

Problem set 1 - FYS3110

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I. THEORY

We're going to solve the differential equation

$$-u''(x) = f(x), \quad x \in (0, 1), \quad u(0) = u(1) = 0. \quad (1)$$

We'll approximate this differential equation by a function $v(x) \approx u(x)$ by the approximation scheme

$$-\frac{-v_{i+1} + v_{i-1} - 2v_i}{h^2} = f_i, \quad i = 1, 2, \dots, n, \quad (2)$$

which may be rearranged into

$$2v_i - v_{i+1} - v_{i-1} = f_i h^2 \equiv b_i. \quad (3)$$

From 3 we can write

$$\begin{pmatrix} 2v_1 - v_2 \\ -v_1 + 2v_2 - v_3 \\ \vdots \\ 2v_n - v_{n-1} \end{pmatrix} = \begin{pmatrix} 2 & -1 & 0 & \cdots & 0 \\ -1 & 2 & -1 & \cdots & 0 \\ \vdots & & & & \\ 0 & \cdots & 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} \quad (4)$$