Ønsker tilbakemelding :)

Matte 3, Øving 1

1,1,15

a) $-3\left(\frac{i}{2}\right) = \underline{-\frac{3}{2}i} \quad (a=0)$

b) $(8+i)-(5+i) = 8-5+i-i = \underline{3} \quad (b=0)$

c) $\frac{2}{i} = \frac{2}{i}\cdot\frac{i}{i} = \frac{2i}{-1} = -2i$

1,1,9

$$\frac{2+3i}{1+2i} - \frac{8+i}{6-i}$$

$$= \frac{(2+3i)(1-2i)}{(1+2i)(1-2i)} - \frac{(8+i)(6+i)}{(6-i)(6+i)}$$

$$= \frac{2-4i+3i-6i^2}{1^2+2^2} - \frac{(48+8i+6i+i^2)}{6^2+1^2}$$

$$= \frac{2+6-i}{5} - \frac{48-1+14i}{37}$$

$$= \frac{8}{5} - \frac{1}{5}i - \frac{47}{37} - \frac{14}{37}i$$

$$= \frac{61}{185} - \frac{107}{185}i$$

$$\underline{\approx 0{,}33 - 0{,}58i}$$

1.1.12 $(2+i)(-1-i)(3-2i)$

$= -(2+i)(1+i)(3-2i)$

$= -(2+3i+i^2)(3-2i)$

$= -(2-1+3i)(3-2i)$

$= -(1+3i)(3-2i)$

$= -(3-2i+9i-6i^2)$

$= -3-7i-6$

$\underline{= -9-7i}$

1.1.12 $i^{4k+t} = i^{4k} \cdot i^t$

$= (i^4)^k \cdot i^t$

$= ((i^2)^2)^k \cdot i^t$

$= ((-1)^2)^k \cdot i^t$

$= 1^k \cdot i^t$

$= i^t$

So $i^{4k+t} = i^t$.

$t=0: \quad i^{4k} = i^{0} = 1$

$t=1: \quad i^{4k+1} = i^{1} = i$

$t=2: \quad i^{4k+2} = i^{2} = -1$

$t=3: \quad i^{4k+3} = i^{3} = i^{2} \cdot i = -i$ $\quad \blacksquare$

1.1.17 $\quad i^{11} = i^{3} = -i$

$i^{-20} = 1$

$i^{-1} = i^{3} = -i$

$\therefore \quad 3i^{11} + 6i^{3} + \frac{8}{i^{20}} + i^{-1} = -3i - 6i + 8 - i$

$= \underline{8 - 10i}$

1.1.20 a) $iz = 4 - zi$

$\Leftrightarrow \quad 2zi = 4$

$\Leftrightarrow \quad z = \frac{4}{2i} = \frac{2}{i} = \underline{-2i}$

b) $z^{2} + 16 = 0$

$\Leftrightarrow \quad z^{2} = -16$

$\Leftrightarrow \quad z = \sqrt{-16}$

$\Leftrightarrow \quad z = \sqrt{16} \cdot \sqrt{-1}$

$\Leftrightarrow \quad \underline{z = 4i}$

1.2.3 The one with greatest modulus is farthest away.

$$|i| = 1$$

$$|2-i| = \sqrt{2^2+(-1)^2} = \sqrt{5} \approx 2{,}23.$$

$$|-3| = 3$$

$$|-3| > |2-i| > |i|$$

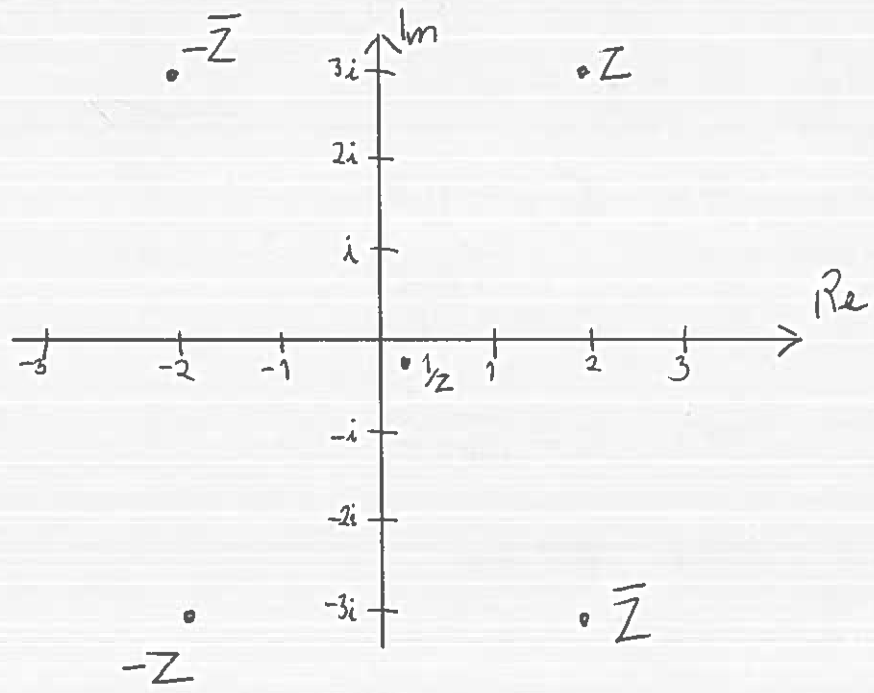
$\Rightarrow$ -3 is farthest away.

1.2.4

$$z = 2+3i \qquad \bar{z} = 2-3i$$

$$-z = -2-3i \qquad -\bar{z} = -2+3i$$

$$\frac{1}{z} = \frac{1 \cdot (2-3i)}{(2+3i)(2-3i)} = \frac{2-3i}{2^2+3^2} = \frac{2}{13} - \frac{3}{13}i$$



1.2.7 a) $\operatorname{Im} Z = -2$

A straight line parallell to the real-axis with an imaginary component $-2i$

b) $|Z-1+i| = 3 \iff |Z-(1-i)| = 3$

A circle around the complex number $1-i$ with radius 3.

c) $|2Z - i| = 4 \iff |Z - \frac{i}{2}| = 2$

A circle centered on $\frac{i}{2}$ with radius 2

d) $|Z| = 3|Z-1|$ (∗)

$Z = a+bi$

$|Z| = |a+bi|$

and $|Z-1| = |(a-1)+bi|$

$(*) \iff \sqrt{a^2+b^2} = 3\sqrt{(a-1)^2+b^2}$

$a^2+b^2 = 9(a-1)^2+9b^2$

$a^2+b^2 = 9a^2-18a-9+9b^2$

$-8a^2+18a-8b^2 = 9 \quad |\div(-8)$

$a^2 - \frac{9}{4}a + \left(\frac{-8}{-8}\right)b^2 = -\frac{9}{8}$

$$\underbrace{a^2 - \frac{9}{4}a + \left(\frac{9}{8}\right)^2} - \left(\frac{9}{8}\right)^2 - b^2 = -\frac{9}{8}$$

$$\left(a - \frac{9}{8}\right)^2 + b^2 = -\frac{9}{8} + \left(\frac{9}{8}\right)^2$$

$$\left(a - \frac{9}{8}\right)^2 + (b - 0)^2 = \frac{9}{64} = \left(\frac{3}{8}\right)^2$$

A circle centered on $\frac{9}{8}$ (no imaginary part) with radius $\frac{3}{8}$

j) $|z| > 6$

All points (numbers) outside the circle with radius 6 and centered on origo.