ate^{-2t} + 2Ae^{-2t}$$

Innsatt i (1): (A og $e^{-2t}$ er i alle ledd)

$$A\cdot e^{-2t}\left[(4t^2 - 8t + 2) + 4(-2t^2 + 2t) + 4t^2\right] = e^{-2t}$$

$$A\left[\cancel{8t^2} - \cancel{8t^2} - \cancel{8t} + \cancel{8t} + 2\right] = 1$$
$$2A = 1$$
$$A = 1/2$$

$$\underline{y_{1p} = \frac{1}{2}t^2e^{-2t}}$$

$$y_{2p} = A\cos 2t + B\sin 2t$$

$$y_{2p}' = -2A\cdot\sin 2t + 2B\cos 2t$$

$$y_{2p}'' = -4A\cos 2t - 4B\sin 2t$$

Innsatt i (2):

$$-\cancel{4A\cos 2t} - \cancel{4B\sin 2t} - 8A\sin 2t + 8B\cos t$$
$$+ \cancel{4A\cos 2t} + \cancel{4B\sin 2t} = \sin 2t$$
$$-8A = 1 \Leftrightarrow A = -1/8$$
$$8B = 0 \Leftrightarrow B = 0$$

$$y_{2p} = -\frac{1}{8}\cos 2t$$

$$y_p = y_{1p} + y_{2p}$$

$$y_p = \frac{1}{2}t^2 e^{-2t} - \frac{1}{8}\cos 2t$$

4.6

1) $y'' + 9y = \tan(3t)$

$\Downarrow$

$r^2 + 9 = 0$

$\Downarrow$

$y_1 = \cos 3t \quad y_2 = \sin 3t$

(i) $v_1' y_1 + v_2' y_2 = 0$

(ii) $v_1' y_1' + v_2' y_2' = f(t)$

(i) $\Rightarrow v_1' = -v_2' \cdot \frac{y_2}{y_1}$

$\Rightarrow \frac{y_2}{y_1} y_1' (-v_2') + v_2' y_2' = f(t) \quad | \cdot y_1$

$v_2'(-y_2 y_1' + y_2' y_1) = f(t) y_1$

$v_2' = \frac{y_1 \cdot f(t)}{W(t)}$

$\Rightarrow v_1' = -\frac{y_2}{y_1} \cdot \frac{y_1 f(t)}{W(t)} = \frac{-y_2 f(t)}{W(t)}$

Vi har $y_1 = \cos 3t$, $y_2 = \sin 3t$, og dermed $y_1' = -3\sin(3t)$, $y_2' = 3\cos 3t$

$W(t) = \begin{vmatrix} \cos 3t & \sin 3t \\ -3\sin 3t & 3\cos 3t \end{vmatrix} = 3\cos^2(3t) + 3\sin^2(3t) = 3$

$f(t) = \tan(3t)$

$v_1' = -\frac{\sin(3t) \cdot \tan(3t)}{3} = -\frac{\sin^2(3t)}{3\cos(3t)}$

$v_2' = \frac{\cos 3t \cdot \tan 3t}{3} = \frac{1}{3}\sin(3t)$

$$V_1 = -\frac{1}{3}\int \frac{\sin^2(3t)}{\cos(3t)}\,dt$$

$$= \frac{1}{9}\sin(3t) - \frac{1}{9}\ln(\sec(3t) + \tan(3t)) \qquad (\text{"Maple"})$$

$$V_2 = \int \frac{1}{3}\sin(3t)\,dt = -\frac{1}{9}\cos(3t)$$

$$y_p = V_1 y_1 + V_2 y_2$$

$$y_p = \left[\frac{1}{9}\sin(3t) - \frac{1}{9}\ln(\sec(3t) + \tan(3t))\right]\cdot\cos(3t)$$
$$-\frac{1}{9}\cos(3t)\cdot\sin(3t)$$

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4) $$x'' - 2x' - 3x = 4e^{3t}$$

$$\Downarrow$$

$$r^2 - 2r - 3 = 0$$

$$\Leftrightarrow r = \frac{2 \pm \sqrt{4+4\cdot 3}}{2} = 1 \pm 2$$

$$\Rightarrow x_1 = e^{-t}, \quad x_2 = e^{3t}$$

$$x_p = V_1 x_1 + V_2 x_2$$

$$x_1' = -e^{-t}$$

$$x_2' = 3e^{3t}$$

$$W(t) = \begin{vmatrix} e^{-t} & e^{3t} \\ -e^{-t} & 3e^{3t} \end{vmatrix} = 3e^{2t} + e^{2t} = 4e^{2t}$$

$$f(t) = 4e^{3t}$$

$$V_1' = \frac{-x_2 \cdot f(t)}{w(t)} = \frac{-e^{3t} \cdot 4e^{3t}}{4e^{2t}}$$

$$= -e^{(3+3-2)t} = -e^{4t}$$

$$V_2' = \frac{x_1 f(t)}{w(t)} = \frac{e^{-t} \cdot 4e^{3t}}{4e^{2t}}$$

$$= e^{(-1+3-2)t} = 1$$

$$\Rightarrow V_1 = \int_0^t -e^{4\bar{t}} \, d\bar{t} = -\frac{1}{4} e^{4t}$$

$$V_2 = \int_0^t 1 \, d\bar{t} = t$$

$$x_p = e^{-t} \cdot \left(-\frac{1}{4} e^{4t}\right) + t \cdot e^{3t}$$

$$= -\frac{1}{4} e^{3t} + t \cdot e^{3t}$$

5) $y'' - 2y' + y = e^t$

$\Downarrow$

$r^2 - 2r + 1 = 0$

$\Rightarrow r = \frac{2 \pm \sqrt{4-4}}{2} = 1$

$\Rightarrow y_1 = e^t \quad , \quad y_2 = te^t$

$\Rightarrow y_1' = e^t \quad , \quad y_2' = e^t + te^t = (t+1)e^t$

$$W(t) = \begin{vmatrix} e^t & te^t \\ e^t & (t+1)e^t \end{vmatrix}$$

$= (t+1)e^{2t} - te^{2t}$

$= e^{2t}$

$f(t) = e^t$

$v_1' = \frac{-y_2 \cdot f(t)}{W(t)} = \frac{-te^t \cdot e^t}{e^{2t}} = -t$

$v_2' = \frac{y_1 f(t)}{W(t)} = \frac{e^t \cdot e^t}{e^{2t}} = 1$

$V_1 = \int_0^t -\bar{t} \cdot d\bar{t} = -\frac{1}{2}t^2$

$V_2 = \int_0^t 1 \, d\bar{t} = t$

$y_p = v_1 y_1 + v_2 y_2$

$y_p = -\frac{1}{2}t^2 e^t + t \cdot te^t$

$= \frac{1}{2}t^2 e^t$

13) $t^2 y'' + 3t y' - 3y = 0$

$\Leftrightarrow \quad y'' + \frac{3}{t} y' - \frac{3}{t^2} y = 0 \qquad (*)$

$y_1 = t$:

Innsatt i $(*)$

$$(t)'' + \frac{3}{t}(t)' - \frac{3}{t^2} t$$

$$= 0 + \frac{3}{t} - \frac{3}{t} = 0 \quad \checkmark$$

$y_2 = t^{-3}$:

Innsatt i $(*)$:

$$(t^{-3})'' + \frac{3}{t} \cdot (t^{-3})' - \frac{3}{t^2} \cdot t^{-3}$$

$$= (-3)\cdot(-4)\cdot t^{-5} + \frac{3}{t} \cdot (-3) t^{-4} - \frac{3}{t^5}$$

$$= \frac{12}{t^5} - \frac{9}{t^5} - \frac{3}{t^5}$$

$$= 0 \quad \checkmark$$

$y_1 = t$ og $y_2 = t^{-3}$, er løsninger

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$y_1' = 1$, $y_2' = -3t^{-4}$

$y_p = v_1 y_1 + v_2 y_2$

hvor $v_1' = \frac{-y_2 f(t)}{W(t)}$

$v_2' = \frac{y_1 f(t)}{W(t)}$

$f(t) = 1/t$

$$W(t) = \begin{vmatrix} t & t^{-3} \\ 1 & -3t^{-4} \end{vmatrix} = -3t^{-3} - t^{-3} = -4t^{-3}$$

$$v_1' = \frac{-t^{-3} \cdot 1/t}{-4t^{-3}} = \frac{1}{4} t^{-3-1+3} = \frac{1}{4} t^{-1}$$

$$v_2' = \frac{t \cdot 1/t}{-4t^{-3}} = -\frac{t^3}{4}$$

$$v_1 = \int \frac{1}{4} t^{-1}\, dt = \frac{1}{4} \ln|t|$$

$$v_2 = \int -\frac{t^3}{4}\, dt = -\frac{1}{4} \cdot \frac{1}{4} \cdot t^4 = -\frac{t^4}{16}$$

$$y_p = \frac{1}{4} \ln|t| \cdot t - \frac{1}{16} \cdot t^4 \cdot t^{-3}$$

$$= \frac{t}{16}(4\ln|t| - 1)$$

Så den generelle løsningen blir:

$$\underline{\underline{y(t) = \frac{t}{16}(4\ln|t|-1) + c_1 t + c_2 t^{-3}}}$$

4.7

1) $x'' + \omega_0^2 x = A\cos(\omega t), \quad \omega \neq \omega_0 \quad (*)$

a) $x_p = a\cos\omega t$

$x_p' = -a\omega\sin\omega t$

$x_p'' = -a\omega^2\cos\omega t$

Innsatt i $(*)$:

$$-a\omega^2\cancel{\cos(\omega t)} + a\omega_0^2\cancel{\cos(\omega t)} = A\cancel{\cos(\omega t)}$$

$$a(-\omega^2 + \omega_0^2) = A$$

$$a = \frac{A}{\omega_0^2 - \omega^2}$$

$$\Rightarrow x_p = \frac{A}{\omega_0^2 - \omega^2} \cdot \cos(\omega t) \quad \checkmark$$

b) $x_p = ae^{i\omega t}$

$x_p' = ia\omega e^{i\omega t}$

$x_p'' = -a\omega^2 e^{i\omega t}$

Innsatt i $(*)$:

$$-a\omega^2 e^{i\omega t} + \omega_0^2 a e^{i\omega t} = A\cos(\omega t)$$

$$\underbrace{ae^{i\omega t}}_{=x_p}(-\omega^2 + \omega_0^2) = A\cos(\omega t)$$

$$x_p(\omega_0^2 - \omega^2) = A\cos(\omega t)$$

$$x_p = \frac{A}{\omega_0^2 - \omega^2}\cos(\omega t) \quad \checkmark$$

1) $m = 1$ kg
$k = 4$ kg/$s^2$
$f(t) = 4\cos(\omega t)$ N
$y$ - posisjon $y(0) = y'(0) = 0$

a) Diff. ligningen blir:

$$y'' + \left(\frac{k}{m}\right)y = f(t) \quad , \quad y(0) = y'(0) = 0$$

$$y'' + 4y = 4\cos(\omega t) \qquad (*)$$

Ubestemte koeffisienters metode:

$$y_p := A\cos(\omega t) + B\sin(\omega t)$$

$$\Rightarrow y_p' = -A\omega\sin(\omega t) + B\omega\cos(\omega t)$$

$$y_p'' = -A\omega^2\cos(\omega t) - B\omega^2\sin(\omega t)$$

Innsatt i $(*)$:

$$-A\omega^2\cos(\omega t) - B\omega^2\sin(\omega t) + 4A\cos(\omega t)$$
$$+ 4B\sin(\omega t) = 4\cos(\omega t)$$

$$\Rightarrow B = 0$$

$$-A\omega^2 + 4A = 4$$

$$A(4 - \omega^2) = 4$$

$$A = \frac{4}{4-\omega^2} \qquad (|\omega| \neq 2)$$

$$\Rightarrow y_p = \frac{4}{4-\omega^2}\cos(\omega t)$$

Må løse T.H.L:

$$y'' + 4y = 0$$

$$r^2 + 4 = 0$$

$$r = \pm 2i$$

$$\Rightarrow y_h = C_1 \cos(2t) + C_2 \sin(2t)$$

$$y(t) = y_p + y_h$$

$$y(t) = \frac{4}{4-\omega^2} \cos(\omega t) + C_1 \cos(2t) + C_2 \sin(2t) \quad (\omega \neq 2)$$

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Hvis $\omega = 2$ får vi

$$y_p = At\cos 2t + Bt\sin 2t$$

$$y_p' = A[t(-2\sin 2t) + \cos 2t] + B[t\,2\cos 2t + \sin 2t]$$

$$= -2At\sin(2t) + A\cos(2t) + 2Bt\cos(2t) + B\sin(2t)$$

$$y_p'' = -2A[t\,2\cos(2t) + 1\cdot\sin(2t)] - 2A\sin(2t)$$
$$+ 2B[t(-2\sin 2t) + \cos 2t] + 2B\cos(2t)$$

$$= -4At\cos(2t) - 2A\sin(2\ ) - 2A\sin(2t)$$
$$- 4Bt\sin 2t + 2B\cos(2t) + 2B\cos(2t)$$

$$= [-4At + 4B]\cdot\cos(2t)$$
$$+ [-4A - 4Bt]\cdot\sin(2t)$$

Innsatt i (*):

$$[-4At + 4B]\cos 2t + [-4A - 4Bt]\sin(2t)$$
$$+ 4At\cos 2t + 4Bt\sin 2t = 4\cos(2t)$$

$$\Leftrightarrow \left.\begin{aligned} 4B &= 4 \\ -4A &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} B &= 1 \\ A &= 0 \end{aligned}$$

Da har vi at $y_p = t\sin 2t$
og det gir:

$$y(t) = t\sin(2t) + C_1\cos 2t + C_2 \sin 2t$$

når $\omega = 2$.

b) $\bar{\omega} = \frac{\omega_0+\omega}{2} = \frac{2+\omega}{2}$

$\delta = \frac{\omega_0-\omega}{2} = \frac{2-\omega}{2}$

$$y(t) = \frac{4}{2\bar{\omega}\delta}\sin(\delta t)\cdot\sin(\bar{\omega}t)$$

$$= \frac{2}{\frac{(2+\omega)}{2}\frac{(2-\omega)}{2}}\sin(\delta t)\sin(\bar{\omega}t)$$

$$y(t) = \frac{8}{2^2-\omega^2}\sin(\delta t)\sin(\bar{\omega}t)$$

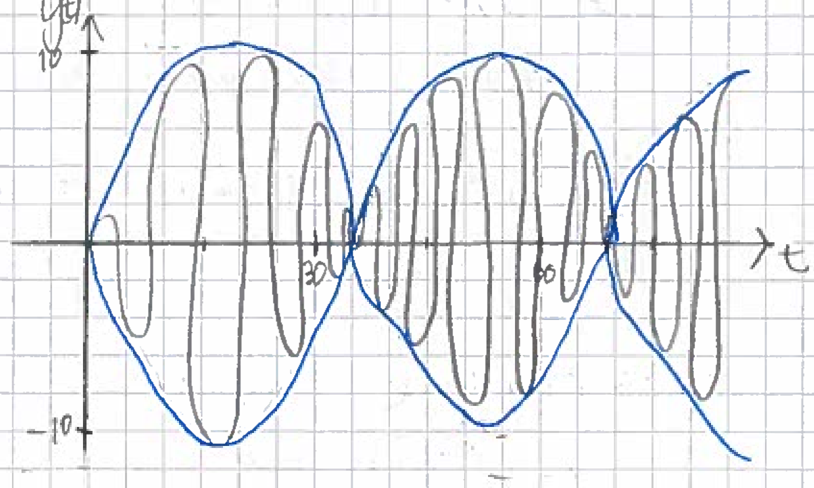
La $\omega = 1{,}8$

$\Rightarrow \bar{\omega} = 1{,}9 \quad , \delta = 0{,}1$

$$\frac{8}{2^2-1{,}8^2} = 10{,}5$$

$$y(t) = 10{,}5\sin(0{,}1t)\sin(1{,}9t)$$

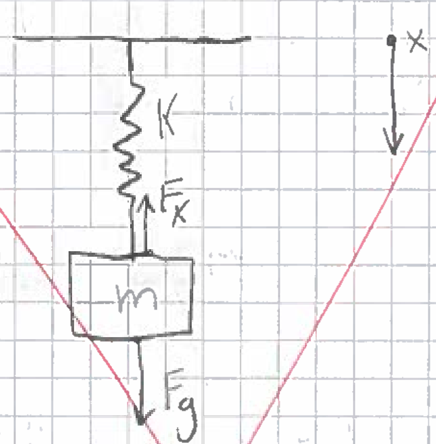
Skisse av $w = 1{,}8$:




4.5) $m = 0{,}05$ kg

$|F_r| = 0{,}01 v = 0{,}01 x'$

$F(t) = 5\cos(4{,}4t)$




Når $t < 0$: $\Sigma F = 0$ (systemet i ro)

$$\Sigma F = F_g - F_K$$

$$F_g = F_K$$

$$m \cdot g = K x_0 \quad \text{(Hookes lov)}$$

$$\Rightarrow K = \frac{mg}{x_0}$$

$$K = \frac{50 \cdot 10^{-3} \text{ Kg} \cdot 9{,}81 \text{ m/s}^2}{10 \cdot 10^{-2} \text{ m}}$$

$$= 4{,}9 \text{ Kg/s}^2$$

Når $t \geq 0$ har vi:

$$ma = -kx + mg - dv + F(t)$$

$$\Rightarrow \quad mx'' - mg + kx + dx' = F(t)$$

$$mx'' + dx' + kx - kx_0 = F(t)$$

siden $mg = kx_0$

$$mx'' + dx' + k(x - x_0) = F(t)$$

$$y = x - x_0$$
$$y' = x'$$
$$y'' = x''$$

$$\Rightarrow \quad my'' + dy' + ky = F(t)$$

T.H.L.: $my'' + dy' + ky = 0$

$$\Updownarrow$$

$$y'' + \left(\frac{d}{m}\right)y' + \left(\frac{k}{m}\right)y = 0$$

$\Downarrow$ setter inn verdier

$$y'' + \frac{0{,}01}{50 \cdot 10^{-3}}y' + \frac{4{,}9}{50 \cdot 10^{-3}}y = 0$$

$$y'' + 0{,}2y' +$$

45) $m = 0{,}05$ kg

$|F_r| = d \cdot x'$ ($x'$ - fart, $d = 0{,}01$)

$F(t) = 5\cos(4.4t)$

$F_K = K \cdot x$ ($x$ - posisjon)

Når $t < 0$: Systemet i ro, $x = x_0$

$$\sum F = 0$$
$$F_g - F_K = 0$$
$$F_g = F_K$$
$$mg = Kx_0 \qquad (1)$$

Når $t \geq 0$: $F(t)$ begynner å virke

$$\sum F = -F_K - F_r + F_g + F(t)$$
$$ma = -Kx - dx' + mg + F(t)$$
$$mx'' = -Kx + mg - dx' + F(t)$$

Siden $mg = Kx_0$ kan vi skrive:

$$mx'' = -K(x - x_0) - dx' + F(t)$$

Sier $y = x - x_0$ og får

$(y' = x'$ og $y'' = x'')$

$$my'' = -Ky - dy' + F(t)$$
$$my'' + dy' + Ky = F(t)$$

Vi har at $m = 0{,}05$ kg

$d = 0{,}01$

$K = \frac{m \cdot g}{x_0} = \frac{0{,}05 \cdot 9{,}81}{0{,}1} = 4{,}9$

$F(t) = 5\cos(4.4t)$

Med verdier blir systemet:

$$0{,}05y'' + 0{,}01y' + 4{,}9y = 5\cos(4{,}4t) \quad (*)$$

$$x(0) = x_0 \Rightarrow y(0) = x_0 - x_0 = 0$$
$$x'(0) = 0 \Rightarrow y'(0) = 0$$

$$(*) \cdot 20 \Rightarrow y'' + 0{,}20y' + 98y = 100\cos(4{,}4t) \quad (**)$$

Vil finne en partikulærløsning $y_p$.
Bruker kompleksmetoden for å finne $z_p$ og da vil $y_p = Re(z_p)$

$$z'' + 0{,}2z' + 98z = 100e^{i4{,}4t}$$

$$z_p = Ae^{i4{,}4t}$$
$$z_p' = iA4{,}4e^{i4{,}4t}$$
$$z_p'' = -4{,}4^2Ae^{i4{,}4t}$$

$$\Rightarrow -4{,}4^2Ae^{i4{,}4t} + 0{,}2 \cdot iA \cdot 4{,}4e^{i4{,}4t} + 98Ae^{i4{,}4t}$$
$$= 100e^{i4{,}4t}$$

$$\Rightarrow (-4{,}4^2 + i0{,}2 \quad + 98)A = 100$$

$$A = \frac{100}{78{,}64 + 0{,}2i}$$

$$A = \frac{100(78{,}64 - 0{,}2i)}{78{,}64^2 + 0{,}2^2}$$

$$A = 1{,}27 - 0{,}0032i$$

$$\Rightarrow z_p = 1{,}27e^{i4{,}4t} - 0{,}0032ie^{i4{,}4t}$$
$$= 1{,}27(\cos(4{,}4t) + i\sin(4{,}4t))$$
$$-0{,}0032i(\cos(4{,}4t) + i\sin(4{,}4t))$$

$$y_p = Re(z_p)$$
$$y_p = 1{,}27\cos(4{,}4t) + 0{,}0032\sin(4{,}4t)$$

Vil nå finne løsningene på THL.

THL: $y'' + 0{,}2y' + 98y = 0$

$$\Rightarrow r^2 + 0{,}2r + 98 = 0$$

$$r = \frac{-0{,}2 \pm \sqrt{0{,}2^2 - 4\cdot 98}}{2}$$
$$= -0{,}1 \pm \frac{\sqrt{-391{,}96}}{2}$$
$$= -0{,}1 \pm 9{,}9i$$
$$\Rightarrow y_h = e^{-0{,}1t}\left(C_1\cos(9{,}9t) + C_2\sin(9{,}9t)\right)$$

$$y(t) = y_p + y_h$$
$$x(t) = y(t) + x_0$$
$$= 1{,}27\cos(4{,}4t) + 0{,}002\sin(4{,}4t)$$
$$+ e^{-0{,}1t}\left[C_1\cos(9{,}9t) + C_2\sin(9{,}9t)\right] + 0{,}1$$
$$x(0) = 0{,}1$$
$$\Rightarrow 0{,}1 = 1{,}27 + C_1 + 0{,}1$$
$$C_1 = -1{,}27$$
$$x'(0) = 0$$
$$x'(t) = -5{,}64\sin(4{,}4t) + 0{,}014\cos(4{,}4t)$$
$$-0{,}1e^{-0{,}1t}\left[C_1\cos(9{,}9t)\ C_2\sin(9{,}9t)\right]$$
$$+ e^{-0{,}1t}\left[9{,}9C_1(-\sin(9{,}9t)) + 9{,}9C_2\cos(9{,}9t)\right)$$
$$x'(0) = 0{,}014 - 0{,}1C_1 + 9{,}9C_2 = 0$$
$$\Rightarrow C_2 = \frac{0{,}1\cdot C_1 - 0{,}014}{9{,}9} = -0{,}014$$

Dette gir da at systemet beskrives av ligningen:

$$x(t) = 1{,}27\cos(4{,}4t) + 0{,}0032\sin(4{,}4t) - e^{-0{,}1t}\left[1{,}27\cos(9{,}9t) + 0{,}014\sin(9{,}9t)\right]$$