INSTITUTO POLITECNICO NACIONAL

UNIDAD PROFESIONAL INTERDICIPLINARIA EN INGENIERÍA Y TECNOLOGÍAS AVANZADAS

SEÑALES Y SISTEMAS

(Problemas 12)

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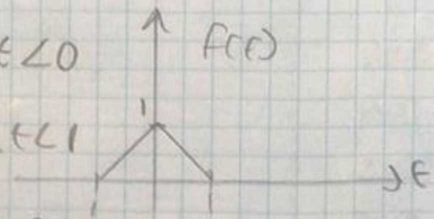
Ian Rojas Gómez 22/11/2020

Señales y Sistemas PR12

Solución 1

$$f(t) = \begin{cases} t+1, & -1 < t < 0 \\ 1-t, & 0 < t < 1 \end{cases}$$



$$S_f(t) = \sum_{n=-\infty}^{\infty} D_n e^{nj\omega_0 t}$$

$T = 2$

$\omega_0 = \frac{2\pi}{T} = \pi$

$$D_n = \frac{1}{T}\int_{\langle T_0 \rangle} f(t)\, e^{-nj\omega_0 t} dt = \frac{1}{2}\left[\int_{-1}^{0} (t+1)\, e^{-nj\omega_0 t} dt + \int_{0}^{1} (1-t)\, e^{-nj\omega_0 t} dt\right] = -\frac{(-1)^{3n} + (-1)^n - 2}{2n^2\pi^2}$$

$n \neq 0$

$D_0 = \frac{1}{2}$

$$\boxed{S_f(t) = \frac{1}{2} e^{nj\omega_0 t} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{-(-1)^{3n} + (-1)^n - 2}{2n^2\pi^2} \cdot e^{nj\omega_0 t}}$$

Notamos que la gráfica es simétrica

$\Rightarrow D_1 = D_{-1}$, $D_{-n} = D_n$

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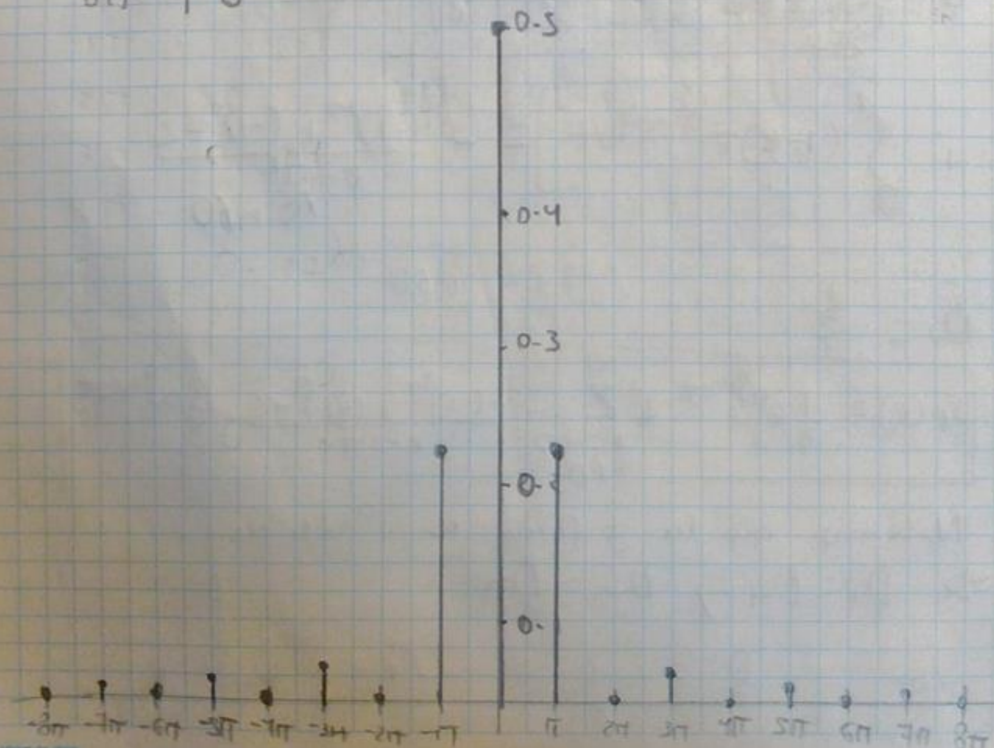
Espectro de Magnitud

w	$\|D_n\|$
-8π	0
-7π	-0.0041
-6π	0
-5π	-0.0081
-4π	0
-3π	-0.0225
-2π	0
$-\pi$	-0.2026
0	0.5
π	0.2026
2π	0
3π	0.0225
4π	0
5π	0.0081
6π	0
7π	0.0041
8π	0



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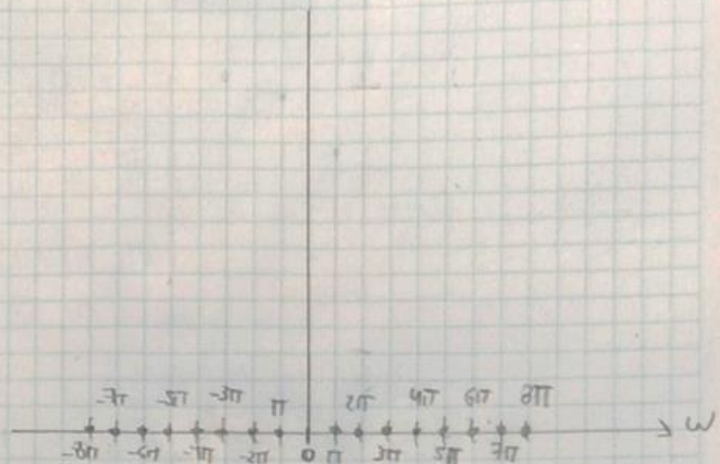
RR12

ω	$\measuredangle D_n$
-8π	0
-7π	0
-6π	0
-5π	0
-4π	0
-3π	0
-2π	0
$-\pi$	0
0	0
π	0
2π	0
3π	0
4π	0
5π	0
6π	0
7π	0
8π	0



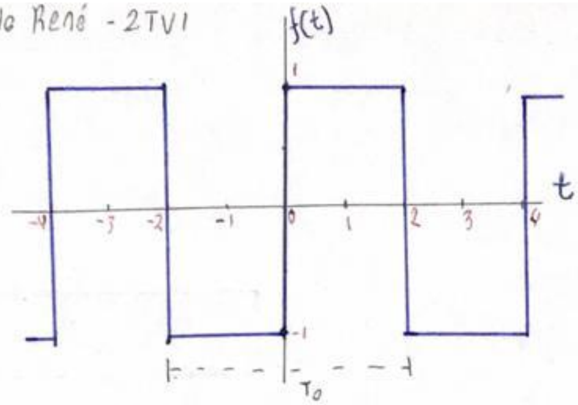
$a_0 = D_0 = \frac{1}{2}$

$a_n = D_n + D_{-n} = \frac{-(-1)^{3n} + (-1)^n - 2}{n^2\pi^2}$

$b_n = D_{-n} - D_n = 0$

PR12 - 2 - Morgado Resendiz Lisardo René - 2TV1

$$f(t)=\begin{cases} -1 & -2<t\leq 0 \\ 1 & 0<t\leq 2 \end{cases}$$



$$W_0=\frac{2\pi}{T_0}=\frac{\pi}{2}$$

$$T_0=4$$

Cálculo de D_n:

$$D_n=\frac{1}{T_0}\int_{t_0}^{t_0+T_0} f(t)e^{-nw_0jt}dt=\frac{1}{4}\int_{-2}^{2} f(t)\,e^{-nw_0jt}dt$$

$$D_0=\frac{1}{4}\left[-\int_{-2}^{0}e^{0}dt+\int_{0}^{2}e^{0}dt\right] \Leftarrow D_n=\frac{1}{4}\left[-\int_{-2}^{0}e^{-nw_0jt}dt+\int_{0}^{2}e^{-nw_0jt}dt\right]$$

$$D_0=\frac{1}{4}\left[-\left[t\Big|_{-2}^{0}\right]+\left[t\Big|_{0}^{2}\right]\right]$$

$$D_n=-\frac{1}{4}\int_{-2}^{0}e^{-nw_0jt}dt+\frac{1}{4}\int_{0}^{2}e^{-nw_0jt}dt$$

$$D_0=\frac{1}{4}\left[-[0+2]+[2-0]\right]$$

$$D_n=-\frac{1}{4}\left[\frac{e^{-nw_0jt}}{-nw_0j}\Bigg|_{-2}^{0}\right]+\frac{1}{4}\left[\frac{e^{-nw_0jt}}{-nw_0j}\Bigg|_{0}^{2}\right]$$

$$D_0=\frac{1}{4}[-2+2]$$

$$D_0=\frac{1}{4}[0]$$

$$D_n=-\frac{1}{4}\left[\frac{e^{0}}{-nw_0j}-\frac{e^{2w_0nj}}{-nw_0j}\right]+\frac{1}{4}\left[\frac{e^{-2w_0nj}}{-nw_0j}-\frac{1}{-nw_0j}\right]$$

$$D_0=0$$

$$D_n=\frac{2-e^{2w_0nj}-e^{-2w_0nj}}{4nw_0j}=\frac{2-e^{\pi nj}-e^{-\pi nj}}{2\pi nj}$$

como $e^{zj}=\cos(z)+j\operatorname{sen}(z)$

$$D_n=\frac{2-\cos(\pi n)+\cancel{j\operatorname{sen}(\pi n)}^{0}-\cos(-\pi n)-\cancel{j\operatorname{sen}(-\pi n)}^{0}}{2\pi nj}=\frac{2-\cos(\pi n)-\cos(-\pi n)}{2\pi nj}$$

$$\int f(t)=\sum_{n=-8}^{n=8}D_n\,e^{nw_0jt}$$

Con matlab obtenemos: $\boxed{* \text{Para } D_1 \Rightarrow \frac{1}{2\pi j}\left(\frac{[illegible]}{2\pi j}\right) = \frac{[illegible]}{2^2\pi^2 j^2} = \frac{[illegible]}{4\pi^2(-1)} = -\frac{[illegible]}{\cancel{8}\pi\cancel{\pi}} = -\frac{2j}{\pi j}} \text{ y así para toda } n$

$$\int f(t) = \left\{ \dots, 0, \frac{2j}{7\pi} e^{-7\frac{\pi}{2}tj}, 0, \frac{2j}{5\pi} e^{-5\frac{\pi}{2}tj}, 0, \frac{2j}{3\pi} e^{-3\frac{\pi}{2}tj}, 0, \frac{2j}{\pi} e^{-1\frac{\pi}{2}tj}, 0, -\frac{2j}{\pi} e^{\frac{\pi}{2}tj}, 0, -\frac{2j}{3\pi} e^{3\frac{\pi}{2}tj}, 0, -\frac{2j}{5\pi} e^{5\frac{\pi}{2}tj}, 0, -\frac{2j}{7\pi} e^{7\frac{\pi}{2}tj}, 0, \dots \right\}$$

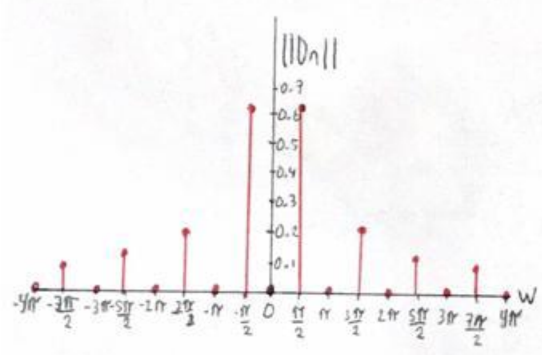
$\boxed{* \text{Para } \|D_1\| \Rightarrow \left|-\frac{2j}{\pi}\right| = \sqrt{0^2 + \left(-\frac{2}{\pi}\right)^2} = 0.637} \text{ y así para toda } n$

$$\|D_n\| = \{0, 0.0909, 0, 0.127, 0, 0.212, 0, 0.637, 0, 0.637, 0, 0.212, 0, 0.127, 0, 0.0909, 0\}$$

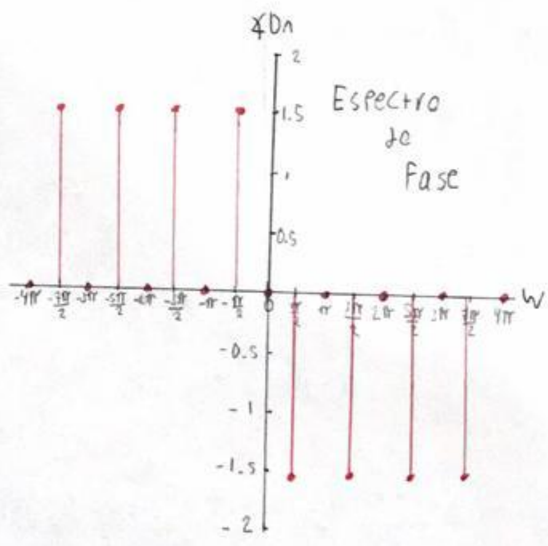
$$\measuredangle D_n \text{ rad} = \{0, 1.57, 0, 1.57, 0, 1.57, 0, 1.57, 0, -1.57, 0, -1.57, 0, -1.57, 0, -1.57, 0\}$$

$\boxed{* (W_0)(n)}$ y así p/toda n

$$W = \left\{-4\pi, -\frac{7}{2}\pi, -3\pi, -\frac{5}{2}\pi, -2\pi, -\frac{3}{2}\pi, -\pi, -\frac{1}{2}\pi, 0, \frac{1}{2}\pi, \pi, \frac{3}{2}\pi, 2\pi, \frac{5}{2}\pi, 3\pi, \frac{7}{2}\pi, 4\pi\right\}$$

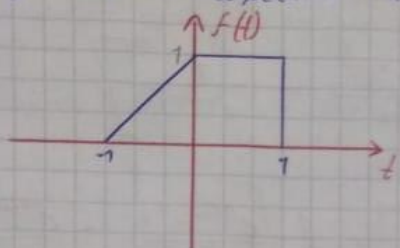


Espectro de magnitud



PROBLEMA 3 PR. 12

Encontrar la serie de Fourier exponencial, así como gráficar su espectro de $n=-8$ hasta $n=8$



$$f(t)=\begin{cases} t+1 & -1<x<0 \\ 1 & 0<x<1 \end{cases}$$

$$D_n = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} f(t)\,e^{-n\omega_0 j t}\,dt$$

$$T_0 = 2 \Rightarrow \omega_0 = \frac{2\pi}{T_0} = \pi$$

$$D_n = \frac{1}{2}\int_{-1}^{1} f(t)\,e^{-n\pi j t}\,dt$$

$$D_n = \frac{1}{2}\int_{-1}^{0}(t+1)\,e^{-n\pi j t} + \frac{1}{2}\int_{0}^{1}(1)e^{-n\pi j t}$$

$$D_n = \underbrace{\frac{1}{2}\int_{-1}^{0} t\,e^{-n\pi j t}dt}_{A} + \underbrace{\frac{1}{2}\int_{-1}^{0} e^{-n\pi j t}\,dt + \frac{1}{2}\int_{0}^{1} e^{-n\pi j t}}_{B}$$

Nota

$$D_n = \frac{1}{2}\int_{-1}^{0} t\,e^{-n\pi j t}\,dt + \underbrace{\frac{1}{2}\int_{-1}^{1} e^{-n\pi j t}\,dt}_{B} \longrightarrow \frac{1}{2}\left[\frac{e^{-n\pi j t}}{-n\pi j}\right]_{-1}^{1}$$

$u = t \qquad v = \frac{e^{-n\pi j t}}{-n\pi j}$

$du = dt \qquad dv = e^{-n\pi j t}$

$$\frac{1}{2}\left[\frac{t e^{-n\pi j t}}{-n\pi j} + \int \frac{e^{-n\pi j t}}{+n\pi j}\,dt\right]_{-1}^{0}$$

$$\frac{1}{2}\left[\frac{t e^{-n\pi j t}}{-n\pi j} + \frac{e^{-n\pi j t}}{-n^2\pi^2 j^2}\right]_{-1}^{0}$$

$$\frac{1}{2}\left[-\frac{t e^{-n\pi j t}}{n\pi j} - \frac{e^{-n\pi j t}}{n^2\pi^2 j^2}\right]_{-1}^{0} \rightarrow \frac{1}{2}\left[-\frac{t}{n\pi j} - \frac{1}{n^2\pi^2 j^2}\right]$$

$$-\frac{1}{2}\left[+\frac{t e^{n\pi j}}{n\pi j} - \frac{e^{n\pi j}}{n^2\pi^2 j^2}\right]$$

$$\frac{1}{2}\left[-\frac{e^{-n\pi j}}{n\pi j}+\frac{e^{n\pi j}}{n\pi j}\right]+\frac{1}{2}\left(-\frac{e^{n\pi j}}{n\pi j}+\frac{e^{n\pi j}}{n^2\pi^2 j^2}-\cancel{\frac{1}{n\pi j}}^{\,0}-\frac{1}{n^2\pi^2 j^2}\right)$$

$$\frac{1}{2}\left[\cancel{\frac{e^{n\pi j}}{n\pi j}}^{\,0}-\frac{e^{-n\pi j}}{n\pi j}+\frac{e^{n\pi j}}{n^2\pi^2 j^2}-\cancel{\frac{e^{n\pi j}}{n\pi j}}^{\,0}-\frac{1}{n^2\pi^2 j^2}\right]$$

$$\frac{1}{2}\left[\frac{e^{n\pi j}}{n^2\pi^2 j^2}-\frac{e^{-n\pi j}}{n\pi j}-\frac{1}{n^2\pi^2 j^2}\right]=\underbrace{\frac{1}{2}\left[\frac{e^{n\pi j}-1}{n^2\pi^2 j^2}-\frac{e^{-n\pi j}}{n\pi j}\right]}_{D_n}$$

considerando $e^{(rj)}=\cos(r)+j\,\text{sen}(r)$

$$D_n=\frac{1}{2}\left[\frac{\cos(n\pi)+j\,\text{sen}(n\pi)-1}{n^2\pi^2 j^2}-\frac{\cos(n\pi)-j\,\text{sen}(n\pi)}{n\pi j}\right]$$

Considerando, que D_n se indetermina en $n=0$

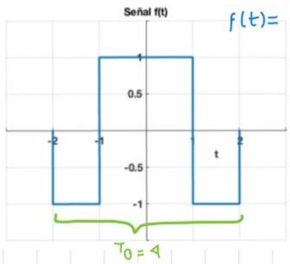
$$D_0=\frac{1}{2}\int_{-1}^{0}e^{-kn\pi j\,0}\,dt+\frac{1}{2}\int_0^1 e^0=\frac{1}{2}\int_{-1}^{0}dt+\frac{1}{2}\int_0^1 dt$$

$$=\frac{1}{2}t\Big|_{-1}^{0}+\frac{1}{2}t\Big|_0^1=-\frac{1}{2}+\frac{1}{2}=\underline{0}$$

Finalmente

$$S_f(t)=\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}\left(\frac{\cos(n\pi)+j\,\text{sen}(n\pi)-1}{n^2\pi^2 j^2}-\frac{\cos(n\pi)-j\,\text{sen}(n\pi)}{n\pi j}\right)+$$

4. Señal $f(t)$ periódica



$$f(t)=\begin{cases} -1 & -2\le t<-1 \\ 1 & 1\le t<1 \\ -1 & 1\le t<2 \end{cases}$$

$$f(t)=\sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

$$\omega_0=\frac{2\pi}{T_0}=\frac{2\pi}{4}=\frac{\pi}{2}$$

$$D_n=\frac{1}{T_0}\int_0^T f(t)\,e^{-jn\omega_0 t}\,dt$$

$$=\frac{1}{4}\left[-\int_{-2}^{-1} e^{-jn\omega_0 t}\,dt+\int_{-1}^{1} e^{-jn\omega_0 t}\,dt-\int_{1}^{2} e^{-jn\omega_0 t}\,dt\right]$$

$$=\frac{1}{4}\left[\frac{1}{jn\omega_0}\,e^{-jn\omega_0 t}\Big|_{-2}^{-1}-\frac{1}{jn\omega_0}\,e^{-jn\omega_0 t}\Big|_{-1}^{1}+\frac{1}{jn\omega_0}\,e^{-jn\omega_0 t}\Big|_{1}^{2}\right]$$

$$=\frac{1\,\cancel{2}}{\cancel{4}jn\pi}\left[e^{jn\frac{\pi}{2}}-e^{\cancel{2}jn\frac{\pi}{\cancel{2}}}-\left[e^{-jn\frac{\pi}{2}}-e^{jn\frac{\pi}{2}}\right]+\left[e^{-jn\pi}-e^{-jn\frac{\pi}{2}}\right]\right]$$

$$=\frac{1}{2jn\pi}\left[2e^{jn\frac{\pi}{2}}-2e^{-jn\frac{\pi}{2}}-e^{jn\pi}+e^{-jn\pi}\right]$$

$$=\frac{1}{\cancel{2j}n\pi}\left[\cancel{2j}\,(2\,\mathrm{sen}\left(\tfrac{n\pi}{2}\right)-\mathrm{sen}\,(n\pi)\right]$$

$$D_n=\frac{2\,\mathrm{sen}\left(\frac{n\pi}{2}\right)-\mathrm{sen}\,(n\pi)}{n\pi}$$

propiedad

$$\mathrm{sen}\ x=\frac{e^{ix}-e^{-ix}}{2i}$$

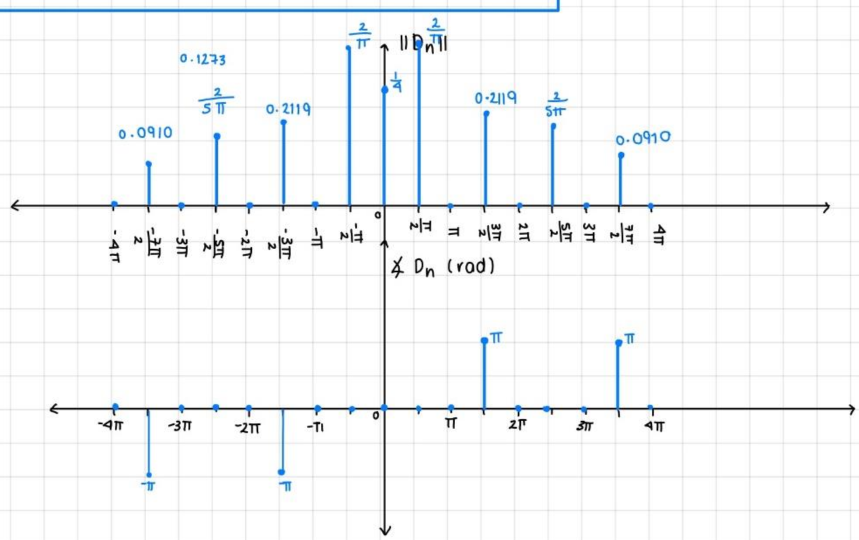
$$2\,\frac{\left[e^{j\left(\frac{n\pi}{2}\right)}-e^{-j\left(\frac{n\pi}{2}\right)}\right]}{(2j)}(2j)=2\,\mathrm{sen}\left(\tfrac{n\pi}{2}\right)(2j)$$

$$\frac{-\left[e^{j(n\pi)}-e^{j(n\pi)}\right]}{(2j)}(2j)=-2\ \mathrm{sen}\,(n\pi)\,;$$

$$D_0=\frac{1}{4}\int_{-2}^{2}dt=\frac{1}{4}\left[-\int_{-2}^{-1}dt+\int_{-1}^{1}dt-\int_{1}^{2}dt\right]$$

$$=\frac{1}{4}\left[-\left(\cancel{-1+2}\right)+(2)-\cancel{(1)}\right]=0$$

$$\boxed{Sf(t)=\frac{1}{\pi}\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}\frac{1}{n}\left[2\,\mathrm{sen}\left(\tfrac{n\pi}{2}\right)-\mathrm{sen}\,(n\pi)\right]e^{\frac{\pi}{2}njt}}$$



Los espectros de Fourier de la serie de Fourier reciben el nombre de espectros discretos ¿Por qué?

El espectro de amplitud es la gráfica D_n vs ω y el espectro de fase es la gráfica del ángulo de fase ($\emptyset_n$ de D_n) vs ω. Puesto que el índice n toma solamente valores enteros, los espectros de amplitud y fase NO SON CURVAS CONTÍNUAS si no que aparece la VARIABLE DISCRETA $n\omega_0$, por consiguiente se les denomina como espectros de frecuencia discreta o espectros de líneas.

PROBLEMA 1

Problemas 12

Solucion 1

```
clear all; clc; close all;
syms t n;

assume(n, 'integer');
T = 2;
w0 = 2 * pi /T;

Dn = @(n) (1/T) *( int((t+1) * exp(-n*j*w0*t), t, -1, 0) + int((1-t) * exp(-n*j*w0*t), t, 0, 1));

simplify(Dn(n))
```

ans =

$$-\frac{(-1)^{3n}+(-1)^n-2}{2\,n^2\,\pi^2}$$

```
Dn(0)
```

ans =

$$\frac{1}{2}$$

```
y = [];
x = []
```

x =

[]

```
for i = 0:1:8
    y(i+1) = round(abs(vpa(Dn(i), 3)), 4);
    x(i+1) = i*w0;
end

x
```

x = 1×9

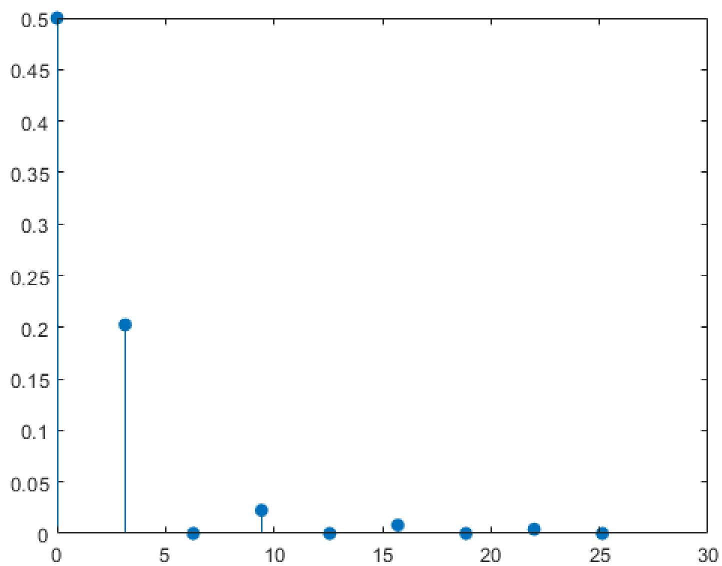
0 3.1416 6.2832 9.4248 12.5664 15.7080 18.8496 ⋯

```
y
```

y = 1×9

    0.5000   0.2026   0   0.0225   0   0.0081   0 ⋯

```
stem(x,y,'filled')
```




```
y = []
```

y =

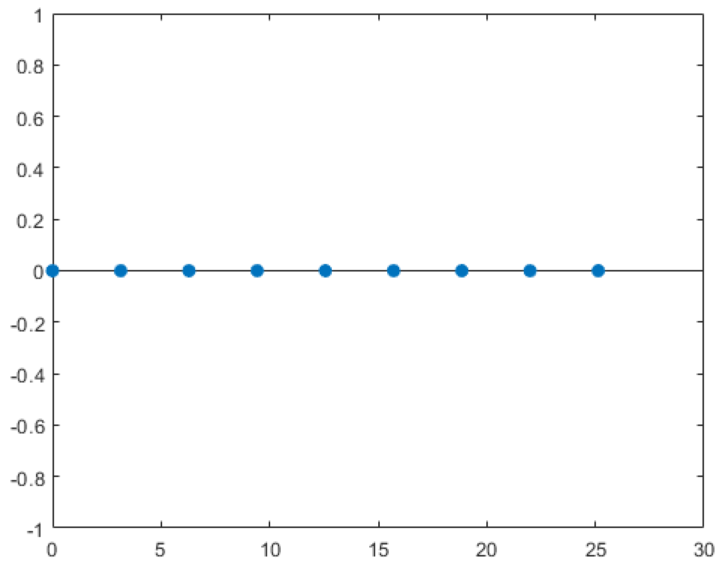
    []

```
x = []
```

x =

    []

```
for i = 0:1:8
    y(i+1) = round(angle(vpa(Dn(i), 3)), 4);
    x(i+1) = i*w0;
end
stem(x,y,'filled')
```




```
round(angle(vpa(Dn(-1), 3)), 4)
```

ans =

0

```
round(angle(vpa(Dn(-2), 3)), 4)
```

ans =

0

```
round(angle(vpa(Dn(-3), 3)), 4)
```

ans =

0

```
x
```

x = 1×9

0   3.1416   6.2832   9.4248   12.5664   15.7080   18.8496 ⋯

```
y
```

y = 1×9

0   0   0   0   0   0   0   0   0

```
a0 = Dn(0)
```

a0 =

$\frac{1}{2}$

```
an = simplify(Dn(n) + Dn(-n))
```

an =

$$-\frac{(-1)^{3n}+(-1)^n-2}{n^2\pi^2}$$

```
bn = simplify((Dn(-n) - Dn(n))/j)
```

bn =

0

PROBLEMA 2

```
clear all;
syms n t;
assume(n, 'integer');

%Variables de la señal
T=4;
w0=(2*pi)/T;

%Definiendo la señal y simplificandola
Dn = @(n) (1/T)*(-int(exp(-n*w0*j*t),t,[-2 0])+int(exp(-n*w0*j*t),t,[0 2]));
D = @(n) simplify(Dn(n));

%Valor de Dn
D(n)

%Valor de D0
D(0)

%Eje de magnitud de Dn
MD=vpa(abs(D(-8:1:8)),3);

%Eje de ángulo de Dn
AD=vpa(angle(D(-8:1:8)),3);

%Eje de los omegas
omegas=w0*(-8:1:8);

%Gráfica del espectro de magnitud
stem(omegas,MD)

%Gráfica del espectro de fase
stem(omegas,AD)
```

```
% *****   Problema 3, Procedimiento en matlab   *****
% _Ramirez Aniceto Lauro Alexis
clear all;

%Declarando las variables

syms n t w0 pi;
assume(n,['integer']);
T = 2;
w0=2*pi;
%Buscando la señal
Dn = @(n) (1/T)*int((t+1)*exp(-j*n*w0*t),t,[-1 0]) + (1/T)*int(exp(-j*n*w0*t)),t,[0 1];
Dn(0)

%Obtencion de la magnitud y del angulo de dn
MDn = vpa(abs(Dn(-8:1:8)),3)

ADn = vpa(angle(Dn(-8:1:8)),3)

omga = w0*(-8:1:8)
%Graficando
stem(omga,MDn
stem(omga,ADn)
```

PROBLEMA 4

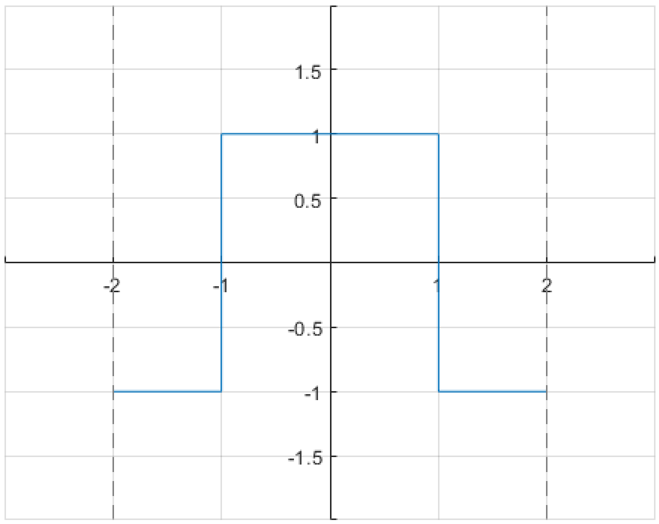
```
syms t pi n
assume(n,["integer"])

T0=2;
w0=2*pi/T0;
x = piecewise(-2<=t & t<-1,-1,t>=-1 & t<1,1, 1<=t & t<2,-1)
```

x =

$$x = \begin{cases} -1 & \text{if } t \in [-2,-1) \\ 1 & \text{if } t \in [-1,1) \\ -1 & \text{if } t \in [1,2) \end{cases}$$

```
fplot(x)
plano = gca;
plano.XAxisLocation = "origin";
plano.YAxisLocation = "origin";
```

```
plano.Box = "off";
grid on
ylim([-2,2])
xlim([-3,3]);
```




```
clc
syms t pi n
assume(n,["integer"])

T0=4;
w0=2*pi/T0;
Dn(n) = (1/T0)*int (x*exp(-j*n*w0*t),t,-2,2);
D(n) = simplify(Dn(n))
```

D(n) =

$$-\frac{\sin(n\,\pi) - 2\sin\left(\frac{n\,\pi}{2}\right)}{n\,\pi}$$

```
MD1= vpa(abs(D(-8:-1)),3)
```

MD1 =

$$\left(\frac{0.125\,|2.0\sin(4.0\,\pi) - 1.0\sin(8.0\,\pi)|}{|\pi|} \quad \frac{0.143\,|\sin(7.0\,\pi) - 2.0\sin(3.5\,\pi)|}{|\pi|} \quad \frac{0.167\,|2.0\sin(3.0\,\pi) - 1.0\sin(6.0\,\pi)|}{|\pi|} \quad \frac{0.2\,|\sin(5.0\,\pi) - 2.0\sin(2.5\,\pi)|}{|\pi|} \quad \frac{0.25\,|2.0\sin(2.0\,\pi) - 1.0\sin(4.0\,\pi)|}{|\pi|} \quad \frac{0.333\,|\sin(3.0\,\pi) - 2.0\sin(1.5\,\pi)|}{|\pi|} \quad \frac{0.5\,|2.0\sin(\pi) - 1.0\sin(2.0\,\pi)|}{|\pi|} \quad \frac{|\sin(\pi) - 2.0\sin(0.5\,\pi)|}{|\pi|}\right)$$

```
MD2=vpa(abs(D(1:8)),3)
```

MD2 =

$$\left(\frac{|\sin(\pi)-2.0\sin(0.5\,\pi)|}{|\pi|}\quad \frac{0.5\,|2.0\sin(\pi)-1.0\sin(2.0\,\pi)|}{|\pi|}\quad \frac{0.333\,|\sin(3.0\,\pi)-2.0\sin(1.5\,\pi)|}{|\pi|}\quad \frac{0.25\,|2.0\sin(2.0\,\pi)-1.0\sin(4.0\,\pi)|}{|\pi|}\quad \frac{0.2\,|\sin(5.0\,\pi)-2.0\sin(2.5\,\pi)|}{|\pi|}\quad \frac{0.167\,|2.0\sin(3.0\,\pi)-1.0\sin(6.0\,\pi)|}{|\pi|}\quad \frac{0.143\,|\sin(7.0\,\pi)-2.0\sin(3.5\,\pi)|}{|\pi|}\quad \frac{0.125\,|2.0\sin(4.0\,\pi)-1.0\sin(8.0\,\pi)|}{|\pi|}\right)$$

```
AD1=vpa(angle(D(-8:-1)),3)
```

AD1 =

$$\left(\operatorname{angle}\left(\frac{2.0\sin(4.0\,\pi)-1.0\sin(8.0\,\pi)}{\pi}\right)\quad \operatorname{angle}\left(\frac{2.0\sin(3.5\,\pi)-\sin(7.0\,\pi)}{\pi}\right)\quad \operatorname{angle}\left(\frac{2.0\sin(3.0\,\pi)-1.0\sin(6.0\,\pi)}{\pi}\right)\quad \operatorname{angle}\left(\frac{2.0\sin(2.5\,\pi)-\sin(5.0\,\pi)}{\pi}\right)\quad \operatorname{angle}\left(\frac{2.0\sin(2.0\,\pi)-1.0\sin(4.0\,\pi)}{\pi}\right)\quad \operatorname{angle}\left(\frac{2.0\sin(1.5\,\pi)-\sin(3.0\,\pi)}{\pi}\right)\quad \operatorname{angle}\left(-\frac{\sin(2.0\,\pi)-2.0\sin(\pi)}{\pi}\right)\quad \operatorname{angle}\left(\frac{2.0\sin(0.5\,\pi)-1.0\sin(\pi)}{\pi}\right)\right)$$

```
AD2=vpa(angle(D(1:8)),3)
```

AD1 =

$$\left(\operatorname{angle}\left(\frac{2.0\sin(0.5\,\pi)-1.0\sin(\pi)}{\pi}\right)\quad \operatorname{angle}\left(-\frac{\sin(2.0\,\pi)-2.0\sin(\pi)}{\pi}\right)\quad \operatorname{angle}\left(\frac{2.0\sin(1.5\,\pi)-\sin(3.0\,\pi)}{\pi}\right)\quad \operatorname{angle}\left(\frac{2.0\sin(2.0\,\pi)-1.0\sin(4.0\,\pi)}{\pi}\right)\quad \operatorname{angle}\left(\frac{2.0\sin(2.5\,\pi)-\sin(5.0\,\pi)}{\pi}\right)\quad \operatorname{angle}\left(\frac{2.0\sin(3.0\,\pi)-1.0\sin(6.0\,\pi)}{\pi}\right)\quad \operatorname{angle}\left(\frac{2.0\sin(3.5\,\pi)-\sin(7.0\,\pi)}{\pi}\right)\quad \operatorname{angle}\left(\frac{2.0\sin(4.0\,\pi)-1.0\sin(8.0\,\pi)}{\pi}\right)\right)$$