Øving 6

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Oppgave 1

$$h_p(s) = \frac{1 - T_1 s}{\left(\frac{s}{\omega_0}\right)^2 + 2\xi \frac{s}{\omega_0} + 1}$$

$$h_r(s) = K_p \frac{1 + T_i s}{T_i s}$$

a) $$h_0 = h_p h_r = \frac{K_p}{T_i} \frac{(1 - T_1 s)(1 + T_i s)}{\left(\left(\frac{s}{\omega_0}\right)^2 + 2\xi \frac{s}{\omega_0} + 1\right) s}$$

Knekkfrekvenser:

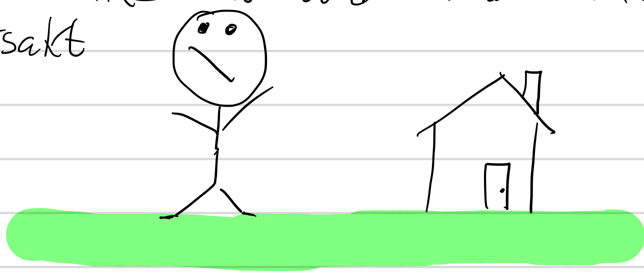
$$\frac{1}{T_1} = 5 \quad , \quad \omega_0 = 2 \quad , \quad \frac{1}{T_i} = 0{,}2$$

b) Vi har at

$$M(s) \approx \begin{cases} 1 , & |h_0| \ll 1 \\ h_0 , & |h_0| \gg 1 \end{cases}$$

og $$N(s) \approx \begin{cases} 1/h_0 , & |h_0| \ll 1 \\ 1 , & |h_0| \gg 1 \end{cases}$$

Skjønner ikke hvordan man skal skissere c og d eksakt



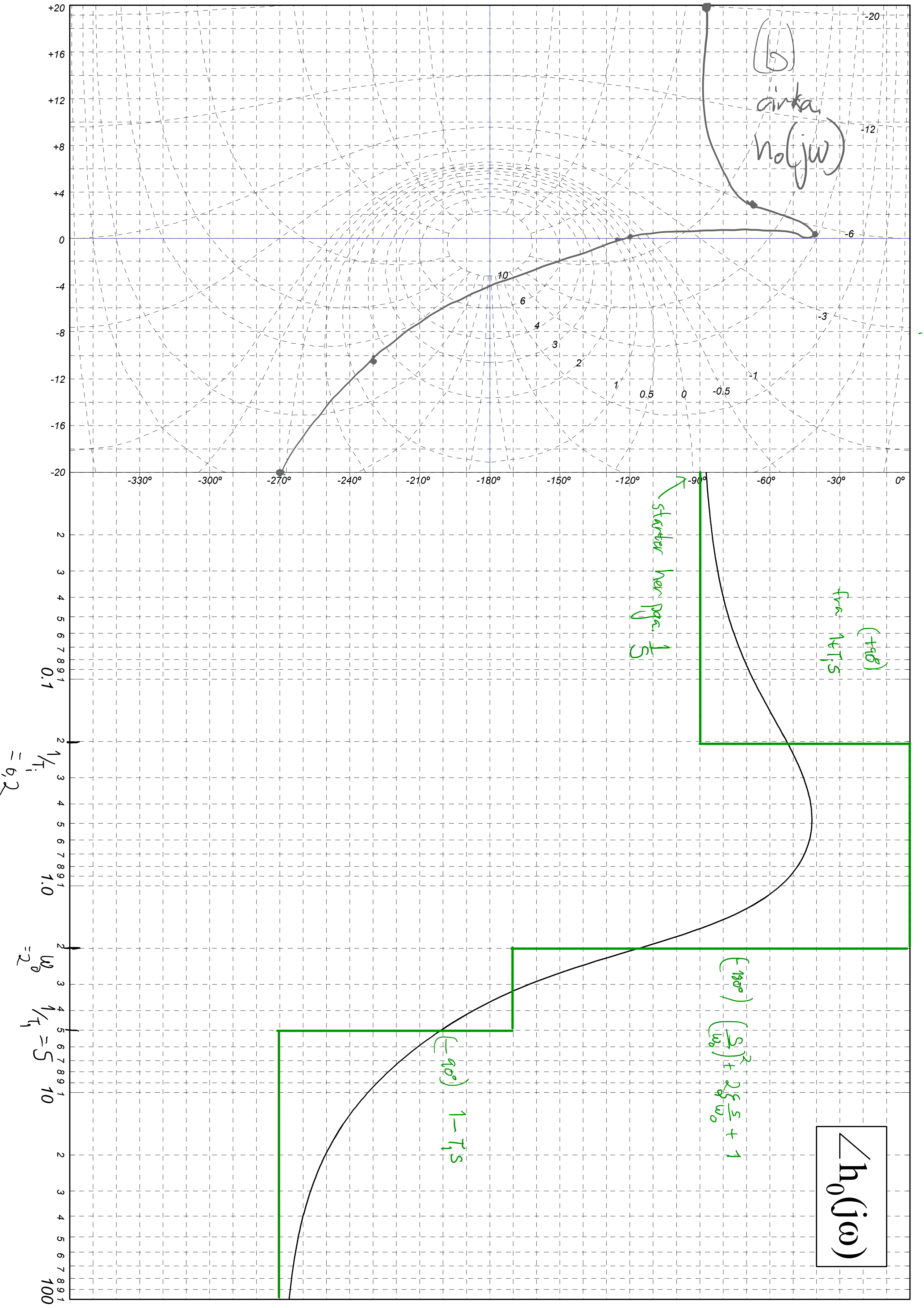
c) $|N(j\omega)| = \left|\frac{1}{1+h_0}\right| = \frac{1}{\sqrt{1+h_0^2}}$

d) Vi har nå at

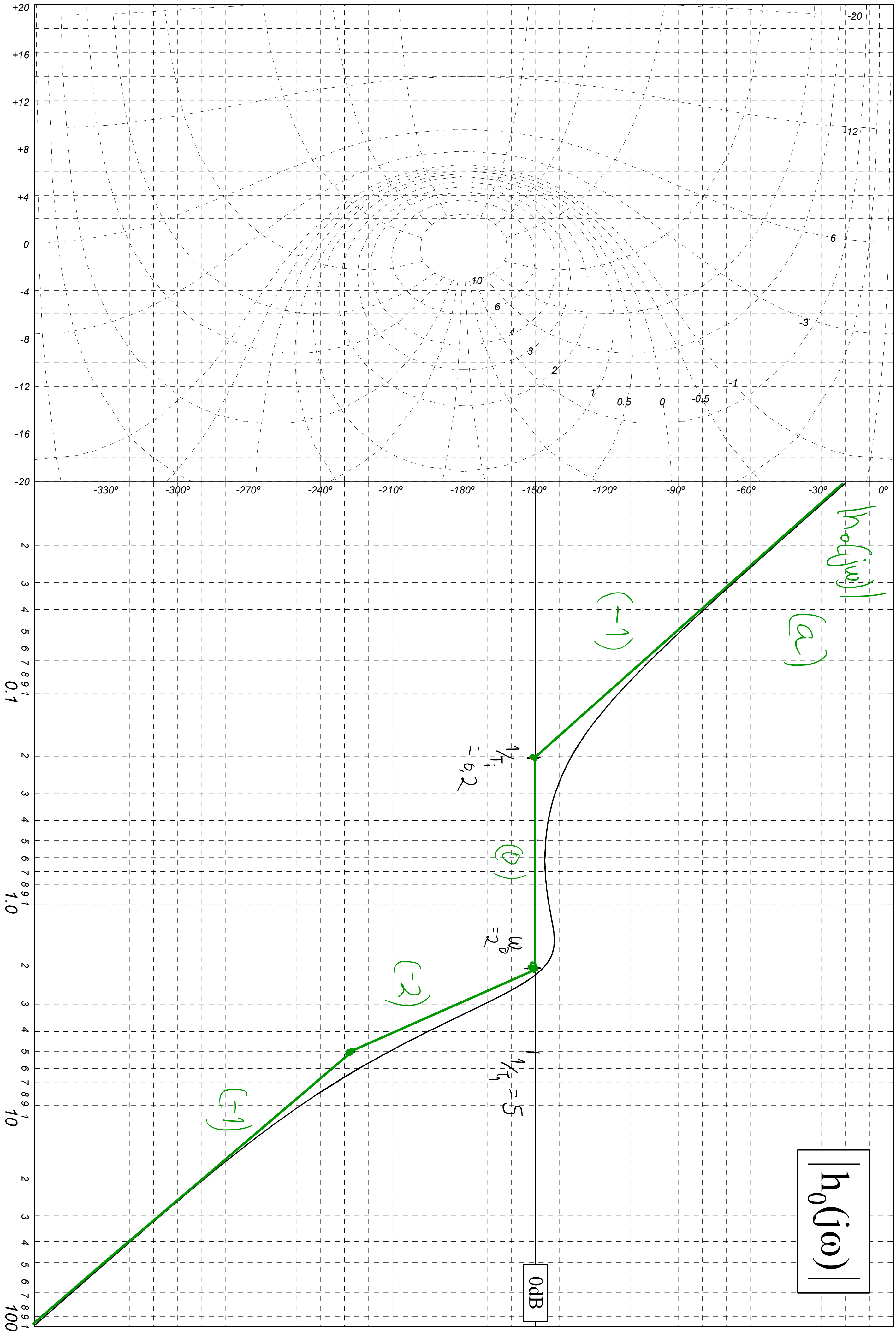
$$h_0(j\omega) = h_p h_r e^{-\tau j\omega}$$

$$\Rightarrow \quad |h_0(j\omega)| = h_p h_r$$

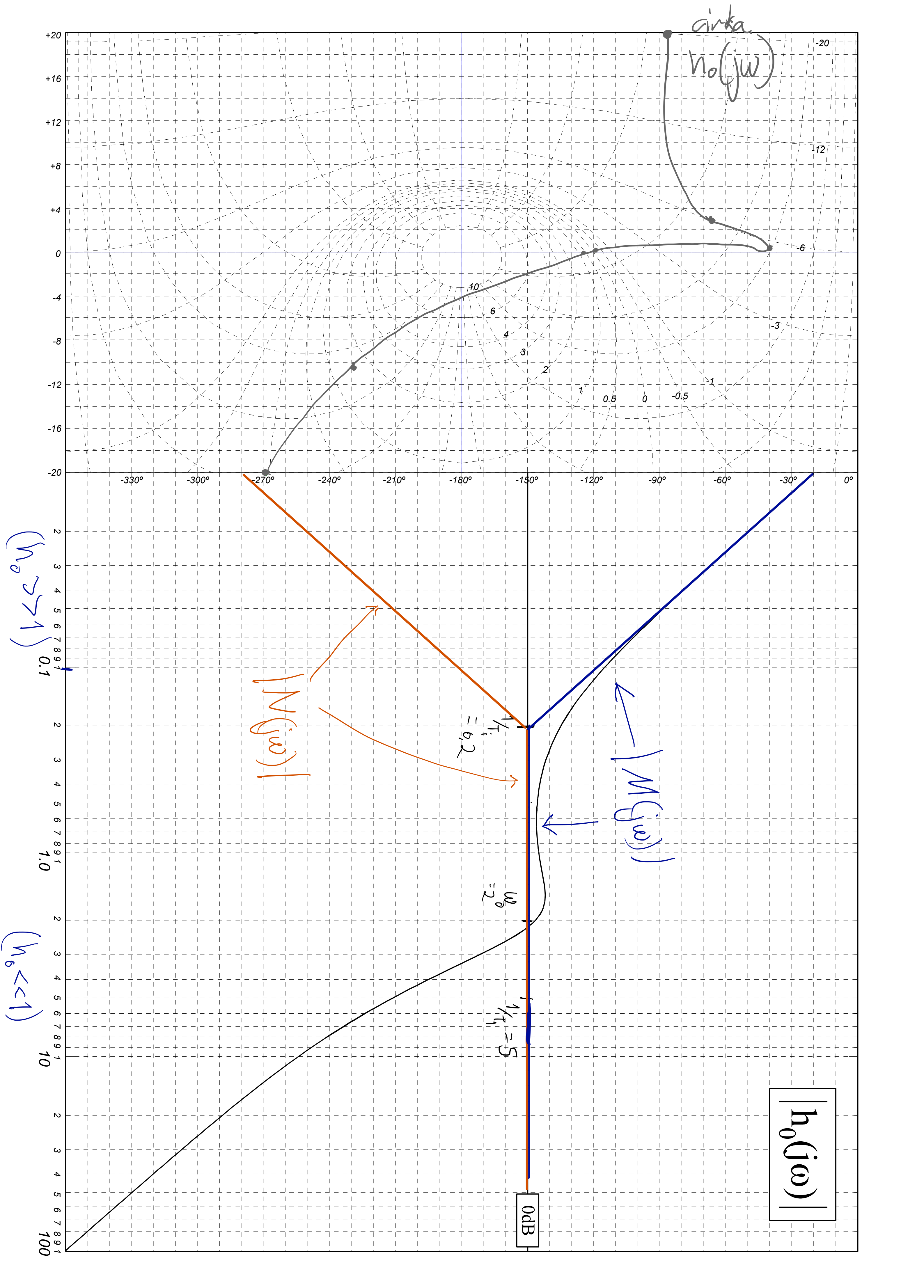
og $\angle h_0(j\omega) = \angle h_p + \angle h_p - \tau\omega$



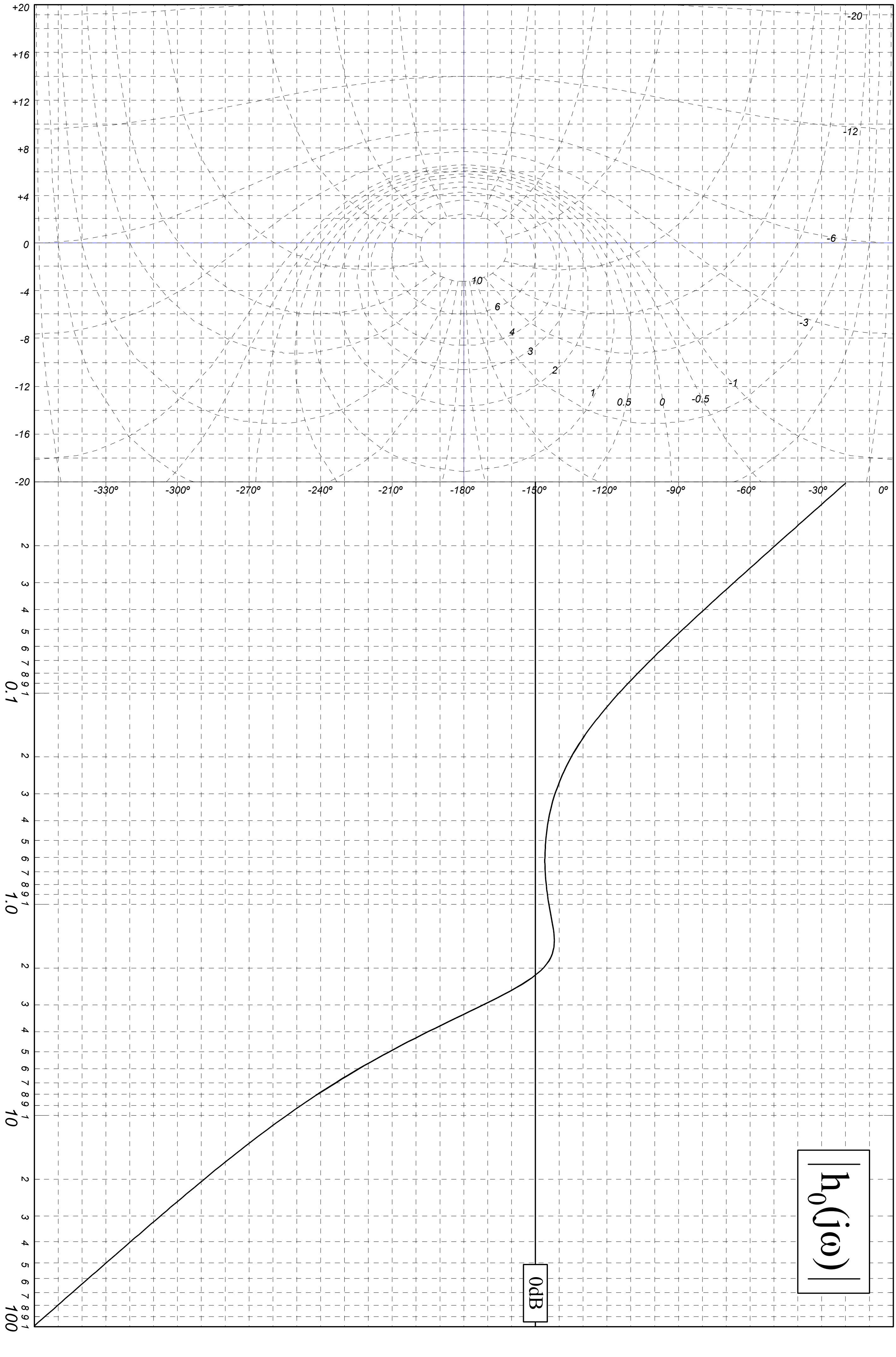
(b)
$h_0(j\omega)$
starter her pga. $\frac{1}{s}$
fra $1+T_i s$
$(+90°)$
$1/T_i = 0{,}2$
$(-180°)$
$\left(\frac{s}{\omega_0}\right)^2 + 2\xi\frac{s}{\omega_0} + 1$
$\omega_0 = 2$
$(-90°)$
$1 - T_1 s$
$1/T_1 = 5$
$\angle h_0(j\omega)$



+20
+16
+12
+8
+4
0
-4
-8
-12
-16
-20
-330°
-300°
-270°
-240°
-210°
-180°
-150°
-120°
-90°
-60°
-30°
0°
10
6
4
3
2
1
0.5
0
-0.5
-1
-3
-6
-12
-20
0.1
1.0
10
100
0dB
|h0(jω)|
|h0(jω)| (a)
(-1)
1/τi = 0,2
(0)
ω0 = 2
(-2)
1/τ1 = 5
(-1)



cirka
h0(jw)
|h0(jω)|
|M(jω)|
|N(jω)|
1/Ti = 0,2
ω0 = 2
1/τ1 = 5
0dB
(h0 >> 1)
(h0 << 1)


+20
+16
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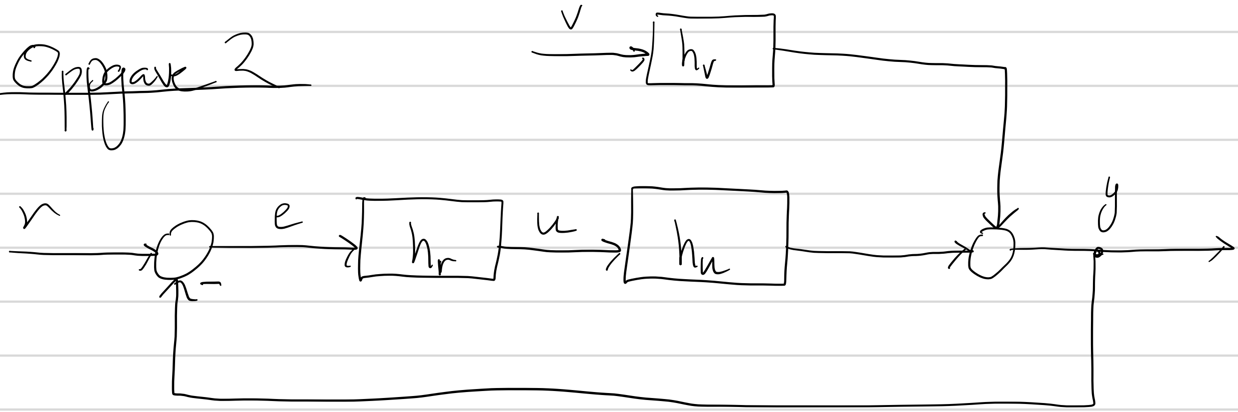
-210°
-180°
-150°
-120°
-90°
-60°
-30°
0°
-20
-12
-6
-3
-1
-0.5
0
0.5
1
2
3
4
6
10
0.1
1.0
10
100
0dB
$|h_0(j\omega)|$

Oppgave 2



a) $u = e h_r$

$= (r - y) h_r$

$= (r - u h_u) h_r \qquad (v = 0)$

$\Rightarrow u = r h_r - u h_0$

$\Rightarrow \dfrac{u}{r} = \dfrac{r h_r}{1 + h_0}$

$u = e h_r$

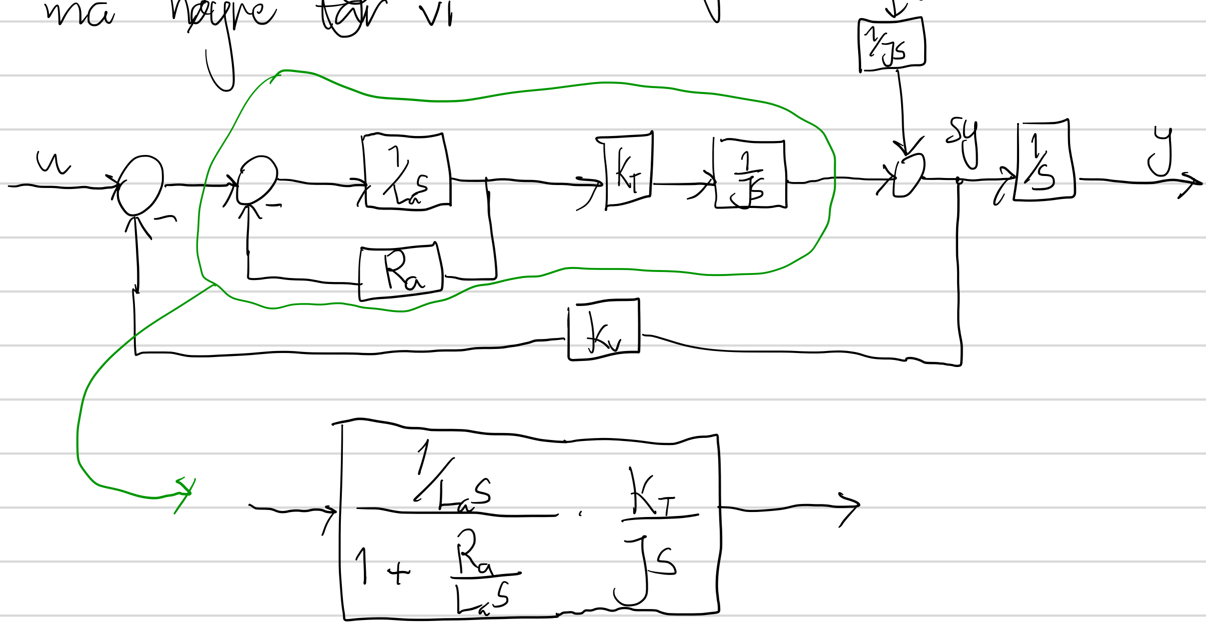
$= -y h_r \qquad (r = 0)$

$= -(u h_u + v h_v) h_r$

$\Rightarrow u(1 + h_0) = v h_v h_r$

$\Leftrightarrow \dfrac{u}{v} = \dfrac{v h_v h_r}{1 + h_0}$

b) Hvis vi flytter summasjonsblokken med v mot høyre får vi



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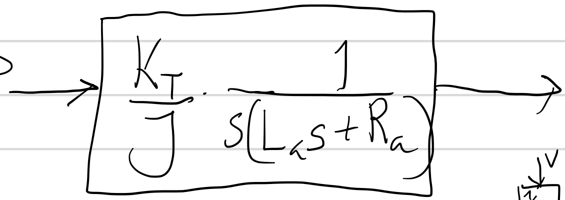

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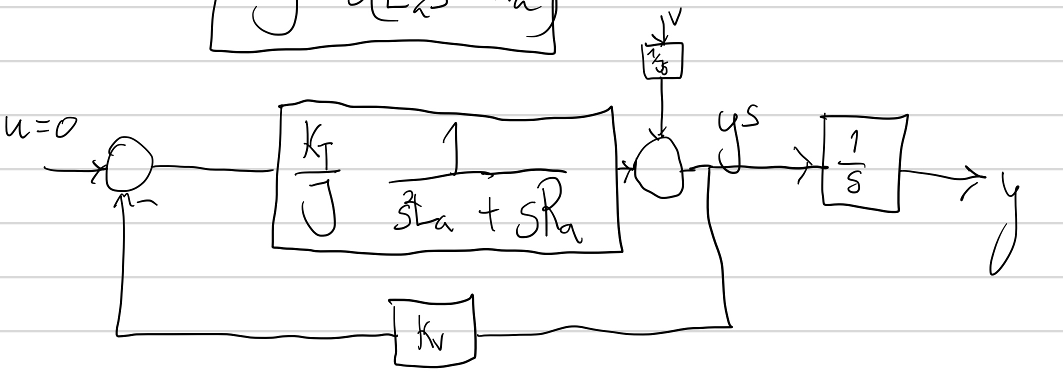

Har da at:

$$ys = v \cdot \frac{1}{Js} - ysK_v \frac{K_T}{J} \frac{1}{s^2L_a + sR_a}$$

$$\Rightarrow \quad y\left(1 + \frac{K_vK_T/J}{s^2L_a + sR_a}\right) = v\,\frac{1}{Js^2}$$

$$\Leftrightarrow \quad \frac{y}{v} = h_v = \frac{1}{Js^2} \cdot \frac{s^2L_a + sR_a}{s^2L_a + sR_a + \frac{K_vK_T}{J}}$$

$$h_v(s) = \frac{1}{Js^{\cancel{2}}} \cdot \frac{s^{\cancel{2}} L_a + \cancel{s} R_a}{s^2 L_a + s R_a + \frac{K_v K_T}{J}}$$

$$= \frac{\frac{sL_a + R_a}{K_v K_T}}{s\left(s^2 \frac{JL_a}{K_v K_T} + s\frac{JR_a}{K_v K_T} + 1\right)}$$

Oppgave 3

$$h(s) = a_2 s^2 + a_1 s + a_0$$

Faktoriserer polynomet til

$$a_2(s-\lambda_1)(s-\lambda_2) = a_2\left(s^2 - (\lambda_1+\lambda_2)s + \lambda_1\lambda_2\right)$$
$$= a_2\left(s^2 + (-\lambda_1-\lambda_2)s + \lambda_1\lambda_2\right)$$

Vi vet også at

$$\lambda = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_2a_0}}{2a_2}$$

Case 1: $\lambda_1 = \overline{\lambda_2} = \alpha + j\beta$

Må vise at $\alpha < 0$. Siden $\alpha = -\frac{a_1}{2a_2}$ og a_1 og a_2 har samme fortegn vi

$$\alpha < 0 \quad \checkmark$$

Case 2: λ_1 og λ_2 reelle. Må da vise at de er negative.

La

$$\lambda_1 = \frac{-a_1 - \sqrt{a_1^2 - 4a_2a_0}}{2a_2}$$

$$\lambda_2 = \frac{-a_1 + \sqrt{a_1^2 - 4a_2a_0}}{2a_2}$$

Siden $\lambda_2 > \lambda_1$ må vi vise at $\lambda_2 < 0$.

$$\lambda_2 = \frac{-a_1 + \sqrt{a_1^2 - 4a_2a_0}}{2a_2}$$

$$\lambda_2 \in \mathbb{R} \Rightarrow a_1^2 - 4a_2a_0 > 0$$

$$\Rightarrow \lambda_2 < \frac{-a_1}{2a_2} < 0$$ dersom a_1 og a_2 har samme fortegn.

Case 2: λ_1 og λ_2 reelle. Må da vise at de er negative.

Polynomet kan skrives som

$$a_2 s^2 + a_1 s + a_0 = a_2(s-\lambda_1)(s-\lambda_2)$$

$$\Rightarrow \quad = a_2 s^2 + a_2(-\lambda_1-\lambda_2)s + a_2\lambda_1\lambda_2$$

$$\Rightarrow a_1 = -a_2(\lambda_1+\lambda_2) \quad , \quad a_0 = a_2\lambda_1\lambda_2$$

Dersom $a_2 > 0$, $a_1 > 0$, $a_0 > 0$ vil

$$\lambda_1 + \lambda_2 < 0 \quad \text{og} \quad \lambda_1\lambda > 0$$

$\lambda_1\lambda_2 > 0 \Rightarrow \lambda_1$ og λ_2 har samme fortegn.
$\lambda_1 + \lambda_2 < 0$ gir da at $\lambda_1 < 0$ og $\lambda_2 < 0$
Så systemet er asymptotisk stabilt.

Dersom $a_2 < 0$, $a_1 < 0$, $a_0 < 0$ så kan vi skrive

$$a_2 s^2 + a_1 s + a_0 = -(a_2' s^2 + a_1' s + a_0') = -h(s).$$

hvor $a_i' = -a_i$.

$h(s)$ er as. stabilt $\Leftrightarrow$ $-h(s)$ er as. stabilt
Så systemet er asymptotisk stabilt.

Oppgave 4

Et system blir ustabilt dersom Nyquist-kurven inneholder -1.

Kurven i høyre halvplan svarer til $h_1(s) = \frac{1}{s+a}$.

Når K_p øker vil kurven gå mer ut i positiv retning, men ikke andre vei.
Så $K_p h_1(s)$ er stabilt for alle $K_p > 0$.

For $h_2(s)$ har vi at kurven krysser x-aksen ($|\omega| \to \infty$) ved $x_0 = \frac{K_p}{-a}$. Siden vi krever $x_0 > -1$ har vi

$$\frac{K_p}{-a} > -1 \iff K_p < a$$