Øving 4, Fysikk

Rendell Cale, gruppe 2

Godkjent
SB

Ønsker tilbakemelding :)

Oppgave 1

a) Når kulen slippes har den potensiell energi $E_p = Mgl$ og like før støtet vil all den potensielle energien ha blitt gjort om til kinetisk energi $E_K = \frac{1}{2}MV^2$

Det gir

$$\frac{1}{2}MV^2 = Mgl$$

$$\Leftrightarrow \quad \underline{\underline{V = \sqrt{2gl}}}$$

R

S = ?

b) Bruker bevaring av bevegelsesmengde:

$$MV + m\underbrace{v}_{=0} = MV' + mv' \qquad (1)$$

Siden det er et elastisk støt har vi bevaring av kinetisk energi:

$$\frac{1}{2}MV^2 + \underbrace{\frac{1}{2}mv^2}_{=0} = \frac{1}{2}MV'^2 + \frac{1}{2}mv'^2 \qquad (2)$$

$$(2) \Leftrightarrow MV'^2 = MV^2 - mv'^2$$

$$\Leftrightarrow V'^2 = V^2 - \frac{m}{M}v'^2 = 2gl - \frac{m}{M}v'^2$$

$$(1) \iff MV' = MV - mv'$$

$$\iff V' = V - \frac{m}{M} v'$$

$$(2) \Rightarrow \left(V - \frac{m}{M} v'\right)^2 = 2gl - \frac{m}{M} v'^2$$

$$\iff V^2 - 2\frac{m}{M} V v' + \left(\frac{m}{M} v'\right)^2 = 2gl - \frac{m}{M} v'^2$$

$$\iff \cancel{2gl} - 2\frac{m}{M}\sqrt{2gl}\, v' + \frac{m^2}{M^2} v'^2 = \cancel{2gl} - \frac{m}{M} v'^2$$

$$\iff \left(-2\cancel{\frac{m}{M}}\sqrt{2gl} + \frac{m^{\cancel{2}}}{M^{\cancel{2}}} v' + \cancel{\frac{m}{M}} v'\right)\cancel{v'} = 0$$

$$\iff \frac{m}{M} v' + v' = 2\sqrt{2gl}$$

$$\iff \frac{m+M}{M} v' = 2\sqrt{2gl}$$

$$\iff v' = \frac{M}{m+M} 2\sqrt{2gl}$$ R

$$(1) \Rightarrow V' = V - \frac{m}{M}\frac{M}{m+M} 2\sqrt{2gl}$$

$$V' = \sqrt{2gl}\left(\frac{M-m}{M+m}\right)$$ R

$M \ll m$:

$$V' \approx \sqrt{2gl}\left(\frac{-m}{m}\right)$$

$$V' \approx -\sqrt{2gl}$$ R

$$v' \approx \frac{M}{m} 2\sqrt{2gl} \approx 0$$ R

$M \gg m$:

$$V' \approx \sqrt{2gl}\left(\frac{M}{M}\right)$$

$$\approx \sqrt{2gl}$$ R

$$v' \approx \frac{\cancel{M}}{\cancel{M}} 2\sqrt{2gl}$$

$$= 2\sqrt{2gl}$$ R

Resultatene i grensetilfellene stemmer godt med forventningene.

c) For den "lille" massen har vi:

$$\sum F_m = m\frac{(v')^2}{\ell}$$

$$\Leftrightarrow \quad S' - mg = m\frac{(v')^2}{\ell}$$

$$\Leftrightarrow \quad S' = m\left[\frac{\left(\frac{M}{m+M}\,2\sqrt{2g\ell}\right)^2}{\ell} + g\right]$$

$$= \frac{m}{\ell}\left(\left(\frac{2M}{m+M}\right)^2 2g\ell + g\ell\right)$$

$$\Leftrightarrow \quad \underline{\underline{S' = mg\left(2\left(\frac{2M}{M+m}\right)^2+1\right)}}$$

For den "store" massen har vi:

$$\sum F_M = M\frac{V^2}{\ell}$$

$$S' - Mg = M\frac{\left(\sqrt{2g\ell}\left(1-\frac{2m}{M+m}\right)\right)^2}{\ell}$$

$$\Leftrightarrow \quad S' = Mg\left(2\left(1-\frac{2m}{M+m}\right)^2+1\right)$$

$$\Leftrightarrow \quad \underline{\underline{S' = Mg\left(2\left(\frac{M-m}{M+m}\right)^2+1\right)}}$$

d) $M = 10{,}0\,g = 0{,}0100\,Kg$
$m = 20{,}0\,g = 0{,}0200\,Kg$
$g = 9{,}81\,m/s^2$
$\ell = 1{,}00\,m$

- $V = \sqrt{2 \cdot 9{,}81\,m/s^2 \cdot 1{,}00\,m}$
$= 4{,}43\,m/s$ R

- $V' = \underbrace{\sqrt{2g\ell}}_{V=4{,}43\,m/s}\left(\frac{10{,}0\,g - 20{,}0\,g}{10{,}0\,g + 20{,}0\,g}\right)$
$= -1{,}48\,m/s$ R

- $v' = \frac{10{,}0\,g}{20{,}0\,g + 10{,}0\,g} \cdot 2 \cdot \sqrt{2g\ell}$
$= 2{,}95\,m/s$ R

- $S = M\frac{V^2}{\ell} + Mg = M(2g + g) = 3Mg$
$= 3 \cdot 0{,}0100\,Kg \cdot 9{,}81\,m/s^2$
$= 0{,}29\,N$ R

- $S' = 0{,}0100\,Kg \cdot 9{,}81\,m/s^2 \cdot \left(2\left(\frac{10-20}{10+20}\right)^2 + 1\right)$ R $= 0{,}120\,N$
$= 0{,}24\,N$ G

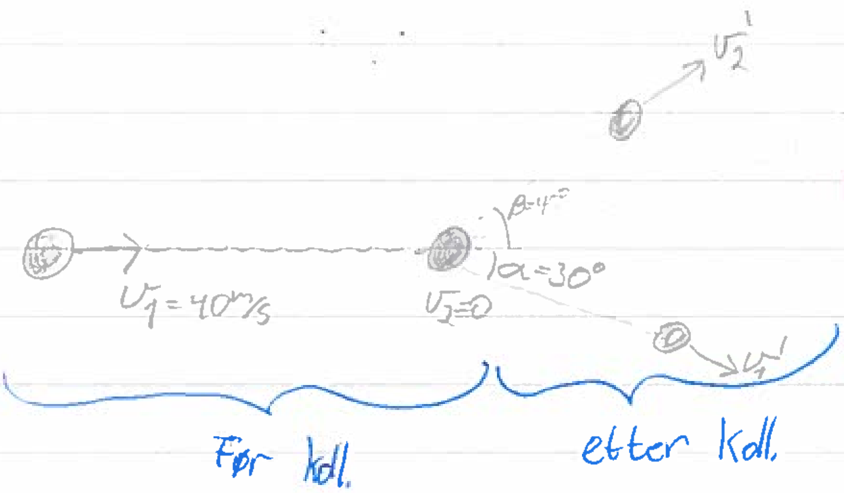
$$S' = 0{,}020\,\text{kg} \cdot 9{,}81\,\text{m/s}^2 \cdot \left(2\left(\frac{2 \cdot 10}{10+20}\right)^2 + 1\right)$$

$$= 0{,}37\ \text{N}$$

R

Oppgave 2

a)



R

b) m: massen til pucken(e)

~~Antar elastisk støt og har da~~

$$\cancel{E_{k,\text{før}} = E_{k,\text{etter}}}$$

$$\cancel{\frac{1}{2} m v_1^2 = \frac{1}{2} m v_1'^2 + \frac{1}{2} m v_2'^2}$$

$$\cancel{\Leftrightarrow \quad v_1^2 = v_1'^2 + v_2'^2}$$

Bevegelsesmengden vil være bevart i x- og y-retning:

(x) $mv_1 = mv_1' \cdot \cos\beta + mv_2' \cdot \cos\alpha$ Husk å dekomponere her!

$\Leftrightarrow \quad v_1 = v_1' + v_2'$

(y) $0 = mv_2' \sin\beta - mv_1' \sin\alpha$

$\Leftrightarrow \quad v_1' = v_2' \dfrac{\sin\beta}{\sin\alpha}$

$\overset{(x)}{\Rightarrow} \quad v_1 = v_2'\left(1 + \dfrac{\sin\beta}{\sin\alpha}\right)$

$\Leftrightarrow \quad v_2' = v_1 \dfrac{\sin\alpha}{\sin\alpha + \sin\beta}$

$= 40(\sqrt{2} - 1)\ \text{m/s}$

$\approx 16{,}57\ \text{m/s}$ G

$\overset{(y)}{\Rightarrow} \quad v_1' \overset{(x)}{=} v_1 - v_2'$

$= 40{,}0 - 16{,}57\ \text{m/s}$

$\approx 23{,}43\ \text{m/s}$ G

Farten til den ene er 23,43 m/s og den andre har fart 16,57 m/s.

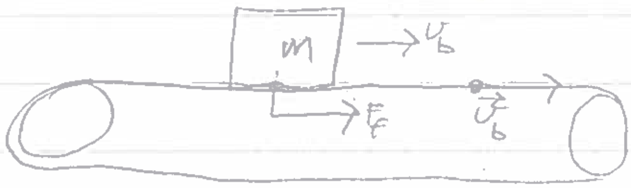
c) $\frac{E_{k,etter}}{E_{k,før}} = \frac{v_1'^2 + v_2'^2}{v_1^2}$

$= \frac{(23{,}43\ m/s)^2 + (16{,}57\ m/s)^2}{(40\ m/s)^2}$

$= 0{,}51$ Følgefeil

49% av den kinetiske energien går tapt.

Oppgave 3



a) $F_f = \mu_k mg$

$W_f = W_{tot} = \Delta E_k = \frac{1}{2} m v_b^2 - 0$

$= \frac{1}{2} m v_b^2$ R

Friksjonskrafta gjør et arbeid $W_f = \frac{1}{2} m v_b^2$ på kartongen.

b) $W_f = \int_0^{x_K} F_f \, ds$ (Kartongen lander i origo)

$= \int_0^{x_K} \mu_K mg \, ds$

$= \mu_K mg \, x_K$

Fra (a) har vi $W_f = \frac{1}{2} m v_b^2$ så

$\mu_K \cancel{m} g \, x_K = \frac{1}{2} \cancel{m} v_b^2$

$\Leftrightarrow \quad \underline{\underline{x_K = \frac{1}{2g\mu_K} v_b^2}}$

~~c)~~ $\cancel{v_b^2 - \underbrace{v_0^2}_{=0} = 2a x_K}$

c) $a = \frac{v_b - v_0}{t} \Leftrightarrow t = \frac{v_b}{a}$, $v_0 = 0$ (*)

$F_f = ma \Leftrightarrow a = \frac{F_f}{m} = \mu_K g$

$\overset{(*)}{\Rightarrow} \quad \underline{\underline{t = \frac{v_b}{\mu_K g}}}$

Det tar $t = \frac{v_b}{\mu_K g}$ tid for kartongen å få fart $\vec{v}_b$.

d) Båndet har beveget seg en avstand

$$s = v_b \cdot t$$

$$= v_b \cdot \frac{v_b}{\mu_K g} = 2 \cdot \frac{v_b^2}{2\mu_K g}$$

$$\underline{\underline{= 2 \cdot x_K}}$$

e) Krafta F_f virker på båndet i en avstand $2x_K$
Arbeidet blir da

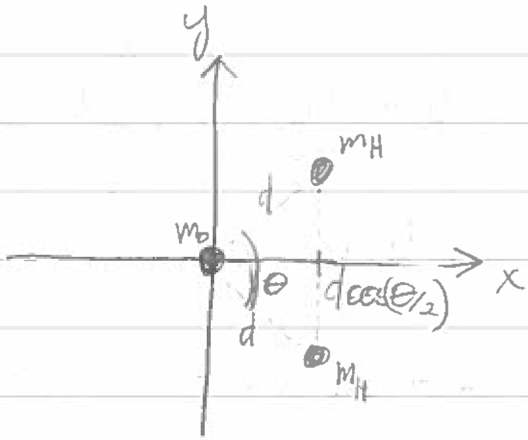
$$W_b = \int_0^{2x_K} F_f \, ds$$

$$= \mu_K m g \, 2x_K$$

$$= \cancel{2} \cdot m \cancel{g} \cdot \cancel{\mu_K} \cdot \frac{v_b^2}{\cancel{2}\cancel{\mu_K g}}$$

$$\underline{\underline{W_b = m v_b^2}}$$

Oppgave 4



, $\theta = 105^\circ$

Av symmetrigrunner vil massesenteret ligge ved $\bar{y} = 0$.

Regner da ut $\bar{x} = \frac{1}{M} \sum m_i r_i$

$$= \frac{\overset{0=}{m_O \cdot 0} + 2m_H \cdot \cos(\frac{\theta}{2}) d}{m_O + 2m_H}$$

$$= \frac{2m_H d \cos(\frac{\theta}{2})}{m_O + 2m_H}$$

$$= \frac{2 \cdot 1u \cdot d \cdot \cos(52{,}5)}{16u + 2 \cdot 1u}$$

$$= \frac{\cos(52{,}5)}{9} d$$

$$\approx \underline{\underline{0{,}0676 \cdot d}}$$

R

Oppgave 5

a) Raketten står i ro iftl. koordinatsystemet så $p_0 = 0$.
Bevaring av bevegelsesmengde gir da

$$0 = -u_{ex}\,dm + (m - dm)\,dv$$

$$\iff \quad u_{ex}\,dm = (m-dm)\,dv$$

$$= m\,dv - \underbrace{\cancel{dm\,dv}}_{\approx 0}$$

$$\iff \quad u_{ex}\,dm = m\,dv$$

R

b) $$dv = \frac{u_{ex}}{m}\,dm$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{u_{ex}}{m} \cdot \frac{dm}{dt}$$

Vi vet at $m(t) = m_0 - \beta t$
og det gir $\frac{dm}{dt} = \cancel{-}\beta$

R

se bort fra markeringene

Så $$\underset{a}{\frac{dv}{dt}} = \cancel{-}\frac{\beta\, u_{ex}}{m}$$

R

Vi har $\beta = 480\ kg/s$, $u_{ex} = -3{,}27\ km/s$

$$m(0) = m_0 = 2{,}55 \cdot 10^5\ kg$$

Som gir $a = \frac{-480\ kg/s \cdot (-3{,}27 \cdot 10^3\ m/s)}{2{,}55 \cdot 10^5\ kg}$

$$= 6{,}16\ m/s^2$$

c) $m(t) = m_0 - \beta t$

$$\Rightarrow \frac{dv}{dt} = -\frac{\beta u_{ex}}{m_0 - \beta t}$$

$$= \frac{\beta u_{ex}}{\beta t - m_0}$$

$$= \frac{u_{ex}}{t - m_0/\beta}$$

Vi ønsker $v(60) - v(0) = v(60)$

$$v(60) = \int_0^{60} \frac{dv}{dt}\, dt = \int_0^{60} \frac{u_{ex}}{t - m_0/\beta}\, dt$$

$x = t - m_0/\beta$, $dx = dt$

$$\Rightarrow v(60) = u_{ex} \int_{t=0}^{t=60} \frac{1}{x}\, dx = u_{ex} \ln|x| = u_{ex} \ln|t - m_0/\beta| \Big|_0^{60}$$

$$\Rightarrow v(60) = u_{ex}\left[\ln\left|60 - \frac{m_0}{\beta}\right| - \ln\left(\frac{m_0}{\beta}\right)\right]$$

$$= u_{ex} \ln\left|\frac{60}{m_0/\beta} - 1\right|$$

$$= u_{ex} \ln\left|\frac{60\beta}{m_0} - 1\right|$$

$$\approx 392 \ \mathrm{m/s}$$

<u>Raketten øker farten sin med 392 m/s</u>

R