Ønsker tilbakemelding :)

Øving 6

1.5

1) $A = \begin{pmatrix} 2 & -5 & 8 \\ -2 & -7 & 1 \\ 4 & 2 & 7 \end{pmatrix}$

Note that $R_1 = R_2 + R_3$

So when reduced there will be a free variable, so the system has non-trivial solutions.

3) $A = \begin{pmatrix} -2 & 5 & -7 \\ -6 & 7 & 1 \end{pmatrix}$

The system has non-trivial solution because there are 2 equations and 3 variables, so at least 1 variable will be free.

5) $A = \begin{pmatrix} 1 & 3 & 1 \\ -4 & -9 & 2 \\ 0 & -3 & -6 \end{pmatrix}$ $R_2 \leftarrow R_2 + 4R_1$

$\Leftrightarrow \begin{pmatrix} 1 & 3 & 1 \\ 0 & 3 & 6 \\ 0 & -3 & -6 \end{pmatrix}$ $R_3 \leftarrow R_3 + R_2$ then $R_2 \leftarrow \frac{1}{3} R_2$

$\Leftrightarrow \begin{pmatrix} 1 & 3 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \overset{R_1 \leftarrow R_1 - 3R_2}{\Longleftrightarrow} \begin{pmatrix} 1 & 0 & -5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$

$$A = \begin{pmatrix} 1 & 0 & -5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\left.\begin{array}{l} x_1 - 5x_3 = 0 \\ x_2 + 2x_3 = 0 \\ x_3 \text{ free} \end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} x_1 = 5x_3 \\ x_2 = 2x_3 \\ x_3 \text{ free} \end{array}\right.$$

As a vector:

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5x_3 \\ 2x_3 \\ x_3 \end{pmatrix} = \underline{\underline{x_3 \cdot \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}}}$$

7)

$$A = \begin{pmatrix} 1 & 3 & -3 & -7 \\ 0 & 1 & -4 & 5 \end{pmatrix} \quad R_1 \leftarrow R_1 - 3R_2$$

$$\Leftrightarrow \begin{pmatrix} 1 & 0 & 9 & -22 \\ 0 & 1 & -4 & 5 \end{pmatrix}$$

x_3 and x_4 are free

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -9x_3 + 22x_4 \\ 4x_3 - 5x_4 \\ x_3 \\ x_4 \end{pmatrix}$$

$$\underline{\underline{\vec{x} = x_3 \cdot \begin{pmatrix} -9 \\ 4 \\ 1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 22 \\ -5 \\ 0 \\ 1 \end{pmatrix}}}$$

10) $A = \begin{pmatrix} 1 & 3 & 0 & -4 \\ 2 & 6 & 0 & -8 \end{pmatrix}$

$\Leftrightarrow \begin{pmatrix} 1 & 3 & 0 & -4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

x_2, x_3 and x_4 are free

$x_1 = -3x_2 + 4x_4$

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -3x_2 + 4x_4 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

$$\underline{\underline{\vec{x} = x_2 \cdot \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_3 \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + x_4 \cdot \begin{pmatrix} 4 \\ 0 \\ 0 \\ 1 \end{pmatrix}}}$$

23)

a) True.
To be inconsistent there m t be a row $[0\ 0 \ldots 0\ 0 \mid a]$ $(a \neq 0)$, but that is impossible in a homogeneous system.

b) False
It's an implicit descriptio .
Solving for $\vec{x}$ would give an explicit description.

c) False
It always has a trivial solution; $\vec{x} = \vec{0}$.

d) False
It describes a line through $\vec{p}$ parallel to $\vec{v}$

e) False
It would be true if $\vec{p}$ was a parti ular solution to $A\vec{x}=\vec{b}$.

35) $(1,1,1)^T = \begin{pmatrix}1\\1\\1\end{pmatrix}$

$$A \cdot \begin{pmatrix}1\\1\\1\end{pmatrix} = \vec{0}$$

$$A = (\vec{a}_1 | \vec{a}_2 | \vec{a}_3) \quad , \; a_i \in \mathbb{R}^3$$

$$\Rightarrow A \begin{pmatrix}1\\1\\1\end{pmatrix} = \vec{a}_1 + \vec{a}_2 + \vec{a}_3$$

$$\Rightarrow \vec{a}_1 + \vec{a}_2 + \vec{a}_3 = \vec{0}$$

For example: $\vec{a}_2 = -\vec{a}_1$, $\vec{a}_1 = \begin{pmatrix}1\\1\\1\end{pmatrix}$, $\vec{a}_3 = \vec{0}$

Then $A = \begin{pmatrix}1 & -1 & 0\\1 & -1 & 0\\1 & -1 & 0\end{pmatrix}$

and $A \cdot \begin{pmatrix}1\\1\\1\end{pmatrix} = 1 \cdot \begin{pmatrix}1\\1\\1\end{pmatrix} + 1 \cdot \begin{pmatrix}-\\-\\-1\end{pmatrix} + 1 \cdot \begin{pmatrix}0\\0\\0\end{pmatrix} = \vec{0}$

So one such matrix is:

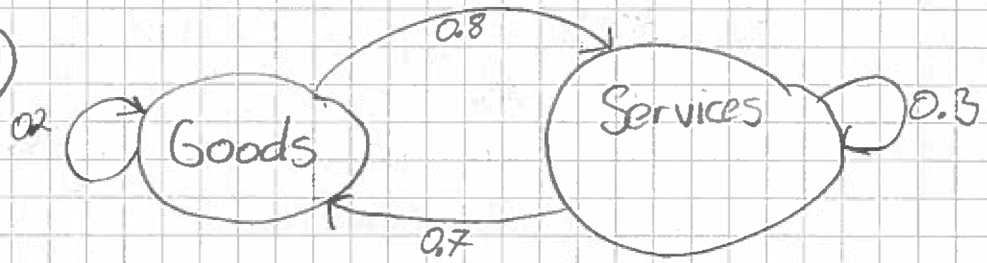
$$A = \begin{pmatrix}1 & -1 & 0\\1 & -1 & 0\\1 & -1 & 0\end{pmatrix}$$

39) By theorem 5: $A(c \cdot \vec{u}) = c \cdot A\vec{u}$

If $A\vec{u} = \vec{0}$ then $c \cdot A\vec{u} = c\vec{0} = \vec{0}$

So $c \cdot \vec{u}$ is also a solution to $A\vec{x} = \vec{0}$

1.6 1)



P_G : price of goods

P_S : — " — services.

Goods: $0{,}7 P_S + \cancel{0{,}2 P_G} = 0{,}8 P_G + \cancel{0{,}2 P_G}$

$0{,}7 P_S - 0{,}8 P_G = 0 \quad (G)$

Services: $0{,}8 P_G + \cancel{0{,}3 P_S} = 0{,}7 P_S + \cancel{0{,}3 P_S}$

$-0{,}7 P_S + 0{,}8 P_G = 0 \quad (S)$

Note that $(G) = -(S)$ so there will be one free variable. Let's make the price of services ($\approx$ human labour) a free variable.

From (G): $0{,}7 P_S - 0{,}8 P_G = 0$

$\Leftrightarrow P_G = \frac{0{,}7}{0{,}8} P_S$

$P_G = \frac{7}{8} P_S$

So when the price of labour is P_S, the system will be in equilibrium when

$P_G = \frac{7}{8} P_S$.

7) $(x_1)\cdot NaHCO_3 + (x_2)\cdot H_3C_6H_5O_7$

$= (x_3)\cdot Na_3C_6H_5O_7 + (x_4)\cdot H_2O + (x_5)\,CO_2$

$$\Rightarrow x_1\cdot\begin{pmatrix}1\\1\\3\\1\end{pmatrix} + x_2\cdot\begin{pmatrix}3+5\\6\\7\\0\end{pmatrix} = x_3\cdot\begin{pmatrix}5\\6\\7\\3\end{pmatrix} + x_4\begin{pmatrix}2\\0\\1\\0\end{pmatrix} + x_5\cdot\begin{pmatrix}0\\1\\2\\0\end{pmatrix}\begin{array}{l}\leftarrow H\\ \leftarrow C\\ \leftarrow O\\ \leftarrow Na\end{array} \quad (*)$$

Want to find the smallest integers $\{x_i\}$ such that they solve the system.

$$(*) \Leftrightarrow x_1\begin{pmatrix}1\\1\\3\\1\end{pmatrix} + x_2\begin{pmatrix}8\\6\\7\\0\end{pmatrix} + x_3\begin{pmatrix}-5\\-6\\-7\\-3\end{pmatrix} + x_4\begin{pmatrix}-2\\0\\-1\\0\end{pmatrix} + x_5\begin{pmatrix}0\\-1\\-2\\0\end{pmatrix} = \vec{0}$$

$$\Leftrightarrow \begin{pmatrix}1 & 8 & -5 & -2 & 0\\ 1 & 6 & -6 & 0 & -1\\ 3 & 7 & -7 & -1 & -2\\ 1 & 0 & -3 & 0 & 0\end{pmatrix}\vec{x} = \vec{0}$$

$$\begin{pmatrix}1 & 8 & -5 & -2 & 0\\ 1 & 6 & -6 & 0 & -1\\ 3 & 7 & -7 & -1 & -2\\ 1 & 0 & -3 & 0 & 0\end{pmatrix}$$

Want to preserve row order

$R_2 \leftarrow R_2 - R_1$, $R_3 \leftarrow R_3 - 3R_1$

$R_4 \leftarrow R_4 - R_1$

$$\Leftrightarrow \begin{pmatrix}1 & 8 & -5 & -2 & 0\\ 0 & -2 & -1 & 2 & -1\\ 0 & -17 & 8 & 5 & -2\\ 0 & -8 & 2 & 2 & 0\end{pmatrix}$$

$R_3 \leftarrow R_3 - \frac{17}{2} R_2$

$R_4 \leftarrow R_4 - 4R_2$

then $R_3 \leftarrow 2\cdot R_3$

$$\Leftrightarrow \begin{pmatrix}1 & 8 & -5 & -2 & 0\\ 0 & -2 & -1 & 2 & -1\\ 0 & 0 & 33 & -24 & 13\\ 0 & 0 & 6 & -6 & 0\end{pmatrix}$$

$R_4 \leftarrow R_4 - \frac{6}{33} R_3$

then $R_4 \leftarrow -\frac{11}{2} R_4$

$$\Leftrightarrow \begin{pmatrix}1 & 8 & -5 & -2 & 0\\ 0 & -2 & -1 & 2 & -1\\ 0 & 0 & 33 & -24 & 13\\ 0 & 0 & 0 & 9 & 13\end{pmatrix}$$

$R_4 \leftarrow \frac{1}{9} R_4$

$$\Leftrightarrow \begin{matrix}1 & 8 & -5 & -2 & 0\\ 0 & -2 & -1 & 2 & -1\\ 0 & 0 & 33 & -24 & 13\\ 0 & 0 & 0 & 1 & \frac{13}{9}\end{matrix}$$

7) $NaHCO_3: \begin{pmatrix}1\\1\\3\\1\end{pmatrix}$, $H_3C_6H_5O_7: \begin{pmatrix}3+5\\6\\7\\0\end{pmatrix} = \begin{pmatrix}8\\6\\7\\0\end{pmatrix}$

$Na_3C_6H_5O_7: \begin{pmatrix}5\\6\\7\\3\end{pmatrix}$, $H_2O: \begin{pmatrix}2\\0\\1\\0\end{pmatrix}$, $CO_2: \begin{pmatrix}0\\1\\2\\0\end{pmatrix}$

All the above vectors are sorted like the periodic table, so $\begin{pmatrix}H\\C\\O\\Na\end{pmatrix}$

To solve the system, we must determine the smallest positive integers x_1, x_2, x_3, x_4, x_5 that solve:

$$(x_1)\cdot NaHCO_3 + (x_2)\cdot H_3C_6H_5O_7 = (x_3)\cdot Na_3C_6H_5O_7 + (x_4)\cdot H_2O + (x_5)\cdot CO_2$$

$$\Leftrightarrow x_1\cdot\begin{pmatrix}1\\1\\3\\1\end{pmatrix} + x_2\cdot\begin{pmatrix}8\\6\\7\\0\end{pmatrix} = x_3\begin{pmatrix}5\\6\\7\\3\end{pmatrix} + x_4\cdot\begin{pmatrix}2\\0\\1\\0\end{pmatrix} + x_5\cdot\begin{pmatrix}0\\1\\2\\0\end{pmatrix}$$

$$\Leftrightarrow \underbrace{\begin{pmatrix}1 & 8 & -5 & -2 & 0\\1 & 6 & -6 & 0 & -1\\3 & 7 & -7 & -1 & -2\\1 & 0 & -3 & 0 & 0\end{pmatrix}}_{A}\cdot\underbrace{\begin{pmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{pmatrix}}_{\vec{x}} = \vec{0}$$

$$A = \begin{pmatrix}1 & 8 & -5 & -2 & 0\\1 & 6 & -6 & 0 & -1\\3 & 7 & -7 & -1 & -2\\1 & 0 & -3 & 0 & 0\end{pmatrix}$$

$R_2 \leftarrow R_2 - R_1$, $R_3 \leftarrow R_3 - 3R_1$, $R_4 \leftarrow R_4 - R_1$

$$\Leftrightarrow \begin{pmatrix} 1 & 8 & -5 & -2 & 0 \\ 0 & -2 & -1 & 2 & -1 \\ 0 & -17 & 8 & 5 & -2 \\ 0 & -8 & 2 & 2 & 0 \end{pmatrix} \quad \begin{array}{l} R_3 \leftarrow R_3 - \frac{17}{2} R_2 \\ R_4 \leftarrow R_4 - 4R_2 \end{array}$$

$$\Leftrightarrow \begin{pmatrix} 1 & 8 & -5 & -2 & 0 \\ 0 & -2 & -1 & 2 & -1 \\ 0 & 0 & 33/2 & -12 & 13/2 \\ 0 & 0 & 6 & -6 & 4 \end{pmatrix} \quad \begin{array}{l} R_2 \leftarrow -R_2 \\ R_3 \leftarrow 2R_3 \\ R_4 \leftarrow \frac{1}{2} R_4 \end{array}$$

$$\Leftrightarrow \begin{pmatrix} 1 & 8 & -5 & -2 & 0 \\ 0 & 2 & 1 & -2 & 1 \\ 0 & 0 & 33 & -24 & 13 \\ 0 & 0 & 3 & -3 & 2 \end{pmatrix} \quad R_4 \leftarrow R_4 - \frac{3}{33} R_3$$

$$\Leftrightarrow \begin{pmatrix} 1 & 8 & -5 & -2 & 0 \\ 0 & 2 & 1 & -2 & 1 \\ 0 & 0 & 33 & -24 & 13 \\ 0 & 0 & 0 & -\frac{9}{11} & \frac{9}{11} \end{pmatrix} \quad R_4 \leftarrow -\frac{11}{9} R_4$$

$$\Leftrightarrow \begin{pmatrix} 1 & 8 & -5 & -2 & 0 \\ 0 & 2 & 1 & -2 & 1 \\ 0 & 0 & 33 & -24 & 13 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix} \quad \begin{array}{l} R_3 \leftarrow R_3 + 24R_4 \\ R_2 \leftarrow R_2 + 2R_4 \\ R_1 \leftarrow R_1 + 2R_4 \end{array}$$

$$\Leftrightarrow \begin{pmatrix} 1 & 8 & -5 & 0 & -2 \\ 0 & 2 & 1 & 0 & -1 \\ 0 & 0 & 33 & 0 & -11 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix} \quad R_3 \leftarrow \frac{1}{33} R_3$$

$$\Leftrightarrow \begin{pmatrix} 1 & 8 & -5 & 0 & -2 \\ 0 & 2 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1/3 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix} \quad \begin{array}{l} R_2 \leftarrow R_2 - R_3 \\ R_1 \leftarrow R_1 + 5R_3 \end{array}$$

$$\Leftrightarrow \begin{pmatrix} 1 & 8 & 0 & 0 & -11/3 \\ 0 & 2 & 0 & 0 & -2/3 \\ 0 & 0 & 1 & 0 & -1/3 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix} \quad \begin{array}{l} R_1 \leftarrow R_1 - 4R_2 \\ \text{then } R_2 \leftarrow \frac{1}{2} R_2 \end{array}$$

$$\Leftrightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1/3 \\ 0 & 0 & 1 & 0 & -1/3 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

So the solution is given by:

$$\begin{cases} x_1 - x_5 = 0 \\ x_2 - \frac{1}{3}x_5 = 0 \\ x_3 - \frac{1}{3}x_5 = 0 \\ x_4 - x_5 = 0 \\ x_5 \text{ free} \end{cases}$$

$$\Leftrightarrow \begin{cases} x_1 = x_5 \\ x_2 = \frac{1}{3}x_5 \\ x_3 = \frac{1}{3}x_5 \\ x_4 = x_5 \\ x_5 \text{ free} \end{cases}$$

$x_5 = 3$ is the lowest integer that makes $x_1, \dots, x_4$ all integers.

So we have $x_1 = 3$, $x_2 = 1$, $x_3 = 1$, $x_4 = 3$, $x_5 = 3$.

The balanced chemical equation is then.

$$3\,NaHCO_3 + H_3C_6H_5O_7 \rightarrow Na_3C_6H_5O_7 + 3H_2O + 3CO_2$$

1.7

1) $x_1 \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 7 \\ 2 \\ -6 \end{pmatrix} + x_3 \cdot \begin{pmatrix} 9 \\ 4 \\ -8 \end{pmatrix} = \vec{0}$ (*)

$$A := \begin{pmatrix} 5 & 7 & 9 \\ 0 & 2 & 4 \\ 0 & -6 & -8 \end{pmatrix} \quad R_3 \leftarrow R_3 + 3R_2$$

$$\Leftrightarrow \begin{pmatrix} 5 & 7 & 9 \\ 0 & 2 & 4 \\ 0 & 0 & 4 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} 5 & 7 & 9 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

There is a pivot in each row, so no variables are free, so there's only one (trivial) solution to (*).
The vectors are independent.

5) $R_4 \leftrightarrow R_1$

$$\begin{pmatrix} 1 & -3 & 2 \\ 3 & -7 & -4 \\ -1 & 5 & -4 \\ 0 & -8 & 5 \end{pmatrix} \quad \begin{matrix} R_2 \leftarrow R_2 - 3R_1 \\ R_3 \leftarrow R_3 + R_1 \end{matrix}$$

$$\Leftrightarrow \begin{pmatrix} 1 & -3 & 2 \\ 0 & 2 & -10 \\ 0 & -2 & -2 \\ 0 & -8 & 5 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} 1 & -3 & 2 \\ 0 & 1 & -5 \\ 0 & 1 & 1 \\ 0 & -8 & 5 \end{pmatrix} \quad \begin{matrix} R_3 \leftarrow R_3 - R_2 \\ R_4 \leftarrow R_4 + 8R_2 \end{matrix}$$

$$\Leftrightarrow \begin{pmatrix} 1 & -3 & 2 \\ 0 & 1 & -5 \\ 0 & 0 & 6 \\ 0 & -0 & -35 \end{pmatrix} \Leftrightarrow \begin{pmatrix} 1 & -3 & 2 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

We can see that $R_4 = R_3$ so there will be at least one free variable and thus the set is linearly dependent.

13) $\vec{v}_1 = \begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} -2 \\ -9 \\ 6 \end{pmatrix}, \quad \vec{v}_3 = \begin{pmatrix} 3 \\ h \\ -9 \end{pmatrix}$

If $h = 15$ then $\vec{v}_3 = 3 \cdot \vec{v}_1$ so they would be linearly dependent.

We can also make $\vec{v}_3$ a multiple of $\vec{v}_2$.

Assume $\vec{v}_3 = x \cdot \vec{v}_2$, $x \in \mathbb{R}$.

Then
$$3 = x \cdot (-2) \qquad (1)$$
$$h = x(-9) \qquad (2)$$
$$-9 = x \cdot 6 \qquad (3)$$

Let $x = -\frac{3}{2}$ such that (1) and (3) are satisfied. This makes

$$h = \left(-\frac{3}{2}\right)(-9) = \frac{27}{2}$$

27) A 7×5-matrix:

$$5\left\{\underbrace{\begin{bmatrix} \blacksquare & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \blacksquare & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \blacksquare & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \blacksquare & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \blacksquare & \cdot & \cdot \end{bmatrix}}_{7}\right.$$

5 collumns must have pivots because ~~lets~~ if there were fewer pivots than rows there would be a free variable.

39) A is an $m \times m$ matrix such that $A\vec{x} = \vec{b}$ has at most one solution. This implies that $A\vec{x} = \vec{0}$ $(\vec{b} = \vec{0})$ has at most one solution, and this solution is always going to be the trivial $\vec{x} = \vec{0}$ solution. A is then by definition linearly independent.