Norwegian University of Science and Technology
Department of Electronics and Telecommunications

TTT4120 Digital Signal Processing
Problem Set 9

The main topic for this problem set is filter design. Relevant chapters from the textbook are 10.2.2, 10.3.3, and 10.3.4. The maximum score for each problem is given in parentheses.

Problem 1 (2.5 points)

In this problem the windowing method will be used to design a causal digital linear phase FIR filter with magnitude response that approximates that of the ideal lowpass filter

$$H_d(f) = \begin{cases} 1 & |f| < f_c \\ 0 & f_c \leq |f| \leq 0.5. \end{cases}$$

The basic idea of this method is to obtain a causal finite impulse response by time-shifting and truncating the impulse response of the ideal filter using a window function.

(a) Find the unit sample response, $h_d(n)$, of the ideal lowpass filter as a function of f_c.

(b) Find the unit sample response of the causal FIR approximation as a function of the window function $w(n)$ and f_c. (Assume that the window length, N, is an odd number.)

(c) Write a Matlab function that computes the filter coefficients given f_c and the window function $w(n)$.

(d) Let $f_c = 0.2$ and the window length $N = 31$. Test your function using a rectangular window and a Hamming window (useful Matlab functions: `rectwin`, `hamming`). Compute and plot the magnitude response of the filter for each window type. Compare the plots and comment the differences between using a rectangular window and a Hamming window.

(e) Use the Matlab function `fir1` to generate the same filters as in (d). Compute and plot the magnitude response of the filter for each window type. Compare the plots to those from (d).

Problem 2 (2.5 points)

The analog filter shown in Figure 1 was considered in Problem Set 4.

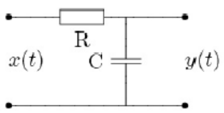

Figure 1: Analog filter

It was found to be a lowpass filter with transfer function

$$H_a(s) = \frac{1/RC}{s + 1/RC}.$$

(a) Show that the filter in Figure 1 has cut-off frequency[1] $\Omega_c = 1/RC$.

(b) Bilinear transform is given by

$$s = \frac{2}{T}\frac{1 - z^{-1}}{1 + z^{-1}}$$

where $s = \sigma + j\Omega$ and $z = re^{j\omega}$. It can be used to design digital filters from known analog filters. An important property of the bilinear transform is that it maps the imaginary axis in the s-plane ($s = j\Omega$) into the unit circle in the z-plane ($z = e^{j\omega}$).

- Show that the bilinear transform results in the following frequency transformation[2]

$$\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right).$$

(c) We wish to design a discrete lowpass filter with cut-off frequency $\omega_c = 0.2\pi$ by applying the bilinear transform to the analog filter shown in Figure 1.

- Show that the transfer function of the resulting discrete filter is given by

$$H(z) = \frac{0.245(1 + z^{-1})}{1 - 0.51z^{-1}}.$$

- Plot the magnitude response of the filter and verify that it satisfies the specifications.
- *Optional:* Plot the magnitude response of the prototype filter and compare it to the resulting digital filter (use Matlab function `freqs`).

[1] Cut-off frequency Ω_c of a filter with frequency response $H_a(\Omega)$ is given by $|H_a(\Omega_c)| = 1/\sqrt{2}$.

[2] You might need the following identity $\frac{\sin\omega}{1+\cos\omega} = \tan(\frac{\omega}{2})$.

Problem 3 (2 points)

Given a 2nd order lowpass Butterworth filter with transfer function

$$H_a(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

(a) Show that $H_a(s)$ is a Butterworth filter with cut-off frequency $\Omega_c = 1$.

(b) Plot the magnitude response of the filter. (Hint: `freqs`)

(c) Find the poles for the filter.

(d) Use the impulse invariance method to convert the analog filter to two digital IIR filters with cut-off frequencies $\omega_{c1} = 0.25$ and $\omega_{c2} = 1.4$, respectively.

(e) Plot the magnitude response of the two digital IIR filters. Compare with the plot in b) and comment.

Problem 4 (noise removal) (3 points)

Due to different reasons i.e impurities of physical recording media and uncontrolled surroundings while recording, unwanted noise may coexist with the intended sound. Different strategies for removing this unwanted noise exist, since the noise removal process in most cases must be specifically tailored to match the characteristics of the noise.

In this exercise you should design a bandstop filter in order to remove bandlimited noise from the provided sound file `pianoise.wav`. The file contains the sound of a piano corrupted by a bandlimited noise with approximate bandwidth 700Hz centered at about 5000Hz. The sample frequency of the sound file is 22050Hz. The filter should have a transition band of max 200Hz, max peak-to-peak passband ripple of 1dB, and a stopband attenuation of minimum 50dB.

The filters should be designed using "Filter Design and Analysis Tool". (Use Matlab command `fdatool` to start the program.)

(a) Design an FIR filter that meets the given specifications using the windowing method with Hamming window.

- Find the lowest filter length that satisfies the specifications. (use zoom to inspect the different parts of the filter characteristics in order to check whether specifications have been meet)

- Plot the magnitude and phase responses of the filter for this filter length. Make a separate plot of the magnitude response in the passband.

(b) Design an FIR filter that meets the above specifications by using optimal equiripple FIR filter design.

- Find the lowest filter length that satisfies the specifications.
- Plot the magnitude and phase responses of the filter for this filter length. Make a separate plot of the magnitude response in the passband.

(c) Design an elliptic IIR digital filter with the given specifications.

- Find the lowest filter order that satisfies the specifications.
- Plot the magnitude and phase responses of the filter for this filter length. Make a separate plot of the magnitude response in the passband.

(d) Compare the plots and the filter orders of the filters designed in (a)-(c), and comment the pros and cons of the different filter design methods.

(e) Use the FIR filter from (a) to filter the given audio file.
(Hint: The filter coefficients can be exported to a workspace variable by choosing "File - Export". The filtering can then be performed from the Matlab command window using `filter`. Use `wavread` to load the sound file.)

- Compare the original file with the filtered file. Can you hear the noise?
- Vary the filter order
  - What effect does the filter order have on the transition band and stopband attenuation of the filter.
  - How well is the noise removed from the sound file?

Problem 5 (Extra assignment. Optional.)

In this problem you should design a digital filter given the following specifications:

- Passband edge frequency $f_p = 0.2$
- Stopband edge frequency $f_s = 0.3$
- Peak-to-peak passband ripple $r_p = 0.4$ dB

- Stopband attenuation $r_s = 50$ dB

(a) Use the Matlab function `fir1` to design an FIR filter with the given specifications using windowing method. The cut-off frequency should be set to $f_c = (f_p + f_s)/2$.

  - Find the lowest filter length that satisfies the specifications in the case of rectangular and Hamming windows.
  - Plot the magnitude and phase responses of the filters for these values of N.

(b) *Optional:* Design an FIR filter that meets the above specifications using optimal equiripple FIR filter design implemented by the Matlab function `firpm`. The function `firpmord` can be used to estimate the filter order.

  - Find the lowest filter length that satisfies the specifications.
  - Plot the magnitude and phase responses of the filter for this value of N.

(c) Design an elliptic IIR digital filter with the given specifications. Use Matlab functions `ellipord` and `ellip`.

  - Find the lowest filter order that satisfies the specifications.
  - Plot the magnitude and phase responses of the filter for this value of N.

(d) Compare the plots and the filter orders of the filters designed in (a)-(c), and comment the pros and cons of the different filter design methods.