Exercise 1

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5-Card poker hands

a) We draw 5 cards from a collection of 52 where order doesn't matter and we don't place any cards back.

$$\Rightarrow \binom{52}{5} = 2598960 \quad \text{5-card hands.}$$

b) The probability of each event is

$$p = \frac{1}{\binom{52}{5}}$$

c) There are four ways a royal straight flush can happen, one for every suit

$$\Rightarrow P(\text{royal straight flush}) = \frac{4}{\binom{52}{5}} \approx 1{,}54 \cdot 10^{-6}$$

In a four card hand, 4-of-a-kind can happen in thirteen ways. By adding a fifth card that can be anything from the remaining $(52-4)$ cards we get

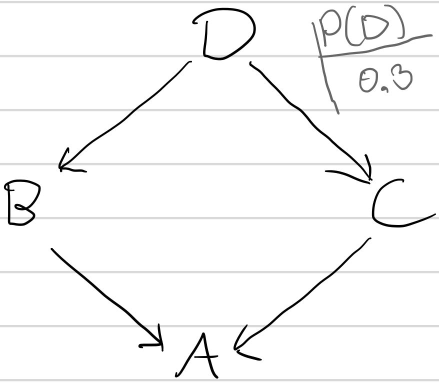
$$P(\text{4-of-a-kind}) = \frac{13 \cdot (52-4)}{\binom{52}{5}} = \underline{\frac{1}{4165}} \approx 0{,}24\,\%$$

Bayesian Network Construction

a) $P(A,B,C,D) = P(A \mid B,C)\,P(B \mid D)\,P(C \mid D)\,P(D)$

parents$(A) = \{B, C\}$

parents(B) = parents$(C) = \{D\}$



$P(D)$
0.3

D	$P(B \mid D)$
T	0.1
F	0.2

D	$P(C \mid D)$
T	0.95
F	0.70

B	C	$P(A \mid B,C)$
F	F	1.0
F	T	0.5
T	F	0.25
T	T	0.1

The independence structure between the variables helps reduce the size of the tables greatly. Take, for instance the table for C. We only had to consider $C \mid D$, and could ignore $C \mid A$ and $C \mid B$, which made the table about four times as small.

b) $p(X_1, \dots, X_n) = p(X_1) \prod_{i=2}^{n} p(X_i \mid X_{i-1})$

$\text{parent}(X_i) = \{X_{i-1}\}$

$$X_1 \rightarrow X_2 \rightarrow X_3 \rightarrow \dots \rightarrow X_{n-1} \rightarrow X_n$$

$P(X_1)$
0.2

X_1	$P(X_2 \mid X_1)$
T	0.4
F	0.6

. . . . . .

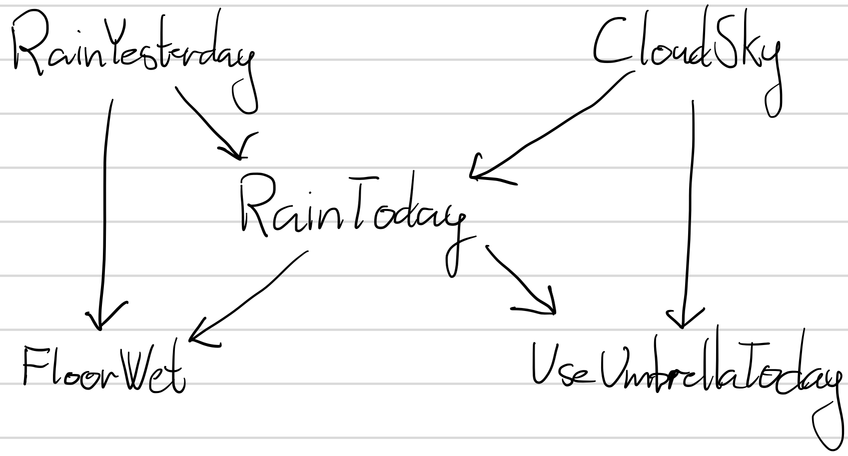
X_{n-1}	$P(X_n \mid X_{n-1})$
T	0.0
F	0.3

c) P(RainToday, RainYesterday, FloorWet, UseUmbrellaToday, CloudSky)

= p(RainYesterday) · p(CloudSky) · P(RainToday | RainYesterday, CloudSky)

· p(FloorWet | RainToday, RainYesterday)

· p(UseUmbrellaToday | RainToday, CloudSky)

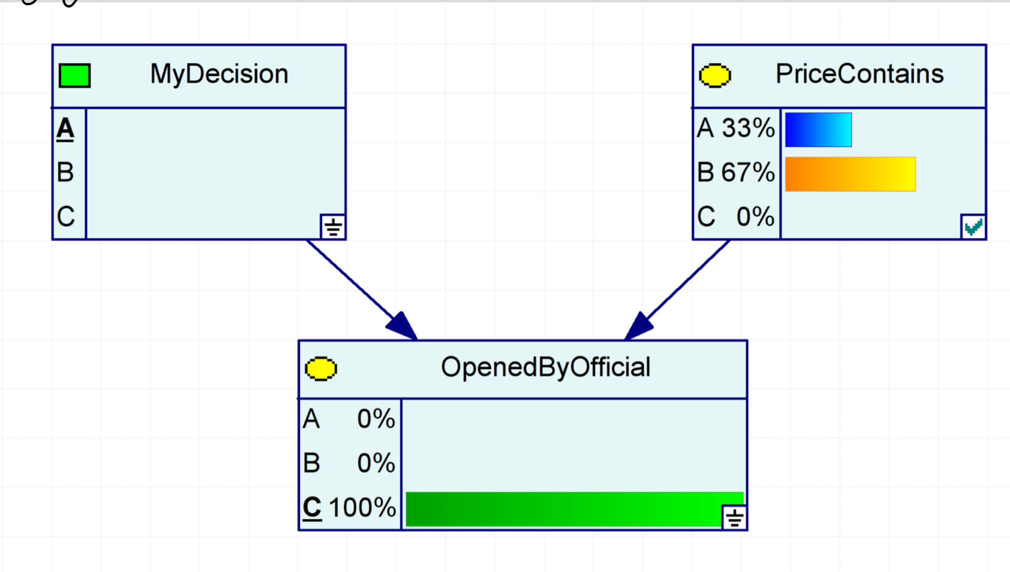


Bayesian Network Aplication

I set up the ContainsPrice to give $P(A)=P(B)=P(C)=\frac{1}{3}$.

For the OpenedByOfficial chance, I set it up so that probabillity of official opening the same as "my decission" or the same as "contains price" was zero, and then I adjusted the remaing values of column to give sum 1.

Result:



Conclusion:
It is advantageous to switch.