	Oving 8 Onsker tiltakemeloling:)
2.1	1) $A = \begin{pmatrix} 2 & 0 & +1 \\ 4 & -5 & 2 \end{pmatrix}, B = \begin{pmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{pmatrix}$
	$C = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$, $D = \begin{pmatrix} 3 & 5 \\ -1 & 4 \end{pmatrix}$
	$a) - 2A = (-4 \ 0 \ 2)$ $(-8 \ 10 \ -4)$
	b) $3-2A = \begin{pmatrix} 7-4 & -5+0 & 1+2 \\ 1-8 & -4+10 & -3-4 \end{pmatrix}$
	= (3 -5 3) (-7 6 -7)
	c) A·C cloesn't make sense because we need # columns of A to be equal to #rows of C.
	$\begin{array}{c} d) \ \ CD = \begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 5 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} -1 & 4 \end{pmatrix} \end{array}$
	$= \left(\begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 1 & 2 \end{pmatrix} \begin{pmatrix} 5 & 1 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 4 & 1 \\ 4 & 1 \end{pmatrix} \right)$
	$= \begin{pmatrix} 1 & 9 \\ -7 & -6 \end{pmatrix}$

10)
$$A = \begin{pmatrix} 2 & -3 \\ -4 & 6 \end{pmatrix}$$
, $B = \begin{pmatrix} 8 & 4 \\ 5 & 5 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -2 \\ 3 & 1 \end{pmatrix}$

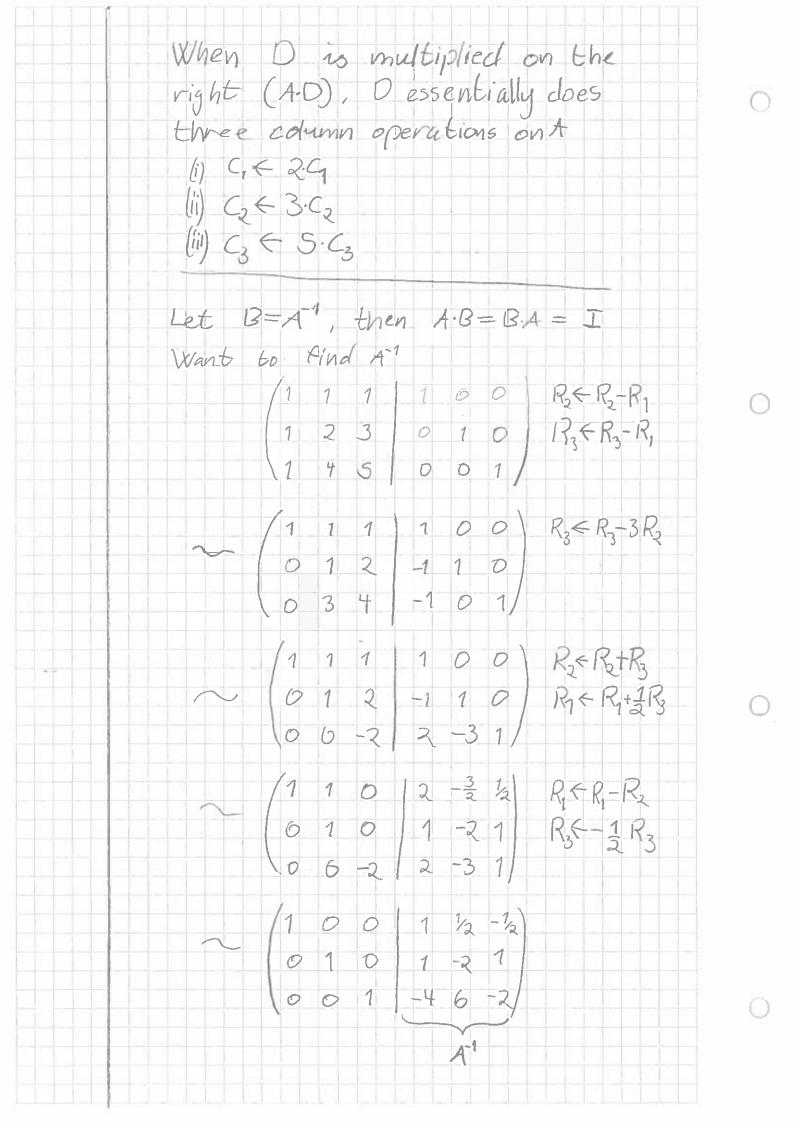
AB: $\begin{pmatrix} 3 & 5 \\ 5 & 5 \end{pmatrix}$.

 $\begin{pmatrix} 2 & -3 \\ 28-35 & 24-35 \\ 4 & 6 \end{pmatrix} + 48+65 - 44+65$
 $\Rightarrow AB = \begin{pmatrix} 1 & -7 \\ -2 & 14 \end{pmatrix}$

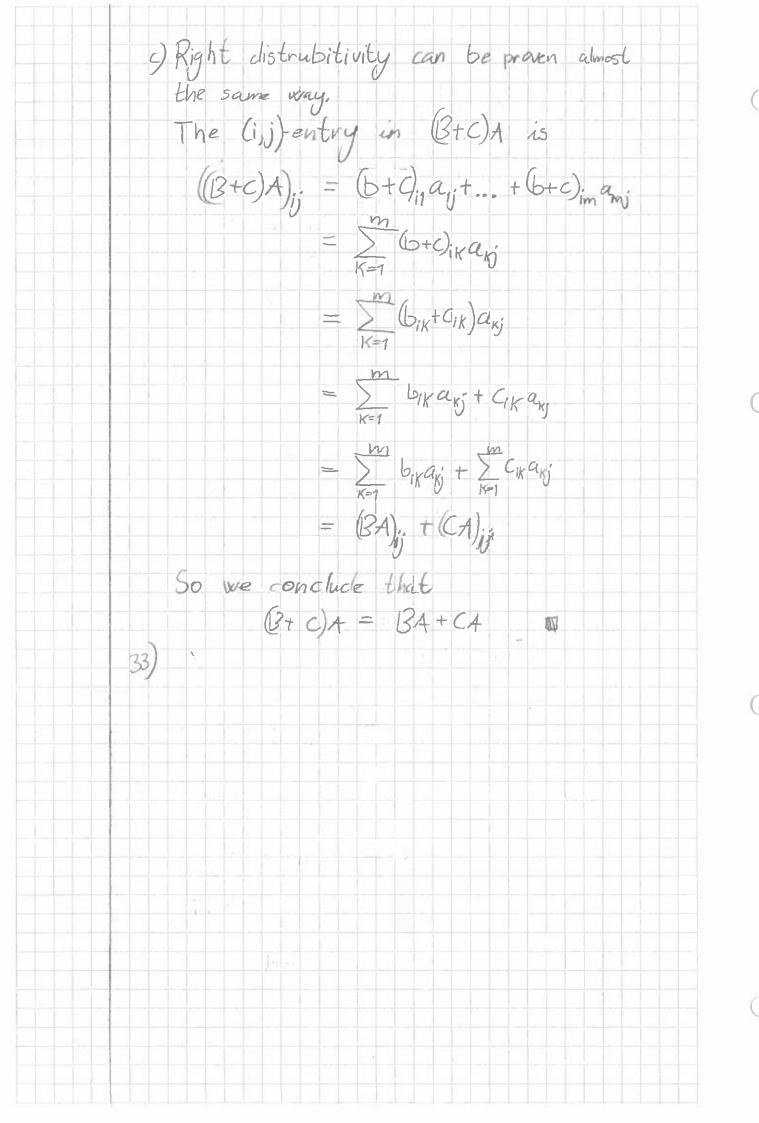
AC: $\begin{pmatrix} 5 & -2 \\ 3 & 1 \end{pmatrix}$
 $\begin{pmatrix} 2 & -3 \\ 25-33 & -22-31 \\ -4 & 6 \end{pmatrix} + 45+63 + 42+61$
 $\Rightarrow AC = \begin{pmatrix} 1 & -7 \\ -2 & 14 \end{pmatrix}$

So $AB = AC = \begin{pmatrix} 1 & -7 \\ -2 & 14 \end{pmatrix}$

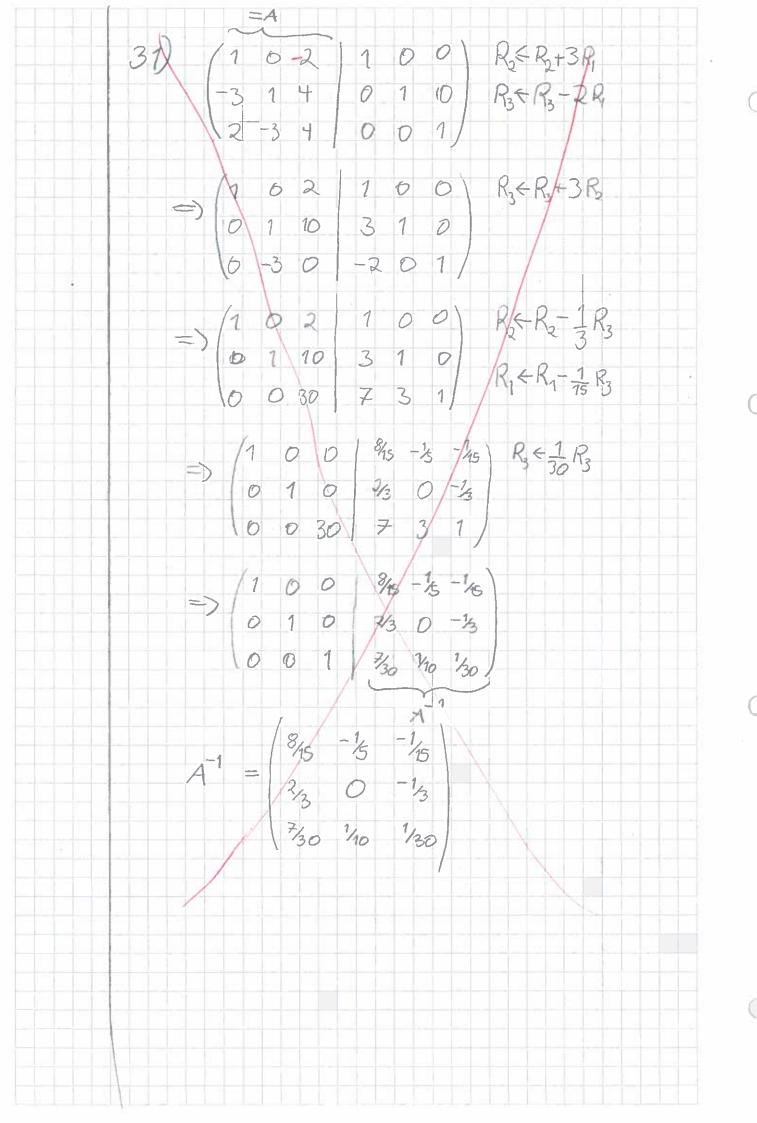
2 3 | , D= | 4 5 | 3 1 1 2 3 2 6 12 25 4D = /2 3 5 6 15 12 25/ /1 1 1 2 1 4 DA: 001 0 30 20 => DA = 5 20 25/ When D is multiplied with A on the left (D-A), this amounts to doing Inree row operations on A (i) R2 = 3.R2 (ii) B ← 5·R3

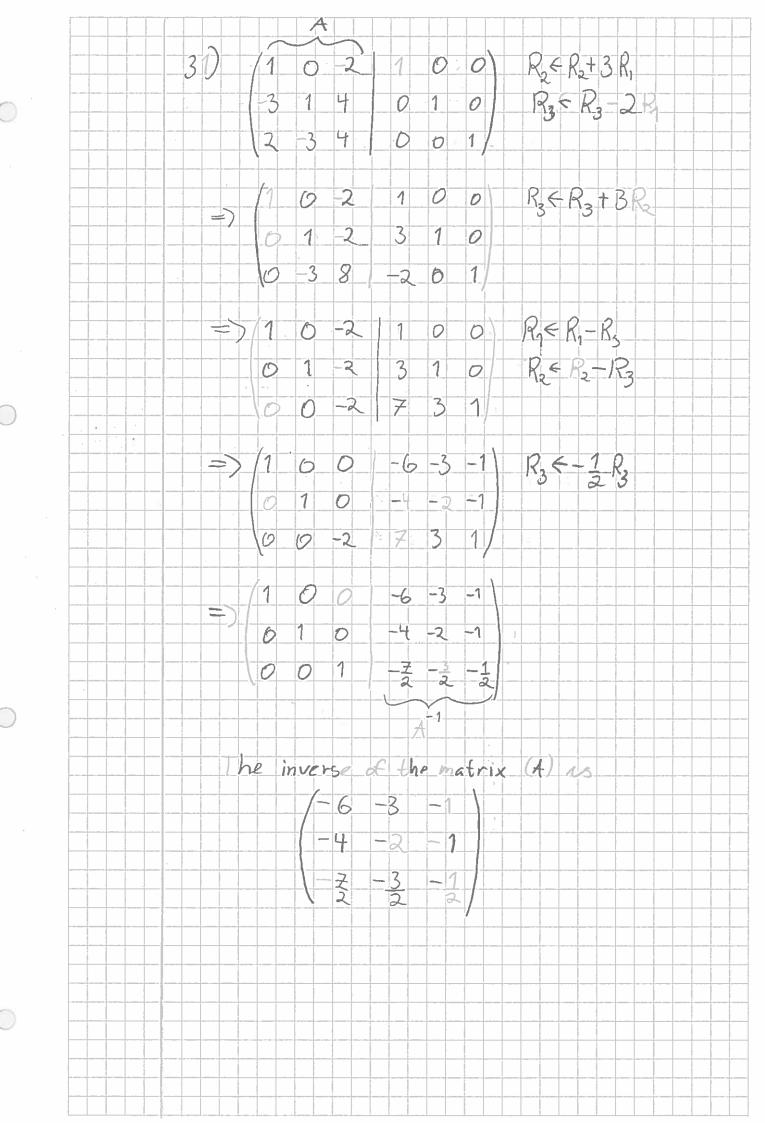


So if $B = A^{-1} = \begin{pmatrix} 1 & 2 & -2 \\ 1 & -2 & 1 \end{pmatrix}$ then AB = BA 29) Let A be an mxn matrix and assume 13 and Care such that AB, AC and A(B+C) are defined. We want to show that (b) A(B+C) = ABTAC and (C) (BtC)A-BACCA. b) Consider the (ij)-entry in AB. It is by definition equal to $(AB)_{ij} = \sum_{k=1}^{n} a_{ik} \cdot b_{kj}$ The (i,j)-entry in AB tAC is then $(AB+AC)_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj} + \sum_{k=1}^{n} a_{ik}c_{kj}$ $= \sum_{K=1}^{n} (a_{ik} b_{kj} + a_{ik} c_{kj})$ $= \sum_{k=1}^{n} a_{ik}(b_{ki} + C_{kj})$ But this is exactly equal to the (i,j)-cutry of A(BtC) SO A(BtC) must be equal to AB+AC.



 $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ The equation Ax = 3 can be writtenias $\begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} x_1 \end{pmatrix} = \begin{pmatrix} 0 \\ d \end{pmatrix}$ Which reveals a system of equations. $\lim_{x \to 0} ax + bx = 0$ (ii) $Cx_1 + dx_2 = 0$ Solving for x and & gives (i) $x_1 = -b x_2$ (ii) (C=a) x2 + dx = 0 (=) adx2 = bcx2 (=) ad = bc (=) ac-bc=0If ad-bc is zero, then there will a ways be some xix such that AX = 3 (non-trivial), and if ad-bc is not zero, then the system has no solutions except the trivial one, \$=8. For square matrices, having a determinant equal to zero is equivalent to not being invertible.





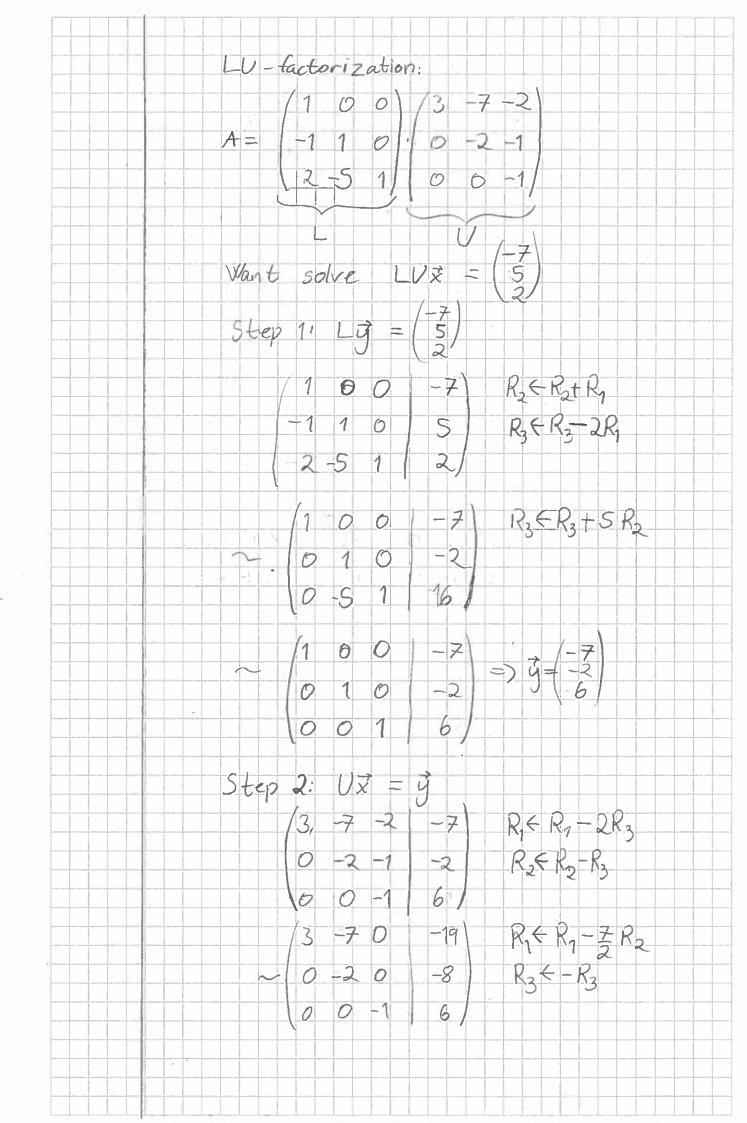
Suppose AX=B has a solution for all BER, (A is mxn). Then A has n pivot positions, so one in every row. This means that reduced echelon form of A is In, the num identity matrix. Because A can be reduced to In, A must be invertible, because the operations that transform A to In, also transform In to 1. (theorem 7) 2.3 2) det(-46) = 4.9-66 = 0 It is not invertible A = /-1 -3 0 1 \ Want to make A $\begin{pmatrix} 3 & 5 & 8 & -3 \end{pmatrix}$ triangular. $\begin{pmatrix} -2 & -6 & 3 & 2 \end{pmatrix}$ $R_2 \leftarrow R_2 + 3R_1$ $\begin{pmatrix} 0 & -1 & 2 & 1 \end{pmatrix}$ $R_3 \leftarrow R_3 - 2R_1$ 30 -1-301) clet A = (-1).(-4).(3).(1) 0 - 486 = 12 $0 0 36 | det A \neq 0 50$ A is invertible

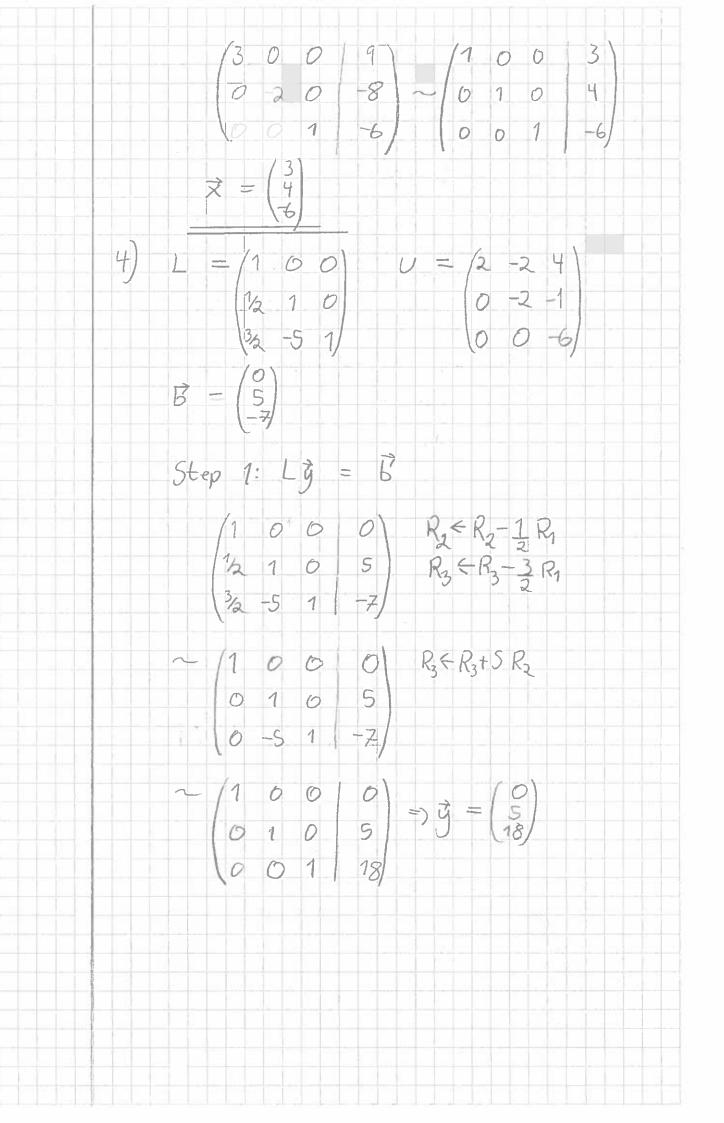
a. True b. True C. False a. True e. True 13) A square triangular matrix is invertible when the cleterminant is non-zero, The determinant is the product of the main diagonal because the matrix is triangular. If no entries in the diagonal are zero, the the product will also be non-zero. This is of course the same as having n pivot positions, so a square triangular matrix is invertible if all entrics in the main diagonal are non-zero.

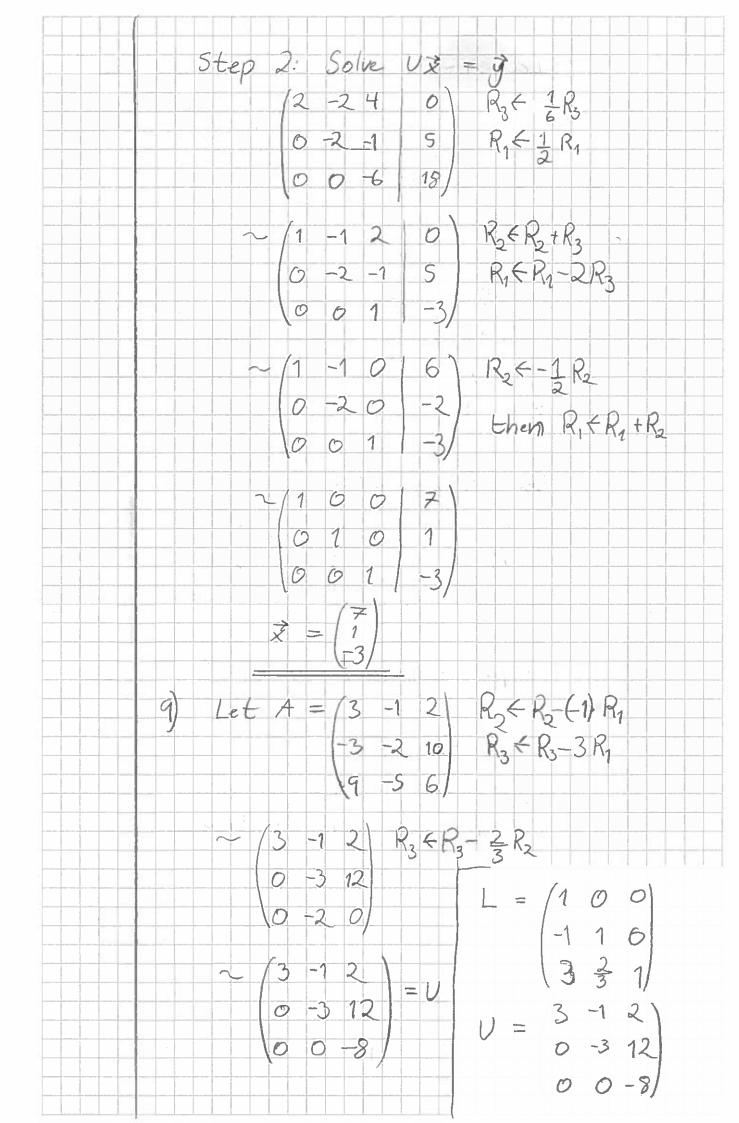
Row reduction: 2.5
 (3 -7 -2 | -7)

 -3 5 1 5

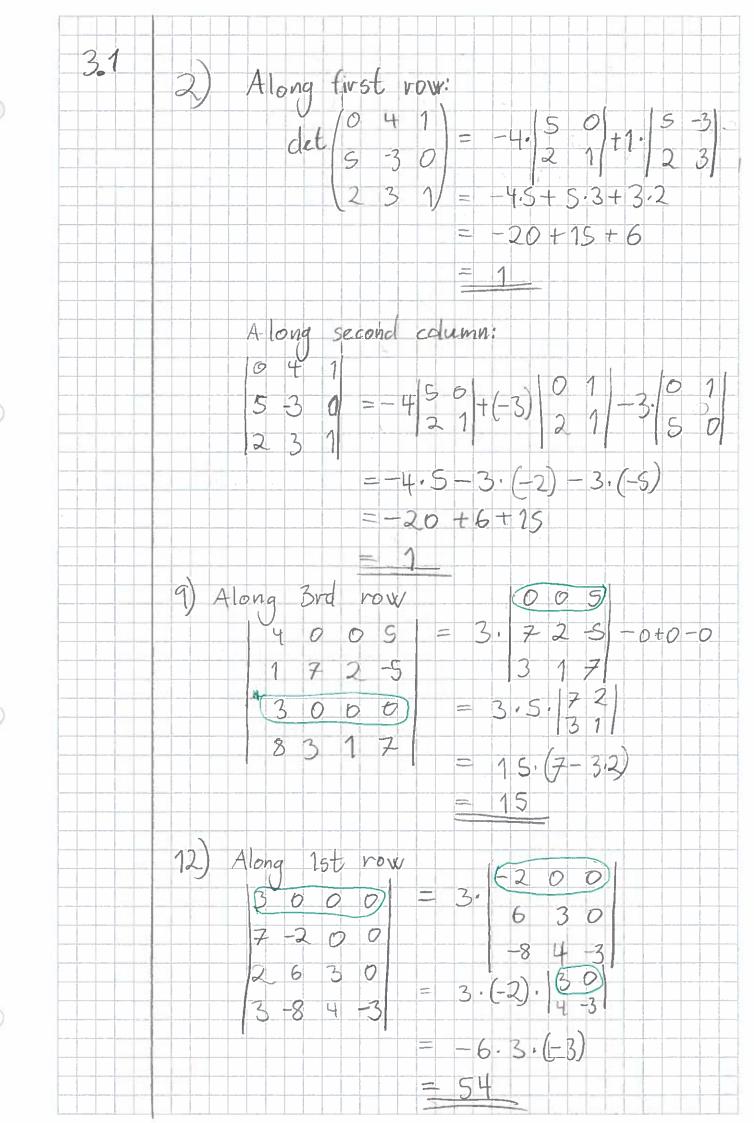
 (6 -4 0 2)
 RS CR2+R3 R3+R3-2R1 $\begin{pmatrix} 3 & -7 & -2 & 1 & -7 \end{pmatrix}$ $R_3 \leftarrow R_3 + 5 R_3$ $\begin{pmatrix} 0 & -2 & -1 & 1 & -2 \\ 0 & 10 & 4 & 16 \end{pmatrix}$ $\begin{pmatrix} 3 & -7 & -2 & | & -7 & | & R_1 \leftarrow R_1 - 2R_3 \\ 0 & -2 & -1 & | & -2 & | & R_2 \leftarrow R_2 - R_3 \\ 0 & 0 & -1 & | & 6 \end{pmatrix}$ $\begin{pmatrix} 1 & -\frac{7}{3} & 0 & -\frac{19}{3} \end{pmatrix}$ $R_1 \leftarrow R_1 + \frac{7}{3}R_2$ 0 1 0 4 0 6 1 -6 0 0 3 0 1 -6 \Rightarrow $\chi = \begin{pmatrix} 3 \\ 4 \\ -6 \end{pmatrix}$







24) Let A = QR, where R is square and upper triangular, and Q is such that $Q \cdot Q^T = I$ (orthogonal) The equation AZ=B can then be unitten as (QR)Z = B I.QT (=) QQ(Rx) = QTB'(=) $R\vec{x} = \vec{O}\vec{B}$ We can view QTB as a linear trus form from R" to R" and thus write Z=QB, CER. This gives RX = Z. By the definition of R, Ris invertible so it must have a pivot in every row. Since R then has n pivots the columns of R must span all of R", and thus the system also has a unique solution, x, for all Cin R". The system RX = QB is a triangular system that can be solved via, for instance backward substitution.



37)
$$A = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$$
, $\begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix} = 3.2 - 4.1 \le 2$
 $5A = \begin{pmatrix} 5.3 & 5.1 \\ 5.4 & 5.2 \end{pmatrix} = \begin{pmatrix} 15 & 5 \\ 20 & 10 \end{pmatrix}$
 $clet(5A) = \begin{vmatrix} 15 \\ 20 & p \end{vmatrix} = 15.10 - 20.5 = 15.0$
 $clearly det5A \neq 5detA$.

In this case det5A = 5^2 detA

38) Let $A = \begin{pmatrix} a & b \\ ka & kd \end{pmatrix}$
 $a constant soaan:$
 $kA = \begin{pmatrix} a & kb \\ ka & kd \end{pmatrix}$
 $detA = ad = bc$
 $det(A = Vakd - Vakb = k^2(ad - cd))$
 $= k^2 detA$
 $so for 2x2 matrices det(A = k^2detA)$

10) Let A= (1 3 -1 0 -2 0 2 -4 -2 -6 -2 -6 2 3 10 1 5 -6 2 -3 0 2 -4 5 9/ Want reduce to echelon form R3 + R3 + 2R1 , R4 + R4-R1 (1 3 -1 0 2) $R_3 \leftrightarrow R_3$ 0 2 -4 +2 -6 R₃ \leftrightarrow R₃
0 0 0 3 -6 then: $R_3 \leftarrow R_3 - R_5$ 0 0 2 -5 2 -1 $R_4 \leftarrow R_4 - R_5$ 2 -4 5 9 (1 3 -1 0 2 $R_3 \leftrightarrow R_4$ 0 2 -4 -2 -6 then: $R_5 \leftarrow R_5 - \frac{1}{3}R_4$ 0 0 9 15 0 -1 4 5 0 0 3 6 3 -1 0 2 Nedid 2 row A = | 0 | 2 | -4 | -2 | -6 | interchanges so A = | 0 | 0 | -1 | 4 | 5 | det $A_E = -(-) \det A$ 0-145 det A= -(-)det A 00919 = clet A 0 0 0 1/ det A= 1.2.ED.9.1 =-18 The determinant is -18

Let A and B be square matrices. Then detAB = detA.detB det 4 and det B are just some numbers, so we can use commutativity to see that det A det B = det B det A But det BdetA is just de BA, so we can conclude det AB = det BA for all square matrices.

