

TFE4101

KRETS- OG DIGITALTEKNIKK

Forenkling av Boolske funksjoner

Gajski:

Kap. 4.1 – 4.3: Forenkling av Boolske funksjoner

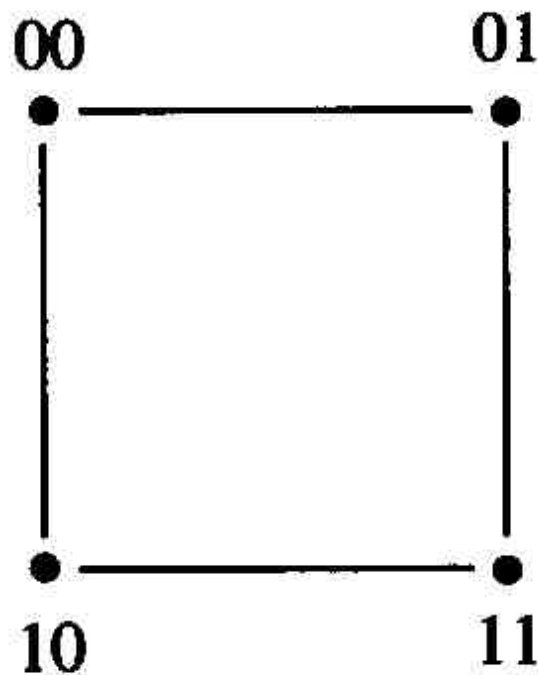
Forenkling av Boolske funksjoner

- Boolske funksjoner kan representeres ved
 - sannhetstabell
 - likning på kanonisk form (unik, mange operatorer)
 - likning på standard form (ikke unik, færre operatorer)
 - likning på ikke-standard form (ikke unik, enda færre operatorer)
- Færre operatorer gir
 - færre transistorer og mindre kretser
 - potensielt langsommere kretser (ikke-standard form)
- Mål ved forenkling
 - redusere antall operatorer totalt
 - minimalisere antall operatorer langs kritisk sti
 - benytte raske operatorer langs kritisk sti

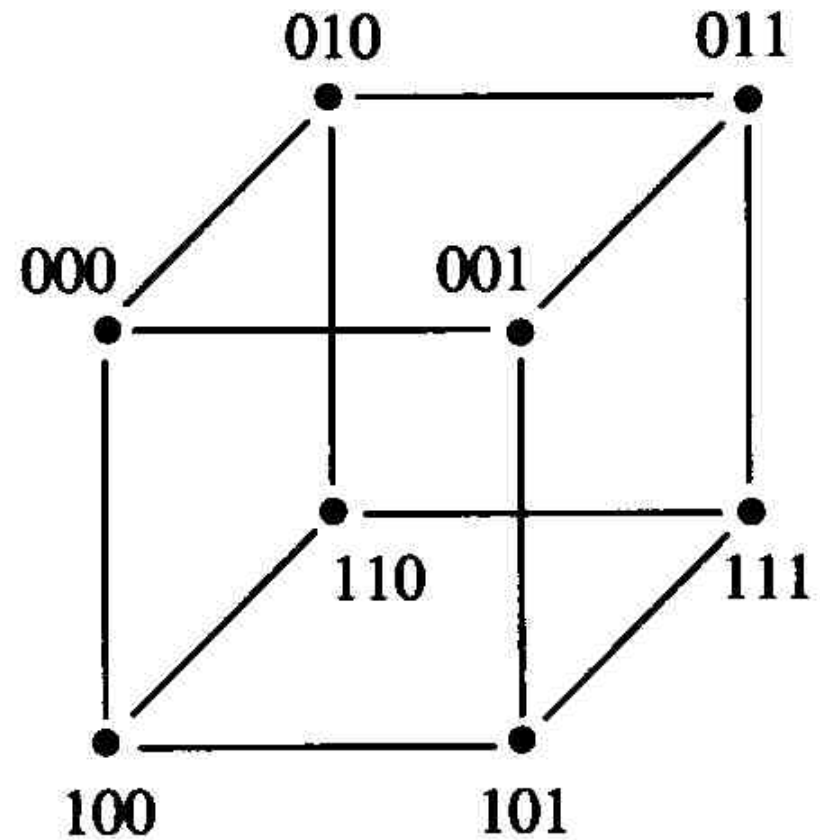
Boolske n-kuber



(a) $n = 1$

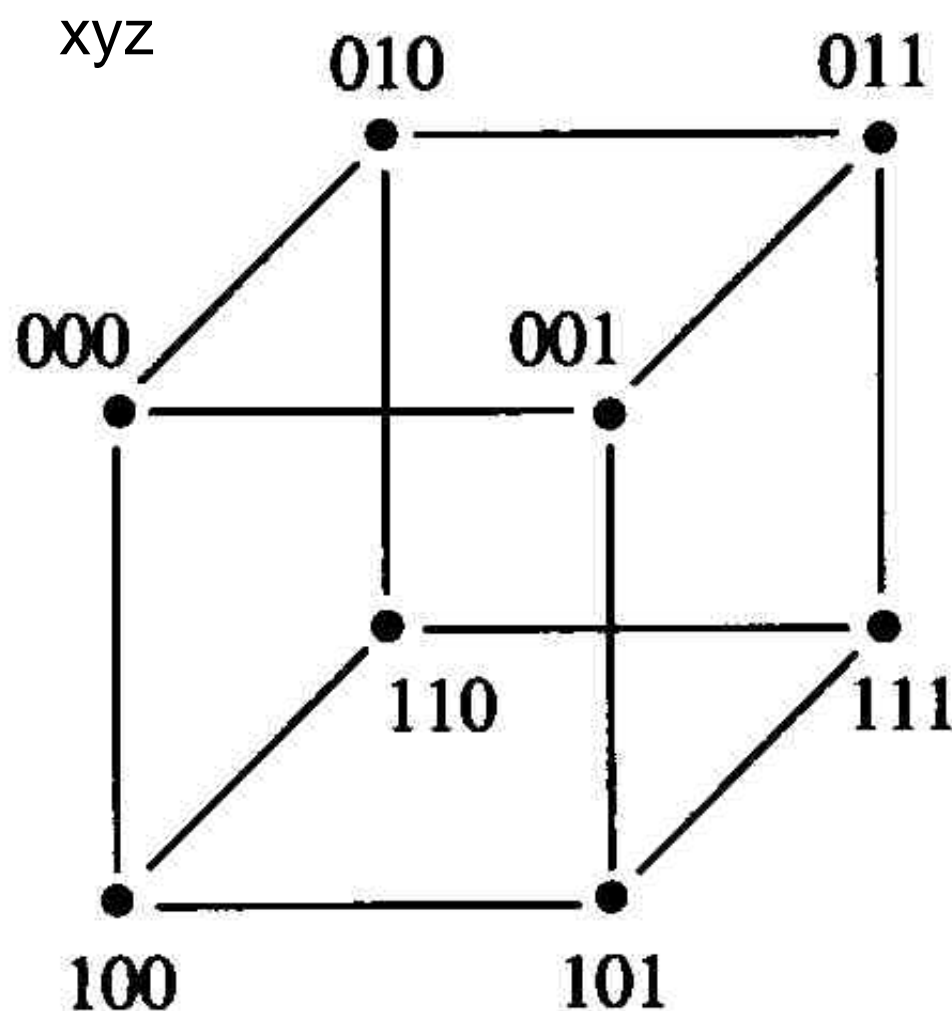


(b) $n = 2$



(c) $n = 3$

Funksjon beskrevet med n-kube

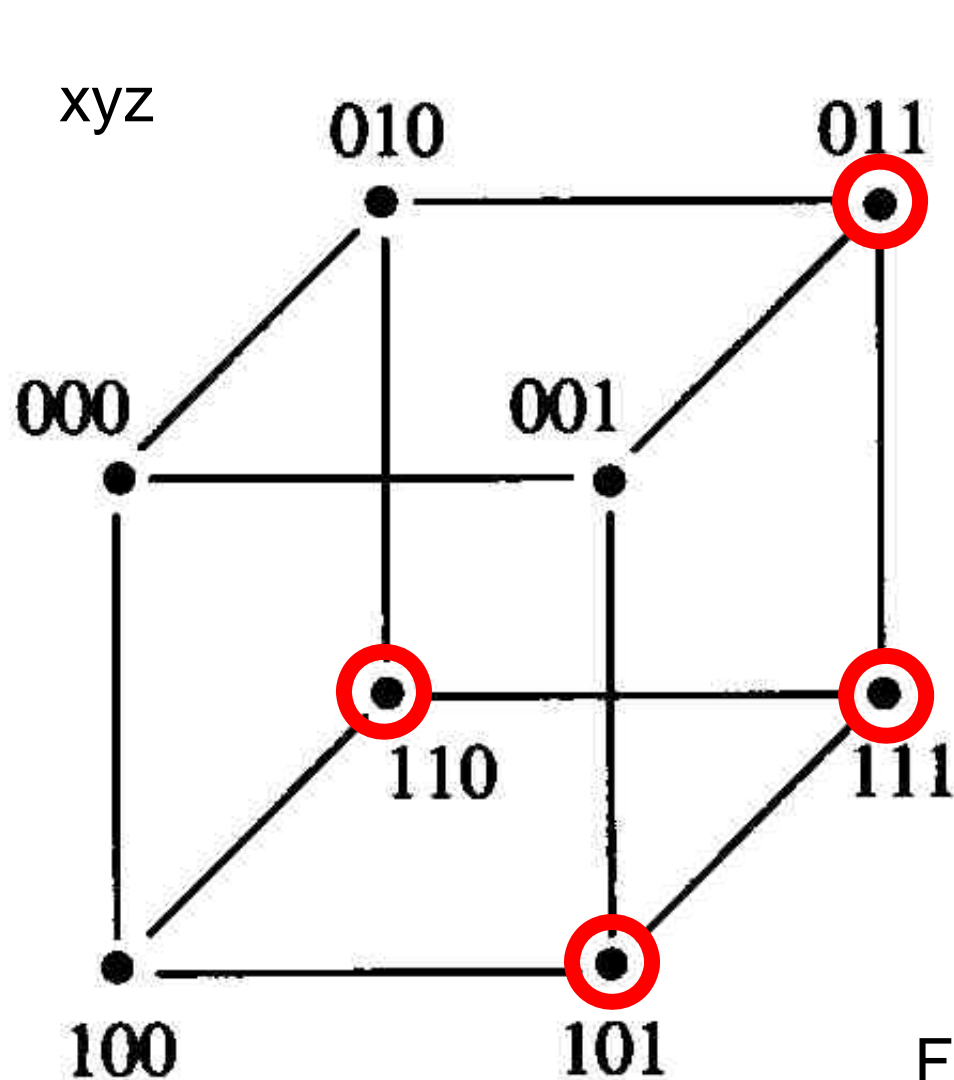


$$F_1 = \bar{x}yz + x\bar{y}z + xy\bar{z} + xyz$$

Ring rundt tilhørende
hjørner i kuben

(c) $n = 3$

Funksjon beskrevet med n-kube

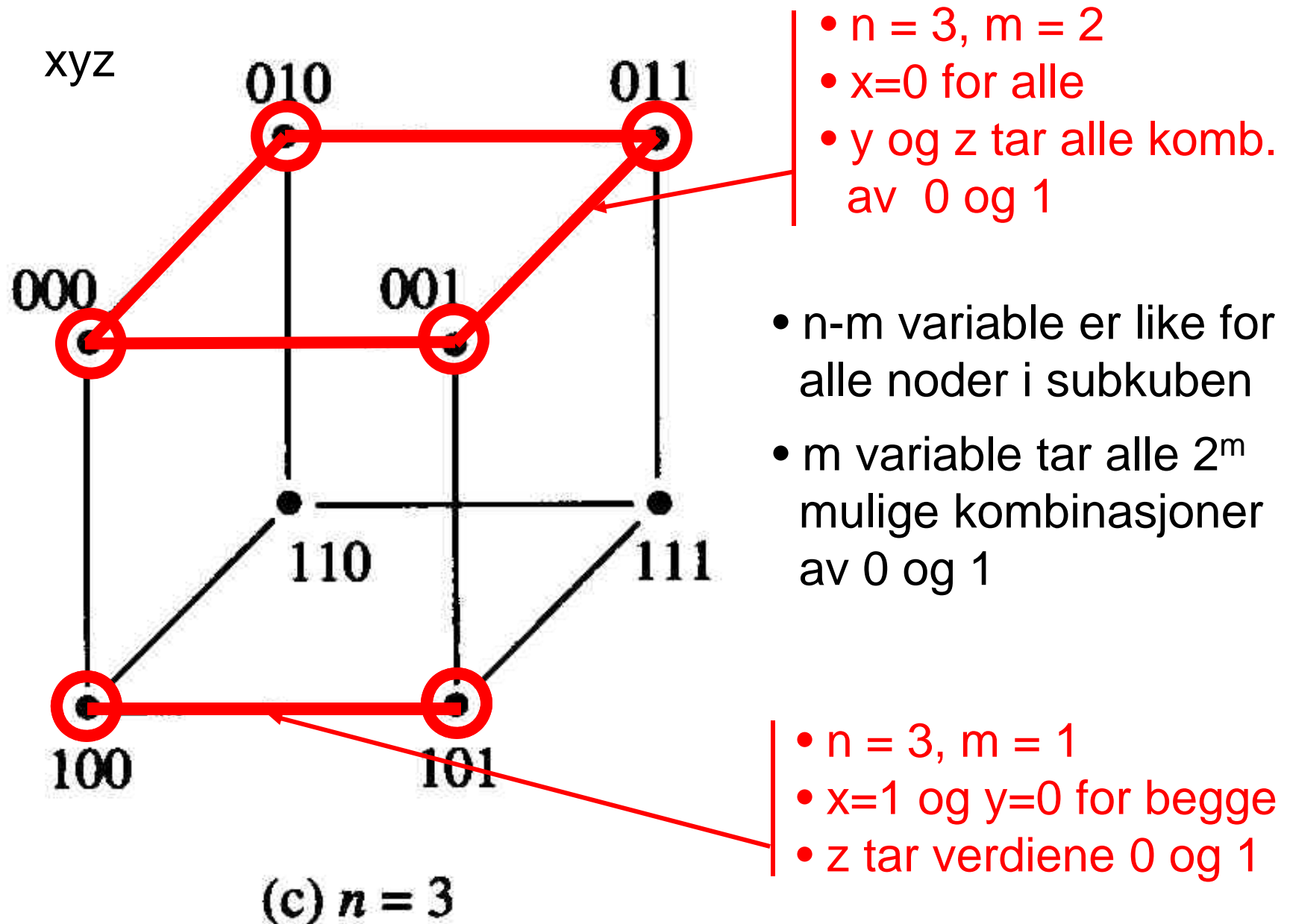


x	y	z	F_1
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

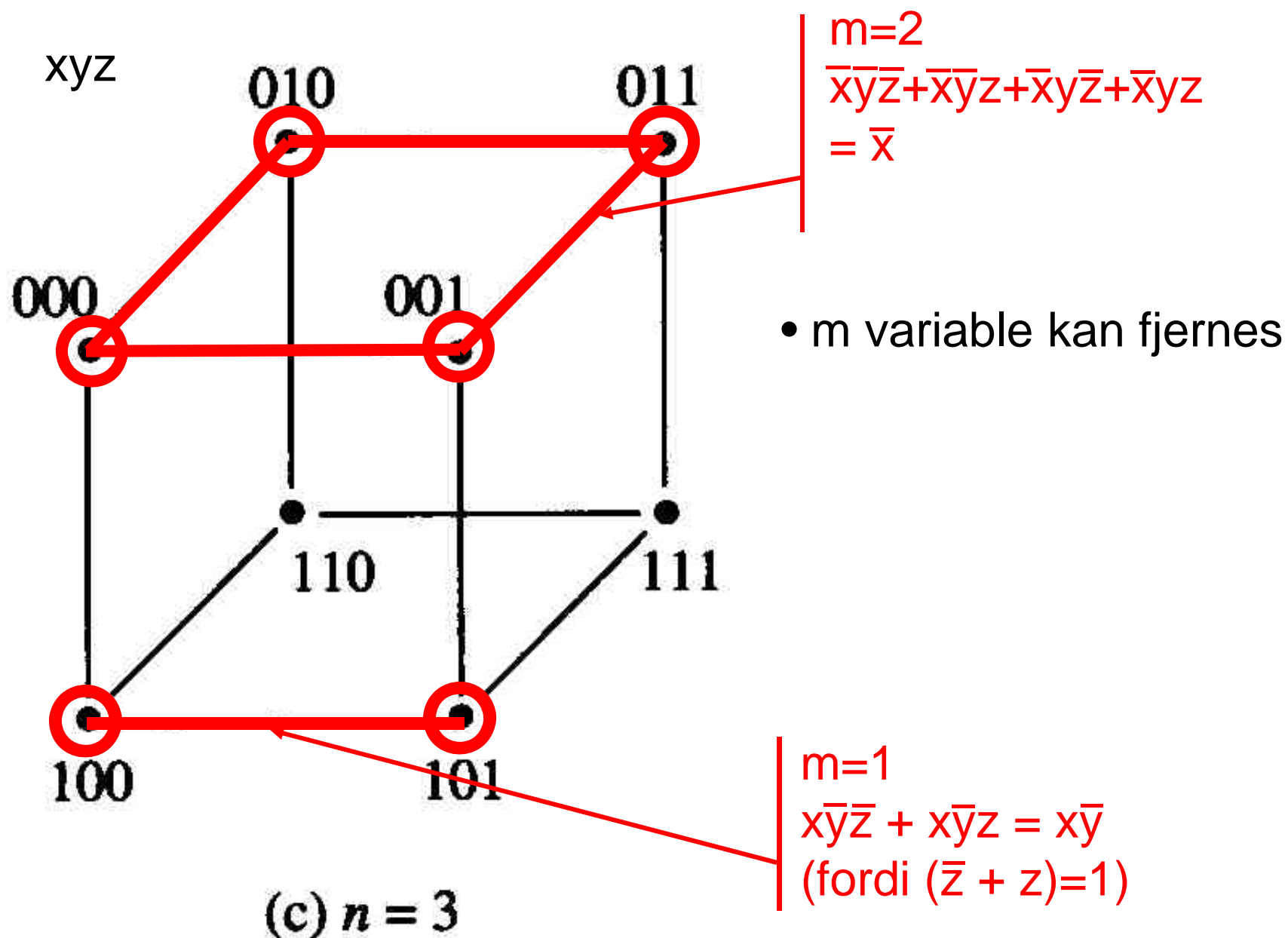
$$F_1 = \bar{x}yz + x\bar{y}z + xy\bar{z} + xyz$$

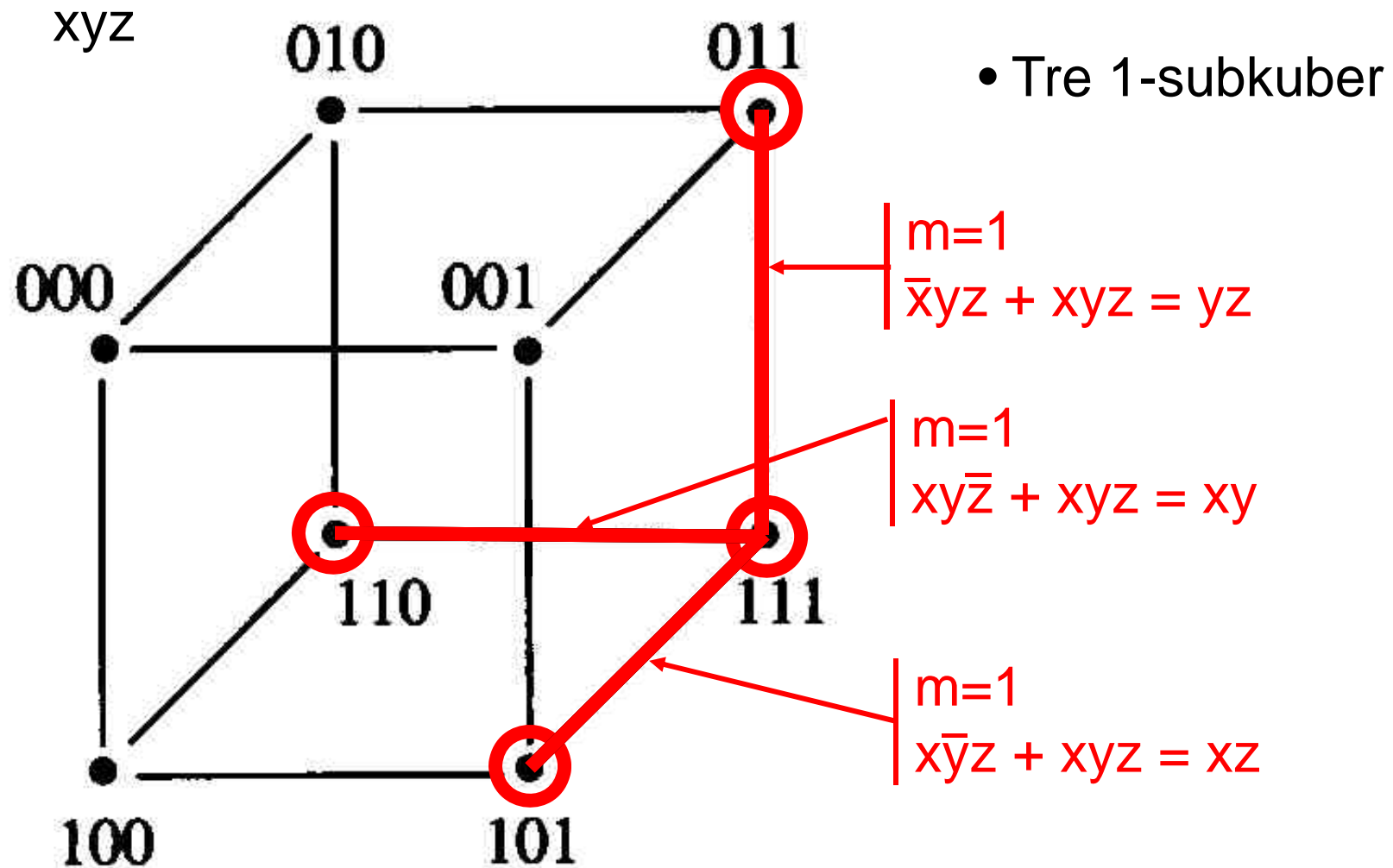
(c) $n = 3$

m-subkuber



Forenkling med m-subkuber

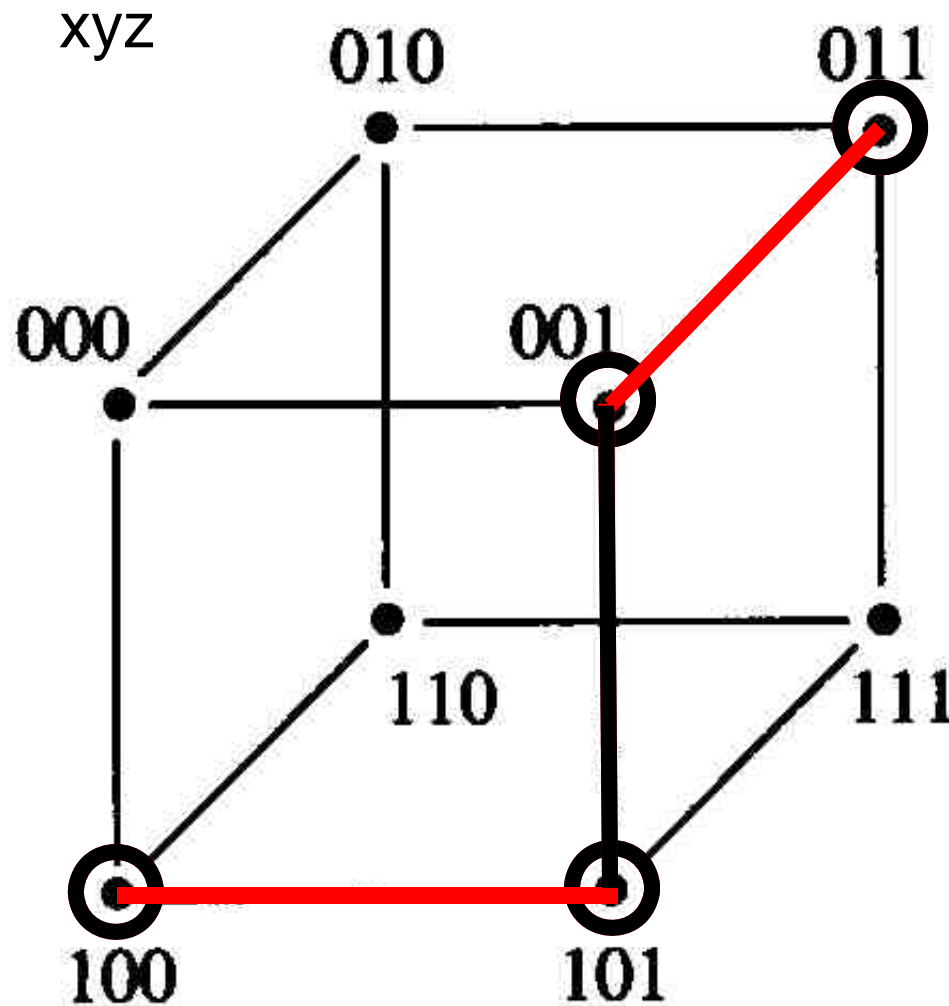


Forenkling av F_1 

$$F_1 = xz + xy + yz$$

(c) $n = 3$

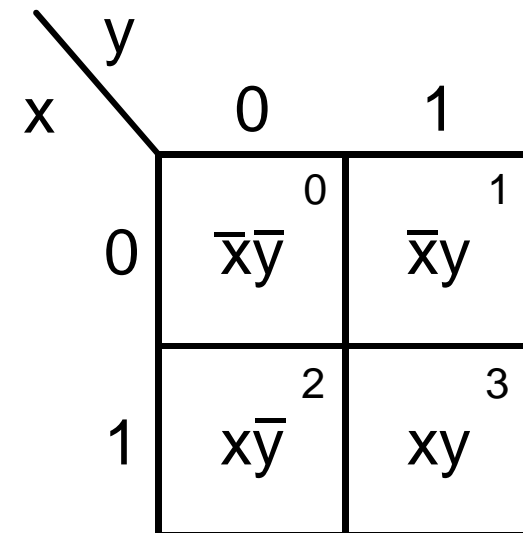
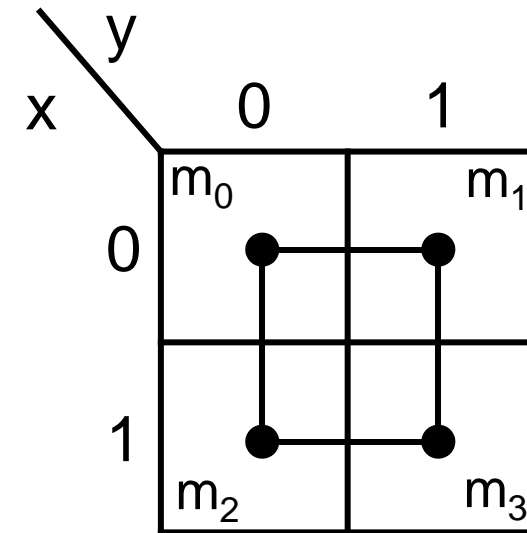
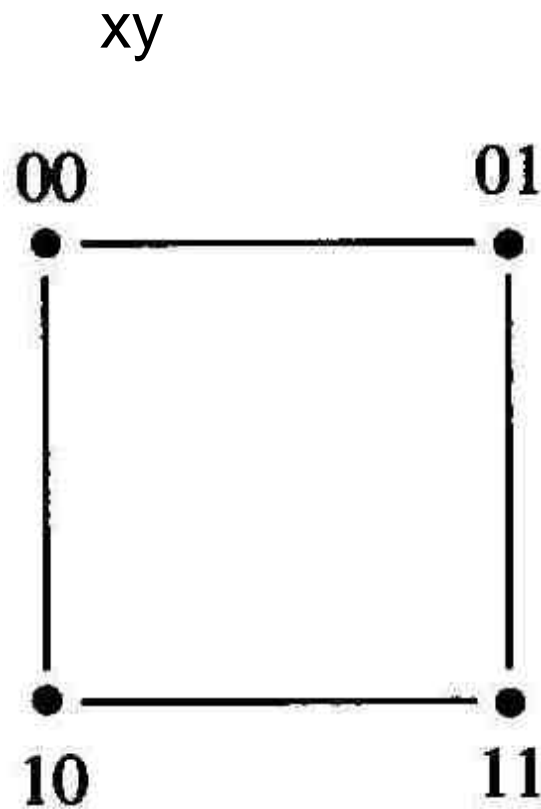
Implikanter



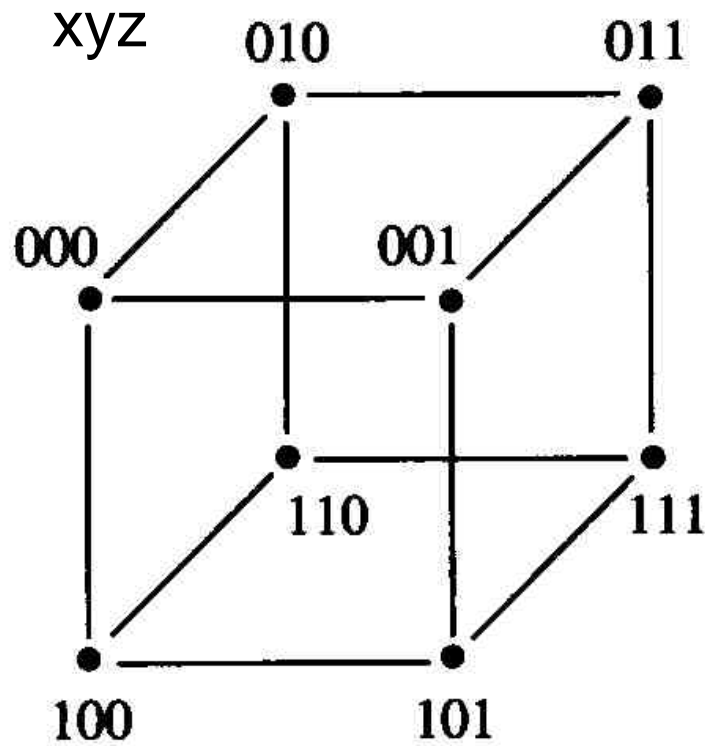
- Implikanter
 - $\Sigma(1,3,4,5)$
 - $x\bar{y}$
 - $\bar{y}z$
 - $\bar{x}z$
- Prim implikanter
 - $x\bar{y}$
 - $\bar{y}z$
 - $\bar{x}z$
- Essensielle prim implikanter
 - $x\bar{y}$
 - $\bar{x}z$

(c) $n = 3$

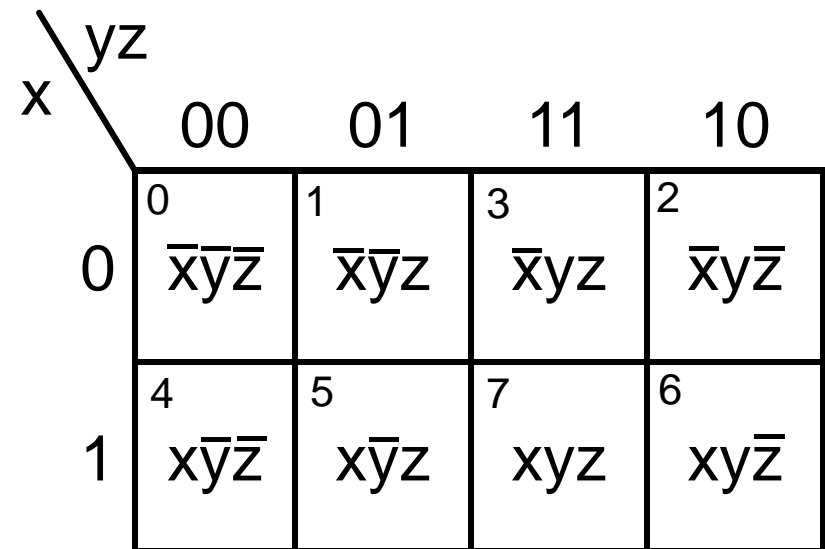
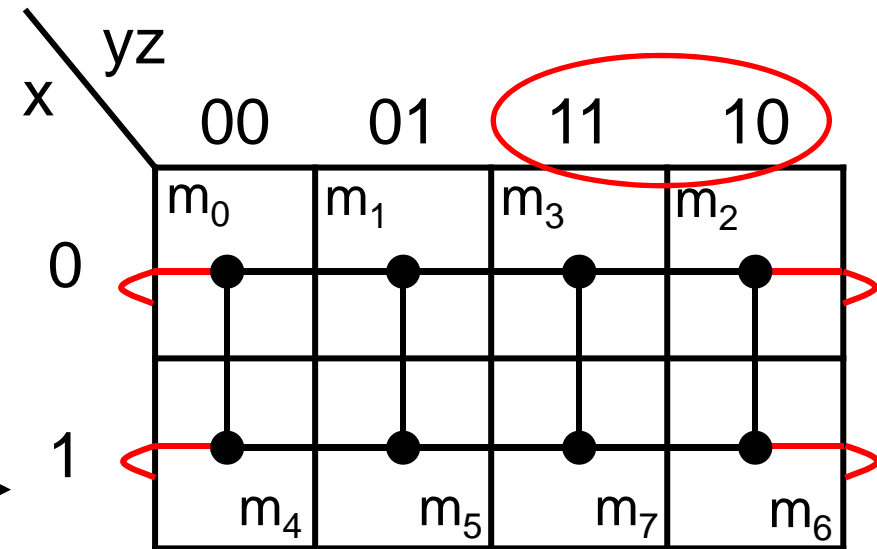
Karnaugh diagram



Karnaugh diagram

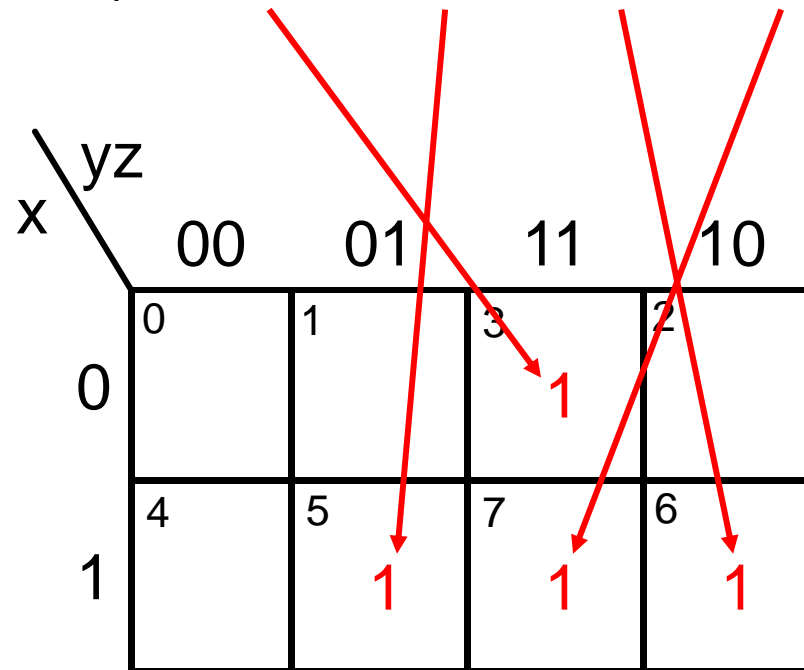


(c) $n = 3$

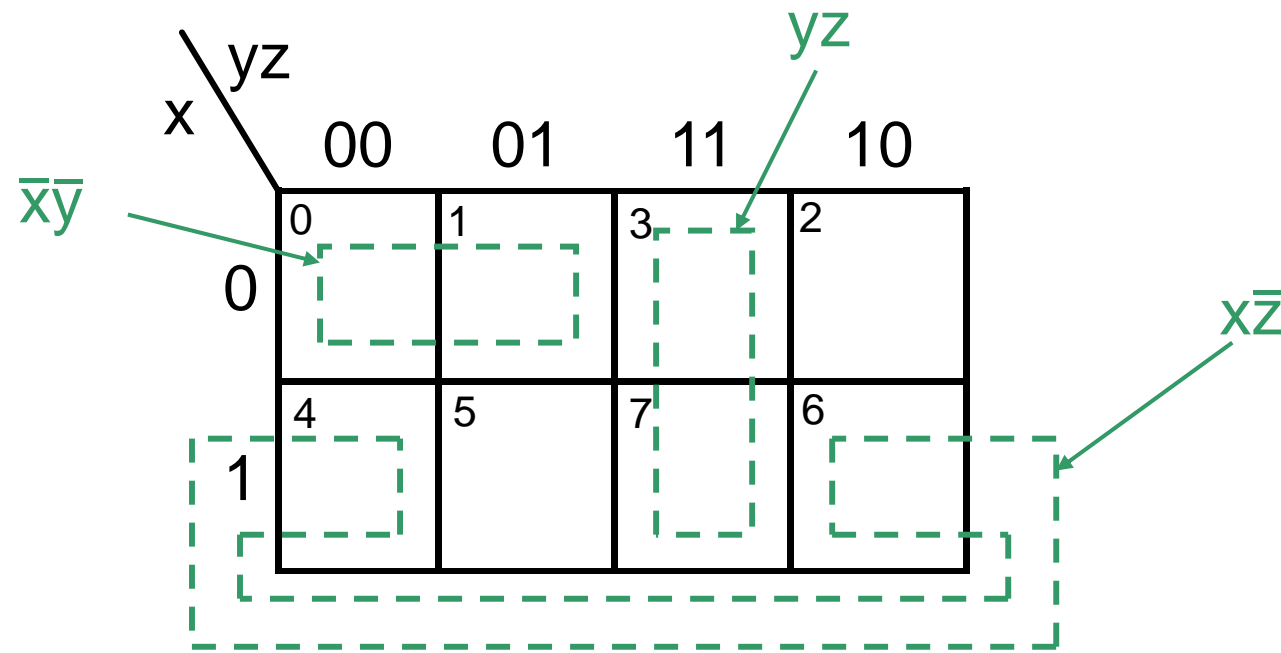


Beskrivelse av F_1 med Karnaugh diagram

$$F_1 = \bar{x}yz + x\bar{y}z + xy\bar{z} + xyz$$

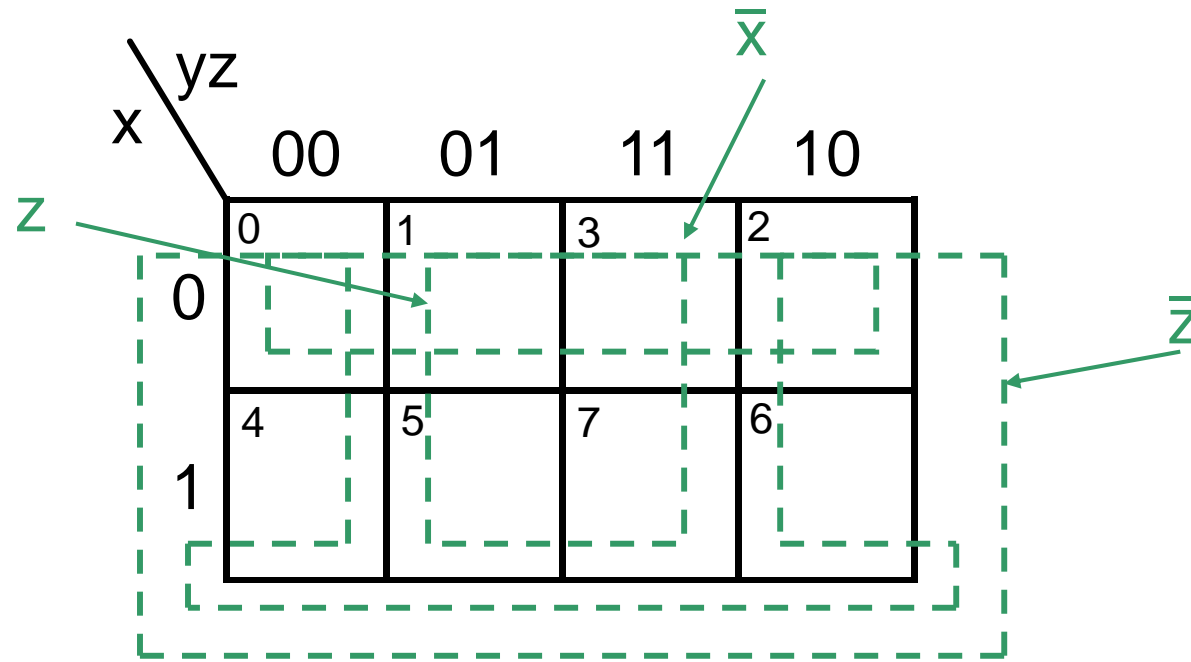


1-subkuber i Karnaugh diagram



		yz			
		00	01	11	10
x	0	0 $\bar{x}\bar{y}\bar{z}$	1 $\bar{x}\bar{y}z$	3 $\bar{x}yz$	2 $\bar{x}y\bar{z}$
	1	4 $x\bar{y}\bar{z}$	5 $x\bar{y}z$	7 xyz	6 $xy\bar{z}$

2-subkuber i Karnaugh diagram



		yz			
		00	01	11	10
x	0	0 $\bar{x}\bar{y}\bar{z}$	1 $\bar{x}\bar{y}z$	3 $\bar{x}yz$	2 $\bar{x}y\bar{z}$
	1	4 $x\bar{y}\bar{z}$	5 $x\bar{y}z$	7 xyz	6 $xy\bar{z}$

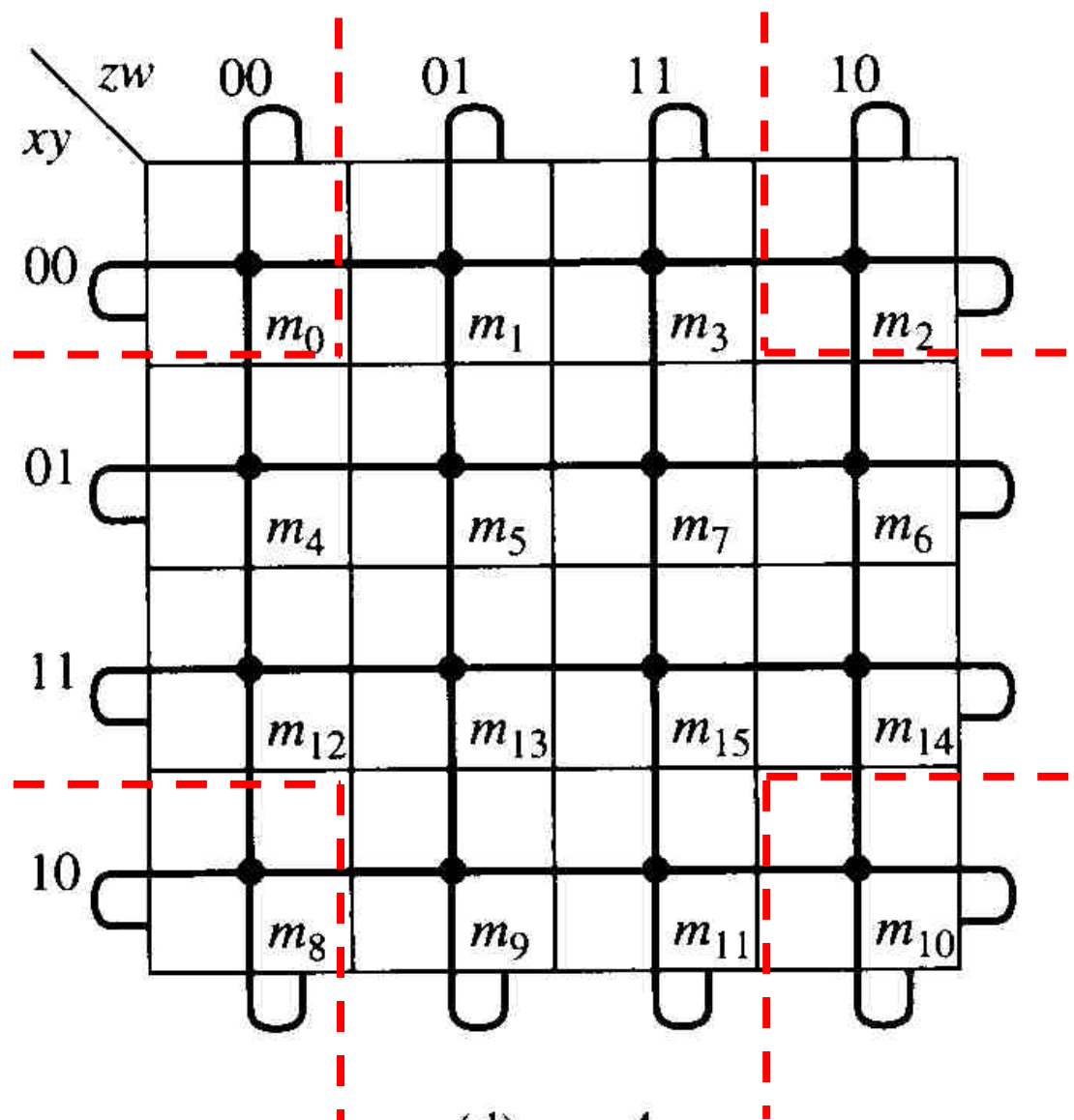
Forenkling av F_1 med Karnaugh diagram

$$F_1 = \bar{x}yz + x\bar{y}z + xy\bar{z} + xyz$$

		yz			
		00	01	11	10
x	0	0	1	3 1	2
	1	4	5 1	7 1	6 1

$$F_1 = xz + xy + yz$$

Karnaugh diagram

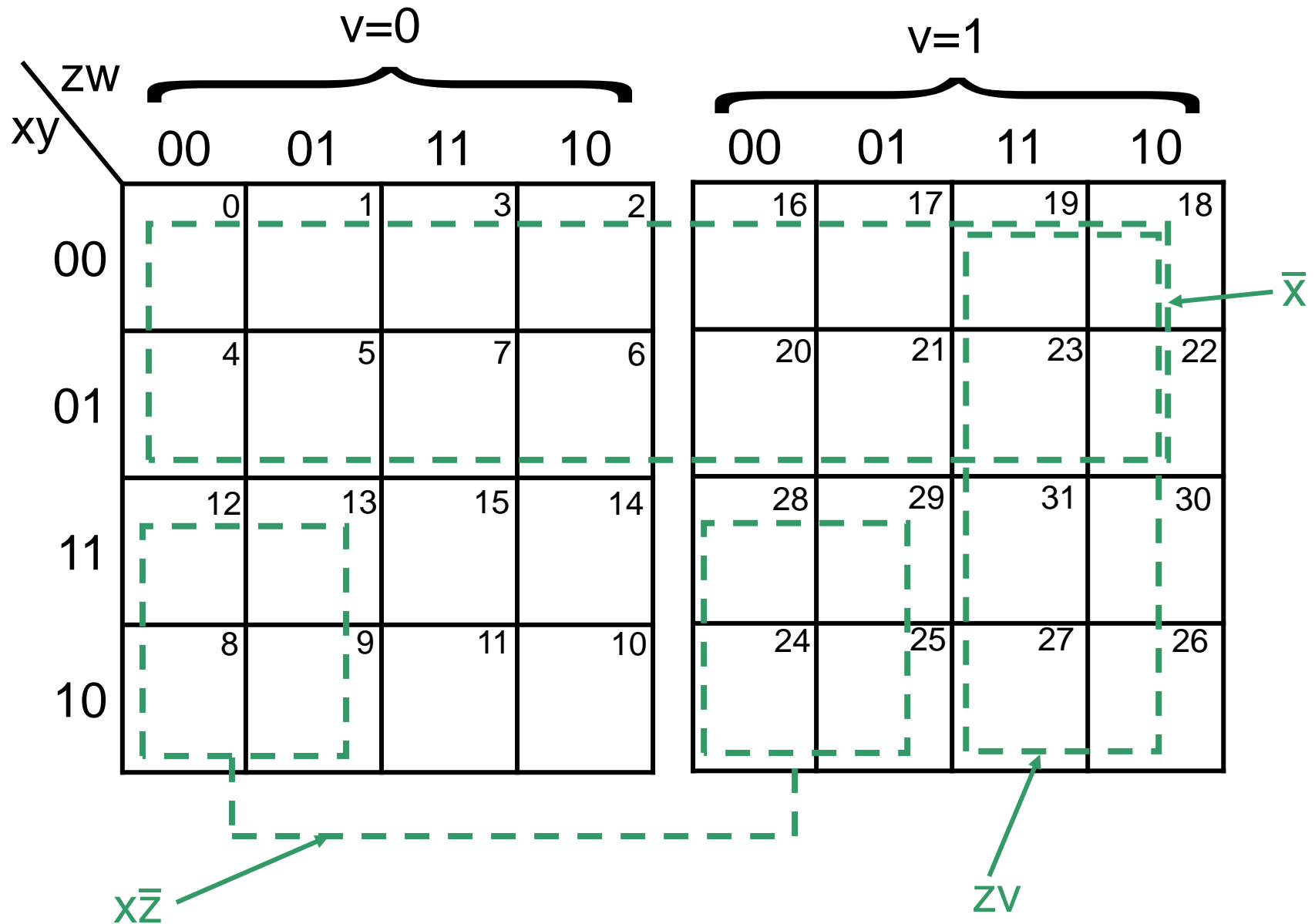


2-minutt-
oppgave

Hva er
dette?

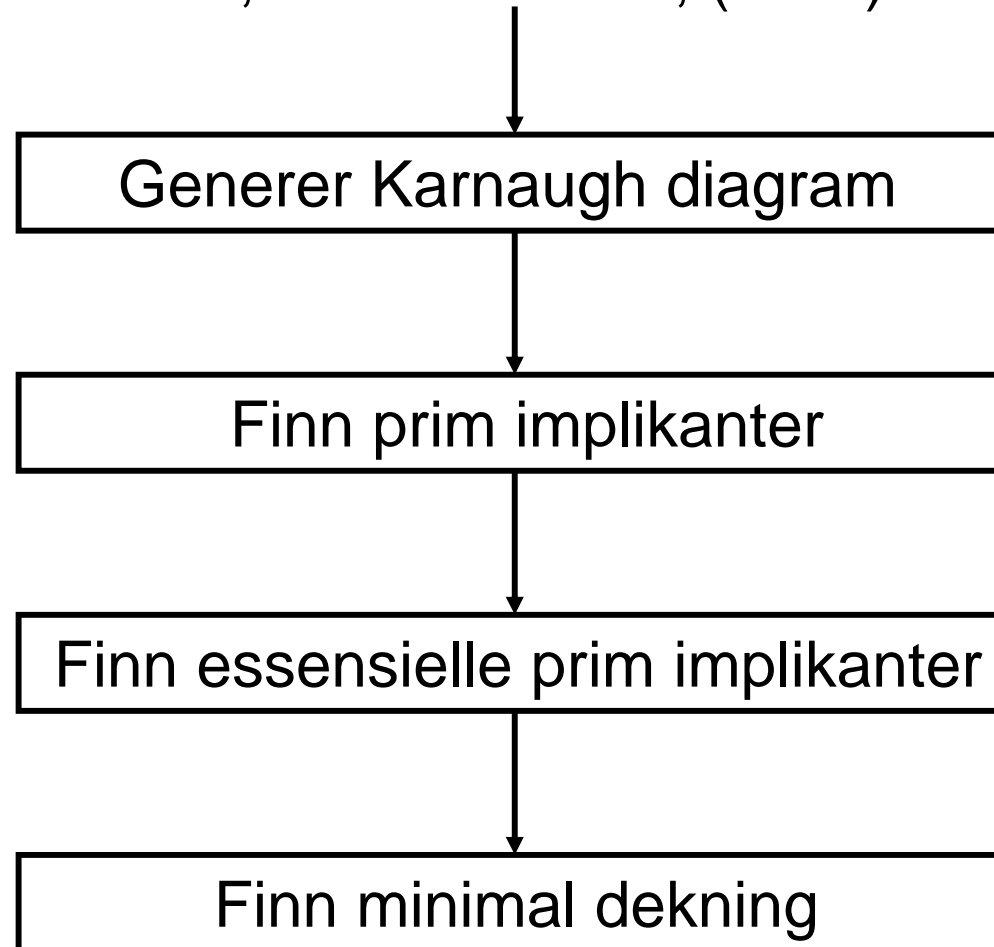
(d) $n = 4$

Karnaugh diagram med fem variable

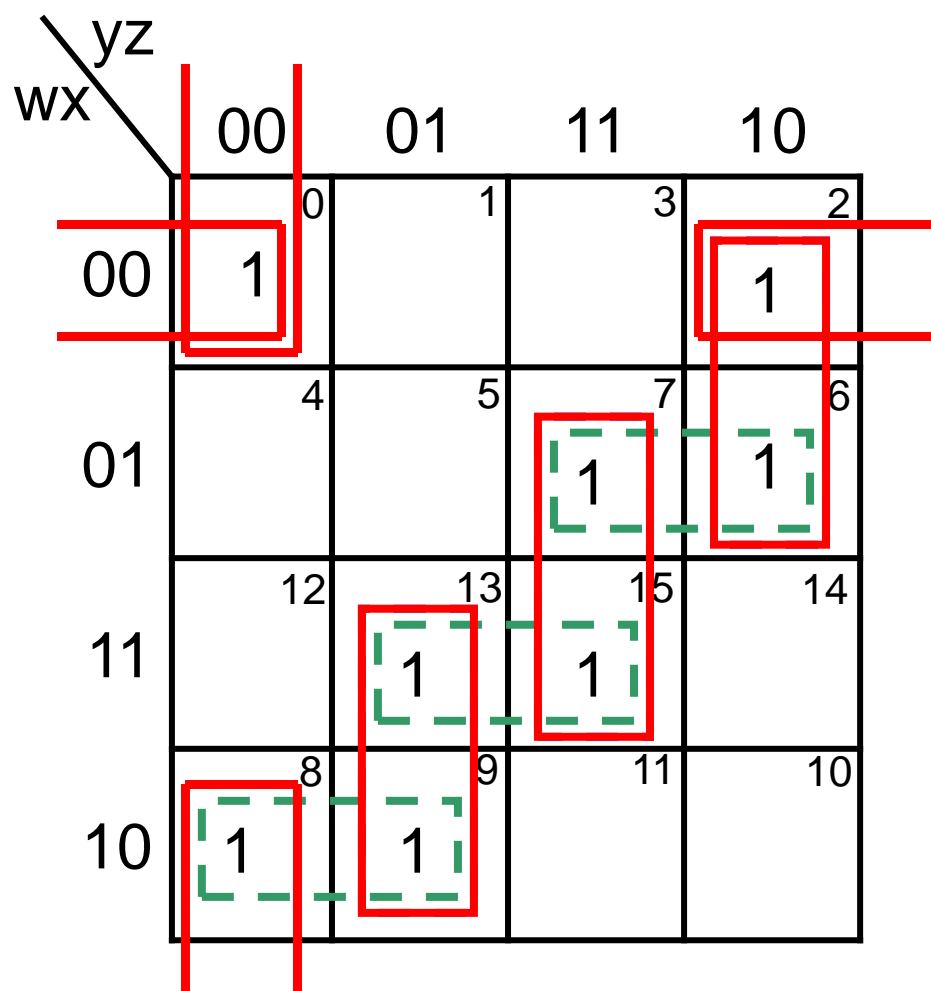


Forenkling med Karnaugh diagram

Sannhetstabell, kanonisk form, (ikke-)standard form



Finn minimal dekning



- Ingen essensielle prim implikanter

$$F = \bar{w}\bar{x}\bar{z}$$

$$+ \bar{w}xy$$

$$+ wxz$$

$$+ w\bar{x}\bar{y}$$

$$F = \bar{w}\bar{x}\bar{z}$$

$$+ xyz$$

$$+ \bar{w}y\bar{z}$$

$$+ w\bar{y}\bar{z}$$

$$+ \bar{x}\bar{y}\bar{z}$$

"Don't care" betingelser

- Ubrukte kombinasjoner av inngangsvariable kan utnyttes ved forenkling
 - Eks: BCD kodete tall
- Deres mintermer merkes med X i Karnaugh diagram
- Ved forenkling kan man sette en X lik 0 eller 1 etter behov
- Det forenklete Boolske uttrykket blir avhengig av de valgte verdier for X

"Don't care" betingelser

Siffer					Sifferkomplement				
Des.	BCD				Des.	BCD			
	x_3	x_2	x_1	x_0		y_3	y_2	y_1	y_0
0	0	0	0	0	9	1	0	0	1
1	0	0	0	1	8	1	0	0	0
2	0	0	1	0	7	0	1	1	1
3	0	0	1	1	6	0	1	1	0
4	0	1	0	0	5	0	1	0	1
5	0	1	0	1	4	0	1	0	0
6	0	1	1	0	3	0	0	1	1
7	0	1	1	1	2	0	0	1	0
8	1	0	0	0	1	0	0	0	1
9	1	0	0	1	0	0	0	0	0
10	1	0	1	0	?	X	X	X	X
...									
15	1	1	1	1	?	X	X	X	X

"Don't care" betingelser

$x_3x_2 \backslash x_1x_0$		x_1x_0			
		00	01	11	10
x_3x_2	00	0 1	1 1	3	2
	01	4	5	7	6
	11	12 X	13 X	15 X	14 X
	10	8	9	11 X	10 X

$$y_3 = \bar{x}_3 \bar{x}_2 \bar{x}_1$$

$x_3x_2 \backslash x_1x_0$		x_1x_0			
		00	01	11	10
x_3x_2	00	0	1	3 1	2 1
	01	4 1	5 1	7	6
	11	12 X	13 X	15 X	14 X
	10	8	9	11 X	10 X

$$y_2 = x_2 \bar{x}_1 + \bar{x}_2 x_1 = x_2 \oplus x_1$$

"Don't care" betingelser

$x_3x_2 \backslash x_1x_0$		x_1x_0			
		00	01	11	10
00	0		1	3	2
				1	1
01	4		5	7	6
				1	1
11	12	X	X	15	14
				X	X
10	8		9	11	10
				X	X

$$y_1 = x_1$$

$x_3x_2 \backslash x_1x_0$		x_1x_0			
		00	01	11	10
00	0		1	3	2
		1			1
01	4		5	7	6
		1			1
11	12	X	X	15	14
				X	X
10	8		9	11	10
		1		X	X

$$y_0 = \bar{x}_0$$

Gruppeoppgave

Forenkle:

$$F = \bar{w} \bar{x} \bar{y} z + \bar{x} \bar{y} \bar{z} + w \bar{y} \bar{z} + w y z + w \bar{x} y$$

Don't care set: $d(w,x,y,z) = \Sigma(6,14)$

yz
wx
