INSTITUTO POLITECNICO NACIONAL

UNIDAD PROFESIONAL INTERDICIPLINARIA EN INGENIERÍA Y TECNOLOGÍAS AVANZADAS

SEÑALES Y SISTEMAS

(Problemas 11)

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Solución 1 $[-t, 2]$

a) $f(t) = \begin{cases} 2 & -1 < t < 0, \\ 1 & 0 < t < 1, \\ 0 & \text{en caso contrario} \end{cases}$

$T = 4$

$w = \frac{2\pi}{T} = \frac{\pi}{2}$

$$D_n = \frac{1}{T}\int_{-1}^{1} f(t)e^{-nwjt}\,dt = \frac{1}{T}\int_{-1}^{0} 2e^{-nwjt}\,dt + \int_{0}^{1} e^{-nwjt}\,dt$$

$$= \frac{(-1)^{\frac{n}{2}+\frac{1}{2}} - 2(-1)^n j + j}{2(-1)^{n/2} n\pi}$$

$\wedge\ n \neq 0$

$D_0 = \frac{3}{4}$

$$\boxed{S_f(t) = \frac{3}{4} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \left(\frac{(-1)^{\frac{n}{2}+\frac{1}{2}} - 2(-1)^n j + j}{2(-1)^{\frac{n}{2}} n\pi} e^{nwjt} \right)}$$

b) $a_0 = \frac{1}{T}\int_{-1}^{0} 2\,dt + \frac{1}{T}\int_{0}^{1} dt = \frac{3}{4}$

$$a_n = \frac{2}{T}\int_{-1}^{0} 2\cos(nwt)\,dt + \frac{2}{T}\int_{0}^{1} \cos(nwt)\,dt$$

$$a_n = \frac{3\sin\left(\frac{\pi n}{2}\right)}{n\pi}$$

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$$b_n = \frac{2}{T}\int_{-1}^{0} 2\,sen(n\omega t)\,dt + \frac{2}{T}\int_{0}^{1} sen(n\omega t)\,dt$$

$$b_n = \frac{-2\sin^2\left(\frac{\pi n}{4}\right)}{n\pi} = \frac{-\frac{2}{2}\left(1-\cos\left(\frac{\pi n}{2}\right)\right)}{n\pi} = \frac{\cos\left(\frac{\pi n}{2}\right)-1}{n\pi}$$

$n \neq 0$

$b_0 = 0$

$$\boxed{Sf(t) = \frac{3}{4} + \sum_{n=1}^{\infty}\left(\frac{3\,sen\left(\frac{\pi n}{2}\right)}{n\pi}\cos(n\omega t) - \frac{2\,sen^2\left(\frac{\pi n}{4}\right)}{n\pi}\,sen(n\omega t)\right)}$$

c) $D_n \Rightarrow a_n, b_n$

$a_0 = D_0 = \frac{3}{4}$

$a_n = D_n + D_{-n} = \frac{3\sin\left(\frac{\pi n}{2}\right)}{n\pi}$

$b_n = \frac{D_n - D_n}{j} =$

$$\frac{e^{-\frac{\pi n i}{2}}\left(e^{\frac{\pi n i}{2}}-1\right)^2}{2n\pi}$$

$$\frac{e^{\pi n i} - 2e^{\frac{\pi n i}{2}} + 1}{2n\pi\, e^{\frac{\pi n i}{2}}}$$

$$= \frac{e^{\frac{\pi n i}{2}}}{2n\pi} - \frac{1}{n\pi} + \frac{1}{2n\pi e^{\frac{\pi n i}{2}}}$$

$$\frac{\cos\frac{\pi n}{2} + \cancel{sen\frac{\pi n}{2} i} + \cos\frac{\pi n}{2} - \cancel{sen\frac{\pi n}{2} i}}{2n\pi}$$

$$-\frac{1}{n\pi}$$

$$= \frac{\cos\frac{\pi n}{2} - 1}{n\pi}$$

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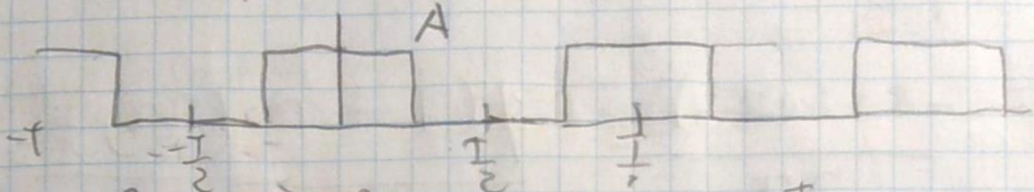
Solución 2.

a)



$T_0 = T \quad \omega_0 = \frac{2\pi}{T}$

$$S_f(t) = \sum_{n=-\infty}^{\infty} D_n \, e^{n\omega_0 j t}$$

$$D_n = \frac{1}{T_0} \int_{\langle T_0 \rangle} f(t) \, e^{-n\omega_0 j t} \, dt$$

$$= \frac{1}{T} \int_{-\frac{T}{4}}^{\frac{T}{4}} Aa \, e^{-\omega_0 j t} \, dt = \frac{Aa \sin\left(\frac{\pi n}{2}\right)}{n\pi}$$

$$\boxed{S_f(t) = \sum_{n=-\infty}^{\infty} \frac{Aa \sin\left(\frac{\pi n}{2}\right)}{n\pi} \cdot e^{n\omega_0 j t}}$$

b) Si $a = 1$

$$\boxed{S_f(t) = \sum_{n=-\infty}^{\infty} \frac{A \sin\left(\frac{\pi n}{2}\right)}{n\pi} \cdot e^{n\omega_0 j t}}$$

c) $a = 1 - \epsilon$

$$S_f(t) = \lim_{\epsilon \to 0} \sum_{n=-\infty}^{\infty} \frac{(1-\epsilon) A \sin\left(\frac{\pi n}{2}\right)}{n\pi} \cdot e^{n\omega_0 j t}$$

$$\boxed{= \frac{A \sin\left(\frac{\pi n}{2}\right)}{n\pi} \cdot e^{n\omega_0 j t}}$$

$a_0 = \frac{Aa}{2}$

$a_n = \frac{2Aa \sin\left(\frac{\pi n}{2}\right)}{n\pi}$

$b_n = 0$

$$S_f(t) = 2Aa + \sum_{n=1}^{\infty} \frac{2Aa \sin(2\pi n)}{n\pi} \cos(n\omega t)$$

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d)

$$|D_2| = 0 = 0$$

$$\|D_3\| = \left|-\frac{Aa}{3\pi}\right| = \frac{A\cancel{a}}{3\pi} = \frac{A}{3\pi}$$

"a" va dando la altura de la señal cuando es "on", en este caso se ve aplicado a la serie de Fourier.

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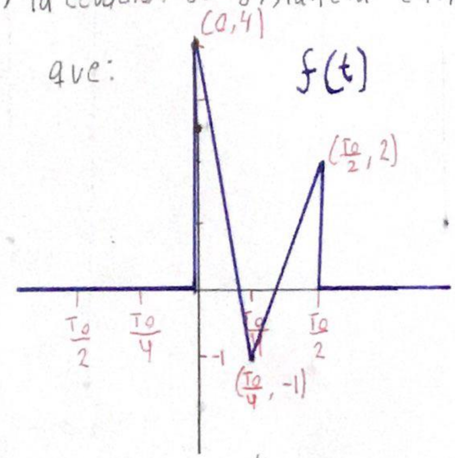
Dados los puntos $(0,4)$, $\left(\frac{T_0}{2}, 2\right)$, $\left(\frac{T_0}{4}, -1\right)$ y la ecuación de distancia entre puntos $y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$ Tenemos que:



$$f(t) = \begin{cases} -\frac{20}{T_0}t + 4 & 0 < t \leq \frac{T_0}{4} \\ \frac{+12t - 3T_0}{T_0} - 1 & \frac{T_0}{4} < t < \frac{T_0}{2} \end{cases}$$

El problema nos indica:

$T_0 = 1$

$W_0 = \frac{2\pi}{T_0} = 2\pi$

$$D_n = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} f(t)\, e^{-nW_0 jt} dt$$

$$Sf(t) = \sum_{n=-\infty}^{\infty} D_n e^{nW_0 jt}$$

$$D_n = \frac{1}{1}\int_{-0.5}^{0.5} f(t)\, e^{-nW_0 jt} = \int_0^{1/2} f(t)\, e^{-nW_0 jt}$$

$$D_n = \int_0^{1/4} (-20t + 4)\, e^{-nW_0 jt} dt + \int_{1/4}^{1/2} (12t - 4)\, e^{-njW_0 t} dt$$

* integrando y simplificando con MATLAB Tenemos que:

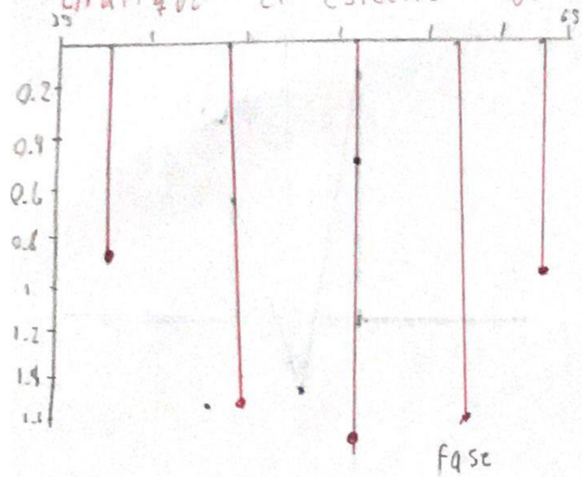
$$D_n = \frac{3(-1)^n - \frac{8}{(-1)^{n/2}} + \pi(-1)^{n+\frac{1}{2}} n + 5 - 2\pi n j}{n^2 \pi^2}$$

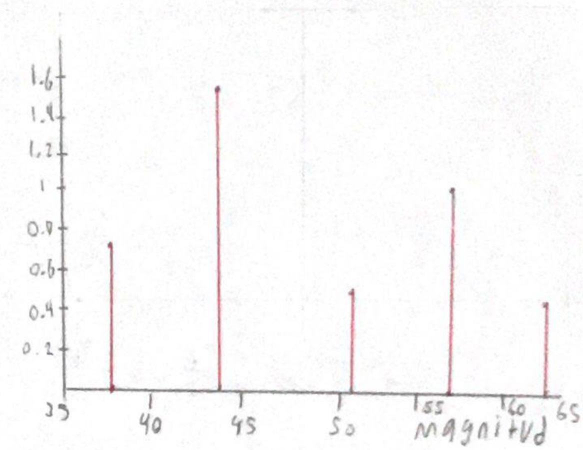
$$D_0 = \frac{1}{2}$$

$$Sf(t) = \sum_{n=6}^{\infty} D_n e^{njtW_0}$$

$$Sf(t) = \left\{(0.045 - 0.053j)e^{6(2\pi)jt} + (0.004 - 0.153j)e^{7(2\pi)jt} - 0.040e^{8(2\pi)jt} + \ldots\right\}$$

Grafique el espectro de fourier





¿Quién es el primer armónico?

$(0.045-0.053j)e^{6(2\pi)jt}$

¿El segundo?

$(0.004-0.153j)e^{7(2\pi)jt}$

Extras: Código utilizado en Matlab

PR11

Solucion 1

a) Calculando Coeficientes de la Serie de Fourier Exponencial

```
clear all; clc; close all;

syms n t pi

assume(n, 'integer')

T = 4;
w = (2 * pi)/T;

Dn = @(n) (1/T) * ( int(2 * exp(-n*w*j*t) , t, -1, 0) + int( exp(-n*w*j*t) , t,
0, 1) );

Dn(n)
```

ans =

$$-\frac{\mathrm{i}}{2\,n\,\pi}-\frac{e^{\frac{n\,\pi\,\mathrm{i}}{2}}\,\mathrm{i}-\mathrm{i}}{n\,\pi}+\frac{e^{-\frac{n\,\pi\,\mathrm{i}}{2}}\,\mathrm{i}}{2\,n\,\pi}$$

```
simplify(Dn(n))
```

ans =

$$\frac{e^{-\frac{n\,\pi\,\mathrm{i}}{2}}\,\mathrm{i}-2\,e^{\frac{n\,\pi\,\mathrm{i}}{2}}\,\mathrm{i}+\mathrm{i}}{2\,n\,\pi}$$

```
Dn(0)
```

ans =

$$\frac{3}{4}$$

Calculando la Serie de Fourier Trigonométrica

```
assume([n, 'positive', 'integer'])
a0 = (1/T) * (int(2, t,-1,0) + int(1, t, 0,1))
```

a0 =

$\frac{3}{4}$

```
an = @(n) (2/T) * (int(2 * cos(n*w*t),t,-1,0) + int( cos(n*w*t), t, 0,1));

an(n)
```

ans =

$$\frac{3\sin\left(\frac{n\pi}{2}\right)}{n\pi}$$

```
bn = @(n) (2/T) * ( int(2 * sin(n*w*t),t,-1,0) + int(sin(n*w*t), t, 0,1) );

bn(n)
```

ans =

$$-\frac{2\sin\left(\frac{n\pi}{4}\right)^2}{n\pi}$$

```
bn(0)
```

ans =

0

Verificacion de resultados a través de las transformaciones

```
a00 = Dn(0)
```

a00 =

$\frac{3}{4}$

```
aN = simplify(Dn(n) + Dn(-n))
```

aN =

$$\frac{3\sin\left(\frac{n\pi}{2}\right)}{n\pi}$$

```
bN = simplify(( Dn(-n) - Dn(n))/j, 'Criterion', 'preferReal', 'Steps', 30)
```

bN =

$$\frac{e^{-\frac{n\pi i}{2}}\left(e^{\frac{n\pi i}{2}}-1\right)^2}{2n\pi}$$

```
simplify(( Dn(-1) - Dn(1))/j)
```

ans =

$$\frac{\cos\left(\frac{\pi}{2}\right)-1}{\pi}$$

```
bn(1)
```

ans =

$$-\frac{2\sin\left(\frac{\pi}{4}\right)^2}{\pi}$$

```
b00 = simplify(( Dn(0) - Dn(0) )/j)
```

b00 =

0

Solución 2

a) Cuando 0 <= a <= 1

```
clear all; clc; close all;

syms T n A a t;

T0 = T
```

T0 =

T

```
w0 = 2 * pi / T0
```

w0 =

$\frac{2\,\pi}{T}$

```
assume(T, 'positive');
assume(A, 'positive')
assume([n, 'integer', 'positive'])

Dn = @(n) (1/T0) * int(A*a * exp(-w0*j*t*n), t, -T0/4, T0/4);

Dn(n)
```

ans =

$\frac{A\,a\,\sin\left(\frac{\pi\,n}{2}\right)}{n\,\pi}$

```
a0 = Dn(0)
```

a0 =

$\frac{A\,a}{2}$

```
an = Dn(n)+Dn(-n)
```

an =

$\frac{2\,A\,a\,\sin\left(\frac{\pi\,n}{2}\right)}{n\,\pi}$

```
bn = (Dn(-n) - Dn(n))/j
```

bn =

0

```
Dn(2)
```

ans =

0

```
Dn(3)
```

ans =

$-\frac{A\,a}{3\,\pi}$