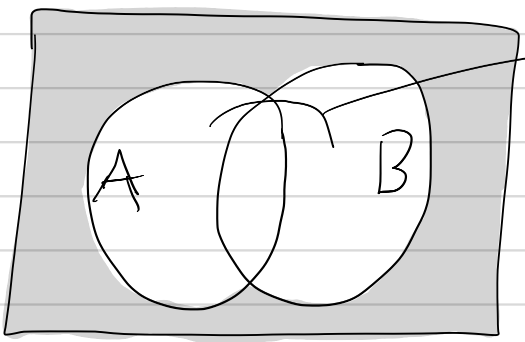
Innlevering 1

Rendell Cale

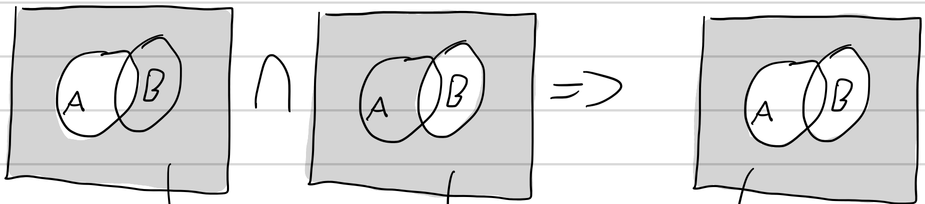
Oppgave 1

1) $(A \cup B)' = A' \cap B'$



hvit = $A \cup B$

Så farget område = $(A \cup B)'$

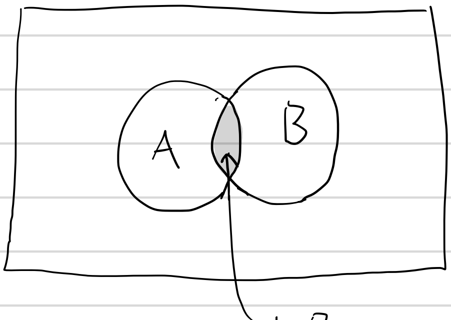


A' B'

grått i A' og B' = $A' \cap B'$

Så $(A \cup B)' = A' \cap B'$

2) $(A \cap B)' = A' \cup B'$

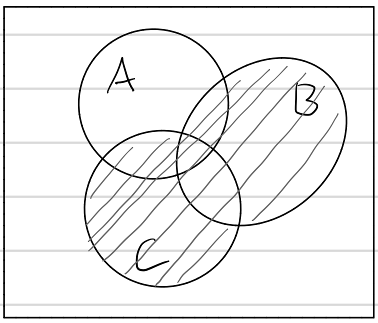


$A \cap B$ => alt utenfor er i $(A \cap B)'$
(uten farger)

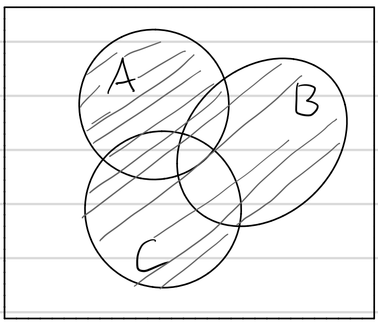
Observer at alt utenfor er enten i A' eller B', og at ingenting i det merkede område $A \cap B$ er i A' eller B'.

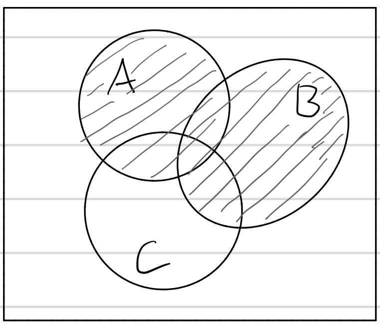
Så vi har
$$(A \cap B)' = \text{"alt utenfor"} = A' \cup B'$$

3) Assosiative lover

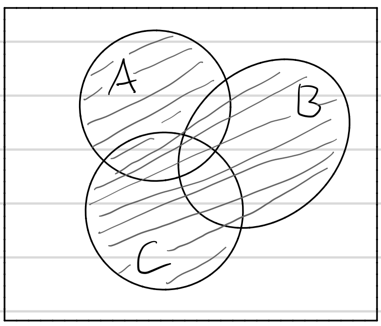


$\cup A =$

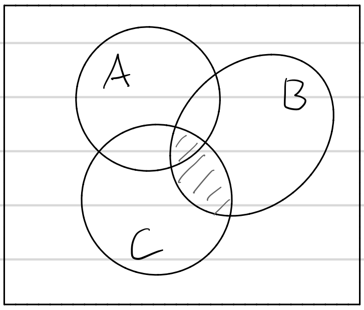




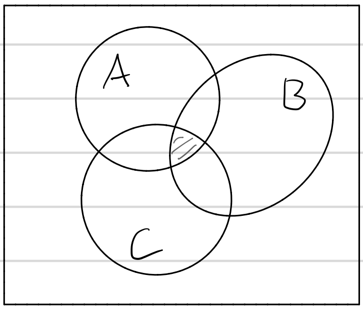
$\cup C =$

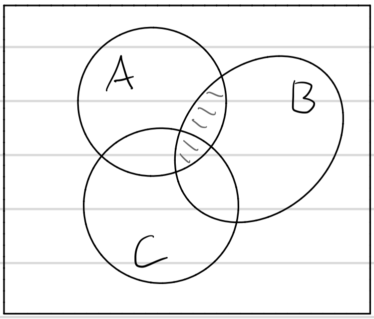


De er like

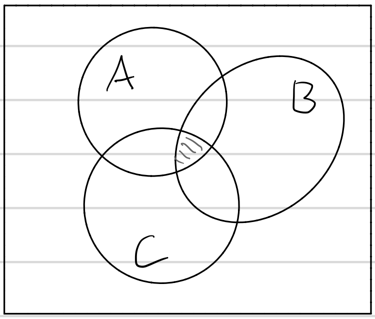


$\cap A =$



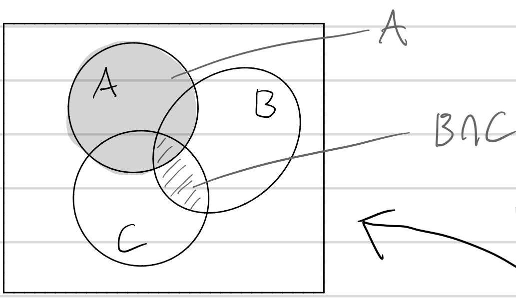


$\cap C =$

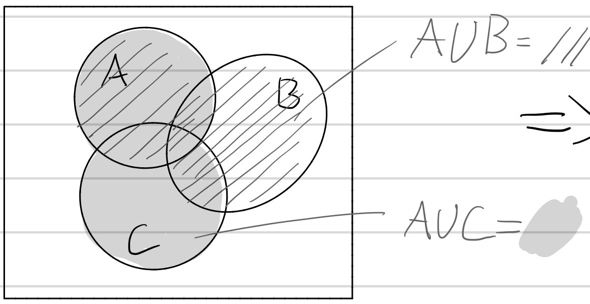


De er like

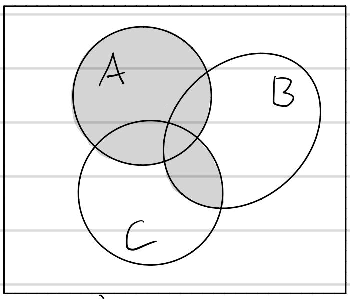
4) Distributive lover



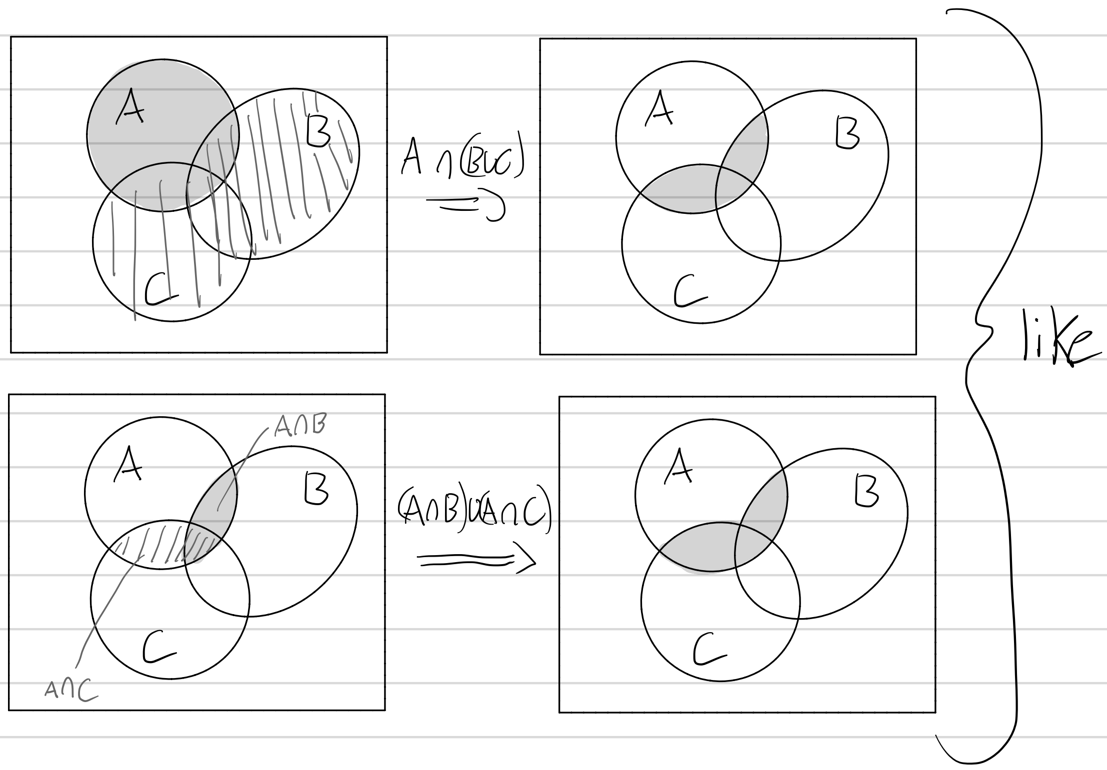
like så $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$



$\Rightarrow$



$(A \cup B) \cap (A \cup C)$



Oppgave 2

Hva er $P(B|A)$?

La E = ekte mynt

F = falsk mynt (2 kroner)

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(A \cap B \mid E)P(E) + P(A \cap B \mid F)P(F)$$

$$= (0{,}5 \cdot 0{,}5) \cdot 0{,}5 + 1 \cdot 0{,}5$$

$$= 0{,}625$$

$$P(A) = P(A|E)P(E) + P(A|F)P(F)$$

$$\Rightarrow \quad P(A) = 0{,}5 \cdot 0{,}5 + 1 \cdot 0{,}5$$

$$= 0{,}75$$

$$P(A \cap B) = \frac{0{,}625}{0{,}75} = 0{,}833$$

$$P(B) = P(A) = 0{,}75.$$

Så $P(A)P(B) = 0{,}75 \cdot 0{,}75 = 0{,}5625 \neq P(A \cap B)$.

Det betyr at A og B er avhengige.

Oppgave 3

nei-seer

$$P(J) = 0{,}8 \quad , \quad P(\bar{J}) = 0{,}2$$

$$P(R \mid J) = 0{,}02 \quad , \quad P(R \mid \bar{J}) = 0{,}05$$

Andelen av innringerne som mener ja vil være

$$P(J \mid R) = \frac{P(J \cap R)}{P(R)} = \frac{P(R \mid J)P(J)}{P(R)}$$

$$= \frac{P(R \mid J)P(J)}{P(R \mid J)P(J) + P(R \mid \bar{J})P(\bar{J})}$$

Så vi får

$$P(J|R) = \frac{0{,}02 \cdot 0{,}8}{0{,}02 \cdot 0{,}8 + 0{,}05 \cdot 0{,}2}$$

$$= \underline{\underline{0{,}615}}$$

Tja.. resultatet gir en tydelig overvekt i antall nei-seere.

Oppgave 4

a) $R = E \cup F$

$$P(R) = P(E \cup F)$$
$$= P(E) + P(F) - P(E \cap F)$$
$$= 0{,}05 + 0{,}05 - 0{,}02$$
$$P(R) = \underline{\underline{0{,}08}}$$

E og F er <u>ikke disjunkte</u> siden da ville $E \cap F = \emptyset$, men vi vet at

$$P(E \cap F) = 0{,}02 \neq 0 = P(\emptyset).$$

E og F er <u>avhengige</u> siden $P(E \cap F) \neq \underbrace{P(E)P(F)}_{0{,}05 \cdot 0{,}05 = 0{,}0025}$

b) $P(V \mid \bar{E} \cap \bar{F}) = 0{,}07$

$P(V \mid E \cup F) = 0{,}5$

Merk at $(E \cup F) \cup (\bar{E} \cap \bar{F}) =$ univers

og $(E \cup F) \cap (\bar{E} \cap \bar{F}) = \emptyset$.

Da kan vi skrive

$$P(V) = P(V \mid E \cup F)\,P(E \cup F) + P(V \mid \bar{E} \cap \bar{F})\,\underbrace{P(\bar{E} \cap \bar{F})}_{1 - P(E \cup F)}$$

$$= 0{,}5 \cdot (0{,}05 + 0{,}05 - 0{,}02) + 0{,}07 \cdot (1 - 0{,}08)$$

$$\underline{\underline{= 0{,}1044}}$$

Ønsker da å vite

$$P(E \cup F \cup V) = 1 - P(\bar{E} \cap \bar{F} \cap \bar{V})$$

$$= 1 - P(\bar{V} \mid \bar{E} \cap \bar{F})\,P(\bar{E} \cap \bar{F})$$

$$P(\bar{V}) = \underbrace{P(\bar{V} \mid \bar{E} \cap \bar{F})\,P(\bar{E} \cap \bar{F})}_{\text{Ønsker å vite dette}} + \underbrace{P(\bar{V} \mid E \cup F)}_{= 0{,}5 \text{ fra oppgaveteksten.}}\,P(E \cup F)$$

$$P(\overline{V} \mid \overline{E} \cap \overline{F})\, P(\overline{E} \cap \overline{F}) = -P(\overline{V} \mid E \cup F)\, P(E \cup F) + P(\overline{V})$$

$$= (1 - 0{,}1044) - 0{,}5 \cdot (0{,}05 + 0{,}05 - 0{,}02)$$

$$= 0{,}8556$$

$$\Rightarrow P(E \cup F \cup V) = 1 - 0{,}8556$$

$$\underline{\underline{= 0{,}1444}}$$

Oppgave 5

$$F(x) = 1 - \exp\left\{-\frac{x^2}{2\alpha}\right\} \ ; \ x>0$$

a) $P(X \leq 3) = F(3) - \cancel{F(0)}^{=0}$

$$= 1 - \exp\left(-\frac{3^2}{2\cdot 5}\right)$$

$$= \underline{\underline{0{,}593}}$$

$$P(X \geq 5) = P(5 < X \leq +\infty)$$

$$= F(\infty) - F(5)$$

$$= 1 - \left(1 - \exp\left(-\frac{5^2}{2\cdot 5}\right)\right)$$

$$= \underline{\underline{0{,}082}}$$

b) $f(x) = \frac{dF(x)}{dx} = -\left(-\frac{2x}{2\alpha}\right)\exp\left(-\frac{x^2}{2\alpha}\right)$

$$= \frac{x}{\alpha}\exp\left(-\frac{x^2}{2\alpha}\right)$$

$f(x)$ har max ved x gitt av

$$f'(x_{max}) = 0$$

$$\frac{d}{dx}\left(\frac{x}{\alpha}\exp\left(\frac{-x^2}{2\alpha}\right)\right) = 0$$

$$\Leftrightarrow \frac{1}{\alpha}\cancel{\exp\left(\frac{-x^2}{2\alpha}\right)} + \left(\frac{-x}{\alpha}\right)\frac{x}{\alpha}\cancel{\exp\left(\frac{-x^2}{2\alpha}\right)} = 0$$

$$\Leftrightarrow \frac{1}{\alpha} - \frac{x^2}{\alpha^2} = 0$$

$$\Leftrightarrow \alpha = x^2$$

$$\Leftrightarrow x = +\sqrt{\alpha}$$

Sannsynlighetstettheten $f(x)$ har maksimum ved

$$\underline{\underline{x = +\sqrt{\alpha}}}$$

c) Før:

Gj. snitt = 2,8910

Median = 1,1170

Std. avvik = 17,0960

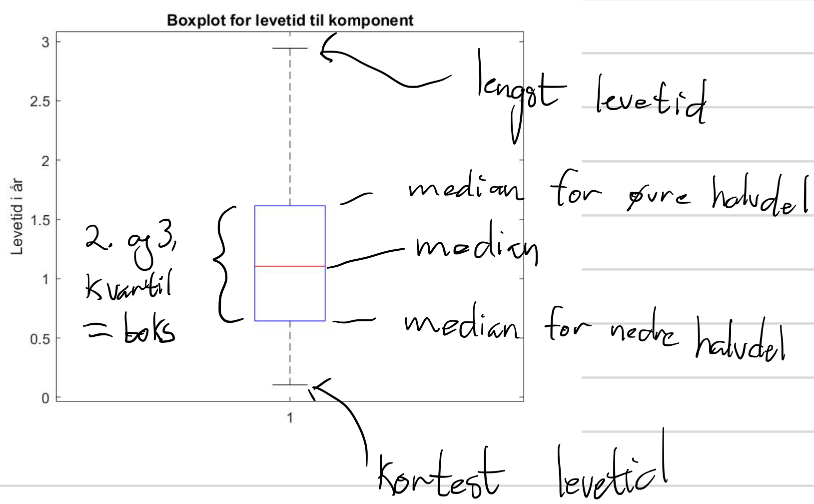
Etter:

Gj. snitt = 1,1826

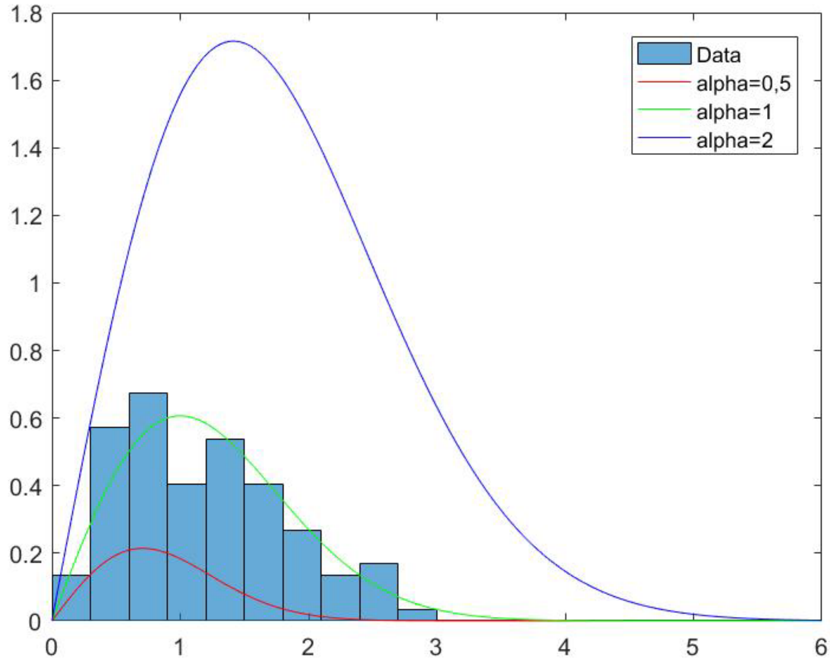
Median = 1,1036

Std. avvik = 0,6510

Medianen endret seg ikke vesentlig, men gjennomsnittet ble mer enn halvert, og std. avviket ble redusert kraftig. Vi ser også nå at medianen og snittet er nesten like.



Boxplot er en god måte å illustrere spredningen i datasettet.



Den sanne verdien for α er nok ≈ 1, siden $\alpha = 1$ følger dataene ganske godt.

Oppgave 6

	$y=0$	$y=1$	$y=2$	sum:
$x=-1$	$1/6$	$1/12$	$1/12$	$\rightarrow 1/3$
$x=0$	$1/12$	$1/6$	$1/12$	$\rightarrow 1/3$
$x=1$	$1/12$	$1/12$	$1/6$	$\rightarrow 1/3$
Sum:	$\downarrow$ $\frac{1}{3}$	$\downarrow$ $1/3$	$\downarrow$ $1/3$	

Vi kan da se at

$$g(x) = \frac{1}{3} \quad \text{og} \quad h(y) = \frac{1}{3}$$

for gyldige verdier av x, y.