Innlevering 2

Oppgave 1

a) Siden Ole velger tilfeldig vil hvert svar være uavhengig av det forrige.
Siden hvert spm. har like mange alternativer vil sannsynligheten for rett være lik hver gang.

Så X er binomisk fordelt med $n=20$ og $p=\frac{1}{m}$.

$$P(X \geq 8) = 1 - P(X \leq 7)$$

$$= 1 - \sum_{i=0}^{7} \binom{20}{i} \left(\frac{1}{m}\right)^{i} \left(\frac{m-1}{m}\right)^{20-i}$$

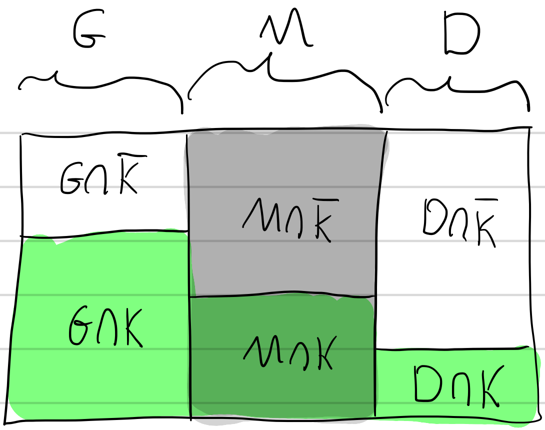
$m=2$: $P(X \geq 8) = 0,87$

$m=4$: $P(X \geq 8) = 0,10$

$m=5$: $P(X \geq 8) = 0,03$

$$E(X) = n \cdot p = \frac{n}{m} = \begin{cases} 10 & , m=2 \\ 5 & , m=4 \\ 4 & , m=5 \end{cases}$$

b)



$$P(K) = P(K|G)P(G) + P(K|M)P(M) + P(K|D)P(D)$$

$$= 0{,}8 \cdot 0{,}3 + 0{,}4 \cdot 0{,}5 + 0{,}2 \cdot 0{,}2$$

$$= 0{,}48$$

$$P(D|K) = \frac{P(K \cap D)}{P(K)} \cdot \frac{P(D)}{P(D)} = \frac{P(K|D)\,P(D)}{P(K)}$$

$$= \frac{0{,}2 \cdot 0{,}2}{0{,}48}$$

$$= 0{,}083$$

Oppgave 2

$$P(X=x) = \frac{\lambda^x}{x!} e^{-\lambda} \quad ; \quad x = 0, 1, 2, \ldots$$

a) $$P(X \geq 1) = 1 - P(X=0)$$

$$= 1 - \frac{\lambda^0}{0!} e^{-\lambda}$$

$$= 1 - e^{-\lambda}$$

$$\underline{\underline{= 0{,}999}}$$ (en eller flere klager)

$$P(X \leq 2 \mid X \geq 1) = \frac{P(1 \leq X \leq 2)}{P(X \geq 1)}$$

$$= \frac{P(X=1) + P(X=2)}{1 - P(X=0)}$$

$$= \frac{\lambda e^{-\lambda} + \frac{1}{2}\lambda^2 e^{-\lambda}}{1 - e^{-\lambda}} = \frac{7 \cdot e^{-7} + \frac{1}{2} 7^2 e^{-7}}{1 - e^{-7}}$$

$$\underline{\underline{= 0{,}029}}$$

b) Ønsker å vite $E(X)$.

$$E(X) = E(X_A)p_A + E(X_B)p_B + E(X_C)p_C$$
$$= \lambda_A p_A + \lambda_B p_B + \lambda_C p_C$$
$$= 5 \cdot 0{,}5 + 15 \cdot 0{,}25 + 20 \cdot 0{,}25$$
$$= 11{,}25$$

Forhandleren kan forvente 11,25 klager.

Oppgave 3

a) $$P(X \leq 4) = F(4) - F(0)$$
$$= \underline{\underline{1 - e^{-\frac{2}{5}} \approx 0{,}33}}$$

$$P(X > 7) = F(\infty) - F(7)$$
$$= \underline{\underline{e^{-7/10} \approx 0{,}50}}$$

$$P(X > 7 \mid X > 4) = \frac{P(X > 7)}{1 - P(X \leq 4)}$$
$$= \frac{e^{-7/10}}{1 - (1 - e^{-2/5})} = e^{-\frac{7}{10} + \frac{2}{5}}$$
$$= \underline{\underline{e^{-3/10} \approx 0{,}74}}$$

b) $P(X > c) = P(c < X < \infty)$

$= F(\infty) - F(c)$

$= 1 - 1 + e^{-c/\beta}$

$= \underline{\underline{e^{-c/\beta}}}$

$P(Y > y) = P(X - c > y)$

$= P(X > y + c)$

$= \underline{\underline{e^{-\frac{c+y}{\beta}}}}$

Vi har at $\underbrace{P(y \leq Y)}_{F_Y(y)} = 1 - P(Y > y)$

$F_Y(y) = 1 - e^{-\frac{c+y}{\beta}}$.

Så Y er også eksponentialfordelt.

Oppgave 4

a) $X \sim N(\mu, \sigma^2)$

$$Z = \frac{X - \mu}{\sigma} \sim N(0,1)$$

1) $x = 1{,}5$ kg svarer til $z = \dfrac{1{,}5 - 2{,}0}{0{,}5} = -1$

Slår opp i tabell

$$P(Z \leq -1) = 0{,}1587$$

$$\Rightarrow \quad P(X \leq 1{,}5) = P(Z \leq -1) = \underline{\underline{0{,}1587}}$$

2) $$P(2 \leq X \leq 2{,}5) = P\left(\frac{2-2}{0{,}5} \leq Z \leq \frac{2{,}5-2}{0{,}5}\right)$$

$$= P(0 \leq Z \leq 1)$$

$$= P(Z \leq 1) - P(Z \leq 0)$$

$$= 0{,}8413 - 0{,}5$$

$$= \underline{\underline{0{,}3413}}$$

b) $$P(2 \leq X \leq 2{,}5 \mid X \geq 1{,}5)$$

$$= \frac{P(2 \leq X \leq 2{,}5 \cap X \geq 1{,}5)}{P(X \geq 1{,}5)} = \frac{P(2 \leq X \leq 2{,}5)}{1 - P(X < 1{,}5)}$$

$$= \frac{0{,}3413}{1 - 0{,}1587} = \underline{\underline{0{,}4057}}$$

$$P(X \leq x \mid x \geq 1{,}5) = \frac{P(1{,}5 \leq x \leq x)}{P(X \leq 1{,}5)}$$

$$= \begin{cases} \dfrac{F_X(x) - F_X(1{,}5)}{F_X(1{,}5)}, & x \geq 1{,}5 \\ 0, & x < 1{,}5 \end{cases}$$

Oppgave 5

a) Trysil:

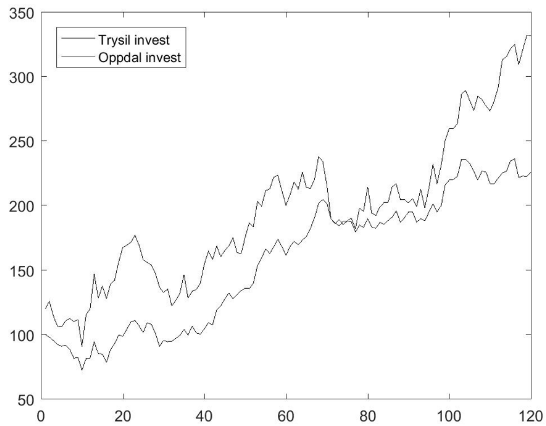
snitt = 156,4

std = 51,0

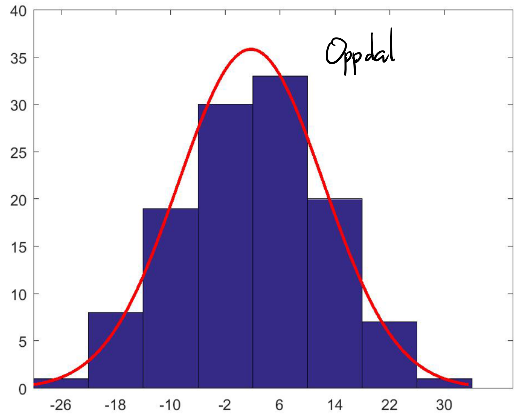
Oppdal:

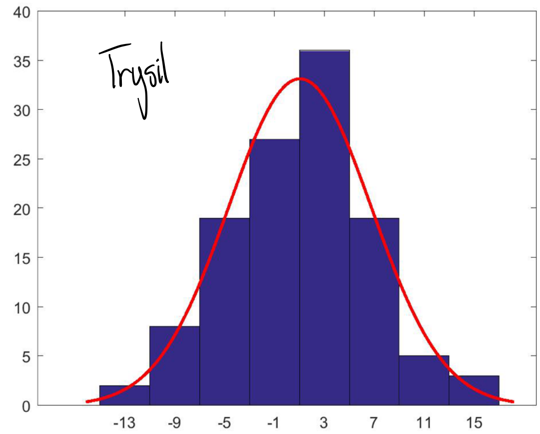
snitt = 195,3

std = 58,3



Veldig sterk positiv korrelasjon siden grafene nærmest følger hverandre.





Dataene ser veldig normalfordelt ut.

Sannsynligheten for at aksjeprisene i Oppdal øker en gitt dag er

$$\underline{\underline{0,5882}} \qquad \text{(fra formlen)}.$$

Bruker formler for betinget sannsynlighet.

Ønsker

$$p = P(\text{Oppdal} > 0 \mid \text{Trysil} < 0)$$

$$= \frac{P((\text{Oppdal} > 0) \cap (\text{Trysil} < 0))}{P(\text{Trysil} < 0)}$$

Som er den oppgitte formlen (minus $\leq$ vs. $<$, som ikke har ~~betydning~~ hvis vi har veldig mange datapunkter).

Formlen gir

$$\underline{\underline{p = 0,3529}}$$