Problem Set 7

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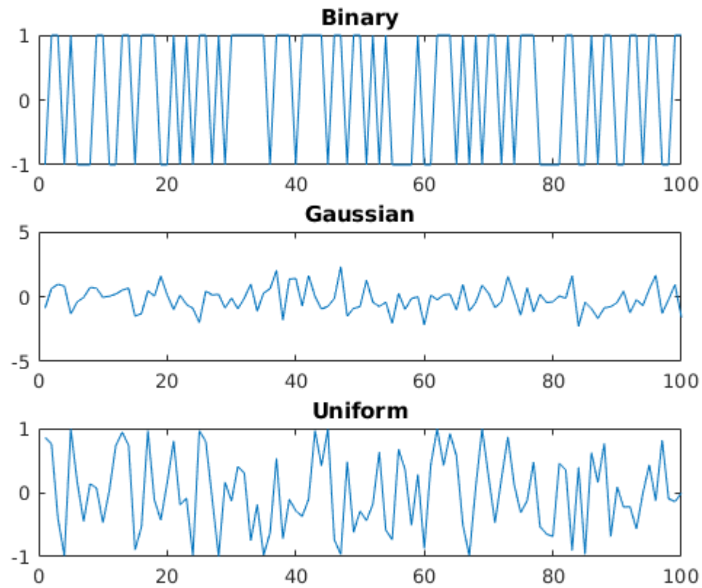
Problem 1



a) The binary and uniform signals are always contained in $[-1, 1]$ whereas the gaussian can in principle be arbitrarily large.
The Gaussian seems like the least sparodic signal of the three.

b) Binary:

$$p(x) = \begin{cases} 0.5 & , x=-1 \text{ or } x=1 \\ 0 & , \text{otherwise} \end{cases}$$

$$E(X) = \int_{-\infty}^{\infty} x p(x)\, dx$$

$$= 0.5(-1) + 0.5(1)$$

$$= 0$$

$$\gamma_{xx}[l] = E(X[n]\, X[n+l])$$

$$= N\, \delta(l)$$

$$\Gamma_{xx}(F) = \sum_{l=-\infty}^{\infty} \gamma_{xx}[l]\, e^{-j2\pi F l}$$

$$= N$$

White Gaussian noise:

$$X \sim p(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

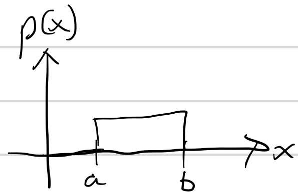
$$E(X) = \mu = 0 \quad \text{for previous prob.}$$

$$\gamma_{xx}[l] = E(X[n+l]\,X[n])$$
$$= \sigma^2 \delta(l)$$

$$\Gamma_{xx}(f) = \sigma^2$$

White uniform noise:

$$p(x) = \frac{1}{b-a}$$



$$E(x) = a + \frac{b-a}{2} = 0 \quad \text{for the previous prob.}$$

$$\gamma_{xx}(l) = E\left(\left(X(n+l) - m_x\right)\left(X(n) - m_x\right)\right), \quad l \neq 0$$
$$= E\left(X(n+l) - m_x\right) E\left(X(n) - m_x\right)$$
$$= 0$$

$$\gamma_{xx}(0) = E(x^2) = \int_a^b x^2 p(x)\, dx$$

$$= \frac{1}{b-a} \left[\frac{x^3}{3} \right]_a^b$$

$$= \frac{1}{3} \frac{b^3 - a^3}{b-a}$$

$$= \frac{1}{3} \quad \text{when } b = -1,\ a = 1$$

$$\Gamma_{xx}(f) = \frac{1}{3} \frac{b^3 - a^3}{b-a} \delta(l)$$

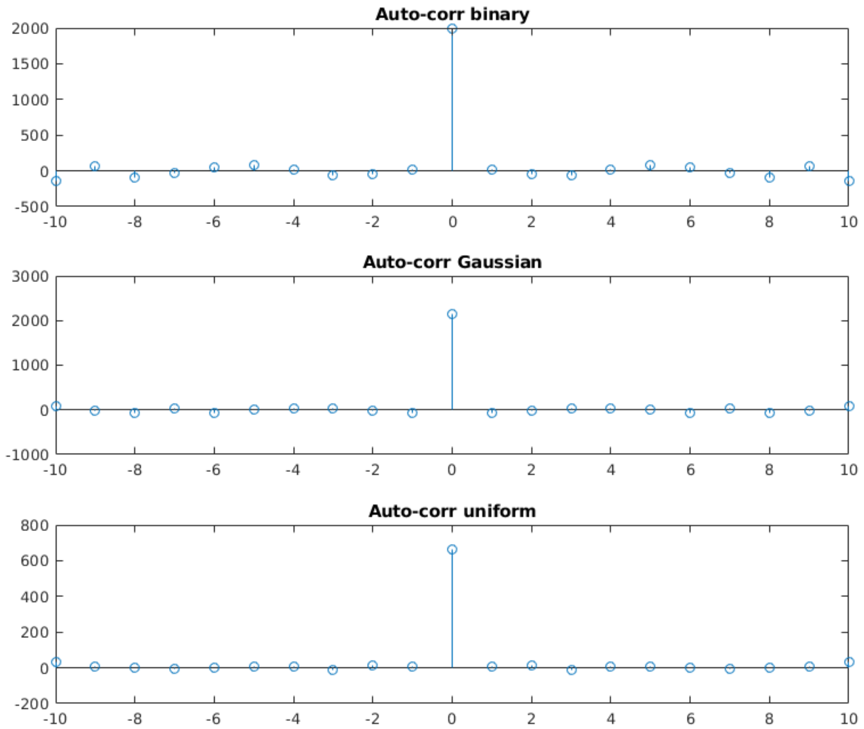
c) Computed means:

binary: 0.008

Gaussian: 0.014898

Uniform: 0.011916

They are all close to their theoretical values.



We didn't compute σ^2, but they all seem to fit the pattern $\sigma^2 \delta(l)$ quite good.

Problem 2

a) Mean:

$$m_x = H(0)\, m_w \quad , \; m_w = 0$$
$$= H(0) \cdot 0$$
$$= 0$$

Autocorrelation:

$$\gamma_{xx}[\ell] = r_{hh}[\ell] * \gamma_{ww}[\ell]$$
$$= r_{hh}[\ell] * \sigma_w^2\, \delta[\ell]$$
$$= \sigma^2\, r_{hh}[\ell] * \delta[\ell]$$
$$= \sigma^2\, r_{hh}[\ell]$$
$$= r_{hh}[\ell] \quad , \; \sigma^2 = 1$$

$$h[n] = \begin{cases} \left(-\frac{1}{2}\right)^n & , n \geq 0 \\ 0 & , n < 0 \end{cases}$$

$$r_{hh}[\ell] = h[\ell] * h[-\ell]$$

$$= \sum_{n=-\infty}^{\infty} h[n]\, h[\ell - n]$$

$$= \sum_{n=0}^{\ell} \left(-\frac{1}{2}\right)^{n} \left(-\frac{1}{2}\right)^{\ell-n}$$

$$= \sum_{n=0}^{\ell} \left(\frac{1}{2}\right)^{\ell}$$

$$= \frac{1 - \left(\frac{1}{2}\right)^{\ell+1}}{1 - \frac{1}{2}} \quad , \ \ell > 0$$

$$= 2 - \left(\frac{1}{2}\right)^{\ell}$$

This gives

$$\gamma_{xx}[\ell] = 2 - \left(\frac{1}{2}\right)^{|\ell|}$$

Power density spectrum:

$$\Gamma_{xx}(f) = |H(f)|^2$$

$$= \frac{1}{\left|1 + \frac{1}{2} e^{-j2\pi f}\right|^2}$$

$$= \frac{1}{\left(1 + \frac{1}{2}\cos(2\pi f)\right)^2 + \left(\frac{1}{2}\sin(2\pi f)\right)^2}$$

$$\Rightarrow \Gamma_{xx}(f) = \frac{1}{\left(\frac{1}{2}\right)^2 + \cos(2\pi f) + 1}$$

$$= \frac{4}{5 + 4\cos(2\pi f)}$$

Power:

$$P = \int_{-0.5}^{0.5} S(f)\, df$$

$$= 4\int_{-1/2}^{1/2} \frac{1}{5 + 4\cos(2\pi f)}\, df$$

$$= \frac{4}{3}$$

b) $$\hat{m}_x = \frac{1}{N} \sum_{n=0}^{N} x[n]$$

$$\hat{\gamma}_{xx}[l] = \frac{1}{2N} \sum_{n=-N}^{N} x[n]x[n+l]$$

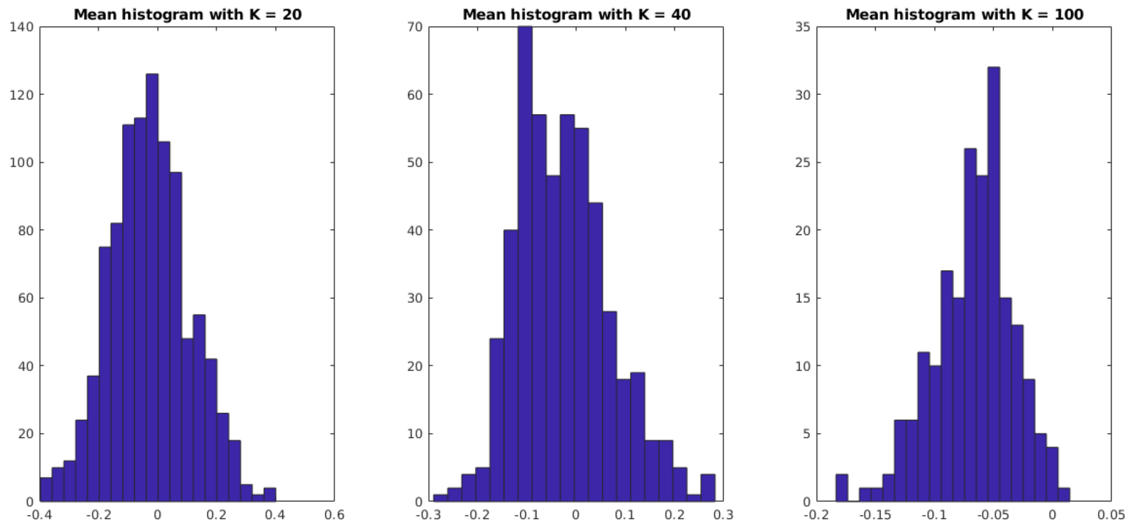
$$\hat{\Gamma}_{xx}(f) = \sum_{l=-N}^{N} \hat{\gamma}_{xx}[l]\, e^{-j2\pi f l}$$

$$\hat{P} = \int_{-0.5}^{0.5} \hat{\Gamma}_{xx}(f)\, df$$

c)

Problem 3

a, b, c, d)



For (c) we get

$$\hat{m}_x = 0.0055052$$

$$\hat{\sigma}_x^2 = 1.3135$$

e) Note that the spread is alot less when K is higher. This is what we expect when computing averages since the varianse goes as roughly $\frac{1}{K}$.